Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/40-1.2.2.3-d+e-x^2-^m-a+b-x^2+c-x^4-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [413]. This is test number [40].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.76 (412)	0.24 (1)
Mathematica	98.06 (405)	1.94 (8)
Maple	96.61 (399)	3.39 (14)
Fricas	80.15 (331)	19.85 (82)
Sympy	45.28 (187)	54.72 (226)
Mupad	44.55 (184)	55.45 (229)
Giac	44.07 (182)	55.93 (231)
Maxima	18.64 (77)	81.36 (336)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

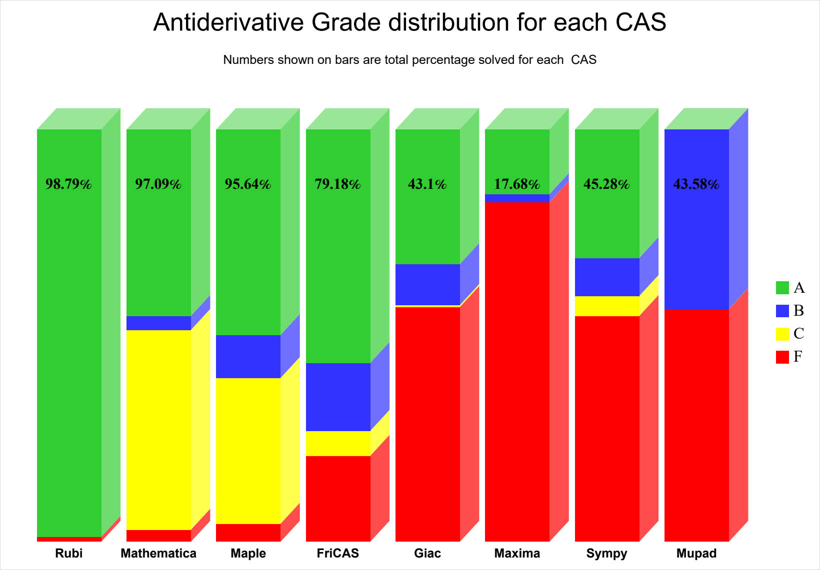
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

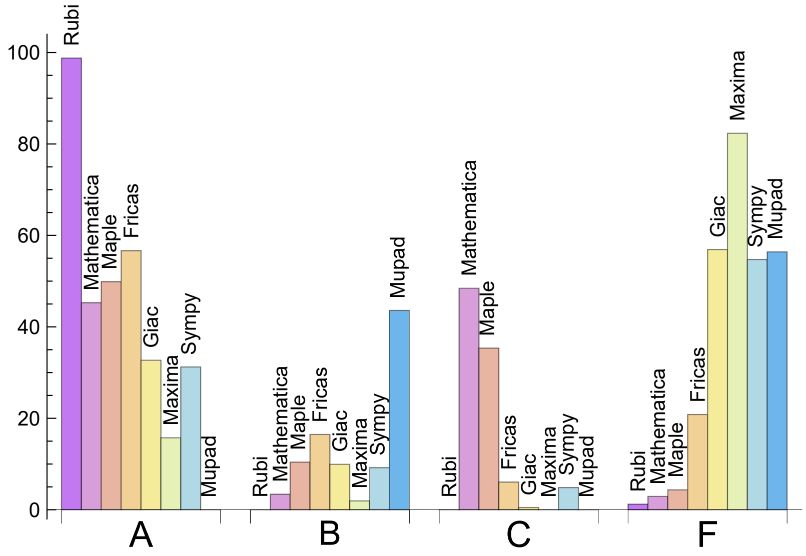
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.789	0.000	0.000	1.211
Fricas	56.659	16.465	6.053	20.823
Maple	49.879	10.412	35.351	4.358
Mathematica	45.278	3.390	48.426	2.906
Giac	32.688	9.927	0.484	56.901
Sympy	31.235	9.201	4.843	54.722
Maxima	15.738	1.937	0.000	82.324
Mupad	0.000	43.584	0.000	56.416

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	8	75.00	25.00	0.00
Maple	14	100.00	0.00	0.00
Fricas	82	79.27	20.73	0.00
Sympy	226	83.19	14.16	2.65
Mupad	229	0.00	100.00	0.00
Giac	231	96.54	0.00	3.46
Maxima	336	85.42	0.60	13.99

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.14
Maxima	0.24
Giac	0.58
Maple	0.97
Fricas	2.03
Sympy	3.19
Mathematica	4.37
Mupad	7.64

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	97.43	1.14	45.00	0.93
Rubi	155.19	1.01	112.50	1.00
Maple	159.83	1.19	119.00	0.93
Mathematica	171.93	1.21	110.00	1.00
Sympy	233.94	2.07	82.00	1.08
Fricas	554.18	2.61	118.00	1.19
Giac	1126.08	3.80	83.00	1.01
Mupad	3780.98	8.37	67.00	0.96

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

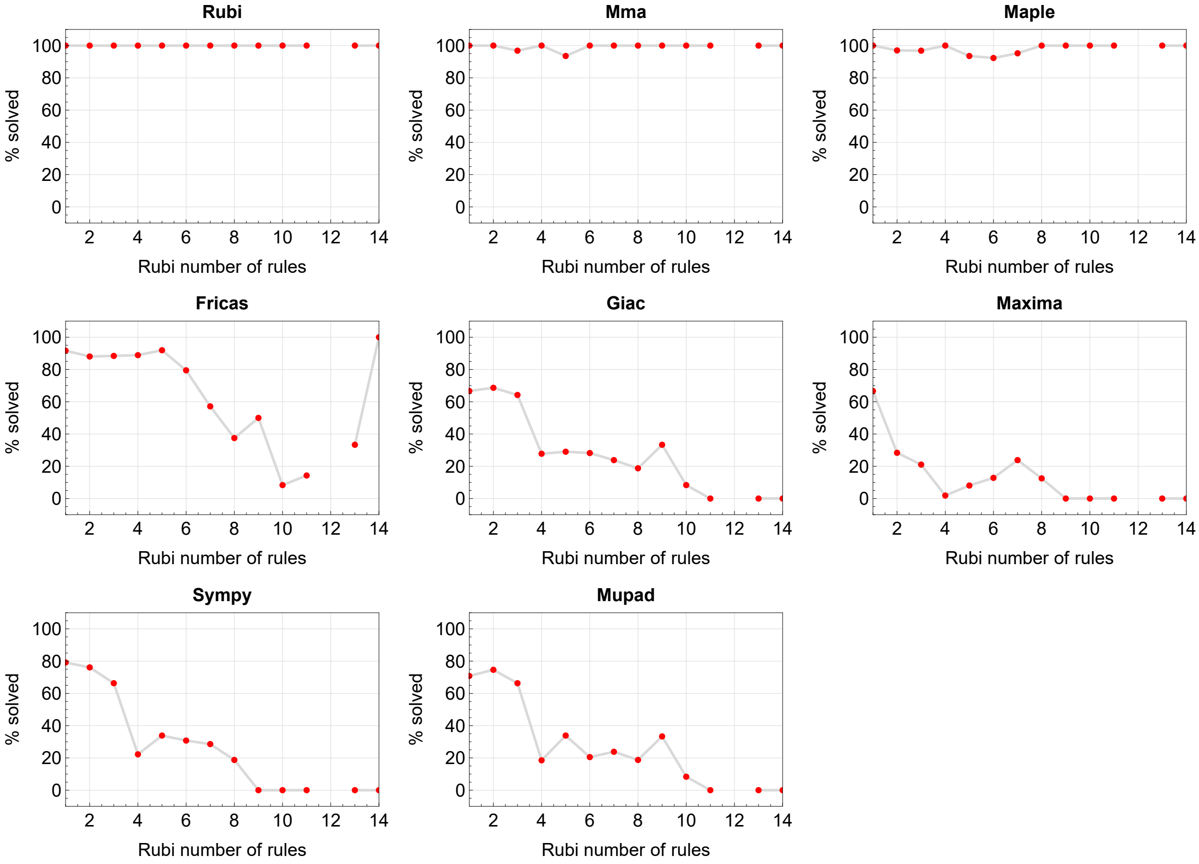

Rubi
Mma
Maple
Fricas
Giac
Maxima
Sympy
Mupad

Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

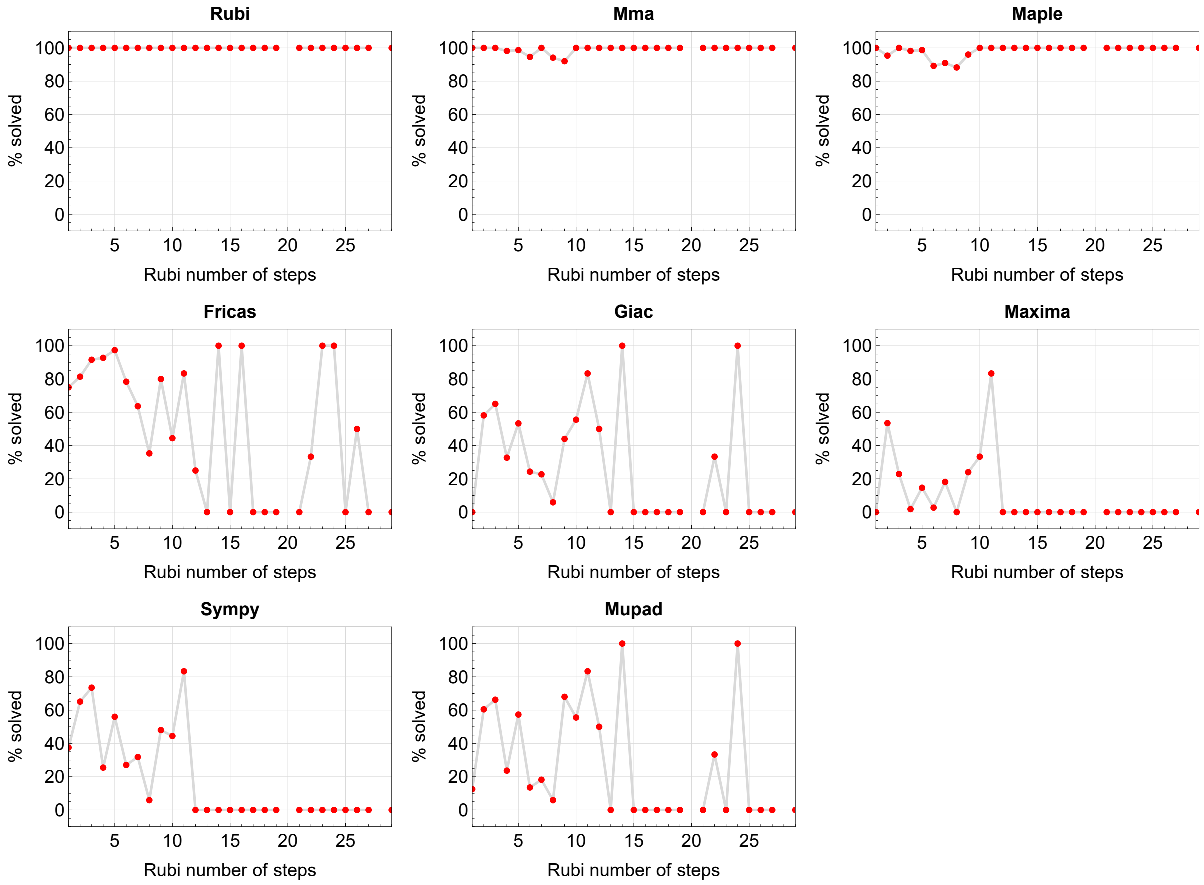
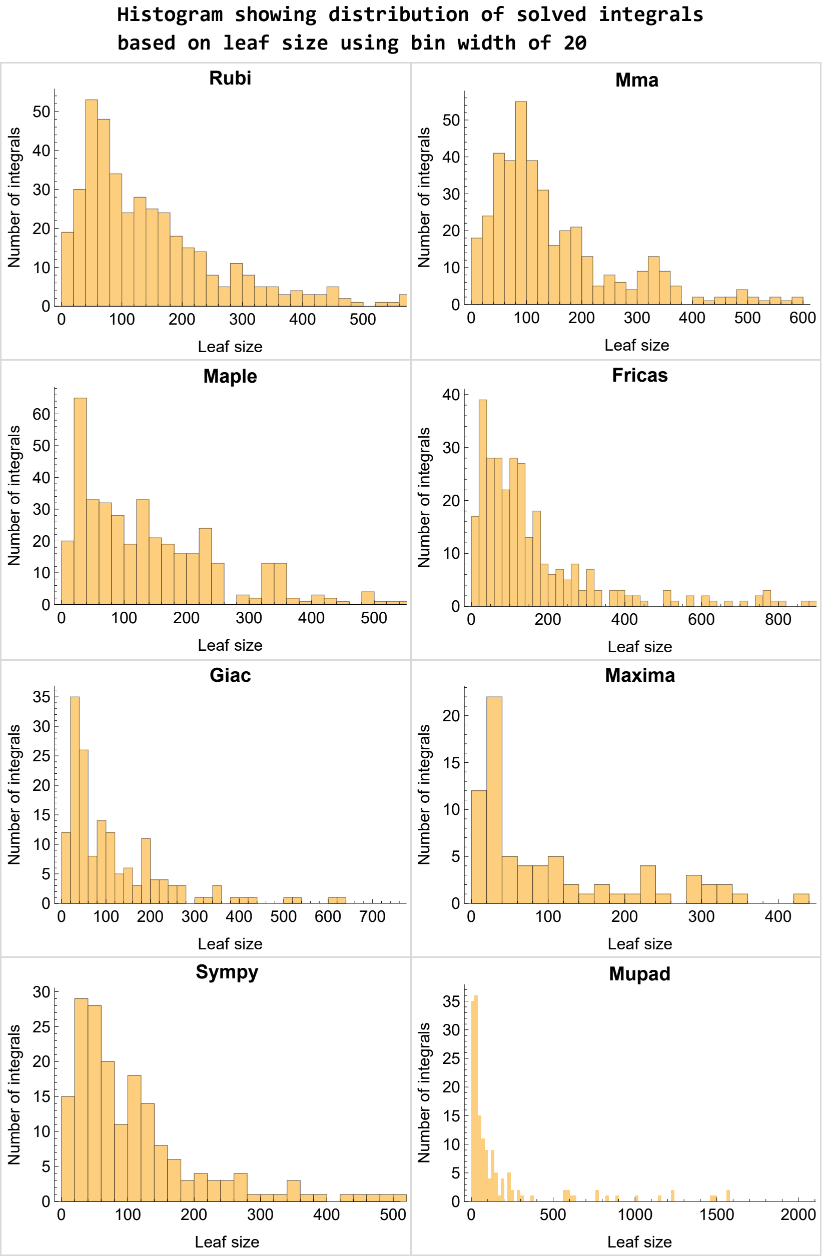


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.



Histogram showing distribution of solved integrals
based on leaf size using bin width of 20

Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

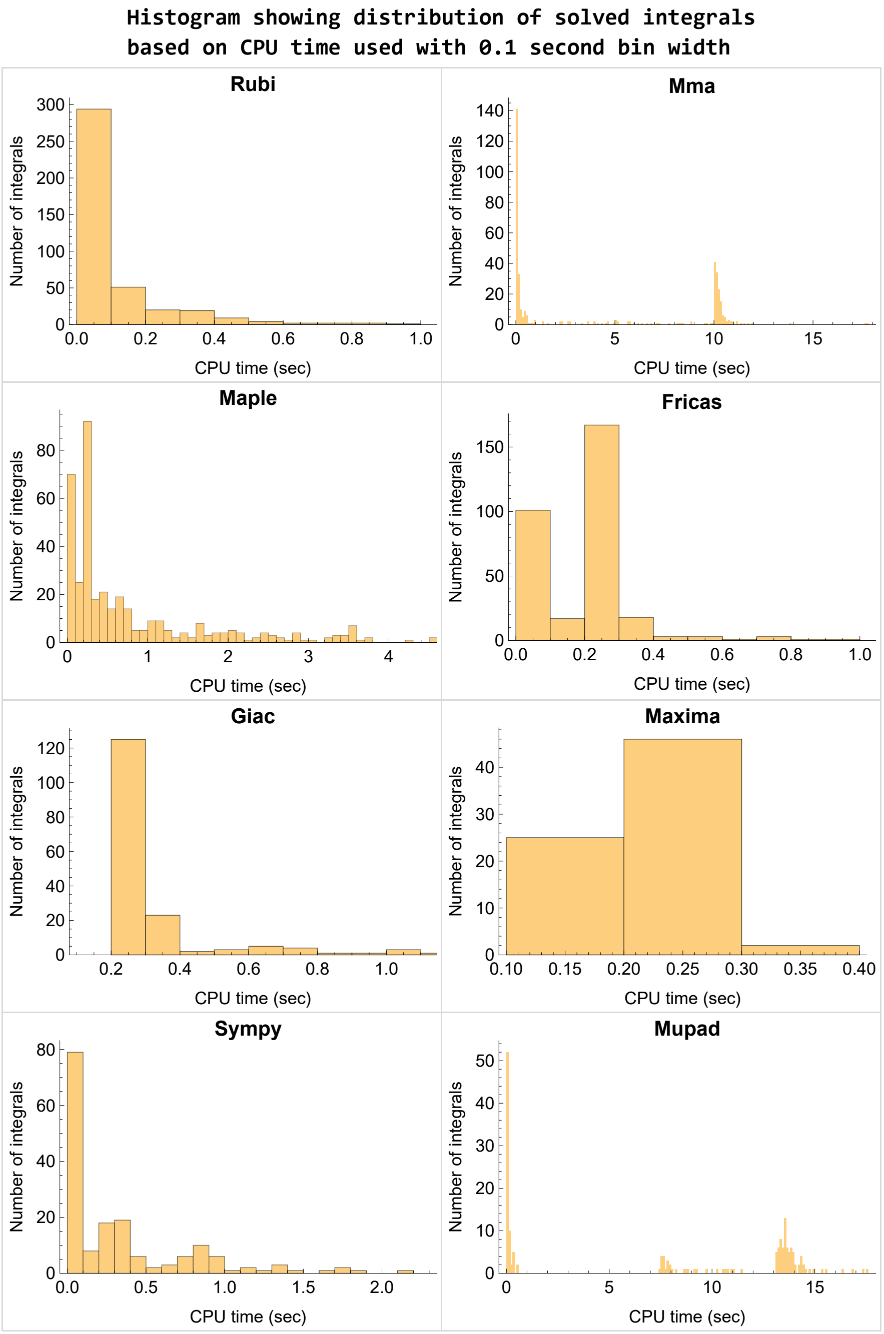


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

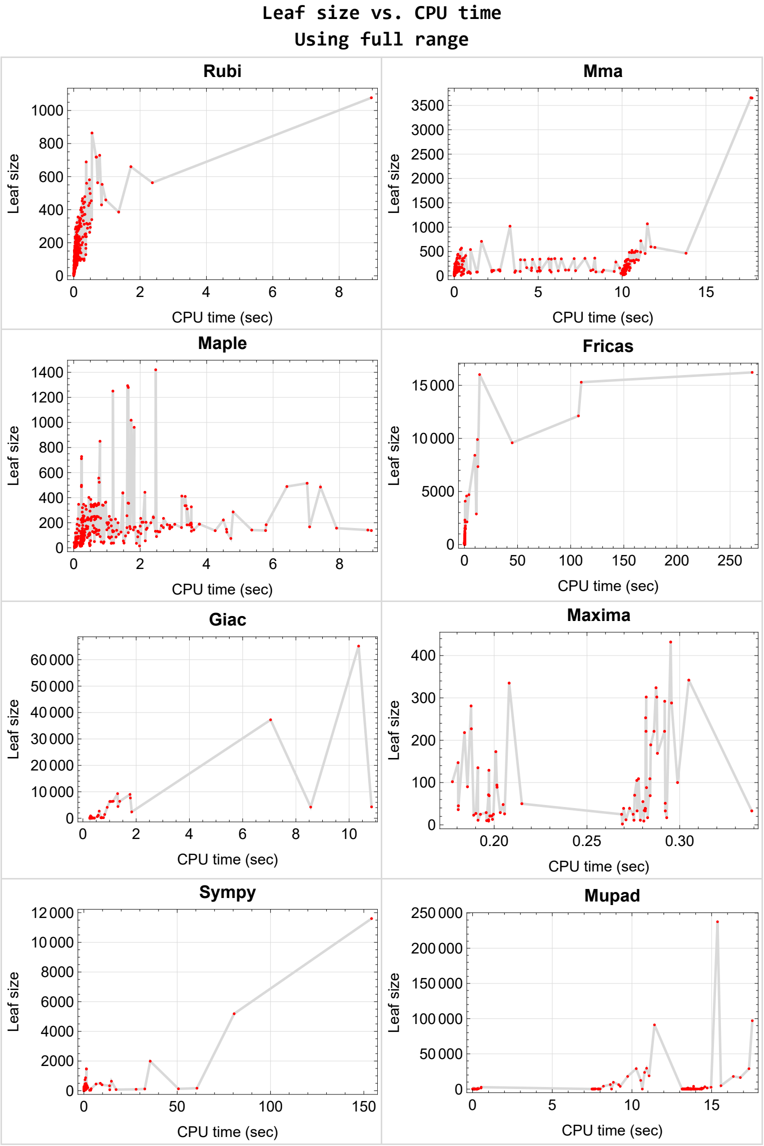


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{175, 399, 404, 405\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{115, 402\}$

Maple $\{16, 17, 20, 21\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

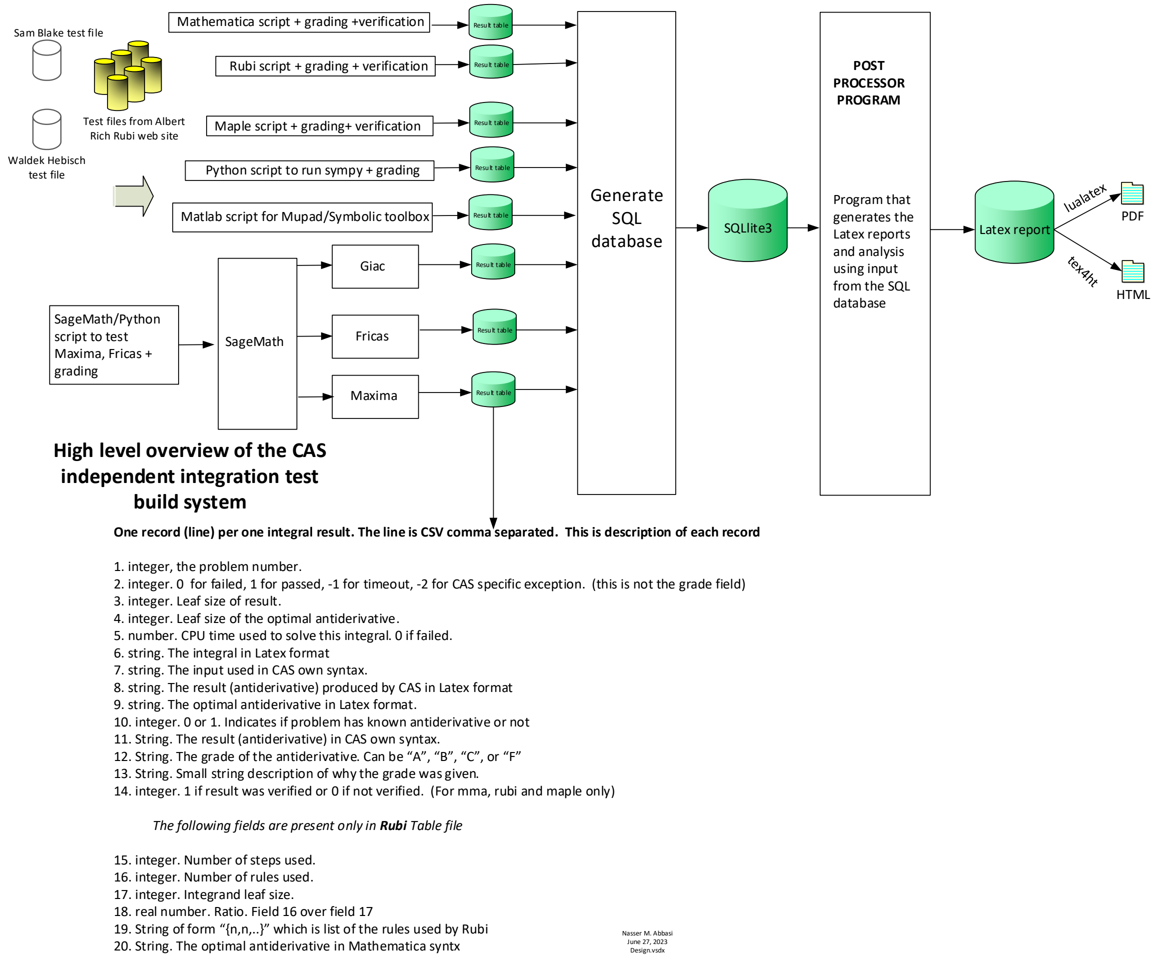
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade { }

C grade { }

F normal fail { 174 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 24, 34, 35, 36, 37, 41, 42, 43, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 94, 95, 96, 97, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 170, 176, 177, 178, 179, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 400, 401, 402, 403, 408, 409, 410, 411 }

B grade { 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 65, 80, 88 }

C grade { 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 44, 45, 46, 48, 49, 73, 92, 93, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 199, 200, 201, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 406, 407, 412, 413 }

F normal fail { 174, 180, 181, 186, 187, 188 }

F(-1) timedout fail { 104, 105 }

F(-2) exception fail { }

Maple

A grade { 5, 6, 7, 8, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 107, 108, 110, 111, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 142, 143, 148, 149, 157, 158, 159, 160, 164, 165, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 316, 317, 323, 324, 325, 326, 331, 338, 339, 340, 347, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 407, 412, 413 }

B grade { 9, 10, 113, 114, 115, 116, 117, 118, 161, 162, 163, 166, 167, 170, 172, 202, 203, 204, 208, 209, 210, 318, 319, 320, 321, 322, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 341, 342, 343, 344, 345, 346, 389 }

C grade { 1, 2, 3, 4, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 97, 100, 101, 102, 103, 104, 105, 106, 109, 112, 137, 138, 139, 140, 141, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 168, 169, 171, 173, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 266, 267, 270, 271, 272, 273, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 394, 395, 396, 397, 398, 406, 408, 409, 410, 411 }

F normal fail { 174, 176, 177, 178, 179, 180, 181, 186, 187, 188, 400, 401, 402, 403 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 98, 99, 108, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 150, 151, 152, 153, 157, 158, 159, 164, 168, 189, 190, 191, 192, 193, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 215, 216, 217, 218, 220, 221, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 332, 333, 334, 335, 339, 340, 341, 342, 343, 344, 348, 349, 350, 351, 352, 356, 357, 358, 359, 360, 364, 365, 366, 367, 371, 372, 373, 374, 375, 376, 380, 381, 382, 385, 386, 387, 390, 392 }

B grade { 1, 2, 3, 4, 7, 14, 15, 22, 23, 24, 25, 65, 88, 94, 95, 96, 97, 100, 101, 104, 105, 106, 107, 109, 110, 111, 112, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 166, 167, 194, 197, 198, 211, 212, 213, 214, 219, 222, 223, 224, 258, 259, 260, 263, 264, 265, 266, 267, 268, 270, 271, 272, 273, 408, 409, 410, 411 }

C grade { 102, 103, 141, 147, 286, 287, 288, 289, 293, 294, 295, 296, 300, 301, 302, 303, 307, 308, 309, 310, 311, 312, 394, 395, 396 }

F normal fail { 154, 160, 165, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 290, 291, 292, 297, 298, 299, 304, 305, 306, 313, 314, 315, 321, 322, 323, 329, 330, 331, 336, 337, 338, 345, 346, 347, 353, 354, 355, 361, 362, 363, 368, 369, 370, 377, 378, 379, 384, 397, 400, 401, 402, 403, 413 }

F(-1) timedout fail { 155, 156, 161, 162, 163, 269, 274, 275, 383, 388, 389, 391, 393, 398, 406, 407, 412 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 13, 37, 42, 43, 44, 47, 51, 52, 56, 57, 58, 61, 66, 72, 73, 74, 76, 77, 84, 85, 86, 89, 92, 93, 94, 96, 98, 99, 108, 120, 121, 122, 123, 128, 129, 130, 131, 137, 138, 139, 140, 141, 144, 145, 146, 147, 244, 245, 246, 247, 252, 253, 254, 255, 284, 285 }

B grade { 7, 11, 12, 65, 88, 95, 282, 283 }

C grade { }

F normal fail { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 45, 46, 48, 49, 50, 53, 54, 55, 59, 60, 62, 63, 64, 67, 68, 69, 70, 71, 75, 78, 79, 80, 81, 82, 83, 87, 90, 91, 97, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 266, 267, 270, 271, 272, 273, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-1) timedout fail { 214, 344 }

F(-2) exception fail { 38, 124, 125, 126, 127, 132, 133, 134, 135, 136, 142, 143, 148, 149, 189, 190, 191, 192, 193, 194, 215, 216, 217, 218, 219, 248, 249, 250, 251, 256, 257, 258, 259, 260, 261, 262, 268, 269, 274, 275, 276, 277, 278, 279, 280, 281, 397 }

Giac

A grade { 1, 2, 5, 6, 8, 11, 12, 37, 38, 41, 42, 43, 44, 45, 46, 48, 49, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 98, 99, 102, 103, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 189, 190, 191, 192, 193, 194, 198, 214, 215, 216, 217, 218, 219, 222, 223, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285 }

B grade { 3, 4, 7, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 47, 50, 53, 65, 88, 95, 96, 100, 101, 195, 196, 197, 224, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

C grade { 106, 109 }

F normal fail { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 39, 40, 54, 68, 80, 104, 105, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-1) timedout fail { }

F(-2) exception fail { 9, 10, 107, 110, 111, 112, 220, 221 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 179, 185, 189, 190, 191, 192, 193, 194, 214, 215, 216, 217, 218, 219, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 282, 283, 284, 285 }

C grade { }

F normal fail { }

F(-1) timedout fail { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 276, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 100, 101, 106, 109, 120, 121, 122, 123, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 157, 158, 159, 164, 166, 189, 190, 191, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 258, 267, 277, 278, 279, 280 }

B grade { 7, 14, 15, 23, 25, 38, 42, 65, 88, 95, 96, 104, 105, 124, 125, 126, 132, 133, 167, 192, 193, 194, 214, 215, 216, 217, 248, 249, 256, 257, 261, 262, 276, 281, 282, 283, 284, 285 }

C grade { 18, 19, 20, 21, 98, 99, 108, 150, 151, 152, 153, 168, 176, 177, 178, 179, 182, 183, 184, 185 }

F normal fail { 22, 24, 113, 114, 115, 116, 117, 118, 119, 154, 155, 156, 160, 161, 162, 163, 165, 169, 170, 171, 172, 173, 174, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-1) timedout fail { 142, 143, 148, 149, 175, 180, 181, 186, 187, 188, 218, 219, 220, 259, 260, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 399, 400, 401, 404, 405 }

F(-2) exception fail { 102, 103, 107, 110, 111, 112 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	183	34	221	767	109	241	599
N.S.	1	1.00	0.74	0.14	0.89	3.11	0.44	0.98	2.43
time (sec)	N/A	0.100	0.057	0.200	0.282	0.260	0.322	0.266	0.376

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	184	35	221	767	110	241	603
N.S.	1	1.00	0.74	0.14	0.89	3.11	0.45	0.98	2.44
time (sec)	N/A	0.092	0.033	0.193	0.286	0.256	0.323	0.284	13.696

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	95	36	109	755	110	230	579
N.S.	1	1.00	1.10	0.42	1.27	8.78	1.28	2.67	6.73
time (sec)	N/A	0.028	0.021	0.198	0.278	0.265	0.361	0.269	0.333

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	95	37	109	755	110	228	579
N.S.	1	1.00	1.10	0.43	1.27	8.78	1.28	2.65	6.73
time (sec)	N/A	0.033	0.020	0.191	0.284	0.281	0.334	0.267	13.680

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	33	35	39	33	41	52	29
N.S.	1	1.00	0.82	0.88	0.98	0.82	1.02	1.30	0.72
time (sec)	N/A	0.014	0.016	0.236	0.273	0.238	0.053	0.276	0.091

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	40	39	42	49	40	21
N.S.	1	1.00	0.86	0.78	0.76	0.82	0.96	0.78	0.41
time (sec)	N/A	0.018	0.011	0.200	0.270	0.258	0.046	0.260	13.563

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	32	13	25	29	32	29	12
N.S.	1	1.00	2.00	0.81	1.56	1.81	2.00	1.81	0.75
time (sec)	N/A	0.002	0.013	0.214	0.269	0.248	0.041	0.272	0.104

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75
time (sec)	N/A	0.002	0.005	0.199	0.271	0.256	0.052	0.291	0.028

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	204	100	148	138	0	57
N.S.	1	1.00	0.80	2.72	1.33	1.97	1.84	0.00	0.76
time (sec)	N/A	0.026	0.017	0.247	0.299	0.283	0.172	0.000	13.896

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	91	204	70	151	131	0	43
N.S.	1	1.00	0.86	1.92	0.66	1.42	1.24	0.00	0.41
time (sec)	N/A	0.031	0.015	0.245	0.276	0.264	0.203	0.000	13.854

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	83	302	137	87	92	57
N.S.	1	1.00	0.80	1.11	4.03	1.83	1.16	1.23	0.76
time (sec)	N/A	0.032	0.025	0.272	0.282	0.263	0.096	0.287	0.070

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	75	75	302	140	80	100	41
N.S.	1	1.00	0.83	0.83	3.36	1.56	0.89	1.11	0.46
time (sec)	N/A	0.029	0.016	0.222	0.288	0.253	0.094	0.267	13.571

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	18	17	17	22	19	9
N.S.	1	1.00	1.92	1.38	1.31	1.31	1.69	1.46	0.69
time (sec)	N/A	0.004	0.006	0.224	0.293	0.262	0.070	0.263	13.492

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	45	36	0	71	70	0	0
N.S.	1	1.00	2.81	2.25	0.00	4.44	4.38	0.00	0.00
time (sec)	N/A	0.010	10.020	1.888	0.000	0.085	0.848	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	45	36	0	69	70	0	0
N.S.	1	1.00	1.29	1.03	0.00	1.97	2.00	0.00	0.00
time (sec)	N/A	0.021	10.020	1.130	0.000	0.074	0.881	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	43	43	74	88	0	56	61	0	0
N.S.	1	1.00	1.72	2.05	0.00	1.30	1.42	0.00	0.00
time (sec)	N/A	0.016	10.031	1.927	0.000	0.076	0.799	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	89	89	74	88	0	53	60	0	0
N.S.	1	1.00	0.83	0.99	0.00	0.60	0.67	0.00	0.00
time (sec)	N/A	0.028	10.026	1.139	0.000	0.074	0.822	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	47	38	0	73	66	0	0
N.S.	1	1.00	0.53	0.43	0.00	0.82	0.74	0.00	0.00
time (sec)	N/A	0.010	10.018	1.514	0.000	0.080	0.786	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	47	38	0	73	66	0	0
N.S.	1	1.00	0.31	0.25	0.00	0.48	0.43	0.00	0.00
time (sec)	N/A	0.019	10.015	0.471	0.000	0.078	0.780	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	90	90	76	90	0	87	70	0	0
N.S.	1	1.00	0.84	1.00	0.00	0.97	0.78	0.00	0.00
time (sec)	N/A	0.009	10.026	2.491	0.000	0.076	0.837	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	156	156	76	90	0	84	71	0	0
N.S.	1	1.00	0.49	0.58	0.00	0.54	0.46	0.00	0.00
time (sec)	N/A	0.022	10.019	0.795	0.000	0.080	0.854	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	0	76	0	0	0
N.S.	1	1.00	1.00	1.50	0.00	7.60	0.00	0.00	0.00
time (sec)	N/A	0.006	0.410	1.981	0.000	0.081	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	47	38	0	65	71	0	0
N.S.	1	1.00	4.70	3.80	0.00	6.50	7.10	0.00	0.00
time (sec)	N/A	0.011	10.021	1.200	0.000	0.076	0.900	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	73	0	0	0
N.S.	1	1.00	1.04	1.22	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.016	0.006	0.477	0.000	0.086	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	47	38	0	62	71	0	0
N.S.	1	1.00	2.04	1.65	0.00	2.70	3.09	0.00	0.00
time (sec)	N/A	0.021	10.020	1.146	0.000	0.080	0.862	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	162	122	189	94
N.S.	1	1.00	2.21	0.87	0.00	1.98	1.49	2.30	1.15
time (sec)	N/A	0.073	0.076	0.090	0.000	0.254	0.286	0.700	14.338

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	162	122	191	98
N.S.	1	1.00	2.21	0.87	0.00	1.98	1.49	2.33	1.20
time (sec)	N/A	0.070	0.075	0.088	0.000	0.259	0.286	0.688	0.100

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	189	75	0	176	110	195	30
N.S.	1	1.00	2.42	0.96	0.00	2.26	1.41	2.50	0.38
time (sec)	N/A	0.066	0.071	0.080	0.000	0.263	0.302	0.714	14.382

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	189	75	0	179	121	197	88
N.S.	1	1.00	2.20	0.87	0.00	2.08	1.41	2.29	1.02
time (sec)	N/A	0.067	0.067	0.079	0.000	0.254	0.293	0.711	0.105

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	182	88	0	172	121	189	99
N.S.	1	1.00	2.33	1.13	0.00	2.21	1.55	2.42	1.27
time (sec)	N/A	0.035	0.083	0.068	0.000	0.261	0.300	0.679	13.987

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	182	69	0	173	110	190	57
N.S.	1	1.00	2.33	0.88	0.00	2.22	1.41	2.44	0.73
time (sec)	N/A	0.031	0.079	0.066	0.000	0.270	0.303	0.699	13.569

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	168	112	199	29
N.S.	1	1.00	2.71	0.87	0.00	2.40	1.60	2.84	0.41
time (sec)	N/A	0.032	0.082	0.063	0.000	0.261	0.312	0.772	0.124

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	168	112	200	29
N.S.	1	1.00	2.71	0.87	0.00	2.40	1.60	2.86	0.41
time (sec)	N/A	0.029	0.084	0.062	0.000	0.258	0.324	0.687	13.198

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	250	121	0	244	158	6331	129
N.S.	1	1.00	1.87	0.90	0.00	1.82	1.18	47.25	0.96
time (sec)	N/A	0.069	0.123	0.224	0.000	0.255	0.362	1.009	0.192

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	248	119	0	232	160	6341	232
N.S.	1	1.00	1.91	0.92	0.00	1.78	1.23	48.78	1.78
time (sec)	N/A	0.109	0.077	0.093	0.000	0.263	0.393	1.041	13.935

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	248	119	0	232	160	6341	232
N.S.	1	1.00	1.91	0.92	0.00	1.78	1.23	48.78	1.78
time (sec)	N/A	0.086	0.030	0.052	0.000	0.262	0.382	1.097	13.832

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	25	25	26	25	12
N.S.	1	1.00	1.00	0.90	0.86	0.86	0.90	0.86	0.41
time (sec)	N/A	0.017	0.016	0.073	0.200	0.287	0.237	0.261	0.113

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	138	52	0	164	117	51	55
N.S.	1	1.00	2.30	0.87	0.00	2.73	1.95	0.85	0.92
time (sec)	N/A	0.045	0.133	0.071	0.000	0.259	0.254	0.269	13.774

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	126	74	0	110	95	0	66
N.S.	1	1.00	2.03	1.19	0.00	1.77	1.53	0.00	1.06
time (sec)	N/A	0.041	0.053	0.122	0.000	0.273	0.208	0.000	13.575

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	134	66	0	120	83	0	24
N.S.	1	1.00	2.03	1.00	0.00	1.82	1.26	0.00	0.36
time (sec)	N/A	0.039	0.047	0.086	0.000	0.253	0.217	0.000	13.198

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	83	35	0	31	42	39	29
N.S.	1	1.00	1.84	0.78	0.00	0.69	0.93	0.87	0.64
time (sec)	N/A	0.043	0.060	0.110	0.000	0.249	0.058	0.265	0.095

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	17	12	11	19	22	11	19
N.S.	1	1.00	1.13	0.80	0.73	1.27	1.47	0.73	1.27
time (sec)	N/A	0.006	0.008	0.064	0.279	0.257	0.062	0.272	13.611

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	11	11	14	11	11
N.S.	1	1.00	1.00	0.86	0.79	0.79	1.00	0.79	0.79
time (sec)	N/A	0.004	0.004	0.036	0.275	0.237	0.044	0.283	0.027

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	97	34	33	33	44	33	29
N.S.	1	1.00	2.55	0.89	0.87	0.87	1.16	0.87	0.76
time (sec)	N/A	0.024	0.153	0.067	0.292	0.237	0.059	0.271	13.745

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	99	35	0	29	42	45	29
N.S.	1	1.00	2.06	0.73	0.00	0.60	0.88	0.94	0.60
time (sec)	N/A	0.028	0.074	0.080	0.000	0.246	0.055	0.278	0.091

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	97	35	0	33	44	52	29
N.S.	1	1.00	2.11	0.76	0.00	0.72	0.96	1.13	0.63
time (sec)	N/A	0.030	0.181	0.101	0.000	0.249	0.061	0.294	13.719

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	16	17	15	14	46	15
N.S.	1	1.00	0.81	0.76	0.81	0.71	0.67	2.19	0.71
time (sec)	N/A	0.009	0.006	0.240	0.283	0.247	0.045	0.262	0.058

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	101	35	0	31	42	52	29
N.S.	1	1.00	2.20	0.76	0.00	0.67	0.91	1.13	0.63
time (sec)	N/A	0.027	0.210	0.079	0.000	0.255	0.062	0.306	13.650

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	99	27	0	26	29	46	21
N.S.	1	1.00	2.25	0.61	0.00	0.59	0.66	1.05	0.48
time (sec)	N/A	0.023	0.073	0.086	0.000	0.246	0.054	0.290	0.061

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	14	16	0	15	12	42	15
N.S.	1	1.00	0.61	0.70	0.00	0.65	0.52	1.83	0.65
time (sec)	N/A	0.019	0.010	0.090	0.000	0.251	0.054	0.300	0.081

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	12	11	12	12	8	12	12
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.73	1.09	1.09
time (sec)	N/A	0.003	0.005	0.027	0.197	0.262	0.040	0.266	0.044

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	29	25	26	33	14
N.S.	1	1.00	0.74	0.67	0.74	0.64	0.67	0.85	0.36
time (sec)	N/A	0.011	0.008	0.043	0.203	0.263	0.051	0.271	0.306

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	39	0	47	46	77	20
N.S.	1	1.00	0.95	0.89	0.00	1.07	1.05	1.75	0.45
time (sec)	N/A	0.024	0.010	0.087	0.000	0.247	0.049	0.310	13.329

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	127	78	0	109	94	0	63
N.S.	1	1.00	1.92	1.18	0.00	1.65	1.42	0.00	0.95
time (sec)	N/A	0.020	0.055	0.060	0.000	0.249	0.218	0.000	0.070

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	34	0	28	39	39	30
N.S.	1	1.00	1.83	0.74	0.00	0.61	0.85	0.85	0.65
time (sec)	N/A	0.019	0.045	0.047	0.000	0.241	0.061	0.276	13.492

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	12	10	9	17	14	9	17
N.S.	1	1.00	1.33	1.11	1.00	1.89	1.56	1.00	1.89
time (sec)	N/A	0.006	0.006	0.048	0.281	0.260	0.066	0.277	0.061

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	11	11	7	11	11
N.S.	1	1.00	1.00	1.00	1.00	1.00	0.64	1.00	1.00
time (sec)	N/A	0.003	0.006	0.025	0.191	0.254	0.041	0.264	0.027

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	25	25	26	25	12
N.S.	1	1.00	1.00	0.83	0.86	0.86	0.90	0.86	0.41
time (sec)	N/A	0.010	0.005	0.031	0.193	0.254	0.052	0.257	0.066

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	34	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.68	0.40
time (sec)	N/A	0.017	0.010	0.037	0.000	0.252	0.048	0.271	0.063

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	43	46	41	20
N.S.	1	1.00	0.84	0.78	0.00	0.86	0.92	0.82	0.40
time (sec)	N/A	0.016	0.010	0.043	0.000	0.251	0.051	0.298	13.207

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	27	27	22	34	15
N.S.	1	1.00	1.00	0.71	0.87	0.87	0.71	1.10	0.48
time (sec)	N/A	0.010	0.004	0.191	0.190	0.245	0.046	0.264	0.071

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	41	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40
time (sec)	N/A	0.015	0.010	0.046	0.000	0.259	0.055	0.297	13.357

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	40	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.80	0.40
time (sec)	N/A	0.016	0.015	0.044	0.000	0.273	0.048	0.271	0.072

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	41	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40
time (sec)	N/A	0.016	0.013	0.052	0.000	0.261	0.048	0.287	0.089

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	32	12	25	29	32	29	11
N.S.	1	1.00	2.29	0.86	1.79	2.07	2.29	2.07	0.79
time (sec)	N/A	0.004	0.012	0.030	0.275	0.239	0.046	0.276	13.368

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	31	26	29	27	29	33	15
N.S.	1	1.00	0.79	0.67	0.74	0.69	0.74	0.85	0.38
time (sec)	N/A	0.011	0.006	0.044	0.196	0.276	0.048	0.269	13.242

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	42	39	0	45	46	77	20
N.S.	1	1.00	0.88	0.81	0.00	0.94	0.96	1.60	0.42
time (sec)	N/A	0.027	0.014	0.050	0.000	0.245	0.050	0.327	0.132

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	124	74	0	101	88	0	73
N.S.	1	1.00	2.00	1.19	0.00	1.63	1.42	0.00	1.18
time (sec)	N/A	0.040	0.043	0.113	0.000	0.252	0.193	0.000	0.070

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	83	35	0	31	41	26	29
N.S.	1	1.00	1.69	0.71	0.00	0.63	0.84	0.53	0.59
time (sec)	N/A	0.059	0.102	0.103	0.000	0.248	0.053	0.271	13.277

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	81	35	0	31	41	26	29
N.S.	1	1.00	1.88	0.81	0.00	0.72	0.95	0.60	0.67
time (sec)	N/A	0.032	0.054	0.096	0.000	0.243	0.051	0.283	0.089

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	83	35	0	31	41	26	29
N.S.	1	1.00	1.69	0.71	0.00	0.63	0.84	0.53	0.59
time (sec)	N/A	0.044	0.065	0.092	0.000	0.278	0.057	0.265	0.086

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.001	0.003	0.032	0.269	0.240	0.059	0.290	13.386

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	99	34	33	31	41	26	29
N.S.	1	1.00	2.61	0.89	0.87	0.82	1.08	0.68	0.76
time (sec)	N/A	0.020	0.131	0.049	0.277	0.243	0.054	0.267	0.085

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	35	39	29	39	39	29
N.S.	1	1.00	0.86	1.00	1.11	0.83	1.11	1.11	0.83
time (sec)	N/A	0.013	0.011	0.217	0.281	0.248	0.049	0.274	13.347

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	12	8	0	7	7	30	7
N.S.	1	1.00	0.52	0.35	0.00	0.30	0.30	1.30	0.30
time (sec)	N/A	0.012	0.007	0.067	0.000	0.239	0.050	0.277	13.276

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	11	10	10	7	11	10
N.S.	1	1.00	0.91	1.00	0.91	0.91	0.64	1.00	0.91
time (sec)	N/A	0.002	0.004	0.027	0.196	0.235	0.036	0.298	13.361

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	29	22	21	21	19	43	12
N.S.	1	1.00	0.45	0.34	0.32	0.32	0.29	0.66	0.18
time (sec)	N/A	0.022	0.005	0.037	0.198	0.257	0.048	0.280	0.269

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	35	0	36	39	39	18
N.S.	1	1.00	0.93	0.81	0.00	0.84	0.91	0.91	0.42
time (sec)	N/A	0.022	0.010	0.076	0.000	0.240	0.048	0.288	13.528

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	0	39	39	39	18
N.S.	1	1.00	0.87	0.76	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.024	0.016	0.077	0.000	0.234	0.050	0.302	0.232

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	125	78	0	100	87	0	76
N.S.	1	1.00	2.02	1.26	0.00	1.61	1.40	0.00	1.23
time (sec)	N/A	0.021	0.060	0.063	0.000	0.236	0.204	0.000	0.062

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	87	35	0	31	42	26	31
N.S.	1	1.00	1.74	0.70	0.00	0.62	0.84	0.52	0.62
time (sec)	N/A	0.028	0.066	0.049	0.000	0.255	0.055	0.284	13.145

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	82	35	0	31	42	26	31
N.S.	1	1.00	1.86	0.80	0.00	0.70	0.95	0.59	0.70
time (sec)	N/A	0.018	0.041	0.050	0.000	0.251	0.067	0.271	0.085

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	10	14	0	13	10	26	13
N.S.	1	1.00	0.26	0.36	0.00	0.33	0.26	0.67	0.33
time (sec)	N/A	0.022	0.006	0.059	0.000	0.247	0.050	0.274	13.513

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	5	7	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.56	0.78	1.00
time (sec)	N/A	0.002	0.004	0.027	0.197	0.258	0.032	0.265	0.032

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	19	35	10
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.76	1.40	0.40
time (sec)	N/A	0.008	0.005	0.032	0.198	0.249	0.051	0.268	0.063

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	34	34	39	34	18
N.S.	1	1.00	0.87	0.76	0.74	0.74	0.85	0.74	0.39
time (sec)	N/A	0.012	0.008	0.218	0.281	0.265	0.042	0.268	13.246

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	0	39	39	39	18
N.S.	1	1.00	0.87	0.76	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.013	0.010	0.047	0.000	0.258	0.047	0.262	0.062

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.001	0.002	0.023	0.199	0.258	0.045	0.266	0.065

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	34	55	39	39	39	18
N.S.	1	1.00	1.05	0.89	1.45	1.03	1.03	1.03	0.47
time (sec)	N/A	0.019	0.009	0.044	0.280	0.254	0.055	0.282	0.119

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	40	35	0	39	39	39	18
N.S.	1	1.00	0.85	0.74	0.00	0.83	0.83	0.83	0.38
time (sec)	N/A	0.023	0.012	0.042	0.000	0.265	0.050	0.296	0.074

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	0	39	39	39	18
N.S.	1	1.00	0.87	0.76	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.026	0.010	0.046	0.000	0.273	0.048	0.285	0.118

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	33	33	46	33	29
N.S.	1	1.00	2.30	0.79	0.77	0.77	1.07	0.77	0.67
time (sec)	N/A	0.023	0.076	0.061	0.281	0.275	0.062	0.289	13.395

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	33	33	46	33	29
N.S.	1	1.00	2.30	0.79	0.77	0.77	1.07	0.77	0.67
time (sec)	N/A	0.021	0.017	0.036	0.339	0.258	0.066	0.278	0.002

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	24	23	34	22	25	17
N.S.	1	1.00	1.29	1.14	1.10	1.62	1.05	1.19	0.81
time (sec)	N/A	0.003	0.010	0.036	0.189	0.251	0.044	0.271	13.453

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	53	26	38	49	53	44	17
N.S.	1	1.00	1.89	0.93	1.36	1.75	1.89	1.57	0.61
time (sec)	N/A	0.009	0.015	0.051	0.282	0.254	0.299	0.271	13.524

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	72	45	51	55	474	57	290
N.S.	1	1.00	2.00	1.25	1.42	1.53	13.17	1.58	8.06
time (sec)	N/A	0.023	0.034	0.067	0.292	0.266	0.793	0.287	13.410

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	40	0	147	46	41	117
N.S.	1	1.00	0.99	0.54	0.00	1.99	0.62	0.55	1.58
time (sec)	N/A	0.032	0.061	0.070	0.000	0.268	0.091	0.295	0.122

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	97	78	69	69	740	69	827
N.S.	1	1.00	1.17	0.94	0.83	0.83	8.92	0.83	9.96
time (sec)	N/A	0.037	0.089	0.122	0.284	0.263	0.630	0.281	13.439

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	147	136	105	185	874	109	897
N.S.	1	1.00	1.24	1.14	0.88	1.55	7.34	0.92	7.54
time (sec)	N/A	0.060	0.163	0.162	0.277	0.286	0.935	0.286	13.587

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	111	38	0	625	122	604	771
N.S.	1	1.00	0.47	0.16	0.00	2.67	0.52	2.58	3.29
time (sec)	N/A	0.162	0.081	0.159	0.000	0.283	0.640	0.540	13.521

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	166	82	0	953	165	1112	1491
N.S.	1	1.00	0.53	0.26	0.00	3.02	0.52	3.52	4.72
time (sec)	N/A	0.204	0.143	0.404	0.000	0.293	0.918	0.573	13.515

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	53	55	0	103	0	122	121
N.S.	1	1.00	0.33	0.34	0.00	0.64	0.00	0.76	0.76
time (sec)	N/A	0.100	0.043	0.238	0.000	0.263	0.000	0.368	13.946

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	53	52	0	103	0	126	121
N.S.	1	1.00	0.31	0.30	0.00	0.60	0.00	0.73	0.70
time (sec)	N/A	0.104	0.032	0.208	0.000	0.258	0.000	0.369	13.955

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	B	B	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	0	44	0	517	1469	0	1227
N.S.	1	1.00	0.00	0.28	0.00	3.23	9.18	0.00	7.67
time (sec)	N/A	0.094	0.000	0.117	0.000	0.265	1.339	0.000	14.434

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	B	B	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	0	42	0	513	1467	0	1227
N.S.	1	1.00	0.00	0.26	0.00	3.21	9.17	0.00	7.67
time (sec)	N/A	0.082	0.000	0.106	0.000	0.283	1.389	0.000	14.357

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	115	48	0	219	27	4217	133
N.S.	1	1.00	1.01	0.42	0.00	1.92	0.24	36.99	1.17
time (sec)	N/A	0.050	0.130	0.076	0.000	0.255	0.116	8.556	13.435

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	115	90	0	251	0	0	159
N.S.	1	1.00	0.94	0.74	0.00	2.06	0.00	0.00	1.30
time (sec)	N/A	0.054	0.103	0.121	0.000	0.267	0.000	0.000	14.209

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	115	87	88	264	143	92	133
N.S.	1	1.00	0.93	0.70	0.71	2.13	1.15	0.74	1.07
time (sec)	N/A	0.051	0.093	0.119	0.282	0.289	0.134	0.299	14.098

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	130	46	0	903	172	4293	1007
N.S.	1	1.00	0.96	0.34	0.00	6.64	1.26	31.57	7.40
time (sec)	N/A	0.075	0.104	0.053	0.000	0.286	0.934	10.843	13.843

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	138	188	0	1141	0	0	1155
N.S.	1	1.00	0.86	1.18	0.00	7.13	0.00	0.00	7.22
time (sec)	N/A	0.084	0.093	0.101	0.000	0.384	0.000	0.000	14.475

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	139	347	0	1457	0	0	3285
N.S.	1	1.00	0.34	0.84	0.00	3.52	0.00	0.00	7.93
time (sec)	N/A	0.312	0.105	0.148	0.000	0.600	0.000	0.000	14.576

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	163	75	0	1469	0	0	1575
N.S.	1	1.00	0.70	0.32	0.00	6.28	0.00	0.00	6.73
time (sec)	N/A	0.117	0.131	0.282	0.000	0.978	0.000	0.000	14.718

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	103	200	0	105	0	0	0
N.S.	1	1.00	1.07	2.08	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.085	10.134	2.321	0.000	0.100	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	19	113	0	51	0	0	0
N.S.	1	1.00	0.76	4.52	0.00	2.04	0.00	0.00	0.00
time (sec)	N/A	0.019	10.052	2.021	0.000	0.091	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	103	204	0	107	0	0	0
N.S.	1	1.00	1.07	2.12	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.118	10.139	2.087	0.000	0.096	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	107	0	0	0
N.S.	1	1.00	1.16	2.22	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.084	10.103	2.176	0.000	0.110	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	35	95	0	37	0	0	0
N.S.	1	1.00	1.30	3.52	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.023	10.090	1.559	0.000	0.089	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	107	0	0	0
N.S.	1	1.00	1.16	2.22	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.120	10.148	2.051	0.000	0.092	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	187	515	0	322	0	0	0
N.S.	1	1.00	0.63	1.74	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.074	10.233	7.023	0.000	0.115	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	96	94	94	110	98	95
N.S.	1	1.00	1.00	0.91	0.89	0.89	1.04	0.92	0.90
time (sec)	N/A	0.053	0.027	0.216	0.201	0.268	0.027	0.288	0.050

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	72	71	71	78	73	71
N.S.	1	1.00	1.00	0.91	0.90	0.90	0.99	0.92	0.90
time (sec)	N/A	0.034	0.013	0.207	0.197	0.271	0.030	0.348	13.896

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	48	48	56	50	49
N.S.	1	1.00	1.00	0.88	0.86	0.86	1.00	0.89	0.88
time (sec)	N/A	0.019	0.008	0.206	0.205	0.249	0.022	0.431	0.025

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.009	0.005	0.063	0.206	0.283	0.018	0.422	0.044

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	47	0	131	104	50	45
N.S.	1	1.00	1.00	0.85	0.00	2.38	1.89	0.91	0.82
time (sec)	N/A	0.022	0.028	0.224	0.000	0.284	0.144	0.346	14.265

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	70	0	222	138	72	68
N.S.	1	1.00	1.05	0.95	0.00	3.00	1.86	0.97	0.92
time (sec)	N/A	0.034	0.040	0.238	0.000	0.264	0.229	0.350	13.732

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	92	92	0	306	219	86	97
N.S.	1	1.00	0.99	0.99	0.00	3.29	2.35	0.92	1.04
time (sec)	N/A	0.045	0.043	0.248	0.000	0.283	0.352	0.327	13.730

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	113	113	0	424	204	111	129
N.S.	1	1.00	0.92	0.92	0.00	3.45	1.66	0.90	1.05
time (sec)	N/A	0.074	0.057	0.253	0.000	0.259	0.440	0.322	13.971

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	133	128	129	129	144	131	127
N.S.	1	1.00	1.00	0.96	0.97	0.97	1.08	0.98	0.95
time (sec)	N/A	0.074	0.021	0.240	0.197	0.243	0.036	0.316	0.060

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	89	89	104	91	89
N.S.	1	1.00	1.00	0.93	0.92	0.92	1.07	0.94	0.92
time (sec)	N/A	0.045	0.016	0.241	0.202	0.266	0.027	0.285	14.069

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	60	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.00	0.83	0.83
time (sec)	N/A	0.018	0.004	0.231	0.215	0.242	0.021	0.282	0.027

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.005	0.003	0.188	0.199	0.233	0.018	0.275	0.030

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	97	104	0	268	236	118	141
N.S.	1	1.00	0.90	0.96	0.00	2.48	2.19	1.09	1.31
time (sec)	N/A	0.046	0.052	0.284	0.000	0.279	0.233	0.275	13.691

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	134	129	0	394	314	144	183
N.S.	1	1.00	1.02	0.98	0.00	3.01	2.40	1.10	1.40
time (sec)	N/A	0.119	0.078	0.254	0.000	0.259	0.412	0.275	13.594

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	154	152	0	516	257	160	164
N.S.	1	1.00	0.99	0.98	0.00	3.33	1.66	1.03	1.06
time (sec)	N/A	0.170	0.077	0.268	0.000	0.267	0.751	0.283	13.184

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	174	179	0	662	292	183	199
N.S.	1	1.00	0.95	0.97	0.00	3.60	1.59	0.99	1.08
time (sec)	N/A	0.190	0.092	0.271	0.000	0.306	1.727	0.269	13.245

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	200	212	0	806	335	216	240
N.S.	1	1.00	0.90	0.95	0.00	3.61	1.50	0.97	1.08
time (sec)	N/A	0.215	0.127	0.285	0.000	0.280	13.765	0.296	13.305

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	444	118	432	2878	500	510	4022
N.S.	1	1.00	1.02	0.27	0.99	6.59	1.14	1.17	9.20
time (sec)	N/A	0.291	0.219	0.217	0.295	11.202	8.810	0.319	13.869

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	360	80	342	2133	350	411	2712
N.S.	1	1.00	0.97	0.22	0.92	5.76	0.95	1.11	7.33
time (sec)	N/A	0.313	0.174	0.230	0.305	2.391	1.440	0.276	0.540

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	269	56	288	1480	238	319	1479
N.S.	1	1.00	0.91	0.19	0.97	4.98	0.80	1.07	4.98
time (sec)	N/A	0.189	0.178	0.221	0.296	0.707	0.729	0.274	13.505

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	183	34	221	767	109	241	599
N.S.	1	1.00	0.74	0.14	0.89	3.11	0.44	0.98	2.43
time (sec)	N/A	0.100	0.035	0.216	0.292	0.296	0.341	0.277	0.354

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	134	27	169	112	20	179	33
N.S.	1	1.00	0.72	0.15	0.91	0.61	0.11	0.97	0.18
time (sec)	N/A	0.074	0.016	0.210	0.288	0.263	0.076	0.269	13.576

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	234	253	0	4084	0	344	4802
N.S.	1	1.00	0.70	0.75	0.00	12.15	0.00	1.02	14.29
time (sec)	N/A	0.186	0.106	0.310	0.000	0.697	0.000	0.286	15.587

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	362	307	0	8409	0	531	16369
N.S.	1	1.00	0.80	0.68	0.00	18.56	0.00	1.17	36.13
time (sec)	N/A	0.255	0.322	0.377	0.000	9.782	0.000	0.308	16.805

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	371	119	292	2116	352	430	2560
N.S.	1	1.00	1.02	0.33	0.80	5.83	0.97	1.18	7.05
time (sec)	N/A	0.255	0.175	0.479	0.292	0.716	1.639	0.290	14.968

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	295	97	324	1596	275	350	1565
N.S.	1	1.00	0.85	0.28	0.93	4.57	0.79	1.00	4.48
time (sec)	N/A	0.192	0.118	0.238	0.287	0.724	1.006	0.281	0.532

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	267	67	253	873	136	268	637
N.S.	1	1.00	0.97	0.24	0.92	3.17	0.49	0.97	2.32
time (sec)	N/A	0.131	0.196	0.237	0.282	0.287	0.450	0.279	14.343

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	183	46	189	183	39	194	58
N.S.	1	1.00	0.91	0.23	0.94	0.91	0.19	0.96	0.29
time (sec)	N/A	0.082	0.073	0.219	0.284	0.288	0.139	0.269	0.090

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	429	334	0	9892	0	621	17945
N.S.	1	1.00	0.62	0.48	0.00	14.36	0.00	0.90	26.04
time (sec)	N/A	0.376	0.198	0.361	0.000	12.224	0.000	0.285	16.364

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	864	864	540	402	0	15292	0	879	28923
N.S.	1	1.00	0.62	0.47	0.00	17.70	0.00	1.02	33.48
time (sec)	N/A	0.550	0.365	0.457	0.000	109.947	0.000	0.289	17.346

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	386	180	286	0	204	214	0	0
N.S.	1	0.99	0.46	0.74	0.00	0.53	0.55	0.00	0.00
time (sec)	N/A	0.263	10.180	4.797	0.000	0.094	2.113	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	140	235	0	166	173	0	0
N.S.	1	1.00	0.43	0.72	0.00	0.51	0.53	0.00	0.00
time (sec)	N/A	0.176	10.104	2.702	0.000	0.088	1.719	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	120	200	0	114	124	0	0
N.S.	1	1.00	0.45	0.76	0.00	0.43	0.47	0.00	0.00
time (sec)	N/A	0.083	10.067	1.333	0.000	0.086	1.297	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	77	169	0	89	78	0	0
N.S.	1	1.00	0.34	0.75	0.00	0.39	0.35	0.00	0.00
time (sec)	N/A	0.046	10.035	0.424	0.000	0.077	0.841	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	95	107	0	0	0	0	0
N.S.	1	1.00	0.28	0.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	10.159	0.474	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	522	556	0	0	0	0	0
N.S.	1	1.00	0.90	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	10.591	0.753	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	729	729	332	1018	0	0	0	0	0
N.S.	1	1.00	0.46	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.782	10.854	1.727	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	141	222	0	167	180	0	0
N.S.	1	1.00	0.66	1.04	0.00	0.78	0.85	0.00	0.00
time (sec)	N/A	0.175	10.115	4.503	0.000	0.102	1.881	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	121	186	0	115	129	0	0
N.S.	1	1.00	0.75	1.15	0.00	0.71	0.80	0.00	0.00
time (sec)	N/A	0.088	10.075	1.023	0.000	0.100	1.389	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	77	154	0	91	82	0	0
N.S.	1	1.00	0.62	1.24	0.00	0.73	0.66	0.00	0.00
time (sec)	N/A	0.054	10.043	0.418	0.000	0.097	0.947	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	91	97	0	0	0	0	0
N.S.	1	1.00	1.26	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	10.190	0.454	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	508	523	0	0	0	0	0
N.S.	1	1.00	1.70	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	10.810	0.768	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	321	961	0	0	0	0	0
N.S.	1	1.00	0.76	2.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	10.976	1.819	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	458	1420	0	0	0	0	0
N.S.	1	1.00	0.81	2.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.721	11.376	2.467	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	78	160	0	87	73	0	0
N.S.	1	1.00	0.62	1.27	0.00	0.69	0.58	0.00	0.00
time (sec)	N/A	0.049	10.040	1.051	0.000	0.090	0.900	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	92	99	0	0	0	0	0
N.S.	1	1.00	1.26	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	10.158	0.453	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	86	158	0	114	70	0	0
N.S.	1	1.00	1.59	2.93	0.00	2.11	1.30	0.00	0.00
time (sec)	N/A	0.027	10.038	0.622	0.000	0.084	0.916	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	85	165	0	131	76	0	0
N.S.	1	1.00	1.63	3.17	0.00	2.52	1.46	0.00	0.00
time (sec)	N/A	0.032	10.034	1.850	0.000	0.081	0.869	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	80	175	0	97	83	0	0
N.S.	1	1.00	0.34	0.74	0.00	0.41	0.35	0.00	0.00
time (sec)	N/A	0.042	10.039	1.132	0.000	0.081	0.867	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	98	110	0	0	0	0	0
N.S.	1	1.00	0.28	0.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	10.151	0.479	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	79	0	0	0	0	0
N.S.	1	1.00	1.00	1.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	10.213	1.084	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	50	86	0	0	0	0	0
N.S.	1	1.00	0.16	0.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	10.137	1.273	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	59	78	0	0	0	0	0
N.S.	1	1.00	1.48	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	10.133	1.084	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	65	86	0	0	0	0	0
N.S.	1	1.00	0.22	0.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	10.123	1.049	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	0	21	21
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.11
time (sec)	N/A	0.006	0.182	0.084	0.243	0.292	0.000	0.364	13.095

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	196	136	0	0	0	167	0	0
N.S.	1	0.96	0.67	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.147	0.586	0.000	0.000	0.000	60.588	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	142	106	0	0	0	119	0	0
N.S.	1	0.95	0.71	0.00	0.00	0.00	0.79	0.00	0.00
time (sec)	N/A	0.086	0.560	0.000	0.000	0.000	32.630	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.78	0.00	0.00
time (sec)	N/A	0.032	0.431	0.000	0.000	0.000	17.250	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	0.93
time (sec)	N/A	0.006	0.059	0.000	0.000	0.000	3.658	0.000	13.653

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	103	86	75	0	0	129	0	0
N.S.	1	0.95	0.80	0.69	0.00	0.00	1.19	0.00	0.00
time (sec)	N/A	0.077	0.868	4.728	0.000	0.000	50.668	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	79	65	56	0	0	94	0	0
N.S.	1	0.92	0.76	0.65	0.00	0.00	1.09	0.00	0.00
time (sec)	N/A	0.045	0.797	2.154	0.000	0.000	27.980	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	0	0	61	0	0
N.S.	1	1.00	1.00	0.88	0.00	0.00	1.45	0.00	0.00
time (sec)	N/A	0.013	0.549	0.891	0.000	0.000	13.868	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	0	0	29	0	15
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.61	0.00	0.83
time (sec)	N/A	0.003	0.047	0.485	0.000	0.000	3.080	0.000	0.073

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	42	0	116	75	54	42
N.S.	1	1.00	1.00	0.82	0.00	2.27	1.47	1.06	0.82
time (sec)	N/A	0.025	0.018	0.249	0.000	0.256	0.101	0.265	0.098

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	0	90	58	42	28
N.S.	1	1.00	1.00	0.82	0.00	2.37	1.53	1.11	0.74
time (sec)	N/A	0.023	0.014	0.223	0.000	0.268	0.086	0.269	13.162

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	0	73	34	23	21
N.S.	1	1.00	1.00	0.76	0.00	2.52	1.17	0.79	0.72
time (sec)	N/A	0.015	0.008	0.496	0.000	0.280	0.072	0.266	10.652

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	68	46	18	16
N.S.	1	1.00	1.00	0.67	0.00	2.83	1.92	0.75	0.67
time (sec)	N/A	0.007	0.005	0.231	0.000	0.264	0.067	0.266	0.035

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	65	54	0	189	226	56	74
N.S.	1	1.00	0.90	0.75	0.00	2.62	3.14	0.78	1.03
time (sec)	N/A	0.034	0.026	0.296	0.000	0.270	0.211	0.275	0.095

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	76	64	0	278	257	66	96
N.S.	1	1.00	0.85	0.72	0.00	3.12	2.89	0.74	1.08
time (sec)	N/A	0.053	0.042	0.313	0.000	0.273	0.274	0.295	0.095

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	70	50	0	199	0	109	0
N.S.	1	1.00	1.13	0.81	0.00	3.21	0.00	1.76	0.00
time (sec)	N/A	0.030	0.087	0.623	0.000	0.264	0.000	0.332	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	50	33	0	138	0	82	0
N.S.	1	1.00	1.32	0.87	0.00	3.63	0.00	2.16	0.00
time (sec)	N/A	0.016	0.058	0.279	0.000	0.260	0.000	0.316	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	69	59	0	209	0	101	0
N.S.	1	1.00	1.13	0.97	0.00	3.43	0.00	1.66	0.00
time (sec)	N/A	0.025	0.112	0.285	0.000	0.270	0.000	0.335	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	69	0	279	0	113	0
N.S.	1	1.00	1.00	0.86	0.00	3.49	0.00	1.41	0.00
time (sec)	N/A	0.044	0.175	0.314	0.000	0.282	0.000	0.316	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	98	96	0	251	0	0	0
N.S.	1	1.00	0.64	0.63	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.036	2.724	0.284	0.000	0.271	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	86	75	0	223	0	0	0
N.S.	1	1.00	0.78	0.68	0.00	2.03	0.00	0.00	0.00
time (sec)	N/A	0.023	2.242	0.259	0.000	0.260	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	54	0	121	0	0	0
N.S.	1	1.00	0.77	0.83	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.014	0.959	0.230	0.000	0.269	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	249	0	152	0	0	0
N.S.	1	1.00	1.00	3.19	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.023	1.375	0.591	0.000	0.248	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	111	488	0	297	0	0	0
N.S.	1	1.00	0.89	3.90	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.033	2.332	0.231	0.000	0.253	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	123	711	0	365	0	0	0
N.S.	1	1.00	0.73	4.23	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.056	2.686	0.233	0.000	0.275	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	94	0	265	0	0	0
N.S.	1	1.00	0.81	0.62	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.035	2.707	0.233	0.000	0.256	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	110	74	0	236	0	0	0
N.S.	1	1.00	1.01	0.68	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.023	2.235	0.235	0.000	0.252	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	67	54	0	125	0	0	0
N.S.	1	1.00	1.05	0.84	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.015	0.934	0.231	0.000	0.261	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	256	0	155	0	0	0
N.S.	1	1.00	1.00	3.32	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.021	1.335	0.284	0.000	0.244	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	110	499	0	302	0	0	0
N.S.	1	1.00	0.89	4.02	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.031	2.351	0.234	0.000	0.254	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	122	728	0	376	0	0	0
N.S.	1	1.00	0.73	4.36	0.00	2.25	0.00	0.00	0.00
time (sec)	N/A	0.055	2.635	0.237	0.000	0.280	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	38	25	0	73	0	0	0
N.S.	1	1.00	1.27	0.83	0.00	2.43	0.00	0.00	0.00
time (sec)	N/A	0.007	0.510	0.231	0.000	0.238	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	40	34	33	0	65	0	0	0
N.S.	1	1.67	1.42	1.38	0.00	2.71	0.00	0.00	0.00
time (sec)	N/A	0.010	0.497	0.228	0.000	0.236	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	72	71	59	0	137	0	0	0
N.S.	1	0.99	0.97	0.81	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.073	3.608	0.252	0.000	0.255	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	121	134	0	446	345	158	182
N.S.	1	1.00	1.00	1.11	0.00	3.69	2.85	1.31	1.50
time (sec)	N/A	0.130	0.057	0.240	0.000	0.256	0.446	0.285	7.957

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	84	81	0	311	275	101	113
N.S.	1	1.00	0.98	0.94	0.00	3.62	3.20	1.17	1.31
time (sec)	N/A	0.106	0.035	0.221	0.000	0.269	0.350	0.290	8.007

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	51	0	210	212	56	52
N.S.	1	1.00	0.98	0.80	0.00	3.28	3.31	0.88	0.81
time (sec)	N/A	0.070	0.040	0.221	0.000	0.264	0.244	0.289	0.041

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	33	0	134	124	38	38
N.S.	1	1.00	0.98	0.67	0.00	2.73	2.53	0.78	0.78
time (sec)	N/A	0.029	0.011	0.051	0.000	0.258	0.113	0.279	8.720

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	133	109	0	895	0	144	3901
N.S.	1	1.00	0.98	0.80	0.00	6.58	0.00	1.06	28.68
time (sec)	N/A	0.143	0.137	0.479	0.000	0.352	0.000	0.291	9.265

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	177	174	0	1765	0	235	6267
N.S.	1	1.00	0.95	0.93	0.00	9.44	0.00	1.26	33.51
time (sec)	N/A	0.239	0.270	0.376	0.000	0.853	0.000	0.285	9.168

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	172	144	0	1079	0	0	0
N.S.	1	1.00	1.24	1.04	0.00	7.76	0.00	0.00	0.00
time (sec)	N/A	0.220	0.393	0.997	0.000	0.540	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	139	100	0	940	0	0	0
N.S.	1	1.00	1.29	0.93	0.00	8.70	0.00	0.00	0.00
time (sec)	N/A	0.094	0.176	0.333	0.000	0.334	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	94	64	0	432	0	88	0
N.S.	1	1.00	1.24	0.84	0.00	5.68	0.00	1.16	0.00
time (sec)	N/A	0.051	0.125	0.267	0.000	0.297	0.000	0.285	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	172	122	0	701	0	129	0
N.S.	1	1.00	1.62	1.15	0.00	6.61	0.00	1.22	0.00
time (sec)	N/A	0.078	0.264	0.303	0.000	0.343	0.000	0.286	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	174	152	0	1063	0	343	0
N.S.	1	1.00	1.17	1.02	0.00	7.13	0.00	2.30	0.00
time (sec)	N/A	0.185	0.435	0.381	0.000	0.513	0.000	0.301	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	169	235	0	131	0	0	0
N.S.	1	1.00	0.92	1.28	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.064	5.055	2.015	0.000	0.080	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	162	230	0	125	0	0	0
N.S.	1	1.00	0.99	1.40	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.042	4.611	0.494	0.000	0.081	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	168	223	0	110	0	0	0
N.S.	1	1.00	1.16	1.54	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.028	4.314	0.430	0.000	0.077	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	293	0	124	0	0	0
N.S.	1	1.00	0.85	2.14	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.052	8.896	0.490	0.000	0.102	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	164	224	0	113	0	0	0
N.S.	1	1.00	3.35	4.57	0.00	2.31	0.00	0.00	0.00
time (sec)	N/A	0.007	10.262	0.627	0.000	0.094	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	176	251	0	164	0	0	0
N.S.	1	1.00	1.89	2.70	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.312	10.217	1.072	0.000	0.102	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	240	341	0	216	0	0	0
N.S.	1	1.00	1.45	2.05	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.366	10.313	0.869	0.000	0.098	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	157	225	0	121	0	0	0
N.S.	1	1.00	0.99	1.42	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.045	10.150	1.115	0.000	0.080	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	143	218	0	113	0	0	0
N.S.	1	1.00	1.04	1.59	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.030	10.117	0.516	0.000	0.080	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	205	0	99	0	0	0
N.S.	1	1.00	0.82	1.78	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.014	10.063	0.271	0.000	0.089	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	72	104	0	55	0	0	0
N.S.	1	1.00	1.04	1.51	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.042	10.079	0.483	0.000	0.095	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	226	328	0	140	0	0	0
N.S.	1	1.00	1.92	2.78	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.090	10.284	0.600	0.000	0.102	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	235	336	0	181	0	0	0
N.S.	1	1.00	1.65	2.37	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.169	10.258	0.687	0.000	0.100	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	136	223	0	153	0	0	0
N.S.	1	1.00	0.94	1.55	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.034	10.137	1.131	0.000	0.080	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	158	225	0	131	0	0	0
N.S.	1	1.00	1.61	2.30	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.016	10.129	0.388	0.000	0.083	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	160	223	0	131	0	0	0
N.S.	1	1.00	1.67	2.32	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.017	9.860	0.292	0.000	0.086	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	204	328	0	168	0	0	0
N.S.	1	1.00	1.23	1.98	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.068	10.158	0.527	0.000	0.098	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	168	339	0	194	0	0	0
N.S.	1	1.00	1.51	3.05	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.176	10.286	0.621	0.000	0.103	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	192	346	0	251	0	0	0
N.S.	1	1.00	1.01	1.82	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.365	10.252	0.746	0.000	0.111	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	135	135	135	135	156	148	131
N.S.	1	1.00	1.00	1.00	1.00	1.00	1.16	1.10	0.97
time (sec)	N/A	0.081	0.035	0.258	0.191	0.260	0.030	0.292	0.036

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	102	102	102	112	111	101
N.S.	1	1.00	1.01	0.99	0.99	0.99	1.09	1.08	0.98
time (sec)	N/A	0.062	0.020	0.261	0.177	0.239	0.027	0.266	7.489

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	69	78	76	70
N.S.	1	1.00	1.00	0.96	0.95	0.95	1.07	1.04	0.96
time (sec)	N/A	0.039	0.014	0.262	0.197	0.241	0.026	0.274	7.514

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	36	39	40	38
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.93	0.95	0.90
time (sec)	N/A	0.017	0.007	0.064	0.181	0.235	0.018	0.294	0.026

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	57	0	159	117	62	57
N.S.	1	1.00	0.98	0.86	0.00	2.41	1.77	0.94	0.86
time (sec)	N/A	0.029	0.040	0.508	0.000	0.248	0.215	0.283	0.047

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	88	79	0	268	153	83	77
N.S.	1	1.05	1.06	0.95	0.00	3.23	1.84	1.00	0.93
time (sec)	N/A	0.057	0.039	0.255	0.000	0.262	0.387	0.274	7.559

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	119	110	107	0	391	196	109	112
N.S.	1	1.03	0.96	0.93	0.00	3.40	1.70	0.95	0.97
time (sec)	N/A	0.074	0.070	0.272	0.000	0.273	0.662	0.290	7.644

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	154	142	130	0	530	241	145	144
N.S.	1	1.03	0.95	0.87	0.00	3.53	1.61	0.97	0.96
time (sec)	N/A	0.124	0.094	0.566	0.000	0.252	1.180	0.282	7.585

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	219	218	218	272	261	220
N.S.	1	1.00	1.00	0.98	0.98	0.98	1.22	1.17	0.99
time (sec)	N/A	0.135	0.058	0.267	0.184	0.255	0.036	0.283	0.049

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	155	147	147	192	181	148
N.S.	1	1.00	1.01	1.00	0.95	0.95	1.24	1.17	0.95
time (sec)	N/A	0.087	0.038	0.260	0.181	0.259	0.030	0.274	7.638

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	91	90	90	107	100	90
N.S.	1	1.00	1.00	0.95	0.94	0.94	1.11	1.04	0.94
time (sec)	N/A	0.045	0.017	0.140	0.186	0.250	0.027	0.284	0.024

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	45	41	48	43	42
N.S.	1	1.00	1.00	0.86	0.92	0.84	0.98	0.88	0.86
time (sec)	N/A	0.016	0.005	0.013	0.181	0.238	0.019	0.287	0.013

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	144	178	0	406	371	205	229
N.S.	1	1.00	1.01	1.24	0.00	2.84	2.59	1.43	1.60
time (sec)	N/A	0.085	0.046	0.583	0.000	0.266	0.486	0.273	0.039

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	183	213	0	600	484	228	293
N.S.	1	1.00	1.10	1.28	0.00	3.61	2.92	1.37	1.77
time (sec)	N/A	0.195	0.069	0.284	0.000	0.254	1.171	0.283	7.678

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	217	237	0	794	398	265	257
N.S.	1	1.00	1.08	1.18	0.00	3.95	1.98	1.32	1.28
time (sec)	N/A	0.266	0.072	0.288	0.000	0.257	9.721	0.290	0.072

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	267	285	0	1016	0	320	308
N.S.	1	1.00	1.07	1.14	0.00	4.06	0.00	1.28	1.23
time (sec)	N/A	0.337	0.099	0.257	0.000	0.274	0.000	0.294	7.718

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	345	347	0	1266	0	393	375
N.S.	1	1.00	1.09	1.09	0.00	3.99	0.00	1.24	1.18
time (sec)	N/A	0.390	0.152	0.296	0.000	0.286	0.000	0.282	7.680

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	88	79	0	268	153	83	77
N.S.	1	1.05	1.06	0.95	0.00	3.23	1.84	1.00	0.93
time (sec)	N/A	0.054	0.031	0.215	0.000	0.251	0.386	0.286	0.001

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	88	79	0	268	153	83	77
N.S.	1	1.00	1.06	0.95	0.00	3.23	1.84	1.00	0.93
time (sec)	N/A	0.057	0.013	0.250	0.000	0.259	0.389	0.283	7.569

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	570	227	0	16218	0	9306	29551
N.S.	1	1.00	1.24	0.49	0.00	35.33	0.00	20.27	64.38
time (sec)	N/A	0.966	0.429	0.244	0.000	270.801	0.000	1.303	10.920

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	402	129	0	9584	0	6418	17954
N.S.	1	1.00	1.27	0.41	0.00	30.33	0.00	20.31	56.82
time (sec)	N/A	0.483	0.334	0.240	0.000	44.923	0.000	1.138	9.730

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	269	76	0	4690	0	4110	9600
N.S.	1	1.00	1.13	0.32	0.00	19.71	0.00	17.27	40.34
time (sec)	N/A	0.354	0.204	0.232	0.000	4.220	0.000	0.912	8.837

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	172	45	0	1525	0	1402	4109
N.S.	1	1.00	0.99	0.26	0.00	8.76	0.00	8.06	23.61
time (sec)	N/A	0.133	0.098	0.061	0.000	0.357	0.000	0.818	8.212

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	129	38	0	613	87	1026	763
N.S.	1	1.00	0.86	0.25	0.00	4.09	0.58	6.84	5.09
time (sec)	N/A	0.059	0.056	0.041	0.000	0.276	0.547	0.579	0.305

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	274	215	0	16013	0	7664	23640
N.S.	1	1.00	1.08	0.85	0.00	63.04	0.00	30.17	93.07
time (sec)	N/A	0.363	0.182	0.343	0.000	14.270	0.000	1.783	10.793

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	354	345	0	0	0	2394	91169
N.S.	1	1.00	0.83	0.80	0.00	0.00	0.00	5.58	212.52
time (sec)	N/A	0.839	0.495	0.518	0.000	0.000	0.000	1.833	11.414

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	540	320	0	12117	0	8992	29030
N.S.	1	1.00	0.96	0.57	0.00	21.52	0.00	15.97	51.56
time (sec)	N/A	2.371	0.966	0.262	0.000	107.283	0.000	1.768	10.269

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	415	224	0	7338	0	6390	18785
N.S.	1	1.00	1.08	0.58	0.00	19.01	0.00	16.55	48.67
time (sec)	N/A	1.355	0.679	0.218	0.000	12.700	0.000	1.381	11.076

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	310	178	0	4573	0	4425	12350
N.S.	1	1.00	1.06	0.61	0.00	15.61	0.00	15.10	42.15
time (sec)	N/A	0.501	0.465	0.163	0.000	1.979	0.000	1.317	10.551

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	243	151	0	2309	0	2682	6404
N.S.	1	1.00	0.96	0.60	0.00	9.16	0.00	10.64	25.41
time (sec)	N/A	0.349	0.266	0.105	0.000	0.407	0.000	0.611	8.642

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	660	660	708	850	0	0	0	37254	237586
N.S.	1	1.00	1.07	1.29	0.00	0.00	0.00	56.45	359.98
time (sec)	N/A	1.724	1.621	0.791	0.000	0.000	0.000	7.050	15.371

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1077	1077	1020	1250	0	0	0	65158	97073
N.S.	1	1.00	0.95	1.16	0.00	0.00	0.00	60.50	90.13
time (sec)	N/A	8.970	3.321	1.179	0.000	0.000	0.000	10.357	17.554

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	181	149	0	370	439	196	0
N.S.	1	1.00	0.84	0.69	0.00	1.72	2.04	0.91	0.00
time (sec)	N/A	0.099	0.320	0.350	0.000	0.351	0.521	0.290	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	148	124	0	304	274	155	0
N.S.	1	1.00	0.85	0.71	0.00	1.74	1.57	0.89	0.00
time (sec)	N/A	0.080	0.231	0.337	0.000	0.334	0.458	0.286	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	112	96	0	232	153	113	0
N.S.	1	1.00	0.85	0.73	0.00	1.76	1.16	0.86	0.00
time (sec)	N/A	0.067	0.173	0.319	0.000	0.340	0.386	0.287	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	72	0	174	114	81	0
N.S.	1	1.00	0.96	0.74	0.00	1.79	1.18	0.84	0.00
time (sec)	N/A	0.041	0.277	0.271	0.000	0.319	0.341	0.281	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	87	85	0	249	134	82	0
N.S.	1	1.04	0.98	0.96	0.00	2.80	1.51	0.92	0.00
time (sec)	N/A	0.045	0.156	0.303	0.000	0.315	3.904	0.282	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	105	92	95	0	289	450	93	0
N.S.	1	1.04	0.91	0.94	0.00	2.86	4.46	0.92	0.00
time (sec)	N/A	0.049	0.187	0.304	0.000	0.313	6.508	0.286	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	69	58	173	93	639	81	133
N.S.	1	1.00	0.80	0.67	2.01	1.08	7.43	0.94	1.55
time (sec)	N/A	0.070	0.157	0.285	0.201	0.338	14.752	0.308	7.871

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	103	83	227	136	1989	123	154
N.S.	1	1.00	0.82	0.66	1.80	1.08	15.79	0.98	1.22
time (sec)	N/A	0.095	0.215	0.291	0.188	0.336	35.551	0.323	7.881

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	164	132	108	281	177	5187	162	189
N.S.	1	0.99	0.80	0.65	1.70	1.07	31.44	0.98	1.15
time (sec)	N/A	0.136	0.295	0.309	0.188	0.401	80.517	0.304	7.875

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	167	133	335	224	11602	207	226
N.S.	1	1.00	0.80	0.63	1.60	1.07	55.25	0.99	1.08
time (sec)	N/A	0.148	0.463	0.314	0.208	0.442	154.155	0.299	7.908

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	119	138	0	63	0	0	0
N.S.	1	1.00	0.62	0.72	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.070	6.628	5.773	0.000	0.089	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	114	133	0	58	0	0	0
N.S.	1	1.00	0.68	0.79	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.048	5.689	1.932	0.000	0.101	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	109	128	0	53	0	0	0
N.S.	1	1.00	0.73	0.86	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.031	4.596	1.176	0.000	0.086	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	102	121	0	46	0	0	0
N.S.	1	1.00	0.72	0.86	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.028	3.650	0.614	0.000	0.091	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	232	90	138	0	0	0	0	0
N.S.	1	1.30	0.51	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.083	9.531	0.750	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0	0
N.S.	1	1.00	1.00	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	10.255	3.243	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	174	183	0	0	0	0	0
N.S.	1	1.00	0.73	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	10.351	3.411	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	129	148	0	73	0	0	0
N.S.	1	1.00	0.59	0.68	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.084	10.088	4.597	0.000	0.088	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	124	143	0	68	0	0	0
N.S.	1	1.00	0.63	0.72	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.062	8.265	1.796	0.000	0.084	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	119	138	0	63	0	0	0
N.S.	1	1.00	0.66	0.77	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.046	6.819	1.169	0.000	0.088	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	114	133	0	58	0	0	0
N.S.	1	1.00	0.66	0.77	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.041	5.075	0.682	0.000	0.112	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	148	170	0	0	0	0	0
N.S.	1	1.00	0.71	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	10.236	1.688	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	333	213	177	0	0	0	0	0
N.S.	1	1.50	0.96	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	10.281	2.639	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	288	174	183	0	0	0	0	0
N.S.	1	1.25	0.75	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	10.387	3.534	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	106	126	0	53	0	0	0
N.S.	1	1.00	0.68	0.80	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.053	10.154	4.606	0.000	0.101	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	104	121	0	49	0	0	0
N.S.	1	1.00	0.73	0.85	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.037	10.076	1.642	0.000	0.081	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	69	106	0	41	0	0	0
N.S.	1	1.00	0.57	0.88	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.021	10.053	0.693	0.000	0.083	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	0	11	0	0	0
N.S.	1	1.00	1.04	0.96	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.005	10.020	0.213	0.000	0.079	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	55	47	0	0	0	0	0
N.S.	1	1.00	0.52	0.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	10.129	0.826	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0	0
N.S.	1	1.00	1.00	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.122	10.303	2.698	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	186	183	0	0	0	0	0
N.S.	1	1.00	0.78	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.165	10.383	3.481	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	109	138	0	101	0	0	0
N.S.	1	1.00	0.58	0.73	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.081	10.094	8.956	0.000	0.094	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	104	133	0	96	0	0	0
N.S.	1	1.00	0.61	0.78	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.056	10.084	3.541	0.000	0.084	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	99	128	0	91	0	0	0
N.S.	1	1.00	0.66	0.86	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.037	10.085	2.590	0.000	0.085	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	99	128	0	84	0	0	0
N.S.	1	1.00	0.66	0.86	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.037	10.069	2.561	0.000	0.085	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	97	126	0	81	0	0	0
N.S.	1	1.00	0.67	0.87	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.028	10.058	1.423	0.000	0.088	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	99	128	0	84	0	0	0
N.S.	1	1.00	0.66	0.86	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.029	5.079	0.645	0.000	0.088	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	207	138	161	0	0	0	0	0
N.S.	1	1.20	0.80	0.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.094	10.192	1.602	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	208	185	0	0	0	0	0
N.S.	1	1.00	0.89	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	10.319	2.812	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	159	193	0	0	0	0	0
N.S.	1	1.00	0.60	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.481	10.486	3.512	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	112	157	0	78	0	0	0
N.S.	1	1.00	0.97	1.35	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.073	10.101	7.908	0.000	0.084	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	107	152	0	73	0	0	0
N.S.	1	1.00	1.13	1.60	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.055	8.137	2.888	0.000	0.082	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	102	147	0	68	0	0	0
N.S.	1	1.00	1.38	1.99	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.039	6.197	2.182	0.000	0.094	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	94	140	0	60	0	0	0
N.S.	1	1.00	2.04	3.04	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.028	4.952	1.292	0.000	0.086	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	90	125	0	56	0	0	0
N.S.	1	1.00	2.05	2.84	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.027	3.930	0.643	0.000	0.088	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	51	141	0	0	0	0	0
N.S.	1	1.00	1.11	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	9.930	0.762	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0	0
N.S.	1	1.00	2.65	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	10.276	3.552	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0	0
N.S.	1	1.00	2.39	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	10.352	3.779	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	122	167	0	89	0	0	0
N.S.	1	1.00	0.86	1.18	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.085	10.096	7.098	0.000	0.088	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	117	162	0	84	0	0	0
N.S.	1	1.00	0.97	1.34	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.066	10.119	2.895	0.000	0.083	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	112	157	0	79	0	0	0
N.S.	1	1.00	1.12	1.57	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.048	10.013	2.207	0.000	0.088	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	107	152	0	74	0	0	0
N.S.	1	1.00	1.32	1.88	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.037	7.229	1.707	0.000	0.092	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	102	147	0	69	0	0	0
N.S.	1	1.00	1.38	1.99	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.033	5.128	0.680	0.000	0.086	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	130	173	0	0	0	0	0
N.S.	1	1.00	1.81	2.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	10.269	2.001	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	201	180	0	0	0	0	0
N.S.	1	1.00	2.16	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	10.323	2.891	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0	0
N.S.	1	1.00	2.39	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	10.392	3.786	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	97	142	0	64	0	0	0
N.S.	1	1.00	1.49	2.18	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.049	10.102	5.358	0.000	0.089	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	92	125	0	59	0	0	0
N.S.	1	1.00	2.00	2.72	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.039	10.091	1.950	0.000	0.084	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	34	110	0	52	0	0	0
N.S.	1	1.00	1.36	4.40	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.023	10.069	0.741	0.000	0.085	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	47	0	9	0	0	0
N.S.	1	1.00	1.90	4.70	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.008	0.011	0.214	0.000	0.074	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	24	48	0	0	0	0	0
N.S.	1	1.00	1.41	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	10.142	0.912	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0	0
N.S.	1	1.00	2.65	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	10.316	2.951	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	108	189	0	0	0	0	0
N.S.	1	1.00	1.06	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	10.428	3.538	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	97	142	0	111	0	0	0
N.S.	1	1.00	1.04	1.53	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.074	10.112	8.850	0.000	0.082	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	92	137	0	106	0	0	0
N.S.	1	1.00	1.24	1.85	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.050	10.090	4.257	0.000	0.080	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	79	132	0	101	0	0	0
N.S.	1	1.00	1.44	2.40	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	0.033	10.090	2.520	0.000	0.086	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	79	132	0	80	0	0	0
N.S.	1	1.00	1.44	2.40	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.033	10.081	2.460	0.000	0.079	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	79	132	0	80	0	0	0
N.S.	1	1.00	1.44	2.40	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.034	10.070	1.204	0.000	0.081	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	79	130	0	77	0	0	0
N.S.	1	1.00	1.44	2.36	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.027	5.773	0.243	0.000	0.080	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	101	164	0	0	0	0	0
N.S.	1	1.00	1.40	2.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	10.225	1.823	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	196	188	0	0	0	0	0
N.S.	1	1.00	1.96	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	10.358	3.040	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	244	198	0	0	0	0	0
N.S.	1	1.00	1.91	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	10.438	3.533	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	354	246	0	138	0	0	0
N.S.	1	1.00	1.46	1.02	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.094	5.991	2.394	0.000	0.082	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	349	241	0	133	0	0	0
N.S.	1	1.00	1.58	1.09	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.073	5.617	0.664	0.000	0.084	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	343	236	0	128	0	0	0
N.S.	1	1.00	1.73	1.19	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.055	5.104	0.524	0.000	0.085	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	338	231	0	123	0	0	0
N.S.	1	1.00	1.91	1.31	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.037	4.658	0.421	0.000	0.089	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	331	224	0	115	0	0	0
N.S.	1	1.00	1.96	1.33	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.032	3.945	0.227	0.000	0.083	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	283	386	0	0	0	0	0
N.S.	1	1.00	0.88	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	9.630	0.514	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	481	339	0	0	0	0	0
N.S.	1	1.00	1.69	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	10.668	0.886	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	308	346	0	0	0	0	0
N.S.	1	1.00	0.99	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.425	10.593	0.738	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	364	256	0	148	0	0	0
N.S.	1	1.00	1.36	0.96	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.107	8.374	1.602	0.000	0.084	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	358	251	0	143	0	0	0
N.S.	1	1.00	1.45	1.02	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.084	7.786	0.630	0.000	0.085	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	354	246	0	138	0	0	0
N.S.	1	1.00	1.57	1.09	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.066	7.145	0.539	0.000	0.085	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	349	241	0	133	0	0	0
N.S.	1	1.00	1.69	1.16	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.048	6.379	0.423	0.000	0.084	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	343	236	0	128	0	0	0
N.S.	1	1.00	1.73	1.19	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.045	5.780	0.256	0.000	0.078	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	477	337	0	0	0	0	0
N.S.	1	1.00	1.68	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	10.562	0.558	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	372	309	346	0	0	0	0	0
N.S.	1	1.22	1.01	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	10.484	0.608	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	309	346	0	0	0	0	0
N.S.	1	1.00	0.70	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	10.546	0.750	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	337	229	0	123	0	0	0
N.S.	1	1.00	1.80	1.22	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.066	10.344	1.257	0.000	0.082	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	331	224	0	118	0	0	0
N.S.	1	1.00	1.95	1.32	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.047	10.309	0.520	0.000	0.080	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	214	209	0	111	0	0	0
N.S.	1	1.00	1.42	1.38	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.028	10.139	0.278	0.000	0.080	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	142	85	0	39	0	0	0
N.S.	1	1.00	2.22	1.33	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.007	10.054	0.117	0.000	0.076	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	159	107	0	0	0	0	0
N.S.	1	1.00	0.95	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	10.162	0.447	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	481	339	0	0	0	0	0
N.S.	1	1.00	1.68	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	10.752	0.608	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	308	346	0	0	0	0	0
N.S.	1	1.00	0.98	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	10.720	0.730	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	339	241	0	187	0	0	0
N.S.	1	1.00	1.55	1.10	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.077	10.362	2.401	0.000	0.091	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	333	236	0	182	0	0	0
N.S.	1	1.00	1.66	1.18	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.059	10.308	0.810	0.000	0.082	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	328	231	0	177	0	0	0
N.S.	1	1.00	1.81	1.28	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.044	10.295	0.661	0.000	0.090	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	329	231	0	160	0	0	0
N.S.	1	1.00	1.82	1.28	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.042	10.280	0.651	0.000	0.088	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	329	231	0	160	0	0	0
N.S.	1	1.00	1.82	1.28	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.035	10.263	0.524	0.000	0.085	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	328	231	0	158	0	0	0
N.S.	1	1.00	1.81	1.28	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.033	4.196	0.123	0.000	0.086	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	483	337	0	0	0	0	0
N.S.	1	1.00	1.70	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	10.435	0.546	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	311	351	0	0	0	0	0
N.S.	1	1.00	1.00	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	10.537	0.645	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	320	356	0	0	0	0	0
N.S.	1	1.00	0.94	1.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	10.677	0.747	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	467	467	584	485	0	566	0	0	0
N.S.	1	1.00	1.25	1.04	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.267	11.958	7.425	0.000	0.103	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	488	410	0	398	0	0	0
N.S.	1	1.00	1.37	1.15	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.131	11.105	3.355	0.000	0.096	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	298	0	0	0
N.S.	1	1.00	1.07	1.28	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.050	10.190	0.955	0.000	0.087	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	214	200	0	0	0	0	0
N.S.	1	1.00	0.53	0.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	10.224	0.997	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	1069	1279	0	0	0	0	0
N.S.	1	1.00	1.49	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.675	11.508	1.643	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	596	489	0	576	0	0	0
N.S.	1	1.00	1.08	0.88	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.856	11.719	6.414	0.000	0.117	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	503	413	0	408	0	0	0
N.S.	1	1.00	1.11	0.91	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.530	10.909	3.253	0.000	0.098	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	293	364	0	310	0	0	0
N.S.	1	1.00	0.76	0.95	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.240	10.192	0.964	0.000	0.105	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	205	201	0	0	0	0	0
N.S.	1	1.00	1.04	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	10.208	1.043	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	464	1293	0	0	0	0	0
N.S.	1	1.00	0.65	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.688	13.811	1.622	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	304	355	0	301	0	0	0
N.S.	1	1.00	0.63	0.74	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.334	10.211	1.653	0.000	0.097	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	216	198	0	0	0	0	0
N.S.	1	1.00	1.06	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	10.208	1.048	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	295	357	0	311	0	0	0
N.S.	1	1.00	1.01	1.22	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.063	10.240	1.636	0.000	0.099	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	207	199	0	0	0	0	0
N.S.	1	1.00	0.50	0.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	10.231	1.013	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	154	184	0	123	0	0	0
N.S.	1	1.00	0.67	0.80	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.098	10.164	5.791	0.000	0.099	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	127	143	0	82	0	0	0
N.S.	1	1.00	0.76	0.85	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.050	10.122	3.614	0.000	0.103	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	73	108	0	45	0	0	0
N.S.	1	1.00	0.60	0.89	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.022	10.065	1.354	0.000	0.093	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	59	55	0	0	0	0	0
N.S.	1	1.00	0.48	0.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	10.129	1.494	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	399	175	443	0	0	0	0	0
N.S.	1	1.26	0.55	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	10.465	2.142	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.009	0.882	0.078	0.237	0.549	0.000	1.438	8.074

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	498	498	373	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.497	0.597	0.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	358	345	303	0	0	0	0	0	0
N.S.	1	0.96	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.551	0.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	232	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.476	0.000	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.009	0.896	0.051	0.224	0.267	0.000	0.302	7.809

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	37	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.54	0.00	1.08	1.08
time (sec)	N/A	0.007	0.862	0.065	0.213	0.280	0.000	0.292	8.059

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	446	446	270	251	0	0	0	0	0
N.S.	1	1.00	0.61	0.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	10.747	0.701	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	719	247	0	0	0	0	0
N.S.	1	1.00	3.30	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	11.115	0.741	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	77	327	0	323	0	0	0
N.S.	1	1.00	1.18	5.03	0.00	4.97	0.00	0.00	0.00
time (sec)	N/A	0.096	0.069	3.539	0.000	0.400	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	77	311	0	112	0	0	0
N.S.	1	1.00	1.22	4.94	0.00	1.78	0.00	0.00	0.00
time (sec)	N/A	0.091	0.054	3.415	0.000	0.388	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	336	0	328	0	0	0
N.S.	1	1.00	1.12	4.67	0.00	4.56	0.00	0.00	0.00
time (sec)	N/A	0.080	8.821	3.381	0.000	0.391	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	81	336	0	114	0	0	0
N.S.	1	1.00	1.16	4.80	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.082	8.440	3.349	0.000	0.395	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	3652	437	0	0	0	0	0
N.S.	1	1.00	6.52	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	17.740	1.477	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	3658	439	0	0	0	0	0
N.S.	1	1.00	6.94	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.478	17.676	1.478	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [147] had the largest ratio of [.7778000000000000047]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	6	1.00	17	0.353
2	A	9	6	1.00	18	0.333
3	A	3	3	1.00	18	0.167
4	A	3	3	1.00	19	0.158
5	A	5	3	1.00	17	0.176
6	A	3	2	1.00	17	0.118
7	A	2	2	1.00	17	0.118
8	A	2	2	1.00	17	0.118
9	A	5	3	1.00	27	0.111
10	A	3	2	1.00	28	0.071
11	A	5	3	1.00	21	0.143
12	A	3	2	1.00	22	0.091
13	A	3	3	1.00	15	0.200
14	A	2	2	1.00	22	0.091
15	A	5	5	1.00	23	0.217
16	A	3	3	1.00	21	0.143
17	A	6	6	1.00	22	0.273
18	A	1	1	1.00	22	0.045
19	A	3	3	1.00	21	0.143
20	A	1	1	1.00	23	0.043
21	A	3	3	1.00	22	0.136
22	A	1	1	1.00	28	0.036
23	A	2	2	1.00	24	0.083
24	A	4	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	5	1.00	25	0.200
26	A	5	3	1.00	26	0.115
27	A	5	3	1.00	26	0.115
28	A	5	3	1.00	27	0.111
29	A	5	3	1.00	27	0.111
30	A	3	2	1.00	27	0.074
31	A	3	2	1.00	27	0.074
32	A	3	2	1.00	28	0.071
33	A	3	2	1.00	28	0.071
34	A	3	2	1.00	30	0.067
35	A	5	3	1.00	29	0.103
36	A	6	4	1.00	29	0.138
37	A	3	2	1.00	32	0.062
38	A	5	3	1.00	31	0.097
39	A	5	3	1.00	22	0.136
40	A	5	3	1.00	23	0.130
41	A	3	2	1.00	22	0.091
42	A	3	2	1.00	22	0.091
43	A	3	3	1.00	22	0.136
44	A	5	3	1.00	22	0.136
45	A	5	3	1.00	22	0.136
46	A	5	3	1.00	20	0.150
47	A	5	3	1.00	17	0.176
48	A	5	3	1.00	22	0.136
49	A	5	3	1.00	22	0.136
50	A	5	3	1.00	22	0.136
51	A	2	2	1.00	22	0.091
52	A	7	3	1.00	22	0.136
53	A	5	3	1.00	22	0.136
54	A	3	2	1.00	22	0.091
55	A	3	2	1.00	22	0.091
56	A	3	2	1.00	22	0.091
57	A	2	2	1.00	22	0.091
58	A	3	2	1.00	22	0.091
59	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	2	1.00	20	0.100
61	A	3	2	1.00	17	0.118
62	A	3	2	1.00	22	0.091
63	A	3	2	1.00	22	0.091
64	A	3	2	1.00	22	0.091
65	A	3	3	1.00	22	0.136
66	A	7	3	1.00	22	0.136
67	A	5	3	1.00	22	0.136
68	A	5	3	1.00	18	0.167
69	A	3	2	1.00	18	0.111
70	A	3	2	1.00	18	0.111
71	A	3	2	1.00	18	0.111
72	A	2	2	1.00	18	0.111
73	A	5	3	1.00	16	0.188
74	A	5	3	1.00	13	0.231
75	A	5	3	1.00	18	0.167
76	A	2	2	1.00	18	0.111
77	A	7	3	1.00	18	0.167
78	A	5	3	1.00	18	0.167
79	A	5	3	1.00	18	0.167
80	A	3	2	1.00	20	0.100
81	A	3	2	1.00	20	0.100
82	A	3	2	1.00	20	0.100
83	A	3	2	1.00	20	0.100
84	A	2	2	1.00	20	0.100
85	A	3	2	1.00	18	0.111
86	A	3	2	1.00	15	0.133
87	A	3	2	1.00	20	0.100
88	A	3	3	1.00	20	0.150
89	A	5	3	1.00	20	0.150
90	A	5	3	1.00	20	0.150
91	A	5	3	1.00	20	0.150
92	A	5	3	1.00	22	0.136
93	A	5	3	1.00	22	0.136
94	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	2	1.00	22	0.091
96	A	3	2	1.00	22	0.091
97	A	3	2	1.00	18	0.111
98	A	9	5	1.00	18	0.278
99	A	10	6	1.00	18	0.333
100	A	9	5	1.00	18	0.278
101	A	10	6	1.00	18	0.333
102	A	9	5	1.00	29	0.172
103	A	9	5	1.00	26	0.192
104	A	9	5	1.00	24	0.208
105	A	9	5	1.00	22	0.227
106	A	9	5	1.00	25	0.200
107	A	9	5	1.00	31	0.161
108	A	9	5	1.00	32	0.156
109	A	9	5	1.00	23	0.217
110	A	9	5	1.00	25	0.200
111	A	9	5	1.00	29	0.172
112	A	9	5	1.00	32	0.156
113	A	4	4	1.00	22	0.182
114	A	5	5	1.00	24	0.208
115	A	4	4	1.00	24	0.167
116	A	4	4	1.00	24	0.167
117	A	4	4	1.00	24	0.167
118	A	4	4	1.00	24	0.167
119	A	3	3	1.00	39	0.077
120	A	2	1	1.00	17	0.059
121	A	2	1	1.00	17	0.059
122	A	2	1	1.00	17	0.059
123	A	2	1	1.00	15	0.067
124	A	3	2	1.00	17	0.118
125	A	3	3	1.00	17	0.176
126	A	3	3	1.00	17	0.176
127	A	4	4	1.00	17	0.235
128	A	2	1	1.00	19	0.053
129	A	2	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	1	1.00	17	0.059
131	A	2	1	1.00	9	0.111
132	A	3	2	1.00	19	0.105
133	A	4	3	1.00	19	0.158
134	A	5	4	1.00	19	0.210
135	A	5	5	1.00	19	0.263
136	A	5	5	1.00	19	0.263
137	A	11	7	1.00	19	0.368
138	A	11	7	1.00	19	0.368
139	A	11	7	1.00	19	0.368
140	A	9	6	1.00	17	0.353
141	A	9	6	1.00	9	0.667
142	A	12	8	1.00	19	0.421
143	A	14	9	1.00	19	0.474
144	A	11	8	1.00	19	0.421
145	A	11	8	1.00	19	0.421
146	A	10	7	1.00	17	0.412
147	A	10	7	1.00	9	0.778
148	A	22	9	1.00	19	0.474
149	A	24	10	1.00	19	0.526
150	A	6	5	0.99	21	0.238
151	A	5	5	1.00	21	0.238
152	A	4	4	1.00	21	0.190
153	A	3	3	1.00	19	0.158
154	A	3	3	1.00	21	0.143
155	A	6	6	1.00	21	0.286
156	A	7	7	1.00	21	0.333
157	A	8	8	1.00	22	0.364
158	A	7	7	1.00	22	0.318
159	A	6	6	1.00	20	0.300
160	A	2	2	1.00	22	0.091
161	A	10	10	1.00	22	0.454
162	A	11	11	1.00	22	0.500
163	A	12	11	1.00	22	0.500
164	A	6	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	2	2	1.00	23	0.087
166	A	3	3	1.00	29	0.103
167	A	3	3	1.00	29	0.103
168	A	3	3	1.00	22	0.136
169	A	3	3	1.00	24	0.125
170	A	2	2	1.00	21	0.095
171	A	3	3	1.00	21	0.143
172	A	1	1	1.00	22	0.045
173	A	3	3	1.00	21	0.143
174	F	0	0	N/A	0.000	N/A
175	N/A	0	0	1.00	19	0.000
176	A	9	6	0.96	19	0.316
177	A	7	6	0.95	19	0.316
178	A	6	5	1.00	17	0.294
179	A	2	2	1.00	9	0.222
180	A	6	5	1.00	19	0.263
181	A	8	5	1.00	19	0.263
182	A	6	4	0.95	19	0.210
183	A	5	4	0.92	19	0.210
184	A	4	3	1.00	17	0.176
185	A	1	1	1.00	9	0.111
186	A	4	3	1.00	19	0.158
187	A	5	3	1.00	19	0.158
188	A	6	3	1.00	19	0.158
189	A	4	3	1.00	24	0.125
190	A	4	3	1.00	24	0.125
191	A	3	3	1.00	24	0.125
192	A	2	2	1.00	22	0.091
193	A	5	5	1.00	24	0.208
194	A	6	6	1.00	24	0.250
195	A	6	6	1.00	26	0.231
196	A	3	3	1.00	26	0.115
197	A	4	4	1.00	26	0.154
198	A	6	6	1.00	26	0.231
199	A	5	5	1.00	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	4	4	1.00	28	0.143
201	A	3	3	1.00	28	0.107
202	A	3	3	1.00	28	0.107
203	A	4	4	1.00	28	0.143
204	A	6	6	1.00	28	0.214
205	A	5	5	1.00	29	0.172
206	A	4	4	1.00	29	0.138
207	A	3	3	1.00	29	0.103
208	A	3	3	1.00	29	0.103
209	A	4	4	1.00	29	0.138
210	A	6	6	1.00	29	0.207
211	A	2	2	1.00	19	0.105
212	A	3	3	1.67	19	0.158
213	A	7	5	0.99	31	0.161
214	A	4	3	1.00	39	0.077
215	A	4	3	1.00	39	0.077
216	A	3	3	1.00	39	0.077
217	A	2	2	1.00	37	0.054
218	A	5	5	1.00	39	0.128
219	A	6	6	1.00	39	0.154
220	A	7	7	1.00	41	0.171
221	A	6	6	1.00	41	0.146
222	A	3	3	1.00	41	0.073
223	A	4	4	1.00	41	0.098
224	A	6	6	1.00	41	0.146
225	A	6	6	1.00	20	0.300
226	A	5	5	1.00	20	0.250
227	A	4	4	1.00	18	0.222
228	A	8	7	1.00	20	0.350
229	A	1	1	1.00	20	0.050
230	A	23	13	1.00	20	0.650
231	A	26	14	1.00	20	0.700
232	A	5	5	1.00	20	0.250
233	A	4	4	1.00	20	0.200
234	A	3	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	4	4	1.00	20	0.200
236	A	8	8	1.00	20	0.400
237	A	9	9	1.00	20	0.450
238	A	4	4	1.00	20	0.200
239	A	2	2	1.00	20	0.100
240	A	2	2	1.00	18	0.111
241	A	9	8	1.00	20	0.400
242	A	16	11	1.00	20	0.550
243	A	23	14	1.00	20	0.700
244	A	2	1	1.00	22	0.045
245	A	2	1	1.00	22	0.045
246	A	2	1	1.00	22	0.045
247	A	2	1	1.00	20	0.050
248	A	3	2	1.00	22	0.091
249	A	3	3	1.05	22	0.136
250	A	3	3	1.03	22	0.136
251	A	4	4	1.03	22	0.182
252	A	2	1	1.00	24	0.042
253	A	2	1	1.00	24	0.042
254	A	2	1	1.00	22	0.045
255	A	2	1	1.00	14	0.071
256	A	3	2	1.00	24	0.083
257	A	4	3	1.00	24	0.125
258	A	5	4	1.00	24	0.167
259	A	5	4	1.00	24	0.167
260	A	5	4	1.00	24	0.167
261	A	3	3	1.05	22	0.136
262	A	3	3	1.00	23	0.130
263	A	5	3	1.00	24	0.125
264	A	5	3	1.00	24	0.125
265	A	5	3	1.00	24	0.125
266	A	3	2	1.00	22	0.091
267	A	3	2	1.00	14	0.143
268	A	6	3	1.00	24	0.125
269	A	8	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	4	3	1.00	24	0.125
271	A	4	3	1.00	24	0.125
272	A	4	3	1.00	22	0.136
273	A	4	3	1.00	14	0.214
274	A	10	4	1.00	24	0.167
275	A	12	5	1.00	24	0.208
276	A	7	5	1.00	24	0.208
277	A	6	5	1.00	24	0.208
278	A	5	5	1.00	24	0.208
279	A	4	4	1.00	24	0.167
280	A	4	4	1.04	24	0.167
281	A	4	4	1.04	24	0.167
282	A	4	4	1.00	24	0.167
283	A	5	5	1.00	24	0.208
284	A	6	5	0.99	24	0.208
285	A	7	5	1.00	24	0.208
286	A	6	6	1.00	24	0.250
287	A	5	5	1.00	24	0.208
288	A	4	4	1.00	22	0.182
289	A	4	4	1.00	14	0.286
290	A	8	7	1.30	24	0.292
291	A	8	7	1.00	24	0.292
292	A	25	10	1.00	24	0.417
293	A	7	6	1.00	24	0.250
294	A	6	5	1.00	24	0.208
295	A	5	4	1.00	22	0.182
296	A	5	5	1.00	14	0.357
297	A	13	8	1.00	24	0.333
298	A	21	10	1.50	24	0.417
299	A	27	10	1.25	24	0.417
300	A	5	5	1.00	24	0.208
301	A	4	4	1.00	24	0.167
302	A	3	3	1.00	22	0.136
303	A	1	1	1.00	14	0.071
304	A	4	4	1.00	24	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	9	8	1.00	24	0.333
306	A	10	9	1.00	24	0.375
307	A	6	5	1.00	24	0.208
308	A	5	5	1.00	24	0.208
309	A	4	4	1.00	24	0.167
310	A	4	4	1.00	24	0.167
311	A	4	4	1.00	22	0.182
312	A	4	4	1.00	14	0.286
313	A	9	8	1.20	24	0.333
314	A	19	10	1.00	24	0.417
315	A	29	11	1.00	24	0.458
316	A	8	7	1.00	24	0.292
317	A	7	7	1.00	24	0.292
318	A	6	6	1.00	24	0.250
319	A	5	5	1.00	22	0.227
320	A	5	5	1.00	14	0.357
321	A	7	7	1.00	24	0.292
322	A	7	7	1.00	24	0.292
323	A	21	10	1.00	24	0.417
324	A	9	7	1.00	24	0.292
325	A	8	7	1.00	24	0.292
326	A	7	6	1.00	24	0.250
327	A	6	5	1.00	22	0.227
328	A	6	6	1.00	14	0.429
329	A	13	8	1.00	24	0.333
330	A	21	13	1.00	24	0.542
331	A	27	13	1.00	24	0.542
332	A	6	6	1.00	24	0.250
333	A	5	5	1.00	24	0.208
334	A	4	4	1.00	22	0.182
335	A	2	2	1.00	14	0.143
336	A	2	2	1.00	24	0.083
337	A	8	8	1.00	24	0.333
338	A	9	9	1.00	24	0.375
339	A	7	6	1.00	24	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	6	6	1.00	24	0.250
341	A	5	5	1.00	24	0.208
342	A	5	5	1.00	24	0.208
343	A	5	5	1.00	22	0.227
344	A	5	5	1.00	14	0.357
345	A	8	8	1.00	24	0.333
346	A	17	10	1.00	24	0.417
347	A	26	11	1.00	24	0.458
348	A	7	6	1.00	24	0.250
349	A	6	6	1.00	24	0.250
350	A	5	5	1.00	24	0.208
351	A	4	4	1.00	22	0.182
352	A	4	4	1.00	14	0.286
353	A	7	6	1.00	24	0.250
354	A	7	6	1.00	24	0.250
355	A	18	9	1.00	24	0.375
356	A	8	6	1.00	24	0.250
357	A	7	6	1.00	24	0.250
358	A	6	5	1.00	24	0.208
359	A	5	4	1.00	22	0.182
360	A	5	5	1.00	14	0.357
361	A	12	7	1.00	24	0.292
362	A	19	11	1.22	24	0.458
363	A	22	10	1.00	24	0.417
364	A	5	5	1.00	24	0.208
365	A	4	4	1.00	24	0.167
366	A	3	3	1.00	22	0.136
367	A	1	1	1.00	14	0.071
368	A	3	3	1.00	24	0.125
369	A	6	6	1.00	24	0.250
370	A	7	7	1.00	24	0.292
371	A	6	5	1.00	24	0.208
372	A	5	5	1.00	24	0.208
373	A	4	4	1.00	24	0.167
374	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	4	4	1.00	22	0.182
376	A	4	4	1.00	14	0.286
377	A	8	7	1.00	24	0.292
378	A	15	10	1.00	24	0.417
379	A	22	11	1.00	24	0.458
380	A	5	5	1.00	26	0.192
381	A	4	4	1.00	26	0.154
382	A	3	3	1.00	24	0.125
383	A	3	3	1.00	26	0.115
384	A	6	6	1.00	26	0.231
385	A	6	6	1.00	27	0.222
386	A	5	5	1.00	27	0.185
387	A	4	4	1.00	25	0.160
388	A	2	2	1.00	27	0.074
389	A	8	8	1.00	27	0.296
390	A	5	5	1.00	26	0.192
391	A	2	2	1.00	28	0.071
392	A	3	3	1.00	27	0.111
393	A	3	3	1.00	29	0.103
394	A	5	5	1.00	24	0.208
395	A	4	4	1.00	24	0.167
396	A	3	3	1.00	22	0.136
397	A	4	4	1.00	24	0.167
398	A	9	8	1.26	24	0.333
399	N/A	0	0	1.00	24	0.000
400	A	8	7	1.00	24	0.292
401	A	7	6	0.96	24	0.250
402	A	6	5	1.00	22	0.227
403	A	2	2	1.00	14	0.143
404	N/A	0	0	1.00	24	0.000
405	N/A	0	0	1.00	24	0.000
406	A	8	8	1.00	24	0.333
407	A	10	10	1.00	26	0.385
408	A	2	2	1.00	40	0.050
409	A	2	2	1.00	40	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	2	2	1.00	46	0.043
411	A	2	2	1.00	46	0.043
412	A	8	8	1.00	29	0.276
413	A	10	10	1.00	31	0.323

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{c+dx^2}{a+bx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 247

$$
\begin{aligned}
\int \frac{c+dx^2}{a+bx^4}\,dx = & -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\
& +\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\
& -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\
& +\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}
$$

```
[Out] -1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-d*a^(1/2)+c*b^(1/
2))/a^(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^
(1/2))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)
*x*2^(1/2)/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arcta
n(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{c+dx^2}{a+bx^4}\,dx = &-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}d+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}\\&+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}d+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}\\&-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}\\&+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}\end{aligned}$$

```
[In] Int[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] -1/2*((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqr
t[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*d)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/
4)) + ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}\\
&= \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}\\
&\quad -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad+\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad-\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.74

$$
\int\frac{c+dx^2}{a+bx^4}\,dx
$$

$$
=\frac{-2\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c-\sqrt{a}d\right)\Big(\log\Big(}{4\sqrt{2}a^{3/4}b^{3/4}}
$$

```
[In] Integrate[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c -
 Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^(3
/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4} b+a\right)}\frac{\left(\_R^{2} d+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\sqrt{2}\,(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arctan}}{8b(\frac{a}{b}}$

```
[In] int((d*x^2+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*sum((_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. 2(166) = 332.

Time = 0.26 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.11

$$\begin{aligned}
\int \frac{c+dx^2}{a+bx^4}\,dx = {} & -\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& + \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
& + \frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& - \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
& + \frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& + \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg) \\
& - \frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& - \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg)
\end{aligned}$$

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*
d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^
2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^
2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a
*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log
(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d
```

```
^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt(-(b^2*
c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a
^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^
3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 +
a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - 2*c*d)/(a*b)))
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int \frac{c+dx^2}{a+bx^4}\,dx$$
$$= \operatorname{RootSum}\Bigg(256t^4a^3b^3 + 64t^2a^2b^2cd + a^2d^4 + 2abc^2d^2 + b^2c^4, \Bigg(t \mapsto t\log\Bigg(x + \frac{64t^3a^3b^2d + 12ta^2bcd^2 - 4tab}{a^2d^4 - b^2c^4}$$

```
[In] integrate((d*x**2+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c*
*2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d + 12*_t*a*
*2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{c+dx^2}{a+bx^4}\,dx = {} & \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} \\ & + \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} \\ & + \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ & - \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}\end{aligned}$$

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)
*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b
)*c - sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{c+dx^2}{a+bx^4}\,dx = {} & \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} \\ & + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} \\ & + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} \\ & - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}\end{aligned}$$

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*log(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^2*c - (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.43

$$
\begin{aligned}
\int \frac{c+dx^2}{a+bx^4}\,dx = {} & -2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right. \\
& \left.-\frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{-\frac{b\,c^2\,\sqrt{-a^3\,b^3}-a\,d^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}} \\
& -2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right. \\
& \left.-\frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{-a^3\,b^3}-b\,c^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}
\end{aligned}
$$

```
[In] int((c + d*x^2)/(a + b*x^4),x)
```

```
[Out] - 2*atanh((8*b^3*c^2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b^
3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (
2*b*c^3*(-a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^
2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b
^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^
(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a))*(-(b*c^2*(-a^3*b^3)^(1/2) - a*d
^2*(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((8*b^3*c
^2*x*((c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-a^3*b^3)
^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)
^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c^2*(-a^3*b^
3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3
))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)^(1/2))/a^2 + (2*c*
d^2*(-a^3*b^3)^(1/2))/a))*(-(a*d^2*(-a^3*b^3)^(1/2) - b*c^2*(-a^3*b^3)^(1/2
) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

3.2 $\int \frac{c-dx^2}{a+bx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 247

$$\begin{aligned}\int \frac{c-dx^2}{a+bx^4}\,dx = &-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}\\ &+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}\\ &-\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}\\ &+\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}\end{aligned}$$

```
[Out] 1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(
3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))
/a^(3/4)/b^(3/4)*2^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1
/2))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{c-dx^2}{a+bx^4}\,dx = &-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c-\sqrt{a}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}\\ &+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}c-\sqrt{a}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}\\ &-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}\\ &+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}\end{aligned}$$

```
[In] Int[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] -1/2*((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqr
t[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c + Sqrt[a]*d)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/
4)) + ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}\\
&= \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}\\
&\quad -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad+\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad-\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\
&\quad+\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.74

$$
\int\frac{c-dx^2}{a+bx^4}\,dx
$$

$$
= \frac{\left(-2\sqrt{b}c+2\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c+\sqrt{a}d\right)\left(\log\left(\sqrt{}\right.\right.}{4\sqrt{2}a^{3/4}b^{3/4}}
$$

```
[In] Integrate[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] ((-2*Sqrt[b]*c + 2*Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c
 + Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^
(3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4} b+a\right)} \frac{\left(-\_R^{2} d+c\right) \ln\left(x-\_R\right)}{\_R^{3}}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{d \sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arct}\right.}{8b(}$

```
[In] int((-d*x^2+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*sum((-_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. 2(166) = 332.

Time = 0.26 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.11

$$
\begin{aligned}
\int \frac{c-dx^2}{a+bx^4}\,dx = {} & -\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& +\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
& +\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& -\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
& +\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& +\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg) \\
& -\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
& -\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg)
\end{aligned}
$$

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d
)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*
c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*
sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(
b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)
```

```
/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4
 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*
d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) -
a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^
4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*
d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3
*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 + a
^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - 2*c*d)/(a*b)))
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.45

$$\int \frac{c-dx^2}{a+bx^4}\,dx = -\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd+a^2d^4+2abc^2d^2+b^2c^4,\ \left(t\mapsto t\log\left(x+\frac{64t^3a^3b^2d-12ta^2bcd^2+}{a^2d^4-b^2c^4}\right.\right.\right.$$

```
[In] integrate((-d*x**2+c)/(b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c
**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d - 12*_t*a
**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{c-dx^2}{a+bx^4}\,dx &= \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} \\ &\quad + \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} \\ &\quad + \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ &\quad - \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}\end{aligned}$$

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)
*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b
)*c + sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{c-dx^2}{a+bx^4}\,dx &= \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}\\ &\quad+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}\\ &\quad+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}\\ &\quad-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}\end{aligned}$$

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*log(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^2*c + (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b^3)
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.44

$$\int \frac{c-dx^2}{a+bx^4}\,dx = 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{\frac{a\,d^2\,\sqrt{-a^3\,b^3}-b\,c^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}} + 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{\frac{b\,c^2\,\sqrt{-a^3\,b^3}-a\,d^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

```
[In] int((c - d*x^2)/(a + b*x^4),x)
```

```
[Out] 2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) +
 (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*
b*c^3*(-a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*
x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(-a^3*b^3)^(1
/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)^(1
/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a))*((a*d^2*(-a^3*b^3)^(1/2) - b*c^2*
(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b^3*c^2*
x*((c*d)/(8*a*b) + (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3*b^3)^(1
/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^(1
/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c*d)/(8*a*b) +
(c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3))^
(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^(1/2))/a^2 - (2*c*d^2
*(-a^3*b^3)^(1/2))/a))*((b*c^2*(-a^3*b^3)^(1/2) - a*d^2*(-a^3*b^3)^(1/2) +
2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

3.3 $\int \frac{c+dx^2}{a-bx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{c+dx^2}{a-bx^4}\,dx=\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

```
[Out] 1/2*arctan(b^(1/4)*x/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*ar
ctanh(b^(1/4)*x/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1181, 211, 214}

$$\int \frac{c+dx^2}{a-bx^4}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c-\sqrt{a}d\right)}{2a^{3/4}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}d+\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}$$

```
[In] Int[(c + d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\left(-\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int\frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int\frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx\\&=\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int\frac{c+dx^2}{a-bx^4}\,dx=\frac{2\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c+\sqrt{a}d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}$$

```
[In] Integrate[(c + d*x^2)/(a - b*x^4),x]
[Out] (2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + Sqrt[
a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(
3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4} b-a\right)}\frac{\left(\_R^{2} d+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$	36
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{d\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	104

```
[In] int((d*x^2+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*sum((_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(58) = 116$.

Time = 0.26 (sec) , antiderivative size = 755, normalized size of antiderivative = 8.78

$$
\begin{aligned}
&\int \frac{c+dx^2}{a-bx^4}\,dx \\
&= \frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
&\quad+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
&\quad-\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
&\quad-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\Bigg) \\
&\quad-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
&\quad+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg) \\
&\quad+\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\Bigg(-\left(b^2c^4-a^2d^4\right)x \\
&\quad-\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\Bigg)
\end{aligned}
$$

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/
(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^
2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4
+ 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt
```

```
((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c
^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*
b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 +
a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x +
 (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3
 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^
3)) - 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt
((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)*
sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*
b)))
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{c+dx^2}{a-bx^4}\,dx = -\operatorname{RootSum}\Bigg(256t^4a^3b^3 - 64t^2a^2b^2cd - a^2d^4 + 2abc^2d^2 - b^2c^4, \Bigg(t \mapsto t\log\Bigg(x + \frac{-64t^3a^3b^2d + 12ta^2bcd^2 +}{a^2d^4 - b^2c^4}$$

```
[In] integrate((d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c
**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d + 12*_t*
a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27

$$\int \frac{c+dx^2}{a-bx^4}\,dx = \frac{\left(\sqrt{b}c - \sqrt{a}d\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{b}c + \sqrt{a}d\right)\log\left(\frac{\sqrt{b}x - \sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x + \sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(b)*c - sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c + sqrt(a)*d)*log((sqrt(b)
*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(58) = 116$.

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.67

$$
\begin{aligned}
\int \frac{c+dx^2}{a-bx^4}\,dx = &-\frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} \\
&-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} \\
&-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} \\
&+\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}
\end{aligned}
$$

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a
/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c - sqrt(-a*b)*b
*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^
(3/4) - 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*log(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 579, normalized size of antiderivative = 6.73

$$\int \frac{c+dx^2}{a-bx^4}\,dx$$

$$\begin{aligned}
&= 2\operatorname{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right.\\
&\quad\left.+\frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}-2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}\\
&\quad+2\operatorname{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right.\\
&\quad\left.+\frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}
\end{aligned}$$

```
[In] int((c + d*x^2)/(a - b*x^4),x)
```

```
[Out] 2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) -
(d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*
c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((
c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3)^(1/2))/(
16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a^
2 - (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3
)^(1/2) - 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b^3*c^2*x*((c*d)/
(8*a*b) + (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2))/(16*a^
2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (
2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c*d)/(8*a*b) + (c^2*(a^3*b^3
)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c
^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2)
)/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*
a^3*b^3))^(1/2)
```

3.4 $\int \frac{c-dx^2}{a-bx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{c-dx^2}{a-bx^4}\,dx=\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

```
[Out] 1/2*arctanh(b^(1/4)*x/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*a
rctan(b^(1/4)*x/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1181, 211, 214}

$$\int \frac{c-dx^2}{a-bx^4}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}d+\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c-\sqrt{a}d\right)}{2a^{3/4}b^{3/4}}$$

```
[In] Int[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\left(-\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int\frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int\frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx\\ &= \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\begin{aligned}&\int\frac{c-dx^2}{a-bx^4}\,dx\\ &= \frac{2\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c-\sqrt{a}d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}\end{aligned}$$

```
[In] Integrate[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c - Sqrt[
a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(
3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.43

method	result	size
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4} b-a\right)}\frac{\left(-\_R^{2} d+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$	37
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	104

```
[In] int((-d*x^2+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
[Out] -1/4/b*sum((-_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(58) = 116$.

Time $= 0.28$ (sec) , antiderivative size $= 755$, normalized size of antiderivative $= 8.78$

$$
\begin{aligned}
&\int \frac{c-dx^{2}}{a-bx^{4}}\,dx \\
&= \frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+2\,cd}{ab}}\log\left(-(b^{2}c^{4}-a^{2}d^{4})x\right. \\
&\quad \left.+\left(a^{3}b^{2}d\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-ab^{2}c^{3}-a^{2}bcd^{2}\right)\sqrt{-\frac{ab\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+2\,cd}{ab}}\right) \\
&\quad -\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+2\,cd}{ab}}\log\left(-(b^{2}c^{4}-a^{2}d^{4})x\right. \\
&\quad \left.-\left(a^{3}b^{2}d\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-ab^{2}c^{3}-a^{2}bcd^{2}\right)\sqrt{-\frac{ab\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+2\,cd}{ab}}\right) \\
&\quad -\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-2\,cd}{ab}}\log\left(-(b^{2}c^{4}-a^{2}d^{4})x\right. \\
&\quad \left.+\left(a^{3}b^{2}d\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+ab^{2}c^{3}+a^{2}bcd^{2}\right)\sqrt{\frac{ab\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-2\,cd}{ab}}\right) \\
&\quad +\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-2\,cd}{ab}}\log\left(-(b^{2}c^{4}-a^{2}d^{4})x\right. \\
&\quad \left.-\left(a^{3}b^{2}d\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+ab^{2}c^{3}+a^{2}bcd^{2}\right)\sqrt{\frac{ab\sqrt{\frac{b^{2}c^{4}+2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-2\,cd}{ab}}\right)
\end{aligned}
$$

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)
/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d
^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^
4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*s
```

```
qrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^
2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a
^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^
2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*
a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*
x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*
c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*
b^3)) - 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqr
t((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)
*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*
b)))
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{c-dx^2}{a-bx^4}\,dx$$
$$=\operatorname{RootSum}\left(256t^4a^3b^3+64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d-12ta^2bcd^2-}{a^2d^4-b^2c^4}\right.\right.\right.$$

```
[In] integrate((-d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c*
*2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d - 12*_t*a
**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27

$$\int \frac{c-dx^2}{a-bx^4}\,dx=\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(b)*c + sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c - sqrt(a)*d)*log((sqrt(b)
*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(58) = 116$.

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.65

$$\int \frac{c-dx^2}{a-bx^4}\,dx = -\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}$$

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a
/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + sqrt(-a*b)*b
*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^
(3/4) - 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*log(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 579, normalized size of antiderivative = 6.73

$$\int \frac{c-dx^2}{a-bx^4}\,dx = -2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}} - 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}-2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

```
[In] int((c - d*x^2)/(a - b*x^4),x)
```

```
[Out] - 2*atanh((8*b^3*c^2*x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2
) - (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (
2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*
x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3)^(1
/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/
2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a
^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((8*b^3*c^2*x*(
(c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^(1/2))/
(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a
^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c^2*(a^3*b^3)^(1/2))/(
16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2
*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)
^(1/2))/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) - 2*a^2*b^2*c*d
)/(16*a^3*b^3))^(1/2)
```

3.5 $\int \frac{2+3x^2}{4+9x^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{2+3x^2}{4+9x^4} dx = -\frac{\arctan\left(1-\sqrt{3}x\right)}{2\sqrt{3}} + \frac{\arctan\left(1+\sqrt{3}x\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctan(-1+x*3^(1/2))*3^(1/2)+1/6*arctan(1+x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1176, 631, 210}

$$\int \frac{2+3x^2}{4+9x^4} dx = \frac{\arctan\left(\sqrt{3}x+1\right)}{2\sqrt{3}} - \frac{\arctan\left(1-\sqrt{3}x\right)}{2\sqrt{3}}$$

```
[In] Int[(2 + 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[3]*x]/Sqrt[3] + ArcTan[1 + Sqrt[3]*x]/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}\int \frac{1}{\frac{2}{3}-\frac{2x}{\sqrt{3}}+x^2}\,dx+\frac{1}{6}\int \frac{1}{\frac{2}{3}+\frac{2x}{\sqrt{3}}+x^2}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{3}x\right)}{2\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{3}x\right)}{2\sqrt{3}}\\ &= -\frac{\tan^{-1}\left(1-\sqrt{3}x\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(1+\sqrt{3}x\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{4+9x^4}\,dx=\frac{-\arctan\left(1-\sqrt{3}x\right)+\arctan\left(1+\sqrt{3}x\right)}{2\sqrt{3}}$$

```
[In] Integrate[(2 + 3*x^2)/(4 + 9*x^4),x]
[Out] (-ArcTan[1 - Sqrt[3]*x] + ArcTan[1 + Sqrt[3]*x])/(2*Sqrt[3])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{2}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{3x^{3}\sqrt{3}}{4}+\frac{x\sqrt{3}}{2}\right)}{6}$
default	$\frac{\sqrt{6}\,\sqrt{2}\left(\ln\left(\frac{x^{2}+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^{2}-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+1\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}-1\right)\right)}{48}+\frac{\sqrt{6}\,\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^{2}+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+\right.\right.}{48}$
meijerg	$\frac{\sqrt{6}\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}+\frac{3\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}}{2-\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}+\frac{3\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}}{2+\sqrt{3}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}\right)}{24}+-$

```
[In] int((3*x^2+2)/(9*x^4+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*3^(1/2)*arctan(1/2*x*3^(1/2))+1/6*3^(1/2)*arctan(3/4*x^3*3^(1/2)+1/2*x*
3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{4+9x^4}\,dx=\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{4}\,\sqrt{3}\left(3\,x^{3}+2\,x\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{2}\,\sqrt{3}x\right)$$

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/4*sqrt(3)*(3*x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/2*sqrt
(3)*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{2+3x^2}{4+9x^4}\,dx=\frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{2}\right)+2\operatorname{atan}\left(\frac{3\sqrt{3}x^{3}}{4}+\frac{\sqrt{3}x}{2}\right)\right)}{12}$$

```
[In] integrate((3*x**2+2)/(9*x**4+4),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/2) + 2*atan(3*sqrt(3)*x**3/4 + sqrt(3)*x/2))/12
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{2+3x^2}{4+9x^4}\,dx=\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(3\,x+\sqrt{3}\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(3\,x-\sqrt{3}\right)\right)$$

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(3*x + sqrt(3))) + 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(3*x - sqrt(3)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{2+3x^2}{4+9x^4}\,dx=\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{9}{8}\,\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{9}{8}\,\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)$$

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x + sqrt(2)*(4/9)^(1/4))) + 1
/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x - sqrt(2)*(4/9)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{2+3x^2}{4+9x^4}\,dx=\frac{\sqrt{3}\,\left(\mathrm{atan}\left(\frac{3\,\sqrt{3}\,x^3}{4}+\frac{\sqrt{3}\,x}{2}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\,x}{2}\right)\right)}{6}$$

```
[In] int((3*x^2 + 2)/(9*x^4 + 4),x)
[Out] (3^(1/2)*(atan((3^(1/2)*x)/2 + (3*3^(1/2)*x^3)/4) + atan((3^(1/2)*x)/2)))/6
```

3.6 $\int \frac{2-3x^2}{4+9x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{2-3x^2}{4+9x^4}\,dx = -\frac{\log\left(2-2\sqrt{3}x+3x^2\right)}{4\sqrt{3}} + \frac{\log\left(2+2\sqrt{3}x+3x^2\right)}{4\sqrt{3}}$$

```
[Out] -1/12*ln(2+3*x^2-2*x*3^(1/2))*3^(1/2)+1/12*ln(2+3*x^2+2*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1179, 642}

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{\log\left(3x^2+2\sqrt{3}x+2\right)}{4\sqrt{3}} - \frac{\log\left(3x^2-2\sqrt{3}x+2\right)}{4\sqrt{3}}$$

```
[In] Int[(2 - 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] -1/4*Log[2 - 2*Sqrt[3]*x + 3*x^2]/Sqrt[3] + Log[2 + 2*Sqrt[3]*x + 3*x^2]/(4
*Sqrt[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{2}{\sqrt{3}}+2x}{-\frac{2}{3}-\frac{2x}{\sqrt{3}}-x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\frac{2}{\sqrt{3}}-2x}{-\frac{2}{3}+\frac{2x}{\sqrt{3}}-x^2}\,dx}{4\sqrt{3}}\\ &= -\frac{\log\left(2-2\sqrt{3}x+3x^2\right)}{4\sqrt{3}}+\frac{\log\left(2+2\sqrt{3}x+3x^2\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{2-3x^2}{4+9x^4}\,dx=\frac{-\log\left(-2+2\sqrt{3}x-3x^2\right)+\log\left(2+2\sqrt{3}x+3x^2\right)}{4\sqrt{3}}$$

```
[In] Integrate[(2 - 3*x^2)/(4 + 9*x^4),x]
[Out] (-Log[-2 + 2*Sqrt[3]*x - 3*x^2] + Log[2 + 2*Sqrt[3]*x + 3*x^2])/(4*Sqrt[3])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\ln\left(2+3x^2-2x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(2+3x^2+2x\sqrt{3}\right)\sqrt{3}}{12}$
default	$\frac{\sqrt{6}\,\sqrt{2}\,\left(\ln\left(\frac{x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+1\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}-1\right)\right)}{48}-\frac{\sqrt{6}\,\sqrt{2}\,\left(\ln\left(\frac{x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}\right.\right.}{48}$
meijerg	$\frac{\sqrt{6}\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{3}\left(x^4\right)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{3}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{3}\left(x^4\right)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\left(x^4\right)^{\frac{1}{4}}}{2+\sqrt{3}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}\right)}{24}-$

```
[In] int((-3*x^2+2)/(9*x^4+4),x,method=_RETURNVERBOSE)
[Out] -1/12*ln(2+3*x^2-2*x*3^(1/2))*3^(1/2)+1/12*ln(2+3*x^2+2*x*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{1}{12}\,\sqrt{3}\log\left(\frac{9\,x^4+24\,x^2+4\,\sqrt{3}(3\,x^3+2\,x)+4}{9\,x^4+4}\right)$$

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((9*x^4 + 24*x^2 + 4*sqrt(3)*(3*x^3 + 2*x) + 4)/(9*x^4 + 4)
)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{2-3x^2}{4+9x^4}\,dx = -\frac{\sqrt{3}\log\left(x^2-\frac{2\sqrt{3}x}{3}+\frac{2}{3}\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\frac{2\sqrt{3}x}{3}+\frac{2}{3}\right)}{12}$$

```
[In] integrate((-3*x**2+2)/(9*x**4+4),x)
```

```
[Out] -sqrt(3)*log(x**2 - 2*sqrt(3)*x/3 + 2/3)/12 + sqrt(3)*log(x**2 + 2*sqrt(3)*
x/3 + 2/3)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{1}{12}\,\sqrt{3}\log\left(3\,x^2+2\,\sqrt{3}x+2\right)-\frac{1}{12}\,\sqrt{3}\log\left(3\,x^2-2\,\sqrt{3}x+2\right)$$

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*log(3*x^2 + 2*sqrt(3)*x + 2) - 1/12*sqrt(3)*log(3*x^2 - 2*sqrt
(3)*x + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)$$

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="giac")
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(2)*(4/9)^(1/4)*x + 2/3) - 1/12*sqrt(3)*log(x^2
- sqrt(2)*(4/9)^(1/4)*x + 2/3)
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.41

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\,x^2+2}\right)}{6}$$

```
[In] int(-(3*x^2 - 2)/(9*x^4 + 4),x)
[Out] (3^(1/2)*atanh((2*3^(1/2)*x)/(3*x^2 + 2)))/6
```

3.7 $\int \frac{2+3x^2}{4-9x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{2+3x^2}{4-9x^4} \, dx = \frac{\operatorname{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {26, 212}

$$\int \frac{2+3x^2}{4-9x^4} \, dx = \frac{\operatorname{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[In] Int[(2 + 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTanh[Sqrt[3/2]*x]/Sqrt[6]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{2-3x^2}\,dx\\ &= \frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{2+3x^2}{4-9x^4}\,dx = \frac{-\log\left(\sqrt{6}-3x\right)+\log\left(\sqrt{6}+3x\right)}{2\sqrt{6}}$$

```
[In] Integrate[(2 + 3*x^2)/(4 - 9*x^4),x]
[Out] (-Log[Sqrt[6] - 3*x] + Log[Sqrt[6] + 3*x])/(2*Sqrt[6])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result
default	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
risch	$\frac{\sqrt{6}\,\ln\left(3x+\sqrt{6}\right)}{12}-\frac{\sqrt{6}\,\ln\left(3x-\sqrt{6}\right)}{12}$
meijerg	$-\frac{\sqrt{6}\,x\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)\right)}{24\left(x^{4}\right)^{\frac{1}{4}}}-\frac{\sqrt{6}\,x^{3}\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\right.\right.}{24(x}$

```
[In] int((3*x^2+2)/(-9*x^4+4),x,method=_RETURNVERBOSE)
[Out] 1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time $= 0.25$ (sec) , antiderivative size $= 29$, normalized size of antiderivative $= 1.81$

$$\int \frac{2+3x^2}{4-9x^4}\,dx = \frac{1}{12}\,\sqrt{6}\log\left(\frac{3\,x^2+2\,\sqrt{6}x+2}{3\,x^2-2}\right)$$

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time $= 0.04$ (sec) , antiderivative size $= 32$, normalized size of antiderivative $= 2.00$

$$\int \frac{2+3x^2}{4-9x^4}\,dx = -\frac{\sqrt{6}\log\left(x-\frac{\sqrt{6}}{3}\right)}{12}+\frac{\sqrt{6}\log\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

```
[In] integrate((3*x**2+2)/(-9*x**4+4),x)
```

```
[Out] -sqrt(6)*log(x - sqrt(6)/3)/12 + sqrt(6)*log(x + sqrt(6)/3)/12
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time $= 0.27$ (sec) , antiderivative size $= 25$, normalized size of antiderivative $= 1.56$

$$\int \frac{2+3x^2}{4-9x^4}\,dx = -\frac{1}{12}\,\sqrt{6}\log\left(\frac{3\,x-\sqrt{6}}{3\,x+\sqrt{6}}\right)$$

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{2+3x^2}{4-9x^4}\,dx = \frac{1}{12}\,\sqrt{6}\log\left(\left|x+\frac{1}{3}\,\sqrt{6}\right|\right) - \frac{1}{12}\,\sqrt{6}\log\left(\left|x-\frac{1}{3}\,\sqrt{6}\right|\right)$$

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*log(abs(x + 1/3*sqrt(6))) - 1/12*sqrt(6)*log(abs(x - 1/3*sqrt(
6)))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2+3x^2}{4-9x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

```
[In] int(-(3*x^2 + 2)/(9*x^4 - 4),x)
```

```
[Out] (6^(1/2)*atanh((6^(1/2)*x)/2))/6
```

3.8 $\int \frac{2-3x^2}{4-9x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{2-3x^2}{4-9x^4}\,dx=\frac{\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {26, 209}

$$\int \frac{2-3x^2}{4-9x^4}\,dx=\frac{\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[In] Int[(2 - 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\text{integral} = \int \frac{1}{2+3x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[In] Integrate[(2 - 3*x^2)/(4 - 9*x^4),x]
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result
default	$\frac{\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
risch	$\frac{\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
meijerg	$-\frac{\sqrt{6}\,x\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)\right)}{24\left(x^{4}\right)^{\frac{1}{4}}}+\frac{\sqrt{6}\,x^{3}\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\sqrt{3}\right.\right.}{24(x}$

```
[In] int((-3*x^2+2)/(-9*x^4+4),x,method=_RETURNVERBOSE)
[Out] 1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)$$

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

```
[In] integrate((-3*x**2+2)/(-9*x**4+4),x)
[Out] sqrt(6)*atan(sqrt(6)*x/2)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)$$

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)$$

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

```
[In] int((3*x^2 - 2)/(9*x^4 - 4),x)
[Out] (6^(1/2)*atan((6^(1/2)*x)/2))/6
```

3.9 $\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx=-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

```
[Out] 1/2*b^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)*2^(1/2)+1/2*b^(1/4
)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1176, 631, 210}

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx=\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

```
[In] Int[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))) + (b
^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx + \frac{1}{2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx$$

$$= \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}} + \frac{\sqrt[4]{b}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a}\sqrt{b} + bx^2}{a + bx^4}\,dx = \frac{\sqrt[4]{b}\left(-\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + \arctan\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right)}{\sqrt{2}\sqrt[4]{a}}$$

```
[In] Integrate[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] (b^(1/4)*(-ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)]))/(Sqrt[2]*a^(1/4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(51) = 102$.

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.72

method	result
default	$\frac{\sqrt{b}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8\sqrt{a}}+\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arcta}\right.}{8(}$

```
[In] int((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/(a/b)^(1/4)*2^(1/2)*(ln((x^
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.97

$$\int\frac{\sqrt{a}\sqrt{b}+bx^{2}}{a+bx^{4}}\,dx$$

$$=\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^{4}-4\sqrt{a}\sqrt{b}x^{2}+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^{3}-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^{4}+a}\right),\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{\right.\right.$$

$$\left.\left.+\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^{3}+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{a}\right)\right]$$

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*log((b*x^4 - 4*sqrt(a)*sqrt(b)*x^2 +
4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a)) + a)/(b*x^4
+ a)), sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(sqrt(b)/sqrt
(a))) + sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)*x
^3 + a*x)*sqrt(sqrt(b)/sqrt(a))/a)]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4} + \frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

```
[In] integrate((b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a
))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log
(sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)
/4
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}$$

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 1/2*sqrt(2)*sqrt(b)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/sqrt(sqrt(a)*sqrt(b))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\,b^{1/4}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{3/4}\,x^3}{2\,a^{3/4}}+\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)\right)}{4\,a^{1/4}}$$

```
[In] int((b*x^2 + a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

```
[Out] (2^(1/2)*b^(1/4)*(2*atan((2^(1/2)*b^(1/4)*x)/(2*a^(1/4))) + 2*atan((2^(1/2)
*b^(3/4)*x^3)/(2*a^(3/4)) + (2^(1/2)*b^(1/4)*x)/(2*a^(1/4)))))/(4*a^(1/4))
```

3.10 $\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 106

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4} \, dx = -\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

```
[Out] -1/4*b^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)*2^(
1/2)+1/4*b^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1179, 642}

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4} \, dx = \frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

```
[In] Int[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

```
[Out] -1/2*(b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(Sqrt
[2]*a^(1/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(2*Sqrt[2]*a^(1/4))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}}\\ &= -\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \frac{\sqrt[4]{b}\left(-\log\left(-\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x-\sqrt{b}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)\right)}{2\sqrt{2}\sqrt[4]{a}}$$

```
[In] Integrate[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

```
[Out] (b^(1/4)*(-Log[-Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x - Sqrt[b]*x^2] + Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(2*Sqrt[2]*a^(1/4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(70) = 140.

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.92

method	result
default	$\frac{\sqrt{b}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8\sqrt{a}}-\frac{\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arc}\right.}{8}$

```
[In] int((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8/(a/b)^(1/4)*2^(1/2)*(ln((x^
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4+4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right),\right.$$
$$-\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\right)$$
$$\left.+\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{a}\right)\right]$$

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*log((b*x^4 + 4*sqrt(a)*sqrt(b)*x^2 + 4
*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a)) + a)/(b*x^4 +
a)), -sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(-sqrt(b)/sqr
t(a))) + sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)
*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a))/a)]
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = -\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4} + \frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

```
[In] integrate((-b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))
/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(sq
rt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}} - \frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}}$$

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/
a^(1/4) - 1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/a^(1/4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.85 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx=\frac{\sqrt{2}\,b^{1/4}\,\mathrm{atanh}\left(\frac{2\,\sqrt{2}\,a^{1/4}\,b^{11/4}\,x}{2\,\sqrt{a}\,b^{5/2}+2\,b^3\,x^2}\right)}{2\,a^{1/4}}$$

```
[In] int(-(b*x^2 - a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

```
[Out] (2^(1/2)*b^(1/4)*atanh((2*2^(1/2)*a^(1/4)*b^(11/4)*x)/(2*a^(1/2)*b^(5/2) +
2*b^3*x^2)))/(2*a^(1/4))
```

3.11 $\int \frac{d+ex^2}{d^2+e^2x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[Out] 1/2*arctan(-1+x*2^(1/2)*e^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)+1/2*arctan
(1+x*2^(1/2)*e^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1176, 631, 210}

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[In] Int[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])) + ArcT
an[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+x^2}\,dx}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx=\frac{-\arctan\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)+\arctan\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[In] Integrate[(d + e*x^2)/(d^2 + e^2*x^4),x]
[Out] (-ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]] + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)/
Sqrt[d]])/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{2}\,\ln\left(e\,x^{2}\sqrt{-ed}-dex\sqrt{2}-d\sqrt{-ed}\right)}{4\sqrt{-ed}}+\frac{\sqrt{2}\,\ln\left(e\,x^{2}\sqrt{-ed}+dex\sqrt{2}-d\sqrt{-ed}\right)}{4\sqrt{-ed}}$
default	$\frac{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8d}+\frac{\sqrt{2}\bigg(\ln\left(\frac{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right)+}{}$

```
[In] int((e*x^2+d)/(e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)/(-e*d)^(1/2)*ln(e*x^2*(-e*d)^(1/2)-d*e*x*2^(1/2)-d*(-e*d)^(1/2
))+1/4*2^(1/2)/(-e*d)^(1/2)*ln(e*x^2*(-e*d)^(1/2)+d*e*x*2^(1/2)-d*(-e*d)^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.83

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx$$
$$=\left[-\frac{\sqrt{2}\sqrt{-de}\log\left(\frac{e^2x^4-4\,dex^2-2\,\sqrt{2}(ex^3-dx)\sqrt{-de}+d^2}{e^2x^4+d^2}\right)}{4\,de},\frac{\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}\sqrt{de}x}{2\,d}\right)+\sqrt{2}\sqrt{de}\arctan\bigg(\frac{\sqrt{2}(ex^3+dx)\sqrt{d}{2\,d^2}}{2\,de}\right.$$

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(2)*sqrt(-d*e)*log((e^2*x^4 - 4*d*e*x^2 - 2*sqrt(2)*(e*x^3 - d*x)
*sqrt(-d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), 1/2*(sqrt(2)*sqrt(d*e)*arctan(1/
2*sqrt(2)*sqrt(d*e)*x/d) + sqrt(2)*sqrt(d*e)*arctan(1/2*sqrt(2)*(e*x^3 + d*
x)*sqrt(d*e)/d^2))/(d*e)]
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4} + \frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}$$

```
[In] integrate((e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(-1/(d*e))*log(-sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4 + s
qrt(2)*sqrt(-1/(d*e))*log(sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(51) = 102.

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 4.03

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} + \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} - \frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}}$$

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(e + sqrt(e^2))*log((2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1/4
) - sqrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1/4
) + sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) + 1/8*sqrt(
2)*(e + sqrt(e^2))*log((2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) - sqrt(
2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) + sqrt(
2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) - 1/8*sqrt(2)*(e - s
qrt(e^2))*log(sqrt(e^2)*x^2 + sqrt(2)*sqrt(d)*(e^2)^(1/4)*x + d)/(sqrt(d)*(
e^2)^(3/4)) + 1/8*sqrt(2)*(e - sqrt(e^2))*log(sqrt(e^2)*x^2 - sqrt(2)*sqrt(
d)*(e^2)^(1/4)*x + d)/(sqrt(d)*(e^2)^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\sqrt{-de}\log\left(x^2+\sqrt{2}x\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{e^2}}\right)}{4\,de} - \frac{\sqrt{2}\sqrt{-de}\log\left(x^2-\sqrt{2}x\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{e^2}}\right)}{4\,de}$$

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(-d*e)*log(x^2 + sqrt(2)*x*(d^2/e^2)^(1/4) + sqrt(d^2/e^2))
/(d*e) - 1/4*sqrt(2)*sqrt(-d*e)*log(x^2 - sqrt(2)*x*(d^2/e^2)^(1/4) + sqrt(
d^2/e^2))/(d*e)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{3/2}\,x^3}{2\,d^{3/2}}+\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)\right)}{4\,\sqrt{d}\,\sqrt{e}}$$

```
[In] int((d + e*x^2)/(d^2 + e^2*x^4),x)
```

```
[Out] (2^(1/2)*(2*atan((2^(1/2)*e^(1/2)*x)/(2*d^(1/2))) + 2*atan((2^(1/2)*e^(3/2)
*x^3)/(2*d^(3/2)) + (2^(1/2)*e^(1/2)*x)/(2*d^(1/2)))))/(4*d^(1/2)*e^(1/2))
```

3.12 $\int \frac{d-ex^2}{d^2+e^2x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx=-\frac{\log \left(d-\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\log \left(d+\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[Out] -1/4*ln(d+e*x^2-x*2^(1/2)*d^(1/2)*e^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)+1/4*ln(d
+e*x^2+x*2^(1/2)*d^(1/2)*e^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1179, 642}

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx=\frac{\log \left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\log \left(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[In] Int[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[e]) +
 Log[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{e}}+2x}{-\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2}\,dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{e}}-2x}{-\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2}\,dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}\\ &= -\frac{\log\left(d-\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\log\left(d+\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx=\frac{-\log\left(-d+\sqrt{2}\sqrt{d}\sqrt{e}x-ex^2\right)+\log\left(d+\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[In] Integrate[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] (-Log[-d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x - e*x^2] + Log[d + Sqrt[2]*Sqrt[d]*Sqr
t[e]*x + e*x^2])/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\sqrt{2}\,\ln\left(e\,x^{2}\sqrt{ed}+dex\sqrt{2}+d\sqrt{ed}\right)}{4\sqrt{ed}}-\frac{\sqrt{2}\,\ln\left(e\,x^{2}\sqrt{ed}-dex\sqrt{2}+d\sqrt{ed}\right)}{4\sqrt{ed}}$
default	$\frac{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8d}-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right)+\right.}{}$

```
[In] int((-e*x^2+d)/(e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)/(e*d)^(1/2)*ln(e*x^2*(e*d)^(1/2)+d*e*x*2^(1/2)+d*(e*d)^(1/2))-1
/4*2^(1/2)/(e*d)^(1/2)*ln(e*x^2*(e*d)^(1/2)-d*e*x*2^(1/2)+d*(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.56

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \left[\frac{\sqrt{2}\sqrt{de}\log\left(\frac{e^2x^4+4\,dex^2+2\,\sqrt{2}(ex^3+dx)\sqrt{de}+d^2}{e^2x^4+d^2}\right)}{4\,de}, -\frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\sqrt{-de}x}{2\,d}\right)-\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}(ex^3-dx)\sqrt{-de}}{2\,d^2}\right)}{2\,de}\right]$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(d*e)*log((e^2*x^4 + 4*d*e*x^2 + 2*sqrt(2)*(e*x^3 + d*x)*s
qrt(d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), -1/2*(sqrt(2)*sqrt(-d*e)*arctan(1/2
*sqrt(2)*sqrt(-d*e)*x/d) - sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(e*x^3 - d
*x)*sqrt(-d*e)/d^2))/(d*e)]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = -\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}$$

```
[In] integrate((-e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(1/(d*e))*log(-sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4 + sqr
t(2)*sqrt(1/(d*e))*log(sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.36

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = -\frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} - \frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} + \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}}$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(e - sqrt(e^2))*log((2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1/
4) - sqrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1/
4) + sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) - 1/8*sqrt
(2)*(e - sqrt(e^2))*log((2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) - sqrt
(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) + sqrt
(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) + 1/8*sqrt(2)*(e +
sqrt(e^2))*log(sqrt(e^2)*x^2 + sqrt(2)*sqrt(d)*(e^2)^(1/4)*x + d)/(sqrt(d)*
(e^2)^(3/4)) - 1/8*sqrt(2)*(e + sqrt(e^2))*log(sqrt(e^2)*x^2 - sqrt(2)*sqrt
(d)*(e^2)^(1/4)*x + d)/(sqrt(d)*(e^2)^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\right)}{2\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}\right)}{2de} + \frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\right)}{2\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}\right)}{2de}$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(d^2/e^2)^(1/4))/(
d^2/e^2)^(1/4))/(d*e) + 1/2*sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(d^2/e^2)^(1/4))/(d^2/e^2)^(1/4))/(d*e)
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.46

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx=\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{2\,\sqrt{2}\,\sqrt{d}\,e^{7/2}\,x}{2\,e^4\,x^2+2\,d\,e^3}\right)}{2\,\sqrt{d}\,\sqrt{e}}$$

```
[In] int((d - e*x^2)/(d^2 + e^2*x^4),x)
```

```
[Out] (2^(1/2)*atanh((2*2^(1/2)*d^(1/2)*e^(7/2)*x)/(2*d*e^3 + 2*e^4*x^2)))/(2*d^(
1/2)*e^(1/2))
```

3.13 $\int \frac{5+2x^2}{-1+x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3\arctan(x)}{2} - \frac{7\text{arctanh}(x)}{2}$$

```
[Out] -3/2*arctan(x)-7/2*arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1181, 213, 209}

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3\arctan(x)}{2} - \frac{7\text{arctanh}(x)}{2}$$

```
[In] Int[(5 + 2*x^2)/(-1 + x^4),x]
```

```
[Out] (-3*ArcTan[x])/2 - (7*ArcTanh[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{3}{2}\int\frac{1}{1+x^2}\,dx\right)+\frac{7}{2}\int\frac{1}{-1+x^2}\,dx\\ &= -\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int\frac{5+2x^2}{-1+x^4}\,dx=-\frac{3\arctan(x)}{2}+\frac{7}{4}\log(1-x)-\frac{7}{4}\log(1+x)$$

```
[In] Integrate[(5 + 2*x^2)/(-1 + x^4),x]
[Out] (-3*ArcTan[x])/2 + (7*Log[1 - x])/4 - (7*Log[1 + x])/4
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result	si
default	$\frac{7\ln(x-1)}{4}-\frac{7\ln(x+1)}{4}-\frac{3\arctan(x)}{2}$	18
risch	$\frac{7\ln(x-1)}{4}-\frac{7\ln(x+1)}{4}-\frac{3\arctan(x)}{2}$	18
parallelrisch	$\frac{7\ln(x-1)}{4}+\frac{3i\ln(x-i)}{4}-\frac{3i\ln(x+i)}{4}-\frac{7\ln(x+1)}{4}$	30
meijerg	$\frac{5x\left(\ln\left(1-\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^4\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^4\right)^{\frac{1}{4}}\right)\right)}{4\left(x^4\right)^{\frac{1}{4}}}+\frac{x^3\left(\ln\left(1-\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^4\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^4\right)^{\frac{1}{4}}\right)\right)}{2\left(x^4\right)^{\frac{3}{4}}}$	78

```
[In] int((2*x^2+5)/(x^4-1),x,method=_RETURNVERBOSE)
[Out] 7/4*ln(x-1)-7/4*ln(x+1)-3/2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3}{2}\arctan{(x)} - \frac{7}{4}\log{(x+1)} + \frac{7}{4}\log{(x-1)}$$

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="fricas")
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{5+2x^2}{-1+x^4}\,dx = \frac{7\log{(x-1)}}{4} - \frac{7\log{(x+1)}}{4} - \frac{3\,\mathrm{atan}\,(x)}{2}$$

```
[In] integrate((2*x**2+5)/(x**4-1),x)
[Out] 7*log(x - 1)/4 - 7*log(x + 1)/4 - 3*atan(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3}{2}\arctan{(x)} - \frac{7}{4}\log{(x+1)} + \frac{7}{4}\log{(x-1)}$$

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="maxima")
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3}{2}\arctan{(x)} - \frac{7}{4}\log{(|x+1|)} + \frac{7}{4}\log{(|x-1|)}$$

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="giac")
[Out] -3/2*arctan(x) - 7/4*log(abs(x + 1)) + 7/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3\,\mathrm{atan}(x)}{2} - \frac{7\,\mathrm{atanh}(x)}{2}$$

```
[In] int((2*x^2 + 5)/(x^4 - 1),x)
[Out] - (3*atan(x))/2 - (7*atanh(x))/2
```

3.14 $\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx=\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[Out] EllipticE(x*b^(1/2),I)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1213, 435}

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx=\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[In] Int[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
```

```
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx \\ &= \frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\begin{aligned}\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx &= x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right) \\ &\quad + \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\end{aligned}$$

```
[In] Integrate[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + (b*x^3*Hypergeometric2F1[1/2,
 3/4, 7/4, b^2*x^4])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

method	result	size
meijerg	$\frac{b\,x^{3}{}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^{2}x^{4}\right)}{3}+x\,{}_{2}F_{1}\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^{2}x^{4}\right)$	36
default	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,F\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}-\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(F\left(\sqrt{b}\,x,i\right)-E\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}$	100
elliptic	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,F\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}-\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(F\left(\sqrt{b}\,x,i\right)-E\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}$	100

```
[In] int((b*x^2+1)/(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*x^3*hypergeom([1/2,3/4],[7/4],b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],b^
2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(11) = 22$.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.44

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \frac{\frac{\sqrt{-b^2}(b+1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}} - \frac{\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}} - \sqrt{-b^2x^4+1}b}{b^2x}$$

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b^2)*(b + 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - sqrt(
-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - sqrt(-b^2*x^4 + 1)*
b)/(b^2*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(12) = 24$.

Time = 0.85 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.38

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(2
*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{b\,x^2+1}{\sqrt{1-b^2\,x^4}}\,dx$$

```
[In] int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2),x)
[Out] int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2), x)
```

3.15 $\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}$$

```
[Out] -EllipticE(x*b^(1/2),I)/b^(1/2)+2*EllipticF(x*b^(1/2),I)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1213, 434, 435, 254, 227}

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[In] Int[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*x
], -1])/Sqrt[b]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}}\,dx \\
&= 2\int \frac{1}{\sqrt{1-bx^2}\sqrt{1+bx^2}}\,dx - \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx \\
&= -\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}} + 2\int \frac{1}{\sqrt{1-b^2x^4}}\,dx \\
&= -\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}} + \frac{2F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right) - \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)$$

```
[In] Integrate[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[1/2,
 3/4, 7/4, b^2*x^4])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
meijerg	$-\frac{b\,x^{3}{}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^{2}x^{4}\right)}{3}+x\,{}_{2}F_{1}\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^{2}x^{4}\right)$	36
default	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,F\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}+\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(F\left(\sqrt{b}\,x,i\right)-E\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}$	99
elliptic	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,F\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}+\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(F\left(\sqrt{b}\,x,i\right)-E\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}$	99

```
[In] int((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b*x^3*hypergeom([1/2,3/4],[7/4],b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],b
^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(25) = 50.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = \frac{\frac{\sqrt{-b^2}(b-1)xF(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1)}{\sqrt{b}} + \frac{\sqrt{-b^2}xE(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1)}{\sqrt{b}} + \sqrt{-b^2x^4+1}b}{b^2x}$$

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b^2)*(b - 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + sqrt(
-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + sqrt(-b^2*x^4 + 1)*
b)/(b^2*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.

Time $= 0.88$ (sec) , antiderivative size $= 70$, normalized size of antiderivative $= 2.00$

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((-b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(
2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=-\int \frac{b\,x^2-1}{\sqrt{1-b^2\,x^4}}\,dx$$

```
[In] int(-(b*x^2 - 1)/(1 - b^2*x^4)^(1/2),x)
```

```
[Out] -int((b*x^2 - 1)/(1 - b^2*x^4)^(1/2), x)
```

3.16 $\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx=\frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}$$

```
[Out] EllipticE(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1214, 1213, 435}

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx=\frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

```
[In] Int[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] (Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2
*x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
```

```
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-b^2x^4}\int\frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}} \\
&= \frac{\sqrt{1-b^2x^4}\int\frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx}{\sqrt{-1+b^2x^4}} \\
&= \frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$
\int\frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx
= \frac{\sqrt{1-b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\right)}{3\sqrt{-1+b^2x^4}}
$$

```
[In] Integrate[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
[Out] (Sqrt[1 - b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b*x^3*H
ypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[-1 + b^2*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.93 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

method	result	size
meijerg	$\frac{b\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x^3{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)}{3\sqrt{\operatorname{signum}(b^2x^4-1)}}+\frac{\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)}{\sqrt{\operatorname{signum}(b^2x^4-1)}}$	88
default	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,F\left(x\sqrt{-b},i\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(F\left(x\sqrt{-b},i\right)-E\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	107
elliptic	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,F\left(x\sqrt{-b},i\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(F\left(x\sqrt{-b},i\right)-E\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	107

```
[In] int((b*x^2+1)/(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)*x^3*hypergeom([1/2
,3/4],[7/4],b^2*x^4)+1/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)*x
*hypergeom([1/4,1/2],[5/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\frac{(b+1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}}-\frac{xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}}-\sqrt{b^2x^4-1}}{bx}$$

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -((b + 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - x*elliptic_e(ar
csin(1/(sqrt(b)*x)), -1)/sqrt(b) - sqrt(b^2*x^4 - 1))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int \frac{b\,x^2+1}{\sqrt{b^2\,x^4-1}}\,dx$$

```
[In] int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2),x)
```

```
[Out] int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2), x)
```

3.17 $\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{2\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}$$

```
[Out] -EllipticE(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)+2*Elli
pticF(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1214, 1213, 434, 435, 254, 227}

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \frac{2\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}} - \frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

```
[In] Int[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] -((Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b
^2*x^4])) + (2*Sqrt[1 - b^2*x^4]*EllipticF[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]
*Sqrt[-1 + b^2*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-b^2x^4}\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}\\ &= \frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}}\,dx}{\sqrt{-1+b^2x^4}}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx}{\sqrt{-1+b^2x^4}} + \frac{\left(2\sqrt{1-b^2x^4}\right)\int \frac{1}{\sqrt{1-bx^2}\sqrt{1+bx^2}}\,dx}{\sqrt{-1+b^2x^4}} \\
&= -\frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{\left(2\sqrt{1-b^2x^4}\right)\int \frac{1}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}} \\
&= -\frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{2\sqrt{1-b^2x^4}F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$
\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\sqrt{1-b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\right)}{3\sqrt{-1+b^2x^4}}
$$

```
[In] Integrate[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] -1/3*(Sqrt[1 - b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/Sqrt[-1 + b^2*x^4]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

method	result	size
meijerg	$-\frac{b\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x^3{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)}{3\sqrt{\operatorname{signum}(b^2x^4-1)}}+\frac{\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)}{\sqrt{\operatorname{signum}(b^2x^4-1)}}$	88
default	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,F\left(x\sqrt{-b},i\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}-\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(F\left(x\sqrt{-b},i\right)-E\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	108
elliptic	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,F\left(x\sqrt{-b},i\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}-\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(F\left(x\sqrt{-b},i\right)-E\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	108

```
[In] int((-b*x^2+1)/(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)*x^3*hypergeom([1/
2,3/4],[7/4],b^2*x^4)+1/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)*
x*hypergeom([1/4,1/2],[5/4],b^2*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\frac{(b-1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid -1\right)}{\sqrt{b}} + \frac{xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid -1\right)}{\sqrt{b}} + \sqrt{b^2x^4-1}}{bx}$$

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -((b - 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + x*elliptic_e(ar
csin(1/(sqrt(b)*x)), -1)/sqrt(b) + sqrt(b^2*x^4 - 1))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((-b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) - I
*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\int \frac{b\,x^2-1}{\sqrt{b^2\,x^4-1}}\,dx$$

```
[In] int(-(b*x^2 - 1)/(b^2*x^4 - 1)^(1/2),x)
[Out] -int((b*x^2 - 1)/(b^2*x^4 - 1)^(1/2), x)
```

3.18 $\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx=-\frac{x\sqrt{1+b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

```
[Out] -x*(b^2*x^4+1)^(1/2)/(b*x^2+1)+(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)
/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*(
(b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1210}

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx=\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

```
[In] Int[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] -((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 +
 b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rubi steps

$$\text{integral} = -\frac{x\sqrt{1+b^2x^4}}{1+bx^2} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)\left|\frac{1}{2}\right.\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right) - \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)$$

```
[In] Integrate[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] - (b*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.43

method	result	size
meijerg	$-\frac{b\,x^3{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)}{3}+x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)$	38
default	$\frac{\sqrt{-i b\,x^2+1}\,\sqrt{i b\,x^2+1}\,F\left(x\sqrt{i b},i\right)}{\sqrt{i b}\,\sqrt{b^2x^4+1}}-\frac{i\sqrt{-i b\,x^2+1}\,\sqrt{i b\,x^2+1}\,\left(F\left(x\sqrt{i b},i\right)-E\left(x\sqrt{i b},i\right)\right)}{\sqrt{i b}\,\sqrt{b^2x^4+1}}$	120
elliptic	$\frac{\sqrt{-i b\,x^2+1}\,\sqrt{i b\,x^2+1}\,F\left(x\sqrt{i b},i\right)}{\sqrt{i b}\,\sqrt{b^2x^4+1}}-\frac{i\sqrt{-i b\,x^2+1}\,\sqrt{i b\,x^2+1}\,\left(F\left(x\sqrt{i b},i\right)-E\left(x\sqrt{i b},i\right)\right)}{\sqrt{i b}\,\sqrt{b^2x^4+1}}$	120

```
[In] int((-b*x^2+1)/(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b*x^3*hypergeom([1/2,3/4],[7/4],-b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],
-b^2*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\frac{bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-(b^2+b)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\sqrt{b^2x^4+1}}{bx}$$

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) - (b^2 + b)*x
*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1) + sqrt(b^2*x^4 + 1
))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((-b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*
pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\int \frac{b\,x^2-1}{\sqrt{b^2\,x^4+1}}\,dx$$

```
[In] int(-(b*x^2 - 1)/(b^2*x^4 + 1)^(1/2),x)
[Out] -int((b*x^2 - 1)/(b^2*x^4 + 1)^(1/2), x)
```

3.19 $\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \frac{x\sqrt{1+b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\,|\,\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

```
[Out] x*(b^2*x^4+1)^(1/2)/(b*x^2+1)-(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)/
cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*((
b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)^(1/2)+(b*x^2+1)*(cos(2*ar
ctan(x*b^(1/2)))^2)^(1/2)/cos(2*arctan(x*b^(1/2)))*EllipticF(sin(2*arctan(x
*b^(1/2))),1/2*2^(1/2))*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)
^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1212, 226, 1210}

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}} - \frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,E\left(2\arctan\left(\sqrt{b}x\right)\,|\,\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}} + \frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

```
[In] Int[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] (x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*
x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]) +
((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x
], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{\sqrt{1+b^2x^4}}\,dx - \int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx\\ &= \frac{x\sqrt{1+b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}\\ &\quad + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.31

$$\begin{aligned}\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx &= x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)\\ &\quad + \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\end{aligned}$$

```
[In] Integrate[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + (b*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
meijerg	$\frac{b\,x^{3}{}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^{2}x^{4}\right)}{3}+x\,{}_{2}F_{1}\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^{2}x^{4}\right)$	38
default	$\frac{\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\,F\left(x\sqrt{i b},i\right)}{\sqrt{i b}\,\sqrt{b^{2}x^{4}+1}}+\frac{i\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\,\left(F\left(x\sqrt{i b},i\right)-E\left(x\sqrt{i b},i\right)\right)}{\sqrt{i b}\,\sqrt{b^{2}x^{4}+1}}$	120
elliptic	$\frac{\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\,F\left(x\sqrt{i b},i\right)}{\sqrt{i b}\,\sqrt{b^{2}x^{4}+1}}+\frac{i\sqrt{-i b\,x^{2}+1}\,\sqrt{i b\,x^{2}+1}\,\left(F\left(x\sqrt{i b},i\right)-E\left(x\sqrt{i b},i\right)\right)}{\sqrt{i b}\,\sqrt{b^{2}x^{4}+1}}$	120

```
[In] int((b*x^2+1)/(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*x^3*hypergeom([1/2,3/4],[7/4],-b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],-
b^2*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.48

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \frac{bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+(b^2-b)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+\sqrt{b^2x^4+1}}{bx}$$

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) + (b^2 - b)*x*
(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1) + sqrt(b^2*x^4 + 1)
)/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*g
amma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*p
i))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx=\int \frac{b\,x^2+1}{\sqrt{b^2\,x^4+1}}\,dx$$

```
[In] int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2),x)
[Out] int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2), x)
```

3.20 $\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

```
[Out] x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)+(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)
/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*(
(b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1210}

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}$$

```
[In] Int[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b
*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{-1-b^2x^4}}{1+bx^2} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)\mid\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{\sqrt{1+b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\right)}{3\sqrt{-1-b^2x^4}}$$

```
[In] Integrate[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
[Out] -1/3*(Sqrt[1 + b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)]
+ b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/Sqrt[-1 - b^2*x^4]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result	size
meijerg	$-\frac{b\sqrt{\operatorname{signum}(b^2x^4+1)}\,x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)}{3\sqrt{-\operatorname{signum}(b^2x^4+1)}}+\frac{\sqrt{\operatorname{signum}(b^2x^4+1)}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)}{\sqrt{-\operatorname{signum}(b^2x^4+1)}}$	90
default	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,F\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(F\left(x\sqrt{-ib},i\right)-E\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122
elliptic	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,F\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(F\left(x\sqrt{-ib},i\right)-E\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122

```
[In] int((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*b/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2)*x^3*hypergeom([1/
2,3/4],[7/4],-b^2*x^4)+1/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2)
*x*hypergeom([1/4,1/2],[5/4],-b^2*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx =$$

$$-\frac{\sqrt{-b^2}(b+1)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) | -1\right) - \sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) | -1\right) - \sqrt{-b^2x}}{bx}$$

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-b^2)*(b + 1)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x),
-1) - sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1)
- sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((-b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar
(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

Giac [F]

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=\int -\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=-\int \frac{b\,x^2-1}{\sqrt{-b^2\,x^4-1}}\,dx$$

```
[In] int(-(b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2),x)
```

```
[Out] -int((b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2), x)
```

3.21 $\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{x\sqrt{-1-b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

```
[Out] -x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)-(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2
)/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*
((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^4-1)^(1/2)+(b*x^2+1)*(cos(2
*arctan(x*b^(1/2)))^2)^(1/2)/cos(2*arctan(x*b^(1/2)))*EllipticF(sin(2*arcta
n(x*b^(1/2))),1/2*2^(1/2))*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^
4-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1212, 226, 1210}

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = \frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}} - \frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}} - \frac{x\sqrt{-b^2x^4-1}}{bx^2+1}$$

```
[In] Int[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] -((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1
+ b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4
]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{\sqrt{-1-b^2x^4}}\,dx - \int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx \\ &= -\frac{x\sqrt{-1-b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}} \\ &\quad + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.49

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = \frac{\sqrt{1+b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\right)}{3\sqrt{-1-b^2x^4}}$$

```
[In] Integrate[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (Sqrt[1 + b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + b*x^
3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[-1 - b^2*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

method	result	size
meijerg	$\frac{b\sqrt{\operatorname{signum}(b^2x^4+1)}\, x^3\, {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)}{3\sqrt{-\operatorname{signum}(b^2x^4+1)}}+\frac{\sqrt{\operatorname{signum}(b^2x^4+1)}\, x\, {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)}{\sqrt{-\operatorname{signum}(b^2x^4+1)}}$	90
default	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,F\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}-\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(F\left(x\sqrt{-ib},i\right)-E\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122
elliptic	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,F\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}-\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(F\left(x\sqrt{-ib},i\right)-E\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122

```
[In] int((b*x^2+1)/(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2)*x^3*hypergeom([1/2
,3/4],[7/4],-b^2*x^4)+1/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2)*
x*hypergeom([1/4,1/2],[5/4],-b^2*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.54

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{\sqrt{-b^2}(b-1)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + \sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + \sqrt{-b^2x}}{bx}$$

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-b^2)*(b - 1)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x),
-1) + sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1)
+ sqrt(-b^2*x^4 - 1))/(b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_pola
r(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

Giac [F]

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx=\int \frac{b\,x^2+1}{\sqrt{-b^2\,x^4-1}}\,dx$$

```
[In] int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2),x)
[Out] int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2), x)
```

3.22 $\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 10

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx=\frac{E(\arcsin(cx)|-1)}{c}$$

```
[Out] EllipticE(c*x,I)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {435}

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx=\frac{E(\arcsin(cx)|-1)}{c}$$

```
[In] Int[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral}=\frac{E(\sin^{-1}(cx)|-1)}{c}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \frac{E(\arcsin(cx)|-1)}{c}$$

```
[In] Integrate[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.98 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{E(x\ \mathrm{csgn}(c)c,i)\,\mathrm{csgn}(c)}{c}$	15
elliptic	$\frac{\sqrt{-c^4x^4+1}\left(\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}-\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}\right)}{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}}$	154

```
[In] int((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] EllipticE(x*csgn(c)*c,I)*csgn(c)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 7.60

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx$$
$$= -\frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}c^3 - \sqrt{-c^4}\left((c^2+1)xF\left(\arcsin\left(\frac{1}{cx}\right)|-1\right) - xE\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)\right)}{c^5x}$$

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*c^3 - sqrt(-c^4)*((c^2 + 1)*x*ellipt
ic_f(arcsin(1/(c*x)), -1) - x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

Sympy [F]

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

```
[In] integrate((c**2*x**2+1)**(1/2)/(-c**2*x**2+1)**(1/2),x)
[Out] Integral(sqrt(c**2*x**2 + 1)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{\sqrt{1-c^2\,x^2}}\,dx$$

```
[In] int((c^2*x^2 + 1)^(1/2)/(1 - c^2*x^2)^(1/2),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(1 - c^2*x^2)^(1/2), x)
```

3.23 $\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 10

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \frac{E(\arcsin(cx)|-1)}{c}$$

```
[Out] EllipticE(c*x,I)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1213, 435}

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \frac{E(\arcsin(cx)|-1)}{c}$$

```
[In] Int[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx \\
&= \frac{E\left(\sin^{-1}(cx)|-1\right)}{c}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 4.70

$$
\begin{aligned}
\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx &= x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^4x^4\right) \\
&\quad + \frac{1}{3}c^2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^4x^4\right)
\end{aligned}
$$

```
[In] Integrate[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] + (c^2*x^3*Hypergeometric2F1[1/
2, 3/4, 7/4, c^4*x^4])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.80

method	result	size
meijerg	$\frac{c^2x^3{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)}{3}+x{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right)$	38
default	$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}-\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$	118
elliptic	$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}-\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$	118

```
[In] int((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c^2*x^3*hypergeom([1/2,3/4],[7/4],c^4*x^4)+x*hypergeom([1/4,1/2],[5/4],
c^4*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(9) = 18$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 6.50

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{-c^4x^4+1}c^3-\sqrt{-c^4}\left((c^2+1)xF\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)-xE\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)\right)}{c^5x}$$

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-c^4*x^4 + 1)*c^3 - sqrt(-c^4)*((c^2 + 1)*x*elliptic_f(arcsin(1/(c*x
)), -1) - x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(5) = 10$.

Time = 0.90 (sec) , antiderivative size = 71, normalized size of antiderivative = 7.10

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \frac{c^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi))
/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \int \frac{c^2x^2+1}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx=\int \frac{c^2x^2+1}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx=\int \frac{c^2\,x^2+1}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((c^2*x^2 + 1)/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((c^2*x^2 + 1)/(1 - c^4*x^4)^(1/2), x)
```

3.24 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 23

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = -\frac{E(\arcsin(cx)|-1)}{c} + \frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}$$

```
[Out] -EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {434, 435, 254, 227}

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c} - \frac{E(\arcsin(cx)|-1)}{c}$$

```
[In] Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
```

```
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}\,dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + 2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + \frac{2F(\sin^{-1}(cx)|-1)}{c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$
\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{-c^2}x\right)\middle|-1\right)}{\sqrt{-c^2}}
$$

```
[In] Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{(2F(x\ \mathrm{csgn}(c)c,i)-E(x\ \mathrm{csgn}(c)c,i))\,\mathrm{csgn}(c)}{c}$	28
elliptic	$\frac{\sqrt{-c^4x^4+1}\left(\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}\right)}{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}}$	153

```
[In] int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}c^3+\sqrt{-c^4}\left((c^2-1)xF(\arcsin\left(\frac{1}{cx}\right)\mid-1)+xE(\arcsin\left(\frac{1}{cx}\right)\mid-1)\right)}{c^5x}$$

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*c^3 + sqrt(-c^4)*((c^2 - 1)*x*ellipti
c_f(arcsin(1/(c*x)), -1) + x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2),x)
[Out] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2), x)
```

3.25 $\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 23

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx=-\frac{E(\arcsin(cx)|-1)}{c}+\frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}$$

```
[Out] -EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1213, 434, 435, 254, 227}

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx=\frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}-\frac{E(\arcsin(cx)|-1)}{c}$$

```
[In] Int[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
```

```
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx \\
&= 2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}\,dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + 2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + \frac{2F(\sin^{-1}(cx)|-1)}{c}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\begin{aligned}
\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx = {} & x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^4x^4\right) \\
& - \frac{1}{3}c^2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^4x^4\right)
\end{aligned}$$

```
[In] Integrate[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] - (c^2*x^3*Hypergeometric2F1[1/
2, 3/4, 7/4, c^4*x^4])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

method	result	size
meijerg	$-\frac{c^{2} x^{3} {}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^{4} x^{4}\right)}{3}+x {}_{2}F_{1}\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^{4} x^{4}\right)$	38
default	$\frac{\sqrt{-c^{2} x^{2}+1}\, \sqrt{c^{2} x^{2}+1}\, F\left(x \sqrt{c^{2}}, i\right)}{\sqrt{c^{2}}\, \sqrt{-c^{4} x^{4}+1}}+\frac{\sqrt{-c^{2} x^{2}+1}\, \sqrt{c^{2} x^{2}+1}\, \left(F\left(x \sqrt{c^{2}}, i\right)-E\left(x \sqrt{c^{2}}, i\right)\right)}{\sqrt{c^{2}}\, \sqrt{-c^{4} x^{4}+1}}$	117
elliptic	$\frac{\sqrt{-c^{2} x^{2}+1}\, \sqrt{c^{2} x^{2}+1}\, F\left(x \sqrt{c^{2}}, i\right)}{\sqrt{c^{2}}\, \sqrt{-c^{4} x^{4}+1}}+\frac{\sqrt{-c^{2} x^{2}+1}\, \sqrt{c^{2} x^{2}+1}\, \left(F\left(x \sqrt{c^{2}}, i\right)-E\left(x \sqrt{c^{2}}, i\right)\right)}{\sqrt{c^{2}}\, \sqrt{-c^{4} x^{4}+1}}$	117

```
[In] int((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c^2*x^3*hypergeom([1/2,3/4],[7/4],c^4*x^4)+x*hypergeom([1/4,1/2],[5/4]
,c^4*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.70

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \frac{\sqrt{-c^4x^4+1}c^3+\sqrt{-c^4}\left((c^2-1)xF\left(\arcsin\left(\frac{1}{cx}\right)\mid -1\right)+xE\left(\arcsin\left(\frac{1}{cx}\right)\mid -1\right)\right)}{c^5x}$$

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-c^4*x^4 + 1)*c^3 + sqrt(-c^4)*((c^2 - 1)*x*elliptic_f(arcsin(1/(c*x)
), -1) + x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(14) = 28$.

Time = 0.86 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx = -\frac{c^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((-c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] -c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi)
)/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_pol
ar(2*I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \int -\frac{c^2x^2-1}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

Giac [F]

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \int -\frac{c^2x^2-1}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx = -\int \frac{c^2\,x^2-1}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int(-(c^2*x^2 - 1)/(1 - c^4*x^4)^(1/2),x)
[Out] -int((c^2*x^2 - 1)/(1 - c^4*x^4)^(1/2), x)
```

3.26 $\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{-b+2de}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}+\frac{\arctan\left(\frac{\sqrt{-b+2de}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

```
[Out] -arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((2
*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1175, 632, 210}

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}-\frac{\arctan\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

```
[In] Int[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[-b + 2*d*e] - 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]) + Arc
Tan[(Sqrt[-b + 2*d*e] + 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{-b+2de}\,x}{e}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{-b+2de}\,x}{e}+x^2}\,dx}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{b+2de}{e^2}-x^2}\,dx,x,-\frac{\sqrt{-b+2de}}{e}+2x\right)}{e}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{b+2de}{e^2}-x^2}\,dx,x,\frac{\sqrt{-b+2de}}{e}+2x\right)}{e}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}+\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(82) = 164.

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$
\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx = \frac{\frac{\left(-b+2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}+\frac{\left(b-2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}
$$

```
[In] Integrate[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((-b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b
^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] + ((b - 2*d*e + Sqrt[b^2
 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[
b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{-2ex+\sqrt{2ed-b}}{\sqrt{2ed+b}}\right)}{\sqrt{2ed+b}}+\frac{\arctan\left(\frac{2ex+\sqrt{2ed-b}}{\sqrt{2ed+b}}\right)}{\sqrt{2ed+b}}$	71
risch	$-\frac{\ln\left(-e\,x^{2}\sqrt{-2ed-b}+(2ed+b)x+d\sqrt{-2ed-b}\right)}{2\sqrt{-2ed-b}}+\frac{\ln\left(-e\,x^{2}\sqrt{-2ed-b}+(-2ed-b)x+d\sqrt{-2ed-b}\right)}{2\sqrt{-2ed-b}}$	104

```
[In] int((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((2
*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.98

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx = \left[-\frac{\sqrt{-2\,de-b}\log\left(\frac{e^2x^4-(4\,de+b)x^2+d^2-2\,(ex^3-dx)\sqrt{-2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\,(2\,de+b)}, \frac{\sqrt{2\,de+b}\arctan\left(\frac{ex}{\sqrt{2\,de+b}}\right)+\sqrt{2\,de+b}\arctan\Bigg(}{2\,de+b}\right.$$

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e - b)*log((e^2*x^4 - (4*d*e + b)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e - b))/(e^2*x^4 + b*x^2 + d^2))/(2*d*e + b), (sqrt(2*d*e + b)
*arctan(e*x/sqrt(2*d*e + b)) + sqrt(2*d*e + b)*arctan((e^2*x^3 + (d*e + b)*
x)*sqrt(2*d*e + b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx = -\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b+2de}}-2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b+2de}}+2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

```
[In] integrate((e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(-1/(b + 2*d*e)) - 2*d*e*
sqrt(-1/(b + 2*d*e)))/e)/2 + sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(b*sq
rt(-1/(b + 2*d*e)) + 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2
```

Maxima [F]

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx=\int \frac{ex^2+d}{e^2x^4+bx^2+d^2}\,dx$$

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 + b*x^2 + d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(73) = 146$.

Time = 0.70 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.30

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx=\frac{\left(2\,d^2e^3+de^4-bde^2\right)\sqrt{2\,de+b}\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b+\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5+bde^4-b^2de^2}+\frac{\left(2\,d^2e^3+de^4-bde^2\right)\sqrt{2\,de+b}\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b-\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5+bde^4-b^2de^2}$$

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")
```

```
[Out] (2*d^2*e^3 + d*e^4 - b*d*e^2)*sqrt(2*d*e + b)*arctan(2*sqrt(1/2)*x/sqrt((b
+ sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 + b*d*e^4 - b^2*d*e^
2) + (2*d^2*e^3 + d*e^4 - b*d*e^2)*sqrt(2*d*e + b)*arctan(2*sqrt(1/2)*x/sqr
t((b - sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 + b*d*e^4 - b^2
*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx=\frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{b+2\,d\,e}}\right)+\mathrm{atan}\left(\frac{b^2\,x-\frac{x\,{\left(b+2\,d\,e\right)}^2}{2}+\frac{b\,x\,\left(b+2\,d\,e\right)}{2}+2\,b\,e^2\,x^3-e^2\,x^3\,\left(b+2\,d\,e\right)}{\left(b\,d-2\,d^2\,e\right)\,\sqrt{b+2\,d\,e}}\right)}{\sqrt{b+2\,d\,e}}$$

```
[In] int((d + e*x^2)/(b*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((e*x)/(b + 2*d*e)^(1/2)) + atan((b^2*x - (x*(b + 2*d*e)^2)/2 + (b*x*(
b + 2*d*e))/2 + 2*b*e^2*x^3 - e^2*x^3*(b + 2*d*e))/((b*d - 2*d^2*e)*(b + 2*
d*e)^(1/2))))/(b + 2*d*e)^(1/2)
```

3.27 $\int \frac{d+ex^2}{d^2+fx^2+e^2x^4} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4} \, dx = -\frac{\arctan\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} + \frac{\arctan\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

```
[Out] -arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((2
*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1175, 632, 210}

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4} \, dx = \frac{\arctan\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} - \frac{\arctan\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

```
[In] Int[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2*d*e - f] - 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]) + ArcT
an[(Sqrt[2*d*e - f] + 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de-f}x}{e}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de-f}x}{e}+x^2}\,dx}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2}\,dx, x, -\frac{\sqrt{2de-f}}{e}+2x\right)}{e}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2}\,dx, x, \frac{\sqrt{2de-f}}{e}+2x\right)}{e}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. $2(82) = 164$.

Time = 0.07 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$
\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\frac{\left(2de-f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}+\frac{\left(-2de+f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\sqrt{-4d^2e^2+f^2}}
$$

```
[In] Integrate[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e - f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-
4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e + f + Sqrt[
-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/
Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{-2ex+\sqrt{2ed-f}}{\sqrt{2ed+f}}\right)}{\sqrt{2ed+f}}+\frac{\arctan\left(\frac{2ex+\sqrt{2ed-f}}{\sqrt{2ed+f}}\right)}{\sqrt{2ed+f}}$	71
risch	$-\frac{\ln\left(e\,x^{2}\sqrt{-2ed-f}+(-2ed-f)x-d\sqrt{-2ed-f}\right)}{2\sqrt{-2ed-f}}+\frac{\ln\left(e\,x^{2}\sqrt{-2ed-f}+(2ed+f)x-d\sqrt{-2ed-f}\right)}{2\sqrt{-2ed-f}}$	104

```
[In] int((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((2
*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.98

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = \left[-\frac{\sqrt{-2\,de-f}\log\left(\frac{e^{2}x^{4}-(4\,de+f)x^{2}+d^{2}-2\,(ex^{3}-dx)\sqrt{-2\,de-f}}{e^{2}x^{4}+fx^{2}+d^{2}}\right)}{2\,(2\,de+f)}, \frac{\sqrt{2\,de+f}\arctan\left(\frac{ex}{\sqrt{2\,de+f}}\right)+\sqrt{2\,de+f}\,\mathrm{arcta}}{2\,de+f}\right.$$

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e - f)*log((e^2*x^4 - (4*d*e + f)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e - f))/(e^2*x^4 + f*x^2 + d^2))/(2*d*e + f), (sqrt(2*d*e + f)
*arctan(e*x/sqrt(2*d*e + f)) + sqrt(2*d*e + f)*arctan((e^2*x^3 + (d*e + f)*
x)*sqrt(2*d*e + f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = -\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^{2}+\frac{x\left(-2de\sqrt{-\frac{1}{2de+f}}-f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^{2}+\frac{x\left(2de\sqrt{-\frac{1}{2de+f}}+f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

```
[In] integrate((e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e + f)) - f*
sqrt(-1/(2*d*e + f)))/e)/2 + sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(2*d*
e*sqrt(-1/(2*d*e + f)) + f*sqrt(-1/(2*d*e + f)))/e)/2
```

Maxima [F]

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx=\int \frac{ex^2+d}{e^2x^4+fx^2+d^2}\,dx$$

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 + f*x^2 + d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(73) = 146$.

Time = 0.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.33

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx=-\frac{(2\,d^2e^3+de^4-de^2f)\sqrt{2\,de+f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{f+\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5+de^4f-de^2f^2}-\frac{(2\,d^2e^3+de^4-de^2f)\sqrt{2\,de+f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{f-\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5+de^4f-de^2f^2}$$

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(2*d^2*e^3 + d*e^4 - d*e^2*f)*sqrt(2*d*e + f)*arctan(2*sqrt(1/2)*x/sqrt((f
 + sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 + d*e^4*f - d*e^2*f
^2) - (2*d^2*e^3 + d*e^4 - d*e^2*f)*sqrt(2*d*e + f)*arctan(2*sqrt(1/2)*x/sq
rt((f - sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 + d*e^4*f - d*
e^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\operatorname{atan}\left(\frac{f^2\,x-\frac{x\,(f+2\,d\,e)^2}{2}+\frac{f\,x\,(f+2\,d\,e)}{2}+2\,e^2\,f\,x^3-e^2\,x^3\,(f+2\,d\,e)}{(2\,d\,f-d\,(f+2\,d\,e))\,\sqrt{f+2\,d\,e}}\right)+\operatorname{atan}\left(\frac{e\,x}{\sqrt{f+2\,d\,e}}\right)}{\sqrt{f+2\,d\,e}}$$

```
[In] int((d + e*x^2)/(f*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((f^2*x - (x*(f + 2*d*e)^2)/2 + (f*x*(f + 2*d*e))/2 + 2*e^2*f*x^3 - e^
2*x^3*(f + 2*d*e))/((2*d*f - d*(f + 2*d*e))*(f + 2*d*e)^(1/2))) + atan((e*x
)/(f + 2*d*e)^(1/2)))/(f + 2*d*e)^(1/2)
```

3.28 $\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

```
[Out] arctanh((-2*e*x+(2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2)-arctanh
((2*e*x+(2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1175, 632, 212}

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

```
[In] Int[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[b + 2*d*e] - 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e] - ArcTan
h[(Sqrt[b + 2*d*e] + 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{b+2de}x}{e}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{b+2de}x}{e}+x^2}\,dx}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2}-x^2}\,dx, x, -\frac{\sqrt{b+2de}}{e}+2x\right)}{e}-\frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2}-x^2}\,dx, x, \frac{\sqrt{b+2de}}{e}+2x\right)}{e}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. $2(78) = 156$.

Time $= 0.07$ (sec) , antiderivative size $= 189$, normalized size of antiderivative $= 2.42$

$$
\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx = \frac{\frac{\left(b+2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}+\frac{\left(-b-2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}
$$

```
[In] Integrate[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt[b
^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]] + ((-b - 2*d*e + Sqrt[b
^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])/Sq
rt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\arctan\left(\frac{2ex+\sqrt{2ed+b}}{\sqrt{2ed-b}}\right)}{\sqrt{2ed-b}}-\frac{\arctan\left(\frac{-2ex+\sqrt{2ed+b}}{\sqrt{2ed-b}}\right)}{\sqrt{2ed-b}}$	75
risch	$\frac{\ln\left(e\,x^{2}\sqrt{-2ed+b}+(2ed-b)x-d\sqrt{-2ed+b}\right)}{2\sqrt{-2ed+b}}-\frac{\ln\left(e\,x^{2}\sqrt{-2ed+b}+(-2ed+b)x-d\sqrt{-2ed+b}\right)}{2\sqrt{-2ed+b}}$	92

```
[In] int((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2*d*e-b)^(1/2)*arctan((2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))-1/(2*d*e-
b)^(1/2)*arctan((-2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.26

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx$$
$$=\left[-\frac{\sqrt{-2\,de+b}\log\left(\frac{e^2x^4-(4\,de-b)x^2+d^2-2\,(ex^3-dx)\sqrt{-2\,de+b}}{e^2x^4-bx^2+d^2}\right)}{2\,(2\,de-b)},\frac{\sqrt{2\,de-b}\arctan\left(\frac{ex}{\sqrt{2\,de-b}}\right)+\sqrt{2\,de-b}\arctan\left(\right.}{2\,de-b}\right.$$

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e + b)*log((e^2*x^4 - (4*d*e - b)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e + b))/(e^2*x^4 - b*x^2 + d^2))/(2*d*e - b), (sqrt(2*d*e - b)
*arctan(e*x/sqrt(2*d*e - b)) + sqrt(2*d*e - b)*arctan((e^2*x^3 + (d*e - b)*
x)*sqrt(2*d*e - b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx = \frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b-2de}}+2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2} - \frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b-2de}}-2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

```
[In] integrate((e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(1/(b - 2*d*e)) + 2*d*e*sqr
t(1/(b - 2*d*e)))/e)/2 - sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(b*sqrt(1/
(b - 2*d*e)) - 2*d*e*sqrt(1/(b - 2*d*e)))/e)/2
```

Maxima [F]

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\int \frac{ex^2+d}{e^2x^4-bx^2+d^2}\,dx$$

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 - b*x^2 + d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(69) = 138.

Time = 0.71 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.50

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{(2\,d^2e^3+de^4+bde^2)\sqrt{2\,de-b}\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{b+\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5-bde^4-b^2de^2}+\frac{(2\,d^2e^3+de^4+bde^2)\sqrt{2\,de-b}\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{b-\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5-bde^4-b^2de^2}$$

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

```
[Out] (2*d^2*e^3 + d*e^4 + b*d*e^2)*sqrt(2*d*e - b)*arctan(2*sqrt(1/2)*x/sqrt(-(b
 + sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 - b*d*e^4 - b^2*d*e
^2) + (2*d^2*e^3 + d*e^4 + b*d*e^2)*sqrt(2*d*e - b)*arctan(2*sqrt(1/2)*x/sq
rt(-(b - sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 - b*d*e^4 - b
^2*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.38

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{b-2\,d\,e}}{d-e\,x^2}\right)}{\sqrt{b-2\,d\,e}}$$

```
[In] int((d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x)
[Out] atanh((x*(b - 2*d*e)^(1/2))/(d - e*x^2))/(b - 2*d*e)^(1/2)
```

3.29 $\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}+\frac{\arctan\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

```
[Out] -arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((2
*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1175, 632, 210}

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}-\frac{\arctan\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

```
[In] Int[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2*d*e + f] - 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]) + ArcT
an[(Sqrt[2*d*e + f] + 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de+f}x}{e}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de+f}x}{e}+x^2}\,dx}{2e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2}\,dx,x,-\frac{\sqrt{2de+f}}{e}+2x\right)}{e}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2}\,dx,x,\frac{\sqrt{2de+f}}{e}+2x\right)}{e}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(86) = 172.

Time = 0.07 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.20

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx$$

$$= \frac{\frac{\left(2de+f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}}+\frac{\left(-2de-f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\sqrt{-4d^2e^2+f^2}}$$

```
[In] Integrate[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sqrt[
-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e - f + Sqr
t[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]]
])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{-2ex+\sqrt{2ed+f}}{\sqrt{2ed-f}}\right)}{\sqrt{2ed-f}}+\frac{\arctan\left(\frac{2ex+\sqrt{2ed+f}}{\sqrt{2ed-f}}\right)}{\sqrt{2ed-f}}$	75
risch	$-\frac{\ln\left(e\,x^{2}\sqrt{-2ed+f}+(-2ed+f)x-d\sqrt{-2ed+f}\right)}{2\sqrt{-2ed+f}}+\frac{\ln\left(e\,x^{2}\sqrt{-2ed+f}+(2ed-f)x-d\sqrt{-2ed+f}\right)}{2\sqrt{-2ed+f}}$	92

```
[In] int((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((2
*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.08

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx$$
$$=\left[-\frac{\sqrt{-2\,de+f}\log\left(\frac{e^2x^4-(4\,de-f)x^2+d^2-2\,(ex^3-dx)\sqrt{-2\,de+f}}{e^2x^4-fx^2+d^2}\right)}{2\,(2\,de-f)},\right.$$
$$\left.-\frac{\sqrt{2\,de-f}\arctan\left(-\frac{ex}{\sqrt{2\,de-f}}\right)+\sqrt{2\,de-f}\arctan\left(-\frac{(e^2x^3+(de-f)x)\sqrt{2\,de-f}}{2\,d^2e-df}\right)}{2\,de-f}\right]$$

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e + f)*log((e^2*x^4 - (4*d*e - f)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e + f))/(e^2*x^4 - f*x^2 + d^2))/(2*d*e - f), -(sqrt(2*d*e - f
)*arctan(-e*x/sqrt(2*d*e - f)) + sqrt(2*d*e - f)*arctan(-(e^2*x^3 + (d*e -
f)*x)*sqrt(2*d*e - f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = -\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de-f}}+f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de-f}}-f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

```
[In] integrate((e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
[Out] -sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e - f)) + f*
sqrt(-1/(2*d*e - f)))/e)/2 + sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(2*d*
e*sqrt(-1/(2*d*e - f)) - f*sqrt(-1/(2*d*e - f)))/e)/2
```

Maxima [F]

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = \int \frac{ex^2+d}{e^2x^4-fx^2+d^2}\,dx$$

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/(e^2*x^4 - f*x^2 + d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(77) = 154$.

Time = 0.71 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.29

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = -\frac{\left(2\,d^2e^3+de^4+de^2f\right)\sqrt{2\,de-f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{f+\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5-de^4f-de^2f^2} - \frac{\left(2\,d^2e^3+de^4+de^2f\right)\sqrt{2\,de-f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{f-\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5-de^4f-de^2f^2}$$

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
[Out] -(2*d^2*e^3 + d*e^4 + d*e^2*f)*sqrt(2*d*e - f)*arctan(2*sqrt(1/2)*x/sqrt(-(
f + sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 - d*e^4*f - d*e^2*
f^2) - (2*d^2*e^3 + d*e^4 + d*e^2*f)*sqrt(2*d*e - f)*arctan(2*sqrt(1/2)*x/s
qrt(-(f - sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 - d*e^4*f -
d*e^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx=-\frac{\mathrm{atan}\left(\frac{e^2\,x^3\,\sqrt{2\,d\,e-f}-f\,x\,\sqrt{2\,d\,e-f}+d\,e\,x\,\sqrt{2\,d\,e-f}}{d\,\left(f-2\,d\,e\right)}\right)-\mathrm{atan}\left(\frac{e\,x}{\sqrt{2\,d\,e-f}}\right)}{\sqrt{2\,d\,e-f}}$$

```
[In] int((d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x)
```

```
[Out] -(atan((e^2*x^3*(2*d*e - f)^(1/2) - f*x*(2*d*e - f)^(1/2) + d*e*x*(2*d*e -
f)^(1/2))/(d*(f - 2*d*e))) - atan((e*x)/(2*d*e - f)^(1/2)))/(2*d*e - f)^(1/
2)
```

3.30 $\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx=-\frac{\log\left(d-\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}}+\frac{\log\left(d+\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e-b)^(1/2))/(2*d*e-b)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e-
b)^(1/2))/(2*d*e-b)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1178, 642}

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx=\frac{\log\left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}-\frac{\log\left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

```
[In] Int[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/Sqrt[-b + 2*d*e] + Log[d + Sqrt[-b
 + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{\sqrt{-b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{-b+2de}x}{e}-x^2}\,dx}{2\sqrt{-b+2de}} - \frac{\int \frac{\frac{\sqrt{-b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{-b+2de}x}{e}-x^2}\,dx}{2\sqrt{-b+2de}}\\ &= -\frac{\log\left(d-\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}} + \frac{\log\left(d+\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. $2(78) = 156$.

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.33

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx = \frac{\frac{\left(b+2de-\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}} - \frac{\left(b+2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

```
[In] Integrate[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e - Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b^
2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] - ((b + 2*d*e + Sqrt[b^2
- 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b
 + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\sqrt{2ed-b}\,\ln\left(-e\,x^2+x\sqrt{2ed-b}-d\right)}{-4ed+2b}-\frac{\sqrt{2ed-b}\,\ln\left(d+e\,x^2+x\sqrt{2ed-b}\right)}{-4ed+2b}$	88
risch	$-\frac{\ln\left(-e\,x^2\sqrt{2ed-b}+(2ed-b)x-d\sqrt{2ed-b}\right)}{2\sqrt{2ed-b}}+\frac{\ln\left(-e\,x^2\sqrt{2ed-b}+(-2ed+b)x-d\sqrt{2ed-b}\right)}{2\sqrt{2ed-b}}$	106

```
[In] int((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-4*d*e+2*b)*(2*d*e-b)^(1/2)*ln(-e*x^2+x*(2*d*e-b)^(1/2)-d)-1/(-4*d*e+2*b
)*(2*d*e-b)^(1/2)*ln(d+e*x^2+x*(2*d*e-b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.21

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx = \left[\frac{\log\left(\frac{e^2x^4+(4\,de-b)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\,\sqrt{2\,de-b}}, -\frac{\sqrt{-2\,de+b}\arctan\left(\frac{\sqrt{-2\,de+b}ex}{2\,de-b}\right)-\sqrt{-2\,de+b}\arctan\left(\frac{(e^2x^3-(de-b)x)\sqrt{-2\,de+b}}{2\,d^2e-bd}\right)}{2\,de-b}\right]$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e - b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - b)
)/(e^2*x^4 + b*x^2 + d^2))/sqrt(2*d*e - b), -(sqrt(-2*d*e + b)*arctan(sqrt(
-2*d*e + b)*e*x/(2*d*e - b)) - sqrt(-2*d*e + b)*arctan((e^2*x^3 - (d*e - b)
*x)*sqrt(-2*d*e + b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.55

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx = \frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b-2de}}+2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2} - \frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b-2de}}-2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

```
[In] integrate((-e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(-1/(b - 2*d*e)) + 2*d*e*sq
rt(-1/(b - 2*d*e)))/e)/2 - sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(b*sqrt(
-1/(b - 2*d*e)) - 2*d*e*sqrt(-1/(b - 2*d*e)))/e)/2
```

Maxima [F]

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx=\int -\frac{ex^2-d}{e^2x^4+bx^2+d^2}\,dx$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
[Out] -integrate((e*x^2 - d)/(e^2*x^4 + b*x^2 + d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(66)=132$.

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.42

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx=\frac{(2\,d^2e^3-de^4+bde^2)\sqrt{-2\,de+b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b+\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5+bde^4-b^2de^2}+\frac{(2\,d^2e^3-de^4+bde^2)\sqrt{-2\,de+b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b-\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5+bde^4-b^2de^2}$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")
[Out] (2*d^2*e^3 - d*e^4 + b*d*e^2)*sqrt(-2*d*e + b)*arctan(2*sqrt(1/2)*x/sqrt((b
 + sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 + b*d*e^4 - b^2*d*e
^2) + (2*d^2*e^3 - d*e^4 + b*d*e^2)*sqrt(-2*d*e + b)*arctan(2*sqrt(1/2)*x/s
qrt((b - sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 + b*d*e^4 - b
^2*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx=\frac{\mathrm{atan}\left(\frac{b\,x\,(b-2\,d\,e)+2\,b\,e^2\,x^3+4\,d^2\,e^2\,x-e^2\,x^3\,(b-2\,d\,e)+3\,d\,e\,x\,(b-2\,d\,e)}{(2\,e\,d^2+b\,d)\,\sqrt{b-2\,d\,e}}\right)-\mathrm{atan}\left(\frac{e\,x}{\sqrt{b-2\,d\,e}}\right)}{\sqrt{b-2\,d\,e}}$$

```
[In] int((d - e*x^2)/(b*x^2 + d^2 + e^2*x^4),x)
[Out] (atan((b*x*(b - 2*d*e) + 2*b*e^2*x^3 + 4*d^2*e^2*x - e^2*x^3*(b - 2*d*e) +
3*d*e*x*(b - 2*d*e))/((b*d + 2*d^2*e)*(b - 2*d*e)^(1/2))) - atan((e*x)/(b -
 2*d*e)^(1/2)))/(b - 2*d*e)^(1/2)
```

3.31 $\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx=-\frac{\log\left(d-\sqrt{2de-f}x+ex^2\right)}{2\sqrt{2de-f}}+\frac{\log\left(d+\sqrt{2de-f}x+ex^2\right)}{2\sqrt{2de-f}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e-
f)^(1/2))/(2*d*e-f)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1178, 642}

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx=\frac{\log\left(x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}-\frac{\log\left(-x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}$$

```
[In] Int[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[2*d*e - f]*x + e*x^2]/Sqrt[2*d*e - f] + Log[d + Sqrt[2*d*
e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\frac{\int \frac{\frac{\sqrt{2de-f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de-f}x}{e}-x^2}\,dx}{2\sqrt{2de-f}}-\frac{\int \frac{\frac{\sqrt{2de-f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de-f}x}{e}-x^2}\,dx}{2\sqrt{2de-f}}\\&=-\frac{\log\left(d-\sqrt{2de-f}x+ex^2\right)}{2\sqrt{2de-f}}+\frac{\log\left(d+\sqrt{2de-f}x+ex^2\right)}{2\sqrt{2de-f}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(78) = 156.

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.33

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\frac{\left(2de+f-\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}-\frac{\left(2de+f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\sqrt{-4d^2e^2+f^2}}$$

```
[In] Integrate[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-
4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] - ((2*d*e + f + Sqrt[-
4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/S
qrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\ln\left(d+e\,x^{2}+x\sqrt{2ed-f}\right)}{2\sqrt{2ed-f}}-\frac{\ln\left(-e\,x^{2}+x\sqrt{2ed-f}-d\right)}{2\sqrt{2ed-f}}$	69
risch	$\frac{\ln\left(\sqrt{2ed-f}\,e\,x^{2}+\left(2ed-f\right)x+\sqrt{2ed-f}\,d\right)}{2\sqrt{2ed-f}}-\frac{\ln\left(\sqrt{2ed-f}\,e\,x^{2}+\left(-2ed+f\right)x+\sqrt{2ed-f}\,d\right)}{2\sqrt{2ed-f}}$	102

```
[In] int((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)-1/2/(2*d*e-f)^(1/2)*ln(-e
*x^2+x*(2*d*e-f)^(1/2)-d)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.22

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx$$
$$=\left[\frac{\log\left(\frac{e^2x^4+(4\,de-f)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de-f}}{e^2x^4+fx^2+d^2}\right)}{2\,\sqrt{2\,de-f}},\frac{\sqrt{-2\,de+f}\arctan\left(-\frac{\sqrt{-2\,de+f}ex}{2\,de-f}\right)-\sqrt{-2\,de+f}\arctan\left(-\right.}{2\,de-f}\right.$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e - f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - f)
)/(e^2*x^4 + f*x^2 + d^2))/sqrt(2*d*e - f), (sqrt(-2*d*e + f)*arctan(-sqrt(
-2*d*e + f)*e*x/(2*d*e - f)) - sqrt(-2*d*e + f)*arctan(-(e^2*x^3 - (d*e - f
)*x)*sqrt(-2*d*e + f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx=-\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{\frac{1}{2de-f}}+f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{\frac{1}{2de-f}}-f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

```
[In] integrate((-e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e - f)) + f*sqr
t(1/(2*d*e - f)))/e)/2 + sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(2*d*e*sqrt
(1/(2*d*e - f)) - f*sqrt(1/(2*d*e - f)))/e)/2
```

Maxima [F]

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = \int -\frac{ex^2-d}{e^2x^4+fx^2+d^2}\,dx$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 + f*x^2 + d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(66) = 132$.

Time = 0.70 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.44

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = -\frac{(2\,d^2e^3-de^4+de^2f)\sqrt{-2\,de+f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{f+\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5+de^4f-de^2f^2} + \frac{(2\,d^2e^3-de^4+de^2f)\sqrt{-2\,de+f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{f-\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5+de^4f-de^2f^2}$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(2*d^2*e^3 - d*e^4 + d*e^2*f)*sqrt(-2*d*e + f)*arctan(2*sqrt(1/2)*x/sqrt((
f + sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 + d*e^4*f - d*e^2*
f^2) + (2*d^2*e^3 - d*e^4 + d*e^2*f)*sqrt(-2*d*e + f)*arctan(2*sqrt(1/2)*x/
sqrt((f - sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 + d*e^4*f -
d*e^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\mathrm{atan}\left(\frac{f\,x\,1\mathrm{i}-d\,e\,x\,2\mathrm{i}}{d\,\sqrt{2\,d\,e-f}+e\,x^2\,\sqrt{2\,d\,e-f}}\right)\,1\mathrm{i}}{\sqrt{2\,d\,e-f}}$$

```
[In] int((d - e*x^2)/(f*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((f*x*1i - d*e*x*2i)/(d*(2*d*e - f)^(1/2) + e*x^2*(2*d*e - f)^(1/2)))*
1i)/(2*d*e - f)^(1/2)
```

3.32 $\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx=-\frac{\log\left(d-\sqrt{b+2de}x+ex^2\right)}{2\sqrt{b+2de}}+\frac{\log\left(d+\sqrt{b+2de}x+ex^2\right)}{2\sqrt{b+2de}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e+
b)^(1/2))/(2*d*e+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1178, 642}

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{\log\left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}-\frac{\log\left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

```
[In] Int[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/Sqrt[b + 2*d*e] + Log[d + Sqrt[b +
2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{\sqrt{b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{b+2de}x}{e}-x^2}\,dx}{2\sqrt{b+2de}}-\frac{\int \frac{\frac{\sqrt{b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{b+2de}x}{e}-x^2}\,dx}{2\sqrt{b+2de}}\\ &= -\frac{\log\left(d-\sqrt{b+2de}x+ex^2\right)}{2\sqrt{b+2de}}+\frac{\log\left(d+\sqrt{b+2de}x+ex^2\right)}{2\sqrt{b+2de}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(70) = 140.

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.71

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx = \frac{-\frac{\left(b-2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}+\frac{\left(b-2de-\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

```
[In] Integrate[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (-(((b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt
[b^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]]) + ((b - 2*d*e - Sqrt
[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])/
Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\ln\left(d+e\,x^{2}+x\sqrt{2ed+b}\right)}{2\sqrt{2ed+b}}-\frac{\ln\left(-e\,x^{2}+x\sqrt{2ed+b}-d\right)}{2\sqrt{2ed+b}}$	61
risch	$\frac{\ln\left(\sqrt{2ed+b}\,e\,x^{2}+\left(2ed+b\right)x+\sqrt{2ed+b}\,d\right)}{2\sqrt{2ed+b}}-\frac{\ln\left(\sqrt{2ed+b}\,e\,x^{2}+\left(-2ed-b\right)x+\sqrt{2ed+b}\,d\right)}{2\sqrt{2ed+b}}$	90

```
[In] int((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)-1/2/(2*d*e+b)^(1/2)*ln(-e
*x^2+x*(2*d*e+b)^(1/2)-d)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.40

$$
\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx = \left[\frac{\log\left(\frac{e^2x^4+(4\,de+b)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de+b}}{e^2x^4-bx^2+d^2}\right)}{2\,\sqrt{2\,de+b}},\right.
\left.-\frac{\sqrt{-2\,de-b}\arctan\left(\frac{\sqrt{-2\,de-b}ex}{2\,de+b}\right)-\sqrt{-2\,de-b}\arctan\left(\frac{(e^2x^3-(de+b)x)\sqrt{-2\,de-b}}{2\,d^2e+bd}\right)}{2\,de+b}\right]
$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e + b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e + b)
)/(e^2*x^4 - b*x^2 + d^2))/sqrt(2*d*e + b), -(sqrt(-2*d*e - b)*arctan(sqrt(
-2*d*e - b)*e*x/(2*d*e + b)) - sqrt(-2*d*e - b)*arctan((e^2*x^3 - (d*e + b)
*x)*sqrt(-2*d*e - b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.60

$$
\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx = -\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b+2de}}-2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}
+\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b+2de}}+2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}
$$

```
[In] integrate((-e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] -sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(1/(b + 2*d*e)) - 2*d*e*sqr
t(1/(b + 2*d*e)))/e)/2 + sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(b*sqrt(1/(
b + 2*d*e)) + 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2
```

Maxima [F]

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx=\int -\frac{ex^2-d}{e^2x^4-bx^2+d^2}\,dx$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 - b*x^2 + d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(58) = 116$.

Time = 0.77 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.84

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{(2\,d^2e^3-de^4-bde^2)\sqrt{-2\,de-b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{b+\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5-bde^4-b^2de^2}+\frac{(2\,d^2e^3-de^4-bde^2)\sqrt{-2\,de-b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{b-\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5-bde^4-b^2de^2}$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

```
[Out] (2*d^2*e^3 - d*e^4 - b*d*e^2)*sqrt(-2*d*e - b)*arctan(2*sqrt(1/2)*x/sqrt(-(
b + sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 - b*d*e^4 - b^2*d*
e^2) + (2*d^2*e^3 - d*e^4 - b*d*e^2)*sqrt(-2*d*e - b)*arctan(2*sqrt(1/2)*x/
sqrt(-(b - sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 - b*d*e^4 -
 b^2*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{\operatorname{atanh}\left(\frac{x\sqrt{b+2\,d\,e}}{e\,x^2+d}\right)}{\sqrt{b+2\,d\,e}}$$

```
[In] int((d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x)
```

```
[Out] atanh((x*(b + 2*d*e)^(1/2))/(d + e*x^2))/(b + 2*d*e)^(1/2)
```

3.33 $\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx=-\frac{\log\left(d-\sqrt{2de+f}x+ex^2\right)}{2\sqrt{2de+f}}+\frac{\log\left(d+\sqrt{2de+f}x+ex^2\right)}{2\sqrt{2de+f}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e+
f)^(1/2))/(2*d*e+f)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1178, 642}

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx=\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}-\frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

```
[In] Int[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[2*d*e + f]*x + e*x^2]/Sqrt[2*d*e + f] + Log[d + Sqrt[2*d*
e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{\sqrt{2de+f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de+f}x}{e}-x^2}\,dx}{2\sqrt{2de+f}} - \frac{\int \frac{\frac{\sqrt{2de+f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de+f}x}{e}-x^2}\,dx}{2\sqrt{2de+f}}\\ &= -\frac{\log\left(d-\sqrt{2de+f}x+ex^2\right)}{2\sqrt{2de+f}} + \frac{\log\left(d+\sqrt{2de+f}x+ex^2\right)}{2\sqrt{2de+f}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(70) = 140.

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.71

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = \frac{-\frac{\left(-2de+f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}} + \frac{\left(-2de+f-\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\sqrt{-4d^2e^2+f^2}}$$

```
[In] Integrate[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (-(((-2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sq
rt[-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]]) + ((-2*d*e + f -
 Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f
^2]]])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\ln\left(d+e\,x^2+x\sqrt{2ed+f}\right)}{2\sqrt{2ed+f}}-\frac{\ln\left(-e\,x^2+x\sqrt{2ed+f}-d\right)}{2\sqrt{2ed+f}}$	61
risch	$\frac{\ln\left(\sqrt{2ed+f}\,e\,x^2+(2ed+f)x+\sqrt{2ed+f}\,d\right)}{2\sqrt{2ed+f}}-\frac{\ln\left(\sqrt{2ed+f}\,e\,x^2+(-2ed-f)x+\sqrt{2ed+f}\,d\right)}{2\sqrt{2ed+f}}$	90

```
[In] int((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)-1/2/(2*d*e+f)^(1/2)*ln(-e
*x^2+x*(2*d*e+f)^(1/2)-d)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.40

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = \left[\frac{\log\left(\frac{e^2x^4+(4\,de+f)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de+f}}{e^2x^4-fx^2+d^2}\right)}{2\,\sqrt{2\,de+f}}, -\frac{\sqrt{-2\,de-f}\arctan\left(\frac{\sqrt{-2\,de-f}ex}{2\,de+f}\right)-\sqrt{-2\,de-f}\arctan\left(\frac{(e^2x^3-(de+f)x)\sqrt{-2\,de-f}}{2\,d^2e+df}\right)}{2\,de+f}\right]$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e + f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e + f)
)/(e^2*x^4 - f*x^2 + d^2))/sqrt(2*d*e + f), -(sqrt(-2*d*e - f)*arctan(sqrt(
-2*d*e - f)*e*x/(2*d*e + f)) - sqrt(-2*d*e - f)*arctan((e^2*x^3 - (d*e + f)
*x)*sqrt(-2*d*e - f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.60

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = -\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{\frac{1}{2de+f}}-f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{\frac{1}{2de+f}}+f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

```
[In] integrate((-e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e + f)) - f*sqr
t(1/(2*d*e + f)))/e)/2 + sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(2*d*e*sqrt
(1/(2*d*e + f)) + f*sqrt(1/(2*d*e + f)))/e)/2
```

Maxima [F]

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = \int -\frac{ex^2-d}{e^2x^4-fx^2+d^2}\,dx$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 - f*x^2 + d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(58) = 116$.

Time $= 0.69$ (sec) , antiderivative size $= 200$, normalized size of antiderivative $= 2.86$

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = \frac{(2\,d^2e^3-de^4-de^2f)\sqrt{-2\,de-f}\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{f+\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5-de^4f-de^2f^2} - \frac{(2\,d^2e^3-de^4-de^2f)\sqrt{-2\,de-f}\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{f-\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5-de^4f-de^2f^2}$$

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
```

```
[Out] (2*d^2*e^3 - d*e^4 - d*e^2*f)*sqrt(-2*d*e - f)*arctan(2*sqrt(1/2)*x/sqrt(-(
f + sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 - d*e^4*f - d*e^2*
f^2) - (2*d^2*e^3 - d*e^4 - d*e^2*f)*sqrt(-2*d*e - f)*arctan(2*sqrt(1/2)*x/
sqrt(-(f - sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 - d*e^4*f -
 d*e^2*f^2)
```

Mupad [B] (verification not implemented)

Time $= 13.20$ (sec) , antiderivative size $= 29$, normalized size of antiderivative $= 0.41$

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{x\,\sqrt{f+2\,d\,e}}{e\,x^2+d}\right)}{\sqrt{f+2\,d\,e}}$$

```
[In] int((d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x)
```

```
[Out] atanh((x*(f + 2*d*e)^(1/2))/(d + e*x^2))/(f + 2*d*e)^(1/2)
```

3.34 $\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 134

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = -\frac{e^{3/2}\log\left(\sqrt{c}d-\sqrt{e}\sqrt{2cd-be}x+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}} + \frac{e^{3/2}\log\left(\sqrt{c}d+\sqrt{e}\sqrt{2cd-be}x+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}$$

```
[Out] -1/2*e^(3/2)*ln(d*c^(1/2)+e*x^2*c^(1/2)-x*e^(1/2)*(-b*e+2*c*d)^(1/2))/c^(1/
2)/(-b*e+2*c*d)^(1/2)+1/2*e^(3/2)*ln(d*c^(1/2)+e*x^2*c^(1/2)+x*e^(1/2)*(-b*
e+2*c*d)^(1/2))/c^(1/2)/(-b*e+2*c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1178, 642}

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{e^{3/2}\log\left(\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}} - \frac{e^{3/2}\log\left(-\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}$$

```
[In] Int[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(e^(3/2)*Log[Sqrt[c]*d - Sqrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e*x^2])
/(Sqrt[c]*Sqrt[2*c*d - b*e]) + (e^(3/2)*Log[Sqrt[c]*d + Sqrt[e]*Sqrt[2*c*d
- b*e]*x + Sqrt[c]*e*x^2])/(2*Sqrt[c]*Sqrt[2*c*d - b*e])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^{3/2}\int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x}{-\frac{d}{e}-\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}-x^2}\,dx}{2\sqrt{c}\sqrt{2cd-be}}-\frac{e^{3/2}\int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}-2x}{-\frac{d}{e}+\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}-x^2}\,dx}{2\sqrt{c}\sqrt{2cd-be}}\\ &= -\frac{e^{3/2}\log\left(\sqrt{c}d-\sqrt{e}\sqrt{2cd-be}x+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}+\frac{e^{3/2}\log\left(\sqrt{c}d+\sqrt{e}\sqrt{2cd-be}x+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.87

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{e^{3/2}\left(-\frac{\left(-2cd-be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}-\frac{\left(2cd+be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4c^2d^2+b^2e^2}}$$

```
[In] Integrate[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(-(((-2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sq
rt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-
4*c^2*d^2 + b^2*e^2]]) - ((2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[
b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*
e^2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\sqrt{-c(be-2cd)e}\,e\ln\left(-c\,x^{2}e+\sqrt{-c(be-2cd)e}\,x-cd\right)}{2c(be-2cd)}-\frac{\sqrt{-c(be-2cd)e}\,e\ln\left(-c\,x^{2}e-\sqrt{-c(be-2cd)e}\,x-cd\right)}{2c(be-2cd)}$
default	$4e^{4}c\left(\frac{\left(-b\,e^{2}-2dce-\sqrt{e^{2}(be-2cd)(be+2cd)}\right)\sqrt{2}\,\arctan\left(\frac{cxe\sqrt{2}}{\sqrt{c\left(b\,e^{2}+\sqrt{e^{2}(be-2cd)(be+2cd)}\right)}}\right)}{8\sqrt{e^{2}(be-2cd)(be+2cd)}\,c\,e^{2}\sqrt{c\left(b\,e^{2}+\sqrt{e^{2}(be-2cd)(be+2cd)}\right)}}-\frac{\left(b\,e^{2}+2dce-\sqrt{e^{2}(be-2cd)(be+2cd}\right.}{8\sqrt{e^{2}(be-2cd)(be+2cd)}}\right.$

```
[In] int((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-c*(b*e-2*c*d)*e)^(1/2)/c/(b*e-2*c*d)*e*ln(-c*x^2*e+(-c*(b*e-2*c*d)*e)
^(1/2)*x-c*d)-1/2*(-c*(b*e-2*c*d)*e)^(1/2)/c/(b*e-2*c*d)*e*ln(-c*x^2*e-(-c*
(b*e-2*c*d)*e)^(1/2)*x-c*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.82

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$$

$$=\left[\frac{1}{2}\,e\sqrt{\frac{e}{2\,c^2d-bce}}\log\left(\frac{ce^2x^4+cd^2+(4\,cde-be^2)x^2+2\,((2\,c^2de-bce^2)x^3+(2\,c^2d^2-bcde)x)\sqrt{\frac{e}{2\,c^2d-bce}}}{ce^2x^4+be^2x^2+cd^2}\right.\right.$$

$$-e\sqrt{-\frac{e}{2\,c^2d-bce}}\arctan\left(cx\sqrt{-\frac{e}{2\,c^2d-bce}}\right)$$

$$\left.+\,e\sqrt{-\frac{e}{2\,c^2d-bce}}\arctan\left(\frac{(cex^3-(cd-be)x)\sqrt{-\frac{e}{2\,c^2d-bce}}}{d}\right)\right]$$

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(e/(2*c^2*d - b*c*e))*log((c*e^2*x^4 + c*d^2 + (4*c*d*e - b*e^2)
*x^2 + 2*((2*c^2*d*e - b*c*e^2)*x^3 + (2*c^2*d^2 - b*c*d*e)*x)*sqrt(e/(2*c^
2*d - b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), -e*sqrt(-e/(2*c^2*d - b*c*
e))*arctan(c*x*sqrt(-e/(2*c^2*d - b*c*e))) + e*sqrt(-e/(2*c^2*d - b*c*e))*a
rctan((c*e*x^3 - (c*d - b*e)*x)*sqrt(-e/(2*c^2*d - b*c*e))/d)]
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be-2cd)}}+2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2} - \frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be-2cd)}}-2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}$$

```
[In] integrate((-e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] sqrt(-e**3/(c*(b*e - 2*c*d)))*log(d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e -
 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2 - sqrt(-e**3/(c*(b
*e - 2*c*d)))*log(d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e - 2*c*d))) - 2*c*d
*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2
```

Maxima [F]

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \int -\frac{ex^2-d}{cx^4+bx^2+\frac{cd^2}{e^2}}\,dx$$

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6331 vs. $2(102) = 204$.

Time = 1.01 (sec) , antiderivative size = 6331, normalized size of antiderivative = 47.25

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*e
 + b*c*e^2)*b^2*c^2*e^7*sgn(c)*sgn(e) - 12*b*c^4*d^2*e^6 + 3*b^3*c^2*e^8 +
4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4*sgn(c)
*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*
e^4*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^
```

```
3*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*
b^3*e^6*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e
^2)*b^2*c*e^6*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e +
 b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d^2*
e^5 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6 + sqrt(-4*c^2*d^2 + b^2*e^2)
*b^3*c*e^7 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^7 - sqrt(-2*c^2*d*e + b
*c*e^2)*b^2*c^2*e^7 - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2
)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^5*sgn(c)*sgn(e) + 4*sqrt(-4*c^2*d^2 + b^
2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4 - sqrt(-4*c^2*d^2 + b^2*e^2
)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^6 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2
*c^2*d*e + b*c*e^2)*b^2*c*e^6 + (4*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^2*e^3*sg
n(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^4*sgn(c)*sgn(e) + sqrt(
2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5*sgn(c)*sgn(e) - 24*c^5*d^3*e^3 + 12*b*c^4*
d^2*e^4 + 6*b^2*c^3*d*e^5 - 3*b^3*c^2*e^6 + 8*sqrt(-4*c^2*d^2 + b^2*e^2)*sq
rt(2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(c)*sgn(e) + 8*sqrt(2*c^2*d*e + b*c*e^
2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 +
 b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2*sgn(c)*sgn(e) - 4*sqrt(2*
c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2*sgn(c)*sgn(e) -
 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^2*c*d*e^3*sgn(c)*
sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*d*e^3
*sgn(c)*sgn(e) + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c
^2*d*e^3*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*
e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e
 + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*
e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^
2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt
(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(e) - 8*sqrt(-4*c^2*d^2 + b^2*e^
2)*c^4*d^3*e^2 + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d^2*e^3 - 4*sqrt(-2*c^2
*d*e + b*c*e^2)*c^4*d^2*e^3 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*d*e^4 -
4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^4 - sqrt(-4*c^2*d^2 + b^2*e^2)*b^3*c
*e^5 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^5 + sqrt(-2*c^2*d*e + b*c*e^2
)*b^2*c^2*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt
(-2*c^2*d*e + b*c*e^2)*c^2*d*e^2*sgn(c)*sgn(e) + 3*sqrt(-4*c^2*d^2 + b^2*e^
2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^3*sgn(c)*sgn(
e) + 8*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*e - 4*
sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2 - 2*sqr
t(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*d*e^3 + 4*sqrt(-4*
c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3 + sqrt(-4*c^2*d^2
 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4 - 2*sqrt(-4*c^2*d^2 + b^2*e^
2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4)*e^2 + 2*(8*sqrt(2*c^2*d*e + b*c*e^
2)*c^4*d^4*e^2*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d^3*e^3*sg
n(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3*sgn(c)*sgn(e) - 2*sqr
t(2*c^2*d*e + b*c*e^2)*b^2*c^2*d^2*e^4*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b
*c*e^2)*c^4*d^2*e^4*sgn(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5*s
gn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*d*e^5*sgn(c)*sgn(e) + sqrt
```

```
(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^5*sgn(c)*sgn(e) - 24*c^5*d^4*e^3 + 12*b*c^4
*d^3*e^4 + 6*b^2*c^3*d^2*e^5 - 3*b^3*c^2*d*e^6 + 2*sqrt(-4*c^2*d^2 + b^2*e^
2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^2*e^3*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e +
 b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^2*e^3*sgn(c)*sgn(e) - sqrt(-4*c^
2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e) - sqrt
(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e)
+ 8*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^4*e^2 - 4*sqrt(-2*c^2*d*e + b*c*e^2)*b
*c^3*d^3*e^3 - 4*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3 - 2*sqrt(-2*c^2*d*e
 + b*c*e^2)*b^2*c^2*d^2*e^4 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d^2*e^4 -
2*sqrt(-4*c^2*d^2 + b^2*e^2)*c^4*d^2*e^4 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*c^4
*d^2*e^4 + sqrt(-2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5 - sqrt(-2*c^2*d*e + b*c*e
^2)*b^2*c^2*d*e^5 + sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^5 + sqrt(-2*c^2*d*
e + b*c*e^2)*b*c^3*d*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*
c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^2*d^2*e^2*sgn(c)*sgn(e) + 3*sqrt(-4*c^2
*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*d*
e^3*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)
*c^3*d^2*e^3 - sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*
d*e^4)*abs(e))*arctan(2*sqrt(1/2)*x/sqrt((b*e^2 + sqrt(-4*c^2*d^2*e^2 + b^2
*e^4))/(c*e^2)))/((8*c^5*d^5 - 4*b*c^4*d^4*e - 2*b^2*c^3*d^3*e^2 + 4*b*c^4*
d^3*e^2 - 2*c^5*d^3*e^2 + b^3*c^2*d^2*e^3 - 2*b^2*c^3*d^2*e^3 + b*c^4*d^2*e
^3)*e^2*abs(c)) - 1/8*(2*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6*sgn(c)*sgn(e
) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^7*sgn(c)*sgn(e) - 12*b*c^4*d^2*e^6
+ 3*b^3*c^2*e^8 - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*
c^2*d^2*e^4*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b
*c*e^2)*b*c^2*d^2*e^4*sgn(c)*sgn(e) + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2
*d*e + b*c*e^2)*b^3*e^6*sgn(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c
^2*d*e + b*c*e^2)*b^3*e^6*sgn(c)*sgn(e) - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt
(2*c^2*d*e + b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)
*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) + 4*sqrt(-4*c^2*d^2 + b
^2*e^2)*b*c^3*d^2*e^5 + 2*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6 - sqrt(-4*
c^2*d^2 + b^2*e^2)*b^3*c*e^7 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^7 + s
qrt(-2*c^2*d*e + b*c*e^2)*b^2*c^2*e^7 - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2
*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^5*sgn(c)*sgn(e) + 4*sq
rt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4 - sqrt(-4
*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^6 + 2*sqrt(-4*c^2*d^2
+ b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^6 + (4*sqrt(2*c^2*d*e + b*c*e
^2)*c^4*d^2*e^3*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^4*sgn
(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5*sgn(c)*sgn(e) - 24*c^5*d
^3*e^3 + 12*b*c^4*d^2*e^4 + 6*b^2*c^3*d*e^5 - 3*b^3*c^2*e^6 - 8*sqrt(-4*c^2
*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(c)*sgn(e) - 8*sqrt(
2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(c)*sgn(e) + 4
*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2*sgn(c)*
sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e
^2*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b
^2*c*d*e^3*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*
```

```
c*e^2)*b^2*c*d*e^3*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*
d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt
(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*e^
2)*sqrt(2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e
^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 +
b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d
*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(e) + 8*sqrt(-
4*c^2*d^2 + b^2*e^2)*c^4*d^3*e^2 - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d^2*e
^3 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^2*e^3 - 2*sqrt(-4*c^2*d^2 + b^2*e^2
)*b^2*c^2*d*e^4 + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^4 + sqrt(-4*c^2*d^
2 + b^2*e^2)*b^3*c*e^5 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^5 - sqrt(-2
*c^2*d*e + b*c*e^2)*b^2*c^2*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d
*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^2*d*e^2*sgn(c)*sgn(e) + 3*sqrt(-
4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b
*c*e^3*sgn(c)*sgn(e) + 8*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e
^2)*c^3*d^3*e - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c
^2*d^2*e^2 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*
d*e^3 + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3
 + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4 - 2*sqrt(-
4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4)*e^2 + 2*(8*sqrt(
2*c^2*d*e + b*c*e^2)*c^4*d^4*e^2*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2
)*b*c^3*d^3*e^3*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3*sgn
(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*d^2*e^4*sgn(c)*sgn(e) - 2*
sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^2*e^4*sgn(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*
e^2)*b^3*c*d*e^5*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*d*e^5*sg
n(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^5*sgn(c)*sgn(e) - 24*c^5*
d^4*e^3 + 12*b*c^4*d^3*e^4 + 6*b^2*c^3*d^2*e^5 - 3*b^3*c^2*d*e^6 - 2*sqrt(-
4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^2*e^3*sgn(c)*sgn(e) -
2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^2*e^3*sgn(c)*s
gn(e) + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d*e^4*sg
n(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*
e^4*sgn(c)*sgn(e) - 8*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^4*e^2 + 4*sqrt(-2*c^
2*d*e + b*c*e^2)*b*c^3*d^3*e^3 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3 +
 2*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c^2*d^2*e^4 - 4*sqrt(-2*c^2*d*e + b*c*e^2
)*b*c^3*d^2*e^4 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*c^4*d^2*e^4 + 2*sqrt(-2*c^2*
d*e + b*c*e^2)*c^4*d^2*e^4 - sqrt(-2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5 + sqrt(
-2*c^2*d*e + b*c*e^2)*b^2*c^2*d*e^5 - sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^
5 - sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*s
qrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^2*d^2*e^2*sgn(c)*sgn(
e) + 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e
 + b*c*e^2)*b*c*d*e^3*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*
c^2*d*e + b*c*e^2)*c^3*d^2*e^3 - sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e
 + b*c*e^2)*b*c^2*d*e^4)*abs(e))*arctan(2*sqrt(1/2)*x/sqrt((b*e^2 - sqrt(-4
*c^2*d^2*e^2 + b^2*e^4))/(c*e^2)))/((8*c^5*d^5 - 4*b*c^4*d^4*e - 2*b^2*c^3*
d^3*e^2 + 4*b*c^4*d^3*e^2 - 2*c^5*d^3*e^2 + b^3*c^2*d^2*e^3 - 2*b^2*c^3*d^2
```

```
*e^3 + b*c^4*d^2*e^3)*e^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.96

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = -\frac{e^{3/2}\left(\mathrm{atan}\left(\frac{\sqrt{e}\,x\,\sqrt{b\,c\,e-2\,c^2\,d}}{b\,e-2\,c\,d}\right)+\mathrm{atan}\left(\frac{c\,e^{3/2}\,x^3\,\sqrt{b\,c\,e-2\,c^2\,d}+b\,e^{3/2}\,x\,\sqrt{b\,c\,e-2\,c^2\,d}-c\,d\,\sqrt{e}\,x\,\sqrt{b\,c\,e-2\,c^2\,d}}{d\,\left(2\,c^2\,d-b\,c\,e\right)}\right)\right)}{\sqrt{b\,c\,e-2\,c^2\,d}}$$

```
[In] int((d - e*x^2)/(b*x^2 + c*x^4 + (c*d^2)/e^2),x)
```

```
[Out] -(e^(3/2)*(atan((e^(1/2)*x*(b*c*e - 2*c^2*d)^(1/2))/(b*e - 2*c*d)) + atan((
c*e^(3/2)*x^3*(b*c*e - 2*c^2*d)^(1/2) + b*e^(3/2)*x*(b*c*e - 2*c^2*d)^(1/2)
 - c*d*e^(1/2)*x*(b*c*e - 2*c^2*d)^(1/2))/(d*(2*c^2*d - b*c*e)))))/(b*c*e -
 2*c^2*d)^(1/2)
```

3.35 $\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = -\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

```
[Out] -e^(3/2)*arctan((-2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d)^(1/2))/(b*e+2*c*d)^(1/2)
)/c^(1/2)/(b*e+2*c*d)^(1/2)+e^(3/2)*arctan((2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d
)^(1/2))/(b*e+2*c*d)^(1/2))/c^(1/2)/(b*e+2*c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1175, 632, 210}

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

```
[In] Int[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c}+\frac{e\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c}\\
&=-\frac{e\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx,x,-\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c}-\frac{e\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx,x,\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c}\\
&=-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.91

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$$

$$=\frac{e^{3/2}\left(\frac{\left(2cd-be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}+\frac{\left(-2cd+be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4c^2d^2+b^2e^2}}$$

```
[In] Integrate[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c
```

```
^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e
 + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2
])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\sqrt{-c(be+2cd)e}\,e\ln\left(-c\,x^2e-\sqrt{-c(be+2cd)e}\,x+cd\right)}{2c(be+2cd)}-\frac{\sqrt{-c(be+2cd)e}\,e\ln\left(-c\,x^2e+\sqrt{-c(be+2cd)e}\,x+cd\right)}{2c(be+2cd)}$
default	$4e^4c\left(\frac{\left(b\,e^2-2dce+\sqrt{e^2(be-2cd)(be+2cd)}\right)\sqrt{2}\,\arctan\left(\frac{cxe\sqrt{2}}{\sqrt{c\left(b\,e^2+\sqrt{e^2(be-2cd)(be+2cd)}\right)}}\right)}{8\sqrt{e^2(be-2cd)(be+2cd)}\,c\,e^2\sqrt{c\left(b\,e^2+\sqrt{e^2(be-2cd)(be+2cd)}\right)}}-\frac{\left(-b\,e^2+2dce+\sqrt{e^2(be-2cd)(be+2cd}\right.}{8\sqrt{e^2(be-2cd)(be+2cd)}\,c}\right.$

```
[In] int((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-c*(b*e+2*c*d)*e)^(1/2)/c/(b*e+2*c*d)*e*ln(-c*x^2*e-(-c*(b*e+2*c*d)*e)
^(1/2)*x+c*d)-1/2*(-c*(b*e+2*c*d)*e)^(1/2)/c/(b*e+2*c*d)*e*ln(-c*x^2*e+(-c*
(b*e+2*c*d)*e)^(1/2)*x+c*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

$$\int\frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$$
$$=\left[\frac{1}{2}\,e\sqrt{-\frac{e}{2\,c^2d+bce}}\log\left(\frac{ce^2x^4+cd^2-(4\,cde+be^2)x^2+2\,((2\,c^2de+bce^2)x^3-(2\,c^2d^2+bcde)x)\sqrt{-\frac{e}{2\,c^2d-}}}{ce^2x^4+be^2x^2+cd^2}\right.\right.$$
$$\left.+\,e\sqrt{\frac{e}{2\,c^2d+bce}}\arctan\left(\frac{(cex^3+(cd+be)x)\sqrt{\frac{e}{2\,c^2d+bce}}}{d}\right)\right]$$

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^2
)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(2*
c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), e*sqrt(e/(2*c^2*d + b*c*
e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))*arc
tan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.23

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = -\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2} + \frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

```
[In] integrate((e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e
 + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*
(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*
c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

Maxima [F]

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \int \frac{ex^2+d}{cx^4+bx^2+\frac{cd^2}{e^2}}\,dx$$

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6341 vs. 2(105) = 210.

Time = 1.04 (sec) , antiderivative size = 6341, normalized size of antiderivative = 48.78

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*e
 + b*c*e^2)*b^2*c^2*e^7*sgn(c)*sgn(e) - 12*b*c^4*d^2*e^6 + 3*b^3*c^2*e^8 +
4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4*sgn(c)
*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*
e^4*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^
```

```
3*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*
b^3*e^6*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e
^2)*b^2*c*e^6*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e +
 b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d^2*
e^5 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6 + sqrt(-4*c^2*d^2 + b^2*e^2)
*b^3*c*e^7 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^7 - sqrt(-2*c^2*d*e + b
*c*e^2)*b^2*c^2*e^7 - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2
)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^5*sgn(c)*sgn(e) + 4*sqrt(-4*c^2*d^2 + b^
2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4 - sqrt(-4*c^2*d^2 + b^2*e^2
)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^6 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2
*c^2*d*e + b*c*e^2)*b^2*c*e^6 - (4*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^2*e^3*sg
n(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5*sgn(c)*sgn(e) - 24*c^5*
d^3*e^3 - 12*b*c^4*d^2*e^4 + 6*b^2*c^3*d*e^5 + 3*b^3*c^2*e^6 + 8*sqrt(-4*c^
2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(c)*sgn(e) + 8*sqrt
(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(c)*sgn(e) +
4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2*sgn(c)
*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*
e^2*sgn(c)*sgn(e) - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*
b^2*c*d*e^3*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b
*c*e^2)*b^2*c*d*e^3*sgn(c)*sgn(e) + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2
*d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqr
t(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*e
^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*
e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 +
 b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(e) + 2*sqrt(2*c^2*
d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(e) - 8*sqrt(
-4*c^2*d^2 + b^2*e^2)*c^4*d^3*e^2 - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d^2*
e^3 - 4*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^2*e^3 + 2*sqrt(-4*c^2*d^2 + b^2*e^
2)*b^2*c^2*d*e^4 - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^4 - 4*sqrt(-2*c^2
*d*e + b*c*e^2)*b*c^3*d*e^4 + sqrt(-4*c^2*d^2 + b^2*e^2)*b^3*c*e^5 - 2*sqrt
(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^5 - sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5
 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e +
 b*c*e^2)*c^2*d*e^2*sgn(c)*sgn(e) - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2
*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^3*sgn(c)*sgn(e) + 8*sqrt(-
4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*e + 4*sqrt(-4*c^2*d
^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2 - 2*sqrt(-4*c^2*d^2
+ b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*d*e^3 + 4*sqrt(-4*c^2*d^2 + b^2
*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3 - sqrt(-4*c^2*d^2 + b^2*e^2)*s
qrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^
2*d*e + b*c*e^2)*b^2*c*e^4)*e^2 + 2*(8*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^4*e^
2*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d^3*e^3*sgn(c)*sgn(e) +
 4*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e +
 b*c*e^2)*b^2*c^2*d^2*e^4*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3
*d^2*e^4*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^2*e^4*sgn(c)*sgn
(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5*sgn(c)*sgn(e) + sqrt(2*c^2*d*e
```

```
+ b*c*e^2)*b^2*c^2*d*e^5*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*
e^5*sgn(c)*sgn(e) - 24*c^5*d^4*e^3 - 12*b*c^4*d^3*e^4 + 6*b^2*c^3*d^2*e^5 +
 3*b^3*c^2*d*e^6 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c
^3*d^2*e^3*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*
c*e^2)*c^3*d^2*e^3*sgn(c)*sgn(e) + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*
e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*
c^2*d*e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e) + 8*sqrt(-2*c^2*d*e + b*c*e^2)
*c^4*d^4*e^2 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d^3*e^3 - 4*sqrt(-2*c^2*d
*e + b*c*e^2)*c^4*d^3*e^3 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c^2*d^2*e^4 -
2*sqrt(-4*c^2*d^2 + b^2*e^2)*c^4*d^2*e^4 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*c^4
*d^2*e^4 - sqrt(-2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5 + sqrt(-2*c^2*d*e + b*c*e
^2)*b^2*c^2*d*e^5 - sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^5 - sqrt(-2*c^2*d*
e + b*c*e^2)*b*c^3*d*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*
c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^2*d^2*e^2*sgn(c)*sgn(e) - 3*sqrt(-4*c^2
*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*d*
e^3*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)
*c^3*d^2*e^3 + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*
d*e^4)*abs(e))*arctan(2*sqrt(1/2)*x/sqrt((b*e^2 + sqrt(-4*c^2*d^2*e^2 + b^2
*e^4))/(c*e^2)))/((8*c^5*d^5 + 4*b*c^4*d^4*e - 2*b^2*c^3*d^3*e^2 + 4*b*c^4*
d^3*e^2 - 2*c^5*d^3*e^2 - b^3*c^2*d^2*e^3 + 2*b^2*c^3*d^2*e^3 - b*c^4*d^2*e
^3)*e^2*abs(c)) - 1/8*(2*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6*sgn(c)*sgn(e
) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^7*sgn(c)*sgn(e) - 12*b*c^4*d^2*e^6
+ 3*b^3*c^2*e^8 - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*
c^2*d^2*e^4*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b
*c*e^2)*b*c^2*d^2*e^4*sgn(c)*sgn(e) + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2
*d*e + b*c*e^2)*b^3*e^6*sgn(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c
^2*d*e + b*c*e^2)*b^3*e^6*sgn(c)*sgn(e) - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt
(2*c^2*d*e + b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)
*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) + 4*sqrt(-4*c^2*d^2 + b
^2*e^2)*b*c^3*d^2*e^5 + 2*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6 - sqrt(-4*
c^2*d^2 + b^2*e^2)*b^3*c*e^7 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^7 + s
qrt(-2*c^2*d*e + b*c*e^2)*b^2*c^2*e^7 - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2
*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^5*sgn(c)*sgn(e) + 4*sq
rt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4 - sqrt(-4
*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^6 + 2*sqrt(-4*c^2*d^2
+ b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^6 - (4*sqrt(2*c^2*d*e + b*c*e
^2)*c^4*d^2*e^3*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5*sgn(c
)*sgn(e) - 24*c^5*d^3*e^3 - 12*b*c^4*d^2*e^4 + 6*b^2*c^3*d*e^5 + 3*b^3*c^2*
e^6 - 8*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(
c)*sgn(e) - 8*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*
e*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*
c^2*d^2*e^2*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b
*c*e^2)*b*c^2*d^2*e^2*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c
^2*d*e + b*c*e^2)*b^2*c*d*e^3*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*s
qrt(-2*c^2*d*e + b*c*e^2)*b^2*c*d*e^3*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b
```

```
^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) - 4*sqrt(2*c^2*
d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) + sqrt(
-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) + sqr
t(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) - 2
*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(
e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c
)*sgn(e) + 8*sqrt(-4*c^2*d^2 + b^2*e^2)*c^4*d^3*e^2 + 4*sqrt(-4*c^2*d^2 + b
^2*e^2)*b*c^3*d^2*e^3 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^2*e^3 - 2*sqrt(-
4*c^2*d^2 + b^2*e^2)*b^2*c^2*d*e^4 + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e
^4 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^4 - sqrt(-4*c^2*d^2 + b^2*e^2)*
b^3*c*e^5 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^5 + sqrt(-2*c^2*d*e + b*
c*e^2)*b^2*c^2*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)
*sqrt(-2*c^2*d*e + b*c*e^2)*c^2*d*e^2*sgn(c)*sgn(e) - 3*sqrt(-4*c^2*d^2 + b
^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^3*sgn(c)
*sgn(e) + 8*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*e
 + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2 -
2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*d*e^3 + 4*sqr
t(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3 - sqrt(-4*c^
2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4 + 2*sqrt(-4*c^2*d^2 + b
^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4)*e^2 + 2*(8*sqrt(2*c^2*d*e + b
*c*e^2)*c^4*d^4*e^2*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d^3*e
^3*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3*sgn(c)*sgn(e) -
2*sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*d^2*e^4*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*
e + b*c*e^2)*b*c^3*d^2*e^4*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*c^4*
d^2*e^4*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5*sgn(c)*sgn(e)
 + sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*d*e^5*sgn(c)*sgn(e) - sqrt(2*c^2*d*e +
 b*c*e^2)*b*c^3*d*e^5*sgn(c)*sgn(e) - 24*c^5*d^4*e^3 - 12*b*c^4*d^3*e^4 + 6
*b^2*c^3*d^2*e^5 + 3*b^3*c^2*d*e^6 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^
2*d*e + b*c*e^2)*c^3*d^2*e^3*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*sq
rt(-2*c^2*d*e + b*c*e^2)*c^3*d^2*e^3*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*
e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e) - sqrt(2*c^2*d*e +
 b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e) - 8*sqrt(-2*
c^2*d*e + b*c*e^2)*c^4*d^4*e^2 - 4*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d^3*e^3
 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3 + 2*sqrt(-2*c^2*d*e + b*c*e^2)*
b^2*c^2*d^2*e^4 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*c^4*d^2*e^4 + 2*sqrt(-2*c^2*
d*e + b*c*e^2)*c^4*d^2*e^4 + sqrt(-2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5 - sqrt(
-2*c^2*d*e + b*c*e^2)*b^2*c^2*d*e^5 + sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^
5 + sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*s
qrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^2*d^2*e^2*sgn(c)*sgn(
e) - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e
 + b*c*e^2)*b*c*d*e^3*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*
c^2*d*e + b*c*e^2)*c^3*d^2*e^3 + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e
 + b*c*e^2)*b*c^2*d*e^4)*abs(e))*arctan(2*sqrt(1/2)*x/sqrt((b*e^2 - sqrt(-4
*c^2*d^2*e^2 + b^2*e^4))/(c*e^2)))/((8*c^5*d^5 + 4*b*c^4*d^4*e - 2*b^2*c^3*
d^3*e^2 + 4*b*c^4*d^3*e^2 - 2*c^5*d^3*e^2 - b^3*c^2*d^2*e^3 + 2*b^2*c^3*d^2
```

```
*e^3 - b*c^4*d^2*e^3)*e^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{e^{3/2}\left(\operatorname{atan}\left(\frac{c\sqrt{e}\,x}{\sqrt{c\,(b\,e+2\,c\,d)}}\right)-\operatorname{atan}\left(\frac{(2\,d\,c^2+b\,e\,c)\left(x\left(\frac{\sqrt{e}\left(c\,d\,e^7-\frac{4\,c^3\,d^2\,e^7}{2\,d\,c^2+b\,e\,c}\right)}{d\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}+\frac{e^{3/2}\left(2\,c^2\,d\,e^6-b\,c\,e^7\right)}{c\,d\sqrt{2\,d\,c^2+b\,e\,c}\,(b\,e-2\,c\,d)}\right)+\frac{\sqrt{e}\,x^3\left(c\,e^8-\frac{2\,b\,c^2\,e^9}{2\,d\,c^2+b\,e\,c}\right)}{d\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}\right)}{c\,e^7}\right)\right)}{\sqrt{2\,d\,c^2+b\,e\,c}}$$

```
[In] int((d + e*x^2)/(b*x^2 + c*x^4 + (c*d^2)/e^2),x)
```

```
[Out] (e^(3/2)*(atan((c*e^(1/2)*x)/(c*(b*e + 2*c*d))^(1/2)) - atan(((2*c^2*d + b*
c*e)*(x*((e^(1/2)*(c*d*e^7 - (4*c^3*d^2*e^7)/(2*c^2*d + b*c*e)))/(d*(c*(b*e
 + 2*c*d))^(1/2)*(b*e - 2*c*d)) + (e^(3/2)*(2*c^2*d*e^6 - b*c*e^7))/(c*d*(2
*c^2*d + b*c*e)^(1/2)*(b*e - 2*c*d))) + (e^(1/2)*x^3*(c*e^8 - (2*b*c^2*e^9)
/(2*c^2*d + b*c*e)))/(d*(c*(b*e + 2*c*d))^(1/2)*(b*e - 2*c*d))))/(c*e^7))))
/(2*c^2*d + b*c*e)^(1/2)
```

3.36 $\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)} \, dx = -\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

```
[Out] -e^(3/2)*arctan((-2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d)^(1/2))/(b*e+2*c*d)^(1/2)
)/c^(1/2)/(b*e+2*c*d)^(1/2)+e^(3/2)*arctan((2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d
)^(1/2))/(b*e+2*c*d)^(1/2))/c^(1/2)/(b*e+2*c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2015, 1175, 632, 210}

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)} \, dx = \frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

```
[In] Int[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 2015

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rubi steps

$$\text{integral} = \int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$$

$$= \frac{e\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c} + \frac{e\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c}$$

$$= -\frac{e\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c} - \frac{e\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, \frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c}$$

$$= -\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.91

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx = \frac{e^{3/2}\left(\frac{\left(2cd-be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}+\frac{\left(-2cd+be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4c^2d^2+b^2e^2}}$$

```
[In] Integrate[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

```
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c
^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e
 + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2
])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\sqrt{-c(be+2cd)e}\,e\ln\left(-c\,x^2e-\sqrt{-c(be+2cd)e}\,x+cd\right)}{2c(be+2cd)}-\frac{\sqrt{-c(be+2cd)e}\,e\ln\left(-c\,x^2e+\sqrt{-c(be+2cd)e}\,x+cd\right)}{2c(be+2cd)}$
default	$4e^4c\left(\frac{\left(b\,e^2-2dce+\sqrt{e^2(be-2cd)(be+2cd)}\right)\sqrt{2}\arctan\left(\frac{cxe\sqrt{2}}{\sqrt{c\left(b\,e^2+\sqrt{e^2(be-2cd)(be+2cd)}\right)}}\right)}{8\sqrt{e^2(be-2cd)(be+2cd)}\,c\,e^2\sqrt{c\left(b\,e^2+\sqrt{e^2(be-2cd)(be+2cd)}\right)}}-\frac{\left(-b\,e^2+2dce+\sqrt{e^2(be-2cd)(be+2cd}\right.}{8\sqrt{e^2(be-2cd)(be+2cd)}\,c}\right.$

```
[In] int((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-c*(b*e+2*c*d)*e)^(1/2)/c/(b*e+2*c*d)*e*ln(-c*x^2*e-(-c*(b*e+2*c*d)*e)
^(1/2)*x+c*d)-1/2*(-c*(b*e+2*c*d)*e)^(1/2)/c/(b*e+2*c*d)*e*ln(-c*x^2*e+(-c*
(b*e+2*c*d)*e)^(1/2)*x+c*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx$$
$$=\left[\frac{1}{2}\,e\sqrt{-\frac{e}{2\,c^2d+bce}}\log\left(\frac{ce^2x^4+cd^2-(4\,cde+be^2)x^2+2\,((2\,c^2de+bce^2)x^3-(2\,c^2d^2+bcde)x)\sqrt{-\frac{}{2\,c^2}}}{ce^2x^4+be^2x^2+cd^2}\right.\right.$$
$$\left.+\,e\sqrt{\frac{e}{2\,c^2d+bce}}\arctan\left(\frac{(cex^3+(cd+be)x)\sqrt{\frac{e}{2\,c^2d+bce}}}{d}\right)\right]$$

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^2
)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(2*
c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), e*sqrt(e/(2*c^2*d + b*c*
e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))*arc
tan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.23

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx=-\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$
$$+\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

```
[In] integrate((e*x**2+d)/(b*x**2+c*(d**2/e**2+x**4)),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e
 + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*
(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*
c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

Maxima [F]

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx=\int \frac{ex^2+d}{bx^2+\left(x^4+\frac{d^2}{e^2}\right)c}\,dx$$

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(b*x^2 + (x^4 + d^2/e^2)*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6341 vs. 2(105) = 210.

Time = 1.10 (sec) , antiderivative size = 6341, normalized size of antiderivative = 48.78

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*e
 + b*c*e^2)*b^2*c^2*e^7*sgn(c)*sgn(e) - 12*b*c^4*d^2*e^6 + 3*b^3*c^2*e^8 +
4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4*sgn(c)
*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*
e^4*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^
3*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*
b^3*e^6*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e
^2)*b^2*c*e^6*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e +
 b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d^2*
e^5 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6 + sqrt(-4*c^2*d^2 + b^2*e^2)
*b^3*c*e^7 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^7 - sqrt(-2*c^2*d*e + b
*c*e^2)*b^2*c^2*e^7 - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2
)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^5*sgn(c)*sgn(e) + 4*sqrt(-4*c^2*d^2 + b^
2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4 - sqrt(-4*c^2*d^2 + b^2*e^2
)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^6 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2
*c^2*d*e + b*c*e^2)*b^2*c*e^6 - (4*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^2*e^3*sg
n(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5*sgn(c)*sgn(e) - 24*c^5*
d^3*e^3 - 12*b*c^4*d^2*e^4 + 6*b^2*c^3*d*e^5 + 3*b^3*c^2*e^6 + 8*sqrt(-4*c^
2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(c)*sgn(e) + 8*sqrt
(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(c)*sgn(e) +
4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2*sgn(c)
*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*
e^2*sgn(c)*sgn(e) - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*
b^2*c*d*e^3*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b
```

```
*c*e^2)*b^2*c*d*e^3*sgn(c)*sgn(e) + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2
*d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqr
t(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*e
^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*
e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 +
 b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(e) + 2*sqrt(2*c^2*
d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(e) - 8*sqrt(
-4*c^2*d^2 + b^2*e^2)*c^4*d^3*e^2 - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d^2*
e^3 - 4*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^2*e^3 + 2*sqrt(-4*c^2*d^2 + b^2*e^
2)*b^2*c^2*d*e^4 - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^4 - 4*sqrt(-2*c^2
*d*e + b*c*e^2)*b*c^3*d*e^4 + sqrt(-4*c^2*d^2 + b^2*e^2)*b^3*c*e^5 - 2*sqrt
(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^5 - sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5
 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e +
 b*c*e^2)*c^2*d*e^2*sgn(c)*sgn(e) - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2
*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^3*sgn(c)*sgn(e) + 8*sqrt(-
4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*e + 4*sqrt(-4*c^2*d
^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2 - 2*sqrt(-4*c^2*d^2
+ b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*d*e^3 + 4*sqrt(-4*c^2*d^2 + b^2
*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3 - sqrt(-4*c^2*d^2 + b^2*e^2)*s
qrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^
2*d*e + b*c*e^2)*b^2*c*e^4)*e^2 + 2*(8*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^4*e^
2*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d^3*e^3*sgn(c)*sgn(e) +
 4*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e +
 b*c*e^2)*b^2*c^2*d^2*e^4*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3
*d^2*e^4*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^2*e^4*sgn(c)*sgn
(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5*sgn(c)*sgn(e) + sqrt(2*c^2*d*e
+ b*c*e^2)*b^2*c^2*d*e^5*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*
e^5*sgn(c)*sgn(e) - 24*c^5*d^4*e^3 - 12*b*c^4*d^3*e^4 + 6*b^2*c^3*d^2*e^5 +
 3*b^3*c^2*d*e^6 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c
^3*d^2*e^3*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*
c*e^2)*c^3*d^2*e^3*sgn(c)*sgn(e) + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*
e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*
c^2*d*e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e) + 8*sqrt(-2*c^2*d*e + b*c*e^2)
*c^4*d^4*e^2 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d^3*e^3 - 4*sqrt(-2*c^2*d
*e + b*c*e^2)*c^4*d^3*e^3 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c^2*d^2*e^4 -
2*sqrt(-4*c^2*d^2 + b^2*e^2)*c^4*d^2*e^4 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*c^4
*d^2*e^4 - sqrt(-2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5 + sqrt(-2*c^2*d*e + b*c*e
^2)*b^2*c^2*d*e^5 - sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^5 - sqrt(-2*c^2*d*
e + b*c*e^2)*b*c^3*d*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*
c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^2*d^2*e^2*sgn(c)*sgn(e) - 3*sqrt(-4*c^2
*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*d*
e^3*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)
*c^3*d^2*e^3 + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*
d*e^4)*abs(e))*arctan(2*sqrt(1/2)*x/sqrt((b*e^2 + sqrt(-4*c^2*d^2*e^2 + b^2
*e^4))/(c*e^2)))/((8*c^5*d^5 + 4*b*c^4*d^4*e - 2*b^2*c^3*d^3*e^2 + 4*b*c^4*
```

```
d^3*e^2 - 2*c^5*d^3*e^2 - b^3*c^2*d^2*e^3 + 2*b^2*c^3*d^2*e^3 - b*c^4*d^2*e
^3)*e^2*abs(c)) - 1/8*(2*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6*sgn(c)*sgn(e
) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^7*sgn(c)*sgn(e) - 12*b*c^4*d^2*e^6
 + 3*b^3*c^2*e^8 - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*
c^2*d^2*e^4*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b
*c*e^2)*b*c^2*d^2*e^4*sgn(c)*sgn(e) + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2
*d*e + b*c*e^2)*b^3*e^6*sgn(c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c
^2*d*e + b*c*e^2)*b^3*e^6*sgn(c)*sgn(e) - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt
(2*c^2*d*e + b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)
*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) + 4*sqrt(-4*c^2*d^2 + b
^2*e^2)*b*c^3*d^2*e^5 + 2*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6 - sqrt(-4*
c^2*d^2 + b^2*e^2)*b^3*c*e^7 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^7 + s
qrt(-2*c^2*d*e + b*c*e^2)*b^2*c^2*e^7 - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2
*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^5*sgn(c)*sgn(e) + 4*sq
rt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4 - sqrt(-4
*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^6 + 2*sqrt(-4*c^2*d^2
 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^6 - (4*sqrt(2*c^2*d*e + b*c*e
^2)*c^4*d^2*e^3*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5*sgn(c
)*sgn(e) - 24*c^5*d^3*e^3 - 12*b*c^4*d^2*e^4 + 6*b^2*c^3*d*e^5 + 3*b^3*c^2*
e^6 - 8*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(
c)*sgn(e) - 8*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*
e*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*
c^2*d^2*e^2*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b
*c*e^2)*b*c^2*d^2*e^2*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c
^2*d*e + b*c*e^2)*b^2*c*d*e^3*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*s
qrt(-2*c^2*d*e + b*c*e^2)*b^2*c*d*e^3*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b
^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) - 4*sqrt(2*c^2*
d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3*sgn(c)*sgn(e) + sqrt(
-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) + sqr
t(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4*sgn(c)*sgn(e) - 2
*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c)*sgn(
e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4*sgn(c
)*sgn(e) + 8*sqrt(-4*c^2*d^2 + b^2*e^2)*c^4*d^3*e^2 + 4*sqrt(-4*c^2*d^2 + b
^2*e^2)*b*c^3*d^2*e^3 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^2*e^3 - 2*sqrt(-
4*c^2*d^2 + b^2*e^2)*b^2*c^2*d*e^4 + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e
^4 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^4 - sqrt(-4*c^2*d^2 + b^2*e^2)*
b^3*c*e^5 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^5 + sqrt(-2*c^2*d*e + b*
c*e^2)*b^2*c^2*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)
*sqrt(-2*c^2*d*e + b*c*e^2)*c^2*d*e^2*sgn(c)*sgn(e) - 3*sqrt(-4*c^2*d^2 + b
^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^3*sgn(c)
*sgn(e) + 8*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3*d^3*e
 + 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^2 -
2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*d*e^3 + 4*sqr
t(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^3 - sqrt(-4*c^
2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^4 + 2*sqrt(-4*c^2*d^2 + b
```

```
^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^4)*e^2 + 2*(8*sqrt(2*c^2*d*e + b
*c*e^2)*c^4*d^4*e^2*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d^3*e
^3*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3*sgn(c)*sgn(e) -
2*sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*d^2*e^4*sgn(c)*sgn(e) + 4*sqrt(2*c^2*d*
e + b*c*e^2)*b*c^3*d^2*e^4*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*c^4*
d^2*e^4*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5*sgn(c)*sgn(e)
 + sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*d*e^5*sgn(c)*sgn(e) - sqrt(2*c^2*d*e +
 b*c*e^2)*b*c^3*d*e^5*sgn(c)*sgn(e) - 24*c^5*d^4*e^3 - 12*b*c^4*d^3*e^4 + 6
*b^2*c^3*d^2*e^5 + 3*b^3*c^2*d*e^6 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^
2*d*e + b*c*e^2)*c^3*d^2*e^3*sgn(c)*sgn(e) - 2*sqrt(2*c^2*d*e + b*c*e^2)*sq
rt(-2*c^2*d*e + b*c*e^2)*c^3*d^2*e^3*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*
e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e) - sqrt(2*c^2*d*e +
 b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d*e^4*sgn(c)*sgn(e) - 8*sqrt(-2*
c^2*d*e + b*c*e^2)*c^4*d^4*e^2 - 4*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d^3*e^3
 + 4*sqrt(-2*c^2*d*e + b*c*e^2)*c^4*d^3*e^3 + 2*sqrt(-2*c^2*d*e + b*c*e^2)*
b^2*c^2*d^2*e^4 + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*c^4*d^2*e^4 + 2*sqrt(-2*c^2*
d*e + b*c*e^2)*c^4*d^2*e^4 + sqrt(-2*c^2*d*e + b*c*e^2)*b^3*c*d*e^5 - sqrt(
-2*c^2*d*e + b*c*e^2)*b^2*c^2*d*e^5 + sqrt(-4*c^2*d^2 + b^2*e^2)*b*c^3*d*e^
5 + sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^5 - 6*sqrt(-4*c^2*d^2 + b^2*e^2)*s
qrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^2*d^2*e^2*sgn(c)*sgn(
e) - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e
 + b*c*e^2)*b*c*d*e^3*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*
c^2*d*e + b*c*e^2)*c^3*d^2*e^3 + sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e
 + b*c*e^2)*b*c^2*d*e^4)*abs(e))*arctan(2*sqrt(1/2)*x/sqrt((b*e^2 - sqrt(-4
*c^2*d^2*e^2 + b^2*e^4))/(c*e^2)))/((8*c^5*d^5 + 4*b*c^4*d^4*e - 2*b^2*c^3*
d^3*e^2 + 4*b*c^4*d^3*e^2 - 2*c^5*d^3*e^2 - b^3*c^2*d^2*e^3 + 2*b^2*c^3*d^2
*e^3 - b*c^4*d^2*e^3)*e^2*abs(c))
```

Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx = \frac{e^{3/2}\,\Bigg(\mathrm{atan}\left(\frac{c\,\sqrt{e}\,x}{\sqrt{c\,(b\,e+2\,c\,d)}}\right)-\mathrm{atan}\Bigg(\frac{(2\,d\,c^2+b\,e\,c)\,\Bigg(x\,\Bigg(\frac{\sqrt{e}\,\left(c\,d\,e^7-\frac{4\,c^3\,d^2\,e^7}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}+\frac{e^{3/2}\,\left(2\,c^2\,d\,e^6-b\,c\,e^7\right)}{c\,d\,\sqrt{2\,d\,c^2+b\,e\,c}\,(b\,e-2\,c\,d)}\Bigg)+\frac{\sqrt{e}\,x^3\,\Big(c\,e^8-\frac{2\,b\,c^2\,e^9}{2\,d\,c^2+b\,e\,c}}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d}}{c\,e^7}}{\sqrt{2\,d\,c^2+b\,e\,c}}$$

```
[In] int((d + e*x^2)/(b*x^2 + c*(x^4 + d^2/e^2)),x)
```

```
[Out] (e^(3/2)*(atan((c*e^(1/2)*x)/(c*(b*e + 2*c*d))^(1/2)) - atan(((2*c^2*d + b*
c*e)*(x*((e^(1/2)*(c*d*e^7 - (4*c^3*d^2*e^7)/(2*c^2*d + b*c*e)))/(d*(c*(b*e
```

```
 + 2*c*d))^(1/2)*(b*e - 2*c*d)) + (e^(3/2)*(2*c^2*d*e^6 - b*c*e^7))/(c*d*(2
*c^2*d + b*c*e)^(1/2)*(b*e - 2*c*d))) + (e^(1/2)*x^3*(c*e^8 - (2*b*c^2*e^9)
/(2*c^2*d + b*c*e)))/(d*(c*(b*e + 2*c*d))^(1/2)*(b*e - 2*c*d))))/(c*e^7))))
/(2*c^2*d + b*c*e)^(1/2)
```

3.37 $\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 29

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=-\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right)$$

```
[Out] -1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1178, 642}

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

```
[In] Int[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*Log[a - x + b*x^2] + Log[a + x + b*x^2]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int \frac{\frac{1}{b}+2x}{-\frac{a}{b}-\frac{x}{b}-x^2}\,dx\right)-\frac{1}{2}\int \frac{\frac{1}{b}-2x}{-\frac{a}{b}+\frac{x}{b}-x^2}\,dx\\ &= -\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = -\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right)$$

```
[In] Integrate[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
[Out] -1/2*Log[a - x + b*x^2] + Log[a + x + b*x^2]/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26
norman	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26
risch	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26
parallelrisch	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26

```
[In] int((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=-\frac{\log\left(\frac{a}{b}+x^2-\frac{x}{b}\right)}{2}+\frac{\log\left(\frac{a}{b}+x^2+\frac{x}{b}\right)}{2}$$

```
[In] integrate((-b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
[Out] -log(a/b + x**2 - x/b)/2 + log(a/b + x**2 + x/b)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \operatorname{atanh}\left(\frac{x}{b\,x^2+a}\right)$$

```
[In] int((a - b*x^2)/(x^2*(2*a*b - 1) + a^2 + b^2*x^4),x)
[Out] atanh(x/(a + b*x^2))
```

3.38 $\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 60

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\operatorname{arctanh}\left(\frac{1+2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

```
[Out] arctanh((-2*b*x+1)/(-4*a*b+1)^(1/2))/(-4*a*b+1)^(1/2)-arctanh((2*b*x+1)/(-4
*a*b+1)^(1/2))/(-4*a*b+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1175, 632, 212}

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\operatorname{arctanh}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

```
[In] Int[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] ArcTanh[(1 - 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b] - ArcTanh[(1 + 2*b*x)/
Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\frac{a}{b}-\frac{x}{b}+x^2}\,dx}{2b}+\frac{\int \frac{1}{\frac{a}{b}+\frac{x}{b}+x^2}\,dx}{2b}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2}\,dx, x, -\frac{1}{b}+2x\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2}\,dx, x, \frac{1}{b}+2x\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{1+2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 138 vs. $2(60) = 120$.

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.30

$$
\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \frac{\frac{\left(1+\sqrt{1-4ab}\right)\arctan\left(\frac{bx}{\sqrt{-\frac{1}{2}+ab-\frac{1}{2}\sqrt{1-4ab}}}\right)}{\sqrt{-1+2ab-\sqrt{1-4ab}}}+\frac{\left(-1+\sqrt{1-4ab}\right)\arctan\left(\frac{\sqrt{2}bx}{\sqrt{-1+2ab+\sqrt{1-4ab}}}\right)}{\sqrt{-1+2ab+\sqrt{1-4ab}}}}{\sqrt{2-8ab}}
$$

```
[In] Integrate[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] (((1 + Sqrt[1 - 4*a*b])*ArcTan[(b*x)/Sqrt[-1/2 + a*b - Sqrt[1 - 4*a*b]/2]])
/Sqrt[-1 + 2*a*b - Sqrt[1 - 4*a*b]] + ((-1 + Sqrt[1 - 4*a*b])*ArcTan[(Sqrt[
2]*b*x)/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a*b]]])/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a
*b]])/Sqrt[2 - 8*a*b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\arctan\left(\frac{2bx+1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}+\frac{\arctan\left(\frac{2bx-1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}$	52
risch	$-\frac{\ln\left(b\,x^{2}\sqrt{-4ab+1}+(-4ab+1)x-a\sqrt{-4ab+1}\right)}{2\sqrt{-4ab+1}}+\frac{\ln\left(b\,x^{2}\sqrt{-4ab+1}+x(4ab-1)-a\sqrt{-4ab+1}\right)}{2\sqrt{-4ab+1}}$	90

```
[In] int((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(4*a*b-1)^(1/2)*arctan((2*b*x+1)/(4*a*b-1)^(1/2))+1/(4*a*b-1)^(1/2)*arcta
n((2*b*x-1)/(4*a*b-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.73

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$$
$$=\left[-\frac{\sqrt{-4\,ab+1}\log\left(\frac{b^2x^4-(6\,ab-1)x^2+a^2-2\,(bx^3-ax)\sqrt{-4\,ab+1}}{b^2x^4+(2\,ab-1)x^2+a^2}\right)}{2\,(4\,ab-1)},\frac{\sqrt{4\,ab-1}\arctan\left(\frac{bx}{\sqrt{4\,ab-1}}\right)+\sqrt{4\,ab-1}\arctan}{4\,ab-1}\right.$$

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-4*a*b + 1)*log((b^2*x^4 - (6*a*b - 1)*x^2 + a^2 - 2*(b*x^3 - a*
x)*sqrt(-4*a*b + 1))/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2))/(4*a*b - 1), (sqrt(
4*a*b - 1)*arctan(b*x/sqrt(4*a*b - 1)) + sqrt(4*a*b - 1)*arctan((b^2*x^3 +
(3*a*b - 1)*x)*sqrt(4*a*b - 1)/(4*a^2*b - a)))/(4*a*b - 1)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.95

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=-\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(-4ab\sqrt{-\frac{1}{4ab-1}}+\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}$$
$$+\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(4ab\sqrt{-\frac{1}{4ab-1}}-\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}$$

```
[In] integrate((b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

```
[Out] -sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(-4*a*b*sqrt(-1/(4*a*b - 1)) + sq
rt(-1/(4*a*b - 1)))/b)/2 + sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(4*a*b*
sqrt(-1/(4*a*b - 1)) - sqrt(-1/(4*a*b - 1)))/b)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*b-0.25>0)', see `assume?` for mor
e detai
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \frac{\arctan\left(\frac{2\,bx+1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}} + \frac{\arctan\left(\frac{2\,bx-1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}}$$

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
```

```
[Out] arctan((2*b*x + 1)/sqrt(4*a*b - 1))/sqrt(4*a*b - 1) + arctan((2*b*x - 1)/sq
rt(4*a*b - 1))/sqrt(4*a*b - 1)
```

Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \frac{\mathrm{atan}\left(\frac{b\,x}{\sqrt{4\,a\,b-1}}\right)+\mathrm{atan}\left(\frac{\frac{3\,x\,(4\,a\,b-1)}{4}-\frac{x}{4}+b^2\,x^3}{a\,\sqrt{4\,a\,b-1}}\right)}{\sqrt{4\,a\,b-1}}$$

```
[In] int((a + b*x^2)/(x^2*(2*a*b - 1) + a^2 + b^2*x^4),x)
```

```
[Out] (atan((b*x)/(4*a*b - 1)^(1/2)) + atan(((3*x*(4*a*b - 1))/4 - x/4 + b^2*x^3)
/(a*(4*a*b - 1)^(1/2))))/(4*a*b - 1)^(1/2)
```

3.39 $\int \frac{1+2x^2}{1+bx^2+4x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1+2x^2}{1+bx^2+4x^4} dx = -\frac{\arctan\left(\frac{\sqrt{4-b}-4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}} + \frac{\arctan\left(\frac{\sqrt{4-b}+4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}$$

```
[Out] -arctan((-4*x+(4-b)^(1/2))/(4+b)^(1/2))/(4+b)^(1/2)+arctan((4*x+(4-b)^(1/2)
)/(4+b)^(1/2))/(4+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\int \frac{1+2x^2}{1+bx^2+4x^4} dx = \frac{\arctan\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}} - \frac{\arctan\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

```
[In] Int[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[4 - b] - 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(Sqrt[4 - b
] + 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{1}{2}\sqrt{4-b}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{1}{2}\sqrt{4-b}x+x^2}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b)-x^2}\,dx,x,-\frac{\sqrt{4-b}}{2}+2x\right)\right)\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b)-x^2}\,dx,x,\frac{\sqrt{4-b}}{2}+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 126 vs. $2(62) = 124$.

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.03

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = \frac{\frac{\left(4-b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{-16+b^2}}}\right)}{\sqrt{b-\sqrt{-16+b^2}}}+\frac{\left(-4+b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{b+\sqrt{-16+b^2}}}\right)}{\sqrt{b+\sqrt{-16+b^2}}}}{\sqrt{2}\sqrt{-16+b^2}}$$

```
[In] Integrate[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]]
)/Sqrt[b - Sqrt[-16 + b^2]] + ((-4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]
*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-1
6 + b^2])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{\ln\left(-2x^{2}\sqrt{-4-b}+x(4+b)+\sqrt{-4-b}\right)}{2\sqrt{-4-b}}+\frac{\ln\left(-2x^{2}\sqrt{-4-b}+(-4-b)x+\sqrt{-4-b}\right)}{2\sqrt{-4-b}}$	74
default	$\frac{\left(-4+\sqrt{(b-4)(4+b)}+b\right)\arctan\left(\frac{4x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{2\sqrt{(b-4)(4+b)}+2b}}+\frac{\left(4+\sqrt{(b-4)(4+b)}-b\right)\arctan\left(\frac{4x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}$	124

```
[In] int((2*x^2+1)/(4*x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-4-b)^(1/2)*ln(-2*x^2*(-4-b)^(1/2)+x*(4+b)+(-4-b)^(1/2))+1/2/(-4-b)^(
1/2)*ln(-2*x^2*(-4-b)^(1/2)+(-4-b)*x+(-4-b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx$$
$$=\left[-\frac{\sqrt{-b-4}\log\left(\frac{4\,x^{4}-(b+8)x^{2}-2\,(2\,x^{3}-x)\sqrt{-b-4}+1}{4\,x^{4}+bx^{2}+1}\right)}{2\,(b+4)},\frac{\sqrt{b+4}\arctan\left(\frac{4\,x^{3}+(b+2)x}{\sqrt{b+4}}\right)+\sqrt{b+4}\arctan\left(\frac{2\,x}{\sqrt{b+4}}\right)}{b+4}\right]$$

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b - 4)*log((4*x^4 - (b + 8)*x^2 - 2*(2*x^3 - x)*sqrt(-b - 4) +
1)/(4*x^4 + b*x^2 + 1))/(b + 4), (sqrt(b + 4)*arctan((4*x^3 + (b + 2)*x)/sq
rt(b + 4)) + sqrt(b + 4)*arctan(2*x/sqrt(b + 4)))/(b + 4)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx=-\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^{2}+x\left(-\frac{b\sqrt{-\frac{1}{b+4}}}{2}-2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}+\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^{2}+x\left(\frac{b\sqrt{-\frac{1}{b+4}}}{2}+2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}$$

```
[In] integrate((2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] -sqrt(-1/(b + 4))*log(x**2 + x*(-b*sqrt(-1/(b + 4))/2 - 2*sqrt(-1/(b + 4)))
 - 1/2)/2 + sqrt(-1/(b + 4))*log(x**2 + x*(b*sqrt(-1/(b + 4))/2 + 2*sqrt(-1
/(b + 4))) - 1/2)/2
```

Maxima [F]

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4+bx^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1), x)
```

Giac [F]

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4+bx^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{-b^3\,x-4\,b^2\,x^3-2\,b^2\,x+16\,b\,x+64\,x^3+32\,x}{\left(b^2-16\right)\,\sqrt{b+4}}\right)-\operatorname{atan}\left(\frac{2\,x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

```
[In] int((2*x^2 + 1)/(b*x^2 + 4*x^4 + 1),x)
```

```
[Out] -(atan((32*x + 16*b*x - 2*b^2*x - b^3*x + 64*x^3 - 4*b^2*x^3)/((b^2 - 16)*(
b + 4)^(1/2))) - atan((2*x)/(b + 4)^(1/2)))/(b + 4)^(1/2)
```

3.40 $\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}+\frac{\arctan\left(\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

```
[Out] -arctan((-4*x+(4+b)^(1/2))/(4-b)^(1/2))/(4-b)^(1/2)+arctan((4*x+(4+b)^(1/2)
)/(4-b)^(1/2))/(4-b)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1175, 632, 210}

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = \frac{\arctan\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}-\frac{\arctan\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

```
[In] Int[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[4 + b] - 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(Sqrt[4 + b
] + 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{1}{2}\sqrt{4+b}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{1}{2}\sqrt{4+b}\,x+x^2}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)-x^2}\,dx,x,-\frac{\sqrt{4+b}}{2}+2x\right)\right)\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)-x^2}\,dx,x,\frac{\sqrt{4+b}}{2}+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}+\frac{\tan^{-1}\left(\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 134 vs. 2(66) = 132.

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.03

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = \frac{\frac{\left(4+b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{-b-\sqrt{-16+b^2}}}\right)}{\sqrt{-b-\sqrt{-16+b^2}}}+\frac{\left(-4-b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{-b+\sqrt{-16+b^2}}}\right)}{\sqrt{-b+\sqrt{-16+b^2}}}}{\sqrt{2}\sqrt{-16+b^2}}$$

```
[In] Integrate[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[-b - Sqrt[-16 + b^2]]
])/Sqrt[-b - Sqrt[-16 + b^2]] + ((-4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[
2]*x)/Sqrt[-b + Sqrt[-16 + b^2]]])/Sqrt[-b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqr
t[-16 + b^2])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\ln\left(2x^{2}\sqrt{b-4}+(4-b)x-\sqrt{b-4}\right)}{2\sqrt{b-4}}-\frac{\ln\left(2x^{2}\sqrt{b-4}+x(b-4)-\sqrt{b-4}\right)}{2\sqrt{b-4}}$	66
default	$\frac{\left(4+\sqrt{(b-4)(4+b)}+b\right)\arctan\left(\frac{4x}{\sqrt{-2\sqrt{(b-4)(4+b)}-2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{-2\sqrt{(b-4)(4+b)}-2b}}+\frac{\left(-4+\sqrt{(b-4)(4+b)}-b\right)\arctan\left(\frac{4x}{\sqrt{2\sqrt{(b-4)(4+b)}-2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{2\sqrt{(b-4)(4+b)}-2b}}$	124

```
[In] int((2*x^2+1)/(4*x^4-b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(b-4)^(1/2)*ln(2*x^2*(b-4)^(1/2)+(4-b)*x-(b-4)^(1/2))-1/2/(b-4)^(1/2)*l
n(2*x^2*(b-4)^(1/2)+x*(b-4)-(b-4)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.82

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx$$
$$=\left[\frac{\log\left(\frac{4\,x^{4}+(b-8)x^{2}-2\,(2\,x^{3}-x)\sqrt{b-4}+1}{4\,x^{4}-bx^{2}+1}\right)}{2\,\sqrt{b-4}},\frac{\sqrt{-b+4}\arctan\left(\frac{(4\,x^{3}-(b-2)x)\sqrt{-b+4}}{b-4}\right)+\sqrt{-b+4}\arctan\left(\frac{2\,\sqrt{-b+4}x}{b-4}\right)}{b-4}\right]$$

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/2*log((4*x^4 + (b - 8)*x^2 - 2*(2*x^3 - x)*sqrt(b - 4) + 1)/(4*x^4 - b*x
^2 + 1))/sqrt(b - 4), (sqrt(-b + 4)*arctan((4*x^3 - (b - 2)*x)*sqrt(-b + 4)
/(b - 4)) + sqrt(-b + 4)*arctan(2*sqrt(-b + 4)*x/(b - 4)))/(b - 4)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx=\frac{\sqrt{\frac{1}{b-4}}\log\left(x^{2}+x\left(-\frac{b\sqrt{\frac{1}{b-4}}}{2}+2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}-\frac{\sqrt{\frac{1}{b-4}}\log\left(x^{2}+x\left(\frac{b\sqrt{\frac{1}{b-4}}}{2}-2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}$$

```
[In] integrate((2*x**2+1)/(4*x**4-b*x**2+1),x)
```

```
[Out] sqrt(1/(b - 4))*log(x**2 + x*(-b*sqrt(1/(b - 4))/2 + 2*sqrt(1/(b - 4))) - 1
/2)/2 - sqrt(1/(b - 4))*log(x**2 + x*(b*sqrt(1/(b - 4))/2 - 2*sqrt(1/(b - 4
))) - 1/2)/2
```

Maxima [F]

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-bx^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1), x)
```

Giac [F]

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-bx^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.36

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = -\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{b-4}}{2\,x^2-1}\right)}{\sqrt{b-4}}$$

```
[In] int((2*x^2 + 1)/(4*x^4 - b*x^2 + 1),x)
```

```
[Out] -atanh((x*(b - 4)^(1/2))/(2*x^2 - 1))/(b - 4)^(1/2)
```

3.41 $\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx=\frac{\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

```
[Out] 1/10*arctan(2*x/(1/2*10^(1/2)-1/2*2^(1/2)))*10^(1/2)+1/10*arctan(2*x/(1/2*1
0^(1/2)+1/2*2^(1/2)))*10^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1177, 209}

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx=\frac{\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

```
[In] Int[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/
Sqrt[10]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\left(5-\sqrt{5}\right)\int\frac{1}{3-\sqrt{5}+4x^2}\,dx+\frac{1}{5}\left(5+\sqrt{5}\right)\int\frac{1}{3+\sqrt{5}+4x^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.84

$$\int\frac{1+2x^2}{1+6x^2+4x^4}\,dx=\frac{\left(-1+\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{5\left(3-\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

```
[In] Integrate[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
[Out] ((-1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]])/(2*Sqrt[5*(3 - Sqrt[5])])
+ ((1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(2*Sqrt[5*(3 + Sqrt[5])])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{\sqrt{10}\,\arctan\left(\frac{\sqrt{10}\,x}{5}\right)}{10}+\frac{\sqrt{10}\,\arctan\left(\frac{2\sqrt{10}\,x^3}{5}+\frac{4\sqrt{10}\,x}{5}\right)}{10}$	35
default	$\frac{2\left(\sqrt{5}+1\right)\sqrt{5}\,\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}+\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}$	82

```
[In] int((2*x^2+1)/(4*x^4+6*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/10*10^(1/2)*arctan(1/5*10^(1/2)*x)+1/10*10^(1/2)*arctan(2/5*10^(1/2)*x^3+
4/5*10^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{1}{10}\,\sqrt{10}\arctan\left(\frac{2}{5}\,\sqrt{10}(x^3+2\,x)\right)+\frac{1}{10}\,\sqrt{10}\arctan\left(\frac{1}{5}\,\sqrt{10}x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
[Out] 1/10*sqrt(10)*arctan(2/5*sqrt(10)*(x^3 + 2*x)) + 1/10*sqrt(10)*arctan(1/5*s
qrt(10)*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{\sqrt{10}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{10}x}{5}\right)+2\,\mathrm{atan}\left(\frac{2\sqrt{10}x^3}{5}+\frac{4\sqrt{10}x}{5}\right)\right)}{20}$$

```
[In] integrate((2*x**2+1)/(4*x**4+6*x**2+1),x)
[Out] sqrt(10)*(2*atan(sqrt(10)*x/5) + 2*atan(2*sqrt(10)*x**3/5 + 4*sqrt(10)*x/5)
)/20
```

Maxima [F]

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4+6\,x^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
[Out] integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{1}{10}\,\sqrt{10}\arctan\left(\frac{4\,x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{10}\,\sqrt{10}\arctan\left(\frac{4\,x}{\sqrt{10}-\sqrt{2}}\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
[Out] 1/10*sqrt(10)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/10*sqrt(10)*arctan(4*x/(
sqrt(10) - sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{\sqrt{10}\,\left(\mathrm{atan}\left(\frac{2\,\sqrt{10}\,x^3}{5}+\frac{4\,\sqrt{10}\,x}{5}\right)+\mathrm{atan}\left(\frac{\sqrt{10}\,x}{5}\right)\right)}{10}$$

```
[In] int((2*x^2 + 1)/(6*x^2 + 4*x^4 + 1),x)
[Out] (10^(1/2)*(atan((4*10^(1/2)*x)/5 + (2*10^(1/2)*x^3)/5) + atan((10^(1/2)*x)/
5)))/10
```

3.42 $\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx=\frac{\arctan(x)}{3}+\frac{1}{3}\arctan(2x)$$

```
[Out] 1/3*arctan(x)+1/3*arctan(2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1177, 209}

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx=\frac{\arctan(x)}{3}+\frac{1}{3}\arctan(2x)$$

```
[In] Int[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[x]/3 + ArcTan[2*x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{3}\int \frac{1}{1+4x^2}\,dx + \frac{4}{3}\int \frac{1}{4+4x^2}\,dx \\ &= \frac{1}{3}\tan^{-1}(x) + \frac{1}{3}\tan^{-1}(2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = -\frac{1}{3}\arctan\left(\frac{3x}{-1+2x^2}\right)$$

```
[In] Integrate[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
[Out] -1/3*ArcTan[(3*x)/(-1 + 2*x^2)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{3}+\frac{\arctan(2x)}{3}$	12
risch	$\frac{\arctan\left(\frac{2x}{3}\right)}{3}+\frac{\arctan\left(\frac{4}{3}x^3+\frac{7}{3}x\right)}{3}$	20
parallelrisch	$-\frac{i\ln(x-i)}{6}+\frac{i\ln(x+i)}{6}-\frac{i\ln\left(x-\frac{i}{2}\right)}{6}+\frac{i\ln\left(x+\frac{i}{2}\right)}{6}$	34

```
[In] int((2*x^2+1)/(4*x^4+5*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/3*arctan(x)+1/3*arctan(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{1}{3}\,\arctan\left(\frac{4}{3}\,x^3+\frac{7}{3}\,x\right)+\frac{1}{3}\,\arctan\left(\frac{2}{3}\,x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
[Out] 1/3*arctan(4/3*x^3 + 7/3*x) + 1/3*arctan(2/3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{\operatorname{atan}\left(\frac{2x}{3}\right)}{3}+\frac{\operatorname{atan}\left(\frac{4x^3}{3}+\frac{7x}{3}\right)}{3}$$

```
[In] integrate((2*x**2+1)/(4*x**4+5*x**2+1),x)
[Out] atan(2*x/3)/3 + atan(4*x**3/3 + 7*x/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{1}{3}\,\arctan\left(2\,x\right)+\frac{1}{3}\,\arctan\left(x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{1}{3}\,\arctan\left(2\,x\right) + \frac{1}{3}\,\arctan\left(x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{\operatorname{atan}\left(\frac{2\,x}{3}\right)}{3} + \frac{\operatorname{atan}\left(\frac{4\,x^3}{3}+\frac{7\,x}{3}\right)}{3}$$

```
[In] int((2*x^2 + 1)/(5*x^2 + 4*x^4 + 1),x)
```

```
[Out] atan((2*x)/3)/3 + atan((7*x)/3 + (4*x^3)/3)/3
```

3.43 $\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx=\frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {28, 21, 209}

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx=\frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[In] Int[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= 4\int \frac{1+2x^2}{(2+4x^2)^2}\,dx \\ &= \int \frac{1}{1+2x^2}\,dx \\ &= \frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx = \frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[In] Integrate[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	12
risch	$\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	12

```
[In] int((2*x^2+1)/(4*x^4+4*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx=\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx=\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

```
[In] integrate((2*x**2+1)/(4*x**4+4*x**2+1),x)
[Out] sqrt(2)*atan(sqrt(2)*x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx=\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx=\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\right)}{2}$$

```
[In] int((2*x^2 + 1)/(4*x^2 + 4*x^4 + 1),x)
[Out] (2^(1/2)*atan(2^(1/2)*x))/2
```

3.44 $\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx=-\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\arctan\left(\frac{1+4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -1/7*arctan(1/7*(1-4*x)*7^(1/2))*7^(1/2)+1/7*arctan(1/7*(1+4*x)*7^(1/2))*7^
(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1175, 632, 210}

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx=\frac{\arctan\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[In] Int[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(1 - 4*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4*x)/Sqrt[7]]/Sqrt[7]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{x}{2}+x^2}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{7}{4}-x^2}\,dx,x,-\frac{1}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{7}{4}-x^2}\,dx,x,\frac{1}{2}+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\frac{1+4x}{\sqrt{7}}\right)}{\sqrt{7}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.55

$$
\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx=\frac{\left(-i+\sqrt{7}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3-i\sqrt{7}\right)}}\right)}{\sqrt{42-14i\sqrt{7}}}+\frac{\left(i+\sqrt{7}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3+i\sqrt{7}\right)}}\right)}{\sqrt{42+14i\sqrt{7}}}
$$

```
[In] Integrate[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 - I*Sqrt[7])/2]])/Sqrt[42 - (14*I)*Sqr
t[7]] + ((I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 + I*Sqrt[7])/2]])/Sqrt[42 + (14
*I)*Sqrt[7]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{7}\,\arctan\left(\frac{(4x-1)\sqrt{7}}{7}\right)}{7}+\frac{\arctan\left(\frac{(1+4x)\sqrt{7}}{7}\right)\sqrt{7}}{7}$	34
risch	$\frac{\sqrt{7}\,\arctan\left(\frac{2x\sqrt{7}}{7}\right)}{7}+\frac{\sqrt{7}\,\arctan\left(\frac{4x^3\sqrt{7}}{7}+\frac{5x\sqrt{7}}{7}\right)}{7}$	35

```
[In] int((2*x^2+1)/(4*x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*7^(1/2)*arctan(1/7*(4*x-1)*7^(1/2))+1/7*arctan(1/7*(1+4*x)*7^(1/2))*7^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx=\frac{1}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x^3+5\,x)\right)+\frac{1}{7}\,\sqrt{7}\arctan\left(\frac{2}{7}\,\sqrt{7}x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x^3 + 5*x)) + 1/7*sqrt(7)*arctan(2/7*sqrt
(7)*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx=\frac{\sqrt{7}\cdot\left(2\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}\right)+2\,\mathrm{atan}\left(\frac{4\sqrt{7}x^3}{7}+\frac{5\sqrt{7}x}{7}\right)\right)}{14}$$

```
[In] integrate((2*x**2+1)/(4*x**4+3*x**2+1),x)
```

```
[Out] sqrt(7)*(2*atan(2*sqrt(7)*x/7) + 2*atan(4*sqrt(7)*x**3/7 + 5*sqrt(7)*x/7))/
14
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x+1)\right)+\frac{1}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*
(4*x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x+1)\right)+\frac{1}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*
(4*x - 1))
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx = \frac{\sqrt{7}\left(\mathrm{atan}\left(\frac{4\sqrt{7}\,x^3}{7}+\frac{5\sqrt{7}\,x}{7}\right)+\mathrm{atan}\left(\frac{2\sqrt{7}\,x}{7}\right)\right)}{7}$$

```
[In] int((2*x^2 + 1)/(3*x^2 + 4*x^4 + 1),x)
[Out] (7^(1/2)*(atan((5*7^(1/2)*x)/7 + (4*7^(1/2)*x^3)/7) + atan((2*7^(1/2)*x)/7)
))/7
```

3.45 $\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = -\frac{\arctan\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} + \frac{\arctan\left(\frac{1+2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -1/6*arctan(1/3*(1-2*x*2^(1/2))*3^(1/2))*6^(1/2)+1/6*arctan(1/3*(1+2*x*2^(1
/2))*3^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \frac{\arctan\left(\frac{2\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}} - \frac{\arctan\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[In] Int[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(1 - 2*Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTan[(1 + 2*Sqrt[2]*x)/Sqr
t[3]]/Sqrt[6]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{x}{\sqrt{2}}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{x}{\sqrt{2}}+x^2}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-x^2}\,dx,x,-\frac{1}{\sqrt{2}}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-x^2}\,dx,x,\frac{1}{\sqrt{2}}+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.06

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx=\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{2x}{\sqrt{1-i\sqrt{3}}}\right)}{2\sqrt{3\left(1-i\sqrt{3}\right)}}+\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{2x}{\sqrt{1+i\sqrt{3}}}\right)}{2\sqrt{3\left(1+i\sqrt{3}\right)}}$$

```
[In] Integrate[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 - I*Sqrt[3]]])/(2*Sqrt[3*(1 - I*Sqrt[3]
)]) + ((I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 + I*Sqrt[3]]])/(2*Sqrt[3*(1 + I*Sq
rt[3])])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{\sqrt{6}\,\arctan\left(\frac{x\sqrt{6}}{3}\right)}{6}+\frac{\sqrt{6}\,\arctan\left(\frac{2x^{3}\sqrt{6}}{3}+\frac{2x\sqrt{6}}{3}\right)}{6}$	35
default	$\frac{\sqrt{6}\,\arctan\left(\frac{\left(4x-\sqrt{2}\right)\sqrt{6}}{6}\right)}{6}+\frac{\sqrt{6}\,\arctan\left(\frac{\left(4x+\sqrt{2}\right)\sqrt{6}}{6}\right)}{6}$	40

```
[In] int((2*x^2+1)/(4*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*6^(1/2)*arctan(1/3*x*6^(1/2))+1/6*6^(1/2)*arctan(2/3*x^3*6^(1/2)+2/3*x*
6^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx=\frac{1}{6}\,\sqrt{6}\arctan\left(\frac{2}{3}\,\sqrt{6}(x^3+x)\right)+\frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{3}\,\sqrt{6}x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(2/3*sqrt(6)*(x^3 + x)) + 1/6*sqrt(6)*arctan(1/3*sqrt(6)*
x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx=\frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}\right)+2\,\mathrm{atan}\left(\frac{2\sqrt{6}x^3}{3}+\frac{2\sqrt{6}x}{3}\right)\right)}{12}$$

```
[In] integrate((2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3) + 2*atan(2*sqrt(6)*x**3/3 + 2*sqrt(6)*x/3))/12
```

Maxima [F]

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4+2\,x^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{4}{3}\,\sqrt{3}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x+\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{4}{3}\,\sqrt{3}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x-\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arctan(4/3*sqrt(3)*(1/4)^(3/4)*(2*x + (1/4)^(1/4))) + 1/6*sqrt(
6)*arctan(4/3*sqrt(3)*(1/4)^(3/4)*(2*x - (1/4)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \frac{\sqrt{6}\,\left(\mathrm{atan}\left(\frac{2\,\sqrt{6}\,x^3}{3}+\frac{2\,\sqrt{6}\,x}{3}\right)+\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}\right)\right)}{6}$$

```
[In] int((2*x^2 + 1)/(2*x^2 + 4*x^4 + 1),x)
```

```
[Out] (6^(1/2)*(atan((2*6^(1/2)*x)/3 + (2*6^(1/2)*x^3)/3) + atan((6^(1/2)*x)/3)))
/6
```

3.46 $\int \frac{1+2x^2}{1+x^2+4x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\arctan\left(\frac{\sqrt{3}+4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -1/5*arctan(1/5*(-4*x+3^(1/2))*5^(1/2))*5^(1/2)+1/5*arctan(1/5*(4*x+3^(1/2)
)*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1175, 632, 210}

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx=\frac{\arctan\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\arctan\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[In] Int[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[3] - 4*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4*x)/Sqrt[5]
]/Sqrt[5]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{3}x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{3}x}{2}+x^2}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{5}{4}-x^2}\,dx,x,-\frac{\sqrt{3}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{5}{4}-x^2}\,dx,x,\frac{\sqrt{3}}{2}+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}+4x}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \frac{\left(-3i+\sqrt{15}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{15}\right)}}\right)}{\sqrt{30-30i\sqrt{15}}}+\frac{\left(3i+\sqrt{15}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{15}\right)}}\right)}{\sqrt{30+30i\sqrt{15}}}$$

```
[In] Integrate[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 - I*Sqrt[15])/2]])/Sqrt[30 - (30*I)
*Sqrt[15]] + ((3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 + I*Sqrt[15])/2]])/Sqrt
[30 + (30*I)*Sqrt[15]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{5}\,\arctan\left(\frac{2x\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\,\arctan\left(\frac{4x^{3}\sqrt{5}}{5}+\frac{3x\sqrt{5}}{5}\right)}{5}$	35
default	$\frac{\arctan\left(\frac{\left(4x+\sqrt{3}\right)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\sqrt{5}\,\arctan\left(\frac{\left(4x-\sqrt{3}\right)\sqrt{5}}{5}\right)}{5}$	40

```
[In] int((2*x^2+1)/(4*x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*5^(1/2)*arctan(2/5*x*5^(1/2))+1/5*5^(1/2)*arctan(4/5*x^3*5^(1/2)+3/5*x*
5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx=\frac{1}{5}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}\left(4\,x^{3}+3\,x\right)\right)+\frac{1}{5}\,\sqrt{5}\arctan\left(\frac{2}{5}\,\sqrt{5}x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(4*x^3 + 3*x)) + 1/5*sqrt(5)*arctan(2/5*sqrt
(5)*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx=\frac{\sqrt{5}\cdot\left(2\operatorname{atan}\left(\frac{2\sqrt{5}x}{5}\right)+2\operatorname{atan}\left(\frac{4\sqrt{5}x^{3}}{5}+\frac{3\sqrt{5}x}{5}\right)\right)}{10}$$

```
[In] integrate((2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] sqrt(5)*(2*atan(2*sqrt(5)*x/5) + 2*atan(4*sqrt(5)*x**3/5 + 3*sqrt(5)*x/5))/
10
```

Maxima [F]

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4+x^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x+\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x-\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*arctan(2/5*sqrt(10)*(1/4)^(3/4)*(4*x + sqrt(6)*(1/4)^(1/4))) +
1/5*sqrt(5)*arctan(2/5*sqrt(10)*(1/4)^(3/4)*(4*x - sqrt(6)*(1/4)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \frac{\sqrt{5}\left(\mathrm{atan}\left(\frac{4\sqrt{5}\,x^3}{5}+\frac{3\sqrt{5}\,x}{5}\right)+\mathrm{atan}\left(\frac{2\sqrt{5}\,x}{5}\right)\right)}{5}$$

```
[In] int((2*x^2 + 1)/(x^2 + 4*x^4 + 1),x)
```

```
[Out] (5^(1/2)*(atan((3*5^(1/2)*x)/5 + (4*5^(1/2)*x^3)/5) + atan((2*5^(1/2)*x)/5)
))/5
```

3.47 $\int \frac{1+2x^2}{1+4x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1+2x^2}{1+4x^4}\,dx=-\frac{1}{2}\arctan(1-2x)+\frac{1}{2}\arctan(1+2x)$$

```
[Out] 1/2*arctan(-1+2*x)+1/2*arctan(1+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1176, 631, 210}

$$\int \frac{1+2x^2}{1+4x^4}\,dx=\frac{1}{2}\arctan(2x+1)-\frac{1}{2}\arctan(1-2x)$$

```
[In] Int[(1 + 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -1/2*ArcTan[1 - 2*x] + ArcTan[1 + 2*x]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+x+x^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-2x\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+2x\right)\\
&= -\frac{1}{2}\tan^{-1}(1-2x)+\frac{1}{2}\tan^{-1}(1+2x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x^2}{1+4x^4}\,dx=-\frac{1}{2}\arctan\left(\frac{2x}{-1+2x^2}\right)$$

```
[In] Integrate[(1 + 2*x^2)/(1 + 4*x^4),x]
[Out] -1/2*ArcTan[(2*x)/(-1 + 2*x^2)]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result
risch	$\frac{\arctan(x)}{2}+\frac{\arctan(2x^3+x)}{2}$
default	$\frac{\arctan(2x-1)}{2}+\frac{\arctan(1+2x)}{2}$
parallelrisch	$-\frac{i\ln(x-\frac{1}{2}-\frac{i}{2})}{4}+\frac{i\ln(x-\frac{1}{2}+\frac{i}{2})}{4}-\frac{i\ln(x+\frac{1}{2}-\frac{i}{2})}{4}+\frac{i\ln(x+\frac{1}{2}+\frac{i}{2})}{4}$
meijerg	$\frac{\sqrt{2}\left(\frac{x^3\sqrt{2}\,\ln\left(1-2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\left(x^4\right)^{\frac{1}{4}}}{1-\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\left(x^4\right)^{\frac{1}{4}}}{1+\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}\right)}{8}+-$

```
[In] int((2*x^2+1)/(4*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/2*arctan(2*x^3+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+2x^2}{1+4x^4}\,dx=\frac{1}{2}\,\arctan\left(2\,x^3+x\right)+\frac{1}{2}\,\arctan\left(x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
[Out] 1/2*arctan(2*x^3 + x) + 1/2*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1+2x^2}{1+4x^4}\,dx=\frac{\operatorname{atan}(x)}{2}+\frac{\operatorname{atan}(2x^3+x)}{2}$$

```
[In] integrate((2*x**2+1)/(4*x**4+1),x)
[Out] atan(x)/2 + atan(2*x**3 + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x^2}{1+4x^4}\,dx = \frac{1}{2}\,\arctan\left(2\,x+1\right)+\frac{1}{2}\,\arctan\left(2\,x-1\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(2*x + 1) + 1/2*arctan(2*x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{1+2x^2}{1+4x^4}\,dx = \frac{1}{2}\,\arctan\left(2\,\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{2}\,\arctan\left(2\,\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x + sqrt(2)*(1/4)^(1/4))) + 1/2*arctan(
2*sqrt(2)*(1/4)^(3/4)*(2*x - sqrt(2)*(1/4)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+2x^2}{1+4x^4}\,dx = \frac{\mathrm{atan}(2\,x^3+x)}{2}+\frac{\mathrm{atan}(x)}{2}$$

```
[In] int((2*x^2 + 1)/(4*x^4 + 1),x)
```

```
[Out] atan(x + 2*x^3)/2 + atan(x)/2
```

3.48 $\int \frac{1+2x^2}{1-x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{5}+4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(1/3*(-4*x+5^(1/2))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(4*x+5^(1/2)
)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1175, 632, 210}

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{\arctan\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[5] - 4*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4*x)/Sqrt[3]
]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}x}{2}+x^2}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx,x,-\frac{\sqrt{5}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx,x,\frac{\sqrt{5}}{2}+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{5}+4x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.20

$$
\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{\left(-5i+\sqrt{15}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1-i\sqrt{15}\right)}} + \frac{\left(5i+\sqrt{15}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1+i\sqrt{15}\right)}}
$$

```
[In] Integrate[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] ((-5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 - I*Sqrt[15])/2]])/Sqrt[30*(-1 - I
*Sqrt[15])] + ((5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 + I*Sqrt[15])/2]])/Sq
rt[30*(-1 + I*Sqrt[15])]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{2x\sqrt{3}}{3}\right)}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{4x^{3}\sqrt{3}}{3}+\frac{x\sqrt{3}}{3}\right)}{3}$	35
default	$\frac{\arctan\left(\frac{\left(4x+\sqrt{5}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(4x-\sqrt{5}\right)\sqrt{3}}{3}\right)}{3}$	40

```
[In] int((2*x^2+1)/(4*x^4-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*arctan(2/3*x*3^(1/2))+1/3*3^(1/2)*arctan(4/3*x^3*3^(1/2)+1/3*x*
3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx=\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(4\,x^3+x)\right)+\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(4*x^3 + x)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3
)*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx=\frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{4\sqrt{3}x^3}{3}+\frac{\sqrt{3}x}{3}\right)\right)}{6}$$

```
[In] integrate((2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] sqrt(3)*(2*atan(2*sqrt(3)*x/3) + 2*atan(4*sqrt(3)*x**3/3 + sqrt(3)*x/3))/6
```

Maxima [F]

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-x^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{6}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x+\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{6}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x-\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(6)*(1/4)^(3/4)*(4*x + sqrt(10)*(1/4)^(1/4))) +
1/3*sqrt(3)*arctan(2/3*sqrt(6)*(1/4)^(3/4)*(4*x - sqrt(10)*(1/4)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{4\sqrt{3}\,x^3}{3}+\frac{\sqrt{3}\,x}{3}\right)+\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}\right)\right)}{3}$$

```
[In] int((2*x^2 + 1)/(4*x^4 - x^2 + 1),x)
```

```
[Out] (3^(1/2)*(atan((3^(1/2)*x)/3 + (4*3^(1/2)*x^3)/3) + atan((2*3^(1/2)*x)/3)))
/3
```

3.49 $\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx=-\frac{\arctan\left(\sqrt{3}-2\sqrt{2}x\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{3}+2\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(2*x*2^(1/2)-3^(1/2))*2^(1/2)+1/2*arctan(2*x*2^(1/2)+3^(1/2))*2^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1175, 632, 210}

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx=\frac{\arctan\left(2\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{3}-2\sqrt{2}x\right)}{\sqrt{2}}$$

```
[In] Int[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[Sqrt[3] - 2*Sqrt[2]*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2*Sqrt[2]*x]/Sq
rt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\sqrt{\frac{3}{2}}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\sqrt{\frac{3}{2}}x+x^2}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2}\,dx,x,-\sqrt{\frac{3}{2}}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2}\,dx,x,\sqrt{\frac{3}{2}}+2x\right)\\
&= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{3}+2\sqrt{2}x\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.25

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx=\frac{\left(-3i+\sqrt{3}\right)\arctan\left(\frac{2x}{\sqrt{-1-i\sqrt{3}}}\right)}{2\sqrt{3\left(-1-i\sqrt{3}\right)}}+\frac{\left(3i+\sqrt{3}\right)\arctan\left(\frac{2x}{\sqrt{-1+i\sqrt{3}}}\right)}{2\sqrt{3\left(-1+i\sqrt{3}\right)}}$$

```
[In] Integrate[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 - I*Sqrt[3]]])/(2*Sqrt[3*(-1 - I*Sqr
t[3])]) + ((3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 + I*Sqrt[3]]])/(2*Sqrt[3*(-
1 + I*Sqrt[3])])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}+\frac{\sqrt{2}\,\arctan\left(2x^{3}\sqrt{2}\right)}{2}$	27
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\left(4x-\sqrt{6}\right)\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{\left(4x+\sqrt{6}\right)\sqrt{2}}{2}\right)}{2}$	40

```
[In] int((2*x^2+1)/(4*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x*2^(1/2))*2^(1/2)+1/2*2^(1/2)*arctan(2*x^3*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx=\frac{1}{2}\,\sqrt{2}\arctan\left(2\,\sqrt{2}x^{3}\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(2*sqrt(2)*x^3) + 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx=\frac{\sqrt{2}\cdot\left(2\operatorname{atan}\left(\sqrt{2}x\right)+2\operatorname{atan}\left(2\sqrt{2}x^{3}\right)\right)}{4}$$

```
[In] integrate((2*x**2+1)/(4*x**4-2*x**2+1),x)
[Out] sqrt(2)*(2*atan(sqrt(2)*x) + 2*atan(2*sqrt(2)*x**3))/4
```

Maxima [F]

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-2\,x^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{2}\,\sqrt{2}\arctan\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(4*(1/4)^(3/4)*(2*x + sqrt(3)*(1/4)^(1/4))) + 1/2*sqrt(2)
*arctan(4*(1/4)^(3/4)*(2*x - sqrt(3)*(1/4)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx = \frac{\sqrt{2}\,\left(\mathrm{atan}\left(\sqrt{2}\,x\right)+\mathrm{atan}\left(2\,\sqrt{2}\,x^3\right)\right)}{2}$$

```
[In] int((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1),x)
```

```
[Out] (2^(1/2)*(atan(2^(1/2)*x) + atan(2*2^(1/2)*x^3)))/2
```

3.50 $\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 23

$$\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx = -\arctan\left(\sqrt{7}-4x\right)+\arctan\left(\sqrt{7}+4x\right)$$

```
[Out] arctan(4*x-7^(1/2))+arctan(4*x+7^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1175, 632, 210}

$$\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx = \arctan\left(4x+\sqrt{7}\right)-\arctan\left(\sqrt{7}-4x\right)$$

```
[In] Int[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[Sqrt[7] - 4*x] + ArcTan[Sqrt[7] + 4*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{7}x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{7}x}{2}+x^2}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}-x^2}\,dx,x,-\frac{\sqrt{7}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}-x^2}\,dx,x,\frac{\sqrt{7}}{2}+2x\right) \\
&= -\tan^{-1}\left(\sqrt{7}-4x\right)+\tan^{-1}\left(\sqrt{7}+4x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = -\arctan\left(\frac{x}{-1+2x^2}\right)$$

```
[In] Integrate[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
[Out] -ArcTan[x/(-1 + 2*x^2)]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
risch	$\arctan\left(4x^3-x\right)+\arctan\left(2x\right)$	16
default	$\arctan\left(4x-\sqrt{7}\right)+\arctan\left(4x+\sqrt{7}\right)$	20
parallelrisch	$-\frac{i\ln\left(x^2-\frac{1}{2}ix-\frac{1}{2}\right)}{2}+\frac{i\ln\left(x^2+\frac{1}{2}ix-\frac{1}{2}\right)}{2}$	28

```
[In] int((2*x^2+1)/(4*x^4-3*x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(4*x^3-x)+arctan(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \arctan\left(4\,x^3-x\right)+\arctan\left(2\,x\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
[Out] arctan(4*x^3 - x) + arctan(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \operatorname{atan}(2x)+\operatorname{atan}\left(4x^3-x\right)$$

```
[In] integrate((2*x**2+1)/(4*x**4-3*x**2+1),x)
[Out] atan(2*x) + atan(4*x**3 - x)
```

Maxima [F]

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-3\,x^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
[Out] integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \arctan\left(2\,\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x+\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \arctan\left(2\,\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x-\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
[Out] arctan(2*sqrt(2)*(1/4)^(3/4)*(4*x + sqrt(14)*(1/4)^(1/4))) + arctan(2*sqrt(
2)*(1/4)^(3/4)*(4*x - sqrt(14)*(1/4)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \operatorname{atan}(2\,x) - \operatorname{atan}\left(x-4\,x^3\right)$$

```
[In] int((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1),x)
[Out] atan(2*x) - atan(x - 4*x^3)
```

3.51 $\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 11

$$\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx = \frac{x}{1-2x^2}$$

```
[Out] x/(-2*x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 391}

$$\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx = \frac{x}{1-2x^2}$$

```
[In] Int[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 - 2*x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=4\int\frac{1+2x^2}{\left(-2+4x^2\right)^2}\,dx\\&=\frac{x}{1-2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int\frac{1+2x^2}{1-4x^2+4x^4}\,dx=-\frac{x}{-1+2x^2}$$

```
[In] Integrate[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
[Out] -(x/(-1 + 2*x^2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{x}{2\left(x^2-\frac{1}{2}\right)}$	11
risch	$-\frac{x}{2\left(x^2-\frac{1}{2}\right)}$	11
gosper	$-\frac{x}{2x^2-1}$	13
norman	$-\frac{x}{2x^2-1}$	13
parallelrisch	$-\frac{x}{2x^2-1}$	13

```
[In] int((2*x^2+1)/(4*x^4-4*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*x/(x^2-1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx=-\frac{x}{2\,x^2-1}$$

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
[Out] -x/(2*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx=-\frac{x}{2x^2-1}$$

```
[In] integrate((2*x**2+1)/(4*x**4-4*x**2+1),x)
[Out] -x/(2*x**2 - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx=-\frac{x}{2\,x^2-1}$$

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
[Out] -x/(2*x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx=-\frac{x}{2\,x^2-1}$$

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
[Out] -x/(2*x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx = -\frac{x}{2\,\left(x^2-\frac{1}{2}\right)}$$

```
[In] int((2*x^2 + 1)/(4*x^4 - 4*x^2 + 1),x)
```

```
[Out] -x/(2*(x^2 - 1/2))
```

3.52 $\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = -\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(1+x)+\frac{1}{2}\log(1+2x)$$

```
[Out] -1/2*ln(1-2*x)+1/2*ln(1-x)-1/2*ln(1+x)+1/2*ln(1+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1175, 630, 31}

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = -\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(2x+1)$$

```
[In] Int[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - 2*x] + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2*x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{3x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{3x}{2}+x^2}\,dx\\ &= \frac{1}{2}\int \frac{1}{-1+x}\,dx-\frac{1}{2}\int \frac{1}{-\frac{1}{2}+x}\,dx+\frac{1}{2}\int \frac{1}{\frac{1}{2}+x}\,dx-\frac{1}{2}\int \frac{1}{1+x}\,dx\\ &= -\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(1+x)+\frac{1}{2}\log(1+2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx=-\frac{1}{2}\log\left(1-x-2x^2\right)+\frac{1}{2}\log\left(1+x-2x^2\right)$$

```
[In] Integrate[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
[Out] -1/2*Log[1 - x - 2*x^2] + Log[1 + x - 2*x^2]/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\ln(2x^2-x-1)}{2}-\frac{\ln(2x^2+x-1)}{2}$	26
parallelrisch	$\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}-\frac{\ln(x-\frac{1}{2})}{2}+\frac{\ln(x+\frac{1}{2})}{2}$	26
default	$\frac{\ln(1+2x)}{2}-\frac{\ln(2x-1)}{2}-\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	30
norman	$\frac{\ln(1+2x)}{2}-\frac{\ln(2x-1)}{2}-\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	30

```
[In] int((2*x^2+1)/(4*x^4-5*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(2*x^2-x-1)-1/2*ln(2*x^2+x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = -\frac{1}{2}\,\log\left(2\,x^2+x-1\right)+\frac{1}{2}\,\log\left(2\,x^2-x-1\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x^2 + x - 1) + 1/2*log(2*x^2 - x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = \frac{\log\left(x^2-\frac{x}{2}-\frac{1}{2}\right)}{2}-\frac{\log\left(x^2+\frac{x}{2}-\frac{1}{2}\right)}{2}$$

```
[In] integrate((2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] log(x**2 - x/2 - 1/2)/2 - log(x**2 + x/2 - 1/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = \frac{1}{2}\,\log\left(2\,x+1\right)-\frac{1}{2}\,\log\left(2\,x-1\right)-\frac{1}{2}\,\log\left(x+1\right)+\frac{1}{2}\,\log\left(x-1\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x + 1) - 1/2*log(2*x - 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = \frac{1}{2}\,\log\left(|2\,x+1|\right) - \frac{1}{2}\,\log\left(|2\,x-1|\right) - \frac{1}{2}\,\log\left(|x+1|\right) + \frac{1}{2}\,\log\left(|x-1|\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
[Out] 1/2*log(abs(2*x + 1)) - 1/2*log(abs(2*x - 1)) - 1/2*log(abs(x + 1)) + 1/2*l
og(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.36

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = -\mathrm{atanh}\left(\frac{x}{2\,x^2-1}\right)$$

```
[In] int((2*x^2 + 1)/(4*x^4 - 5*x^2 + 1),x)
[Out] -atanh(x/(2*x^2 - 1))
```

3.53 $\int \frac{1+2x^2}{1-6x^2+4x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \frac{1+2x^2}{1-6x^2+4x^4} \, dx = \frac{\operatorname{arctanh}\left(\sqrt{5}-2\sqrt{2}x\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\sqrt{5}+2\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(2*x*2^(1/2)-5^(1/2))*2^(1/2)-1/2*arctanh(2*x*2^(1/2)+5^(1/2))*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1175, 632, 212}

$$\int \frac{1+2x^2}{1-6x^2+4x^4} \, dx = \frac{\operatorname{arctanh}\left(\sqrt{5}-2\sqrt{2}x\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(2\sqrt{2}x+\sqrt{5}\right)}{\sqrt{2}}$$

```
[In] Int[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTanh[Sqrt[5] - 2*Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2*Sqrt[2]*x]/Sqr
t[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\sqrt{\frac{5}{2}}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\sqrt{\frac{5}{2}}x+x^2}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}-x^2}\,dx,x,-\sqrt{\frac{5}{2}}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}-x^2}\,dx,x,\sqrt{\frac{5}{2}}+2x\right) \\
&= \frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{5}+2\sqrt{2}x\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$
\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx=\frac{\log\left(1+\sqrt{2}x-2x^2\right)-\log\left(-1+\sqrt{2}x+2x^2\right)}{2\sqrt{2}}
$$

```
[In] Integrate[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
[Out] (Log[1 + Sqrt[2]*x - 2*x^2] - Log[-1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\sqrt{2}\,\ln\left(-x\sqrt{2}+2x^{2}-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(x\sqrt{2}+2x^{2}-1\right)}{4}$	39
default	$-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}-\frac{2\sqrt{5}\,\left(5+\sqrt{5}\right)\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$	82

```
[In] int((2*x^2+1)/(4*x^4-6*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*ln(-x*2^(1/2)+2*x^2-1)-1/4*2^(1/2)*ln(x*2^(1/2)+2*x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx=\frac{1}{4}\,\sqrt{2}\log\left(\frac{4\,x^{4}-2\,x^{2}-2\,\sqrt{2}(2\,x^{3}-x)+1}{4\,x^{4}-6\,x^{2}+1}\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((4*x^4 - 2*x^2 - 2*sqrt(2)*(2*x^3 - x) + 1)/(4*x^4 - 6*x^2
+ 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx=\frac{\sqrt{2}\log\left(x^{2}-\frac{\sqrt{2}x}{2}-\frac{1}{2}\right)}{4}-\frac{\sqrt{2}\log\left(x^{2}+\frac{\sqrt{2}x}{2}-\frac{1}{2}\right)}{4}$$

```
[In] integrate((2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x/2 - 1/2)/4 - sqrt(2)*log(x**2 + sqrt(2)*x/2 -
1/2)/4
```

Maxima [F]

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-6\,x^2+1}\,dx$$

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(35) = 70$.

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.75

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx = -\frac{1}{4}\,\sqrt{2}\log\left(\left|x+\frac{1}{4}\,\sqrt{10}+\frac{1}{4}\,\sqrt{2}\right|\right)+\frac{1}{4}\,\sqrt{2}\log\left(\left|x+\frac{1}{4}\,\sqrt{10}-\frac{1}{4}\,\sqrt{2}\right|\right) -\frac{1}{4}\,\sqrt{2}\log\left(\left|x-\frac{1}{4}\,\sqrt{10}+\frac{1}{4}\,\sqrt{2}\right|\right)+\frac{1}{4}\,\sqrt{2}\log\left(\left|x-\frac{1}{4}\,\sqrt{10}-\frac{1}{4}\,\sqrt{2}\right|\right)$$

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs
(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) +
1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2\,x^2-1}\right)}{2}$$

```
[In] int((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*x)/(2*x^2 - 1)))/2
```

3.54 $\int \frac{1-2x^2}{1+bx^2+4x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{1-2x^2}{1+bx^2+4x^4} \, dx = -\frac{\log \left(1-\sqrt{4-b}x+2x^2\right)}{2\sqrt{4-b}}+\frac{\log \left(1+\sqrt{4-b}x+2x^2\right)}{2\sqrt{4-b}}$$

```
[Out] -1/2*ln(1+2*x^2-x*(4-b)^(1/2))/(4-b)^(1/2)+1/2*ln(1+2*x^2+x*(4-b)^(1/2))/(4
-b)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1178, 642}

$$\int \frac{1-2x^2}{1+bx^2+4x^4} \, dx = \frac{\log \left(\sqrt{4-b}x+2x^2+1\right)}{2\sqrt{4-b}}-\frac{\log \left(-\sqrt{4-b}x+2x^2+1\right)}{2\sqrt{4-b}}$$

```
[In] Int[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[4 - b]*x + 2*x^2]/Sqrt[4 - b] + Log[1 + Sqrt[4 - b]*x + 2
*x^2]/(2*Sqrt[4 - b])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{\sqrt{4-b}}{2}+2x}{-\frac{1}{2}-\frac{1}{2}\sqrt{4-b}x-x^2}\,dx}{2\sqrt{4-b}}-\frac{\int \frac{\frac{\sqrt{4-b}}{2}-2x}{-\frac{1}{2}+\frac{1}{2}\sqrt{4-b}x-x^2}\,dx}{2\sqrt{4-b}}\\ &= -\frac{\log\left(1-\sqrt{4-b}x+2x^2\right)}{2\sqrt{4-b}}+\frac{\log\left(1+\sqrt{4-b}x+2x^2\right)}{2\sqrt{4-b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.92

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx=\frac{\frac{\left(4+b-\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{-16+b^2}}}\right)}{\sqrt{b-\sqrt{-16+b^2}}}-\frac{\left(4+b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{b+\sqrt{-16+b^2}}}\right)}{\sqrt{b+\sqrt{-16+b^2}}}}{\sqrt{2}\sqrt{-16+b^2}}$$

```
[In] Integrate[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b - Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]]
)/Sqrt[b - Sqrt[-16 + b^2]] - ((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*
x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-16
 + b^2])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{\ln\left(-2x^2\sqrt{4-b}+(4-b)x-\sqrt{4-b}\right)}{2\sqrt{4-b}}+\frac{\ln\left(-2x^2\sqrt{4-b}+x(b-4)-\sqrt{4-b}\right)}{2\sqrt{4-b}}$	78
default	$\frac{\left(-4-\sqrt{(b-4)(4+b)}-b\right)\arctan\left(\frac{4x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{2\sqrt{(b-4)(4+b)}+2b}}+\frac{\left(4-\sqrt{(b-4)(4+b)}+b\right)\arctan\left(\frac{4x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}$	128

```
[In] int((-2*x^2+1)/(4*x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(4-b)^(1/2)*ln(-2*x^2*(4-b)^(1/2)+(4-b)*x-(4-b)^(1/2))+1/2/(4-b)^(1/2)
*ln(-2*x^2*(4-b)^(1/2)+x*(b-4)-(4-b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.65

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = \left[-\frac{\sqrt{-b+4}\log\left(\frac{4\,x^4-(b-8)x^2+2\,(2\,x^3+x)\sqrt{-b+4}+1}{4\,x^4+bx^2+1}\right)}{2\,(b-4)},\ \frac{\sqrt{b-4}\arctan\left(\frac{4\,x^3+(b-2)x}{\sqrt{b-4}}\right)-\sqrt{b-4}\arctan\left(\frac{2\,x}{\sqrt{b-4}}\right)}{b-4}\right]$$

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b + 4)*log((4*x^4 - (b - 8)*x^2 + 2*(2*x^3 + x)*sqrt(-b + 4) +
1)/(4*x^4 + b*x^2 + 1))/(b - 4), (sqrt(b - 4)*arctan((4*x^3 + (b - 2)*x)/sq
rt(b - 4)) - sqrt(b - 4)*arctan(2*x/sqrt(b - 4)))/(b - 4)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = \frac{\sqrt{-\frac{1}{b-4}}\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b-4}}}{2}+2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2} - \frac{\sqrt{-\frac{1}{b-4}}\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b-4}}}{2}-2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}$$

```
[In] integrate((-2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 4))*log(x**2 + x*(-b*sqrt(-1/(b - 4))/2 + 2*sqrt(-1/(b - 4)))
+ 1/2)/2 - sqrt(-1/(b - 4))*log(x**2 + x*(b*sqrt(-1/(b - 4))/2 - 2*sqrt(-1/
(b - 4))) + 1/2)/2
```

Maxima [F]

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+bx^2+1}\,dx$$

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + b*x^2 + 1), x)
```

Giac [F]

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+bx^2+1}\,dx$$

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{2\,x}{\sqrt{b-4}}\right)-\operatorname{atan}\left(\frac{b^3\,x+4\,b^2\,x^3-2\,b^2\,x-16\,b\,x-64\,x^3+32\,x}{{\left(b-4\right)}^{3/2}\,\left(b+4\right)}\right)}{\sqrt{b-4}}$$

```
[In] int(-(2*x^2 - 1)/(b*x^2 + 4*x^4 + 1),x)
```

```
[Out] -(atan((2*x)/(b - 4)^(1/2)) - atan((32*x - 16*b*x - 2*b^2*x + b^3*x - 64*x^
3 + 4*b^2*x^3)/((b - 4)^(3/2)*(b + 4))))/(b - 4)^(1/2)
```

3.55 $\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx=\frac{\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(2*x/(1/2*10^(1/2)-1/2*2^(1/2)))*2^(1/2)-1/2*arctan(2*x/(1/2*10^(
1/2)+1/2*2^(1/2)))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1177, 209}

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx=\frac{\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

```
[In] Int[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/S
qrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(-1-\sqrt{5}\right)\int \frac{1}{3+\sqrt{5}+4x^2}\,dx+\left(-1+\sqrt{5}\right)\int \frac{1}{3-\sqrt{5}+4x^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = \frac{-\left(\left(-5+\sqrt{5}\right)\sqrt{3+\sqrt{5}}\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)\right)-\sqrt{3-\sqrt{5}}\left(5+\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{4\sqrt{5}}$$

```
[In] Integrate[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
[Out] (-((-5 + Sqrt[5])*Sqrt[3 + Sqrt[5]]*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]) - Sqrt
[3 - Sqrt[5]]*(5 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(4*Sqrt[5])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}+\frac{\sqrt{2}\,\arctan\left(2x^3\sqrt{2}+2x\sqrt{2}\right)}{2}$	34
default	$-\frac{2\sqrt{5}\,\left(5+\sqrt{5}\right)\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}$	82

```
[In] int((-2*x^2+1)/(4*x^4+6*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(x*2^(1/2))*2^(1/2)+1/2*2^(1/2)*arctan(2*x^3*2^(1/2)+2*x*2^(1/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(2\,\sqrt{2}(x^3+x)\right) - \frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(2*sqrt(2)*(x^3 + x)) - 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = -\frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\sqrt{2}x\right) - 2\,\mathrm{atan}\left(2\sqrt{2}x^3 + 2\sqrt{2}x\right)\right)}{4}$$

```
[In] integrate((-2*x**2+1)/(4*x**4+6*x**2+1),x)
[Out] -sqrt(2)*(2*atan(sqrt(2)*x) - 2*atan(2*sqrt(2)*x**3 + 2*sqrt(2)*x))/4
```

Maxima [F]

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+6\,x^2+1}\,dx$$

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 6*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{4\,x}{\sqrt{10}+\sqrt{2}}\right) + \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{4\,x}{\sqrt{10}-\sqrt{2}}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
[Out] -1/2*sqrt(2)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/2*sqrt(2)*arctan(4*x/(sqr
t(10) - sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx=\frac{\sqrt{2}\,\left(\mathrm{atan}\left(2\,\sqrt{2}\,x^3+2\,\sqrt{2}\,x\right)-\mathrm{atan}\left(\sqrt{2}\,x\right)\right)}{2}$$

```
[In] int(-(2*x^2 - 1)/(6*x^2 + 4*x^4 + 1),x)
[Out] (2^(1/2)*(atan(2*2^(1/2)*x + 2*2^(1/2)*x^3) - atan(2^(1/2)*x)))/2
```

3.56 $\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 9

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = -\arctan(x)+\arctan(2x)$$

```
[Out] -arctan(x)+arctan(2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1177, 209}

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = \arctan(2x)-\arctan(x)$$

```
[In] Int[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[x] + ArcTan[2*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{1+4x^2}\,dx-4\int \frac{1}{4+4x^2}\,dx\\ &= -\tan^{-1}(x)+\tan^{-1}(2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx=\arctan\left(\frac{x}{1+2x^2}\right)$$

```
[In] Integrate[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
[Out] ArcTan[x/(1 + 2*x^2)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$-\arctan\left(x\right)+\arctan\left(2x\right)$	10
risch	$-\arctan\left(2x\right)+\arctan\left(4x^3+3x\right)$	18
parallelrisch	$\frac{i\ln(x-i)}{2}-\frac{i\ln(x+i)}{2}-\frac{i\ln(x-\frac{i}{2})}{2}+\frac{i\ln(x+\frac{i}{2})}{2}$	34

```
[In] int((-2*x^2+1)/(4*x^4+5*x^2+1),x,method=_RETURNVERBOSE)
[Out] -arctan(x)+arctan(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = \arctan\left(4\,x^3+3\,x\right)-\arctan\left(2\,x\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
[Out] arctan(4*x^3 + 3*x) - arctan(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = -\operatorname{atan}\left(2x\right)+\operatorname{atan}\left(4x^3+3x\right)$$

```
[In] integrate((-2*x**2+1)/(4*x**4+5*x**2+1),x)
[Out] -atan(2*x) + atan(4*x**3 + 3*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = \arctan\left(2\,x\right)-\arctan\left(x\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
[Out] arctan(2*x) - arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = \arctan\left(2\,x\right)-\arctan\left(x\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
[Out] arctan(2*x) - arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = \operatorname{atan}\left(4\,x^3+3\,x\right)-\operatorname{atan}(2\,x)$$

```
[In] int(-(2*x^2 - 1)/(5*x^2 + 4*x^4 + 1),x)
```

```
[Out] atan(3*x + 4*x^3) - atan(2*x)
```

3.57 $\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 11

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{1+2x^2}$$

```
[Out] x/(2*x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {28, 391}

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2x^2+1}$$

```
[In] Int[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 + 2*x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=4\int\frac{1-2x^2}{\left(2+4x^2\right)^2}\,dx\\&=\frac{x}{1+2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int\frac{1-2x^2}{1+4x^2+4x^4}\,dx=\frac{x}{1+2x^2}$$

```
[In] Integrate[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
[Out] x/(1 + 2*x^2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{x}{2x^2+1}$	11
risch	$\frac{x}{2x^2+1}$	11
gosper	$\frac{x}{2x^2+1}$	12
norman	$\frac{x}{2x^2+1}$	12
parallelrisch	$\frac{x}{2x^2+1}$	12

```
[In] int((-2*x^2+1)/(4*x^4+4*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x/(x^2+1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2\,x^2+1}$$

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
[Out] x/(2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2x^2+1}$$

```
[In] integrate((-2*x**2+1)/(4*x**4+4*x**2+1),x)
[Out] x/(2*x**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2\,x^2+1}$$

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
[Out] x/(2*x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2\,x^2+1}$$

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
[Out] x/(2*x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2\,\left(x^2+\frac{1}{2}\right)}$$

```
[In] int(-(2*x^2 - 1)/(4*x^2 + 4*x^4 + 1),x)
[Out] x/(2*(x^2 + 1/2))
```

3.58 $\int \frac{1-2x^2}{1+3x^2+4x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{1-2x^2}{1+3x^2+4x^4} \, dx = -\frac{1}{2}\log\left(1-x+2x^2\right)+\frac{1}{2}\log\left(1+x+2x^2\right)$$

```
[Out] -1/2*ln(2*x^2-x+1)+1/2*ln(2*x^2+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1178, 642}

$$\int \frac{1-2x^2}{1+3x^2+4x^4} \, dx = \frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

```
[In] Int[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - x + 2*x^2] + Log[1 + x + 2*x^2]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
```

```
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int \frac{\frac{1}{2}+2x}{-\frac{1}{2}-\frac{x}{2}-x^2}\,dx\right)-\frac{1}{2}\int \frac{\frac{1}{2}-2x}{-\frac{1}{2}+\frac{x}{2}-x^2}\,dx\\ &= -\frac{1}{2}\log\left(1-x+2x^2\right)+\frac{1}{2}\log\left(1+x+2x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = -\frac{1}{2}\log\left(1-x+2x^2\right)+\frac{1}{2}\log\left(1+x+2x^2\right)$$

```
[In] Integrate[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
[Out] -1/2*Log[1 - x + 2*x^2] + Log[1 + x + 2*x^2]/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$-\frac{\ln\left(x^2-\frac{1}{2}x+\frac{1}{2}\right)}{2}+\frac{\ln\left(x^2+\frac{1}{2}x+\frac{1}{2}\right)}{2}$	24
default	$-\frac{\ln\left(2x^2-x+1\right)}{2}+\frac{\ln\left(2x^2+x+1\right)}{2}$	26
norman	$-\frac{\ln\left(2x^2-x+1\right)}{2}+\frac{\ln\left(2x^2+x+1\right)}{2}$	26
risch	$-\frac{\ln\left(2x^2-x+1\right)}{2}+\frac{\ln\left(2x^2+x+1\right)}{2}$	26

```
[In] int((-2*x^2+1)/(4*x^4+3*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^2-1/2*x+1/2)+1/2*ln(x^2+1/2*x+1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{2}\,\log\left(2\,x^2+x+1\right)-\frac{1}{2}\,\log\left(2\,x^2-x+1\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = -\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{2}+\frac{\log\left(x^2+\frac{x}{2}+\frac{1}{2}\right)}{2}$$

```
[In] integrate((-2*x**2+1)/(4*x**4+3*x**2+1),x)
[Out] -log(x**2 - x/2 + 1/2)/2 + log(x**2 + x/2 + 1/2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{2}\,\log\left(2\,x^2+x+1\right)-\frac{1}{2}\,\log\left(2\,x^2-x+1\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{2}\,\log\left(2\,x^2+x+1\right)-\frac{1}{2}\,\log\left(2\,x^2-x+1\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = \operatorname{atanh}\left(\frac{x}{2\,x^2+1}\right)$$

```
[In] int(-(2*x^2 - 1)/(3*x^2 + 4*x^4 + 1),x)
[Out] atanh(x/(2*x^2 + 1))
```

3.59 $\int \frac{1-2x^2}{1+2x^2+4x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{1-2x^2}{1+2x^2+4x^4} \, dx = -\frac{\log \left(1-\sqrt{2}x+2x^2\right)}{2\sqrt{2}}+\frac{\log \left(1+\sqrt{2}x+2x^2\right)}{2\sqrt{2}}$$

```
[Out] -1/4*ln(1+2*x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+2*x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1178, 642}

$$\int \frac{1-2x^2}{1+2x^2+4x^4} \, dx = \frac{\log \left(2x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log \left(2x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[In] Int[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[2]*x + 2*x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + 2*x^2]/(2*Sqr
t[2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{1}{\sqrt{2}}+2x}{-\frac{1}{2}-\frac{x}{\sqrt{2}}-x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{\frac{1}{\sqrt{2}}-2x}{-\frac{1}{2}+\frac{x}{\sqrt{2}}-x^2}\,dx}{2\sqrt{2}}\\ &= -\frac{\log\left(1-\sqrt{2}x+2x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+2x^2\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx = \frac{-\log\left(-1+\sqrt{2}x-2x^2\right)+\log\left(1+\sqrt{2}x+2x^2\right)}{2\sqrt{2}}$$

```
[In] Integrate[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[2]*x - 2*x^2] + Log[1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+2x^2+x\sqrt{2}\right)\sqrt{2}}{4}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+2x^2+x\sqrt{2}\right)\sqrt{2}}{4}$	39

```
[In] int((-2*x^2+1)/(4*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(1+2*x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+2*x^2+x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(\frac{4\,x^4+6\,x^2+2\,\sqrt{2}(2\,x^3+x)+1}{4\,x^4+2\,x^2+1}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((4*x^4 + 6*x^2 + 2*sqrt(2)*(2*x^3 + x) + 1)/(4*x^4 + 2*x^2
+ 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx = -\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}$$

```
[In] integrate((-2*x**2+1)/(4*x**4+2*x**2+1),x)
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x/2 + 1/2)/4 + sqrt(2)*log(x**2 + sqrt(2)*x/2 +
 1/2)/4
```

Maxima [F]

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+2\,x^2+1}\,dx$$

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(x^2+\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{4}\,\sqrt{2}\log\left(x^2-\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(x^2 + (1/4)^(1/4)*x + 1/2) - 1/4*sqrt(2)*log(x^2 - (1/4)^(1
/4)*x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx=\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2\,x^2+1}\right)}{2}$$

```
[In] int(-(2*x^2 - 1)/(2*x^2 + 4*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*x)/(2*x^2 + 1)))/2
```

3.60 $\int \frac{1-2x^2}{1+x^2+4x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = -\frac{\log\left(1-\sqrt{3}x+2x^2\right)}{2\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+2x^2\right)}{2\sqrt{3}}$$

```
[Out] -1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1178, 642}

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = \frac{\log\left(2x^2+\sqrt{3}x+1\right)}{2\sqrt{3}} - \frac{\log\left(2x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

```
[In] Int[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[3]*x + 2*x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + 2*x^2]/(2*Sqr
t[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{\sqrt{3}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{3}x}{2}-x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{\frac{\sqrt{3}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{3}x}{2}-x^2}\,dx}{2\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}x+2x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+2x^2\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx=\frac{-\log\left(-1+\sqrt{3}x-2x^2\right)+\log\left(1+\sqrt{3}x+2x^2\right)}{2\sqrt{3}}$$

```
[In] Integrate[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[3]*x - 2*x^2] + Log[1 + Sqrt[3]*x + 2*x^2])/(2*Sqrt[3])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+2x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+2x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	39

```
[In] int((-2*x^2+1)/(4*x^4+x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time $= 0.25$ (sec) , antiderivative size $= 43$, normalized size of antiderivative $= 0.86$

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(\frac{4\,x^4+7\,x^2+2\,\sqrt{3}(2\,x^3+x)+1}{4\,x^4+x^2+1}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log((4*x^4 + 7*x^2 + 2*sqrt(3)*(2*x^3 + x) + 1)/(4*x^4 + x^2 +
1))
```

Sympy [A] (verification not implemented)

Time $= 0.05$ (sec) , antiderivative size $= 46$, normalized size of antiderivative $= 0.92$

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = -\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}$$

```
[In] integrate((-2*x**2+1)/(4*x**4+x**2+1),x)
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x/2 + 1/2)/6 + sqrt(3)*log(x**2 + sqrt(3)*x/2 +
 1/2)/6
```

Maxima [F]

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+x^2+1}\,dx$$

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 + x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time $= 0.30$ (sec) , antiderivative size $= 41$, normalized size of antiderivative $= 0.82$

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(x^2+\frac{1}{2}\,\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{6}\,\sqrt{3}\log\left(x^2-\frac{1}{2}\,\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*log(x^2 + 1/2*sqrt(6)*(1/4)^(1/4)*x + 1/2) - 1/6*sqrt(3)*log(x^
2 - 1/2*sqrt(6)*(1/4)^(1/4)*x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{2\,x^2+1}\right)}{3}$$

```
[In] int(-(2*x^2 - 1)/(x^2 + 4*x^4 + 1),x)
[Out] (3^(1/2)*atanh((3^(1/2)*x)/(2*x^2 + 1)))/3
```

3.61 $\int \frac{1-2x^2}{1+4x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{1-2x^2}{1+4x^4} \, dx = -\frac{1}{4} \log \left(1-2x+2x^2\right)+\frac{1}{4} \log \left(1+2x+2x^2\right)$$

```
[Out] -1/4*ln(2*x^2-2*x+1)+1/4*ln(2*x^2+2*x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1179, 642}

$$\int \frac{1-2x^2}{1+4x^4} \, dx = \frac{1}{4} \log \left(2x^2+2x+1\right)-\frac{1}{4} \log \left(2x^2-2x+1\right)$$

```
[In] Int[(1 - 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -1/4*Log[1 - 2*x + 2*x^2] + Log[1 + 2*x + 2*x^2]/4
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{4}\int \frac{1+2x}{-\frac{1}{2}-x-x^2}\,dx\right)-\frac{1}{4}\int \frac{1-2x}{-\frac{1}{2}+x-x^2}\,dx\\
&= -\frac{1}{4}\log\left(1-2x+2x^2\right)+\frac{1}{4}\log\left(1+2x+2x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^4}\,dx = -\frac{1}{4}\log\left(1-2x+2x^2\right)+\frac{1}{4}\log\left(1+2x+2x^2\right)$$

```
[In] Integrate[(1 - 2*x^2)/(1 + 4*x^4),x]
[Out] -1/4*Log[1 - 2*x + 2*x^2] + Log[1 + 2*x + 2*x^2]/4
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result
parallelrisch	$-\frac{\ln\left(x^2-x+\frac{1}{2}\right)}{4}+\frac{\ln\left(x^2+x+\frac{1}{2}\right)}{4}$
default	$-\frac{\ln\left(2x^2-2x+1\right)}{4}+\frac{\ln\left(2x^2+2x+1\right)}{4}$
norman	$-\frac{\ln\left(2x^2-2x+1\right)}{4}+\frac{\ln\left(2x^2+2x+1\right)}{4}$
risch	$-\frac{\ln\left(2x^2-2x+1\right)}{4}+\frac{\ln\left(2x^2+2x+1\right)}{4}$
meijerg	$-\frac{\sqrt{2}\left(\frac{x^3\sqrt{2}\,\ln\left(1-2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\left(x^4\right)^{\frac{1}{4}}}{1-\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\left(x^4\right)^{\frac{1}{4}}}{1+\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}\right)}{8}+$

```
[In] int((-2*x^2+1)/(4*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(x^2-x+1/2)+1/4*ln(x^2+x+1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1-2x^2}{1+4x^4}\,dx = \frac{1}{4}\,\log\left(2\,x^2+2\,x+1\right)-\frac{1}{4}\,\log\left(2\,x^2-2\,x+1\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1-2x^2}{1+4x^4}\,dx = -\frac{\log\left(x^2-x+\frac{1}{2}\right)}{4}+\frac{\log\left(x^2+x+\frac{1}{2}\right)}{4}$$

```
[In] integrate((-2*x**2+1)/(4*x**4+1),x)
[Out] -log(x**2 - x + 1/2)/4 + log(x**2 + x + 1/2)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1-2x^2}{1+4x^4}\,dx = \frac{1}{4}\,\log\left(2\,x^2+2\,x+1\right)-\frac{1}{4}\,\log\left(2\,x^2-2\,x+1\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1-2x^2}{1+4x^4}\,dx = \frac{1}{4}\,\log\left(x^2+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{4}\,\log\left(x^2-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="giac")
[Out] 1/4*log(x^2 + sqrt(2)*(1/4)^(1/4)*x + 1/2) - 1/4*log(x^2 - sqrt(2)*(1/4)^(1
/4)*x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{1-2x^2}{1+4x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{2\,x}{2\,x^2+1}\right)}{2}$$

```
[In] int(-(2*x^2 - 1)/(4*x^4 + 1),x)
```

```
[Out] atanh((2*x)/(2*x^2 + 1))/2
```

3.62 $\int \frac{1-2x^2}{1-x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = -\frac{\log\left(1-\sqrt{5}x+2x^2\right)}{2\sqrt{5}}+\frac{\log\left(1+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}$$

```
[Out] -1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1178, 642}

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = \frac{\log\left(2x^2+\sqrt{5}x+1\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\sqrt{5}x+1\right)}{2\sqrt{5}}$$

```
[In] Int[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[5]*x + 2*x^2]/Sqrt[5] + Log[1 + Sqrt[5]*x + 2*x^2]/(2*Sqr
t[5])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{\sqrt{5}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{5}x}{2}-x^2}\,dx}{2\sqrt{5}}-\frac{\int \frac{\frac{\sqrt{5}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{5}x}{2}-x^2}\,dx}{2\sqrt{5}}\\ &= -\frac{\log\left(1-\sqrt{5}x+2x^2\right)}{2\sqrt{5}}+\frac{\log\left(1+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = \frac{-\log\left(-1+\sqrt{5}x-2x^2\right)+\log\left(1+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}$$

```
[In] Integrate[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[5]*x - 2*x^2] + Log[1 + Sqrt[5]*x + 2*x^2])/(2*Sqrt[5])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(1+2x^2+x\sqrt{5}\right)\sqrt{5}}{10}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(1+2x^2+x\sqrt{5}\right)\sqrt{5}}{10}$	39

```
[In] int((-2*x^2+1)/(4*x^4-x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = \frac{1}{10}\,\sqrt{5}\log\left(\frac{4\,x^4+9\,x^2+2\,\sqrt{5}(2\,x^3+x)+1}{4\,x^4-x^2+1}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
[Out] 1/10*sqrt(5)*log((4*x^4 + 9*x^2 + 2*sqrt(5)*(2*x^3 + x) + 1)/(4*x^4 - x^2 +
 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = -\frac{\sqrt{5}\log\left(x^2-\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}$$

```
[In] integrate((-2*x**2+1)/(4*x**4-x**2+1),x)
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x/2 + 1/2)/10 + sqrt(5)*log(x**2 + sqrt(5)*x/2
+ 1/2)/10
```

Maxima [F]

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4-x^2+1}\,dx$$

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{1-2x^2}{1-x^2+4x^4}\,dx &= \frac{1}{10}\,\sqrt{5}\log\left(x^2+\frac{1}{2}\,\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)\\ &\quad-\frac{1}{10}\,\sqrt{5}\log\left(x^2-\frac{1}{2}\,\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)\end{aligned}$$

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(x^2 + 1/2*sqrt(10)*(1/4)^(1/4)*x + 1/2) - 1/10*sqrt(5)*log
(x^2 - 1/2*sqrt(10)*(1/4)^(1/4)*x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,x}{2\,x^2+1}\right)}{5}$$

```
[In] int(-(2*x^2 - 1)/(4*x^4 - x^2 + 1),x)
```

```
[Out] (5^(1/2)*atanh((5^(1/2)*x)/(2*x^2 + 1)))/5
```

3.63 $\int \frac{1-2x^2}{1-2x^2+4x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{1-2x^2}{1-2x^2+4x^4} \, dx = -\frac{\log \left(1-\sqrt{6}x+2x^2\right)}{2\sqrt{6}}+\frac{\log \left(1+\sqrt{6}x+2x^2\right)}{2\sqrt{6}}$$

```
[Out] -1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1178, 642}

$$\int \frac{1-2x^2}{1-2x^2+4x^4} \, dx = \frac{\log \left(2x^2+\sqrt{6}x+1\right)}{2\sqrt{6}}-\frac{\log \left(2x^2-\sqrt{6}x+1\right)}{2\sqrt{6}}$$

```
[In] Int[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[6]*x + 2*x^2]/Sqrt[6] + Log[1 + Sqrt[6]*x + 2*x^2]/(2*Sqr
t[6])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\sqrt{\frac{3}{2}}+2x}{-\frac{1}{2}-\sqrt{\frac{3}{2}}x-x^2}\,dx}{2\sqrt{6}}-\frac{\int \frac{\sqrt{\frac{3}{2}}-2x}{-\frac{1}{2}+\sqrt{\frac{3}{2}}x-x^2}\,dx}{2\sqrt{6}}\\ &= -\frac{\log\left(1-\sqrt{6}x+2x^2\right)}{2\sqrt{6}}+\frac{\log\left(1+\sqrt{6}x+2x^2\right)}{2\sqrt{6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx=\frac{-\log\left(-1+\sqrt{6}x-2x^2\right)+\log\left(1+\sqrt{6}x+2x^2\right)}{2\sqrt{6}}$$

```
[In] Integrate[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[6]*x - 2*x^2] + Log[1 + Sqrt[6]*x + 2*x^2])/(2*Sqrt[6])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(1+2x^2+x\sqrt{6}\right)\sqrt{6}}{12}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(1+2x^2+x\sqrt{6}\right)\sqrt{6}}{12}$	39

```
[In] int((-2*x^2+1)/(4*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = \frac{1}{12}\,\sqrt{6}\log\left(\frac{4\,x^4+10\,x^2+2\,\sqrt{6}(2\,x^3+x)+1}{4\,x^4-2\,x^2+1}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*log((4*x^4 + 10*x^2 + 2*sqrt(6)*(2*x^3 + x) + 1)/(4*x^4 - 2*x^
2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = -\frac{\sqrt{6}\log\left(x^2-\frac{\sqrt{6}x}{2}+\frac{1}{2}\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\frac{\sqrt{6}x}{2}+\frac{1}{2}\right)}{12}$$

```
[In] integrate((-2*x**2+1)/(4*x**4-2*x**2+1),x)
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x/2 + 1/2)/12 + sqrt(6)*log(x**2 + sqrt(6)*x/2
+ 1/2)/12
```

Maxima [F]

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4-2\,x^2+1}\,dx$$

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = \frac{1}{12}\,\sqrt{6}\log\left(x^2+\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{12}\,\sqrt{6}\log\left(x^2-\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
[Out] 1/12*sqrt(6)*log(x^2 + sqrt(3)*(1/4)^(1/4)*x + 1/2) - 1/12*sqrt(6)*log(x^2
- sqrt(3)*(1/4)^(1/4)*x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{2\,x^2+1}\right)}{6}$$

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1),x)
[Out] (6^(1/2)*atanh((6^(1/2)*x)/(2*x^2 + 1)))/6
```

3.64 $\int \frac{1-2x^2}{1-3x^2+4x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{1-2x^2}{1-3x^2+4x^4} \, dx = -\frac{\log \left(1-\sqrt{7}x+2x^2\right)}{2\sqrt{7}}+\frac{\log \left(1+\sqrt{7}x+2x^2\right)}{2\sqrt{7}}$$

```
[Out] -1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1178, 642}

$$\int \frac{1-2x^2}{1-3x^2+4x^4} \, dx = \frac{\log \left(2x^2+\sqrt{7}x+1\right)}{2\sqrt{7}}-\frac{\log \left(2x^2-\sqrt{7}x+1\right)}{2\sqrt{7}}$$

```
[In] Int[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[7]*x + 2*x^2]/Sqrt[7] + Log[1 + Sqrt[7]*x + 2*x^2]/(2*Sqr
t[7])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\frac{\sqrt{7}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{7}x}{2}-x^2}\,dx}{2\sqrt{7}}-\frac{\int \frac{\frac{\sqrt{7}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{7}x}{2}-x^2}\,dx}{2\sqrt{7}}\\ &= -\frac{\log\left(1-\sqrt{7}x+2x^2\right)}{2\sqrt{7}}+\frac{\log\left(1+\sqrt{7}x+2x^2\right)}{2\sqrt{7}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = \frac{-\log\left(-1+\sqrt{7}x-2x^2\right)+\log\left(1+\sqrt{7}x+2x^2\right)}{2\sqrt{7}}$$

```
[In] Integrate[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[7]*x - 2*x^2] + Log[1 + Sqrt[7]*x + 2*x^2])/(2*Sqrt[7])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{7}\right)\sqrt{7}}{14}+\frac{\ln\left(1+2x^2+x\sqrt{7}\right)\sqrt{7}}{14}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{7}\right)\sqrt{7}}{14}+\frac{\ln\left(1+2x^2+x\sqrt{7}\right)\sqrt{7}}{14}$	39

```
[In] int((-2*x^2+1)/(4*x^4-3*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = \frac{1}{14}\,\sqrt{7}\log\left(\frac{4\,x^4+11\,x^2+2\,\sqrt{7}(2\,x^3+x)+1}{4\,x^4-3\,x^2+1}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((4*x^4 + 11*x^2 + 2*sqrt(7)*(2*x^3 + x) + 1)/(4*x^4 - 3*x^
2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = -\frac{\sqrt{7}\log\left(x^2-\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}$$

```
[In] integrate((-2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x/2 + 1/2)/14 + sqrt(7)*log(x**2 + sqrt(7)*x/2
+ 1/2)/14
```

Maxima [F]

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4-3\,x^2+1}\,dx$$

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = \frac{1}{14}\,\sqrt{7}\log\left(x^2+\frac{1}{2}\,\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right) - \frac{1}{14}\,\sqrt{7}\log\left(x^2-\frac{1}{2}\,\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*log(x^2 + 1/2*sqrt(14)*(1/4)^(1/4)*x + 1/2) - 1/14*sqrt(7)*log
(x^2 - 1/2*sqrt(14)*(1/4)^(1/4)*x + 1/2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx=\frac{\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}\,x}{2\,x^2+1}\right)}{7}$$

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1),x)
```

```
[Out] (7^(1/2)*atanh((7^(1/2)*x)/(2*x^2 + 1)))/7
```

3.65 $\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {28, 21, 212}

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[In] Int[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTanh[Sqrt[2]*x]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 4\int \frac{1-2x^2}{(-2+4x^2)^2}\,dx\\ &= \int \frac{1}{1-2x^2}\,dx\\ &= \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = \frac{-\log\left(\sqrt{2}-2x\right)+\log\left(\sqrt{2}+2x\right)}{2\sqrt{2}}$$

```
[In] Integrate[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
[Out] (-Log[Sqrt[2] - 2*x] + Log[Sqrt[2] + 2*x])/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\operatorname{arctanh}\left(x\sqrt{2}\right)\sqrt{2}}{2}$	12
risch	$\frac{\sqrt{2}\,\ln\left(2x+\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(2x-\sqrt{2}\right)}{4}$	30

```
[In] int((-2*x^2+1)/(4*x^4-4*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(x*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{2\,x^2+2\,\sqrt{2}x+1}{2\,x^2-1}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((2*x^2 + 2*sqrt(2)*x + 1)/(2*x^2 - 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = -\frac{\sqrt{2}\log\left(x-\frac{\sqrt{2}}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x+\frac{\sqrt{2}}{2}\right)}{4}$$

```
[In] integrate((-2*x**2+1)/(4*x**4-4*x**2+1),x)
[Out] -sqrt(2)*log(x - sqrt(2)/2)/4 + sqrt(2)*log(x + sqrt(2)/2)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = -\frac{1}{4}\sqrt{2}\log\left(\frac{2\,x-\sqrt{2}}{2\,x+\sqrt{2}}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
[Out] -1/4*sqrt(2)*log((2*x - sqrt(2))/(2*x + sqrt(2)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right) - \frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(x + 1/2*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/2*sqrt(2)
))
```

Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,x\right)}{2}$$

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 4*x^2 + 1),x)
```

```
[Out] (2^(1/2)*atanh(2^(1/2)*x))/2
```

3.66 $\int \frac{1-2x^2}{1-5x^2+4x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{1-2x^2}{1-5x^2+4x^4} \, dx = -\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(1+x)+\frac{1}{6}\log(1+2x)$$

```
[Out] -1/6*ln(1-2*x)-1/6*ln(1-x)+1/6*ln(1+x)+1/6*ln(1+2*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1175, 630, 31}

$$\int \frac{1-2x^2}{1-5x^2+4x^4} \, dx = -\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(x+1)+\frac{1}{6}\log(2x+1)$$

```
[In] Int[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -1/6*Log[1 - 2*x] - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2*x]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{4}\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx\right)-\frac{1}{4}\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx\\ &= -\left(\frac{1}{6}\int \frac{1}{-1+x}\,dx\right)-\frac{1}{6}\int \frac{1}{-\frac{1}{2}+x}\,dx+\frac{1}{6}\int \frac{1}{\frac{1}{2}+x}\,dx+\frac{1}{6}\int \frac{1}{1+x}\,dx\\ &= -\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(1+x)+\frac{1}{6}\log(1+2x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx = -\frac{1}{6}\log\left(1-3x+2x^2\right)+\frac{1}{6}\log\left(1+3x+2x^2\right)$$

```
[In] Integrate[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
[Out] -1/6*Log[1 - 3*x + 2*x^2] + Log[1 + 3*x + 2*x^2]/6
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln(x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln(x-\frac{1}{2})}{6}+\frac{\ln(x+\frac{1}{2})}{6}$	26
risch	$-\frac{\ln(2x^2-3x+1)}{6}+\frac{\ln(2x^2+3x+1)}{6}$	28
default	$\frac{\ln(1+2x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln(x-1)}{6}$	30
norman	$\frac{\ln(1+2x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln(x-1)}{6}$	30

```
[In] int((-2*x^2+1)/(4*x^4-5*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(x-1)+1/6*ln(x+1)-1/6*ln(x-1/2)+1/6*ln(x+1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx=\frac{1}{6}\,\log\left(2\,x^2+3\,x+1\right)-\frac{1}{6}\,\log\left(2\,x^2-3\,x+1\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
[Out] 1/6*log(2*x^2 + 3*x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx=-\frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

```
[In] integrate((-2*x**2+1)/(4*x**4-5*x**2+1),x)
[Out] -log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx=\frac{1}{6}\,\log\left(2\,x+1\right)-\frac{1}{6}\,\log\left(2\,x-1\right)+\frac{1}{6}\,\log\left(x+1\right)-\frac{1}{6}\,\log\left(x-1\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
[Out] 1/6*log(2*x + 1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx = \frac{1}{6}\,\log\left(|2\,x+1|\right) - \frac{1}{6}\,\log\left(|2\,x-1|\right) + \frac{1}{6}\,\log\left(|x+1|\right) - \frac{1}{6}\,\log\left(|x-1|\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*log(abs(2*x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*l
og(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 13.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{3\,x}{2\,x^2+1}\right)}{3}$$

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 5*x^2 + 1),x)
```

```
[Out] atanh((3*x)/(2*x^2 + 1))/3
```

3.67 $\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}+\frac{\operatorname{arctanh}\left(\frac{1+2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

```
[Out] -1/10*arctanh(1/5*(1-2*x*2^(1/2))*5^(1/2))*10^(1/2)+1/10*arctanh(1/5*(1+2*x
*2^(1/2))*5^(1/2))*10^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1175, 632, 212}

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{2}x+1}{\sqrt{5}}\right)}{\sqrt{10}}-\frac{\operatorname{arctanh}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

```
[In] Int[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*Sqrt[2]*x)/Sqrt[5]]/Sqrt[10]) + ArcTanh[(1 + 2*Sqrt[2]*x)/
Sqrt[5]]/Sqrt[10]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{4}\int \frac{1}{-\frac{1}{2}-\frac{x}{\sqrt{2}}+x^2}\,dx\right)-\frac{1}{4}\int \frac{1}{-\frac{1}{2}+\frac{x}{\sqrt{2}}+x^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2}\,dx, x, -\frac{1}{\sqrt{2}}+2x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2}\,dx, x, \frac{1}{\sqrt{2}}+2x\right)\\
&= -\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}+\frac{\tanh^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx = \frac{-\log\left(-1+\sqrt{10}x-2x^2\right)+\log\left(1+\sqrt{10}x+2x^2\right)}{2\sqrt{10}}$$

```
[In] Integrate[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[10]*x - 2*x^2] + Log[1 + Sqrt[10]*x + 2*x^2])/(2*Sqrt[10])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\sqrt{10}\,\ln\left(\sqrt{10}\,x+2x^{2}+1\right)}{20}-\frac{\sqrt{10}\,\ln\left(-\sqrt{10}\,x+2x^{2}+1\right)}{20}$	39
default	$\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}+\frac{2\left(\sqrt{5}+1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$	82

```
[In] int((-2*x^2+1)/(4*x^4-6*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*10^(1/2)*ln(10^(1/2)*x+2*x^2+1)-1/20*10^(1/2)*ln(-10^(1/2)*x+2*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx=\frac{1}{20}\,\sqrt{10}\log\left(\frac{4\,x^4+14\,x^2+2\,\sqrt{10}(2\,x^3+x)+1}{4\,x^4-6\,x^2+1}\right)$$

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(10)*log((4*x^4 + 14*x^2 + 2*sqrt(10)*(2*x^3 + x) + 1)/(4*x^4 - 6*
x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx=-\frac{\sqrt{10}\log\left(x^2-\frac{\sqrt{10}x}{2}+\frac{1}{2}\right)}{20}+\frac{\sqrt{10}\log\left(x^2+\frac{\sqrt{10}x}{2}+\frac{1}{2}\right)}{20}$$

```
[In] integrate((-2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] -sqrt(10)*log(x**2 - sqrt(10)*x/2 + 1/2)/20 + sqrt(10)*log(x**2 + sqrt(10)*
x/2 + 1/2)/20
```

Maxima [F]

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4-6\,x^2+1}\,dx$$

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 6*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\begin{aligned}\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx = {} & \frac{1}{20}\sqrt{10}\log\left(\left|x+\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right) \\ & +\frac{1}{20}\sqrt{10}\log\left(\left|x+\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right) \\ & -\frac{1}{20}\sqrt{10}\log\left(\left|x-\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right) \\ & -\frac{1}{20}\sqrt{10}\log\left(\left|x-\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)\end{aligned}$$

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/20*sqrt(10)*log(
abs(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(1
0) + 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.42

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx = \frac{\sqrt{10}\,\mathrm{atanh}\left(\frac{\sqrt{10}\,x}{2\,x^2+1}\right)}{10}$$

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 6*x^2 + 1),x)
```

```
[Out] (10^(1/2)*atanh((10^(1/2)*x)/(2*x^2 + 1)))/10
```

3.68 $\int \frac{1+x^2}{1+bx^2+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}+\frac{\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}$$

```
[Out] -arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))/(2+b)^(1/2)+arctan((2*x+(2-b)^(1/2)
)/(2+b)^(1/2))/(2+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1175, 632, 210}

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\arctan\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

```
[In] Int[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(Sqrt[2 - b
] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{2-b}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2-b}x+x^2}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx,x,-\sqrt{2-b}+2x\right)\\
&\quad -\text{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx,x,\sqrt{2-b}+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{-\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.00

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx=\frac{\frac{\left(2-b+\sqrt{-4+b^2}\right)\arctan\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{-4+b^2}}}\right)}{\sqrt{b-\sqrt{-4+b^2}}}+\frac{\left(-2+b+\sqrt{-4+b^2}\right)\arctan\left(\frac{\sqrt{2}x}{\sqrt{b+\sqrt{-4+b^2}}}\right)}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\sqrt{-4+b^2}}$$

```
[In] Integrate[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2 - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sq
rt[b - Sqrt[-4 + b^2]] + ((-2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt
[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{\ln\left(-x^{2}\sqrt{-2-b}+x(2+b)+\sqrt{-2-b}\right)}{2\sqrt{-2-b}}+\frac{\ln\left(-x^{2}\sqrt{-2-b}+(-2-b)x+\sqrt{-2-b}\right)}{2\sqrt{-2-b}}$	74
default	$\frac{\left(-2+\sqrt{(b-2)(2+b)}+b\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(2+\sqrt{(b-2)(2+b)}-b\right)\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$	124

```
[In] int((x^2+1)/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2/(-2-b)^(1/2)*ln(-x^2*(-2-b)^(1/2)+x*(2+b)+(-2-b)^(1/2))+1/2/(-2-b)^(1/
2)*ln(-x^2*(-2-b)^(1/2)+(-2-b)*x+(-2-b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx = \left[-\frac{\sqrt{-b-2}\log\left(\frac{x^4-(b+4)x^2-2\,(x^3-x)\sqrt{-b-2}+1}{x^4+bx^2+1}\right)}{2\,(b+2)}, \frac{\sqrt{b+2}\arctan\left(\frac{x^3+(b+1)x}{\sqrt{b+2}}\right)+\sqrt{b+2}\arctan\left(\frac{x}{\sqrt{b+2}}\right)}{b+2}\right]$$

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
[Out] [-1/2*sqrt(-b - 2)*log((x^4 - (b + 4)*x^2 - 2*(x^3 - x)*sqrt(-b - 2) + 1)/(
x^4 + b*x^2 + 1))/(b + 2), (sqrt(b + 2)*arctan((x^3 + (b + 1)*x)/sqrt(b + 2
)) + sqrt(b + 2)*arctan(x/sqrt(b + 2)))/(b + 2)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.42

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx = -\frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b+2}}-2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2} + \frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b+2}}+2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}$$

```
[In] integrate((x**2+1)/(x**4+b*x**2+1),x)
[Out] -sqrt(-1/(b + 2))*log(x**2 + x*(-b*sqrt(-1/(b + 2)) - 2*sqrt(-1/(b + 2))) -
 1)/2 + sqrt(-1/(b + 2))*log(x**2 + x*(b*sqrt(-1/(b + 2)) + 2*sqrt(-1/(b +
2))) - 1)/2
```

Maxima [F]

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx=\int \frac{x^2+1}{x^4+bx^2+1}\,dx$$

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + b*x^2 + 1), x)
```

Giac [F]

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx=\int \frac{x^2+1}{x^4+bx^2+1}\,dx$$

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx=\frac{\operatorname{atan}\left(\frac{x}{\sqrt{b+2}}\right)+\operatorname{atan}\left((b+2)\,\left(x\,\left(\frac{1}{\sqrt{b+2}}+\frac{\frac{4}{b+2}-1}{(b-2)\,\sqrt{b+2}}\right)+\frac{x^3\,\left(\frac{2\,b}{b+2}-1\right)}{(b-2)\,\sqrt{b+2}}\right)\right)}{\sqrt{b+2}}$$

```
[In] int((x^2 + 1)/(b*x^2 + x^4 + 1),x)
```

```
[Out] (atan(x/(b + 2)^(1/2)) + atan((b + 2)*(x*(1/(b + 2)^(1/2) + (4/(b + 2) - 1)
/((b - 2)*(b + 2)^(1/2))) + (x^3*((2*b)/(b + 2) - 1))/((b - 2)*(b + 2)^(1/2
)))))/(b + 2)^(1/2)
```

3.69 $\int \frac{1+x^2}{1+5x^2+x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{1+x^2}{1+5x^2+x^4} \, dx = \frac{\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}} + \frac{\arctan\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{7}}$$

```
[Out] 1/7*arctan(x*2^(1/2)/(5+21^(1/2))^(1/2))*7^(1/2)+1/7*arctan(x*(1/2*7^(1/2)+
1/2*3^(1/2)))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1177, 209}

$$\int \frac{1+x^2}{1+5x^2+x^4} \, dx = \frac{\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}} + \frac{\arctan\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{7}}$$

```
[In] Int[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[21])/2]*x]
/Sqrt[7]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{14}\left(7-\sqrt{21}\right)\int\frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2}\,dx+\frac{1}{14}\left(7+\sqrt{21}\right)\int\frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2}\,dx\\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int\frac{1+x^2}{1+5x^2+x^4}\,dx=\frac{\left(-3+\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}}+\frac{\left(3+\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

```
[In] Integrate[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ((-3 + Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])]
+ ((3 + Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21])]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{\sqrt{7}\,\arctan\left(\frac{x\sqrt{7}}{7}\right)}{7}+\frac{\sqrt{7}\,\arctan\left(\frac{x^{3}\sqrt{7}}{7}+\frac{6x\sqrt{7}}{7}\right)}{7}$	35
default	$\frac{2\left(3+\sqrt{21}\right)\sqrt{21}\,\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}+\frac{2\left(-3+\sqrt{21}\right)\sqrt{21}\,\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

```
[In] int((x^2+1)/(x^4+5*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*7^(1/2)*arctan(1/7*x*7^(1/2))+1/7*7^(1/2)*arctan(1/7*x^3*7^(1/2)+6/7*x*
7^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx = \frac{1}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(x^3+6\,x)\right)+\frac{1}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}x\right)$$

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(x^3 + 6*x)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7
)*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx = \frac{\sqrt{7}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{7}x}{7}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{7}x^3}{7}+\frac{6\sqrt{7}x}{7}\right)\right)}{14}$$

```
[In] integrate((x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] sqrt(7)*(2*atan(sqrt(7)*x/7) + 2*atan(sqrt(7)*x**3/7 + 6*sqrt(7)*x/7))/14
```

Maxima [F]

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx = \int \frac{x^2+1}{x^4+5\,x^2+1}\,dx$$

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 5*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx = \frac{1}{14}\,\sqrt{7}\left(\pi\mathrm{sgn}(x)+2\,\arctan\left(\frac{\sqrt{7}(x^2-1)}{7\,x}\right)\right)$$

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*(pi*sgn(x) + 2*arctan(1/7*sqrt(7)*(x^2 - 1)/x))
```

Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx=\frac{\sqrt{7}\left(\operatorname{atan}\left(\frac{\sqrt{7}\,x^3}{7}+\frac{6\,\sqrt{7}\,x}{7}\right)+\operatorname{atan}\left(\frac{\sqrt{7}\,x}{7}\right)\right)}{7}$$

```
[In] int((x^2 + 1)/(5*x^2 + x^4 + 1),x)
```

```
[Out] (7^(1/2)*(atan((6*7^(1/2)*x)/7 + (7^(1/2)*x^3)/7) + atan((7^(1/2)*x)/7)))/7
```

3.70 $\int \frac{1+x^2}{1+4x^2+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/6*arctan(x/(1/2*6^(1/2)+1
/2*2^(1/2)))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1177, 209}

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

```
[In] Int[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[6]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}\left(3-\sqrt{3}\right)\int\frac{1}{2-\sqrt{3}+x^2}\,dx+\frac{1}{6}\left(3+\sqrt{3}\right)\int\frac{1}{2+\sqrt{3}+x^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int\frac{1+x^2}{1+4x^2+x^4}\,dx=\frac{\left(-1+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\left(1+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[In] Integrate[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[3])*ArcTan[x/Sqrt[2 - Sqrt[3]]])/(2*Sqrt[3*(2 - Sqrt[3])]) + ((
1 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3*(2 + Sqrt[3])])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\sqrt{6}\,\arctan\left(\frac{x\sqrt{6}}{6}\right)}{6}+\frac{\sqrt{6}\,\arctan\left(\frac{x^3\sqrt{6}}{6}+\frac{5x\sqrt{6}}{6}\right)}{6}$	35
default	$\frac{\left(1+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}}+\frac{\left(\sqrt{3}-1\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}$	70

```
[In] int((x^2+1)/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*6^(1/2)*arctan(1/6*x*6^(1/2))+1/6*6^(1/2)*arctan(1/6*x^3*6^(1/2)+5/6*x*
6^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{6}\,\sqrt{6}(x^3+5\,x)\right) + \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{6}\,\sqrt{6}x\right)$$

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(1/6*sqrt(6)*(x^3 + 5*x)) + 1/6*sqrt(6)*arctan(1/6*sqrt(6
)*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{6}\right)+2\operatorname{atan}\left(\frac{\sqrt{6}x^3}{6}+\frac{5\sqrt{6}x}{6}\right)\right)}{12}$$

```
[In] integrate((x**2+1)/(x**4+4*x**2+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/6) + 2*atan(sqrt(6)*x**3/6 + 5*sqrt(6)*x/6))/12
```

Maxima [F]

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \int \frac{x^2+1}{x^4+4\,x^2+1}\,dx$$

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 4*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \frac{1}{12}\,\sqrt{6}\left(\pi\operatorname{sgn}(x) + 2\,\arctan\left(\frac{\sqrt{6}(x^2-1)}{6\,x}\right)\right)$$

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*(pi*sgn(x) + 2*arctan(1/6*sqrt(6)*(x^2 - 1)/x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx=\frac{\sqrt{6}\left(\operatorname{atan}\left(\frac{\sqrt{6}\,x^3}{6}+\frac{5\,\sqrt{6}\,x}{6}\right)+\operatorname{atan}\left(\frac{\sqrt{6}\,x}{6}\right)\right)}{6}$$

```
[In] int((x^2 + 1)/(4*x^2 + x^4 + 1),x)
```

```
[Out] (6^(1/2)*(atan((5*6^(1/2)*x)/6 + (6^(1/2)*x^3)/6) + atan((6^(1/2)*x)/6)))/6
```

3.71 $\int \frac{1+x^2}{1+3x^2+x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{1+x^2}{1+3x^2+x^4} \, dx = \frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}} + \frac{\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt{5}}$$

```
[Out] 1/5*arctan(x*2^(1/2)/(3+5^(1/2))^(1/2))*5^(1/2)+1/5*arctan(x*(1/2+1/2*5^(1/
2)))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1177, 209}

$$\int \frac{1+x^2}{1+3x^2+x^4} \, dx = \frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}} + \frac{\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt{5}}$$

```
[In] Int[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]/S
qrt[5]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx+\frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx\\ &= \frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx=\frac{\left(-1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3-\sqrt{5}}}x\right)}{\sqrt{10\left(3-\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

```
[In] Integrate[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[5])*ArcTan[Sqrt[2/(3 - Sqrt[5])]*x])/Sqrt[10*(3 - Sqrt[5])] + (
(1 + Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/Sqrt[10*(3 + Sqrt[5])]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\sqrt{5}\,\arctan\left(\frac{x^{3}\sqrt{5}}{5}+\frac{4x\sqrt{5}}{5}\right)}{5}$	35
default	$\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}+\frac{2\left(\sqrt{5}+1\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}$	66

```
[In] int((x^2+1)/(x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*arctan(1/5*x*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctan(1/5*x^3*5^(1/2)+4/5*x*
5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \frac{1}{5}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}(x^3+4\,x)\right)+\frac{1}{5}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}x\right)$$

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(x^3 + 4*x)) + 1/5*sqrt(5)*arctan(1/5*sqrt(5
)*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \frac{\sqrt{5}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{5}x^3}{5}+\frac{4\sqrt{5}x}{5}\right)\right)}{10}$$

```
[In] integrate((x**2+1)/(x**4+3*x**2+1),x)
[Out] sqrt(5)*(2*atan(sqrt(5)*x/5) + 2*atan(sqrt(5)*x**3/5 + 4*sqrt(5)*x/5))/10
```

Maxima [F]

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \int \frac{x^2+1}{x^4+3\,x^2+1}\,dx$$

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 + 3*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \frac{1}{10}\,\sqrt{5}\left(\pi\mathrm{sgn}(x)+2\,\arctan\left(\frac{\sqrt{5}(x^2-1)}{5\,x}\right)\right)$$

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
[Out] 1/10*sqrt(5)*(pi*sgn(x) + 2*arctan(1/5*sqrt(5)*(x^2 - 1)/x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx=\frac{\sqrt{5}\,\left(\mathrm{atan}\left(\frac{\sqrt{5}\,x^3}{5}+\frac{4\,\sqrt{5}\,x}{5}\right)+\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)\right)}{5}$$

```
[In] int((x^2 + 1)/(3*x^2 + x^4 + 1),x)
[Out] (5^(1/2)*(atan((4*5^(1/2)*x)/5 + (5^(1/2)*x^3)/5) + atan((5^(1/2)*x)/5)))/5
```

3.72 $\int \frac{1+x^2}{1+2x^2+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 2

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx=\arctan(x)$$

```
[Out] arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {28, 209}

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx=\arctan(x)$$

```
[In] Int[(1 + x^2)/(1 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{1+x^2}\,dx\\ &= \tan^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \arctan(x)$$

```
[In] Integrate[(1 + x^2)/(1 + 2*x^2 + x^4),x]
[Out] ArcTan[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\arctan(x)$	3
risch	$\arctan(x)$	3
parallelrisch	$\frac{i\ln(x+i)}{2}-\frac{i\ln(x-i)}{2}$	18

```
[In] int((x^2+1)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \arctan(x)$$

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \operatorname{atan}(x)$$

```
[In] integrate((x**2+1)/(x**4+2*x**2+1),x)
[Out] atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \arctan(x)$$

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \arctan(x)$$

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] arctan(x)
```

Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \operatorname{atan}(x)$$

```
[In] int((x^2 + 1)/(2*x^2 + x^4 + 1),x)
[Out] atan(x)
```

3.73 $\int \frac{1+x^2}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{1+x^2}{1+x^2+x^4} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1175, 632, 210}

$$\int \frac{1+x^2}{1+x^2+x^4} \, dx = \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(1 + x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= -\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.61

$$
\int \frac{1+x^2}{1+x^2+x^4}\,dx=\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1-i\sqrt{3}\right)}}+\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1+i\sqrt{3}\right)}}
$$

```
[In] Integrate[(1 + x^2)/(1 + x^2 + x^4),x]
[Out] ((-I + Sqrt[3])*ArcTan[x/Sqrt[(1 - I*Sqrt[3])/2]])/Sqrt[6*(1 - I*Sqrt[3])]
+ ((I + Sqrt[3])*ArcTan[x/Sqrt[(1 + I*Sqrt[3])/2]])/Sqrt[6*(1 + I*Sqrt[3])]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	34
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{3}\right)}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{3}$	35

```
[In] int((x^2+1)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx=\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^3+2\,x)\right)+\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)$$

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx=\frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{6}$$

```
[In] integrate((x**2+1)/(x**4+x**2+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)$$

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(\frac{\sqrt{3}(x^2-1)}{3\,x}\right)\right)$$

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*(x^2 - 1)/x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\left(\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\,\sqrt{3}\,x}{3}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{3}$$

```
[In] int((x^2 + 1)/(x^2 + x^4 + 1),x)
```

```
[Out] (3^(1/2)*(atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + atan((3^(1/2)*x)/3)))/3
```

3.74 $\int \frac{1+x^2}{1+x^4} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1+x^2}{1+x^4} \, dx = -\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/2*arctan(1+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1176, 631, 210}

$$\int \frac{1+x^2}{1+x^4} \, dx = \frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

```
[In] Int[(1 + x^2)/(1 + x^4),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}x\right)}{\sqrt{2}}\\ &= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{-\arctan\left(1-\sqrt{2}x\right)+\arctan\left(1+\sqrt{2}x\right)}{\sqrt{2}}$$

```
[In] Integrate[(1 + x^2)/(1 + x^4),x]
[Out] (-ArcTan[1 - Sqrt[2]*x] + ArcTan[1 + Sqrt[2]*x])/Sqrt[2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{x^3\sqrt{2}}{2}+\frac{x\sqrt{2}}{2}\right)}{2}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}$
meijerg	$\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2+\sqrt{2}\left(x^4\right)}\right.}{4(x^4)^{\frac{3}{4}}}$

```
[In] int((x^2+1)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*arctan(1/2*x*2^(1/2))+1/2*2^(1/2)*arctan(1/2*x^3*2^(1/2)+1/2*x*
2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^3+x)\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)$$

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + x)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*
x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{\sqrt{2}x}{2}\right)\right)}{4}$$

```
[In] integrate((x**2+1)/(x**4+1),x)
```

```
[Out] sqrt(2)*(2*atan(sqrt(2)*x/2) + 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/2))/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{\sqrt{2}\,\left(\mathrm{atan}\left(\frac{\sqrt{2}\,x^3}{2}+\frac{\sqrt{2}\,x}{2}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)\right)}{2}$$

```
[In] int((x^2 + 1)/(x^4 + 1),x)
```

```
[Out] (2^(1/2)*(atan((2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) + atan((2^(1/2)*x)/2)))/2
```

3.75 $\int \frac{1+x^2}{1-x^2+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = -\arctan\left(\sqrt{3}-2x\right)+\arctan\left(\sqrt{3}+2x\right)$$

```
[Out] arctan(2*x-3^(1/2))+arctan(2*x+3^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1175, 632, 210}

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)$$

```
[In] Int[(1 + x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx \\ &= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x\right) - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2x\right) \\ &= -\tan^{-1}\left(\sqrt{3}-2x\right) + \tan^{-1}\left(\sqrt{3}+2x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = -\arctan\left(\frac{x}{-1+x^2}\right)$$

```
[In] Integrate[(1 + x^2)/(1 - x^2 + x^4),x]
[Out] -ArcTan[x/(-1 + x^2)]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.35

method	result	size
risch	$\arctan\left(x^3\right)+\arctan\left(x\right)$	8
default	$\arctan\left(2x-\sqrt{3}\right)+\arctan\left(2x+\sqrt{3}\right)$	20
parallelrisch	$\frac{i\ln\left(x^2+ix-1\right)}{2}-\frac{i\ln\left(x^2-ix-1\right)}{2}$	28

```
[In] int((x^2+1)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(x^3)+arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx=\arctan\left(x^3\right)+\arctan(x)$$

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
[Out] arctan(x^3) + arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx=\operatorname{atan}(x)+\operatorname{atan}\left(x^3\right)$$

```
[In] integrate((x**2+1)/(x**4-x**2+1),x)
[Out] atan(x) + atan(x**3)
```

Maxima [F]

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx=\int \frac{x^2+1}{x^4-x^2+1}\,dx$$

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx=\frac{1}{4}\,\pi\operatorname{sgn}(x)+\frac{1}{2}\,\arctan\left(\frac{x^4-3\,x^2+1}{2\,(x^3-x)}\right)$$

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) + 1/2*arctan(1/2*(x^4 - 3*x^2 + 1)/(x^3 - x))
```

Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \operatorname{atan}\left(x^3\right)+\operatorname{atan}(x)$$

```
[In] int((x^2 + 1)/(x^4 - x^2 + 1),x)
[Out] atan(x^3) + atan(x)
```

3.76 $\int \frac{1+x^2}{1-2x^2+x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \frac{1+x^2}{1-2x^2+x^4} \, dx = \frac{x}{1-x^2}$$

```
[Out] x/(-x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {28, 391}

$$\int \frac{1+x^2}{1-2x^2+x^4} \, dx = \frac{x}{1-x^2}$$

```
[In] Int[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] x/(1 - x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1+x^2}{\left(-1+x^2\right)^2}\,dx \\ &= \frac{x}{1-x^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{x}{-1+x^2}$$

```
[In] Integrate[(1 + x^2)/(1 - 2*x^2 + x^4),x]
[Out] -(x/(-1 + x^2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{x}{x^2-1}$	11
norman	$-\frac{x}{x^2-1}$	11
risch	$-\frac{x}{x^2-1}$	11
parallelrisch	$-\frac{x}{x^2-1}$	11
default	$-\frac{1}{2(x+1)}-\frac{1}{2(x-1)}$	16

```
[In] int((x^2+1)/(x^4-2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -x/(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{x}{x^2-1}$$

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
[Out] -x/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{x}{x^2-1}$$

```
[In] integrate((x**2+1)/(x**4-2*x**2+1),x)
[Out] -x/(x**2 - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{x}{x^2-1}$$

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
[Out] -x/(x^2 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{1}{x-\frac{1}{x}}$$

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
[Out] -1/(x - 1/x)
```

Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{x}{x^2-1}$$

```
[In] int((x^2 + 1)/(x^4 - 2*x^2 + 1),x)
[Out] -x/(x^2 - 1)
```

3.77 $\int \frac{1+x^2}{1-3x^2+x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{1+x^2}{1-3x^2+x^4} \, dx = \frac{1}{2}\log\left(1-\sqrt{5}-2x\right)+\frac{1}{2}\log\left(1+\sqrt{5}-2x\right) -\frac{1}{2}\log\left(1-\sqrt{5}+2x\right)-\frac{1}{2}\log\left(1+\sqrt{5}+2x\right)$$

```
[Out] 1/2*ln(1-2*x-5^(1/2))-1/2*ln(1+2*x-5^(1/2))+1/2*ln(1-2*x+5^(1/2))-1/2*ln(1+
2*x+5^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1175, 630, 31}

$$\int \frac{1+x^2}{1-3x^2+x^4} \, dx = \frac{1}{2}\log\left(-2x-\sqrt{5}+1\right)+\frac{1}{2}\log\left(-2x+\sqrt{5}+1\right) -\frac{1}{2}\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x+\sqrt{5}+1\right)$$

```
[In] Int[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] Log[1 - Sqrt[5] - 2*x]/2 + Log[1 + Sqrt[5] - 2*x]/2 - Log[1 - Sqrt[5] + 2*x
]/2 - Log[1 + Sqrt[5] + 2*x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{5}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{5}x+x^2}\,dx\\
&= \frac{1}{2}\int \frac{1}{\frac{1}{2}\left(-1-\sqrt{5}\right)+x}\,dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}\left(1-\sqrt{5}\right)+x}\,dx\\
&\quad +\frac{1}{2}\int \frac{1}{\frac{1}{2}\left(-1+\sqrt{5}\right)+x}\,dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}\left(1+\sqrt{5}\right)+x}\,dx\\
&= \frac{1}{2}\log\left(1-\sqrt{5}-2x\right)+\frac{1}{2}\log\left(1+\sqrt{5}-2x\right)-\frac{1}{2}\log\left(1-\sqrt{5}+2x\right)-\frac{1}{2}\log\left(1+\sqrt{5}\right.\\
&\qquad \left.+2x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx=-\frac{1}{2}\log\left(1-x-x^2\right)+\frac{1}{2}\log\left(1+x-x^2\right)$$

```
[In] Integrate[(1 + x^2)/(1 - 3*x^2 + x^4),x]
[Out] -1/2*Log[1 - x - x^2] + Log[1 + x - x^2]/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.34

method	result	size
default	$-\frac{\ln(x^2+x-1)}{2}+\frac{\ln(x^2-x-1)}{2}$	22
norman	$-\frac{\ln(x^2+x-1)}{2}+\frac{\ln(x^2-x-1)}{2}$	22
risch	$-\frac{\ln(x^2+x-1)}{2}+\frac{\ln(x^2-x-1)}{2}$	22
parallelrisch	$-\frac{\ln(x^2+x-1)}{2}+\frac{\ln(x^2-x-1)}{2}$	22

```
[In] int((x^2+1)/(x^4-3*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^2+x-1)+1/2*ln(x^2-x-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.32

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = -\frac{1}{2}\log\left(x^2+x-1\right)+\frac{1}{2}\log\left(x^2-x-1\right)$$

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.29

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = \frac{\log\left(x^2-x-1\right)}{2}-\frac{\log\left(x^2+x-1\right)}{2}$$

```
[In] integrate((x**2+1)/(x**4-3*x**2+1),x)
[Out] log(x**2 - x - 1)/2 - log(x**2 + x - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.32

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = -\frac{1}{2}\log\left(x^2+x-1\right)+\frac{1}{2}\log\left(x^2-x-1\right)$$

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = -\frac{1}{4}\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}-2\right|\right)$$

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(x + 1/(x - 1/x) - 1/x + 2)) + 1/4*log(abs(x + 1/(x - 1/x) - 1/
x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.18

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = -\mathrm{atanh}\left(\frac{x}{x^2-1}\right)$$

```
[In] int((x^2 + 1)/(x^4 - 3*x^2 + 1),x)
```

```
[Out] -atanh(x/(x^2 - 1))
```

3.78 $\int \frac{1+x^2}{1-4x^2+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\sqrt{3}+\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(x*2^(1/2)-3^(1/2))*2^(1/2)-1/2*arctanh(x*2^(1/2)+3^(1/2))*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1175, 632, 212}

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}$$

```
[In] Int[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] ArcTanh[Sqrt[3] - Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]*x]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{6}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{6}x+x^2}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,-\sqrt{6}+2x\right)-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{6}+2x\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{3}+\sqrt{2}x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx=\frac{\log\left(1+\sqrt{2}x-x^2\right)-\log\left(-1+\sqrt{2}x+x^2\right)}{2\sqrt{2}}$$

```
[In] Integrate[(1 + x^2)/(1 - 4*x^2 + x^4),x]
[Out] (Log[1 + Sqrt[2]*x - x^2] - Log[-1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\sqrt{2}\,\ln\left(x^{2}-x\sqrt{2}-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(x^{2}+x\sqrt{2}-1\right)}{4}$	35
default	$-\frac{\sqrt{3}\left(\sqrt{3}+3\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}-\sqrt{2}\right)}$	70

```
[In] int((x^2+1)/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*ln(x^2-x*2^(1/2)-1)-1/4*2^(1/2)*ln(x^2+x*2^(1/2)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(\frac{x^4-2\,\sqrt{2}(x^3-x)+1}{x^4-4\,x^2+1}\right)$$

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^4 - 2*sqrt(2)*(x^3 - x) + 1)/(x^4 - 4*x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = \frac{\sqrt{2}\log\left(x^2-\sqrt{2}x-1\right)}{4} - \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x-1\right)}{4}$$

```
[In] integrate((x**2+1)/(x**4-4*x**2+1),x)
[Out] sqrt(2)*log(x**2 - sqrt(2)*x - 1)/4 - sqrt(2)*log(x**2 + sqrt(2)*x - 1)/4
```

Maxima [F]

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = \int \frac{x^2+1}{x^4-4\,x^2+1}\,dx$$

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 - 4*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}-\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{2}-\frac{2}{x}\right|}\right)$$

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2/x)/abs(2*x + 2*sqrt(2) - 2/x))
```

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.42

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{x^2-1}\right)}{2}$$

```
[In] int((x^2 + 1)/(x^4 - 4*x^2 + 1),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*x)/(x^2 - 1)))/2
```

3.79 $\int \frac{1+x^2}{1-5x^2+x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \frac{1+x^2}{1-5x^2+x^4} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{7}+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctanh(1/3*(-2*x+7^(1/2))*3^(1/2))*3^(1/2)-1/3*arctanh(1/3*(2*x+7^(1/2
))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1175, 632, 212}

$$\int \frac{1+x^2}{1-5x^2+x^4} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] ArcTanh[(Sqrt[7] - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2*x)/Sqrt[3]]
/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{1-\sqrt{7}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{7}x+x^2}\,dx \\
&= -\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, -\sqrt{7}+2x\right)-\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \sqrt{7}+2x\right) \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}+2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$
\int \frac{1+x^2}{1-5x^2+x^4}\,dx=\frac{\log\left(1+\sqrt{3}x-x^2\right)-\log\left(-1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}
$$

```
[In] Integrate[(1 + x^2)/(1 - 5*x^2 + x^4),x]
[Out] (Log[1 + Sqrt[3]*x - x^2] - Log[-1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{3}\,\ln\left(x^{2}-x\sqrt{3}-1\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^{2}+x\sqrt{3}-1\right)}{6}$	35
default	$-\frac{2\sqrt{21}\,\left(7+\sqrt{21}\right)\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}-\frac{2\left(-7+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

```
[In] int((x^2+1)/(x^4-5*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*3^(1/2)*ln(x^2-x*3^(1/2)-1)-1/6*3^(1/2)*ln(x^2+x*3^(1/2)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{x^4+x^2-2\sqrt{3}(x^3-x)+1}{x^4-5\,x^2+1}\right)$$

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((x^4 + x^2 - 2*sqrt(3)*(x^3 - x) + 1)/(x^4 - 5*x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x-1\right)}{6}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x-1\right)}{6}$$

```
[In] integrate((x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x - 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*x - 1)/6
```

Maxima [F]

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = \int \frac{x^2+1}{x^4-5\,x^2+1}\,dx$$

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 - 5*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(\frac{\left|2\,x-2\,\sqrt{3}-\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{3}-\frac{2}{x}\right|}\right)$$

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) - 2/x)/abs(2*x + 2*sqrt(3) - 2/x))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = -\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{x^2-1}\right)}{3}$$

```
[In] int((x^2 + 1)/(x^4 - 5*x^2 + 1),x)
[Out] -(3^(1/2)*atanh((3^(1/2)*x)/(x^2 - 1)))/3
```

3.80 $\int \frac{1-x^2}{1+bx^2+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = -\frac{\log\left(1-\sqrt{2-b}x+x^2\right)}{2\sqrt{2-b}}+\frac{\log\left(1+\sqrt{2-b}x+x^2\right)}{2\sqrt{2-b}}$$

```
[Out] -1/2*ln(1+x^2-x*(2-b)^(1/2))/(2-b)^(1/2)+1/2*ln(1+x^2+x*(2-b)^(1/2))/(2-b)^
(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1178, 642}

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \frac{\log\left(\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}-\frac{\log\left(-\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}$$

```
[In] Int[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[2 - b]*x + x^2]/Sqrt[2 - b] + Log[1 + Sqrt[2 - b]*x + x^2
]/(2*Sqrt[2 - b])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\sqrt{2-b}+2x}{-1-\sqrt{2-b}x-x^2}\,dx}{2\sqrt{2-b}} - \frac{\int \frac{\sqrt{2-b}-2x}{-1+\sqrt{2-b}x-x^2}\,dx}{2\sqrt{2-b}}\\ &= -\frac{\log\left(1-\sqrt{2-b}x+x^2\right)}{2\sqrt{2-b}} + \frac{\log\left(1+\sqrt{2-b}x+x^2\right)}{2\sqrt{2-b}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 125 vs. 2(62) = 124.

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.02

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \frac{\frac{\left(2+b-\sqrt{-4+b^2}\right)\arctan\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{-4+b^2}}}\right)}{\sqrt{b-\sqrt{-4+b^2}}} - \frac{\left(2+b+\sqrt{-4+b^2}\right)\arctan\left(\frac{\sqrt{2}x}{\sqrt{b+\sqrt{-4+b^2}}}\right)}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\sqrt{-4+b^2}}$$

```
[In] Integrate[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2 + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sq
rt[b - Sqrt[-4 + b^2]] - ((2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[
b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

method	result	size
risch	$-\frac{\ln\left(-x^2\sqrt{2-b}+(2-b)x-\sqrt{2-b}\right)}{2\sqrt{2-b}}+\frac{\ln\left(-x^2\sqrt{2-b}+x(b-2)-\sqrt{2-b}\right)}{2\sqrt{2-b}}$	78
default	$\frac{\left(-2-\sqrt{(b-2)(2+b)}-b\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(2-\sqrt{(b-2)(2+b)}+b\right)\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$	128

```
[In] int((-x^2+1)/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(2-b)^(1/2)*ln(-x^2*(2-b)^(1/2)+(2-b)*x-(2-b)^(1/2))+1/2/(2-b)^(1/2)*l
n(-x^2*(2-b)^(1/2)+x*(b-2)-(2-b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \left[-\frac{\sqrt{-b+2}\log\left(\frac{x^4-(b-4)x^2+2\,(x^3+x)\sqrt{-b+2}+1}{x^4+bx^2+1}\right)}{2\,(b-2)}, \frac{\sqrt{b-2}\arctan\left(\frac{x^3+(b-1)x}{\sqrt{b-2}}\right)-\sqrt{b-2}\arctan\left(\frac{x}{\sqrt{b-2}}\right)}{b-2}\right]$$

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b + 2)*log((x^4 - (b - 4)*x^2 + 2*(x^3 + x)*sqrt(-b + 2) + 1)/(
x^4 + b*x^2 + 1))/(b - 2), (sqrt(b - 2)*arctan((x^3 + (b - 1)*x)/sqrt(b - 2
)) - sqrt(b - 2)*arctan(x/sqrt(b - 2)))/(b - 2)]
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b-2}}+2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2} - \frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b-2}}-2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}$$

```
[In] integrate((-x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 2))*log(x**2 + x*(-b*sqrt(-1/(b - 2)) + 2*sqrt(-1/(b - 2))) +
1)/2 - sqrt(-1/(b - 2))*log(x**2 + x*(b*sqrt(-1/(b - 2)) - 2*sqrt(-1/(b - 2
))) + 1)/2
```

Maxima [F]

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \int -\frac{x^2-1}{x^4+bx^2+1}\,dx$$

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(x^4 + b*x^2 + 1), x)
```

Giac [F]

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \int -\frac{x^2-1}{x^4+bx^2+1}\,dx$$

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{x}{\sqrt{b-2}}\right)-\operatorname{atan}\left((b-2)\left(x\left(\frac{1}{\sqrt{b-2}}+\frac{\frac{4}{b-2}+1}{\sqrt{b-2}\,(b+2)}\right)+\frac{x^3\left(\frac{2\,b}{b-2}-1\right)}{\sqrt{b-2}\,(b+2)}\right)\right)}{\sqrt{b-2}}$$

```
[In] int(-(x^2 - 1)/(b*x^2 + x^4 + 1),x)
[Out] -(atan(x/(b - 2)^(1/2)) - atan((b - 2)*(x*(1/(b - 2)^(1/2) + (4/(b - 2) + 1
)/((b - 2)^(1/2)*(b + 2))) + (x^3*((2*b)/(b - 2) - 1))/((b - 2)^(1/2)*(b +
2)))))/(b - 2)^(1/2)
```

3.81 $\int \frac{1-x^2}{1+5x^2+x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{1-x^2}{1+5x^2+x^4} \, dx = -\frac{\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}} + \frac{\arctan\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(x*2^(1/2)/(5+21^(1/2))^(1/2))*3^(1/2)+1/3*arctan(x*(1/2*7^(1/2)
+1/2*3^(1/2)))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1177, 209}

$$\int \frac{1-x^2}{1+5x^2+x^4} \, dx = \frac{\arctan\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{3}} - \frac{\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}$$

```
[In] Int[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sqrt[21])/2]
*x]/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}\left(-3+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2}\,dx-\frac{1}{6}\left(3+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2}\,dx\\ &= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.74

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx=\frac{\left(7-\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}}+\frac{\left(-7-\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

```
[In] Integrate[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ((7 - Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])] +
 ((-7 - Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21])]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{3}\right)}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{4x\sqrt{3}}{3}\right)}{3}$	35
default	$-\frac{2\sqrt{21}\,\left(7+\sqrt{21}\right)\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}-\frac{2\left(-7+\sqrt{21}\right)\sqrt{21}\,\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

```
[In] int((-x^2+1)/(x^4+5*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*3^(1/2)*arctan(1/3*x*3^(1/2))+1/3*3^(1/2)*arctan(1/3*x^3*3^(1/2)+4/3*x
*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^3+4\,x)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)$$

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 4*x)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = -\frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)-2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{4\sqrt{3}x}{3}\right)\right)}{6}$$

```
[In] integrate((-x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] -sqrt(3)*(2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 4*sqrt(3)*x/3))/6
```

Maxima [F]

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4+5\,x^2+1}\,dx$$

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 5*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\left(\pi\operatorname{sgn}(x) - 2\,\arctan\left(\frac{\sqrt{3}(x^2+1)}{3\,x}\right)\right)$$

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(pi*sgn(x) - 2*arctan(1/3*sqrt(3)*(x^2 + 1)/x))
```

Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{4\,\sqrt{3}\,x}{3}\right)-\operatorname{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{3}$$

```
[In] int(-(x^2 - 1)/(5*x^2 + x^4 + 1),x)
```

```
[Out] (3^(1/2)*(atan((4*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) - atan((3^(1/2)*x)/3)))/3
```

3.82 $\int \frac{1-x^2}{1+4x^2+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1/2)-1/2*arctan(x/(1/2*6^(1/2)+1
/2*2^(1/2)))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1177, 209}

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

```
[In] Int[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\left(-1-\sqrt{3}\right)\int \frac{1}{2+\sqrt{3}+x^2}\,dx+\frac{1}{2}\left(-1+\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+x^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \frac{-\left(\left(-3+\sqrt{3}\right)\sqrt{2+\sqrt{3}}\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)\right)-\sqrt{2-\sqrt{3}}\left(3+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

```
[In] Integrate[(1 - x^2)/(1 + 4*x^2 + x^4),x]
[Out] (-((-3 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*ArcTan[x/Sqrt[2 - Sqrt[3]]]) - Sqrt[2 -
 Sqrt[3]]*(3 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{x^{3}\sqrt{2}}{2}+\frac{3x\sqrt{2}}{2}\right)}{2}$	35
default	$-\frac{\sqrt{3}\,\left(\sqrt{3}+3\right)\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}-\sqrt{2}\right)}$	70

```
[In] int((-x^2+1)/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*2^(1/2)*arctan(1/2*x*2^(1/2))+1/2*2^(1/2)*arctan(1/2*x^3*2^(1/2)+3/2*x
*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^3+3\,x)\right) - \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)$$

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + 3*x)) - 1/2*sqrt(2)*arctan(1/2*sqrt(2
)*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = -\frac{\sqrt{2}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right) - 2\operatorname{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{3\sqrt{2}x}{2}\right)\right)}{4}$$

```
[In] integrate((-x**2+1)/(x**4+4*x**2+1),x)
[Out] -sqrt(2)*(2*atan(sqrt(2)*x/2) - 2*atan(sqrt(2)*x**3/2 + 3*sqrt(2)*x/2))/4
```

Maxima [F]

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4+4\,x^2+1}\,dx$$

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(x^4 + 4*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \frac{1}{4}\,\sqrt{2}\left(\pi\mathrm{sgn}(x) - 2\,\arctan\left(\frac{\sqrt{2}(x^2+1)}{2\,x}\right)\right)$$

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")
[Out] 1/4*sqrt(2)*(pi*sgn(x) - 2*arctan(1/2*sqrt(2)*(x^2 + 1)/x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx=\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\,x^3}{2}+\frac{3\,\sqrt{2}\,x}{2}\right)-\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)\right)}{2}$$

```
[In] int(-(x^2 - 1)/(4*x^2 + x^4 + 1),x)
[Out] (2^(1/2)*(atan((3*2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) - atan((2^(1/2)*x)/2)))/2
```

3.83 $\int \frac{1-x^2}{1+3x^2+x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{1-x^2}{1+3x^2+x^4} \, dx = -\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)$$

```
[Out] -arctan(x*2^(1/2)/(3+5^(1/2))^(1/2))+arctan(x*(1/2+1/2*5^(1/2)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1177, 209}

$$\int \frac{1-x^2}{1+3x^2+x^4} \, dx = \arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right) - \arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[In] Int[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\left(-1-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx+\frac{1}{2}\left(-1+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx\\ &= -\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)+\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.26

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=\arctan\left(\frac{x}{1+x^2}\right)$$

```
[In] Integrate[(1 - x^2)/(1 + 3*x^2 + x^4),x]
[Out] ArcTan[x/(1 + x^2)]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.36

method	result	size
risch	$-\arctan\left(x\right)+\arctan\left(x^{3}+2x\right)$	14
parallelrisch	$\frac{i\ln\left(x^{2}-ix+1\right)}{2}-\frac{i\ln\left(x^{2}+ix+1\right)}{2}$	28
default	$-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}-\frac{2\sqrt{5}\,\left(5+\sqrt{5}\right)\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}$	66

```
[In] int((-x^2+1)/(x^4+3*x^2+1),x,method=_RETURNVERBOSE)
[Out] -arctan(x)+arctan(x^3+2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=\arctan\left(x^3+2\,x\right)-\arctan\left(x\right)$$

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
[Out] arctan(x^3 + 2*x) - arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.26

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=-\operatorname{atan}\left(x\right)+\operatorname{atan}\left(x^3+2x\right)$$

```
[In] integrate((-x**2+1)/(x**4+3*x**2+1),x)
[Out] -atan(x) + atan(x**3 + 2*x)
```

Maxima [F]

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=\int -\frac{x^2-1}{x^4+3\,x^2+1}\,dx$$

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(x^4 + 3*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=\frac{1}{4}\,\pi\operatorname{sgn}(x)-\frac{1}{2}\,\arctan\left(\frac{x^4+x^2+1}{2\left(x^3+x\right)}\right)$$

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) - 1/2*arctan(1/2*(x^4 + x^2 + 1)/(x^3 + x))
```

Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx = \operatorname{atan}\left(x^3+2\,x\right)-\operatorname{atan}(x)$$

```
[In] int(-(x^2 - 1)/(3*x^2 + x^4 + 1),x)
```

```
[Out] atan(2*x + x^3) - atan(x)
```

3.84 $\int \frac{1-x^2}{1+2x^2+x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{1-x^2}{1+2x^2+x^4} \, dx = \frac{x}{1+x^2}$$

```
[Out] x/(x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 391}

$$\int \frac{1-x^2}{1+2x^2+x^4} \, dx = \frac{x}{x^2+1}$$

```
[In] Int[(1 - x^2)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1-x^2}{\left(1+x^2\right)^2}\,dx \\
&= \frac{x}{1+x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx=\frac{x}{1+x^2}$$

```
[In] Integrate[(1 - x^2)/(1 + 2*x^2 + x^4),x]
[Out] x/(1 + x^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
gosper	$\frac{x}{x^2+1}$	10
default	$\frac{x}{x^2+1}$	10
norman	$\frac{x}{x^2+1}$	10
risch	$\frac{x}{x^2+1}$	10
parallelrisch	$\frac{x}{x^2+1}$	10

```
[In] int((-x^2+1)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] x/(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx=\frac{x}{x^2+1}$$

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] x/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx=\frac{x}{x^2+1}$$

```
[In] integrate((-x**2+1)/(x**4+2*x**2+1),x)
[Out] x/(x**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx=\frac{x}{x^2+1}$$

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] x/(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx=\frac{1}{x+\frac{1}{x}}$$

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] 1/(x + 1/x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1}$$

```
[In] int(-(x^2 - 1)/(2*x^2 + x^4 + 1),x)
[Out] x/(x^2 + 1)
```

3.85 $\int \frac{1-x^2}{1+x^2+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx = -\frac{1}{2}\log\left(1-x+x^2\right)+\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] -1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1178, 642}

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx = \frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

```
[In] Int[(1 - x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
```

```
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int\frac{1+2x}{-1-x-x^2}\,dx\right)-\frac{1}{2}\int\frac{1-2x}{-1+x-x^2}\,dx\\ &= -\frac{1}{2}\log\left(1-x+x^2\right)+\frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int\frac{1-x^2}{1+x^2+x^4}\,dx=-\frac{1}{2}\log\left(1-x+x^2\right)+\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[In] Integrate[(1 - x^2)/(1 + x^2 + x^4),x]
[Out] -1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\ln\left(x^2-x+1\right)}{2}+\frac{\ln\left(x^2+x+1\right)}{2}$	22
norman	$-\frac{\ln\left(x^2-x+1\right)}{2}+\frac{\ln\left(x^2+x+1\right)}{2}$	22
risch	$-\frac{\ln\left(x^2-x+1\right)}{2}+\frac{\ln\left(x^2+x+1\right)}{2}$	22
parallelrisch	$-\frac{\ln\left(x^2-x+1\right)}{2}+\frac{\ln\left(x^2+x+1\right)}{2}$	22

```
[In] int((-x^2+1)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx=\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx=-\frac{\log\left(x^2-x+1\right)}{2}+\frac{\log\left(x^2+x+1\right)}{2}$$

```
[In] integrate((-x**2+1)/(x**4+x**2+1),x)
[Out] -log(x**2 - x + 1)/2 + log(x**2 + x + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx=\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx=\frac{1}{4}\log\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}+2\right|\right)-\frac{1}{4}\log\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}-2\right|\right)$$

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/4*log(abs(x + 1/(x + 1/x) + 1/x + 2)) - 1/4*log(abs(x + 1/(x + 1/x) + 1/x
 - 2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.40

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{x}{x^2+1}\right)$$

```
[In] int(-(x^2 - 1)/(x^2 + x^4 + 1),x)
[Out] atanh(x/(x^2 + 1))
```

3.86 $\int \frac{1-x^2}{1+x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{1-x^2}{1+x^4}\,dx=-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{2\sqrt{2}}$$

```
[Out] -1/4*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1179, 642}

$$\int \frac{1-x^2}{1+x^4}\,dx=\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[In] Int[(1 - x^2)/(1 + x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]
)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{2\sqrt{2}} \\ &= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1-x^2}{1+x^4}\,dx = \frac{-\log\left(-1+\sqrt{2}x-x^2\right)+\log\left(1+\sqrt{2}x+x^2\right)}{2\sqrt{2}}$$

```
[In] Integrate[(1 - x^2)/(1 + x^4),x]
[Out] (-Log[-1 + Sqrt[2]*x - x^2] + Log[1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\ln\left(1+x^2-x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+x^2+x\sqrt{2}\right)\sqrt{2}}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}-\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}$
meijerg	$-\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right.}{2+\sqrt{2}\left(x\right.}\right.}{4\left(x^4\right)^{\frac{3}{4}}}$

```
[In] int((-x^2+1)/(x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1-x^2}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^4+4\,x^2+2\,\sqrt{2}(x^3+x)+1}{x^4+1}\right)$$

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^4 + 4*x^2 + 2*sqrt(2)*(x^3 + x) + 1)/(x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1+x^4}\,dx = -\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{4}$$

```
[In] integrate((-x**2+1)/(x**4+1),x)
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1-x^2}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1-x^2}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{1-x^2}{1+x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{x^2+1}\right)}{2}$$

```
[In] int(-(x^2 - 1)/(x^4 + 1),x)
[Out] (2^(1/2)*atanh((2^(1/2)*x)/(x^2 + 1)))/2
```

3.87 $\int \frac{1-x^2}{1-x^2+x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{1-x^2}{1-x^2+x^4} \, dx = -\frac{\log \left(1-\sqrt{3}x+x^2\right)}{2\sqrt{3}}+\frac{\log \left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

```
[Out] -1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1178, 642}

$$\int \frac{1-x^2}{1-x^2+x^4} \, dx = \frac{\log \left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log \left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

```
[In] Int[(1 - x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[3]*x + x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]
)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{2\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = \frac{-\log\left(-1+\sqrt{3}x-x^2\right)+\log\left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

```
[In] Integrate[(1 - x^2)/(1 - x^2 + x^4),x]
[Out] (-Log[-1 + Sqrt[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	35
risch	$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	35

```
[In] int((-x^2+1)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(\frac{x^4+5\,x^2+2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)$$

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = -\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

```
[In] integrate((-x**2+1)/(x**4-x**2+1),x)
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
```

Maxima [F]

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4-x^2+1}\,dx$$

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = -\frac{1}{6}\,\sqrt{3}\log\left(\frac{\left|2\,x-2\,\sqrt{3}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{3}+\frac{2}{x}\right|}\right)$$

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="giac")
[Out] -1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3) + 2/x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{x^2+1}\right)}{3}$$

```
[In] int(-(x^2 - 1)/(x^4 - x^2 + 1),x)
[Out] (3^(1/2)*atanh((3^(1/2)*x)/(x^2 + 1)))/3
```

3.88 $\int \frac{1-x^2}{1-2x^2+x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 2

$$\int \frac{1-x^2}{1-2x^2+x^4} \, dx = \operatorname{arctanh}(x)$$

```
[Out] arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 21, 213}

$$\int \frac{1-x^2}{1-2x^2+x^4} \, dx = \operatorname{arctanh}(x)$$

```
[In] Int[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] ArcTanh[x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1-x^2}{(-1+x^2)^2}\,dx\\ &= -\int \frac{1}{-1+x^2}\,dx\\ &= \tanh^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(2) = 4$.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 9.50

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = -\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

```
[In] Integrate[(1 - x^2)/(1 - 2*x^2 + x^4),x]
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
risch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
parallelrisch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14

```
[In] int((-x^2+1)/(x^4-2*x^2+1),x,method=_RETURNVERBOSE)
[Out] arctanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(2) = 4.

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \frac{1}{2}\log\left(x+1\right) - \frac{1}{2}\log\left(x-1\right)$$

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(2) = 4.

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = -\frac{\log\left(x-1\right)}{2} + \frac{\log\left(x+1\right)}{2}$$

```
[In] integrate((-x**2+1)/(x**4-2*x**2+1),x)
[Out] -log(x - 1)/2 + log(x + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(2) = 4.

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \frac{1}{2}\log\left(x+1\right) - \frac{1}{2}\log\left(x-1\right)$$

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 7.50

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \frac{1}{2}\,\log\left(|x+1|\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \operatorname{atanh}(x)$$

```
[In] int(-(x^2 - 1)/(x^4 - 2*x^2 + 1),x)
[Out] atanh(x)
```

3.89 $\int \frac{1-x^2}{1-3x^2+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}} + \frac{\operatorname{arctanh}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -1/5*arctanh(1/5*(1-2*x)*5^(1/2))*5^(1/2)+1/5*arctanh(1/5*(1+2*x)*5^(1/2))*
5^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1175, 632, 212}

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = \frac{\operatorname{arctanh}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{\operatorname{arctanh}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[In] Int[(1 - x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2*x)/Sqrt[5]]/Sqrt[5]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{-1-x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+x+x^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, -1+2x\right)+\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, 1+2x\right)\\ &= \frac{\tanh^{-1}\left(\frac{-1+2x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\tanh^{-1}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = \frac{-\log\left(-1+\sqrt{5}x-x^2\right)+\log\left(1+\sqrt{5}x+x^2\right)}{2\sqrt{5}}$$

```
[In] Integrate[(1 - x^2)/(1 - 3*x^2 + x^4),x]
[Out] (-Log[-1 + Sqrt[5]*x - x^2] + Log[1 + Sqrt[5]*x + x^2])/(2*Sqrt[5])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}$	34
risch	$\frac{\sqrt{5}\,\ln\left(x^2+x\sqrt{5}+1\right)}{10}-\frac{\sqrt{5}\,\ln\left(x^2-x\sqrt{5}+1\right)}{10}$	35

```
[In] int((-x^2+1)/(x^4-3*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = \frac{1}{10}\sqrt{5}\log\left(\frac{x^4+7\,x^2+2\,\sqrt{5}(x^3+x)+1}{x^4-3\,x^2+1}\right)$$

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((x^4 + 7*x^2 + 2*sqrt(5)*(x^3 + x) + 1)/(x^4 - 3*x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = -\frac{\sqrt{5}\log\left(x^2-\sqrt{5}x+1\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\sqrt{5}x+1\right)}{10}$$

```
[In] integrate((-x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x + 1)/10 + sqrt(5)*log(x**2 + sqrt(5)*x + 1)/1
0
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}-1}{2\,x+\sqrt{5}-1}\right)$$

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/10*sqrt(5)*l
og((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2\,x-2\,\sqrt{5}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{5}+\frac{2}{x}\right|}\right)$$

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
[Out] -1/10*sqrt(5)*log(abs(2*x - 2*sqrt(5) + 2/x)/abs(2*x + 2*sqrt(5) + 2/x))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.47

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,x}{x^2+1}\right)}{5}$$

```
[In] int(-(x^2 - 1)/(x^4 - 3*x^2 + 1),x)
[Out] (5^(1/2)*atanh((5^(1/2)*x)/(x^2 + 1)))/5
```

3.90 $\int \frac{1-x^2}{1-4x^2+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\operatorname{arctanh}\left(\frac{1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -1/6*arctanh(1/3*(1-x*2^(1/2))*3^(1/2))*6^(1/2)+1/6*arctanh(1/3*(1+x*2^(1/2
))*3^(1/2))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1175, 632, 212}

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\operatorname{arctanh}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[In] Int[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2]*x)/Sqrt[
3]]/Sqrt[6]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{2}x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{2}x+x^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{6-x^2}\,dx, x, -\sqrt{2}+2x\right)+\text{Subst}\left(\int \frac{1}{6-x^2}\,dx, x, \sqrt{2}+2x\right)\\ &= \frac{\tanh^{-1}\left(\frac{-1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\tanh^{-1}\left(\frac{1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = \frac{-\log\left(-1+\sqrt{6}x-x^2\right)+\log\left(1+\sqrt{6}x+x^2\right)}{2\sqrt{6}}$$

```
[In] Integrate[(1 - x^2)/(1 - 4*x^2 + x^4),x]
[Out] (-Log[-1 + Sqrt[6]*x - x^2] + Log[1 + Sqrt[6]*x + x^2])/(2*Sqrt[6])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{\sqrt{6}\,\ln\left(x^2+x\sqrt{6}+1\right)}{12}-\frac{\sqrt{6}\,\ln\left(x^2-x\sqrt{6}+1\right)}{12}$	35
default	$\frac{\left(1+\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}}+\frac{\left(\sqrt{3}-1\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}$	70

```
[In] int((-x^2+1)/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*6^(1/2)*ln(x^2+x*6^(1/2)+1)-1/12*6^(1/2)*ln(x^2-x*6^(1/2)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = \frac{1}{12}\,\sqrt{6}\log\left(\frac{x^4+8\,x^2+2\,\sqrt{6}(x^3+x)+1}{x^4-4\,x^2+1}\right)$$

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((x^4 + 8*x^2 + 2*sqrt(6)*(x^3 + x) + 1)/(x^4 - 4*x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = -\frac{\sqrt{6}\log\left(x^2-\sqrt{6}x+1\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\sqrt{6}x+1\right)}{12}$$

```
[In] integrate((-x**2+1)/(x**4-4*x**2+1),x)
```

```
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x + 1)/12 + sqrt(6)*log(x**2 + sqrt(6)*x + 1)/1
2
```

Maxima [F]

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4-4\,x^2+1}\,dx$$

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - 4*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = -\frac{1}{12}\,\sqrt{6}\log\left(\frac{\left|2\,x-2\,\sqrt{6}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{6}+\frac{2}{x}\right|}\right)$$

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
[Out] -1/12*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2/x)/abs(2*x + 2*sqrt(6) + 2/x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{x^2+1}\right)}{6}$$

```
[In] int(-(x^2 - 1)/(x^4 - 4*x^2 + 1),x)
[Out] (6^(1/2)*atanh((6^(1/2)*x)/(x^2 + 1)))/6
```

3.91 $\int \frac{1-x^2}{1-5x^2+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -1/7*arctanh(1/7*(-2*x+3^(1/2))*7^(1/2))*7^(1/2)+1/7*arctanh(1/7*(2*x+3^(1/
2))*7^(1/2))*7^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1175, 632, 212}

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[In] Int[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[3] - 2*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] + 2*x)/Sqrt[
7]]/Sqrt[7]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\int\frac{1}{-1-\sqrt{3}x+x^2}\,dx\right)-\frac{1}{2}\int\frac{1}{-1+\sqrt{3}x+x^2}\,dx\\ &= \text{Subst}\left(\int\frac{1}{7-x^2}\,dx,x,-\sqrt{3}+2x\right)+\text{Subst}\left(\int\frac{1}{7-x^2}\,dx,x,\sqrt{3}+2x\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}+2x}{\sqrt{7}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int\frac{1-x^2}{1-5x^2+x^4}\,dx=\frac{-\log\left(-1+\sqrt{7}x-x^2\right)+\log\left(1+\sqrt{7}x+x^2\right)}{2\sqrt{7}}$$

```
[In] Integrate[(1 - x^2)/(1 - 5*x^2 + x^4),x]
[Out] (-Log[-1 + Sqrt[7]*x - x^2] + Log[1 + Sqrt[7]*x + x^2])/(2*Sqrt[7])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{7}\,\ln\left(x^{2}+x\sqrt{7}+1\right)}{14}-\frac{\sqrt{7}\,\ln\left(x^{2}-x\sqrt{7}+1\right)}{14}$	35
default	$\frac{2\left(3+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}+\frac{2\left(-3+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

```
[In] int((-x^2+1)/(x^4-5*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*7^(1/2)*ln(x^2+x*7^(1/2)+1)-1/14*7^(1/2)*ln(x^2-x*7^(1/2)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx=\frac{1}{14}\,\sqrt{7}\log\left(\frac{x^{4}+9\,x^{2}+2\,\sqrt{7}(x^{3}+x)+1}{x^{4}-5\,x^{2}+1}\right)$$

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((x^4 + 9*x^2 + 2*sqrt(7)*(x^3 + x) + 1)/(x^4 - 5*x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx=-\frac{\sqrt{7}\log\left(x^{2}-\sqrt{7}x+1\right)}{14}+\frac{\sqrt{7}\log\left(x^{2}+\sqrt{7}x+1\right)}{14}$$

```
[In] integrate((-x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x + 1)/14 + sqrt(7)*log(x**2 + sqrt(7)*x + 1)/1
4
```

Maxima [F]

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx=\int -\frac{x^{2}-1}{x^{4}-5\,x^{2}+1}\,dx$$

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - 5*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx = -\frac{1}{14}\,\sqrt{7}\log\left(\frac{\left|2\,x-2\,\sqrt{7}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{7}+\frac{2}{x}\right|}\right)$$

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
[Out] -1/14*sqrt(7)*log(abs(2*x - 2*sqrt(7) + 2/x)/abs(2*x + 2*sqrt(7) + 2/x))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx = \frac{\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}\,x}{x^2+1}\right)}{7}$$

```
[In] int(-(x^2 - 1)/(x^4 - 5*x^2 + 1),x)
[Out] (7^(1/2)*atanh((7^(1/2)*x)/(x^2 + 1)))/7
```

3.92 $\int \frac{-1-3x^2}{1+2x^2+9x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{-1-3x^2}{1+2x^2+9x^4} \, dx = \frac{\arctan\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\arctan\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(1/2*(1-3*x)*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+3*x)*2^(1/2))*2^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\int \frac{-1-3x^2}{1+2x^2+9x^4} \, dx = \frac{\arctan\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\arctan\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[In] Int[(-1 - 3*x^2)/(1 + 2*x^2 + 9*x^4),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{6}\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx\right)-\frac{1}{6}\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,-\frac{2}{3}+2x\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,\frac{2}{3}+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx = -\frac{\left(-i+\sqrt{2}\right)\arctan\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(i+\sqrt{2}\right)\arctan\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

```
[In] Integrate[(-1 - 3*x^2)/(1 + 2*x^2 + 9*x^4),x]
```

```
[Out] -1/2*((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/Sqrt[2*(1 - (2*
I)*Sqrt[2])] - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqr
t[2*(1 + (2*I)*Sqrt[2])])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\sqrt{2}\, \arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\, \arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4}$	34
risch	$-\frac{\sqrt{2}\, \arctan\left(\frac{3x\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\, \arctan\left(\frac{9x^3\sqrt{2}}{4}+\frac{5x\sqrt{2}}{4}\right)}{4}$	35

```
[In] int((-3*x^2-1)/(9*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx=-\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}\big(9\,x^3+5\,x\big)\right)-\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{3}{4}\,\sqrt{2}x\right)$$

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqr
t(2)*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx=-\frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\,\mathrm{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

```
[In] integrate((-3*x**2-1)/(9*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))
/8
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x+1)\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x-1)\right)$$

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="maxima")
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x+1)\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x-1)\right)$$

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="giac")
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx = -\frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)+\mathrm{atan}\left(\frac{3\sqrt{2}\,x}{4}\right)\right)}{4}$$

```
[In] int(-(3*x^2 + 1)/(2*x^2 + 9*x^4 + 1),x)
[Out] -(2^(1/2)*(atan((5*2^(1/2)*x)/4 + (9*2^(1/2)*x^3)/4) + atan((3*2^(1/2)*x)/4
)))/4
```

3.93 $\int \frac{1+3x^2}{-1-2x^2-9x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{1+3x^2}{-1-2x^2-9x^4} \, dx = \frac{\arctan\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\arctan\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(1/2*(1-3*x)*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+3*x)*2^(1/2))*2^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\int \frac{1+3x^2}{-1-2x^2-9x^4} \, dx = \frac{\arctan\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\arctan\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[In] Int[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{6}\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx\right)-\frac{1}{6}\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,-\frac{2}{3}+2x\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,\frac{2}{3}+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx = -\frac{(-i+\sqrt{2})\arctan\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{(i+\sqrt{2})\arctan\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

```
[In] Integrate[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] -1/2*((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/Sqrt[2*(1 - (2*
I)*Sqrt[2])] - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqr
t[2*(1 + (2*I)*Sqrt[2])])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4}$	34
risch	$-\frac{\sqrt{2}\,\arctan\left(\frac{3x\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{9x^3\sqrt{2}}{4}+\frac{5x\sqrt{2}}{4}\right)}{4}$	35

```
[In] int((3*x^2+1)/(-9*x^4-2*x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx=-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(9\,x^3+5\,x)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqr
t(2)*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx=-\frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\,\mathrm{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

```
[In] integrate((3*x**2+1)/(-9*x**4-2*x**2-1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))
/8
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx = -\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x+1)\right) - \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x-1)\right)$$

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="maxima")
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx = -\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x+1)\right) - \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x-1)\right)$$

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="giac")
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx = -\frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{9\,\sqrt{2}\,x^3}{4}+\frac{5\,\sqrt{2}\,x}{4}\right)+\mathrm{atan}\left(\frac{3\,\sqrt{2}\,x}{4}\right)\right)}{4}$$

```
[In] int(-(3*x^2 + 1)/(2*x^2 + 9*x^4 + 1),x)
[Out] -(2^(1/2)*(atan((5*2^(1/2)*x)/4 + (9*2^(1/2)*x^3)/4) + atan((3*2^(1/2)*x)/4
)))/4
```

3.94 $\int \frac{3+2x^2}{1-2x^2+x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{3+2x^2}{1-2x^2+x^4} \, dx = \frac{5x}{2(1-x^2)} + \frac{\operatorname{arctanh}(x)}{2}$$

```
[Out] 5/2*x/(-x^2+1)+1/2*arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {28, 393, 213}

$$\int \frac{3+2x^2}{1-2x^2+x^4} \, dx = \frac{\operatorname{arctanh}(x)}{2} + \frac{5x}{2(1-x^2)}$$

```
[In] Int[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] (5*x)/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{3+2x^2}{(-1+x^2)^2}\,dx\\ &= \frac{5x}{2(1-x^2)} - \frac{1}{2}\int \frac{1}{-1+x^2}\,dx\\ &= \frac{5x}{2(1-x^2)} + \frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = \frac{1}{4}\left(-\frac{10x}{-1+x^2} - \log(1-x) + \log(1+x)\right)$$

```
[In] Integrate[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]
[Out] ((-10*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

method	result	size
norman	$-\frac{5x}{2(x^2-1)} - \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4}$	24
risch	$-\frac{5x}{2(x^2-1)} - \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4}$	24
default	$-\frac{5}{4(x+1)} + \frac{\ln(x+1)}{4} - \frac{5}{4(x-1)} - \frac{\ln(x-1)}{4}$	28
parallelrisch	$-\frac{\ln(x-1)x^2-\ln(x+1)x^2-\ln(x-1)+\ln(x+1)+10x}{4(x^2-1)}$	41

```
[In] int((2*x^2+3)/(x^4-2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -5/2*x/(x^2-1)-1/4*ln(x-1)+1/4*ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = \frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-10\,x}{4\,(x^2-1)}$$

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="fricas")
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 10*x)/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = -\frac{5x}{2x^2-2} - \frac{\log(x-1)}{4} + \frac{\log(x+1)}{4}$$

```
[In] integrate((2*x**2+3)/(x**4-2*x**2+1),x)
[Out] -5*x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = -\frac{5\,x}{2\,(x^2-1)} + \frac{1}{4}\,\log(x+1) - \frac{1}{4}\,\log(x-1)$$

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="maxima")
[Out] -5/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = -\frac{5\,x}{2\,(x^2-1)} + \frac{1}{4}\,\log(|x+1|) - \frac{1}{4}\,\log(|x-1|)$$

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="giac")
[Out] -5/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx=\frac{\operatorname{atanh}(x)}{2}-\frac{5\,x}{2\,\left(x^2-1\right)}$$

```
[In] int((2*x^2 + 3)/(x^4 - 2*x^2 + 1),x)
```

```
[Out] atanh(x)/2 - (5*x)/(2*(x^2 - 1))
```

3.95 $\int \frac{2+3x^2}{5-8x^2+3x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{2+3x^2}{5-8x^2+3x^4} \, dx = \frac{5\text{arctanh}(x)}{2} - \frac{7}{2}\sqrt{\frac{3}{5}}\text{arctanh}\left(\sqrt{\frac{3}{5}}x\right)$$

```
[Out] 5/2*arctanh(x)-7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1180, 213}

$$\int \frac{2+3x^2}{5-8x^2+3x^4} \, dx = \frac{5\text{arctanh}(x)}{2} - \frac{7}{2}\sqrt{\frac{3}{5}}\text{arctanh}\left(\sqrt{\frac{3}{5}}x\right)$$

```
[In] Int[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{15}{2}\int \frac{1}{-3+3x^2}\,dx\right)+\frac{21}{2}\int \frac{1}{-5+3x^2}\,dx\\ &= \frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = \frac{1}{20}\Big(7\sqrt{15}\log\left(\sqrt{15}-3x\right)-25\log(1-x)+25\log(1+x) - 7\sqrt{15}\log\left(\sqrt{15}+3x\right)\Big)$$

```
[In] Integrate[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (7*Sqrt[15]*Log[Sqrt[15] - 3*x] - 25*Log[1 - x] + 25*Log[1 + x] - 7*Sqrt[15
]*Log[Sqrt[15] + 3*x])/20
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{5\ln(x+1)}{4}-\frac{7\,\mathrm{arctanh}\left(\frac{x\sqrt{15}}{5}\right)\sqrt{15}}{10}-\frac{5\ln(x-1)}{4}$	26
risch	$-\frac{5\ln(x-1)}{4}+\frac{7\sqrt{15}\,\ln\left(3x-\sqrt{15}\right)}{20}-\frac{7\sqrt{15}\,\ln\left(3x+\sqrt{15}\right)}{20}+\frac{5\ln(x+1)}{4}$	42

```
[In] int((3*x^2+2)/(3*x^4-8*x^2+5),x,method=_RETURNVERBOSE)
```

```
[Out] 5/4*ln(x+1)-7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)-5/4*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = \frac{7}{20}\,\sqrt{5}\sqrt{3}\log\left(-\frac{2\,\sqrt{5}\sqrt{3}x-3\,x^2-5}{3\,x^2-5}\right)+\frac{5}{4}\,\log\left(x+1\right)-\frac{5}{4}\,\log\left(x-1\right)$$

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

```
[Out] 7/20*sqrt(5)*sqrt(3)*log(-(2*sqrt(5)*sqrt(3)*x - 3*x^2 - 5)/(3*x^2 - 5)) +
5/4*log(x + 1) - 5/4*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\begin{aligned}\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx &= -\frac{5\log\left(x-1\right)}{4}+\frac{5\log\left(x+1\right)}{4}\\ &\quad+\frac{7\sqrt{15}\log\left(x-\frac{\sqrt{15}}{3}\right)}{20}-\frac{7\sqrt{15}\log\left(x+\frac{\sqrt{15}}{3}\right)}{20}\end{aligned}$$

```
[In] integrate((3*x**2+2)/(3*x**4-8*x**2+5),x)
```

```
[Out] -5*log(x - 1)/4 + 5*log(x + 1)/4 + 7*sqrt(15)*log(x - sqrt(15)/3)/20 - 7*sq
rt(15)*log(x + sqrt(15)/3)/20
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = \frac{7}{20}\,\sqrt{15}\log\left(\frac{3\,x-\sqrt{15}}{3\,x+\sqrt{15}}\right)+\frac{5}{4}\,\log\left(x+1\right)-\frac{5}{4}\,\log\left(x-1\right)$$

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

```
[Out] 7/20*sqrt(15)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 5/4*log(x + 1) - 5/4
*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = \frac{7}{20}\,\sqrt{15}\log\left(\frac{|6\,x-2\,\sqrt{15}|}{|6\,x+2\,\sqrt{15}|}\right)+\frac{5}{4}\,\log\left(|x+1|\right)-\frac{5}{4}\,\log\left(|x-1|\right)$$

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

```
[Out] 7/20*sqrt(15)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15))) + 5/4*log(ab
s(x + 1)) - 5/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = \frac{5\,\mathrm{atanh}(x)}{2}-\frac{7\,\sqrt{15}\,\mathrm{atanh}\left(\frac{\sqrt{15}\,x}{5}\right)}{10}$$

```
[In] int((3*x^2 + 2)/(3*x^4 - 8*x^2 + 5),x)
```

```
[Out] (5*atanh(x))/2 - (7*15^(1/2)*atanh((15^(1/2)*x)/5))/10
```

3.96 $\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx=\frac{1}{2}(d+e)\text{arctanh}(x)-\frac{(3d+5e)\text{arctanh}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

```
[Out] 1/2*(d+e)*arctanh(x)-1/30*(3*d+5*e)*arctanh(1/5*x*15^(1/2))*15^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1180, 213}

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx=\frac{1}{2}\text{arctanh}(x)(d+e)-\frac{\text{arctanh}\left(\sqrt{\frac{3}{5}}x\right)(3d+5e)}{2\sqrt{15}}$$

```
[In] Int[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] ((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sqrt[15])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}(3(d+e))\int\frac{1}{-3+3x^2}\,dx\right)+\frac{1}{2}(3d+5e)\int\frac{1}{-5+3x^2}\,dx\\ &= \frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.00

$$\int\frac{d+ex^2}{5-8x^2+3x^4}\,dx=\frac{1}{60}\left(\sqrt{15}(3d+5e)\log\left(\sqrt{15}-3x\right)-15(d+e)\log(1-x)\right.$$
$$\left.+15(d+e)\log(1+x)-\sqrt{15}(3d+5e)\log\left(\sqrt{15}+3x\right)\right)$$

```
[In] Integrate[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] - 3*x] - 15*(d + e)*Log[1 - x] + 15*(d +
 e)*Log[1 + x] - Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] + 3*x])/60
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

method	result
default	$\left(\frac{d}{4}+\frac{e}{4}\right)\ln\left(x+1\right)-\frac{\left(\frac{3d}{2}+\frac{5e}{2}\right)\operatorname{arctanh}\left(\frac{x\sqrt{15}}{5}\right)\sqrt{15}}{15}+\left(-\frac{d}{4}-\frac{e}{4}\right)\ln\left(x-1\right)$
risch	$\frac{\sqrt{15}\,\ln\left(-x\sqrt{15}+5\right)d}{20}+\frac{\sqrt{15}\,\ln\left(-x\sqrt{15}+5\right)e}{12}-\frac{\sqrt{15}\,\ln\left(x\sqrt{15}+5\right)d}{20}-\frac{\sqrt{15}\,\ln\left(x\sqrt{15}+5\right)e}{12}+\frac{\ln(x+1)d}{4}+\frac{\ln(x+1)e}{4}-\ln($

```
[In] int((e*x^2+d)/(3*x^4-8*x^2+5),x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*d+1/4*e)*ln(x+1)-1/15*(3/2*d+5/2*e)*arctanh(1/5*x*15^(1/2))*15^(1/2)+(
-1/4*d-1/4*e)*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx = \frac{1}{60}\sqrt{15}(3\,d+5\,e)\log\left(\frac{3\,x^2-2\,\sqrt{15}x+5}{3\,x^2-5}\right) + \frac{1}{4}\,(d+e)\log\left(x+1\right) - \frac{1}{4}\,(d+e)\log\left(x-1\right)$$

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x^2 - 2*sqrt(15)*x + 5)/(3*x^2 - 5)) + 1/4
*(d + e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(31) = 62$.

Time = 0.79 (sec) , antiderivative size = 474, normalized size of antiderivative = 13.17

$$\begin{aligned}
&\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx \\
&= \frac{(d+e)\log\left(x+\frac{-51d^3(d+e)-180d^2e(d+e)-225de^2(d+e)+60d(d+e)^3-100e^3(d+e)+75e(d+e)^3}{9d^4+24d^3e-40de^3-25e^4}\right)}{4} \\
&\quad - \frac{(d+e)\log\left(x+\frac{51d^3(d+e)+180d^2e(d+e)+225de^2(d+e)-60d(d+e)^3+100e^3(d+e)-75e(d+e)^3}{9d^4+24d^3e-40de^3-25e^4}\right)}{4} \\
&\quad + \frac{\sqrt{15}\cdot(3d+5e)\log\left(x+\frac{-\frac{17\sqrt{15}d^3\cdot(3d+5e)}{5}-12\sqrt{15}d^2e(3d+5e)-15\sqrt{15}de^2\cdot(3d+5e)+\frac{4\sqrt{15}d(3d+5e)^3}{15}-\frac{20\sqrt{15}e^3\cdot(3d+5e)}{3}+\frac{\sqrt{15}e(3d+5e)}{3}}{9d^4+24d^3e-40de^3-25e^4}\right.}{60} \\
&\quad - \frac{\sqrt{15}\cdot(3d+5e)\log\left(x+\frac{\frac{17\sqrt{15}d^3\cdot(3d+5e)}{5}+12\sqrt{15}d^2e(3d+5e)+15\sqrt{15}de^2\cdot(3d+5e)-\frac{4\sqrt{15}d(3d+5e)^3}{15}+\frac{20\sqrt{15}e^3\cdot(3d+5e)}{3}-\frac{\sqrt{15}e(3d+5e)^3}{3}}{9d^4+24d^3e-40de^3-25e^4}\right.}{60}
\end{aligned}$$

```
[In] integrate((e*x**2+d)/(3*x**4-8*x**2+5),x)
```

```
[Out] (d + e)*log(x + (-51*d**3*(d + e) - 180*d**2*e*(d + e) - 225*d*e**2*(d + e)
 + 60*d*(d + e)**3 - 100*e**3*(d + e) + 75*e*(d + e)**3)/(9*d**4 + 24*d**3*
e - 40*d*e**3 - 25*e**4))/4 - (d + e)*log(x + (51*d**3*(d + e) + 180*d**2*e
*(d + e) + 225*d*e**2*(d + e) - 60*d*(d + e)**3 + 100*e**3*(d + e) - 75*e*(
d + e)**3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/4 + sqrt(15)*(3*d +
5*e)*log(x + (-17*sqrt(15)*d**3*(3*d + 5*e)/5 - 12*sqrt(15)*d**2*e*(3*d + 5
*e) - 15*sqrt(15)*d*e**2*(3*d + 5*e) + 4*sqrt(15)*d*(3*d + 5*e)**3/15 - 20*
sqrt(15)*e**3*(3*d + 5*e)/3 + sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**
```

```
3*e - 40*d*e**3 - 25*e**4))/60 - sqrt(15)*(3*d + 5*e)*log(x + (17*sqrt(15)*
d**3*(3*d + 5*e)/5 + 12*sqrt(15)*d**2*e*(3*d + 5*e) + 15*sqrt(15)*d*e**2*(3
*d + 5*e) - 4*sqrt(15)*d*(3*d + 5*e)**3/15 + 20*sqrt(15)*e**3*(3*d + 5*e)/3
 - sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))
/60
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx = \frac{1}{60}\,\sqrt{15}(3\,d+5\,e)\log\left(\frac{3\,x-\sqrt{15}}{3\,x+\sqrt{15}}\right) + \frac{1}{4}\,(d+e)\log\left(x+1\right) - \frac{1}{4}\,(d+e)\log\left(x-1\right)$$

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 1/4*(d +
 e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx = \frac{1}{60}\,\sqrt{15}(3\,d+5\,e)\log\left(\frac{|6\,x-2\,\sqrt{15}|}{|6\,x+2\,\sqrt{15}|}\right) + \frac{1}{4}\,(d+e)\log\left(|x+1|\right) - \frac{1}{4}\,(d+e)\log\left(|x-1|\right)$$

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15)))
+ 1/4*(d + e)*log(abs(x + 1)) - 1/4*(d + e)*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 290, normalized size of antiderivative = 8.06

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx$$

$$= \frac{\sqrt{15}\,\mathrm{atanh}\left(\frac{54\sqrt{15}\,d^3\,x}{25\left(-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3\right)}-\frac{6\sqrt{15}\,e^3\,x}{-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3}-\frac{18\sqrt{15}\,d\,e^2\,x}{5\left(-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3\right)}+}{30}$$

$$-\mathrm{atanh}\left(\frac{18\,d^3\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}-\frac{30\,e^3\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}\right.$$
$$\left.-\frac{30\,d\,e^2\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}+\frac{18\,d^2\,e\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}\right)\left(\frac{d}{2}+\frac{e}{2}\right)$$

```
[In] int((d + e*x^2)/(3*x^4 - 8*x^2 + 5),x)
```

```
[Out] (15^(1/2)*atanh((54*15^(1/2)*d^3*x)/(25*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 +
 30*e^3)) - (6*15^(1/2)*e^3*x)/(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)
- (18*15^(1/2)*d*e^2*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)) + (
18*15^(1/2)*d^2*e*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)))*(3*d
+ 5*e))/30 - atanh((18*d^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) - (30
*e^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) - (30*d*e^2*x)/(30*d*e^2 -
18*d^2*e - 18*d^3 + 30*e^3) + (18*d^2*e*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 +
30*e^3))*(d/2 + e/2)
```

3.97 $\int \frac{3+x^2}{1+3x^2+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx=-\frac{1}{10}\sqrt{180-80\sqrt{5}}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}$$

```
[Out] 1/20*arctan(x*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)-1/10*arctan(x*2
^(1/2)/(3+5^(1/2))^(1/2))*(10-4*5^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1180, 209}

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx=\frac{\left(3+\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}-\frac{1}{10}\sqrt{180-80\sqrt{5}}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[In] Int[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -1/10*(Sqrt[180 - 80*Sqrt[5]]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]) + ((3 + Sqrt
[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx\\ &= -\frac{1}{5}\sqrt{45-20\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = \frac{-\left(3-\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)+\left(3+\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}$$

```
[In] Integrate[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] (-((3 - Sqrt[5])^(3/2)*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]) + (3 + Sqrt[5])^(3/
2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+3\_Z^{2}+1\right)}\frac{\left(\_R^{2}+3\right)\ln\left(x-\_R\right)}{2\_R^{3}+3\_R}\right)}{2}$	40
default	$\frac{2\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}+\frac{2\sqrt{5}\,\left(\sqrt{5}-3\right)\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}$	66

```
[In] int((x^2+3)/(x^4+3*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*sum((_R^2+3)/(2*_R^3+3*_R)*ln(x-_R),_R=RootOf(_Z^4+3*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(44) = 88.

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

$$\begin{aligned}\int\frac{3+x^2}{1+3x^2+x^4}\,dx=&-\frac{1}{10}\sqrt{5}\sqrt{4\sqrt{5}-9}\log\left(\sqrt{4\sqrt{5}-9}\left(3\sqrt{5}+7\right)+2x\right)\\&+\frac{1}{10}\sqrt{5}\sqrt{4\sqrt{5}-9}\log\left(-\sqrt{4\sqrt{5}-9}\left(3\sqrt{5}+7\right)+2x\right)\\&-\frac{1}{10}\sqrt{5}\sqrt{-4\sqrt{5}-9}\log\left(\left(3\sqrt{5}-7\right)\sqrt{-4\sqrt{5}-9}+2x\right)\\&+\frac{1}{10}\sqrt{5}\sqrt{-4\sqrt{5}-9}\log\left(-\left(3\sqrt{5}-7\right)\sqrt{-4\sqrt{5}-9}+2x\right)\end{aligned}$$

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="fricas")
[Out] -1/10*sqrt(5)*sqrt(4*sqrt(5) - 9)*log(sqrt(4*sqrt(5) - 9)*(3*sqrt(5) + 7) +
 2*x) + 1/10*sqrt(5)*sqrt(4*sqrt(5) - 9)*log(-sqrt(4*sqrt(5) - 9)*(3*sqrt(5
) + 7) + 2*x) - 1/10*sqrt(5)*sqrt(-4*sqrt(5) - 9)*log((3*sqrt(5) - 7)*sqrt(
-4*sqrt(5) - 9) + 2*x) + 1/10*sqrt(5)*sqrt(-4*sqrt(5) - 9)*log(-(3*sqrt(5)
- 7)*sqrt(-4*sqrt(5) - 9) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = 2\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2x}{-1+\sqrt{5}}\right) - 2\cdot\left(\frac{1}{2}-\frac{\sqrt{5}}{5}\right)\operatorname{atan}\left(\frac{2x}{1+\sqrt{5}}\right)$$

```
[In] integrate((x**2+3)/(x**4+3*x**2+1),x)
[Out] 2*(sqrt(5)/5 + 1/2)*atan(2*x/(-1 + sqrt(5))) - 2*(1/2 - sqrt(5)/5)*atan(2*x
/(1 + sqrt(5)))
```

Maxima [F]

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = \int \frac{x^2+3}{x^4+3\,x^2+1}\,dx$$

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 3)/(x^4 + 3*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = \frac{1}{5}\left(2\sqrt{5}-5\right)\arctan\left(\frac{2\,x}{\sqrt{5}+1}\right) + \frac{1}{5}\left(2\sqrt{5}+5\right)\arctan\left(\frac{2\,x}{\sqrt{5}-1}\right)$$

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="giac")
[Out] 1/5*(2*sqrt(5) - 5)*arctan(2*x/(sqrt(5) + 1)) + 1/5*(2*sqrt(5) + 5)*arctan(
2*x/(sqrt(5) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\begin{aligned}\int \frac{3+x^2}{1+3x^2+x^4}\,dx = {} & 2\operatorname{atanh}\left(\frac{80\,x\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}-56} - \frac{48\sqrt{5}\,x\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}-56}\right)\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}} \\ & - 2\operatorname{atanh}\left(\frac{80\,x\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}+56} + \frac{48\sqrt{5}\,x\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}+56}\right)\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}\end{aligned}$$

```
[In] int((x^2 + 3)/(3*x^2 + x^4 + 1),x)
```

```
[Out] 2*atanh((80*x*(5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56) - (48*5^(1/2)*x*(
5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56))*(5^(1/2)/5 - 9/20)^(1/2) - 2*at
anh((80*x*(- 5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56) + (48*5^(1/2)*x*(-
5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56))*(- 5^(1/2)/5 - 9/20)^(1/2)
```

3.98 $\int \frac{a+bx^2}{1+x^2+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx=-\frac{(a+b)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(a-b)\log\left(1-x+x^2\right)+\frac{1}{4}(a-b)\log\left(1+x+x^2\right)$$

```
[Out] -1/4*(a-b)*ln(x^2-x+1)+1/4*(a-b)*ln(x^2+x+1)-1/6*(a+b)*arctan(1/3*(1-2*x)*3
^(1/2))*3^(1/2)+1/6*(a+b)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1183, 648, 632, 210, 642}

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx=-\frac{(a+b)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)$$

```
[In] Int[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -1/2*((a + b)*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + ((a + b)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) - ((a - b)*Log[1 - x + x^2])/4 + ((a - b)*Log[1 + x
 + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{a-(a-b)x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{a+(a-b)x}{1+x+x^2}\,dx \\
&= \frac{1}{4}(a-b)\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}(-a+b)\int \frac{-1+2x}{1-x+x^2}\,dx \\
&\quad + \frac{1}{4}(a+b)\int \frac{1}{1-x+x^2}\,dx + \frac{1}{4}(a+b)\int \frac{1}{1+x+x^2}\,dx \\
&= -\frac{1}{4}(a-b)\log\left(1-x+x^2\right) + \frac{1}{4}(a-b)\log\left(1+x+x^2\right) \\
&\quad + \frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) \\
&\quad + \frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)
\end{aligned}$$

$$= -\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}(a-b)\log\left(1-x+x^2\right) + \frac{1}{4}(a-b)\log\left(1+x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx = \frac{\left(2ia+\left(-i+\sqrt{3}\right)b\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}} + \frac{\left(-2ia+\left(i+\sqrt{3}\right)b\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] (((2*I)*a + (-I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*
Sqrt[3]] + (((-2*I)*a + (I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[
6 - (6*I)*Sqrt[3]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result
default	$\frac{(-a+b)\ln\left(x^2-x+1\right)}{4} + \frac{\left(\frac{a}{2}+\frac{b}{2}\right)\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} + \frac{(a-b)\ln\left(x^2+x+1\right)}{4} + \frac{\left(\frac{a}{2}+\frac{b}{2}\right)\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\frac{\sqrt{3}\,a\arctan\left(\frac{2b^2\sqrt{3}\,x}{3\left(a^2-ab+b^2\right)}+\frac{b\sqrt{3}\,a}{3a^2-3ab+3b^2}+\frac{2\sqrt{3}\,a^2x}{3\left(a^2-ab+b^2\right)}-\frac{2\sqrt{3}\,abx}{3\left(a^2-ab+b^2\right)}-\frac{\sqrt{3}\,a^2}{3\left(a^2-ab+b^2\right)}-\frac{\sqrt{3}\,b^2}{3\left(a^2-ab+b^2\right)}\right)}{6} +$

```
[In] int((b*x^2+a)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a+b)*ln(x^2-x+1)+1/3*(1/2*a+1/2*b)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2)
)+1/4*(a-b)*ln(x^2+x+1)+1/3*(1/2*a+1/2*b)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{4}\,(a-b)\log\left(x^2+x+1\right) - \frac{1}{4}\,(a-b)\log\left(x^2-x+1\right)$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 740, normalized size of antiderivative = 8.92

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx = \Bigg(-\frac{a}{4}+\frac{b}{4}$$

$$-\frac{\sqrt{3}i(a+b)}{12}\Bigg)\log\Bigg(x+\frac{2a^3\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)+6a^2b\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)-12ab^2\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right.}{a^4-a^3}$$

$$+\Bigg(-\frac{a}{4}+\frac{b}{4}$$

$$+\frac{\sqrt{3}i(a+b)}{12}\Bigg)\log\Bigg(x+\frac{2a^3\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)+6a^2b\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)-12ab^2\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right.}{a^4-a^3}$$

$$+\Bigg(\frac{a}{4}-\frac{b}{4}$$

$$-\frac{\sqrt{3}i(a+b)}{12}\Bigg)\log\Bigg(x+\frac{2a^3\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)+6a^2b\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)-12ab^2\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)+2}{a^4-a^3b+a}$$

$$+\Bigg(\frac{a}{4}-\frac{b}{4}$$

$$+\frac{\sqrt{3}i(a+b)}{12}\Bigg)\log\Bigg(x+\frac{2a^3\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)+6a^2b\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)-12ab^2\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)+2}{a^4-a^3b+a}$$

```
[In] integrate((b*x**2+a)/(x**4+x**2+1),x)
```

```
[Out] (-a/4 + b/4 - sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 - sqrt(3)*I
*(a + b)/12) + 6*a**2*b*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a
/4 + b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12)
**3 + 2*b**3*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 - sqrt(
3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4)) + (-a/4 + b/4 + sqrt(
3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) + 6*a*
*2*b*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a/4 + b/4 + sqrt(3)*
I*(a + b)/12) + 24*a*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(-a/4
+ b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3
)/(a**4 - a**3*b + a*b**3 - b**4)) + (a/4 - b/4 - sqrt(3)*I*(a + b)/12)*log
(x + (2*a**3*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 - sqr
t(3)*I*(a + b)/12) - 12*a*b**2*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(a
/4 - b/4 - sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(a/4 - b/4 - sqrt(3)*I*(a + b)
/12) - 48*b*(a/4 - b/4 - sqrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3
```

```
- b**4)) + (a/4 - b/4 + sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(a/4 - b/4 +
sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) - 12*a*
b**2*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) + 24*a*(a/4 - b/4 + sqrt(3)*I*(a +
b)/12)**3 + 2*b**3*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(a/4 - b/4 + s
qrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{a+bx^2}{1+x^2+x^4}\,dx &= \frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\\ &\quad+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\ &\quad+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)\end{aligned}$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{a+bx^2}{1+x^2+x^4}\,dx &= \frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\\ &\quad+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\ &\quad+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)\end{aligned}$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 827, normalized size of antiderivative = 9.96

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^2)/(x^2 + x^4 + 1),x)
```

```
[Out] - atan(((x*(4*a*b - 4*a^2 + 2*b^2) + (12*a + 24*x*(b/4 - a/4 + (3^(1/2)*a*1
i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)
/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i + (x*(4*a*b -
4*a^2 + 2*b^2) - (12*a - 24*x*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1
i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (
3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i)/((x*(4*a*b - 4*a^2 + 2*b^2) + (12
*a + 24*x*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 +
 (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (
3^(1/2)*b*1i)/12) - (x*(4*a*b - 4*a^2 + 2*b^2) - (12*a - 24*x*(b/4 - a/4 +
(3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3
^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12) - 2*a
*b^2 + 2*a^2*b + 2*b^3))*((a*1i)/2 - (b*1i)/2 + (3^(1/2)*a)/6 + (3^(1/2)*b)
/6) - atan(((x*(4*a*b - 4*a^2 + 2*b^2) + (12*a + 24*x*(a/4 - b/4 + (3^(1/2)
*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b
*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i + (x*(4*a*
b - 4*a^2 + 2*b^2) - (12*a - 24*x*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)
*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4
 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i)/((x*(4*a*b - 4*a^2 + 2*b^2) +
 (12*a + 24*x*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b
/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12
 + (3^(1/2)*b*1i)/12) - (x*(4*a*b - 4*a^2 + 2*b^2) - (12*a - 24*x*(a/4 - b/
4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12
+ (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12) -
 2*a*b^2 + 2*a^2*b + 2*b^3))*((b*1i)/2 - (a*1i)/2 + (3^(1/2)*a)/6 + (3^(1/2
)*b)/6)
```

3.99 $\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\begin{aligned}\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2} \, dx = {} & \frac{x(a+b-(a-2b)x^2)}{6\left(1+x^2+x^4\right)} - \frac{(4a+b)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} \\ & + \frac{(4a+b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}} - \frac{1}{8}(2a-b)\log\left(1-x+x^2\right) \\ & + \frac{1}{8}(2a-b)\log\left(1+x+x^2\right)\end{aligned}$$

```
[Out] 1/6*x*(a+b-(a-2*b)*x^2)/(x^4+x^2+1)-1/8*(2*a-b)*ln(x^2-x+1)+1/8*(2*a-b)*ln(
x^2+x+1)-1/36*(4*a+b)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/36*(4*a+b)*arct
an(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1192, 1183, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2} \, dx = {} & -\frac{(4a+b)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4a+b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}} \\ & - \frac{1}{8}(2a-b)\log\left(x^2-x+1\right) \\ & + \frac{1}{8}(2a-b)\log\left(x^2+x+1\right) + \frac{x(-(x^2(a-2b))+a+b)}{6\left(x^4+x^2+1\right)}\end{aligned}$$

```
[In] Int[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) - ((4*a + b)*ArcTan[(1 - 2*
x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*a + b)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[
3]) - ((2*a - b)*Log[1 - x + x^2])/8 + ((2*a - b)*Log[1 + x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}+\frac{1}{6}\int \frac{5a-b+(-a+2b)x^2}{1+x^2+x^4}\,dx \\
&= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}+\frac{1}{12}\int \frac{5a-b-(6a-3b)x}{1-x+x^2}\,dx+\frac{1}{12}\int \frac{5a-b+(6a-3b)x}{1+x+x^2}\,dx \\
&= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}+\frac{1}{8}(2a-b)\int \frac{1+2x}{1+x+x^2}\,dx+\frac{1}{8}(-2a+b)\int \frac{-1+2x}{1-x+x^2}\,dx \\
&\quad +\frac{1}{24}(4a+b)\int \frac{1}{1-x+x^2}\,dx+\frac{1}{24}(4a+b)\int \frac{1}{1+x+x^2}\,dx \\
&= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}-\frac{1}{8}(2a-b)\log\left(1-x+x^2\right) \\
&\quad +\frac{1}{8}(2a-b)\log\left(1+x+x^2\right)+\frac{1}{12}(-4a-b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&\quad +\frac{1}{12}(-4a-b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}} \\
&\quad -\frac{1}{8}(2a-b)\log\left(1-x+x^2\right)+\frac{1}{8}(2a-b)\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.24

$$
\begin{aligned}
\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx &= \frac{x(a+b-ax^2+2bx^2)}{6(1+x^2+x^4)} \\
&\quad -\frac{\left(\left(-11i+\sqrt{3}\right)a-2\left(-2i+\sqrt{3}\right)b\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{6\sqrt{6+6i\sqrt{3}}} \\
&\quad -\frac{\left(\left(11i+\sqrt{3}\right)a-2\left(2i+\sqrt{3}\right)b\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{6\sqrt{6-6i\sqrt{3}}}
\end{aligned}
$$

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - a*x^2 + 2*b*x^2))/(6*(1 + x^2 + x^4)) - (((-11*I + Sqrt[3])*a -
 2*(-2*I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/(6*Sqrt[6 + (6*I)*Sqrt
[3]]) - (((11*I + Sqrt[3])*a - 2*(2*I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x
)/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

method	result
default	$-\frac{\left(\frac{a}{3}-\frac{2b}{3}\right)x-\frac{2a}{3}+\frac{b}{3}}{4(x^2-x+1)}-\frac{(6a-3b)\ln\left(x^2-x+1\right)}{24}-\frac{\left(-2a-\frac{b}{2}\right)\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{18}+\frac{\left(-\frac{a}{3}+\frac{2b}{3}\right)x-\frac{2a}{3}+\frac{b}{3}}{4x^2+4x+4}+\frac{(6a-3b)\ln\left(x^2+x+\right.}{24}$
risch	$\frac{a\ln\left(124a^2x^2-100ab\,x^2+28b^2x^2+124a^2x-100abx+28b^2x+124a^2-100ab+28b^2\right)}{4}-\frac{b\ln\left(124a^2x^2-100ab\,x^2+28b^2x^2+124a^2x-100a\right.}{8}$

```
[In] int((b*x^2+a)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((1/3*a-2/3*b)*x-2/3*a+1/3*b)/(x^2-x+1)-1/24*(6*a-3*b)*ln(x^2-x+1)-1/1
8*(-2*a-1/2*b)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*((-1/3*a+2/3*b)*x-2/
3*a+1/3*b)/(x^2+x+1)+1/24*(6*a-3*b)*ln(x^2+x+1)+1/18*(2*a+1/2*b)*arctan(1/3
*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.55

$$\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2}\,dx =$$

$$-\frac{12\,(a-2\,b)x^3-2\,\sqrt{3}((4\,a+b)x^4+(4\,a+b)x^2+4\,a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,\sqrt{3}((4\,a+b)x^4+(}{}$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(a - 2*b)*x^3 - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x^2 + 4*a +
b)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x^2
 + 4*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(a + b)*x - 9*((2*a - b)*x^4
 + (2*a - b)*x^2 + 2*a - b)*log(x^2 + x + 1) + 9*((2*a - b)*x^4 + (2*a - b)
*x^2 + 2*a - b)*log(x^2 - x + 1))/(x^4 + x^2 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 874, normalized size of antiderivative = 7.34

$$\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x**2+a)/(x**4+x**2+1)**2,x)
```

```
[Out] (x**3*(-a + 2*b) + x*(a + b))/(6*x**4 + 6*x**2 + 6) + (-a/4 + b/8 - sqrt(3)
*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) + 9
48*a**2*b*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(-a/4 + b/8 -
sqrt(3)*I*(4*a + b)/72) + 12096*a*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72)**3
+ 148*b**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(-a/4 + b/8 - sqr
t(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3
- 7*b**4)) + (-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4 +
 b/8 + sqrt(3)*I*(4*a + b)/72) + 948*a**2*b*(-a/4 + b/8 + sqrt(3)*I*(4*a +
b)/72) - 816*a*b**2*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(-a/4 +
 b/8 + sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*(-a/4 + b/8 + sqrt(3)*I*(4*a +
 b)/72) - 8640*b*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*
a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) + (a/4 - b/8 - sqrt(3)*I*(4*a
+ b)/72)*log(x + (76*a**3*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) + 948*a**2*b
*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(a/4 - b/8 - sqrt(3)*I*(
4*a + b)/72) + 12096*a*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*(
a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(a/4 - b/8 - sqrt(3)*I*(4*a +
b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) + (
a/4 - b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(a/4 - b/8 + sqrt(3)*I
*(4*a + b)/72) + 948*a**2*b*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) - 816*a*b*
*2*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(a/4 - b/8 + sqrt(3)*I*(4
*a + b)/72)**3 + 148*b**3*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) - 8640*b*(a/
4 - b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2
 + 11*a*b**3 - 7*b**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2}\,dx = \frac{1}{36}\,\sqrt{3}(4\,a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{36}\,\sqrt{3}(4\,a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{8}\,(2\,a-b)\log\left(x^2+x+1\right) - \frac{1}{8}\,(2\,a-b)\log\left(x^2-x+1\right) - \frac{(a-2\,b)x^3-(a+b)x}{6\,(x^4+x^2+1)}$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(2*
a - b)*log(x^2 - x + 1) - 1/6*((a - 2*b)*x^3 - (a + b)*x)/(x^4 + x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2}\,dx = \frac{1}{36}\,\sqrt{3}(4\,a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{36}\,\sqrt{3}(4\,a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{8}\,(2\,a-b)\log\left(x^2+x+1\right) - \frac{1}{8}\,(2\,a-b)\log\left(x^2-x+1\right) - \frac{ax^3-2\,bx^3-ax-bx}{6\,(x^4+x^2+1)}$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(2*
a - b)*log(x^2 - x + 1) - 1/6*(a*x^3 - 2*b*x^3 - a*x - b*x)/(x^4 + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 897, normalized size of antiderivative = 7.54

$$\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^2)/(x^2 + x^4 + 1)^2,x)
```

```
[Out] atan((((2*b - 10*a + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/7
2))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 -
(19*a*b)/9 + b^2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)*1i
 + ((10*a - 2*b + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))
*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19
*a*b)/9 + b^2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)*1i)/(
(19*a*b^2)/36 - (29*a^2*b)/36 + (31*a^3)/108 - (7*b^3)/54 + ((2*b - 10*a +
24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(b/8 - a/4 + (3^(
1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*
(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - ((10*a - 2*b + 24*x*(
b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(b/8 - a/4 + (3^(1/2)*a
*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(b/8 -
 a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)))*((a*1i)/2 - (b*1i)/4 + (3^(
1/2)*a)/9 + (3^(1/2)*b)/36) + atan((((2*b - 10*a + 24*x*(a/4 - b/8 + (3^(1/
2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)
*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*
1i)/18 + (3^(1/2)*b*1i)/72)*1i + ((10*a - 2*b + 24*x*(a/4 - b/8 + (3^(1/2)*
a*1i)/18 + (3^(1/2)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*
1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*1i)
/18 + (3^(1/2)*b*1i)/72)*1i)/((19*a*b^2)/36 - (29*a^2*b)/36 + (31*a^3)/108
- (7*b^3)/54 + ((2*b - 10*a + 24*x*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2
)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a
^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1
i)/72) - ((10*a - 2*b + 24*x*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i
)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18
 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)
))*((b*1i)/4 - (a*1i)/2 + (3^(1/2)*a)/9 + (3^(1/2)*b)/36) - (x^3*(a/6 - b/3
) - x*(a/6 + b/6))/(x^2 + x^4 + 1)
```

3.100 $\int \frac{a+bx^2}{2+x^2+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 234

$$\begin{aligned}\int \frac{a+bx^2}{2+x^2+x^4}\,dx = & -\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(a+\sqrt{2}b\right)\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)\\ & +\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(a+\sqrt{2}b\right)\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)\\ & -\frac{\left(a-\sqrt{2}b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}\\ & +\frac{\left(a-\sqrt{2}b\right)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}\end{aligned}$$

```
[Out] -1/28*arctan((-2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(a+b*2^(1/2))
*(-14+28*2^(1/2))^(1/2)+1/28*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2)
)^(1/2))*(a+b*2^(1/2))*(-14+28*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(-1+2*2^
(1/2))^(1/2))*(a-b*2^(1/2))/(-2+4*2^(1/2))^(1/2)+1/4*ln(x^2+2^(1/2)+x*(-1+2
*2^(1/2))^(1/2))*(a-b*2^(1/2))/(-2+4*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1183, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{a+bx^2}{2+x^2+x^4}\,dx = &-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}b\right)\arctan\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)\\ &+\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}b\right)\arctan\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)\\ &-\frac{\left(a-\sqrt{2}b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}\\ &+\frac{\left(a-\sqrt{2}b\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}\end{aligned}$$

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] -1/2*(Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]
] - 2*x)/Sqrt[1 + 2*Sqrt[2]]]) + (Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)
*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 - ((a - Sqrt[2
]*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2]
)]) + ((a - Sqrt[2]*b)*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt
[2*(-1 + 2*Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{-1+2\sqrt{2}}a-\left(a-\sqrt{2}b\right)x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\int \frac{\sqrt{-1+2\sqrt{2}}a+\left(a-\sqrt{2}b\right)x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}\\
&= \frac{1}{8}\left(\sqrt{2}a+2b\right)\int \frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx\\
&\quad+\frac{1}{8}\left(\sqrt{2}a+2b\right)\int \frac{1}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx\\
&\quad-\frac{\left(a-\sqrt{2}b\right)\int \frac{-\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\left(a-\sqrt{2}b\right)\int \frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}\\
&= -\frac{\left(a-\sqrt{2}b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}\\
&\quad+\frac{\left(a-\sqrt{2}b\right)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}\\
&\quad-\frac{1}{4}\left(\sqrt{2}a+2b\right)\text{Subst}\left(\int \frac{1}{-1-2\sqrt{2}-x^2}\,dx,x,-\sqrt{-1+2\sqrt{2}}+2x\right)\\
&\quad-\frac{1}{4}\left(\sqrt{2}a+2b\right)\text{Subst}\left(\int \frac{1}{-1-2\sqrt{2}-x^2}\,dx,x,\sqrt{-1+2\sqrt{2}}+2x\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(a+\sqrt{2}b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\left(a+\sqrt{2}b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}} \\
&\quad -\frac{\left(a-\sqrt{2}b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} \\
&\quad +\frac{\left(a-\sqrt{2}b\right)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.47

$$
\begin{aligned}
\int \frac{a+bx^2}{2+x^2+x^4}\,dx &= \frac{\left(-2ia+\left(i+\sqrt{7}\right)b\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{14-14i\sqrt{7}}} \\
&\quad +\frac{\left(2ia+\left(-i+\sqrt{7}\right)b\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{14+14i\sqrt{7}}}
\end{aligned}
$$

```
[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] (((-2*I)*a + (I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[14 -
(14*I)*Sqrt[7]] + (((2*I)*a + (-I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 + I*Sqrt[7
])/2]])/Sqrt[14 + (14*I)*Sqrt[7]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+\_Z^{2}+2\right)}\frac{\left(\_R^{2}b+a\right)\ln\left(x-\_R\right)}{2\_R^{3}+\_R}\right)}{2}$
default	$\frac{\left(\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a-4\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,b+4\sqrt{-1+2\sqrt{2}}\,a-2\sqrt{-1+2\sqrt{2}}\,b\right)\ln\left(x^{2}+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)}{56}+\left(7\sqrt{2}\,a-\frac{\left(\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a-4\sqrt{-}\right.}{}\right.$

```
[In] int((b*x^2+a)/(x^4+x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((_R^2*b+a)/(2*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+_Z^2+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(167) = 334$.

Time = 0.28 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.67

$$\begin{aligned}
&\int \frac{a+bx^2}{2+x^2+x^4}\,dx \\
&= -\frac{1}{28}\sqrt{7}\sqrt{a^2-8\,ab+2\,b^2+\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\log\Bigg(-4\left(a^4-a^3b+2\,ab^3-4\,b^4\right)x \\
&\quad+\sqrt{a^2-8\,ab+2\,b^2+\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\left(\sqrt{7}\left(a^3-2\,ab^2\right)+\sqrt{-a^4+4\,a^2b^2-4\,b^4}(a-4\,b)\right)\Bigg) \\
&\quad+\frac{1}{28}\sqrt{7}\sqrt{a^2-8\,ab+2\,b^2+\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\log\Bigg(-4\left(a^4-a^3b+2\,ab^3-4\,b^4\right)x \\
&\quad-\sqrt{a^2-8\,ab+2\,b^2+\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\left(\sqrt{7}\left(a^3-2\,ab^2\right)+\sqrt{-a^4+4\,a^2b^2-4\,b^4}(a-4\,b)\right)\Bigg) \\
&\quad-\frac{1}{28}\sqrt{7}\sqrt{a^2-8\,ab+2\,b^2-\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\log\Bigg(-4\left(a^4-a^3b+2\,ab^3-4\,b^4\right)x \\
&\quad+\sqrt{a^2-8\,ab+2\,b^2-\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\left(\sqrt{7}\left(a^3-2\,ab^2\right)-\sqrt{-a^4+4\,a^2b^2-4\,b^4}(a-4\,b)\right)\Bigg) \\
&\quad+\frac{1}{28}\sqrt{7}\sqrt{a^2-8\,ab+2\,b^2-\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\log\Bigg(-4\left(a^4-a^3b+2\,ab^3-4\,b^4\right)x \\
&\quad-\sqrt{a^2-8\,ab+2\,b^2-\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\left(\sqrt{7}\left(a^3-2\,ab^2\right)-\sqrt{-a^4+4\,a^2b^2-4\,b^4}(a-4\,b)\right)\Bigg)
\end{aligned}$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="fricas")
```

```
[Out] -1/28*sqrt(7)*sqrt(a^2 - 8*a*b + 2*b^2 + sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*
b^4))*log(-4*(a^4 - a^3*b + 2*a*b^3 - 4*b^4)*x + sqrt(a^2 - 8*a*b + 2*b^2 +
 sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4))*(sqrt(7)*(a^3 - 2*a*b^2) + sqrt(-a
^4 + 4*a^2*b^2 - 4*b^4)*(a - 4*b))) + 1/28*sqrt(7)*sqrt(a^2 - 8*a*b + 2*b^2
```

```
 + sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4))*log(-4*(a^4 - a^3*b + 2*a*b^3 -
4*b^4)*x - sqrt(a^2 - 8*a*b + 2*b^2 + sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4
))*(sqrt(7)*(a^3 - 2*a*b^2) + sqrt(-a^4 + 4*a^2*b^2 - 4*b^4)*(a - 4*b))) -
1/28*sqrt(7)*sqrt(a^2 - 8*a*b + 2*b^2 - sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b
^4))*log(-4*(a^4 - a^3*b + 2*a*b^3 - 4*b^4)*x + sqrt(a^2 - 8*a*b + 2*b^2 -
sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4))*(sqrt(7)*(a^3 - 2*a*b^2) - sqrt(-a^
4 + 4*a^2*b^2 - 4*b^4)*(a - 4*b))) + 1/28*sqrt(7)*sqrt(a^2 - 8*a*b + 2*b^2
- sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4))*log(-4*(a^4 - a^3*b + 2*a*b^3 - 4
*b^4)*x - sqrt(a^2 - 8*a*b + 2*b^2 - sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4)
)*(sqrt(7)*(a^3 - 2*a*b^2) - sqrt(-a^4 + 4*a^2*b^2 - 4*b^4)*(a - 4*b)))
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.52

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx$$
$$= \text{RootSum}\left(1568t^4 + t^2\left(-28a^2 + 224ab - 56b^2\right) + a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4, \left(t \mapsto t\log\left(x + \frac{112t^3}{}\right.\right.\right.$$

```
[In] integrate((b*x**2+a)/(x**4+x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 + _t**2*(-28*a**2 + 224*a*b - 56*b**2) + a**4 - 2*a**3*b
 + 5*a**2*b**2 - 4*a*b**3 + 4*b**4, Lambda(_t, _t*log(x + (112*_t**3*a - 44
8*_t**3*b + 6*_t*a**3 + 12*_t*a**2*b - 48*_t*a*b**2 + 8*_t*b**3)/(a**4 - a*
*3*b + 2*a*b**3 - 4*b**4))))
```

Maxima [F]

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx = \int \frac{bx^2+a}{x^4+x^2+2}\,dx$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(167) = 334.

Time = 0.54 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.58

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx =$$

$$-\frac{1}{896}\sqrt{7}\left(\sqrt{7}2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}+4\right)+3\sqrt{7}2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}-4\right)-3\cdot 2^{\frac{3}{4}}b\left(\sqrt{2}+4\right)\sqrt{-2\sqrt{2}+8}\right.$$

$$-\frac{1}{896}\sqrt{7}\left(\sqrt{7}2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}+4\right)+3\sqrt{7}2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}-4\right)-3\cdot 2^{\frac{3}{4}}b\left(\sqrt{2}+4\right)\sqrt{-2\sqrt{2}+8}\right.$$

$$-\frac{1}{1792}\sqrt{7}\left(3\sqrt{7}2^{\frac{3}{4}}b\left(\sqrt{2}+4\right)\sqrt{-2\sqrt{2}+8}+\sqrt{7}2^{\frac{3}{4}}b\left(\sqrt{2}-4\right)\sqrt{-2\sqrt{2}+8}+2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}+4\right)\right.$$

$$\left.+2\cdot 2^{\frac{1}{4}}x\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}+\sqrt{2}\right)$$

$$+\frac{1}{1792}\sqrt{7}\left(3\sqrt{7}2^{\frac{3}{4}}b\left(\sqrt{2}+4\right)\sqrt{-2\sqrt{2}+8}+\sqrt{7}2^{\frac{3}{4}}b\left(\sqrt{2}-4\right)\sqrt{-2\sqrt{2}+8}+2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}+4\right)\right.$$

$$\left.-2\cdot 2^{\frac{1}{4}}x\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}+\sqrt{2}\right)$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="giac")
```

```
[Out] -1/896*sqrt(7)*(sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*sqr
t(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 3*2^(3/4)*b*(sqrt(2) + 4
)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) - 8*s
qrt(7)*2^(1/4)*a*sqrt(2*sqrt(2) + 8) + 8*2^(1/4)*a*sqrt(-2*sqrt(2) + 8))*ar
ctan(2*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2
) + 4)) - 1/896*sqrt(7)*(sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4
) + 3*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 3*2^(3/4)*b*(sq
rt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) +
 8) - 8*sqrt(7)*2^(1/4)*a*sqrt(2*sqrt(2) + 8) + 8*2^(1/4)*a*sqrt(-2*sqrt(2)
 + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sq
rt(sqrt(2) + 4)) - 1/1792*sqrt(7)*(3*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-
2*sqrt(2) + 8) + sqrt(7)*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 2^(
3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*
(sqrt(2) - 4) - 8*sqrt(7)*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) - 8*2^(1/4)*a*sqrt
(2*sqrt(2) + 8))*log(x^2 + 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2))
```

```
+ 1/1792*sqrt(7)*(3*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) +
sqrt(7)*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 2^(3/4)*b*sqrt(2*sqr
t(2) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 8
*sqrt(7)*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) - 8*2^(1/4)*a*sqrt(2*sqrt(2) + 8))*
log(x^2 - 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2))
```

Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.29

$$
\begin{aligned}
\int \frac{a+bx^2}{2+x^2+x^4}\,dx = {}& -\mathrm{atan}\left(\frac{a^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}\,7{}\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1{}\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1{}\mathrm{i}}\right.\\
&-\frac{b^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}\,14{}\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1{}\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1{}\mathrm{i}}\\
&+\frac{\sqrt{7}\,a^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}}{\frac{\sqrt{7}\,a^3\,1{}\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1{}\mathrm{i}}\\
&\left.-\frac{2\,\sqrt{7}\,b^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}}{\frac{\sqrt{7}\,a^3\,1{}\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1{}\mathrm{i}}\right)\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}\,2{}\mathrm{i}\\
&-2\,\mathrm{atanh}\left(\frac{7\,a^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}}{\frac{\sqrt{7}\,a^3\,1{}\mathrm{i}}{2}+a\,b^2+2\,a^2\,b-\frac{a^3}{2}-4\,b^3-\sqrt{7}\,a\,b^2\,1{}\mathrm{i}}\right.\\
&-\frac{14\,b^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}}{\frac{\sqrt{7}\,a^3\,1{}\mathrm{i}}{2}+a\,b^2+2\,a^2\,b-\frac{a^3}{2}-4\,b^3-\sqrt{7}\,a\,b^2\,1{}\mathrm{i}}\\
&+\frac{\sqrt{7}\,a^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}\,1{}\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1{}\mathrm{i}}{2}+a\,b^2+2\,a^2\,b-\frac{a^3}{2}-4\,b^3-\sqrt{7}\,a\,b^2\,1{}\mathrm{i}}\\
&\left.-\frac{\sqrt{7}\,b^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}\,2{}\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1{}\mathrm{i}}{2}+a\,b^2+2\,a^2\,b-\frac{a^3}{2}-4\,b^3-\sqrt{7}\,a\,b^2\,1{}\mathrm{i}}\right)\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1{}\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1{}\mathrm{i}}{56}}
\end{aligned}
$$

```
[In] int((a + b*x^2)/(x^2 + x^4 + 2),x)
```

```
[Out] - atan((a^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/
112 + b^2/56)^(1/2)*7i)/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b
^3 - 7^(1/2)*a*b^2*1i) - (b^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)
*b^2*1i)/56 + a^2/112 + b^2/56)^(1/2)*14i)/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*
a^2*b + a^3/2 + 4*b^3 - 7^(1/2)*a*b^2*1i) + (7^(1/2)*a^2*x*((7^(1/2)*a^2*1i
)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/112 + b^2/56)^(1/2))/((7^(1/2)
```

```
*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b^3 - 7^(1/2)*a*b^2*1i) - (2*7^(1/
2)*b^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/112 +
 b^2/56)^(1/2))/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b^3 - 7^(
1/2)*a*b^2*1i))*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^
2/112 + b^2/56)^(1/2)*2i - 2*atanh((7*a^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)
*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2))/((7^(1/2)*a^3*1i)/2 + a*
b^2 + 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i) - (14*b^2*x*((7^(1/2)*b^2
*1i)/56 - (7^(1/2)*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2))/((7^(1
/2)*a^3*1i)/2 + a*b^2 + 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i) + (7^(1
/2)*a^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)*a^2*1i)/112 - (a*b)/14 + a^2/112
+ b^2/56)^(1/2)*1i)/((7^(1/2)*a^3*1i)/2 + a*b^2 + 2*a^2*b - a^3/2 - 4*b^3 -
 7^(1/2)*a*b^2*1i) - (7^(1/2)*b^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)*a^2*1i)
/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2)*2i)/((7^(1/2)*a^3*1i)/2 + a*b^2 +
 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i))*((7^(1/2)*b^2*1i)/56 - (7^(1/
2)*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2)
```

3.101 $\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 316

$$
\begin{aligned}
\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx = {} & \frac{x(3a+2b-(a-4b)x^2)}{28\,(2+x^2+x^4)} - \frac{1}{56}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(\left(11-\sqrt{2}\right)a\right. \\
& \left. -\left(2-4\sqrt{2}\right)b\right)\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right) \\
& + \frac{1}{56}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(\left(11-\sqrt{2}\right)a\right. \\
& \left. -\left(2-4\sqrt{2}\right)b\right)\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right) \\
& - \frac{\left(11a+\sqrt{2}(a-4b)-2b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}} \\
& + \frac{\left(\left(11+\sqrt{2}\right)a-2\left(b+2\sqrt{2}b\right)\right)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}}
\end{aligned}
$$

```
[Out] 1/28*x*(3*a+2*b-(a-4*b)*x^2)/(x^4+x^2+2)-1/784*arctan((-2*x+(-1+2*2^(1/2))^
(1/2))/(1+2*2^(1/2))^(1/2))*(-b*(2-4*2^(1/2))+a*(11-2^(1/2)))*(-14+28*2^(1/
2))^(1/2)+1/784*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-b*
(2-4*2^(1/2))+a*(11-2^(1/2)))*(-14+28*2^(1/2))^(1/2)-1/112*ln(x^2+2^(1/2)-x
*(-1+2*2^(1/2))^(1/2))*(11*a-2*b+(a-4*b)*2^(1/2))/(-2+4*2^(1/2))^(1/2)+1/11
2*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(a*(11+2^(1/2))-2*b-4*b*2^(1/2))/(
-2+4*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1192, 1183, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx = & -\frac{1}{56}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(\left(11-\sqrt{2}\right)a\right.\\ & \left.-\left(2-4\sqrt{2}\right)b\right)\arctan\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)\\ & +\frac{1}{56}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(\left(11-\sqrt{2}\right)a\right.\\ & \left.-\left(2-4\sqrt{2}\right)b\right)\arctan\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)\\ & -\frac{\left(\sqrt{2}(a-4b)+11a-2b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}\\ & +\frac{\left(\left(11+\sqrt{2}\right)a-2\left(2\sqrt{2}b+b\right)\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}\\ & +\frac{x(-(x^2(a-4b))+3a+2b)}{28\left(x^4+x^2+2\right)}\end{aligned}$$

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

```
[Out] (x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) - (Sqrt[(-1 + 2*Sqrt[2
])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]]
- 2*x)/Sqrt[1 + 2*Sqrt[2]]])/56 + (Sqrt[(-1 + 2*Sqrt[2])/14]*((11 - Sqrt[2]
)*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqr
t[2]]])/56 - ((11*a + Sqrt[2]*(a - 4*b) - 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sq
rt[2]]*x + x^2])/(112*Sqrt[2*(-1 + 2*Sqrt[2])]) + (((11 + Sqrt[2])*a - 2*(b
 + 2*Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqrt[2*(
-1 + 2*Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\text{integral} = \frac{x(3a+2b-(a-4b)x^2)}{28\left(2+x^2+x^4\right)}+\frac{1}{28}\int\frac{11a-2b+(-a+4b)x^2}{2+x^2+x^4}\,dx$$

$$= \frac{x(3a+2b-(a-4b)x^2)}{28\left(2+x^2+x^4\right)}+\frac{\int\frac{\sqrt{-1+2\sqrt{2}}(11a-2b)-\left(11a-2b-\sqrt{2}(-a+4b)\right)x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{56\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$+\frac{\int\frac{\sqrt{-1+2\sqrt{2}}(11a-2b)+\left(11a-2b-\sqrt{2}(-a+4b)\right)x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{56\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$
\begin{aligned}
&= \frac{x(3a+2b-(a-4b)x^2)}{28\left(2+x^2+x^4\right)} - \frac{\left(11a+\sqrt{2}(a-4b)-2b\right)\int \frac{-\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{112\sqrt{2\left(-1+2\sqrt{2}\right)}} \\
&\quad + \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\int \frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{112\sqrt{2}} \\
&\quad + \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\int \frac{1}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{112\sqrt{2}} \\
&\quad + \frac{\left(\left(11+\sqrt{2}\right)a-2\left(b+2\sqrt{2}b\right)\right)\int \frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{112\sqrt{2\left(-1+2\sqrt{2}\right)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(3a+2b-(a-4b)x^2)}{28\left(2+x^2+x^4\right)} \\
&\quad - \frac{\left(11a+\sqrt{2}(a-4b)-2b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}} \\
&\quad + \frac{\left(\left(11+\sqrt{2}\right)a-2\left(b+2\sqrt{2}b\right)\right)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}} \\
&\quad - \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\text{Subst}\left(\int \frac{1}{-1-2\sqrt{2}-x^2}\,dx, x, -\sqrt{-1+2\sqrt{2}}+2x\right)}{56\sqrt{2}} \\
&\quad - \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\text{Subst}\left(\int \frac{1}{-1-2\sqrt{2}-x^2}\,dx, x, \sqrt{-1+2\sqrt{2}}+2x\right)}{56\sqrt{2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(3a+2b-(a-4b)x^2)}{28\left(2+x^2+x^4\right)} - \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2\left(1+2\sqrt{2}\right)}} \\
&\quad + \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2\left(1+2\sqrt{2}\right)}} \\
&\quad - \frac{\left(11a+\sqrt{2}(a-4b)-2b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}} \\
&\quad + \frac{\left(\left(11+\sqrt{2}\right)a-2\left(b+2\sqrt{2}b\right)\right)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.53

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx = \frac{3ax+2bx-ax^3+4bx^3}{28\,(2+x^2+x^4)} - \frac{\left(\left(23i+\sqrt{7}\right)a-4\left(2i+\sqrt{7}\right)b\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{28\sqrt{14-14i\sqrt{7}}} - \frac{\left(\left(-23i+\sqrt{7}\right)a-4\left(-2i+\sqrt{7}\right)b\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{28\sqrt{14+14i\sqrt{7}}}$$

```
[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

```
[Out] (3*a*x + 2*b*x - a*x^3 + 4*b*x^3)/(28*(2 + x^2 + x^4)) - (((23*I + Sqrt[7])
*a - 4*(2*I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/(28*Sqrt[14 -
(14*I)*Sqrt[7]]) - (((-23*I + Sqrt[7])*a - 4*(-2*I + Sqrt[7])*b)*ArcTan[x/S
qrt[(1 + I*Sqrt[7])/2]])/(28*Sqrt[14 + (14*I)*Sqrt[7]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\frac{b}{7}-\frac{a}{28}\right)x^3+\left(\frac{b}{14}+\frac{3a}{28}\right)x}{x^4+x^2+2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+\_Z^2+2\right)}\frac{\left((-a+4b)\_R^2-2b+11a\right)\ln\left(x-\_R\right)}{2\_R^3+\_R}\right)}{56}$
default	$\frac{\frac{\left(-14a-28\sqrt{2}\,a+112b\sqrt{2}+56b\right)x}{1+2\sqrt{2}}+\frac{\sqrt{-1+2\sqrt{2}}\left(-70a-42\sqrt{2}\,a+56b\sqrt{2}+28b\right)}{1+2\sqrt{2}}}{784x\sqrt{-1+2\sqrt{2}}+784x^2+784\sqrt{2}}+\frac{\left(107\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a-50\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,b+106\sqrt{-1+2\sqrt{2}}\,a-88\sqrt{-}\right.}{2}$

```
[In] int((b*x^2+a)/(x^4+x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((1/7*b-1/28*a)*x^3+(1/14*b+3/28*a)*x)/(x^4+x^2+2)+1/56*sum(((-a+4*b)*_R^2-
2*b+11*a)/(2*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+_Z^2+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(235) = 470.

Time = 0.29 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.02

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx =$$

$$-\frac{28\,(a-4\,b)x^3+\sqrt{7}(x^4+x^2+2)\sqrt{211\,a^2-428\,ab+100\,b^2+7\,\sqrt{7}\sqrt{-289\,a^4+136\,a^3b+120\,a^2b^2-32}}}{}$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/784*(28*(a - 4*b)*x^3 + sqrt(7)*(x^4 + x^2 + 2)*sqrt(211*a^2 - 428*a*b +
 100*b^2 + 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 1
6*b^4))*log(-8*(1139*a^4 - 1169*a^3*b + 318*a^2*b^2 + 124*a*b^3 - 88*b^4)*x
 + sqrt(211*a^2 - 428*a*b + 100*b^2 + 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b +
 120*a^2*b^2 - 32*a*b^3 - 16*b^4))*(sqrt(7)*(187*a^3 - 78*a^2*b - 36*a*b^2
+ 8*b^3) + 3*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4)*(
5*a - 6*b))) - sqrt(7)*(x^4 + x^2 + 2)*sqrt(211*a^2 - 428*a*b + 100*b^2 + 7
*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4))*log(
-8*(1139*a^4 - 1169*a^3*b + 318*a^2*b^2 + 124*a*b^3 - 88*b^4)*x - sqrt(211*
a^2 - 428*a*b + 100*b^2 + 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2
 - 32*a*b^3 - 16*b^4))*(sqrt(7)*(187*a^3 - 78*a^2*b - 36*a*b^2 + 8*b^3) + 3
*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4)*(5*a - 6*b)))
 + sqrt(7)*(x^4 + x^2 + 2)*sqrt(211*a^2 - 428*a*b + 100*b^2 - 7*sqrt(7)*sqr
t(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4))*log(-8*(1139*a^4
 - 1169*a^3*b + 318*a^2*b^2 + 124*a*b^3 - 88*b^4)*x + sqrt(211*a^2 - 428*a*
b + 100*b^2 - 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3
- 16*b^4))*(sqrt(7)*(187*a^3 - 78*a^2*b - 36*a*b^2 + 8*b^3) - 3*sqrt(-289*a
^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4)*(5*a - 6*b))) - sqrt(7)*(
x^4 + x^2 + 2)*sqrt(211*a^2 - 428*a*b + 100*b^2 - 7*sqrt(7)*sqrt(-289*a^4 +
 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4))*log(-8*(1139*a^4 - 1169*a^3*
b + 318*a^2*b^2 + 124*a*b^3 - 88*b^4)*x - sqrt(211*a^2 - 428*a*b + 100*b^2
- 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4))*(
sqrt(7)*(187*a^3 - 78*a^2*b - 36*a*b^2 + 8*b^3) - 3*sqrt(-289*a^4 + 136*a^3
*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4)*(5*a - 6*b))) - 28*(3*a + 2*b)*x)/(x^
4 + x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.52

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx = \frac{x^3(-a+4b)+x(3a+2b)}{28x^4+28x^2+56}$$

$$+\mathrm{RootSum}\left(240945152t^4+t^2\left(-1157968a^2+2348864ab-548800b^2\right)+4489a^4-7102a^3b+5757a^2b^2-\right.$$

```
[In] integrate((b*x**2+a)/(x**4+x**2+2)**2,x)
```

```
[Out] (x**3*(-a + 4*b) + x*(3*a + 2*b))/(28*x**4 + 28*x**2 + 56) + RootSum(240945
152*_t**4 + _t**2*(-1157968*a**2 + 2348864*a*b - 548800*b**2) + 4489*a**4 -
 7102*a**3*b + 5757*a**2*b**2 - 2332*a*b**3 + 484*b**4, Lambda(_t, _t*log(x
 + (2634240*_t**3*a - 3161088*_t**3*b + 11996*_t*a**3 + 12792*_t*a**2*b - 2
1936*_t*a*b**2 + 4384*_t*b**3)/(1139*a**4 - 1169*a**3*b + 318*a**2*b**2 + 1
24*a*b**3 - 88*b**4)))
```

Maxima [F]

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx = \int \frac{bx^2+a}{(x^4+x^2+2)^2}\,dx$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/28*((a - 4*b)*x^3 - (3*a + 2*b)*x)/(x^4 + x^2 + 2) + 1/28*integrate(-((a
 - 4*b)*x^2 - 11*a + 2*b)/(x^4 + x^2 + 2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(235) = 470$.

Time = 0.57 (sec) , antiderivative size = 1112, normalized size of antiderivative = 3.52

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/25088*sqrt(7)*(sqrt(7)*2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) - 4*sq
rt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*sqrt(7)*2^(3/4)*a*sqr
t(2*sqrt(2) + 8)*(sqrt(2) - 4) - 12*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(
sqrt(2) - 4) - 3*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) + 12*2^(3/4)*
b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*a*(sqrt(2) - 4)*sqrt(-2*sqrt
```

```
(2) + 8) + 4*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 88*sqrt(7)*2^(1
/4)*a*sqrt(2*sqrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(2*sqrt(2) + 8) - 88*2
^(1/4)*a*sqrt(-2*sqrt(2) + 8) + 16*2^(1/4)*b*sqrt(-2*sqrt(2) + 8))*arctan(2
*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)
) + 1/25088*sqrt(7)*(sqrt(7)*2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) -
4*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*sqrt(7)*2^(3/4)*a
*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 12*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) +
8)*(sqrt(2) - 4) - 3*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) + 12*2^(3
/4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*a*(sqrt(2) - 4)*sqrt(-2*
sqrt(2) + 8) + 4*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 88*sqrt(7)*
2^(1/4)*a*sqrt(2*sqrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(2*sqrt(2) + 8) -
88*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) + 16*2^(1/4)*b*sqrt(-2*sqrt(2) + 8))*arct
an(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2)
+ 4)) + 1/50176*sqrt(7)*(3*sqrt(7)*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2)
+ 8) - 12*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) + sqrt(7)*2^
(3/4)*a*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) - 4*sqrt(7)*2^(3/4)*b*(sqrt(2) -
 4)*sqrt(-2*sqrt(2) + 8) + 2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) - 4*
2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*a*sqrt(2*sqrt(2) +
8)*(sqrt(2) - 4) - 12*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) + 88*sqrt
(7)*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(-2*sqrt(2) +
 8) + 88*2^(1/4)*a*sqrt(2*sqrt(2) + 8) - 16*2^(1/4)*b*sqrt(2*sqrt(2) + 8))*
log(x^2 + 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2)) - 1/50176*sqrt(7)
*(3*sqrt(7)*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 12*sqrt(7)*2^(3/
4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) + sqrt(7)*2^(3/4)*a*(sqrt(2) - 4)*s
qrt(-2*sqrt(2) + 8) - 4*sqrt(7)*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8
) + 2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) - 4*2^(3/4)*b*sqrt(2*sqrt(2
) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 12*2
^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) + 88*sqrt(7)*2^(1/4)*a*sqrt(-2*s
qrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(-2*sqrt(2) + 8) + 88*2^(1/4)*a*sqrt
(2*sqrt(2) + 8) - 16*2^(1/4)*b*sqrt(2*sqrt(2) + 8))*log(x^2 - 2*2^(1/4)*x*s
qrt(-1/8*sqrt(2) + 1/2) + sqrt(2)) - 1/28*(a*x^3 - 4*b*x^3 - 3*a*x - 2*b*x)
/(x^4 + x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 1491, normalized size of antiderivative = 4.72

$$\int \frac{a+bx^2}{\left(2+x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^2)/(x^2 + x^4 + 2)^2,x)
```

```
[Out] atan((b^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3
136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/(
4*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a
```

```
^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1
/2)*a^2*b*39i)/3136)) - (a^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 -
 (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i
)/3136)^(1/2)*17i)/(16*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3
*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a
*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) + (a*b*x*((7^(1/2)*a^2*17i)/1254
4 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21
952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/(4*((7^(1/2)*a^3*187i)/6272 + (7^(1/
2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^
3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) - (17*7^(1/2)
*a^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 +
 (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2))/(112*((7^
(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/
3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^
2*b*39i)/3136)) + (7^(1/2)*b^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952
 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*
1i)/3136)^(1/2))/(28*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a
*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b
^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) + (7^(1/2)*a*b*x*((7^(1/2)*a^2*17i
)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b
^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2))/(28*((7^(1/2)*a^3*187i)/6272 + (7
^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (
9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)))*((7^(1/2
)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/8780
8 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*2i - atan((a^2*x*((7^(1/2
)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/8780
8 + (25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2)*17i)/(16*((3*a*b^2)/1568
 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*
a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/313
6)) - (b^2*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/2
1952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/
(4*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*
a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(
1/2)*a^2*b*39i)/3136)) - (a*b*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/
12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)*a*b*1
i)/3136)^(1/2)*1i)/(4*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3
*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*
b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/3136)) - (17*7^(1/2)*a^2*x*((7^(1/2)*b^2
*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (
25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2))/(112*((3*a*b^2)/1568 - (7^(1/
2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/627
2 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/3136)) + (7
^(1/2)*b^2*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/2
1952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2))/(28
*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*a^
```

```
2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(1/
2)*a^2*b*39i)/3136)) + (7^(1/2)*a*b*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2
*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)
*a*b*1i)/3136)^(1/2))/(28*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)
*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2
)*a*b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/3136)))*((7^(1/2)*b^2*1i)/3136 - (7^
(1/2)*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 +
 (7^(1/2)*a*b*1i)/3136)^(1/2)*2i - (x^3*(a/28 - b/7) - x*((3*a)/28 + b/14))
/(x^2 + x^4 + 2)
```

3.102 $\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 160

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4} \, dx = -\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} - \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)$$

```
[Out] -1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2+2^
(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)-1/2*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^
(1/2))^(1/2))/(2+2^(1/2))^(1/2)+1/2*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/
2))^(1/2))/(2+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1183, 648, 632, 210, 642}

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4} \, dx = -\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} + \frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} - \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right) + \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)$$

```
[In] Int[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/Sqrt[2 + Sqrt[2]]
+ ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
 - (Sqrt[1 + 1/Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/4 + (Sqrt[1 + 1
/Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx}{2\sqrt{2+\sqrt{2}}} + \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx}{2\sqrt{2+\sqrt{2}}}$$

$$= \frac{1}{4}\sqrt{3-2\sqrt{2}}\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx + \frac{1}{4}\sqrt{3-2\sqrt{2}}\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx$$

$$+ \frac{\left(-1-\sqrt{2}\right)\int \frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}} + \frac{\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx}{4\sqrt{2+\sqrt{2}}}$$

$$= -\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)$$

$$- \frac{1}{2}\sqrt{3-2\sqrt{2}}\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx, x, -\sqrt{2+\sqrt{2}}+2x\right)$$

$$- \frac{1}{2}\sqrt{3-2\sqrt{2}}\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx, x, \sqrt{2+\sqrt{2}}+2x\right)$$

$$= -\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)$$

$$- \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = \frac{\sqrt{-1-i}\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{-1-i}}\right) + \sqrt{-1+i}\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+i}}\right)}{2^{3/4}}$$

```
[In] Integrate[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[-1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[-1 - I]] + Sqrt[-1 + I]*ArcTan[(2^(1/
4)*x)/Sqrt[-1 + I]])/2^(3/4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2\,\mathrm{RootOf}\left(\_Z^2-2,\mathrm{index}=1\right)+1\right)}\frac{\left(\_R^2-\sqrt{2}\right)\ln\left(x-\_R\right)}{-2\_R^3+\_R\sqrt{2}}\right)}{2}$
default	$\frac{\sqrt{2}\left(\frac{\sqrt{2+\sqrt{2}}\,\ln\left(1+x^2+x\sqrt{2+\sqrt{2}}\right)}{2}+\frac{2\left(1-\frac{\sqrt{2}}{2}\right)\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(-\frac{\sqrt{2+\sqrt{2}}\,\ln\left(1+x^2-x\sqrt{2+\sqrt{2}}\right)}{2}+\frac{2\left(1-\frac{\sqrt{2}}{2}\right)\arctan\left(\frac{2x-\sqrt{2-}}{\sqrt{2-}}\right.}{\sqrt{2-\sqrt{2}}}\right.}{4}$

```
[In] int((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((_R^2-2^(1/2))/(-2*_R^3+_R*2^(1/2))*ln(x-_R),_R=RootOf(_Z^4-_Z^2*Ro
otOf(_Z^2-2,index=1)+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.64

$$\begin{aligned}\int\frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx=&\ \frac{1}{4}\sqrt{(i+1)\sqrt{2}}\log\left(2x+\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right)\\&-\frac{1}{4}\sqrt{(i+1)\sqrt{2}}\log\left(2x-\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right)\\&+\frac{1}{4}\sqrt{-(i-1)\sqrt{2}}\log\left(2x+\sqrt{2}\sqrt{-(i-1)\sqrt{2}}\right)\\&-\frac{1}{4}\sqrt{-(i-1)\sqrt{2}}\log\left(2x-\sqrt{2}\sqrt{-(i-1)\sqrt{2}}\right)\end{aligned}$$

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((I + 1)*sqrt(2))*log(2*x + sqrt(2)*sqrt((I + 1)*sqrt(2))) - 1/4*sq
rt((I + 1)*sqrt(2))*log(2*x - sqrt(2)*sqrt((I + 1)*sqrt(2))) + 1/4*sqrt(-(I
 - 1)*sqrt(2))*log(2*x + sqrt(2)*sqrt(-(I - 1)*sqrt(2))) - 1/4*sqrt(-(I - 1
)*sqrt(2))*log(2*x - sqrt(2)*sqrt(-(I - 1)*sqrt(2)))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = \text{Exception raised: PolynomialError}$$

```
[In] integrate((-x**2+2**(1/2))/(1+x**4-x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(128*_t**4 - 16*sqrt(2)*_t**2 + 1) c
ontains an element of the set of generators.
```

Maxima [F]

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = \int -\frac{x^2-\sqrt{2}}{x^4-\sqrt{2}x^2+1}\,dx$$

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.76

$$\begin{aligned}\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx &= \frac{1}{4}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)\\ &\quad+\frac{1}{4}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)\\ &\quad+\frac{1}{8}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)\\ &\quad-\frac{1}{8}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)\end{aligned}$$

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2
)) + 1/4*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/8*
sqrt(2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1)
```

Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = -\mathrm{atan}\left(x\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} - \frac{\sqrt{2}\,\sqrt{8}\,x\,\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} - \mathrm{atan}\left(x\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} + \frac{\sqrt{2}\,\sqrt{8}\,x\,\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i}$$

```
[In] int((2^(1/2) - x^2)/(x^4 - 2^(1/2)*x^2 + 1),x)
```

```
[Out] - atan(x*(2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i - (2^(1/2)*8^(1/2)*x*(2^(1
/2)/16 - (8^(1/2)*1i)/32)^(1/2))/2)*(2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i
 - atan(x*(2^(1/2)/16 + (8^(1/2)*1i)/32)^(1/2)*2i + (2^(1/2)*8^(1/2)*x*(2^(
1/2)/16 + (8^(1/2)*1i)/32)^(1/2))/2)*(2^(1/2)/16 + (8^(1/2)*1i)/32)^(1/2)*2
i
```

3.103 $\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 172

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}} - \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)$$

```
[Out] -1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2-2^
(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/2*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/2*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2))/(2-2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1183, 648, 632, 210, 642}

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}} - \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right) + \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)$$

```
[In] Int[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/Sqrt[2 - Sqrt[2]]
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
 - (Sqrt[1 - 1/Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/4 + (Sqrt[1 - 1
/Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx}{2\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx}{2\sqrt{2-\sqrt{2}}}$$

$$
\begin{aligned}
&= \frac{\left(1-\sqrt{2}\right)\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\left(-1+\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} \\
&\quad + \frac{1}{4}\sqrt{3+2\sqrt{2}}\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx \\
&\quad + \frac{1}{4}\sqrt{3+2\sqrt{2}}\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx \\
&= -\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right) \\
&\quad - \frac{1}{2}\sqrt{3+2\sqrt{2}}\mathrm{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx, x, -\sqrt{2-\sqrt{2}}+2x\right) \\
&\quad - \frac{1}{2}\sqrt{3+2\sqrt{2}}\mathrm{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx, x, \sqrt{2-\sqrt{2}}+2x\right) \\
&= -\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right) \\
&\quad - \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$
\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \frac{\sqrt{1-i}\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1-i}}\right)+\sqrt{1+i}\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+i}}\right)}{2^{3/4}}
$$

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTan[(2^(1/4)*
x)/Sqrt[1 + I]])/2^(3/4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\_Z^4+\_Z^2\,\mathrm{RootOf}\left(\_Z^2-2,\mathrm{index}=1\right)\right)}\frac{\left(\_R^2+\sqrt{2}\right)\ln\left(x-\_R\right)}{2\_R^3+\_R\sqrt{2}}\right)}{2}$
default	$\frac{\sqrt{2}\left(-\frac{\sqrt{2-\sqrt{2}}\,\ln\left(1+x^2-x\sqrt{2-\sqrt{2}}\right)}{2}+\frac{2\left(\frac{\sqrt{2}}{2}+1\right)\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{\sqrt{2+\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{2-\sqrt{2}}\,\ln\left(1+x^2+x\sqrt{2-\sqrt{2}}\right)}{2}+\frac{2\left(\frac{\sqrt{2}}{2}+1\right)\arctan\left(\frac{2x+\sqrt{2-}}{\sqrt{2+}}\right.}{\sqrt{2+\sqrt{2}}}\right.}{4}$

```
[In] int((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((_R^2+2^(1/2))/(2*_R^3+_R*2^(1/2))*ln(x-_R),_R=RootOf(1+_Z^4+_Z^2*R
ootOf(_Z^2-2,index=1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.60

$$
\begin{aligned}
\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = {} & \frac{1}{4}\sqrt{(i-1)\ \sqrt{2}}\log\left(2\,x+\sqrt{2}\sqrt{(i-1)\ \sqrt{2}}\right) \\
& -\frac{1}{4}\sqrt{(i-1)\ \sqrt{2}}\log\left(2\,x-\sqrt{2}\sqrt{(i-1)\ \sqrt{2}}\right) \\
& +\frac{1}{4}\sqrt{-(i+1)\ \sqrt{2}}\log\left(2\,x+\sqrt{2}\sqrt{-(i+1)\ \sqrt{2}}\right) \\
& -\frac{1}{4}\sqrt{-(i+1)\ \sqrt{2}}\log\left(2\,x-\sqrt{2}\sqrt{-(i+1)\ \sqrt{2}}\right)
\end{aligned}
$$

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((I - 1)*sqrt(2))*log(2*x + sqrt(2)*sqrt((I - 1)*sqrt(2))) - 1/4*sq
rt((I - 1)*sqrt(2))*log(2*x - sqrt(2)*sqrt((I - 1)*sqrt(2))) + 1/4*sqrt(-(I
 + 1)*sqrt(2))*log(2*x + sqrt(2)*sqrt(-(I + 1)*sqrt(2))) - 1/4*sqrt(-(I + 1
)*sqrt(2))*log(2*x - sqrt(2)*sqrt(-(I + 1)*sqrt(2)))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \text{Exception raised: PolynomialError}$$

```
[In] integrate((x**2+2**(1/2))/(1+x**4+x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(128*_t**4 + 16*sqrt(2)*_t**2 + 1) c
ontains an element of the set of generators.
```

Maxima [F]

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \int \frac{x^2+\sqrt{2}}{x^4+\sqrt{2}x^2+1}\,dx$$

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.73

$$\begin{aligned}\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = {} & \frac{1}{4}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) \\ & +\frac{1}{4}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) \\ & +\frac{1}{8}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right) \\ & -\frac{1}{8}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)\end{aligned}$$

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/4*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2)
+ 2)) + 1/8*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/8*
sqrt(-2*sqrt(2) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \operatorname{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} + \frac{\sqrt{2}\,\sqrt{8}\,x\,\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} + \operatorname{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} - \frac{\sqrt{2}\,\sqrt{8}\,x\,\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i}$$

```
[In] int((2^(1/2) + x^2)/(2^(1/2)*x^2 + x^4 + 1),x)
```

```
[Out] atan(x*(- 2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i + (2^(1/2)*8^(1/2)*x*(- 2^
(1/2)/16 - (8^(1/2)*1i)/32)^(1/2))/2)*(- 2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2
)*2i + atan(x*((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/2)*2i - (2^(1/2)*8^(1/2)*x*
((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/2))/2)*((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/
2)*2i
```

3.104 $\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4} \, dx = \frac{\left(1-\sqrt{2}\right) \arctan\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1-\sqrt{2}\right) \arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1+\sqrt{2}\right) \log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} + \frac{\left(1+\sqrt{2}\right) \log\left(1+\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}}$$

```
[Out] -1/4*ln(1+x^2-x*(2-b)^(1/2))*(1+2^(1/2))/(2-b)^(1/2)+1/4*ln(1+x^2+x*(2-b)^(
1/2))*(1+2^(1/2))/(2-b)^(1/2)+1/2*arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1
-2^(1/2))/(2+b)^(1/2)-1/2*arctan((2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1-2^(1/2))
/(2+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1183, 648, 632, 210, 642}

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4} \, dx = \frac{\left(1-\sqrt{2}\right) \arctan\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} - \frac{\left(1-\sqrt{2}\right) \arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}} - \frac{\left(1+\sqrt{2}\right) \log\left(-\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}} + \frac{\left(1+\sqrt{2}\right) \log\left(\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}$$

```
[In] Int[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - (
(1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - ((
1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) + ((1 + Sqrt[2])
*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2-b}x+x^2}\,dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2-b}x+x^2}\,dx}{2\sqrt{2-b}}$$

$$
\begin{aligned}
&= \frac{1}{4}\left(-1+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2-b}x+x^2}\,dx + \frac{1}{4}\left(-1+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2-b}x+x^2}\,dx \\
&\quad - \frac{\left(1+\sqrt{2}\right)\int \frac{-\sqrt{2-b}+2x}{1-\sqrt{2-b}x+x^2}\,dx}{4\sqrt{2-b}} + \frac{\left(1+\sqrt{2}\right)\int \frac{\sqrt{2-b}+2x}{1+\sqrt{2-b}x+x^2}\,dx}{4\sqrt{2-b}} \\
&= -\frac{\left(1+\sqrt{2}\right)\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} + \frac{\left(1+\sqrt{2}\right)\log\left(1+\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} \\
&\quad + \frac{1}{2}\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx, x, -\sqrt{2-b}+2x\right) \\
&\quad + \frac{1}{2}\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx, x, \sqrt{2-b}+2x\right) \\
&= \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} \\
&\quad - \frac{\left(1+\sqrt{2}\right)\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} + \frac{\left(1+\sqrt{2}\right)\log\left(1+\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}}
\end{aligned}
$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \$\text{Aborted}$$

```
[In] Integrate[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
[Out] $Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+\_Z^2 b+1\right)}\frac{\left(-\_R^2+\sqrt{2}\right)\ln\left(x-\_R\right)}{2\_R^3+\_R b}\right)}{2}$	44
default	$\frac{\left(-\sqrt{(b-2)(2+b)}-b-2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(-\sqrt{(b-2)(2+b)}+b+2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$	136

```
[In] int((-x^2+2^(1/2))/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*sum((-_R^2+2^(1/2))/(2*_R^3+_R*b)*ln(x-_R),_R=RootOf(_Z^4+_Z^2*b+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(123) = 246$.

Time = 0.26 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.23

$$
\begin{aligned}
\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx &= -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\Bigg(2\,\big(2\,b^2-9\big)x \\
&\quad+\sqrt{\frac{1}{2}}\left(2\,b^3-3\,\sqrt{2}(b^2-4)-8\,b-\frac{2\,b^4-14\,b^2-\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\Bigg) \\
&\quad+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\Bigg(2\,\big(2\,b^2-9\big)x \\
&\quad-\sqrt{\frac{1}{2}}\left(2\,b^3-3\,\sqrt{2}(b^2-4)-8\,b-\frac{2\,b^4-14\,b^2-\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\Bigg) \\
&\quad-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\Bigg(2\,\big(2\,b^2-9\big)x \\
&\quad+\sqrt{\frac{1}{2}}\left(2\,b^3-3\,\sqrt{2}(b^2-4)-8\,b+\frac{2\,b^4-14\,b^2-\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b+4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\Bigg) \\
&\quad+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\Bigg(2\,\big(2\,b^2-9\big)x \\
&\quad-\sqrt{\frac{1}{2}}\left(2\,b^3-3\,\sqrt{2}(b^2-4)-8\,b+\frac{2\,b^4-14\,b^2-\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b+4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\Bigg)
\end{aligned}
$$

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*
b^2 - 9)*x + sqrt(1/2)*(2*b^3 - 3*sqrt(2)*(b^2 - 4) - 8*b - (2*b^4 - 14*b^2
 - sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b + 4*sqrt(2) + sqrt(b
^2 - 4))/(b^2 - 4))) + 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4)
)/(b^2 - 4))*log(2*(2*b^2 - 9)*x - sqrt(1/2)*(2*b^3 - 3*sqrt(2)*(b^2 - 4) -
 8*b - (2*b^4 - 14*b^2 - sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*
b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) - 1/2*sqrt(1/2)*sqrt(-(3*b + 4*s
qrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x + sqrt(1/2)*(2*b^3 -
 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 - sqrt(2)*(b^3 - 4*b) + 24)/sq
rt(b^2 - 4))*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))) + 1/2*sqrt
(1/2)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*
```

```
x - sqrt(1/2)*(2*b^3 - 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 - sqrt(2
)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/
(b^2 - 4)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(128) = 256$.

Time = 1.34 (sec) , antiderivative size = 1469, normalized size of antiderivative = 9.18

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((-x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] -RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 + 16*sqrt(2)*b**
2 - 48*b - 64*sqrt(2)) + 2*b**2 + 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*
(64*b**12/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*
b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729)
 + 672*sqrt(2)*b**11/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b
**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*
b**2 + 729) + 5760*b**10/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(
2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3
402*b**2 + 729) + 12064*sqrt(2)*b**9/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8
+ 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt
(2)*b**3 - 3402*b**2 + 729) + 17744*b**8/(8*b**10 + 88*sqrt(2)*b**9 + 828*b
**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*
sqrt(2)*b**3 - 3402*b**2 + 729) - 27480*sqrt(2)*b**7/(8*b**10 + 88*sqrt(2)*
b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*
b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 154608*b**6/(8*b**10 + 88*sqr
t(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 +
2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 141376*sqrt(2)*b**5/(8*b
**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sq
rt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 69072*b**4/
(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 531
0*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 61704*s
qrt(2)*b**3/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 647
0*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 72
9) + 78192*b**2/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 +
 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2
+ 729) - 2592*sqrt(2)*b/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2
)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 34
02*b**2 + 729) - 15552/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)
*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 340
2*b**2 + 729)) + _t*(16*b**7/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqr
```

```
t(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) + 116*sqrt(2)*b**6/(4*b**6 + 28*s
qrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) +
 668*b**5/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**
2 - 27*sqrt(2)*b - 81) + 942*sqrt(2)*b**4/(4*b**6 + 28*sqrt(2)*b**5 + 152*b
**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) + 1226*b**3/(4*b**6
+ 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b -
 81) + 144*sqrt(2)*b**2/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*
b**3 + 189*b**2 - 27*sqrt(2)*b - 81) - 378*b/(4*b**6 + 28*sqrt(2)*b**5 + 15
2*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) - 108*sqrt(2)/(4*
b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2
)*b - 81)) + x)))
```

Maxima [F]

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \int -\frac{x^2-\sqrt{2}}{x^4+bx^2+1}\,dx$$

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 + b*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \int -\frac{x^2-\sqrt{2}}{x^4+bx^2+1}\,dx$$

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 1227, normalized size of antiderivative = 7.67

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \\ -\mathrm{atan}\Bigg(\frac{x\,\sqrt{\frac{12\,b+16\,\sqrt{2}-4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}}\,32\mathrm{i}-b\,x\,{\left(\frac{12\,b+16\,\sqrt{2}-4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}\right)}^{3/2}\,256\mathrm{i}}{}$$

```
[In] int((2^(1/2) - x^2)/(b*x^2 + x^4 + 1),x)
```

```
[Out] atan((x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b
^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*(-(4*2^(1/2)*b^2 -
16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64
*b^2 + 128))^(3/2)*256i + b^2*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^
3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i -
b^4*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6
 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*(-(4*2^(1/2)*b^2 - 1
6*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(3/2)*128i - b^5*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3
 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3/2)*16i +
2^(1/2)*b*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4
 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - 2^(1/2)*b^3*x*(-(4*
2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(2^(1/2)*b^3 - 4*2^(1/2)*b + 2^(1/2)*(
48*b^2 - 12*b^4 + b^6 - 64)^(1/2) + 2*b^2 - 8))*(-(4*2^(1/2)*b^2 - 16*2^(1/
2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 1
28))^(1/2)*2i - atan((x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b
^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*((12
*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(3/2)*256i + b^2*x*((12*b + 16*2^(1/2) - 4*2^(1/
2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128)
)^(1/2)*8i - b^4*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 -
12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*((12*b +
 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(
8*b^4 - 64*b^2 + 128))^(3/2)*128i - b^5*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b
^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3
/2)*16i + 2^(1/2)*b*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2
 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - 2^(1/2)*b^
3*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 -
64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(4*2^(1/2)*b - 2^(1/2)*b^3 + 2
^(1/2)*(48*b^2 - 12*b^4 + b^6 - 64)^(1/2) - 2*b^2 + 8))*((12*b + 16*2^(1/2)
 - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(1/2)*2i
```

3.105 $\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = -\frac{\left(1+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}}+\frac{\left(1+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}}+\frac{\left(1-\sqrt{2}\right)\log\left(1-\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}}-\frac{\left(1-\sqrt{2}\right)\log\left(1+\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}}$$

```
[Out] 1/4*ln(1+x^2-x*(2-b)^(1/2))*(1-2^(1/2))/(2-b)^(1/2)-1/4*ln(1+x^2+x*(2-b)^(1
/2))*(1-2^(1/2))/(2-b)^(1/2)-1/2*arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1+
2^(1/2))/(2+b)^(1/2)+1/2*arctan((2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1+2^(1/2))/
(2+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1183, 648, 632, 210, 642}

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = -\frac{\left(1+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1-\sqrt{2}\right)\log\left(\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}$$

```
[In] Int[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -1/2*((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/Sqrt[2 + b] +
((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) + (
(1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) - ((1 - Sqrt[2]
)*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-b}x+x^2}\,dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-b}x+x^2}\,dx}{2\sqrt{2-b}}$$

$$
\begin{aligned}
&=\frac{1}{4}\left(1+\sqrt{2}\right)\int\frac{1}{1-\sqrt{2-b}x+x^2}\,dx+\frac{1}{4}\left(1+\sqrt{2}\right)\int\frac{1}{1+\sqrt{2-b}x+x^2}\,dx\\
&\quad+\frac{(1-\sqrt{2})\int\frac{-\sqrt{2-b}+2x}{1-\sqrt{2-b}x+x^2}\,dx}{4\sqrt{2-b}}-\frac{(1-\sqrt{2})\int\frac{\sqrt{2-b}+2x}{1+\sqrt{2-b}x+x^2}\,dx}{4\sqrt{2-b}}\\
&=\frac{(1-\sqrt{2})\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}}-\frac{(1-\sqrt{2})\log\left(1+\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}}\\
&\quad+\frac{1}{2}\left(-1-\sqrt{2}\right)\mathrm{Subst}\left(\int\frac{1}{-2-b-x^2}\,dx,x,-\sqrt{2-b}+2x\right)\\
&\quad+\frac{1}{2}\left(-1-\sqrt{2}\right)\mathrm{Subst}\left(\int\frac{1}{-2-b-x^2}\,dx,x,\sqrt{2-b}+2x\right)\\
&=-\frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}}+\frac{(1+\sqrt{2})\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}}\\
&\quad+\frac{(1-\sqrt{2})\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}}-\frac{(1-\sqrt{2})\log\left(1+\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}}
\end{aligned}
$$

Mathematica [F(-1)]

Timed out.

$$\int\frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx=\$\text{Aborted}$$

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
[Out] $Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2b+1\right)}\frac{\left(\_R^2+\sqrt{2}\right)\ln\left(x-\_R\right)}{2\_R^3+\_Rb}\right)}{2}$	42
default	$\frac{\left(\sqrt{(b-2)(2+b)}+b-2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(\sqrt{(b-2)(2+b)}-b+2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$	132

```
[In] int((x^2+2^(1/2))/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*sum((_R^2+2^(1/2))/(2*_R^3+_R*b)*ln(x-_R),_R=RootOf(_Z^4+_Z^2*b+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(123) = 246$.

Time = 0.28 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.21

$$
\begin{aligned}
\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx &= \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2\left(2\,b^2-9\right)x\right.\\
&\quad \left.+\sqrt{\frac{1}{2}}\left(2\,b^3+3\,\sqrt{2}\left(b^2-4\right)-8\,b-\frac{2\,b^4-14\,b^2+\sqrt{2}\left(b^3-4\,b\right)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)\\
&\quad -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2\left(2\,b^2-9\right)x\right.\\
&\quad \left.-\sqrt{\frac{1}{2}}\left(2\,b^3+3\,\sqrt{2}\left(b^2-4\right)-8\,b-\frac{2\,b^4-14\,b^2+\sqrt{2}\left(b^3-4\,b\right)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)\\
&\quad +\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b-4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2\left(2\,b^2-9\right)x\right.\\
&\quad \left.+\sqrt{\frac{1}{2}}\left(2\,b^3+3\,\sqrt{2}\left(b^2-4\right)-8\,b+\frac{2\,b^4-14\,b^2+\sqrt{2}\left(b^3-4\,b\right)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b-4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right)\\
&\quad -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b-4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2\left(2\,b^2-9\right)x\right.\\
&\quad \left.-\sqrt{\frac{1}{2}}\left(2\,b^3+3\,\sqrt{2}\left(b^2-4\right)-8\,b+\frac{2\,b^4-14\,b^2+\sqrt{2}\left(b^3-4\,b\right)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b-4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right)
\end{aligned}
$$

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b
^2 - 9)*x + sqrt(1/2)*(2*b^3 + 3*sqrt(2)*(b^2 - 4) - 8*b - (2*b^4 - 14*b^2
+ sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^
2 - 4))/(b^2 - 4))) - 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))
/(b^2 - 4))*log(2*(2*b^2 - 9)*x - sqrt(1/2)*(2*b^3 + 3*sqrt(2)*(b^2 - 4) -
8*b - (2*b^4 - 14*b^2 + sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b
 - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) + 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sq
rt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x + sqrt(1/2)*(2*b^3 +
3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 + sqrt(2)*(b^3 - 4*b) + 24)/sqr
t(b^2 - 4))*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))) - 1/2*sqrt(
1/2)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x
```

```
 - sqrt(1/2)*(2*b^3 + 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 + sqrt(2)
*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(
b^2 - 4)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(128) = 256$.

Time = 1.39 (sec) , antiderivative size = 1467, normalized size of antiderivative = 9.17

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 - 16*sqrt(2)*b**2
 - 48*b + 64*sqrt(2)) + 2*b**2 - 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*(
64*b**12/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b
**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729)
- 672*sqrt(2)*b**11/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b*
*7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b
**2 + 729) + 5760*b**10/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2
)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 34
02*b**2 + 729) - 12064*sqrt(2)*b**9/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 -
 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(
2)*b**3 - 3402*b**2 + 729) + 17744*b**8/(8*b**10 - 88*sqrt(2)*b**9 + 828*b*
*8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*s
qrt(2)*b**3 - 3402*b**2 + 729) + 27480*sqrt(2)*b**7/(8*b**10 - 88*sqrt(2)*b
**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b
**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 154608*b**6/(8*b**10 - 88*sqrt
(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2
781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 141376*sqrt(2)*b**5/(8*b*
*10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqr
t(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 69072*b**4/(
8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310
*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 61704*sq
rt(2)*b**3/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470
*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729
) + 78192*b**2/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 +
6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 +
 729) + 2592*sqrt(2)*b/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)
*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 340
2*b**2 + 729) - 15552/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*
b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402
*b**2 + 729)) + _t*(16*b**7/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt
```

```
(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) - 116*sqrt(2)*b**6/(4*b**6 - 28*sq
rt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) +
668*b**5/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2
 + 27*sqrt(2)*b - 81) - 942*sqrt(2)*b**4/(4*b**6 - 28*sqrt(2)*b**5 + 152*b*
*4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) + 1226*b**3/(4*b**6 -
 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b -
81) - 144*sqrt(2)*b**2/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b
**3 + 189*b**2 + 27*sqrt(2)*b - 81) - 378*b/(4*b**6 - 28*sqrt(2)*b**5 + 152
*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) + 108*sqrt(2)/(4*b
**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)
*b - 81)) + x)))
```

Maxima [F]

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \int \frac{x^2+\sqrt{2}}{x^4+bx^2+1}\,dx$$

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \int \frac{x^2+\sqrt{2}}{x^4+bx^2+1}\,dx$$

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 1227, normalized size of antiderivative = 7.67

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = -\mathrm{atan}\left(\frac{x\,\sqrt{\frac{12\,b-16\,\sqrt{2}+4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}}\,32\mathrm{i}-b\,x\,{\left(\frac{12\,b-16\,\sqrt{2}+4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}\right)}^{3/2}\,256}{}\right.$$

```
[In] int((2^(1/2) + x^2)/(b*x^2 + x^4 + 1),x)
```

```
[Out] atan((x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b
^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*(-(16*2^(1/2) - 12*
b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64
*b^2 + 128))^(3/2)*256i + b^2*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^
3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i -
b^4*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6
 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*(-(16*2^(1/2) - 12*b
 - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(3/2)*128i - b^5*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3
 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3/2)*16i -
2^(1/2)*b*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4
 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i + 2^(1/2)*b^3*x*(-(16
*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(2^(1/2)*b^3 - 4*2^(1/2)*b + 2^(1/2)*(
48*b^2 - 12*b^4 + b^6 - 64)^(1/2) - 2*b^2 + 8))*(-(16*2^(1/2) - 12*b - 4*2^
(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 1
28))^(1/2)*2i - atan((x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b
^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*((12
*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(3/2)*256i + b^2*x*((12*b - 16*2^(1/2) + 4*2^(1/
2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128)
)^(1/2)*8i - b^4*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 -
12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*((12*b -
 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(
8*b^4 - 64*b^2 + 128))^(3/2)*128i - b^5*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b
^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3
/2)*16i - 2^(1/2)*b*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2
 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i + 2^(1/2)*b^
3*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 -
64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(4*2^(1/2)*b - 2^(1/2)*b^3 + 2
^(1/2)*(48*b^2 - 12*b^4 + b^6 - 64)^(1/2) + 2*b^2 - 8))*((12*b - 16*2^(1/2)
 + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(1/2)*2i
```

3.106 $\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\arctan\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{3}\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}}$$

```
[Out] 1/2*arctan(-3^(1/2)+2*x/a^(1/2))/a^(1/2)+1/2*arctan(3^(1/2)+2*x/a^(1/2))/a^
(1/2)-1/4*ln(a+x^2-x*3^(1/2)*a^(1/2))*3^(1/2)/a^(1/2)+1/4*ln(a+x^2+x*3^(1/2
)*a^(1/2))*3^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1183, 648, 631, 210, 642}

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\arctan\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2\sqrt{a}} - \frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}$$

```
[In] Int[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]]/Sqrt[a] + ArcTan[Sqrt[3] + (2*x)/Sqrt[
a]]/(2*Sqrt[a]) - (Sqrt[3]*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a]) +
(Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{2\sqrt{3}a^{3/2}-3ax}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{2\sqrt{3}a^{3/2}+3ax}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}} \\
&= \frac{1}{4}\int \frac{1}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx \\
&\quad - \frac{\sqrt{3}\int \frac{-\sqrt{3}\sqrt{a}+2x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{a}} + \frac{\sqrt{3}\int \frac{\sqrt{3}\sqrt{a}+2x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{a}}
\end{aligned}
$$

$$= -\frac{\sqrt{3}\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}} + \frac{\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)}{2\sqrt{3}\sqrt{a}}-\frac{\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2x}{\sqrt{3}\sqrt{a}}\right)}{2\sqrt{3}\sqrt{a}}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{3}\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int\frac{2a-x^2}{a^2-ax^2+x^4}\,dx = \frac{\sqrt[4]{-1}\left(-\sqrt{i+\sqrt{3}}\left(3i+\sqrt{3}\right)\arctan\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt{a}}\right)+\sqrt{-i+\sqrt{3}}\left(-3i+\sqrt{3}\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt{a}}\right)\right)}{2\sqrt{6}\sqrt{a}}$$

```
[In] Integrate[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-
I + Sqrt[3]]*Sqrt[a])]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 +
 I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])]))/(2*Sqrt[6]*Sqrt[a])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.42

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^{4}-a\_Z^{2}+a^{2}\right)}\frac{\left(-\_R^{2}+2a\right)\ln\left(x-\_R\right)}{2\_R^{3}-\_Ra}\right)}{2}$	48
default	$\frac{\frac{\sqrt{3}\,\ln\left(a+x^{2}+x\sqrt{3}\,\sqrt{a}\right)}{2}+\arctan\left(\frac{2x+\sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{-\frac{\sqrt{3}\,\ln\left(x\sqrt{3}\,\sqrt{a}-x^{2}-a\right)}{2}-\arctan\left(\frac{\sqrt{3}\,\sqrt{a}-2x}{\sqrt{a}}\right)}{2\sqrt{a}}$	90

```
[In] int((-x^2+2*a)/(x^4-a*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((-_R^2+2*a)/(2*_R^3-_R*a)*ln(x-_R),_R=RootOf(_Z^4-_Z^2*a+a^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(80) = 160$.

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.92

$$
\begin{aligned}
\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx &= \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}+1}{a}}\log\left(\sqrt{\frac{1}{2}}a\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}+1}{a}}+x\right) \\
&\quad -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}+1}{a}}\log\left(-\sqrt{\frac{1}{2}}a\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}+1}{a}}+x\right) \\
&\quad +\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}-1}{a}}\log\left(\sqrt{\frac{1}{2}}a\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}-1}{a}}+x\right) \\
&\quad -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}-1}{a}}\log\left(-\sqrt{\frac{1}{2}}a\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}-1}{a}}+x\right)
\end{aligned}
$$

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a^2) + 1)/a)*log(sqrt(1/2)*a*sqrt((sq
rt(3)*a*sqrt(-1/a^2) + 1)/a) + x) - 1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a
^2) + 1)/a)*log(-sqrt(1/2)*a*sqrt((sqrt(3)*a*sqrt(-1/a^2) + 1)/a) + x) + 1/
2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a^2) - 1)/a)*log(sqrt(1/2)*a*sqrt(-(sq
rt(3)*a*sqrt(-1/a^2) - 1)/a) + x) - 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/
a^2) - 1)/a)*log(-sqrt(1/2)*a*sqrt(-(sqrt(3)*a*sqrt(-1/a^2) - 1)/a) + x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$
\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = -\operatorname{RootSum}\left(16t^4a^2-4t^2a+1,\left(t\mapsto t\log\left(-2ta+x\right)\right)\right)
$$

```
[In] integrate((-x**2+2*a)/(x**4-a*x**2+a**2),x)
```

```
[Out] -RootSum(16*_t**4*a**2 - 4*_t**2*a + 1, Lambda(_t, _t*log(-2*_t*a + x)))
```

Maxima [F]

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx=\int -\frac{x^2-2\,a}{x^4-ax^2+a^2}\,dx$$

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="maxima")
[Out] -integrate((x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.56 (sec) , antiderivative size = 4217, normalized size of antiderivative = 36.99

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx=\text{Too large to display}$$

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="giac")
[Out] -1/6*sqrt(3)*(3*sqrt(3)*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs
(a))))^2*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3*sin(1/2*real_part(arcc
os(1/2*a/abs(a)))) - sqrt(3)*a^2*abs(a)^(3/2)*cosh(1/2*imag_part(arccos(1/2
*a/abs(a))))^3*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3 - 9*sqrt(3)*a^2*a
bs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_part(a
rccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*i
mag_part(arccos(1/2*a/abs(a)))) + 3*sqrt(3)*a^2*abs(a)^(3/2)*cosh(1/2*imag_
part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3*si
nh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 9*sqrt(3)*a^2*abs(a)^(3/2)*cos(1/
2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_part(arccos(1/2*a/abs(a)
)))*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*
a/abs(a))))^2 - 3*sqrt(3)*a^2*abs(a)^(3/2)*cosh(1/2*imag_part(arccos(1/2*a/
abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*imag_part(arc
cos(1/2*a/abs(a))))^2 - 3*sqrt(3)*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos
(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_p
art(arccos(1/2*a/abs(a))))^3 + sqrt(3)*a^2*abs(a)^(3/2)*sin(1/2*real_part(a
rccos(1/2*a/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 + a^3*s
qrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))^3*cosh(1/2*imag_part(a
rccos(1/2*a/abs(a))))^3 - 3*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a
/abs(a))))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3*sin(1/2*real_part(ar
ccos(1/2*a/abs(a))))^2 - 3*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/
abs(a))))^3*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^2*sinh(1/2*imag_part(
arccos(1/2*a/abs(a)))) + 9*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/
abs(a))))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arc
cos(1/2*a/abs(a))))^2*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 3*a^3*sqr
```

```
t(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))^3*cosh(1/2*imag_part(arc
cos(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^2 - 9*a^3*sqr
t(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arcco
s(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^2*sinh(1/2*imag_
part(arccos(1/2*a/abs(a))))^2 - a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1
/2*a/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 + 3*a^3*sqrt(a
bs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/
2*a/abs(a))))^2*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 - 2*sqrt(3)*a^3
*sqrt(abs(a))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))*sin(1/2*real_part(a
rccos(1/2*a/abs(a)))) + 2*sqrt(3)*a^3*sqrt(abs(a))*sin(1/2*real_part(arccos
(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) - 2*a^2*abs(a)^(
3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arccos(1/2
*a/abs(a)))) + 2*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))*
sinh(1/2*imag_part(arccos(1/2*a/abs(a)))))*arctan((sqrt(abs(a))*cos(1/2*arc
cos(1/2*a/abs(a))) + x)/(sqrt(abs(a))*sin(1/2*arccos(1/2*a/abs(a)))))/a^4 -
 1/12*sqrt(3)*(sqrt(3)*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(
a))))^3*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 - 3*sqrt(3)*a^2*abs(a)^
(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arccos(1/
2*a/abs(a))))^3*sin(1/2*real_part(arccos(1/2*a/abs(a))))^2 - 3*sqrt(3)*a^2*
abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^3*cosh(1/2*imag_part(
arccos(1/2*a/abs(a))))^2*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 9*sqrt
(3)*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag
_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))^2*s
inh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 3*sqrt(3)*a^2*abs(a)^(3/2)*cos(1
/2*real_part(arccos(1/2*a/abs(a))))^3*cosh(1/2*imag_part(arccos(1/2*a/abs(a
))))*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^2 - 9*sqrt(3)*a^2*abs(a)^(3/
2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arccos(1/2*a
/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^2*sinh(1/2*imag_part(ar
ccos(1/2*a/abs(a))))^2 - sqrt(3)*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(
1/2*a/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 + 3*sqrt(3)*a
^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))*sin(1/2*real_part(
arccos(1/2*a/abs(a))))^2*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 - 3*a^
3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_par
t(arccos(1/2*a/abs(a))))^3*sin(1/2*real_part(arccos(1/2*a/abs(a)))) + a^3*s
qrt(abs(a))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3*sin(1/2*real_part(a
rccos(1/2*a/abs(a))))^3 + 9*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a
/abs(a))))^2*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(
arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) - 3*a^3*sq
rt(abs(a))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(ar
ccos(1/2*a/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) - 9*a^3*sq
rt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_part(ar
ccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag
_part(arccos(1/2*a/abs(a))))^2 + 3*a^3*sqrt(abs(a))*cosh(1/2*imag_part(arcc
os(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*imag
_part(arccos(1/2*a/abs(a))))^2 + 3*a^3*sqrt(abs(a))*cos(1/2*real_part(arcco
```

```
s(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_
part(arccos(1/2*a/abs(a))))^3 - a^3*sqrt(abs(a))*sin(1/2*real_part(arccos(1
/2*a/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 - 2*sqrt(3)*a^
3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(
arccos(1/2*a/abs(a)))) + 2*sqrt(3)*a^3*sqrt(abs(a))*cos(1/2*real_part(arcco
s(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 2*a^2*abs(a)^
(3/2)*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/
2*a/abs(a)))) - 2*a^2*abs(a)^(3/2)*sin(1/2*real_part(arccos(1/2*a/abs(a))))
*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))))*log(2*x*sqrt(abs(a))*cos(1/2*ar
ccos(1/2*a/abs(a))) + x^2 + abs(a))/a^4 - 1/48*sqrt(3)*(sqrt(3)*sqrt(14*a^2
 + 13*a*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^3 - 3*sqr
t(3)*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))
))^2*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) - 3*sqrt(3)*sqrt(2*a^2 -
 a*abs(a))*a^4*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sg
n(cos(1/2*arccos(1/2*a/abs(a))))) + 3*sqrt(3)*sqrt(14*a^2 + 13*a*abs(a))*a^
4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arcco
s(1/2*a/abs(a)))))^2 + 3*sqrt(3)*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(s
in(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a))))
)^2 - 6*sqrt(3)*sqrt(2*a^2 - a*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos(1/2
*a/abs(a)))))*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(s
in(1/2*arccos(1/2*a/abs(a))))) + 3*sqrt(3)*sqrt(14*a^2 - 13*a*abs(a))*a^4*i
mag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(sin(1/2*arccos
(1/2*a/abs(a))))) - 6*sqrt(3)*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(cos(
1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*r
eal_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) - 3*sqrt(3)*sqrt(2*a^2 - a*abs
(a))*a^4*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/
2*arccos(1/2*a/abs(a)))))^2 + sqrt(3)*sqrt(14*a^2 - 13*a*abs(a))*a^4*real_p
art(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^3 + 9*sqrt(2*a^2 - a*abs(a))*a^4*im
ag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*imag_part(sgn(sin(1/2*arccos(1/
2*a/abs(a)))))^2 - sqrt(14*a^2 - 13*a*abs(a))*a^4*imag_part(sgn(sin(1/2*arc
cos(1/2*a/abs(a)))))^3 - 3*sqrt(14*a^2 + 13*a*abs(a))*a^4*imag_part(sgn(cos
(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a))))
) + 18*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)
))))*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*ar
ccos(1/2*a/abs(a))))) - sqrt(14*a^2 + 13*a*abs(a))*a^4*real_part(sgn(cos(1/
2*arccos(1/2*a/abs(a)))))^3 + 9*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(co
s(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))
)) + 18*sqrt(2*a^2 - a*abs(a))*a^4*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a
)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*a
rccos(1/2*a/abs(a))))) + 9*sqrt(2*a^2 + a*abs(a))*a^4*real_part(sgn(cos(1/2
*arccos(1/2*a/abs(a)))))^2*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) +
9*sqrt(2*a^2 - a*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*
real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2 - 3*sqrt(14*a^2 - 13*a*abs(
a))*a^4*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2
*arccos(1/2*a/abs(a)))))^2 + 8*sqrt(3)*sqrt(2*a^2 + a*abs(a))*a^4*imag_part
```

```
(sgn(cos(1/2*arccos(1/2*a/abs(a))))) + 8*sqrt(3)*sqrt(2*a^2 - a*abs(a))*a^4
*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) + 8*sqrt(2*a^2 - a*abs(a))*a
^4*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) - 8*sqrt(2*a^2 + a*abs(a))
*a^4*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a))))))*arctan(-1/2*(sqrt(a/abs
(a) + 2)*sqrt(abs(a))*sgn(cos(1/2*arccos(1/2*a/abs(a)))) - 2*x)/(sqrt(-1/4*
a/abs(a) + 1/2)*sqrt(abs(a))*sgn(sin(1/2*arccos(1/2*a/abs(a))))))/(a^4*abs(
a)^(3/2)) - 1/96*sqrt(3)*(3*sqrt(3)*sqrt(2*a^2 - a*abs(a))*a^4*imag_part(sg
n(cos(1/2*arccos(1/2*a/abs(a)))))*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)
))))^2 + sqrt(3)*sqrt(14*a^2 - 13*a*abs(a))*a^4*imag_part(sgn(sin(1/2*arcco
s(1/2*a/abs(a)))))^3 + 3*sqrt(3)*sqrt(14*a^2 + 13*a*abs(a))*a^4*imag_part(s
gn(cos(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(cos(1/2*arccos(1/2*a/abs
(a))))) + 6*sqrt(3)*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos
(1/2*a/abs(a)))))*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*real_part(s
gn(cos(1/2*arccos(1/2*a/abs(a))))) + sqrt(3)*sqrt(14*a^2 + 13*a*abs(a))*a^4
*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^3 - 3*sqrt(3)*sqrt(2*a^2 + a
*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(
sin(1/2*arccos(1/2*a/abs(a))))) - 6*sqrt(3)*sqrt(2*a^2 - a*abs(a))*a^4*imag
_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*
a/abs(a)))))*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) - 3*sqrt(3)*sqrt
(2*a^2 + a*abs(a))*a^4*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2*real
_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) - 3*sqrt(3)*sqrt(2*a^2 - a*abs(a)
)*a^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*a
rccos(1/2*a/abs(a)))))^2 + 3*sqrt(3)*sqrt(14*a^2 - 13*a*abs(a))*a^4*imag_pa
rt(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*arccos(1/2*a/a
bs(a)))))^2 - sqrt(14*a^2 + 13*a*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos(1
/2*a/abs(a)))))^3 + 9*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(cos(1/2*arcc
os(1/2*a/abs(a)))))^2*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) + 9*sqr
t(2*a^2 - a*abs(a))*a^4*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2*rea
l_part(sgn(cos(1/2*arccos(1/2*a/abs(a))))) - 3*sqrt(14*a^2 + 13*a*abs(a))*a
^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arcc
os(1/2*a/abs(a)))))^2 + 9*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(sin(1/2*
arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2 + 1
8*sqrt(2*a^2 - a*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*
imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*arccos(
1/2*a/abs(a))))) + 3*sqrt(14*a^2 - 13*a*abs(a))*a^4*imag_part(sgn(sin(1/2*a
rccos(1/2*a/abs(a)))))^2*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) + 18
*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*r
eal_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*arccos(1
/2*a/abs(a))))) + 9*sqrt(2*a^2 - a*abs(a))*a^4*real_part(sgn(cos(1/2*arccos
(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2 + sqrt(14
*a^2 - 13*a*abs(a))*a^4*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^3 + 8
*sqrt(3)*sqrt(2*a^2 - a*abs(a))*a^4*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(
a))))) + 8*sqrt(3)*sqrt(2*a^2 + a*abs(a))*a^4*real_part(sgn(cos(1/2*arccos(
1/2*a/abs(a))))) - 8*sqrt(2*a^2 + a*abs(a))*a^4*imag_part(sgn(cos(1/2*arcco
s(1/2*a/abs(a))))) - 8*sqrt(2*a^2 - a*abs(a))*a^4*real_part(sgn(sin(1/2*arc
```

```
cos(1/2*a/abs(a))))))*log(-x*sqrt(a/abs(a) + 2)*sqrt(abs(a))*sgn(cos(1/2*ar
ccos(1/2*a/abs(a)))) + x^2 + abs(a))/(a^4*abs(a)^(3/2))
```

Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = -\frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{\frac{1}{8\,a}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\,1\mathrm{i}+\sqrt{3}\,x\,\sqrt{\frac{1}{8\,a}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\right)\,\sqrt{\frac{1+\sqrt{3}\,1\mathrm{i}}{a}}\,1\mathrm{i}}{4} - \frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{\frac{1}{8\,a}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\,1\mathrm{i}-\sqrt{3}\,x\,\sqrt{\frac{1}{8\,a}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\right)\,\sqrt{-\frac{-1+\sqrt{3}\,1\mathrm{i}}{a}}\,1\mathrm{i}}{4}$$

```
[In] int((2*a - x^2)/(a^2 - a*x^2 + x^4),x)
```

```
[Out] - (8^(1/2)*atan(x*((3^(1/2)*1i)/(8*a) + 1/(8*a))^(1/2)*1i + 3^(1/2)*x*((3^(
1/2)*1i)/(8*a) + 1/(8*a))^(1/2))*((3^(1/2)*1i + 1)/a)^(1/2)*1i)/4 - (8^(1/2
)*atan(x*(1/(8*a) - (3^(1/2)*1i)/(8*a))^(1/2)*1i - 3^(1/2)*x*(1/(8*a) - (3^
(1/2)*1i)/(8*a))^(1/2))*(-(3^(1/2)*1i - 1)/a)^(1/2)*1i)/4
```

3.107 $\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 122

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\arctan\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} - \frac{\sqrt{3}\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}$$

```
[Out] 1/2*arctan(2*x/a^(1/4)-3^(1/2))/a^(1/4)+1/2*arctan(2*x/a^(1/4)+3^(1/2))/a^(
1/4)-1/4*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)+1/4*ln(x^2+a^(1/
4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1183, 648, 631, 210, 642}

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\arctan\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2\sqrt[4]{a}} - \frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}$$

```
[In] Int[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - (2*x)/a^(1/4)]/a^(1/4) + ArcTan[Sqrt[3] + (2*x)/a^(1/
4)]/(2*a^(1/4)) - (Sqrt[3]*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/
4)) + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{2\sqrt{3}a^{3/4}-3\sqrt{a}x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}}+\frac{\int \frac{2\sqrt{3}a^{3/4}+3\sqrt{a}x}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}}\\
&= \frac{1}{4}\int \frac{1}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx\\
&\quad -\frac{\sqrt{3}\int \frac{-\sqrt{3}\sqrt[4]{a}+2x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{4\sqrt[4]{a}}+\frac{\sqrt{3}\int \frac{\sqrt{3}\sqrt[4]{a}+2x}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{4\sqrt[4]{a}}
\end{aligned}
$$

$$= -\frac{\sqrt{3}\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}$$

$$+\frac{\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}\sqrt[4]{a}}-\frac{\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}\sqrt[4]{a}}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

$$-\frac{\sqrt{3}\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int\frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx$$

$$=\frac{\sqrt[4]{-1}\left(-\sqrt{i+\sqrt{3}}\left(3i+\sqrt{3}\right)\arctan\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt[4]{a}}\right)+\sqrt{-i+\sqrt{3}}\left(-3i+\sqrt{3}\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt[4]{a}}\right)\right)}{2\sqrt{6}\sqrt[4]{a}}$$

```
[In] Integrate[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-
I + Sqrt[3]]*a^(1/4))]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 +
 I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))]))/(2*Sqrt[6]*a^(1/4))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\frac{\sqrt{3}\,\ln\left(x^2+a^{\frac{1}{4}}x\sqrt{3}+\sqrt{a}\right)}{2}+\arctan\left(\frac{2x+a^{\frac{1}{4}}\sqrt{3}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}}+\frac{-\frac{\sqrt{3}\,\ln\left(x^2-a^{\frac{1}{4}}x\sqrt{3}+\sqrt{a}\right)}{2}+\arctan\left(\frac{2x-a^{\frac{1}{4}}\sqrt{3}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}}$	90

```
[In] int((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^(1/4)*(1/2*3^(1/2)*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))+arctan((2*x+a^(1
/4)*3^(1/2))/a^(1/4)))+1/2/a^(1/4)*(-1/2*3^(1/2)*ln(x^2-a^(1/4)*x*3^(1/2)+a
^(1/2))+arctan((2*x-a^(1/4)*3^(1/2))/a^(1/4)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(84) = 168$.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.06

$$\begin{aligned}\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx &= \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+x\right)\\
&\quad-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+x\right)\\
&\quad+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}+x\right)\\
&\quad-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}+x\right)\end{aligned}$$

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a)*log(sqrt(1/2)*sqrt(a
)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a) + x) - 1/2*sqrt(1/2)*sqrt((sqrt(
3)*a*sqrt(-1/a) + sqrt(a))/a)*log(-sqrt(1/2)*sqrt(a)*sqrt((sqrt(3)*a*sqrt(-
1/a) + sqrt(a))/a) + x) + 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(
a))/a)*log(sqrt(1/2)*sqrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
 - 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a)*log(-sqrt(1/2)*s
qrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = \text{Exception raised: PolynomialError}$$

```
[In] integrate((-x**2+2*a**(1/2))/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(64*_t**4*a - 16*_t**2*sqrt(a) + 1)
contains an element of the set of generators.
```

Maxima [F]

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = \int -\frac{x^2-2\sqrt{a}}{x^4-\sqrt{a}x^2+a}\,dx$$

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.30

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = 2\,\mathrm{atanh}\left(x\sqrt{\frac{1}{8\sqrt{a}}-\frac{\sqrt{-27\,a^3}}{24\,a^2}} - \frac{9\,a^{3/2}\,x\sqrt{\frac{1}{8\sqrt{a}}-\frac{\sqrt{-27\,a^3}}{24\,a^2}}}{\sqrt{-27\,a^3}}\right)\sqrt{\frac{1}{8\sqrt{a}}-\frac{\sqrt{-27\,a^3}}{24\,a^2}} + 2\,\mathrm{atanh}\left(x\sqrt{\frac{\sqrt{-27\,a^3}}{24\,a^2}+\frac{1}{8\sqrt{a}}} + \frac{9\,a^{3/2}\,x\sqrt{\frac{\sqrt{-27\,a^3}}{24\,a^2}+\frac{1}{8\sqrt{a}}}}{\sqrt{-27\,a^3}}\right)\sqrt{\frac{\sqrt{-27\,a^3}}{24\,a^2}+\frac{1}{8\sqrt{a}}}$$

```
[In] int((2*a^(1/2) - x^2)/(a + x^4 - a^(1/2)*x^2),x)
```

```
[Out] 2*atanh(x*(1/(8*a^(1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2) - (9*a^(3/2)*x*(
1/(8*a^(1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2))/(-27*a^3)^(1/2))*(1/(8*a^(
1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2) + 2*atanh(x*((-27*a^3)^(1/2)/(24*a^
2) + 1/(8*a^(1/2)))^(1/2) + (9*a^(3/2)*x*((-27*a^3)^(1/2)/(24*a^2) + 1/(8*a
^(1/2)))^(1/2))/(-27*a^3)^(1/2))*((-27*a^3)^(1/2)/(24*a^2) + 1/(8*a^(1/2)))
^(1/2)
```

3.108 $\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 124

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}$$

```
[Out] -1/4*ln(b^(2/3)-b^(1/3)*x+x^2)/b^(1/3)+1/4*ln(b^(2/3)+b^(1/3)*x+x^2)/b^(1/3
)-1/2*arctan(1/3*(b^(1/3)-2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)+1/2*arctan(
1/3*(b^(1/3)+2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1183, 648, 631, 210, 642}

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}$$

```
[In] Int[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(b^(1/3) - 2*x)/(Sqrt[3]*b^(1/3))])/b^(1/3) + (Sqrt[3]
*ArcTan[(b^(1/3) + 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3)) - Log[b^(2/3) - b^(
1/3)*x + x^2]/(4*b^(1/3)) + Log[b^(2/3) + b^(1/3)*x + x^2]/(4*b^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{2b-b^{2/3}x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx}{2b} + \frac{\int \frac{2b+b^{2/3}x}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx}{2b}$$

$$= \frac{3}{4}\int \frac{1}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx + \frac{3}{4}\int \frac{1}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx - \frac{\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx}{4\sqrt[3]{b}} + \frac{\int \frac{\sqrt[3]{b}+2x}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx}{4\sqrt[3]{b}}$$

$$
\begin{aligned}
&= -\frac{\log \left(b^{2/3}-\sqrt[3]{b} x+x^2\right)}{4 \sqrt[3]{b}}+\frac{\log \left(b^{2/3}+\sqrt[3]{b} x+x^2\right)}{4 \sqrt[3]{b}} \\
&\quad +\frac{3 \text{Subst}\left(\int \frac{1}{-3-x^2} \, dx,x,1-\frac{2 x}{\sqrt[3]{b}}\right)}{2 \sqrt[3]{b}}-\frac{3 \text{Subst}\left(\int \frac{1}{-3-x^2} \, dx,x,1+\frac{2 x}{\sqrt[3]{b}}\right)}{2 \sqrt[3]{b}} \\
&= -\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt[3]{b}-2 x}{\sqrt{3} \sqrt[3]{b}}\right)}{2 \sqrt[3]{b}}+\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt[3]{b}+2 x}{\sqrt{3} \sqrt[3]{b}}\right)}{2 \sqrt[3]{b}} \\
&\quad -\frac{\log \left(b^{2/3}-\sqrt[3]{b} x+x^2\right)}{4 \sqrt[3]{b}}+\frac{\log \left(b^{2/3}+\sqrt[3]{b} x+x^2\right)}{4 \sqrt[3]{b}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$
\int \frac{2 b^{2/3}+x^2}{b^{4/3}+b^{2/3} x^2+x^4} \, dx = \frac{\sqrt[4]{-1} \left(\sqrt{-i+\sqrt{3}} \left(-3 i+\sqrt{3}\right) \arctan \left(\frac{(1+i) x}{\sqrt{i+\sqrt{3}} \sqrt[3]{b}}\right)-\sqrt{i+\sqrt{3}} \left(3 i+\sqrt{3}\right) \text{arctanh}\right.}{2 \sqrt{6} \sqrt[3]{b}}
$$

```
[In] Integrate[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[I
 + Sqrt[3]]*b^(1/3))] - Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTanh[((1 + I)*
x)/(Sqrt[-I + Sqrt[3]]*b^(1/3))]))/(2*Sqrt[6]*b^(1/3))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{-\frac{\ln\left(b^{\frac{2}{3}}-b^{\frac{1}{3}} x+x^{2}\right)}{2}+\sqrt{3}\, \arctan\left(\frac{\left(-b^{\frac{1}{3}}+2 x\right) \sqrt{3}}{3 b^{\frac{1}{3}}}\right)}{2 b^{\frac{1}{3}}}+\frac{\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}} x+x^{2}\right)}{2}+\arctan\left(\frac{\left(b^{\frac{1}{3}}+2 x\right) \sqrt{3}}{3 b^{\frac{1}{3}}}\right) \sqrt{3}}{2 b^{\frac{1}{3}}}$	87

```
[In] int((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^(1/3)*(-1/2*ln(b^(2/3)-b^(1/3)*x+x^2)+3^(1/2)*arctan(1/3*(-b^(1/3)+2*
x)*3^(1/2)/b^(1/3)))+1/2/b^(1/3)*(1/2*ln(b^(2/3)+b^(1/3)*x+x^2)+arctan(1/3*
(b^(1/3)+2*x)/b^(1/3)*3^(1/2))*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.13

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \left[\frac{\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\,x^3+\sqrt{3}\left(2\,b^{\frac{2}{3}}x^2+bx-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3\,b^{\frac{2}{3}}x-b}{x^3+b}\right)+\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\,x^3+\sqrt{3}}{}\right.}{}\right.$$

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 + b*x
- b^(4/3))*sqrt(-1/b^(2/3)) - 3*b^(2/3)*x - b)/(x^3 + b)) + sqrt(3)*b*sqrt(
-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 - b*x - b^(4/3))*sqrt(-1/b^
(2/3)) - 3*b^(2/3)*x + b)/(x^3 - b)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^(2/3
)) - b^(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b, 1/4*(2*sqrt(3)*b^(2/3)*arct
an(1/3*sqrt(3)*(2*x + b^(1/3))/b^(1/3)) - 2*sqrt(3)*b^(2/3)*arctan(-1/3*sqr
t(3)*(2*x - b^(1/3))/b^(1/3)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^(2/3)) - b^
(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \frac{\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)+x\right)+\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)+\right.}{\sqrt[3]{}}$$

```
[In] integrate((2*b**(2/3)+x**2)/(b**(4/3)+b**(2/3)*x**2+x**4),x)
```

```
[Out] ((-1/4 - sqrt(3)*I/4)*log(2*b**(1/3)*(-1/4 - sqrt(3)*I/4) + x) + (-1/4 + sq
rt(3)*I/4)*log(2*b**(1/3)*(-1/4 + sqrt(3)*I/4) + x) + (1/4 - sqrt(3)*I/4)*l
og(2*b**(1/3)*(1/4 - sqrt(3)*I/4) + x) + (1/4 + sqrt(3)*I/4)*log(2*b**(1/3)
*(1/4 + sqrt(3)*I/4) + x))/b**(1/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.71

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{2\,b^{\frac{1}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{2\,b^{\frac{1}{3}}} + \frac{\log\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}} - \frac{\log\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}$$

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/b^(1/3))/b^(1/3) + 1/2*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/b^(1/3))/b^(1/3) + 1/4*log(x^2 + b^(1
/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^(2/3))/b^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+b^{\frac{1}{3}}\right)}{3\,|b|^{\frac{1}{3}}}\right)}{2\,|b|^{\frac{1}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-b^{\frac{1}{3}}\right)}{3\,|b|^{\frac{1}{3}}}\right)}{2\,|b|^{\frac{1}{3}}} + \frac{\log\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}} - \frac{\log\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}$$

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3) +
 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3)
+ 1/4*log(x^2 + b^(1/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^
(2/3))/b^(1/3)
```

Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{-\frac{1}{8\,b^{2/3}}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\,1\mathrm{i}+\sqrt{3}\,x\,\sqrt{-\frac{1}{8\,b^{2/3}}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\right)\,\sqrt{-\frac{1+\sqrt{3}\,1\mathrm{i}}{b^{2/3}}}\,1\mathrm{i}}{4} + \frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{-\frac{1}{8\,b^{2/3}}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\,1\mathrm{i}-\sqrt{3}\,x\,\sqrt{-\frac{1}{8\,b^{2/3}}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\right)\,\sqrt{\frac{-1+\sqrt{3}\,1\mathrm{i}}{b^{2/3}}}\,1\mathrm{i}}{4}$$

```
[In] int((2*b^(2/3) + x^2)/(b^(4/3) + x^4 + b^(2/3)*x^2),x)
```

```
[Out] (8^(1/2)*atan(x*(- (3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(1/2)*1i + 3^(
1/2)*x*(- (3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(1/2))*(-(3^(1/2)*1i +
1)/b^(2/3))^(1/2)*1i)/4 + (8^(1/2)*atan(x*((3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*
b^(2/3)))^(1/2)*1i - 3^(1/2)*x*((3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(
1/2))*((3^(1/2)*1i - 1)/b^(2/3))^(1/2)*1i)/4
```

3.109 $\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx = -\frac{(A+aB)\arctan\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(A+aB)\arctan\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}}$$

```
[Out] 1/2*(B*a+A)*arctan(-3^(1/2)+2*x/a^(1/2))/a^(3/2)+1/2*(B*a+A)*arctan(3^(1/2)
+2*x/a^(1/2))/a^(3/2)-1/12*(-B*a+A)*ln(a+x^2-x*3^(1/2)*a^(1/2))/a^(3/2)*3^(
1/2)+1/12*(-B*a+A)*ln(a+x^2+x*3^(1/2)*a^(1/2))/a^(3/2)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1183, 648, 631, 210, 642}

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx = -\frac{(aB+A)\arctan\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(aB+A)\arctan\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2a^{3/2}} - \frac{(A-aB)\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}}$$

```
[In] Int[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -1/2*((A + a*B)*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]])/a^(3/2) + ((A + a*B)*ArcTa
n[Sqrt[3] + (2*x)/Sqrt[a]])/(2*a^(3/2)) - ((A - a*B)*Log[a - Sqrt[3]*Sqrt[a
]*x + x^2])/(4*Sqrt[3]*a^(3/2)) + ((A - a*B)*Log[a + Sqrt[3]*Sqrt[a]*x + x^
2])/(4*Sqrt[3]*a^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{3}\sqrt{a}A-(A-aB)x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{\sqrt{3}\sqrt{a}A+(A-aB)x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}}$$

$$
\begin{aligned}
&= -\frac{(A-aB)\int \frac{-\sqrt{3}\sqrt{a}+2x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\int \frac{\sqrt{3}\sqrt{a}+2x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{3}a^{3/2}} \\
&\quad + \frac{(A+aB)\int \frac{1}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4a} + \frac{(A+aB)\int \frac{1}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{4a} \\
&= -\frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} \\
&\quad + \frac{(A+aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)}{2\sqrt{3}a^{3/2}} \\
&\quad - \frac{(A+aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2x}{\sqrt{3}\sqrt{a}}\right)}{2\sqrt{3}a^{3/2}} \\
&= -\frac{(A+aB)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{(A+aB)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} \\
&\quad - \frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$
\begin{aligned}
&\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx \\
&= \frac{\sqrt[4]{-1}\left(\frac{\left(-2iA+\left(-i+\sqrt{3}\right)aB\right)\arctan\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt{a}}\right)}{\sqrt{-i+\sqrt{3}}} - \frac{\left(2iA+\left(i+\sqrt{3}\right)aB\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt{a}}\right)}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}a^{3/2}}
\end{aligned}
$$

```
[In] Integrate[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*a*B)*ArcTan[((1 + I)*x)/(Sqrt[-I +
 Sqrt[3]]*Sqrt[a])])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*a*B)*Ar
cTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])])/Sqrt[I + Sqrt[3]]))/(Sqrt[6
]*a^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^2+a^2\right)}\frac{\left(B\_R^2+A\right)\ln\left(x-\_R\right)}{2\_R^3-\_Ra}\right)}{2}$
default	$\frac{\frac{\left(-B\sqrt{3}\,a^2+A\sqrt{3}\,a\right)\ln\left(a+x^2+x\sqrt{3}\,\sqrt{a}\right)}{2}+\frac{2\left(3Aa^{\frac{3}{2}}-\frac{\left(-B\sqrt{3}\,a^2+A\sqrt{3}\,a\right)\sqrt{3}\,\sqrt{a}}{2}\right)\arctan\left(\frac{2x+\sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right)}{\sqrt{a}}}{6a^{\frac{5}{2}}}+ -\frac{\left(-B\sqrt{3}\,a^2+A\sqrt{3}\,a\right)\ln\left(x\sqrt{3}\,\sqrt{a}-$

```
[In] int((B*x^2+A)/(x^4-a*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((B*_R^2+A)/(2*_R^3-_R*a)*ln(x-_R),_R=RootOf(_Z^4-_Z^2*a+a^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(104) = 208$.

Time = 0.29 (sec) , antiderivative size = 903, normalized size of antiderivative = 6.64

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx$$

$$=\frac{1}{2}\sqrt{\frac{1}{6}}\sqrt{-\frac{B^2a^2+3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}+4ABa+A^2}{a^3}}\log\Bigg(2\left(B^4a^4+AB^3a^3-A^3Ba-A^4\right)x$$

$$+3\sqrt{\frac{1}{6}}\left(AB^2a^4-A^3a^2-\sqrt{\frac{1}{3}}\left(2Ba^6+Aa^5\right)\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}\right)\sqrt{-\frac{B^2a^2+3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-}{}}}{a}}$$

$$-\frac{1}{2}\sqrt{\frac{1}{6}}\sqrt{-\frac{B^2a^2+3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}+4ABa+A^2}{a^3}}\log\Bigg(2\left(B^4a^4+AB^3a^3-A^3Ba-A^4\right)x$$

$$-3\sqrt{\frac{1}{6}}\left(AB^2a^4-A^3a^2-\sqrt{\frac{1}{3}}\left(2Ba^6+Aa^5\right)\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}\right)\sqrt{-\frac{B^2a^2+3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-}{}}}{a}}$$

$$+\frac{1}{2}\sqrt{\frac{1}{6}}\sqrt{-\frac{B^2a^2-3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}+4ABa+A^2}{a^3}}\log\Bigg(2\left(B^4a^4+AB^3a^3-A^3Ba-A^4\right)x$$

$$+3\sqrt{\frac{1}{6}}\left(AB^2a^4-A^3a^2+\sqrt{\frac{1}{3}}\left(2Ba^6+Aa^5\right)\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}\right)\sqrt{-\frac{B^2a^2-3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-}{}}}{a}}$$

$$-\frac{1}{2}\sqrt{\frac{1}{6}}\sqrt{-\frac{B^2a^2-3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}+4ABa+A^2}{a^3}}\log\Bigg(2\left(B^4a^4+AB^3a^3-A^3Ba-A^4\right)x$$

$$-3\sqrt{\frac{1}{6}}\left(AB^2a^4-A^3a^2+\sqrt{\frac{1}{3}}\left(2Ba^6+Aa^5\right)\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}\right)\sqrt{-\frac{B^2a^2-3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-}{}}}{a}}$$

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/6)*sqrt(-(B^2*a^2 + 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4 - 2*A^2*B^2*a
^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3)*log(2*(B^4*a^4 + A*B^3*a^3 - A^3*B*a -
 A^4)*x + 3*sqrt(1/6)*(A*B^2*a^4 - A^3*a^2 - sqrt(1/3)*(2*B*a^6 + A*a^5)*sq
rt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt(-(B^2*a^2 + 3*sqrt(1/3)*a^3*
```

```
sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3)) - 1/2*sqr
t(1/6)*sqrt(-(B^2*a^2 + 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^
4)/a^6) + 4*A*B*a + A^2)/a^3)*log(2*(B^4*a^4 + A*B^3*a^3 - A^3*B*a - A^4)*x
 - 3*sqrt(1/6)*(A*B^2*a^4 - A^3*a^2 - sqrt(1/3)*(2*B*a^6 + A*a^5)*sqrt(-(B^
4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt(-(B^2*a^2 + 3*sqrt(1/3)*a^3*sqrt(-(
B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3)) + 1/2*sqrt(1/6)*
sqrt(-(B^2*a^2 - 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6)
 + 4*A*B*a + A^2)/a^3)*log(2*(B^4*a^4 + A*B^3*a^3 - A^3*B*a - A^4)*x + 3*sq
rt(1/6)*(A*B^2*a^4 - A^3*a^2 + sqrt(1/3)*(2*B*a^6 + A*a^5)*sqrt(-(B^4*a^4 -
 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt(-(B^2*a^2 - 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4
 - 2*A^2*B^2*a^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3)) - 1/2*sqrt(1/6)*sqrt(-(
B^2*a^2 - 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + 4*A*
B*a + A^2)/a^3)*log(2*(B^4*a^4 + A*B^3*a^3 - A^3*B*a - A^4)*x - 3*sqrt(1/6)
*(A*B^2*a^4 - A^3*a^2 + sqrt(1/3)*(2*B*a^6 + A*a^5)*sqrt(-(B^4*a^4 - 2*A^2*
B^2*a^2 + A^4)/a^6))*sqrt(-(B^2*a^2 - 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4 - 2*A^
2*B^2*a^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3))
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx$$
$$= \text{RootSum}\Bigg(144t^4a^6+t^2\cdot\left(12A^2a^3+48ABa^4+12B^2a^5\right)+A^4+2A^3Ba+3A^2B^2a^2+2AB^3a^3+B^4a^4,\ \Bigg($$

```
[In] integrate((B*x**2+A)/(x**4-a*x**2+a**2),x)
```

```
[Out] RootSum(144*_t**4*a**6 + _t**2*(12*A**2*a**3 + 48*A*B*a**4 + 12*B**2*a**5)
+ A**4 + 2*A**3*B*a + 3*A**2*B**2*a**2 + 2*A*B**3*a**3 + B**4*a**4, Lambda(
_t, _t*log(x + (24*_t**3*A*a**5 + 48*_t**3*B*a**6 - 2*_t*A**3*a**2 + 6*_t*A
**2*B*a**3 + 12*_t*A*B**2*a**4 + 2*_t*B**3*a**5)/(-A**4 - A**3*B*a + A*B**3
*a**3 + B**4*a**4))))
```

Maxima [F]

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx = \int \frac{Bx^2+A}{x^4-ax^2+a^2}\,dx$$

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.84 (sec) , antiderivative size = 4293, normalized size of antiderivative = 31.57

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(3*sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/ab
s(a))))^2*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3*sin(1/2*real_part(arc
cos(1/2*a/abs(a)))) - sqrt(3)*B*a^2*abs(a)^(3/2)*cosh(1/2*imag_part(arccos(
1/2*a/abs(a))))^3*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3 - 9*sqrt(3)*B*
a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_p
art(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(
1/2*imag_part(arccos(1/2*a/abs(a)))) + 3*sqrt(3)*B*a^2*abs(a)^(3/2)*cosh(1/
2*imag_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))
))^3*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 9*sqrt(3)*B*a^2*abs(a)^(3/
2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_part(arccos(1/2
*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(ar
ccos(1/2*a/abs(a))))^2 - 3*sqrt(3)*B*a^2*abs(a)^(3/2)*cosh(1/2*imag_part(ar
ccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*im
ag_part(arccos(1/2*a/abs(a))))^2 - 3*sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/2*rea
l_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))*si
nh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 + sqrt(3)*B*a^2*abs(a)^(3/2)*sin(
1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2*a/abs(
a))))^3 + B*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))^3*cos
h(1/2*imag_part(arccos(1/2*a/abs(a))))^3 - 3*B*a^3*sqrt(abs(a))*cos(1/2*rea
l_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3*s
in(1/2*real_part(arccos(1/2*a/abs(a))))^2 - 3*B*a^3*sqrt(abs(a))*cos(1/2*re
al_part(arccos(1/2*a/abs(a))))^3*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^
2*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 9*B*a^3*sqrt(abs(a))*cos(1/2*
real_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^
2*sin(1/2*real_part(arccos(1/2*a/abs(a))))^2*sinh(1/2*imag_part(arccos(1/2*
a/abs(a)))) + 3*B*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))
^3*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*
a/abs(a))))^2 - 9*B*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))
))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a
/abs(a))))^2*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^2 - B*a^3*sqrt(abs(a
))*cos(1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2
*a/abs(a))))^3 + 3*B*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a)
)))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^2*sinh(1/2*imag_part(arccos(1/
2*a/abs(a))))^3 + sqrt(3)*A*a^2*sqrt(abs(a))*cosh(1/2*imag_part(arccos(1/2*
a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a)))) - sqrt(3)*A*a^2*sqrt(a
```

```
bs(a))*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1
/2*a/abs(a)))) + A*a*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))*
cosh(1/2*imag_part(arccos(1/2*a/abs(a)))) - A*a*abs(a)^(3/2)*cos(1/2*real_p
art(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))))*arcta
n((sqrt(abs(a))*cos(1/2*arccos(1/2*a/abs(a))) + x)/(sqrt(abs(a))*sin(1/2*ar
ccos(1/2*a/abs(a)))))/a^4 + 1/12*sqrt(3)*(sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/
2*real_part(arccos(1/2*a/abs(a))))^3*cosh(1/2*imag_part(arccos(1/2*a/abs(a)
)))^3 - 3*sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))
))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3*sin(1/2*real_part(arccos(1/2
*a/abs(a))))^2 - 3*sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*
a/abs(a))))^3*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^2*sinh(1/2*imag_par
t(arccos(1/2*a/abs(a)))) + 9*sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/2*real_part(a
rccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^2*sin(1/2*r
eal_part(arccos(1/2*a/abs(a))))^2*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))
 + 3*sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^3*
cosh(1/2*imag_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*a/a
bs(a))))^2 - 9*sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/ab
s(a))))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(
1/2*a/abs(a))))^2*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^2 - sqrt(3)*B*a
^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*imag_pa
rt(arccos(1/2*a/abs(a))))^3 + 3*sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/2*real_par
t(arccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^2*sinh(1/
2*imag_part(arccos(1/2*a/abs(a))))^3 - 3*B*a^3*sqrt(abs(a))*cos(1/2*real_pa
rt(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3*sin
(1/2*real_part(arccos(1/2*a/abs(a)))) + B*a^3*sqrt(abs(a))*cosh(1/2*imag_pa
rt(arccos(1/2*a/abs(a))))^3*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3 + 9*
B*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag
_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sin
h(1/2*imag_part(arccos(1/2*a/abs(a)))) - 3*B*a^3*sqrt(abs(a))*cosh(1/2*imag
_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3*s
inh(1/2*imag_part(arccos(1/2*a/abs(a)))) - 9*B*a^3*sqrt(abs(a))*cos(1/2*rea
l_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))*s
in(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*a/abs
(a))))^2 + 3*B*a^3*sqrt(abs(a))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))*s
in(1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2*a/a
bs(a))))^2 + 3*B*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))^
2*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*a/
abs(a))))^3 - B*a^3*sqrt(abs(a))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3
*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 + sqrt(3)*A*a^2*sqrt(abs(a))*c
os(1/2*real_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arccos(1/2*a/abs
(a)))) - sqrt(3)*A*a^2*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a)))
)*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) - A*a*abs(a)^(3/2)*cosh(1/2*ima
g_part(arccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a)))) + A*
a*abs(a)^(3/2)*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(
arccos(1/2*a/abs(a)))))*log(2*x*sqrt(abs(a))*cos(1/2*arccos(1/2*a/abs(a)))
```

```
+ x^2 + abs(a))/a^4 + 1/48*sqrt(3)*(sqrt(3)*sqrt(14*a^2 + 13*a*abs(a))*B*a^
5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^3 - 3*sqrt(3)*sqrt(2*a^2 +
a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2*imag_part(s
gn(sin(1/2*arccos(1/2*a/abs(a))))) + 3*sqrt(3)*sqrt(2*a^2 - a*abs(a))*B*a^5
*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(cos(1/2*arcc
os(1/2*a/abs(a))))) + 3*sqrt(3)*sqrt(14*a^2 + 13*a*abs(a))*B*a^5*imag_part(
sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(
a)))))^2 + 3*sqrt(3)*sqrt(2*a^2 + a*abs(a))*B*a^5*imag_part(sgn(sin(1/2*arc
cos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2 - 6*sq
rt(3)*sqrt(2*a^2 - a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a
)))))*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*a
rccos(1/2*a/abs(a))))) + 3*sqrt(3)*sqrt(14*a^2 - 13*a*abs(a))*B*a^5*imag_pa
rt(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(sin(1/2*arccos(1/2*a
/abs(a))))) - 6*sqrt(3)*sqrt(2*a^2 + a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*
arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_
part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) + 3*sqrt(3)*sqrt(2*a^2 - a*abs(a))
*B*a^5*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*
arccos(1/2*a/abs(a)))))^2 + sqrt(3)*sqrt(14*a^2 - 13*a*abs(a))*B*a^5*real_p
art(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^3 + 9*sqrt(2*a^2 - a*abs(a))*B*a^5*
imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*imag_part(sgn(sin(1/2*arccos(
1/2*a/abs(a)))))^2 + sqrt(14*a^2 - 13*a*abs(a))*B*a^5*imag_part(sgn(sin(1/2
*arccos(1/2*a/abs(a)))))^3 - 3*sqrt(14*a^2 + 13*a*abs(a))*B*a^5*imag_part(s
gn(cos(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(cos(1/2*arccos(1/2*a/abs
(a))))) + 18*sqrt(2*a^2 + a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*
a/abs(a)))))*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(co
s(1/2*arccos(1/2*a/abs(a))))) - sqrt(14*a^2 + 13*a*abs(a))*B*a^5*real_part(
sgn(cos(1/2*arccos(1/2*a/abs(a)))))^3 + 9*sqrt(2*a^2 + a*abs(a))*B*a^5*imag
_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(sin(1/2*arccos(1/
2*a/abs(a))))) + 18*sqrt(2*a^2 - a*abs(a))*B*a^5*imag_part(sgn(sin(1/2*arcc
os(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part
(sgn(sin(1/2*arccos(1/2*a/abs(a))))) + 9*sqrt(2*a^2 + a*abs(a))*B*a^5*real_
part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(sin(1/2*arccos(1/2
*a/abs(a))))) + 9*sqrt(2*a^2 - a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos
(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2 + 3*sqrt(
14*a^2 - 13*a*abs(a))*B*a^5*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))*r
eal_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2 + 4*sqrt(3)*sqrt(2*a^2 + a*a
bs(a))*A*a^4*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a))))) + 4*sqrt(3)*sqrt
(2*a^2 - a*abs(a))*A*a^4*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) - 4*
sqrt(2*a^2 - a*abs(a))*A*a^4*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))
+ 4*sqrt(2*a^2 + a*abs(a))*A*a^4*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a))
))))*arctan(-1/2*(sqrt(a/abs(a) + 2)*sqrt(abs(a))*sgn(cos(1/2*arccos(1/2*a/
abs(a)))) - 2*x)/(sqrt(-1/4*a/abs(a) + 1/2)*sqrt(abs(a))*sgn(sin(1/2*arccos
(1/2*a/abs(a))))))/(a^5*abs(a)^(3/2)) + 1/96*sqrt(3)*(3*sqrt(3)*sqrt(2*a^2
- a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*imag_part(s
gn(sin(1/2*arccos(1/2*a/abs(a)))))^2 + sqrt(3)*sqrt(14*a^2 - 13*a*abs(a))*B
```

```
*a^5*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^3 + 3*sqrt(3)*sqrt(14*a^
2 + 13*a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2*real
_part(sgn(cos(1/2*arccos(1/2*a/abs(a))))) + 6*sqrt(3)*sqrt(2*a^2 + a*abs(a)
)*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*imag_part(sgn(sin(1/2
*arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a))))) + sq
rt(3)*sqrt(14*a^2 + 13*a*abs(a))*B*a^5*real_part(sgn(cos(1/2*arccos(1/2*a/a
bs(a)))))^3 + 3*sqrt(3)*sqrt(2*a^2 + a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*
arccos(1/2*a/abs(a)))))^2*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) + 6
*sqrt(3)*sqrt(2*a^2 - a*abs(a))*B*a^5*imag_part(sgn(sin(1/2*arccos(1/2*a/ab
s(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/
2*arccos(1/2*a/abs(a))))) + 3*sqrt(3)*sqrt(2*a^2 + a*abs(a))*B*a^5*real_par
t(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(sin(1/2*arccos(1/2*a/
abs(a))))) + 3*sqrt(3)*sqrt(2*a^2 - a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*a
rccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2 + 3*
sqrt(3)*sqrt(14*a^2 - 13*a*abs(a))*B*a^5*imag_part(sgn(sin(1/2*arccos(1/2*a
/abs(a)))))*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2 - sqrt(14*a^2 +
 13*a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^3 + 9*sqr
t(2*a^2 + a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2*i
mag_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) + 9*sqrt(2*a^2 - a*abs(a))*B*a
^5*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2*real_part(sgn(cos(1/2*ar
ccos(1/2*a/abs(a))))) + 3*sqrt(14*a^2 + 13*a*abs(a))*B*a^5*imag_part(sgn(co
s(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))
^2 + 9*sqrt(2*a^2 + a*abs(a))*B*a^5*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(
a)))))*real_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))^2 + 18*sqrt(2*a^2 - a*
abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*imag_part(sgn(s
in(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))
) - 3*sqrt(14*a^2 - 13*a*abs(a))*B*a^5*imag_part(sgn(sin(1/2*arccos(1/2*a/a
bs(a)))))^2*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))) + 18*sqrt(2*a^2 +
 a*abs(a))*B*a^5*imag_part(sgn(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sg
n(cos(1/2*arccos(1/2*a/abs(a)))))*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)
)))) + 9*sqrt(2*a^2 - a*abs(a))*B*a^5*real_part(sgn(cos(1/2*arccos(1/2*a/ab
s(a)))))*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^2 - sqrt(14*a^2 - 13
*a*abs(a))*B*a^5*real_part(sgn(sin(1/2*arccos(1/2*a/abs(a)))))^3 + 4*sqrt(3
)*sqrt(2*a^2 - a*abs(a))*A*a^4*imag_part(sgn(sin(1/2*arccos(1/2*a/abs(a))))
) + 4*sqrt(3)*sqrt(2*a^2 + a*abs(a))*A*a^4*real_part(sgn(cos(1/2*arccos(1/2
*a/abs(a))))) + 4*sqrt(2*a^2 + a*abs(a))*A*a^4*imag_part(sgn(cos(1/2*arccos
(1/2*a/abs(a))))) + 4*sqrt(2*a^2 - a*abs(a))*A*a^4*real_part(sgn(sin(1/2*ar
ccos(1/2*a/abs(a))))))*log(-x*sqrt(a/abs(a) + 2)*sqrt(abs(a))*sgn(cos(1/2*a
rccos(1/2*a/abs(a)))) + x^2 + abs(a))/(a^5*abs(a)^(3/2))
```

Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 1007, normalized size of antiderivative = 7.40

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx=\operatorname{atan}\left(\frac{A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}\,6\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}+\frac{2\,\sqrt{3}\,A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}-\frac{B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}\,6\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}-\frac{2\,\sqrt{3}\,B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}\right)\,\sqrt{-\frac{A^2+B^2\,a^2+4\,A\,B\,a+\sqrt{3}\,A^2\,1\mathrm{i}-\sqrt{3}}{24\,a^3}}$$

$$+\operatorname{atan}\left(\frac{A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}\,6\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}-\frac{2\,\sqrt{3}\,A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}-\frac{B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}\,6\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}+\frac{2\,\sqrt{3}\,B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}\right)\,\sqrt{-\frac{A^2+B^2\,a^2+4\,A\,B\,a-\sqrt{3}\,A^2\,1\mathrm{i}+\sqrt{3}}{24\,a^3}}$$

```
[In] int((A + B*x^2)/(a^2 - a*x^2 + x^4),x)
```

```
[Out] atan((A^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^
3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 +
 (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A*B^2*a*1i) + (2*3^(1/2)*A^2*x*((3^
(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3
) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a
 - A*B^2*a - 3^(1/2)*A*B^2*a*1i) - (B^2*a^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^
2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2)*
6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A
*B^2*a*1i) - (2*3^(1/2)*B^2*a^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (
3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B +
A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A*B^2*a*1i))*(-(
3^(1/2)*A^2*1i + A^2 + B^2*a^2 - 3^(1/2)*B^2*a^2*1i + 4*A*B*a)/(24*a^3))^(1
/2)*2i + atan((A^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3)
```

```
 - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*
B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a*1i) - (2*3^(1/2)*A
^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*
1i)/(24*a) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)*
A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a*1i) - (B^2*a^2*x*((3^(1/2)*A^2*1i)/(2
4*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2
))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a +
3^(1/2)*A*B^2*a*1i) + (2*3^(1/2)*B^2*a^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2
/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2))^(1/2))/(2
*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a
*1i))*(-(A^2 - 3^(1/2)*A^2*1i + B^2*a^2 + 3^(1/2)*B^2*a^2*1i + 4*A*B*a)/(24
*a^3))^(1/2)*2i
```

3.110 $\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = -\frac{\left(A+\sqrt{a}B\right)\arctan\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{\left(A+\sqrt{a}B\right)\arctan\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} - \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}} + \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}}$$

```
[Out] -1/12*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))*(A-B*a^(1/2))/a^(3/4)*3^(1/2)+1/12*
ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*(A-B*a^(1/2))/a^(3/4)*3^(1/2)+1/2*arctan(
2*x/a^(1/4)-3^(1/2))*(A+B*a^(1/2))/a^(3/4)+1/2*arctan(2*x/a^(1/4)+3^(1/2))*
(A+B*a^(1/2))/a^(3/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1183, 648, 631, 210, 642\}$

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = -\frac{\left(\sqrt{a}B+A\right)\arctan\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{\left(\sqrt{a}B+A\right)\arctan\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}} - \frac{\left(A-\sqrt{a}B\right)\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}} + \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}}$$

```
[In] Int[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -1/2*((A + Sqrt[a]*B)*ArcTan[Sqrt[3] - (2*x)/a^(1/4)])/a^(3/4) + ((A + Sqrt
[a]*B)*ArcTan[Sqrt[3] + (2*x)/a^(1/4)])/(2*a^(3/4)) - ((A - Sqrt[a]*B)*Log[
Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4)) + ((A - Sqrt[a]*B)*
Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{3}\sqrt[4]{a}A-(A-\sqrt{a}B)x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}} + \frac{\int \frac{\sqrt{3}\sqrt[4]{a}A+(A-\sqrt{a}B)x}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}} \\
&= \frac{1}{4}\left(\frac{A}{\sqrt{a}}+B\right)\int \frac{1}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx + \frac{1}{4}\left(\frac{A}{\sqrt{a}}+B\right)\int \frac{1}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx \\
&\quad - \frac{\left(A-\sqrt{a}B\right)\int \frac{-\sqrt{3}\sqrt[4]{a}+2x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{4\sqrt{3}a^{3/4}} + \frac{\left(A-\sqrt{a}B\right)\int \frac{\sqrt{3}\sqrt[4]{a}+2x}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{4\sqrt{3}a^{3/4}} \\
&= -\frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}} + \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}} \\
&\quad + \frac{\left(A+\sqrt{a}B\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}a^{3/4}} \\
&\quad - \frac{\left(A+\sqrt{a}B\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}a^{3/4}} \\
&= -\frac{\left(A+\sqrt{a}B\right)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{\left(A+\sqrt{a}B\right)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} \\
&\quad - \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}} + \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86

$$
\begin{aligned}
&\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx \\
&= \frac{\sqrt[4]{-1}\left(\frac{\left(-2iA+\left(-i+\sqrt{3}\right)\sqrt{a}B\right)\arctan\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt[4]{a}}\right)}{\sqrt{-i+\sqrt{3}}} - \frac{\left(2iA+\left(i+\sqrt{3}\right)\sqrt{a}B\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt[4]{a}}\right)}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}a^{3/4}}
\end{aligned}
$$

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*Sqrt[a]*B)*ArcTan[((1 + I)*x)/(Sqr
t[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*S
qrt[a]*B)*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))])/Sqrt[I + Sqrt[3
]]))/(Sqrt[6]*a^(3/4))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

method	result
default	$\frac{\frac{\left(A\sqrt{a}\,\sqrt{3}-B\sqrt{3}\,a\right)\ln\left(x^{2}+a^{\frac{1}{4}}x\sqrt{3}+\sqrt{a}\right)}{2}+\frac{2\left(3A\,a^{\frac{3}{4}}-\frac{\left(A\sqrt{a}\,\sqrt{3}-B\sqrt{3}\,a\right)a^{\frac{1}{4}}\sqrt{3}}{2}\right)\arctan\left(\frac{2x+a^{\frac{1}{4}}\sqrt{3}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}}{6a^{\frac{5}{4}}}+\frac{\left(-A\sqrt{a}\,\sqrt{3}+B\sqrt{3}\,a\right)\ln\left(x^{2}-a^{\frac{1}{4}}x\right.}{2}$

```
[In] int((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^(5/4)*(1/2*(A*a^(1/2)*3^(1/2)-B*3^(1/2)*a)*ln(x^2+a^(1/4)*x*3^(1/2)+a
^(1/2))+2*(3*A*a^(3/4)-1/2*(A*a^(1/2)*3^(1/2)-B*3^(1/2)*a)*a^(1/4)*3^(1/2))
/a^(1/4)*arctan((2*x+a^(1/4)*3^(1/2))/a^(1/4)))+1/6/a^(5/4)*(1/2*(-A*a^(1/2
)*3^(1/2)+B*3^(1/2)*a)*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))+2*(3*A*a^(3/4)+1/2
*(-A*a^(1/2)*3^(1/2)+B*3^(1/2)*a)*a^(1/4)*3^(1/2))/a^(1/4)*arctan((2*x-a^(1
/4)*3^(1/2))/a^(1/4)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(116) = 232$.

Time = 0.38 (sec) , antiderivative size = 1141, normalized size of antiderivative = 7.13

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a
 + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt
(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2
 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sqrt(1/3)*(A*B^2*a^3
- A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*
a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2
)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^
4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^
3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*
a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sq
```

```
rt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqr
t(a))*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/
a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) + 1/2*sqrt(1/6)*sqrt(-(4*A*B*a - 3*sqrt
(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))
/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a + sqrt(1/3)*
(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3
*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2
*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 -
2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt
(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^
2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 -
 A^5*a + sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A
^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(
B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^
2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = \text{Exception raised: PolynomialError}$$

```
[In] integrate((B*x**2+A)/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(64*_t**4*a - 16*_t**2*B**2*sqrt(a)
+ B**4) contains an element of the set of generators.
```

Maxima [F]

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = \int \frac{Bx^2+A}{x^4-\sqrt{a}x^2+a}\,dx$$

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 1155, normalized size of antiderivative = 7.22

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx$$

$$= -2\,\mathrm{atanh}\left(\frac{6\,A^2\,x\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-27\,a^3}}{72\,a^3}-\frac{A^2}{24\,a^{3/2}}-\frac{A\,B}{6\,a}}}{2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3\,\sqrt{-27\,a^3}}{3\,a^2}-\frac{A\,B^2\,\sqrt{-27\,a^3}}{3\,a}}\right.$$
$$-\frac{6\,B^2\,a\,x\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-27\,a^3}}{72\,a^3}-\frac{A^2}{24\,a^{3/2}}-\frac{A\,B}{6\,a}}}{2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3\,\sqrt{-27\,a^3}}{3\,a^2}-\frac{A\,B^2\,\sqrt{-27\,a^3}}{3\,a}}$$
$$-\frac{2\,A^2\,x\,\sqrt{-27\,a^3}\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-27\,a^3}}{72\,a^3}-\frac{A^2}{24\,a^{3/2}}-\frac{A\,B}{6\,a}}}{3\,a^{3/2}\,\left(2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3\,\sqrt{-27\,a^3}}{3\,a^2}-\frac{A\,B^2\,\sqrt{-27\,a^3}}{3\,a}\right)}$$
$$\left.+\frac{2\,B^2\,x\,\sqrt{-27\,a^3}\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-27\,a^3}}{72\,a^3}-\frac{A^2}{24\,a^{3/2}}-\frac{A\,B}{6\,a}}}{3\,\sqrt{a}\,\left(2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3\,\sqrt{-27\,a^3}}{3\,a^2}-\frac{A\,B^2\,\sqrt{-27\,a^3}}{3\,a}\right)}\right)\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-A^2}$$

```
[In] int((A + B*x^2)/(a + x^4 - a^(1/2)*x^2),x)
```

```
[Out] - 2*atanh((6*A^2*x*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^
2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(2*A^2
*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2)
- (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((B^2*(-27*a^3)^(1/2))/(72*a^
2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) -
 (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^
3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (2*A^2*x*(-27
*a^3)^(1/2)*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*
a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a^(3/2)*(2
*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a
```

```
^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a))) + (2*B^2*x*(-27*a^3)^(1/2)*((B^2*(-27
*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) -
 A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a^(1/2)*(2*A^2*B - 2*B^3*a + A^3
/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)
^(1/2))/(3*a))))*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*
(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2) - 2*atanh
((6*A^2*x*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3
/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^
3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*
(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/
(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6
*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^
3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(3*a)) + (2*A^2*x*(-27*a^3)^(1/
2)*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) -
(B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(3*a^(3/2)*(2*A^2*B -
2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*
B^2*(-27*a^3)^(1/2))/(3*a))) - (2*B^2*x*(-27*a^3)^(1/2)*((A^2*(-27*a^3)^(1/
2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/
(72*a^2) - (A*B)/(6*a))^(1/2))/(3*a^(1/2)*(2*A^2*B - 2*B^3*a + A^3/a^(1/2)
- A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(
3*a))))*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2
)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2)
```

3.111 $\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 414

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx = -\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}} - \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\,x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}} + \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\,x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

```
[Out] -1/2*arctan((-2*x*c^(1/2)+(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))/(2*a^(1/2)
*c^(1/2)-(a*c)^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(1/2)/c^(1/2)/(2*a^(1/
2)*c^(1/2)-(a*c)^(1/2))^(1/2)+1/2*arctan((2*x*c^(1/2)+(2*a^(1/2)*c^(1/2)+(a
*c)^(1/2))^(1/2))/(2*a^(1/2)*c^(1/2)-(a*c)^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/
2))/a^(1/2)/c^(1/2)/(2*a^(1/2)*c^(1/2)-(a*c)^(1/2))^(1/2)-1/4*ln(a^(1/2)+x^
2*c^(1/2)-x*(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^
(1/2)/(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2)+1/4*ln(a^(1/2)+x^2*c^(1/2)+x*(2
*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^(1/2)/(2*a^(1/
2)*c^(1/2)+(a*c)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1183, 648, 632, 210, 642}

$$\begin{aligned}\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx = &-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}} \\ &+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}} \\ &-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}} \\ &+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}\end{aligned}$$

```
[In] Int[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

```
[Out] -1/2*((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] -
 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(Sqrt[a]*Sqrt[c]*Sqrt[2
*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sq
rt[a]*Sqrt[c] + Sqrt[a*c]] + 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c
]]])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) - ((A - (Sqrt[
a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c
]*x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + ((A - (Sqrt[a]*B)
/Sqrt[c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2
])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}\,dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}\,dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}\\
&= \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}\,dx}{4c}+\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}\,dx}{4c}\\
&\quad -\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+2x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+2x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}\\
&= -\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}\\
&\quad +\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}\\
&\quad -\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{a}\sqrt{c}-\sqrt{ac}}{c}-x^2}\,dx,x,-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+2x\right)}{2c}\\
&\quad -\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-\frac{2\sqrt{a}\sqrt{c}-\sqrt{ac}}{c}-x^2}\,dx,x,\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+2x\right)}{2c}
\end{aligned}
$$

$$= -\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}} + \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}$$
$$- \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$
$$+ \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx$$
$$= \frac{\text{RootSum}\left[a^2+ac\#1^4+c^2\#1^8\&,\ \frac{aA\log(x-\#1)+aB\log(x-\#1)\#1^2+A\sqrt{ac}\log(x-\#1)\#1^2+Ac\log(x-\#1)\#1^4+B\sqrt{ac}\log}{a\#1^3+2c\#1^7}\right.}{4c}$$

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

```
[Out] RootSum[a^2 + a*c*#1^4 + c^2*#1^8 & , (a*A*Log[x - #1] + a*B*Log[x - #1]*#1
^2 + A*Sqrt[a*c]*Log[x - #1]*#1^2 + A*c*Log[x - #1]*#1^4 + B*Sqrt[a*c]*Log[
x - #1]*#1^4 + B*c*Log[x - #1]*#1^6)/(a*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.84

method	result
default	$\frac{-\frac{\left(-B\sqrt{3}\,(ac)^{\frac{3}{4}}a^{\frac{3}{2}}+A\sqrt{3}\,(ac)^{\frac{3}{4}}\sqrt{c}\,a\right)\ln\left(x\sqrt{3}\,(ac)^{\frac{1}{4}}-x^2\sqrt{c}-\sqrt{a}\right)}{2\sqrt{c}}+\frac{2\left(-3Ac\,a^2+\frac{\left(-B\sqrt{3}\,(ac)^{\frac{3}{4}}a^{\frac{3}{2}}+A\sqrt{3}\,(ac)^{\frac{3}{4}}\sqrt{c}\,a\right)\sqrt{3}\,(ac)^{\frac{1}{4}}}{2\sqrt{c}}\right)\arctan\left(\frac{\sqrt{3}\,(ac}{\sqrt{4\sqrt{a}}}\right.}{\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}}}{6a^{\frac{5}{2}}c}$

```
[In] int((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^(5/2)/c*(-1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c
^(1/2)*a)/c^(1/2)*ln(x*3^(1/2)*(a*c)^(1/4)-x^2*c^(1/2)-a^(1/2))+2*(-3*A*c*a
^2+1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c^(1/2)*a)*3^(
1/2)*(a*c)^(1/4)/c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((3
^(1/2)*(a*c)^(1/4)-2*x*c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))+1
/6/a^(5/2)/c*(1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c^(
```

```
1/2)*a)/c^(1/2)*ln(x^2*c^(1/2)+x*3^(1/2)*(a*c)^(1/4)+a^(1/2))+2*(3*A*c*a^2-
1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c^(1/2)*a)*3^(1/2
)*(a*c)^(1/4)/c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((2*x*
c^(1/2)+3^(1/2)*(a*c)^(1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(289) = 578$.

Time = 0.60 (sec) , antiderivative size = 1457, normalized size of antiderivative = 3.52

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx=\text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log
(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3
)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3
*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*
sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c
^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*s
qrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^
3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A
^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3)*(2*B^3*a^4*c^2
 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A
^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt(-(3*sqrt(1/
3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c
 + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)
*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c -
 (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqr
t(1/6)*(A*B^4*a^3*c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)
*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^
4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(
B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)
*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*s
qrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*
c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 -
2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1
/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^
```

```
2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c
^2)))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A + Bx^2}{a - \sqrt{ac}x^2 + cx^4}\,dx = \text{Exception raised: PolynomialError}$$

```
[In] integrate((B*x**2+A)/(a+c*x**4-x**2*(a*c)**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(64*_t**4*a*c**3 - 16*_t**2*B**2*c*s
qrt(a*c) + B**4) contains an element of the set of generators.
```

Maxima [F]

$$\int \frac{A + Bx^2}{a - \sqrt{ac}x^2 + cx^4}\,dx = \int \frac{Bx^2 + A}{cx^4 - \sqrt{ac}x^2 + a}\,dx$$

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx^2}{a - \sqrt{ac}x^2 + cx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 3285, normalized size of antiderivative = 7.93

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(a + c*x^4 - x^2*(a*c)^(1/2)),x)
```

```
[Out] - atan(((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B
^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) -
A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*
(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c
*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c
^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)
 + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3
*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(
3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/
(72*a^3*c^3))^(1/2)*1i - (((12*A*a)/c^2 + (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*
(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2
*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/
2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(B^2*a*(-27*a^3*c
^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/
2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(7
2*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*
(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2
) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^
2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)*1i)/((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^
(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3
*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^
2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-
(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2)
- A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*
c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a
*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) -
 B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)
^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (((12*A*a)/c^2 + (2
*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) -
A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*
a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^
(1/2))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*
a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2
) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*
c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^
3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A
```

```
^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*(B
^3*a + A^2*B*c + A*B^2*(a*c)^(1/2)))/c^4))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A
^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a
^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(
1/2)*2i - atan(((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/
2))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^
(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^
2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2)
 - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A
*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3
))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*
(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c
*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)
^(1/2))/(72*a^3*c^3))^(1/2)*1i - (((12*A*a)/c^2 + (2*x*(4*c*(a*c)^(3/2) - 1
6*a*c^2*(a*c)^(1/2))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/
2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(
a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(A^2*c*(-
27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(
a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(
1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)
))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a
*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) +
4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)*1i)/((((12*A*a)/c^2 - (2*x*(4*
c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*
(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^
2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))
/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c
)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*
B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*
A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)
^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c
^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (((12*A*a)/
c^2 + (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(A^2*c*(-27*a^3*c^3)^
(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) +
 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^
3*c^3))^(1/2))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2
) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a
*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2
 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a
*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c
^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)
 + (2*(B^3*a + A^2*B*c + A*B^2*(a*c)^(1/2)))/c^4))*(-(A^2*c*(-27*a^3*c^3)^(
1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) +
12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3
*c^3))^(1/2)*2i
```

3.112 $\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 234

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = -\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} - \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}} + \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

```
[Out] -1/12*ln(-a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c^(1/2))*(A-B*a^(1/2)/c^(1/
2))/a^(3/4)/c^(1/4)*3^(1/2)+1/12*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c
^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^(3/4)/c^(1/4)*3^(1/2)+1/2*arctan(2*c^(1/4)*
x/a^(1/4)-3^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(3/4)/c^(3/4)+1/2*arctan(2*c^(1/
4)*x/a^(1/4)+3^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(3/4)/c^(3/4)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1183, 648, 631, 210, 642}

$$\begin{aligned}\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx=&-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}\\&+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}c^{3/4}}\\&-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}\\&+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}\end{aligned}$$

```
[In] Int[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

```
[Out] -1/2*((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] - (2*c^(1/4)*x)/a^(1/4)])/(a^(
3/4)*c^(3/4)) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] + (2*c^(1/4)*x)/a^(
1/4)])/(2*a^(3/4)*c^(3/4)) - ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[
3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4)*c^(1/4)) + ((A - (S
qrt[a]*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/
(4*Sqrt[3]*a^(3/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}}-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{2\sqrt{3}a^{3/4}\sqrt[4]{c}}+\frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}}+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{2\sqrt{3}a^{3/4}\sqrt[4]{c}}\\
&= \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c}+\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c}\\
&\quad+\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{-\frac{\sqrt{3}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{3}a^{3/4}c^{3/4}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{\frac{\sqrt{3}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}\\
&= \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}c^{3/4}}\\
&\quad+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}\\
&\quad+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}a^{3/4}c^{3/4}}\\
&\quad-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}a^{3/4}c^{3/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} \\
&\quad+\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}c^{3/4}} \\
&\quad+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.70

$$
\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \frac{\sqrt[4]{-1}\left(\frac{\left(\left(-i+\sqrt{3}\right)\sqrt{a}B-2iA\sqrt{c}\right)\arctan\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{-i+\sqrt{3}}\sqrt[4]{a}}\right)}{\sqrt{-i+\sqrt{3}}}-\frac{\left(\left(i+\sqrt{3}\right)\sqrt{a}B+2iA\sqrt{c}\right)\operatorname{arctanh}\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{i+\sqrt{3}}\sqrt[4]{a}}\right)}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}a^{3/4}c^{3/4}}
$$

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-I + Sqrt[3])*Sqrt[a]*B - (2*I)*A*Sqrt[c])*ArcTan[((1 + I)*
c^(1/4)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((I + Sqrt[
3])*Sqrt[a]*B + (2*I)*A*Sqrt[c])*ArcTanh[((1 + I)*c^(1/4)*x)/(Sqrt[I + Sqrt
[3]]*a^(1/4))])/Sqrt[I + Sqrt[3]]))/(Sqrt[6]*a^(3/4)*c^(3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4-\_Z^2\operatorname{RootOf}\left(\_Z^2-a,\text{index}=1\right)\operatorname{RootOf}\left(\_Z^2-c,\text{index}=1\right)+a\right)}\frac{\left(-B\_R^2-A\right)\ln\left(x-\_R\right)}{-2c\_R^3+\_R\sqrt{a}\sqrt{c}}\right)}{2}$
default	$\frac{\frac{\left(A\sqrt{a}\sqrt{3}\,c-B\sqrt{c}\sqrt{3}\,a\right)\ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{3}+\sqrt{a}+x^2\sqrt{c}\right)}{2\sqrt{c}}+\frac{2\left(3Ac^{\frac{3}{4}}a^{\frac{3}{4}}-\frac{\left(A\sqrt{a}\sqrt{3}\,c-B\sqrt{c}\sqrt{3}\,a\right)a^{\frac{1}{4}}\sqrt{3}}{2c^{\frac{1}{4}}}\right)\arctan\left(\frac{a^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{3}+2x\sqrt{c}}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}}}{6c^{\frac{3}{4}}a^{\frac{5}{4}}}+\frac{\left(-A\sqrt{a}\sqrt{3}\,c+\right.}{}$

```
[In] int((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((-B*_R^2-A)/(-2*c*_R^3+_R*a^(1/2)*c^(1/2))*ln(x-_R),_R=RootOf(c*_Z^
4-_Z^2*RootOf(_Z^2-a,index=1)*RootOf(_Z^2-c,index=1)+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(160) = 320$.

Time = 0.98 (sec) , antiderivative size = 1469, normalized size of antiderivative = 6.28

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2
))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - (A
^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) - sqrt(1/3)
*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/
(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)))
 + 1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c
^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
(A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt
(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) - sqrt(1/
3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2
)/(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
)) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
 + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*
c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
 (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqr
t(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) + sqrt(1
/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^3*c^3)))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
)) + 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
 + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*
c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
 (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqr
t(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) + sqrt(1
/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
```

```
2)/(a^3*c^3)))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \text{Exception raised: PolynomialError}$$

```
[In] integrate((B*x**2+A)/(a+c*x**4-x**2*a**(1/2)*c**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(64*_t**4*a*c**5 - 16*_t**2*B**2*sqr
t(a)*c**(7/2) + B**4*c**2) contains an element of the set of generators.
```

Maxima [F]

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \int \frac{Bx^2+A}{cx^4-\sqrt{a}\sqrt{c}x^2+a}\,dx$$

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a)*sqrt(c)*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 1575, normalized size of antiderivative = 6.73

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^2)/(a + c*x^4 - a^(1/2)*c^(1/2)*x^2),x)
```

```
[Out] - 2*atanh((6*A^2*x*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2
)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(
24*a^(3/2)*c^(1/2)))^(1/2))/((2*A^2*B)/c - (2*B^3*a)/c^2 + A^3/(a^(1/2)*c^(
1/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^2) - (A*B^2*a^(1/2))/c^(3/2) - (
A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^3)) - (6*B^2*a*x*((B^2*(-27*a^3*c^3)^(1/2
))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*
c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a^(3/2)*c^(1/2)))^(1/2))/(2*A^2*B - (2*B
^3*a)/c + (A^3*c^(1/2))/a^(1/2) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c) - (A*
B^2*a^(1/2))/c^(1/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^2)) - (2*A^2*x*(-
27*a^3*c^3)^(1/2)*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)
*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(2
4*a^(3/2)*c^(1/2)))^(1/2))/(3*a^(3/2)*c^(7/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^
4 + A^3/(a^(1/2)*c^(5/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*
a^(1/2))/c^(7/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))) + (2*B^2*x*(-27*
a^3*c^3)^(1/2)*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^
(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a
^(3/2)*c^(1/2)))^(1/2))/(3*a^(1/2)*c^(9/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^4 +
 A^3/(a^(1/2)*c^(5/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*a^(
1/2))/c^(7/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))))*((B^2*(-27*a^3*c^3
)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-2
7*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a^(3/2)*c^(1/2)))^(1/2) - 2*atanh(
(6*A^2*x*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2))
 - (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72
*a^2*c^3))^(1/2))/((2*A^2*B)/c - (2*B^3*a)/c^2 + A^3/(a^(1/2)*c^(1/2)) - (A
^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^2) - (A*B^2*a^(1/2))/c^(3/2) + (A*B^2*(-27
*a^3*c^3)^(1/2))/(3*a*c^3)) - (6*B^2*a*x*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3
*c^2) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2))
 - (B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3))^(1/2))/(2*A^2*B - (2*B^3*a)/c +
(A^3*c^(1/2))/a^(1/2) - (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c) - (A*B^2*a^(1/2
))/c^(1/2) + (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^2)) + (2*A^2*x*(-27*a^3*c^3
)^(1/2)*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2))
- (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72*
a^2*c^3))^(1/2))/(3*a^(3/2)*c^(7/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^4 + A^3/(a
^(1/2)*c^(5/2)) - (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*a^(1/2))/c
^(7/2) + (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))) - (2*B^2*x*(-27*a^3*c^3)^(
1/2)*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2)) - (
```

```
A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72*a^2
*c^3))^(1/2))/(3*a^(1/2)*c^(9/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^4 + A^3/(a^(1
/2)*c^(5/2)) - (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*a^(1/2))/c^(7
/2) + (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))))*((A^2*(-27*a^3*c^3)^(1/2))/(
72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^
(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3))^(1/2)
```

3.113 $\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = -\sqrt{\frac{1}{2}\left(-1+\sqrt{13}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)\middle|\frac{1}{6}\left(-7-\sqrt{13}\right)\right) + \sqrt{7+2\sqrt{13}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(1+13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))*(
-2+2*13^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(1+13^(1/2))^(1/2),1/6*I*3^(1/2)+1
/6*I*39^(1/2))*(7+2*13^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1194, 538, 435, 430}

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = \sqrt{7+2\sqrt{13}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7-\sqrt{13}\right)\right) - \sqrt{\frac{1}{2}\left(\sqrt{13}-1\right)}E\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)\middle|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

```
[In] Int[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

```
[Out] -(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]*x], (-7 -
 Sqrt[13])/6]) + Sqrt[7 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[13]
)]*x], (-7 - Sqrt[13])/6]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{3-x^2}{\sqrt{1+\sqrt{13}-2x^2}\sqrt{-1+\sqrt{13}+2x^2}}\,dx \\
&= \left(5+\sqrt{13}\right)\int \frac{1}{\sqrt{1+\sqrt{13}-2x^2}\sqrt{-1+\sqrt{13}+2x^2}}\,dx-\int \frac{\sqrt{-1+\sqrt{13}+2x^2}}{\sqrt{1+\sqrt{13}-2x^2}}\,dx \\
&= -\sqrt{\frac{1}{2}\left(-1+\sqrt{13}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right) \\
&\quad +\sqrt{7+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx =$$

$$-\frac{i\Big(\left(1+\sqrt{13}\right)E\Big(i\text{arcsinh}\Big(\sqrt{\tfrac{2}{-1+\sqrt{13}}}x\Big)\,|\tfrac{1}{6}\left(-7+\sqrt{13}\right)\Big)-\left(-5+\sqrt{13}\right)\text{EllipticF}\,\Big(i\text{arcsinh}\Big(\sqrt{\tfrac{2}{-1+\sqrt{13}}}x}{\sqrt{2\left(1+\sqrt{13}\right)}}$$

```
[In] Integrate[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

```
[Out] ((-I)*((1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]*x], (-7 +
 Sqrt[13])/6] - (-5 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]
*x], (-7 + Sqrt[13])/6]))/Sqrt[2*(1 + Sqrt[13])]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(74) = 148.

Time = 2.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.08

method	result
default	$\frac{18\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-6+6\sqrt{13}}}{6},\frac{i\sqrt{3}}{6}+\frac{i\sqrt{39}}{6}\right)}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x^4+x^2+3}}+\frac{36\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\Big(F\Big(\frac{x\sqrt{-}}{}}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x}}$
elliptic	$\frac{18\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-6+6\sqrt{13}}}{6},\frac{i\sqrt{3}}{6}+\frac{i\sqrt{39}}{6}\right)}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x^4+x^2+3}}+\frac{36\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\Big(F\Big(\frac{x\sqrt{-}}{}}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x}}$

```
[In] int((-x^2+3)/(-x^4+x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 18/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^
(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)*EllipticF(1/6*x*(-6+6*13^(1/2))^(1/2),
1/6*I*3^(1/2)+1/6*I*39^(1/2))+36/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2
))*x^2)^(1/2)*(1-(-1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)/(1+13^(1
/2))*(EllipticF(1/6*x*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))-E
llipticE(1/6*x*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = \frac{-2i\sqrt{2}x\sqrt{\sqrt{13}+1}F\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}+1}}{2x}\right)\mid\frac{1}{6}\sqrt{13}-\frac{7}{6}\right)+\left(i\sqrt{13}\sqrt{2}x+i\sqrt{2}x\right)\sqrt{\sqrt{13}+1}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{1}}}{2x}\right.\right.}{4x}$$

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(-2*I*sqrt(2)*x*sqrt(sqrt(13) + 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(s
qrt(13) + 1)/x), 1/6*sqrt(13) - 7/6) + (I*sqrt(13)*sqrt(2)*x + I*sqrt(2)*x)
*sqrt(sqrt(13) + 1)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) + 1)/x), 1/
6*sqrt(13) - 7/6) + 4*sqrt(-x^4 + x^2 + 3))/x
```

Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4+x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4+x^2+3}}\right)dx$$

```
[In] integrate((-x**2+3)/(-x**4+x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + x**2 + 3), x) - Integral(-3/sqrt(-x**4 + x**2 +
 3), x)
```

Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

Giac [F]

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx=\int -\frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx=-\int \frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

```
[In] int(-(x^2 - 3)/(x^2 - x^4 + 3)^(1/2),x)
[Out] -int((x^2 - 3)/(x^2 - x^4 + 3)^(1/2), x)
```

3.114 $\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} \, dx = -E\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)+4\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-3\right)$$

```
[Out] -EllipticE(1/3*x*3^(1/2),I*3^(1/2))+4*EllipticF(1/3*x*3^(1/2),I*3^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1194, 21, 434, 435, 430}

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} \, dx = 4\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-3\right)-E\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

```
[In] Int[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

```
[Out] -EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]], -3]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\int \frac{3-x^2}{\sqrt{6-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \int \frac{\sqrt{6-2x^2}}{\sqrt{2+2x^2}}\,dx \\
&= 8\int \frac{1}{\sqrt{6-2x^2}\sqrt{2+2x^2}}\,dx - \int \frac{\sqrt{2+2x^2}}{\sqrt{6-2x^2}}\,dx \\
&= -E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right) + 4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx = -i\sqrt{3}E\left(i\text{arcsinh}(x)\middle|-\frac{1}{3}\right)$$

```
[In] Integrate[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
[Out] (-I)*Sqrt[3]*EllipticE[I*ArcSinh[x], -1/3]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(31) = 62$.

Time = 2.02 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.52

method	result	size
default	$\frac{\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)}{\sqrt{-x^{4}+2x^{2}+3}}+\frac{\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)-E\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\right)}{3\sqrt{-x^{4}+2x^{2}+3}}$	113
elliptic	$\frac{\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)}{\sqrt{-x^{4}+2x^{2}+3}}+\frac{\sqrt{3}\,\sqrt{-3x^{2}+9}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)-E\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\right)}{3\sqrt{-x^{4}+2x^{2}+3}}$	113

```
[In] int((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x
*3^(1/2),I*3^(1/2))+1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+
3)^(1/2)*(EllipticF(1/3*x*3^(1/2),I*3^(1/2))-EllipticE(1/3*x*3^(1/2),I*3^(1
/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{3-x^{2}}{\sqrt{3+2x^{2}-x^{4}}}\,dx = \frac{3i\sqrt{3}xE\left(\arcsin\left(\frac{\sqrt{3}}{x}\right)\mid -\frac{1}{3}\right)-2i\sqrt{3}xF\left(\arcsin\left(\frac{\sqrt{3}}{x}\right)\mid -\frac{1}{3}\right)+\sqrt{-x^{4}+2\,x^{2}+3}}{x}$$

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] (3*I*sqrt(3)*x*elliptic_e(arcsin(sqrt(3)/x), -1/3) - 2*I*sqrt(3)*x*elliptic
_f(arcsin(sqrt(3)/x), -1/3) + sqrt(-x^4 + 2*x^2 + 3))/x
```

Sympy [F]

$$\int \frac{3-x^{2}}{\sqrt{3+2x^{2}-x^{4}}}\,dx = -\int \frac{x^{2}}{\sqrt{-x^{4}+2x^{2}+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^{4}+2x^{2}+3}}\right)dx$$

```
[In] integrate((-x**2+3)/(-x**4+2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 2*x*
*2 + 3), x)
```

Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

```
[In] int(-(x^2 - 3)/(2*x^2 - x^4 + 3)^(1/2),x)
[Out] int(-(x^2 - 3)/(2*x^2 - x^4 + 3)^(1/2), x)
```

3.115 $\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = -\sqrt{\frac{1}{2}\left(-3+\sqrt{21}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right) + \sqrt{9+2\sqrt{21}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(3+21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))*(-
6+2*21^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(3+21^(1/2))^(1/2),1/2*I*3^(1/2)+1/
2*I*7^(1/2))*(9+2*21^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1194, 538, 435, 430}

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = \sqrt{9+2\sqrt{21}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5-\sqrt{21}\right)\right) - \sqrt{\frac{1}{2}\left(\sqrt{21}-3\right)}E\left(\arcsin\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

```
[In] Int[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

```
[Out] -(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]*x], (-5 -
 Sqrt[21])/2]) + Sqrt[9 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(3 + Sqrt[21]
)]*x], (-5 - Sqrt[21])/2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\int \frac{3-x^2}{\sqrt{3+\sqrt{21}-2x^2}\sqrt{-3+\sqrt{21}+2x^2}}\,dx \\
&= \left(3+\sqrt{21}\right)\int \frac{1}{\sqrt{3+\sqrt{21}-2x^2}\sqrt{-3+\sqrt{21}+2x^2}}\,dx-\int \frac{\sqrt{-3+\sqrt{21}+2x^2}}{\sqrt{3+\sqrt{21}-2x^2}}\,dx \\
&= -\sqrt{\frac{1}{2}\left(-3+\sqrt{21}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right) \\
&\quad+\frac{1}{2}\sqrt{36+8\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx =$$

$$-\frac{i\left(\left(3+\sqrt{21}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\left(-3+\sqrt{21}\right)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)\right.\right.}{\sqrt{2\left(3+\sqrt{21}\right)}}$$

```
[In] Integrate[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

```
[Out] ((-I)*((3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]*x], (-5 +
 Sqrt[21])/2] - (-3 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]
*x], (-5 + Sqrt[21])/2]))/Sqrt[2*(3 + Sqrt[21])]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(74) = 148.

Time = 2.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.12

method	result
default	$\frac{18\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-18+6\sqrt{21}}}{6},\frac{i\sqrt{3}}{2}+\frac{i\sqrt{7}}{2}\right)}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x^4+3x^2+3}}+\frac{36\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{-18}}{6}\right.\right.}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x^4+}}$
elliptic	$\frac{18\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-18+6\sqrt{21}}}{6},\frac{i\sqrt{3}}{2}+\frac{i\sqrt{7}}{2}\right)}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x^4+3x^2+3}}+\frac{36\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{-18}}{6}\right.\right.}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x^4+}}$

```
[In] int((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 18/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21
^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*21^(1/2))^(1
/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))+36/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^
(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)/(3
+21^(1/2))*(EllipticF(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1
/2))-EllipticE(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx$$
$$= \frac{-6i\sqrt{2}x\sqrt{\sqrt{21}+3}F(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{21}+3}}{2x}\right) \mid \frac{1}{2}\sqrt{21}-\frac{5}{2}) + \left(i\sqrt{21}\sqrt{2}x+3i\sqrt{2}x\right)\sqrt{\sqrt{21}+3}E(\arcsin(\sqrt{2}}{4x}$$

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(-6*I*sqrt(2)*x*sqrt(sqrt(21) + 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(s
qrt(21) + 3)/x), 1/2*sqrt(21) - 5/2) + (I*sqrt(21)*sqrt(2)*x + 3*I*sqrt(2)*
x)*sqrt(sqrt(21) + 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(21) + 3)/x),
1/2*sqrt(21) - 5/2) + 4*sqrt(-x^4 + 3*x^2 + 3))/x
```

Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4+3x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4+3x^2+3}}\right)dx$$

```
[In] integrate((-x**2+3)/(-x**4+3*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 3*x*
*2 + 3), x)
```

Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

Giac [F]

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx=\int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx=\int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

```
[In] int(-(x^2 - 3)/(3*x^2 - x^4 + 3)^(1/2),x)
[Out] int(-(x^2 - 3)/(3*x^2 - x^4 + 3)^(1/2), x)
```

3.116 $\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = -\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right) + \sqrt{5+2\sqrt{13}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(-1+13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))*
(2+2*13^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(-1+13^(1/2))^(1/2),1/6*I*39^(1/2)
-1/6*I*3^(1/2))*(5+2*13^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1194, 538, 435, 430}

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = \sqrt{5+2\sqrt{13}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7+\sqrt{13}\right)\right) - \sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

```
[In] Int[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

```
[Out] -(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]*x], (-7 +
 Sqrt[13])/6]) + Sqrt[5 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[13
])]*x], (-7 + Sqrt[13])/6]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{3-x^2}{\sqrt{-1+\sqrt{13}-2x^2}\sqrt{1+\sqrt{13}+2x^2}}\,dx \\
&= \left(7+\sqrt{13}\right)\int \frac{1}{\sqrt{-1+\sqrt{13}-2x^2}\sqrt{1+\sqrt{13}+2x^2}}\,dx-\int \frac{\sqrt{1+\sqrt{13}+2x^2}}{\sqrt{-1+\sqrt{13}-2x^2}}\,dx \\
&= -\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right) \\
&\quad+\sqrt{5+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = -\frac{i\left(\left(-1+\sqrt{13}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)\middle|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)-\left(-7+\sqrt{13}\right)\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right),-\right.\right.}{\sqrt{2\left(-1+\sqrt{13}\right)}}$$

```
[In] Integrate[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

```
[Out] ((-I)*((-1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]*x], -7/6
- Sqrt[13]/6] - (-7 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]*
x], -7/6 - Sqrt[13]/6]))/Sqrt[2*(-1 + Sqrt[13])]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(74) = 148.

Time = 2.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.22

method	result
default	$\frac{18\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\frac{i\sqrt{39}}{6}-\frac{i\sqrt{3}}{6}\right)}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}}+\frac{36\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\right.\right.}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}\,\left(\right.}$
elliptic	$\frac{18\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\frac{i\sqrt{39}}{6}-\frac{i\sqrt{3}}{6}\right)}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}}+\frac{36\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\right.\right.}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}\,\left(\right.}$

```
[In] int((-x^2+3)/(-x^4-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 18/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1/
2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)*EllipticF(1/6*x*(6+6*13^(1/2))^(1/2),1/6*
I*39^(1/2)-1/6*I*3^(1/2))+36/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2
)^(1/2)*(1-(1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)/(-1+13^(1/2))*(
EllipticF(1/6*x*(6+6*13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))-Elliptic
E(1/6*x*(6+6*13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = \frac{2i\sqrt{2}x\sqrt{\sqrt{13}-1}F(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-1}}{2x}\right) | -\frac{1}{6}\sqrt{13}-\frac{7}{6}) + \left(i\sqrt{13}\sqrt{2}x - i\sqrt{2}x\right)\sqrt{\sqrt{13}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{1}}}{2x}\right.}{4x}$$

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*sqrt(2)*x*sqrt(sqrt(13) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sq
rt(13) - 1)/x), -1/6*sqrt(13) - 7/6) + (I*sqrt(13)*sqrt(2)*x - I*sqrt(2)*x)
*sqrt(sqrt(13) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 1)/x), -1
/6*sqrt(13) - 7/6) + 4*sqrt(-x^4 - x^2 + 3))/x
```

Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4-x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4-x^2+3}}\right)dx$$

```
[In] integrate((-x**2+3)/(-x**4-x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - x**2 + 3), x) - Integral(-3/sqrt(-x**4 - x**2 +
 3), x)
```

Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

Giac [F]

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

```
[In] int(-(x^2 - 3)/(3 - x^4 - x^2)^(1/2),x)
[Out] int(-(x^2 - 3)/(3 - x^4 - x^2)^(1/2), x)
```

3.117 $\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx=-\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)+2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)$$

```
[Out] -EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+2*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1194, 538, 435, 430}

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx=2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)$$

```
[In] Int[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] -(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/3
]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{3-x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx\\ &= 12\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx - \int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx\\ &= -\sqrt{3}E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right) + 2\sqrt{3}F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = -i\left(E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right) + 2\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)\right)$$

```
[In] Integrate[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
[Out] (-I)*(EllipticE[I*ArcSinh[x/Sqrt[3]], -3] + 2*EllipticF[I*ArcSinh[x/Sqrt[3]
], -3])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(27) = 54$.

Time = 1.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.52

method	result	size
default	$\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{\sqrt{-x^{4}-2x^{2}+3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{\sqrt{-x^{4}-2x^{2}+3}}$	95
elliptic	$\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{\sqrt{-x^{4}-2x^{2}+3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{3x^{2}+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{\sqrt{-x^{4}-2x^{2}+3}}$	95

```
[In] int((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1/
2))+(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*
3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = \frac{i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|{-3}\right)+2i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|{-3}\right)+\sqrt{-x^4-2\,x^2+3}}{x}$$

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] (I*x*elliptic_e(arcsin(1/x), -3) + 2*I*x*elliptic_f(arcsin(1/x), -3) + sqrt
(-x^4 - 2*x^2 + 3))/x
```

Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4-2x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4-2x^2+3}}\right)dx$$

```
[In] integrate((-x**2+3)/(-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 2*x*
*2 + 3), x)
```

Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

Giac [F]

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

```
[In] int(-(x^2 - 3)/(3 - x^4 - 2*x^2)^(1/2),x)
[Out] int(-(x^2 - 3)/(3 - x^4 - 2*x^2)^(1/2), x)
```

3.118 $\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = -\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right) + \sqrt{3+2\sqrt{21}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5+\sqrt{21}\right)\right)$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(-3+21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))*(
6+2*21^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(-3+21^(1/2))^(1/2),1/2*I*7^(1/2)-1
/2*I*3^(1/2))*(3+2*21^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1194, 538, 435, 430}

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = \sqrt{3+2\sqrt{21}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5+\sqrt{21}\right)\right) - \sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)$$

```
[In] Int[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] -(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21])]*x], (-5 +
 Sqrt[21])/2]) + Sqrt[3 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(-3 + Sqrt[21
])]*x], (-5 + Sqrt[21])/2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{3-x^2}{\sqrt{-3+\sqrt{21}-2x^2}\sqrt{3+\sqrt{21}+2x^2}}\,dx \\
&= \left(9+\sqrt{21}\right)\int \frac{1}{\sqrt{-3+\sqrt{21}-2x^2}\sqrt{3+\sqrt{21}+2x^2}}\,dx-\int \frac{\sqrt{3+\sqrt{21}+2x^2}}{\sqrt{-3+\sqrt{21}-2x^2}}\,dx \\
&= -\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right) \\
&\quad+\sqrt{3+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = -\frac{i\Big(\left(-3+\sqrt{21}\right) E\Big(i\text{arcsinh}\Big(\sqrt{\frac{2}{3+\sqrt{21}}}x\Big)\,|-\frac{5}{2}-\frac{\sqrt{21}}{2}\Big)-\left(-9+\sqrt{21}\right)\text{EllipticF}\left(i\text{arcsinh}\Big(\sqrt{\frac{2}{3+\sqrt{21}}}x\Big)\right.,-\frac{5}{2}}{\sqrt{2\left(-3+\sqrt{21}\right)}}$$

```
[In] Integrate[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] ((-I)*((-3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]*x], -5/2
- Sqrt[21]/2] - (-9 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]*
x], -5/2 - Sqrt[21]/2]))/Sqrt[2*(-3 + Sqrt[21])]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(74) = 148.

Time = 2.05 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.22

method	result
default	$\frac{18\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,F\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{i\sqrt{7}}{2}-\frac{i\sqrt{3}}{2}\right)}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}}+\frac{36\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{i\sqrt{}}{2}\right.\right.}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}\,(-}$
elliptic	$\frac{18\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,F\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{i\sqrt{7}}{2}-\frac{i\sqrt{3}}{2}\right)}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}}+\frac{36\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{i\sqrt{}}{2}\right.\right.}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}\,(-}$

```
[In] int((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 18/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1
/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*21^(1/2))^(1/2),
1/2*I*7^(1/2)-1/2*I*3^(1/2))+36/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))
*x^2)^(1/2)*(1-(1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)/(-3+21^(1
/2))*(EllipticF(1/6*x*(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))-El
lipticE(1/6*x*(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = \frac{6i\sqrt{2}x\sqrt{\sqrt{21}-3}F\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{21}-3}}{2x}\right)\middle|-\frac{1}{2}\sqrt{21}-\frac{5}{2}\right)+\left(i\sqrt{21}\sqrt{2}x-3i\sqrt{2}x\right)\sqrt{\sqrt{21}-3}E(\arcsin}{4x}$$

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*I*sqrt(2)*x*sqrt(sqrt(21) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sq
rt(21) - 3)/x), -1/2*sqrt(21) - 5/2) + (I*sqrt(21)*sqrt(2)*x - 3*I*sqrt(2)*
x)*sqrt(sqrt(21) - 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(21) - 3)/x),
-1/2*sqrt(21) - 5/2) + 4*sqrt(-x^4 - 3*x^2 + 3))/x
```

Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4-3x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4-3x^2+3}}\right)dx$$

```
[In] integrate((-x**2+3)/(-x**4-3*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 3*x*
*2 + 3), x)
```

Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

Giac [F]

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

```
[In] int(-(x^2 - 3)/(3 - x^4 - 3*x^2)^(1/2),x)
[Out] int(-(x^2 - 3)/(3 - x^4 - 3*x^2)^(1/2), x)
```

3.119 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 296

$$
\begin{aligned}
\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2} \\
&\quad - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}} \\
&\quad + \frac{\left(b+2\sqrt{a}\sqrt{c}-\sqrt{b^2-4ac}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

```
[Out] 2*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/(a^(1/2)+x^2*c^(1/2))-2*a^(1/4)*c^(1/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/2)-(-4*a*c+
b^2)^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)
/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1211, 1117, 1209}

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}$$

```
[In] Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) - (2*a^(1/4)*
c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2 - 4*a*c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(2\sqrt{a}\sqrt{c}\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx\right)\\&\quad+\left(b+2\sqrt{a}\sqrt{c}-\sqrt{b^2-4ac}\right)\int\frac{1}{\sqrt{a+bx^2+cx^4}}\,dx\\&=\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\\&\quad-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}}\\&\quad+\frac{\left(b+2\sqrt{a}\sqrt{c}-\sqrt{b^2-4ac}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.63

$$\int\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx=-\frac{2i\sqrt{2}a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^2+cx^4}}$$

```
[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^2 + c*x^4]
```

Maple [A] (verified)

Time = 7.02 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.74

method	result
default	$\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{ca\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-\right.}{}}}{}$
elliptic	$-\frac{\left(-2c\,x^{2}+\sqrt{-4ac+b^{2}}-b\right)\sqrt{-(c\,x^{4}+b\,x^{2}+a)(4ac-b^{2})}\left(\frac{\left(4ac-b^{2}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)x^{2}}{a\left(4ac-b^{2}\right)}}\,\sqrt{4+\frac{2\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}}+4abc-b^{3}\right.}{a\left(4ac-b^{2}\right)}}}{4\sqrt{\frac{\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}}{a\left(4ac-b^{2}\right)}}\,\sqrt{-4}}\right.}{}$

```
[In] int((b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))-1/4*(-4*a*c+b^2)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.09

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{2\sqrt{\frac{1}{2}}\left(acx\sqrt{\frac{b^2-4\,ac}{c^2}}-abx\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right) \mid \frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac}) - \sqrt{\frac{1}{2}}\Bigl(\sqrt{b^2 -}}{}$$

```
[In] integrate((b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/2*(2*sqrt(1/2)*(a*c*x*sqrt((b^2 - 4*a*c)/c^2) - a*b*x)*sqrt(c)*sqrt((c*sq
rt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/
(a*c)) - sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x - (2*a*b + b^2)*x + ((2*a - b)*c*
x + sqrt(b^2 - 4*a*c)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*
a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)
) + 4*sqrt(c*x^4 + b*x^2 + a)*a*c)/(a*c*x)
```

Sympy [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{b+2cx^2-\sqrt{-4ac+b^2}}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((b+2*c*x**2-(-4*a*c+b**2)**(1/2))/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{b+2\,c\,x^2-\sqrt{b^2-4\,a\,c}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.120 $\int (d+ex^2)^4 (a+cx^4)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 106

$$\int (d+ex^2)^4 (a+cx^4)\, dx = ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2(cd^2+6ae^2)\, x^5+\frac{4}{7}de(cd^2+ae^2)\, x^7 +\frac{1}{9}e^2(6cd^2+ae^2)\, x^9+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

```
[Out] a*d^4*x+4/3*a*d^3*e*x^3+1/5*d^2*(6*a*e^2+c*d^2)*x^5+4/7*d*e*(a*e^2+c*d^2)*x
^7+1/9*e^2*(a*e^2+6*c*d^2)*x^9+4/11*c*d*e^3*x^11+1/13*c*e^4*x^13
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1168}

$$\int (d+ex^2)^4 (a+cx^4)\, dx = \frac{1}{9}e^2x^9(ae^2+6cd^2)+\frac{4}{7}dex^7(ae^2+cd^2)+\frac{1}{5}d^2x^5(6ae^2+cd^2) +ad^4x+\frac{4}{3}ad^3ex^3+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

```
[In] Int[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2
 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c
*e^4*x^13)/13
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad^4+4ad^3ex^2+d^2\left(cd^2+6ae^2\right)x^4+4de\left(cd^2+ae^2\right)x^6+e^2\left(6cd^2+ae^2\right)x^8\right.\\&\qquad\left.+4cde^3x^{10}+ce^4x^{12}\right)\,dx\\&= ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2\left(cd^2+6ae^2\right)x^5+\frac{4}{7}de\left(cd^2+ae^2\right)x^7\\&\quad+\frac{1}{9}e^2\left(6cd^2+ae^2\right)x^9+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(d+ex^2\right)^4\left(a+cx^4\right)\,dx &= ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2\left(cd^2+6ae^2\right)x^5+\frac{4}{7}de\left(cd^2+ae^2\right)x^7\\&\quad+\frac{1}{9}e^2\left(6cd^2+ae^2\right)x^9+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}\end{aligned}$$

```
[In] Integrate[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2
 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c
*e^4*x^13)/13
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

method	result
norman	$\frac{c\,e^4x^{13}}{13}+\frac{4cd\,e^3x^{11}}{11}+\left(\frac{1}{9}e^4a+\frac{2}{3}e^2d^2c\right)x^9+\left(\frac{4}{7}d\,e^3a+\frac{4}{7}d^3ec\right)x^7+\left(\frac{6}{5}e^2d^2a+\frac{1}{5}d^4c\right)x^5+\frac{4a\,d^3e\,x^3}{3}$
default	$\frac{c\,e^4x^{13}}{13}+\frac{4cd\,e^3x^{11}}{11}+\frac{\left(e^4a+6e^2d^2c\right)x^9}{9}+\frac{\left(4d\,e^3a+4d^3ec\right)x^7}{7}+\frac{\left(6e^2d^2a+d^4c\right)x^5}{5}+\frac{4a\,d^3e\,x^3}{3}+a\,d^4x$
gosper	$\frac{1}{13}c\,e^4x^{13}+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{2}{3}x^9e^2d^2c+\frac{4}{7}x^7d\,e^3a+\frac{4}{7}x^7d^3ec+\frac{6}{5}x^5e^2d^2a+\frac{1}{5}x^5d^4c+\frac{4}{3}a$
risch	$\frac{1}{13}c\,e^4x^{13}+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{2}{3}x^9e^2d^2c+\frac{4}{7}x^7d\,e^3a+\frac{4}{7}x^7d^3ec+\frac{6}{5}x^5e^2d^2a+\frac{1}{5}x^5d^4c+\frac{4}{3}a$
parallelrisch	$\frac{1}{13}c\,e^4x^{13}+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{2}{3}x^9e^2d^2c+\frac{4}{7}x^7d\,e^3a+\frac{4}{7}x^7d^3ec+\frac{6}{5}x^5e^2d^2a+\frac{1}{5}x^5d^4c+\frac{4}{3}a$

```
[In] int((e*x^2+d)^4*(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c*e^4*x^13+4/11*c*d*e^3*x^11+(1/9*e^4*a+2/3*e^2*d^2*c)*x^9+(4/7*d*e^3*
a+4/7*d^3*e*c)*x^7+(6/5*e^2*d^2*a+1/5*d^4*c)*x^5+4/3*a*d^3*e*x^3+a*d^4*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right) \, dx = \frac{1}{13}\,ce^4x^{13} + \frac{4}{11}\,cde^3x^{11} + \frac{1}{9}\left(6\,cd^2e^2 + ae^4\right)x^9 + \frac{4}{3}\,ad^3ex^3 + \frac{4}{7}\left(cd^3e + ade^3\right)x^7 + ad^4x + \frac{1}{5}\left(cd^4 + 6\,ad^2e^2\right)x^5$$

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/3*a
*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2*e
^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right) \, dx = ad^4x + \frac{4ad^3ex^3}{3} + \frac{4cde^3x^{11}}{11} + \frac{ce^4x^{13}}{13} + x^9\left(\frac{ae^4}{9} + \frac{2cd^2e^2}{3}\right) + x^7 \cdot \left(\frac{4ade^3}{7} + \frac{4cd^3e}{7}\right) + x^5 \cdot \left(\frac{6ad^2e^2}{5} + \frac{cd^4}{5}\right)$$

```
[In] integrate((e*x**2+d)**4*(c*x**4+a),x)
```

```
[Out] a*d**4*x + 4*a*d**3*e*x**3/3 + 4*c*d*e**3*x**11/11 + c*e**4*x**13/13 + x**9
*(a*e**4/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 4*c*d**3*e/7) + x**5*(
6*a*d**2*e**2/5 + c*d**4/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right) \, dx = \frac{1}{13}\,ce^4x^{13} + \frac{4}{11}\,cde^3x^{11} + \frac{1}{9}\left(6\,cd^2e^2 + ae^4\right)x^9 + \frac{4}{3}\,ad^3ex^3 + \frac{4}{7}\left(cd^3e + ade^3\right)x^7 + ad^4x + \frac{1}{5}\left(cd^4 + 6\,ad^2e^2\right)x^5$$

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/3*a
*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2*e
^2)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right)\,dx = \frac{1}{13}ce^4x^{13}+\frac{4}{11}cde^3x^{11}+\frac{2}{3}cd^2e^2x^9+\frac{1}{9}ae^4x^9+\frac{4}{7}cd^3ex^7+\frac{4}{7}ade^3x^7+\frac{1}{5}cd^4x^5+\frac{6}{5}ad^2e^2x^5+\frac{4}{3}ad^3ex^3+ad^4x$$

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 2/3*c*d^2*e^2*x^9 + 1/9*a*e^4*x^9 + 4
/7*c*d^3*e*x^7 + 4/7*a*d*e^3*x^7 + 1/5*c*d^4*x^5 + 6/5*a*d^2*e^2*x^5 + 4/3*
a*d^3*e*x^3 + a*d^4*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right)\,dx = x^5\left(\frac{c\,d^4}{5}+\frac{6\,a\,d^2\,e^2}{5}\right)+x^9\left(\frac{2\,c\,d^2\,e^2}{3}+\frac{a\,e^4}{9}\right)+x^7\left(\frac{4\,c\,d^3\,e}{7}+\frac{4\,a\,d\,e^3}{7}\right)+\frac{c\,e^4\,x^{13}}{13}+a\,d^4\,x+\frac{4\,a\,d^3\,e\,x^3}{3}+\frac{4\,c\,d\,e^3\,x^{11}}{11}$$

```
[In] int((a + c*x^4)*(d + e*x^2)^4,x)
```

```
[Out] x^5*((c*d^4)/5 + (6*a*d^2*e^2)/5) + x^9*((a*e^4)/9 + (2*c*d^2*e^2)/3) + x^7
*((4*a*d*e^3)/7 + (4*c*d^3*e)/7) + (c*e^4*x^13)/13 + a*d^4*x + (4*a*d^3*e*x
^3)/3 + (4*c*d*e^3*x^11)/11
```

3.121 $\int \left(d+ex^2\right)^3 \left(a+cx^4\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 79

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)\,dx = ad^3x+ad^2ex^3+\frac{1}{5}d\left(cd^2+3ae^2\right)x^5 +\frac{1}{7}e\left(3cd^2+ae^2\right)x^7+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x+a*d^2*e*x^3+1/5*d*(3*a*e^2+c*d^2)*x^5+1/7*e*(a*e^2+3*c*d^2)*x^7+1/3
*c*d*e^2*x^9+1/11*c*e^3*x^11
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1168}

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)\,dx = \frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right) +ad^3x+ad^2ex^3+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

```
[In] Int[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*
x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad^3+3ad^2ex^2+d\left(cd^2+3ae^2\right)x^4+e\left(3cd^2+ae^2\right)x^6+3cde^2x^8+ce^3x^{10}\right)\,dx\\ &= ad^3x+ad^2ex^3+\frac{1}{5}d\left(cd^2+3ae^2\right)x^5+\frac{1}{7}e\left(3cd^2+ae^2\right)x^7+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^3\left(a+cx^4\right)\,dx = ad^3x+ad^2ex^3+\frac{1}{5}d\left(cd^2+3ae^2\right)x^5 + \frac{1}{7}e\left(3cd^2+ae^2\right)x^7+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

```
[In] Integrate[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*
x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{c\,e^3x^{11}}{11}+\frac{cd\,e^2x^9}{3}+\frac{(a\,e^3+3c\,d^2e)x^7}{7}+\frac{(3d\,e^2a+d^3c)x^5}{5}+a\,d^2e\,x^3+a\,d^3x$	72
norman	$\frac{c\,e^3x^{11}}{11}+\frac{cd\,e^2x^9}{3}+\left(\frac{1}{7}a\,e^3+\frac{3}{7}c\,d^2e\right)x^7+\left(\frac{3}{5}d\,e^2a+\frac{1}{5}d^3c\right)x^5+a\,d^2e\,x^3+a\,d^3x$	72
gosper	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{1}{5}x^5d^3c+a\,d^2e\,x^3+a\,d^3x$	74
risch	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{1}{5}x^5d^3c+a\,d^2e\,x^3+a\,d^3x$	74
parallelrisch	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{1}{5}x^5d^3c+a\,d^2e\,x^3+a\,d^3x$	74

```
[In] int((e*x^2+d)^3*(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c*e^3*x^11+1/3*c*d*e^2*x^9+1/7*(a*e^3+3*c*d^2*e)*x^7+1/5*(3*a*d*e^2+c*
d^3)*x^5+a*d^2*e*x^3+a*d^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right) \, dx = \frac{1}{11} \, ce^3x^{11} + \frac{1}{3} \, cde^2x^9 + \frac{1}{7} \, \left(3 \, cd^2e + ae^3\right)x^7 + ad^2ex^3 + \frac{1}{5} \, \left(cd^3 + 3 \, ade^2\right)x^5 + ad^3x$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/7*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*e*x
^3 + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right) \, dx = ad^3x + ad^2ex^3 + \frac{cde^2x^9}{3} + \frac{ce^3x^{11}}{11} + x^7\left(\frac{ae^3}{7} + \frac{3cd^2e}{7}\right) + x^5 \cdot \left(\frac{3ade^2}{5} + \frac{cd^3}{5}\right)$$

```
[In] integrate((e*x**2+d)**3*(c*x**4+a),x)
```

```
[Out] a*d**3*x + a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + x**7*(a*e**3
/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + c*d**3/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right) \, dx = \frac{1}{11} \, ce^3x^{11} + \frac{1}{3} \, cde^2x^9 + \frac{1}{7} \, \left(3 \, cd^2e + ae^3\right)x^7 + ad^2ex^3 + \frac{1}{5} \, \left(cd^3 + 3 \, ade^2\right)x^5 + ad^3x$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/7*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*e*x
^3 + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)\,dx = \frac{1}{11}ce^3x^{11} + \frac{1}{3}cde^2x^9 + \frac{3}{7}cd^2ex^7 + \frac{1}{7}ae^3x^7 + \frac{1}{5}cd^3x^5 + \frac{3}{5}ade^2x^5 + ad^2ex^3 + ad^3x$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 3/7*c*d^2*e*x^7 + 1/7*a*e^3*x^7 + 1/5*c
*d^3*x^5 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)\,dx = x^5 \left(\frac{c\,d^3}{5} + \frac{3\,a\,d\,e^2}{5}\right) + x^7 \left(\frac{3\,c\,d^2\,e}{7} + \frac{a\,e^3}{7}\right) + \frac{c\,e^3\,x^{11}}{11} + a\,d^3\,x + a\,d^2\,e\,x^3 + \frac{c\,d\,e^2\,x^9}{3}$$

```
[In] int((a + c*x^4)*(d + e*x^2)^3,x)
```

```
[Out] x^5*((c*d^3)/5 + (3*a*d*e^2)/5) + x^7*((a*e^3)/7 + (3*c*d^2*e)/7) + (c*e^3*
x^11)/11 + a*d^3*x + a*d^2*e*x^3 + (c*d*e^2*x^9)/3
```

3.122 $\int \left(d+ex^2\right)^2 \left(a+cx^4\right) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right) \, dx = ad^2x + \frac{2}{3}adex^3 + \frac{1}{5}\left(cd^2+ae^2\right)x^5 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1168}

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right) \, dx = \frac{1}{5}x^5\left(ae^2+cd^2\right) + ad^2x + \frac{2}{3}adex^3 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

```
[In] Int[(d + e*x^2)^2*(a + c*x^4),x]
```

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*
e^2*x^9)/9
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ad^2 + 2adex^2 + \left(cd^2+ae^2\right)x^4 + 2cdex^6 + ce^2x^8\right) \, dx \\ &= ad^2x + \frac{2}{3}adex^3 + \frac{1}{5}\left(cd^2+ae^2\right)x^5 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^2\left(a+cx^4\right)\,dx=ad^2x+\frac{2}{3}adex^3+\frac{1}{5}\left(cd^2+ae^2\right)x^5+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

```
[In] Integrate[(d + e*x^2)^2*(a + c*x^4),x]
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*
e^2*x^9)/9
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
default	$a\,d^2x+\frac{2ade\,x^3}{3}+\frac{\left(a\,e^2+c\,d^2\right)x^5}{5}+\frac{2cde\,x^7}{7}+\frac{c\,e^2x^9}{9}$	49
norman	$\frac{c\,e^2x^9}{9}+\frac{2cde\,x^7}{7}+\left(\frac{a\,e^2}{5}+\frac{c\,d^2}{5}\right)x^5+\frac{2ade\,x^3}{3}+a\,d^2x$	50
gosper	$\frac{1}{9}c\,e^2x^9+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+a\,d^2x$	51
risch	$\frac{1}{9}c\,e^2x^9+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+a\,d^2x$	51
parallelrisch	$\frac{1}{9}c\,e^2x^9+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+a\,d^2x$	51

```
[In] int((e*x^2+d)^2*(c*x^4+a),x,method=_RETURNVERBOSE)
[Out] a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)^2\left(a+cx^4\right)\,dx=\frac{1}{9}\,ce^2x^9+\frac{2}{7}\,cdex^7+\frac{2}{3}\,adex^3+\frac{1}{5}\,\left(cd^2+ae^2\right)x^5+ad^2x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="fricas")
[Out] 1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^2)*x^5 + a
*d^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^2\left(a+cx^4\right)\,dx=ad^2x+\frac{2adex^3}{3}+\frac{2cdex^7}{7}+\frac{ce^2x^9}{9}+x^5\left(\frac{ae^2}{5}+\frac{cd^2}{5}\right)$$

```
[In] integrate((e*x**2+d)**2*(c*x**4+a),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + 2*c*d*e*x**7/7 + c*e**2*x**9/9 + x**5*(a*e**2/5
 + c*d**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)^2\left(a+cx^4\right)\,dx=\frac{1}{9}\,ce^2x^9+\frac{2}{7}\,cdex^7+\frac{2}{3}\,adex^3+\frac{1}{5}\,\left(cd^2+ae^2\right)x^5+ad^2x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^2)*x^5 + a
*d^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(d+ex^2\right)^2\left(a+cx^4\right)\,dx=\frac{1}{9}\,ce^2x^9+\frac{2}{7}\,cdex^7+\frac{1}{5}\,cd^2x^5+\frac{1}{5}\,ae^2x^5+\frac{2}{3}\,adex^3+ad^2x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 1/5*c*d^2*x^5 + 1/5*a*e^2*x^5 + 2/3*a*d*e*x
^3 + a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \left(d+ex^2\right)^2\left(a+cx^4\right)\,dx=x^5\,\left(\frac{c\,d^2}{5}+\frac{a\,e^2}{5}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x+\frac{2\,a\,d\,e\,x^3}{3}+\frac{2\,c\,d\,e\,x^7}{7}$$

```
[In] int((a + c*x^4)*(d + e*x^2)^2,x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5) + (c*e^2*x^9)/9 + a*d^2*x + (2*a*d*e*x^3)/3 + (
2*c*d*e*x^7)/7
```

3.123 $\int (d+ex^2)(a+cx^4)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

```
[Out] a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1168}

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

```
[In] Int[(d + e*x^2)*(a + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(ad+aex^2+cdx^4+cex^6\right)\,dx\\&=adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

```
[In] Integrate[(d + e*x^2)*(a + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
gosper	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
default	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
norman	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
risch	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
parallelrisch	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27

```
[In] int((e*x^2+d)*(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=\frac{1}{7}\,cex^7+\frac{1}{5}\,cdx^5+\frac{1}{3}\,aex^3+adx$$

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=adx+\frac{aex^3}{3}+\frac{cdx^5}{5}+\frac{cex^7}{7}$$

```
[In] integrate((e*x**2+d)*(c*x**4+a),x)
[Out] a*d*x + a*e*x**3/3 + c*d*x**5/5 + c*e*x**7/7
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=\frac{1}{7}\,cex^7+\frac{1}{5}\,cdx^5+\frac{1}{3}\,aex^3+adx$$

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="maxima")
[Out] 1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=\frac{1}{7}\,cex^7+\frac{1}{5}\,cdx^5+\frac{1}{3}\,aex^3+adx$$

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="giac")
[Out] 1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=\frac{c\,e\,x^7}{7}+\frac{c\,d\,x^5}{5}+\frac{a\,e\,x^3}{3}+a\,d\,x$$

```
[In] int((a + c*x^4)*(d + e*x^2),x)
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

3.124 $\int \frac{a+cx^4}{d+ex^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{a+cx^4}{d+ex^2}\,dx=-\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

```
[Out] -c*d*x/e^2+1/3*c*x^3/e+(a*e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/d^(1
/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1168, 211}

$$\int \frac{a+cx^4}{d+ex^2}\,dx=\frac{(ae^2+cd^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

```
[In] Int[(a + c*x^4)/(d + e*x^2),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{cd}{e^2} + \frac{cx^2}{e} + \frac{cd^2 + ae^2}{e^2\,(d + ex^2)}\right) dx \\
&= -\frac{cdx}{e^2} + \frac{cx^3}{3e} + \left(a + \frac{cd^2}{e^2}\right) \int \frac{1}{d + ex^2}\,dx \\
&= -\frac{cdx}{e^2} + \frac{cx^3}{3e} + \frac{(cd^2 + ae^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{a + cx^4}{d + ex^2}\,dx = -\frac{cdx}{e^2} + \frac{cx^3}{3e} + \frac{(cd^2 + ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

```
[In] Integrate[(a + c*x^4)/(d + e*x^2),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(5/2))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{c\left(-\frac{1}{3}e\,x^3+dx\right)}{e^2}+\frac{\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{e^2\sqrt{ed}}$	47
risch	$\frac{c\,x^3}{3e}-\frac{cdx}{e^2}-\frac{\ln\left(ex+\sqrt{-ed}\right)a}{2\sqrt{-ed}}-\frac{\ln\left(ex+\sqrt{-ed}\right)c\,d^2}{2e^2\sqrt{-ed}}+\frac{\ln\left(-ex+\sqrt{-ed}\right)a}{2\sqrt{-ed}}+\frac{\ln\left(-ex+\sqrt{-ed}\right)c\,d^2}{2e^2\sqrt{-ed}}$	113

```
[In] int((c*x^4+a)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -c/e^2*(-1/3*e*x^3+d*x)+(a*e^2+c*d^2)/e^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.38

$$\int \frac{a+cx^4}{d+ex^2}\,dx$$
$$= \left[\frac{2\,cde^2x^3 - 6\,cd^2ex - 3\,(cd^2+ae^2)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right)}{6\,de^3}, \frac{cde^2x^3 - 3\,cd^2ex + 3\,(cd^2+ae^2)\sqrt{de}\arctan}{3\,de^3}\right.$$

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*d*e^2*x^3 - 6*c*d^2*e*x - 3*(c*d^2 + a*e^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d*e^3), 1/3*(c*d*e^2*x^3 - 3*c*d^2*e*
x + 3*(c*d^2 + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(49) = 98.

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.89

$$\int \frac{a+cx^4}{d+ex^2}\,dx = -\frac{cdx}{e^2} + \frac{cx^3}{3e} - \frac{\sqrt{-\frac{1}{de^5}}(ae^2+cd^2)\log\left(-de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}(ae^2+cd^2)\log\left(de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}$$

```
[In] integrate((c*x**4+a)/(e*x**2+d),x)
```

```
[Out] -c*d*x/e**2 + c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(-d*e**
2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(d*e**2
*sqrt(-1/(d*e**5)) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \frac{(cd^2+ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^2} + \frac{ce^2x^3-3\,cdex}{3\,e^3}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) + 1/3*(c*e^2*x^3 - 3*
c*d*e*x)/e^3
```

Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \frac{c\,x^3}{3\,e} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+a\,e^2\right)}{\sqrt{d}\,e^{5/2}} - \frac{c\,d\,x}{e^2}$$

```
[In] int((a + c*x^4)/(d + e*x^2),x)
```

```
[Out] (c*x^3)/(3*e) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2))/(d^(1/2)*e^(5/2
)) - (c*d*x)/e^2
```

3.125 $\int \frac{a+cx^4}{\left(d+ex^2\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^2} \, dx=\frac{cx}{e^2}+\frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+c*d^2/e^2)*x/d/(e*x^2+d)-1/2*(-a*e^2+3*c*d^2)*arctan(x*e^(1/
2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1172, 396, 211}

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^2} \, dx=-\frac{\left(3cd^2-ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[In] Int[(a + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{\int \frac{-a+\frac{cd^2}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\ &= \frac{cx}{e^2} + \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\,(d+ex^2)} + \frac{\left(a-\frac{3cd^2}{e^2}\right)\int \frac{1}{d+ex^2}\,dx}{2d}\\ &= \frac{cx}{e^2} + \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\,(d+ex^2)} + \frac{\left(a-\frac{3cd^2}{e^2}\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} + \frac{(cd^2+ae^2)\,x}{2de^2\,(d+ex^2)} - \frac{(3cd^2-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{cx}{e^{2}}+\frac{\frac{\left(a\,e^{2}+c\,d^{2}\right)x}{2d\left(e\,x^{2}+d\right)}+\frac{\left(a\,e^{2}-3c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^{2}}$	70
risch	$\frac{cx}{e^{2}}+\frac{\left(a\,e^{2}+c\,d^{2}\right)x}{2d\,e^{2}\left(e\,x^{2}+d\right)}-\frac{\ln\left(ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}+\frac{3d\ln\left(ex+\sqrt{-ed}\right)c}{4e^{2}\sqrt{-ed}}+\frac{\ln\left(-ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}-\frac{3d\ln\left(-ex+\sqrt{-ed}\right)c}{4e^{2}\sqrt{-ed}}$	133

```
[In] int((c*x^4+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2-3*c*d^2)/d/(e*d)^
(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.00

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^2}\,dx = \left[\frac{4\,cd^2e^2x^3+\left(3\,cd^3-ade^2+\left(3\,cd^2e-ae^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right)+2\left(3\,cd^3e+ade^3\right)x}{4\left(d^2e^4x^2+d^3e^3\right)},\ \frac{2\,cd^2e^2x^3-}{}\right.$$

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x^2)*sqrt(
-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e + a*d*e^
3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - a*d*e^2 +
(3*c*d^2*e - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e + a*d
*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(68) = 136$.

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.86

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{cx}{e^2}+\frac{x\left(ae^2+cd^2\right)}{2d^2e^2+2de^3x^2}-\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2-3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2-3cd^2\right)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

```
[In] integrate((c*x**4+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d**
3*e**5))*(a*e**2 - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 + s
qrt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5))
+ x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} - \frac{(3\,cd^2-ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x+ae^2x}{2\,(ex^2+d)de^2}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x/e^2 - 1/2*(3*c*d^2 - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2) + 1
/2*(c*d^2*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx = \frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(a\,e^2-3\,c\,d^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

```
[In] int((a + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2))/(2*d^(3/2)*e^(5/2
)) + (x*(a*e^2 + c*d^2))/(2*d*(d*e^2 + e^3*x^2))
```

3.126 $\int \frac{a+cx^4}{(d+ex^2)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{a+cx^4}{(d+ex^2)^3} \, dx = \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d(d+ex^2)^2}+\frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8(d+ex^2)}+\frac{3(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[Out] 1/4*(a+c*d^2/e^2)*x/d/(e*x^2+d)^2+1/8*(3*a/d^2-5*c/e^2)*x/(e*x^2+d)+3/8*(a*
e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1172, 393, 211}

$$\int \frac{a+cx^4}{(d+ex^2)^3} \, dx = \frac{3(ae^2+cd^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)}{8(d+ex^2)}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}$$

```
[In] Int[(a + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (((3*a)/d^2 - (5*c)/e^2)*x)/(8*
(d + e*x^2)) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e
^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2}-\frac{\int\frac{-3a+\frac{cd^2}{e^2}-\frac{4cdx^2}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}\\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2}+\frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\left(d+ex^2\right)}+\frac{1}{8}\left(3\left(\frac{a}{d^2}+\frac{c}{e^2}\right)\right)\int\frac{1}{d+ex^2}\,dx\\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2}+\frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\left(d+ex^2\right)}+\frac{3(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int\frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx=\frac{ae^2x(5d+3ex^2)-cd^2x(3d+5ex^2)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{3(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (a*e^2*x*(5*d + 3*e*x^2) - c*d^2*x*(3*d + 5*e*x^2))/(8*d^2*e^2*(d + e*x^2)^
2) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\frac{\left(3a\,e^{2}-5c\,d^{2}\right)x^{3}}{8d^{2}e}+\frac{\left(5a\,e^{2}-3c\,d^{2}\right)x}{8d\,e^{2}}}{\left(e\,x^{2}+d\right)^{2}}+\frac{3\left(a\,e^{2}+c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8e^{2}d^{2}\sqrt{ed}}$	92
risch	$\frac{\frac{\left(3a\,e^{2}-5c\,d^{2}\right)x^{3}}{8d^{2}e}+\frac{\left(5a\,e^{2}-3c\,d^{2}\right)x}{8d\,e^{2}}}{\left(e\,x^{2}+d\right)^{2}}-\frac{3\ln\left(ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^{2}}-\frac{3\ln\left(ex+\sqrt{-ed}\right)c}{16\sqrt{-ed}\,e^{2}}+\frac{3\ln\left(-ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^{2}}+\frac{3\ln\left(-ex+\sqrt{-ed}\right)c}{16\sqrt{-ed}\,e^{2}}$	153

```
[In] int((c*x^4+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*e^2-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-3*c*d^2)/d/e^2*x)/(e*x^2+d)^2
+3/8*(a*e^2+c*d^2)/e^2/d^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.29

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx$$

$$=\left[-\frac{2\,(5\,cd^3e^2-3\,ade^4)x^3+3\,(cd^4+ad^2e^2+(cd^2e^2+ae^4)x^4+2\,(cd^3e+ade^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right.}{16\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}\right.$$

$$-\frac{(5\,cd^3e^2-3\,ade^4)x^3-3\,(cd^4+ad^2e^2+(cd^2e^2+ae^4)x^4+2\,(cd^3e+ade^3)x^2)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)+(3\,cd}{8\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - 3*a*d*e^4)*x^3 + 3*(c*d^4 + a*d^2*e^2 + (c*d^2*e^2
 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-
d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*
d^4*e^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - 3*a*d*e^4)*x^3 - 3*(c*d^4 + a*
d^2*e^2 + (c*d^2*e^2 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(d*e)*ar
ctan(sqrt(d*e)*x/d) + (3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4
*x^2 + d^5*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(90) = 180$.

Time = 0.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.35

$$\int \frac{a+cx^4}{(d+ex^2)^3}\,dx = -\frac{3\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)\log\left(-\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)}{3ae^2+3cd^2}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)\log\left(\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)}{3ae^2+3cd^2}+x\right)}{16} + \frac{x^3\cdot(3ae^3-5cd^2e)+x(5ade^2-3cd^3)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

```
[In] integrate((c*x**4+a)/(e*x**2+d)**3,x)
```

```
[Out] -3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(-3*d**3*e**2*sqrt(-1/(d**5*e*
*5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 + 3*sqrt(-1/(d**5*e**5
))*(a*e**2 + c*d**2)*log(3*d**3*e**2*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)
/(3*a*e**2 + 3*c*d**2) + x)/16 + (x**3*(3*a*e**3 - 5*c*d**2*e) + x*(5*a*d*e
**2 - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^4}{(d+ex^2)^3}\,dx = \frac{3\,(cd^2+ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^2} - \frac{5\,cd^2ex^3-3\,ae^3x^3+3\,cd^3x-5\,ade^2x}{8\,(ex^2+d)^2d^2e^2}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 3/8*(c*d^2 + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2) - 1/8*(5*c*d^
2*e*x^3 - 3*a*e^3*x^3 + 3*c*d^3*x - 5*a*d*e^2*x)/((e*x^2 + d)^2*d^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{a+cx^4}{(d+ex^2)^3}\,dx = \frac{\frac{x^3\,(3\,a\,e^2-5\,c\,d^2)}{8\,d^2\,e}+\frac{x\,(5\,a\,e^2-3\,c\,d^2)}{8\,d\,e^2}}{d^2+2\,d\,e\,x^2+e^2\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2+a\,e^2)}{8\,d^{5/2}\,e^{5/2}}$$

```
[In] int((a + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] ((x^3*(3*a*e^2 - 5*c*d^2))/(8*d^2*e) + (x*(5*a*e^2 - 3*c*d^2))/(8*d*e^2))/(
d^2 + e^2*x^4 + 2*d*e*x^2) + (3*atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2))/
(8*d^(5/2)*e^(5/2))
```

3.127 $\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx=\frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\left(d+ex^2\right)^2}+\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\left(d+ex^2\right)}+\frac{\left(cd^2+5ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

```
[Out] 1/6*(a+c*d^2/e^2)*x/d/(e*x^2+d)^3+1/24*(5*a/d^2-7*c/e^2)*x/(e*x^2+d)^2+1/16
*(5*a/d^2+c/e^2)*x/d/(e*x^2+d)+1/16*(5*a*e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2
))/d^(7/2)/e^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1172, 393, 205, 211}

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx=\frac{\left(5ae^2+cd^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)}{16d\left(d+ex^2\right)}+\frac{x\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)}{24\left(d+ex^2\right)^2}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}$$

```
[In] Int[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((5*a)/d^2 - (7*c)/e^2)*x)/(24
*(d + e*x^2)^2) + (((5*a)/d^2 + c/e^2)*x)/(16*d*(d + e*x^2)) + ((c*d^2 + 5*
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\,(d+ex^2)^3} - \frac{\int \frac{-5a+\frac{cd^2}{e^2}-\frac{6cdx^2}{e}}{(d+ex^2)^3}\,dx}{6d} \\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\,(d+ex^2)^3} + \frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\,(d+ex^2)^2} + \frac{1}{8}\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)\int \frac{1}{(d+ex^2)^2}\,dx \\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\,(d+ex^2)^3} + \frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\,(d+ex^2)^2} + \frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\,(d+ex^2)} + \frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)\int \frac{1}{d+ex^2}\,dx}{16d} \\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\,(d+ex^2)^3} + \frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\,(d+ex^2)^2} + \frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\,(d+ex^2)} + \frac{(cd^2+5ae^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx=\frac{x\left(cd^2\left(-3d^2-8dex^2+3e^2x^4\right)+ae^2\left(33d^2+40dex^2+15e^2x^4\right)\right)}{48d^3e^2\left(d+ex^2\right)^3}+\frac{\left(cd^2+5ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + a*e^2*(33*d^2 + 40*d*e*x^2 + 1
5*e^2*x^4)))/(48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

method	result
default	$\frac{\frac{\left(5a\,e^{2}+c\,d^{2}\right)x^{5}}{16d^{3}}+\frac{\left(5a\,e^{2}-c\,d^{2}\right)x^{3}}{6d^{2}e}+\frac{\left(11a\,e^{2}-c\,d^{2}\right)x}{16d\,e^{2}}}{\left(e\,x^{2}+d\right)^{3}}+\frac{\left(5a\,e^{2}+c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{16d^{3}e^{2}\sqrt{ed}}$
risch	$\frac{\frac{\left(5a\,e^{2}+c\,d^{2}\right)x^{5}}{16d^{3}}+\frac{\left(5a\,e^{2}-c\,d^{2}\right)x^{3}}{6d^{2}e}+\frac{\left(11a\,e^{2}-c\,d^{2}\right)x}{16d\,e^{2}}}{\left(e\,x^{2}+d\right)^{3}}-\frac{5\ln\left(ex+\sqrt{-ed}\right)a}{32\sqrt{-ed}\,d^{3}}-\frac{\ln\left(ex+\sqrt{-ed}\right)c}{32\sqrt{-ed}\,e^{2}d}+\frac{5\ln\left(-ex+\sqrt{-ed}\right)a}{32\sqrt{-ed}\,d^{3}}+\frac{\ln(-ex+\sqrt{-}}{32\sqrt{-ed}\,e}$

```
[In] int((c*x^4+a)/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*a*e^2+c*d^2)/d^3*x^5+1/6*(5*a*e^2-c*d^2)/d^2/e*x^3+1/16*(11*a*e^2-
c*d^2)/d/e^2*x)/(e*x^2+d)^3+1/16*(5*a*e^2+c*d^2)/d^3/e^2/(e*d)^(1/2)*arctan
(e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.45

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx$$

$$=\Bigg[\frac{6\,(cd^3e^3+5\,ade^5)x^5-16\,(cd^4e^2-5\,ad^2e^4)x^3-3\,((cd^2e^3+5\,ae^5)x^6+cd^5+5\,ad^3e^2+3\,(cd^3e^2+5\,ade^4}{96\,(d^4e^6x^6+3\,d^5e^5x^4+3\,d^6e^4x^2+}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 - 3
*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^
4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e
)*x - d)/(e*x^2 + d)) - 6*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6 + 3*d^5*
e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 8
*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a
*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*s
qrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6
 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.66

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx = -\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot\left(5ae^2+cd^2\right)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^5}}\cdot\left(5ae^2+cd^2\right)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32} + \frac{x^5\cdot\left(15ae^4+3cd^2e^2\right)+x^3\cdot\left(40ade^3-8cd^3e\right)+x\left(33ad^2e^2-3cd^4\right)}{48d^6e^2+144d^5e^3x^2+144d^4e^4x^4+48d^3e^5x^6}$$

```
[In] integrate((c*x**4+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(-d**4*e**2*sqrt(-1/(d**7*e**5
)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(d**4*e**2*sqrt(-1
/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*c*d**2*e**2) + x**3*(40*a*d*e*
*3 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*
e**3*x**2 + 144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = \frac{(cd^2+5\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^2} + \frac{3\,cd^2e^2x^5+15\,ae^4x^5-8\,cd^3ex^3+40\,ade^3x^3-3\,cd^4x+33\,ad^2e^2x}{48\,(ex^2+d)^3d^3e^2}$$

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + 5*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e^2) + 1/48*(3*
c*d^2*e^2*x^5 + 15*a*e^4*x^5 - 8*c*d^3*e*x^3 + 40*a*d*e^3*x^3 - 3*c*d^4*x +
 33*a*d^2*e^2*x)/((e*x^2 + d)^3*d^3*e^2)
```

Mupad [B] (verification not implemented)

Time = 13.97 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = \frac{\frac{x^5\,(c\,d^2+5\,a\,e^2)}{16\,d^3}+\frac{x^3\,(5\,a\,e^2-c\,d^2)}{6\,d^2\,e}+\frac{x\,(11\,a\,e^2-c\,d^2)}{16\,d\,e^2}}{d^3+3\,d^2\,e\,x^2+3\,d\,e^2\,x^4+e^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2+5\,a\,e^2)}{16\,d^{7/2}\,e^{5/2}}$$

```
[In] int((a + c*x^4)/(d + e*x^2)^4,x)
```

```
[Out] ((x^5*(5*a*e^2 + c*d^2))/(16*d^3) + (x^3*(5*a*e^2 - c*d^2))/(6*d^2*e) + (x*
(11*a*e^2 - c*d^2))/(16*d*e^2))/(d^3 + e^3*x^6 + 3*d^2*e*x^2 + 3*d*e^2*x^4)
 + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2))/(16*d^(7/2)*e^(5/2))
```

3.128 $\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\begin{aligned}\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx = {} & a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad\left(2cd^2+3ae^2\right)x^5 \\ & +\frac{1}{7}ae\left(6cd^2+ae^2\right)x^7+\frac{1}{9}cd\left(cd^2+6ae^2\right)x^9 \\ & +\frac{1}{11}ce\left(3cd^2+2ae^2\right)x^{11}+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}\end{aligned}$$

```
[Out] a^2*d^3*x+a^2*d^2*e*x^3+1/5*a*d*(3*a*e^2+2*c*d^2)*x^5+1/7*a*e*(a*e^2+6*c*d^
2)*x^7+1/9*c*d*(6*a*e^2+c*d^2)*x^9+1/11*c*e*(2*a*e^2+3*c*d^2)*x^11+3/13*c^2
*d*e^2*x^13+1/15*c^2*e^3*x^15
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1168}

$$\begin{aligned}\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx = {} & a^2d^3x+a^2d^2ex^3+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right) \\ & +\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right) \\ & +\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}\end{aligned}$$

```
[In] Int[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2
 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)
*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2d^3+3a^2d^2ex^2+ad\left(2cd^2+3ae^2\right)x^4+ae\left(6cd^2+ae^2\right)x^6+cd\left(cd^2+6ae^2\right)x^8 \right.\\ &\qquad \left. +ce\left(3cd^2+2ae^2\right)x^{10}+3c^2de^2x^{12}+c^2e^3x^{14}\right)\,dx\\ &= a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad\left(2cd^2+3ae^2\right)x^5+\frac{1}{7}ae\left(6cd^2+ae^2\right)x^7\\ &\quad +\frac{1}{9}cd\left(cd^2+6ae^2\right)x^9+\frac{1}{11}ce\left(3cd^2+2ae^2\right)x^{11}+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+cx^4\right)^2\,dx &= a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad\left(2cd^2+3ae^2\right)x^5\\ &\quad +\frac{1}{7}ae\left(6cd^2+ae^2\right)x^7+\frac{1}{9}cd\left(cd^2+6ae^2\right)x^9\\ &\quad +\frac{1}{11}ce\left(3cd^2+2ae^2\right)x^{11}+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}\end{aligned}$$

```
[In] Integrate[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2
 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)
*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

method	result
norman	$a^2d^3x + a^2d^2e\,x^3 + \left(\frac{3}{5}d\,e^2a^2 + \frac{2}{5}d^3ac\right)x^5 + \left(\frac{1}{7}e^3a^2 + \frac{6}{7}d^2eac\right)x^7 + \left(\frac{2}{3}acd\,e^2 + \frac{1}{9}c^2d^3\right)x^9 +$
default	$\frac{c^2e^3x^{15}}{15} + \frac{3c^2d\,e^2x^{13}}{13} + \frac{(2e^3ac+3d^2e\,c^2)x^{11}}{11} + \frac{(6acd\,e^2+c^2d^3)x^9}{9} + \frac{(e^3a^2+6d^2eac)x^7}{7} + \frac{(3d\,e^2a^2+2d^3ac)x^5}{5} + a^2d$
gosper	$a^2d^3x + a^2d^2e\,x^3 + \frac{3}{5}x^5d\,e^2a^2 + \frac{2}{5}x^5d^3ac + \frac{1}{7}x^7e^3a^2 + \frac{6}{7}x^7d^2eac + \frac{2}{3}x^9acd\,e^2 + \frac{1}{9}x^9c^2d^3 + \frac{2}{11}x$
risch	$a^2d^3x + a^2d^2e\,x^3 + \frac{3}{5}x^5d\,e^2a^2 + \frac{2}{5}x^5d^3ac + \frac{1}{7}x^7e^3a^2 + \frac{6}{7}x^7d^2eac + \frac{2}{3}x^9acd\,e^2 + \frac{1}{9}x^9c^2d^3 + \frac{2}{11}x$
parallelrisch	$a^2d^3x + a^2d^2e\,x^3 + \frac{3}{5}x^5d\,e^2a^2 + \frac{2}{5}x^5d^3ac + \frac{1}{7}x^7e^3a^2 + \frac{6}{7}x^7d^2eac + \frac{2}{3}x^9acd\,e^2 + \frac{1}{9}x^9c^2d^3 + \frac{2}{11}x$

```
[In] int((e*x^2+d)^3*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*d^3*x+a^2*d^2*e*x^3+(3/5*d*e^2*a^2+2/5*d^3*a*c)*x^5+(1/7*e^3*a^2+6/7*d^
2*e*a*c)*x^7+(2/3*a*c*d*e^2+1/9*c^2*d^3)*x^9+(2/11*e^3*a*c+3/11*d^2*e*c^2)*
x^11+3/13*c^2*d*e^2*x^13+1/15*c^2*e^3*x^15
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \left(d+ex^2\right)^3\left(a+cx^4\right)^2\,dx = \frac{1}{15}\,c^2e^3x^{15} + \frac{3}{13}\,c^2de^2x^{13} + \frac{1}{11}\left(3\,c^2d^2e + 2\,ace^3\right)x^{11} + \frac{1}{9}\left(c^2d^3 + 6\,acde^2\right)x^9 + a^2d^2ex^3 + \frac{1}{7}\left(6\,acd^2e + a^2e^3\right)x^7 + a^2d^3x + \frac{1}{5}\left(2\,acd^3 + 3\,a^2de^2\right)x^5$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x^
11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e + a
^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

$$\int \left(d+ex^2\right)^3\left(a+cx^4\right)^2\,dx = a^2d^3x + a^2d^2ex^3 + \frac{3c^2de^2x^{13}}{13} + \frac{c^2e^3x^{15}}{15} + x^{11}\cdot\left(\frac{2ace^3}{11} + \frac{3c^2d^2e}{11}\right) + x^9\cdot\left(\frac{2acde^2}{3} + \frac{c^2d^3}{9}\right) + x^7\left(\frac{a^2e^3}{7} + \frac{6acd^2e}{7}\right) + x^5\cdot\left(\frac{3a^2de^2}{5} + \frac{2acd^3}{5}\right)$$

```
[In] integrate((e*x**2+d)**3*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**3*x + a**2*d**2*e*x**3 + 3*c**2*d*e**2*x**13/13 + c**2*e**3*x**15/1
5 + x**11*(2*a*c*e**3/11 + 3*c**2*d**2*e/11) + x**9*(2*a*c*d*e**2/3 + c**2*
d**3/9) + x**7*(a**2*e**3/7 + 6*a*c*d**2*e/7) + x**5*(3*a**2*d*e**2/5 + 2*a
*c*d**3/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+cx^4\right)^2\,dx &= \frac{1}{15}\,c^2e^3x^{15}+\frac{3}{13}\,c^2de^2x^{13}+\frac{1}{11}\,\left(3\,c^2d^2e+2\,ace^3\right)x^{11}\\&\quad+\frac{1}{9}\,\left(c^2d^3+6\,acde^2\right)x^9+a^2d^2ex^3\\&\quad+\frac{1}{7}\,\left(6\,acd^2e+a^2e^3\right)x^7+a^2d^3x+\frac{1}{5}\,\left(2\,acd^3+3\,a^2de^2\right)x^5\end{aligned}$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x^
11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e + a
^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+cx^4\right)^2\,dx &= \frac{1}{15}\,c^2e^3x^{15}+\frac{3}{13}\,c^2de^2x^{13}+\frac{3}{11}\,c^2d^2ex^{11}+\frac{2}{11}\,ace^3x^{11}\\&\quad+\frac{1}{9}\,c^2d^3x^9+\frac{2}{3}\,acde^2x^9+\frac{6}{7}\,acd^2ex^7+\frac{1}{7}\,a^2e^3x^7\\&\quad+\frac{2}{5}\,acd^3x^5+\frac{3}{5}\,a^2de^2x^5+a^2d^2ex^3+a^2d^3x\end{aligned}$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 3/11*c^2*d^2*e*x^11 + 2/11*a*c*e^
3*x^11 + 1/9*c^2*d^3*x^9 + 2/3*a*c*d*e^2*x^9 + 6/7*a*c*d^2*e*x^7 + 1/7*a^2*
e^3*x^7 + 2/5*a*c*d^3*x^5 + 3/5*a^2*d*e^2*x^5 + a^2*d^2*e*x^3 + a^2*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx = x^5 \left(\frac{3\,a^2\,d\,e^2}{5}+\frac{2\,c\,a\,d^3}{5}\right)+x^7 \left(\frac{a^2\,e^3}{7}+\frac{6\,c\,a\,d^2\,e}{7}\right) + x^9 \left(\frac{c^2\,d^3}{9}+\frac{2\,a\,c\,d\,e^2}{3}\right)+x^{11} \left(\frac{3\,c^2\,d^2\,e}{11}+\frac{2\,a\,c\,e^3}{11}\right) + a^2\,d^3\,x+\frac{c^2\,e^3\,x^{15}}{15}+a^2\,d^2\,e\,x^3+\frac{3\,c^2\,d\,e^2\,x^{13}}{13}$$

```
[In] int((a + c*x^4)^2*(d + e*x^2)^3,x)
```

```
[Out] x^5*((3*a^2*d*e^2)/5 + (2*a*c*d^3)/5) + x^7*((a^2*e^3)/7 + (6*a*c*d^2*e)/7)
 + x^9*((c^2*d^3)/9 + (2*a*c*d*e^2)/3) + x^11*((3*c^2*d^2*e)/11 + (2*a*c*e^
3)/11) + a^2*d^3*x + (c^2*e^3*x^15)/15 + a^2*d^2*e*x^3 + (3*c^2*d*e^2*x^13)
/13
```

3.129 $\int (d+ex^2)^2 (a+cx^4)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{5}a\left(2cd^2+ae^2\right)x^5+\frac{4}{7}acdex^7 + \frac{1}{9}c\left(cd^2+2ae^2\right)x^9+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

```
[Out] a^2*d^2*x+2/3*a^2*d*e*x^3+1/5*a*(a*e^2+2*c*d^2)*x^5+4/7*a*c*d*e*x^7+1/9*c*(
2*a*e^2+c*d^2)*x^9+2/11*c^2*d*e*x^11+1/13*c^2*e^2*x^13
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1168}

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right) + \frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

```
[In] Int[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x^
7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)/1
3
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2d^2 + 2a^2dex^2 + a\left(2cd^2 + ae^2\right)x^4 + 4acdex^6 + c\left(cd^2 + 2ae^2\right)x^8 + 2c^2dex^{10} \right. \\
&\qquad \left. + c^2e^2x^{12}\right)\,dx \\
&= a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{5}a\left(2cd^2 + ae^2\right)x^5 + \frac{4}{7}acdex^7 + \frac{1}{9}c\left(cd^2 + 2ae^2\right)x^9 + \frac{2}{11}c^2dex^{11} \\
&\quad + \frac{1}{13}c^2e^2x^{13}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \left(d + ex^2\right)^2\left(a + cx^4\right)^2\,dx &= a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{5}a\left(2cd^2 + ae^2\right)x^5 + \frac{4}{7}acdex^7 \\
&\quad + \frac{1}{9}c\left(cd^2 + 2ae^2\right)x^9 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}
\end{aligned}
$$

```
[In] Integrate[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x^
7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)/1
3
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$\frac{c^2e^2x^{13}}{13}+\frac{2c^2de\,x^{11}}{11}+\frac{(2e^2ac+c^2d^2)x^9}{9}+\frac{4acde\,x^7}{7}+\frac{(e^2a^2+2d^2ac)x^5}{5}+\frac{2a^2de\,x^3}{3}+a^2d^2x$
norman	$\frac{c^2e^2x^{13}}{13}+\frac{2c^2de\,x^{11}}{11}+\left(\frac{2}{9}e^2ac+\frac{1}{9}c^2d^2\right)x^9+\frac{4acde\,x^7}{7}+\left(\frac{1}{5}e^2a^2+\frac{2}{5}d^2ac\right)x^5+\frac{2a^2de\,x^3}{3}+a^2d^2x$
gosper	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9c^2d^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5e^2a^2+\frac{2}{5}x^5d^2ac+\frac{2}{3}a^2de\,x^3+a$
risch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9c^2d^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5e^2a^2+\frac{2}{5}x^5d^2ac+\frac{2}{3}a^2de\,x^3+a$
parallelrisch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9c^2d^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5e^2a^2+\frac{2}{5}x^5d^2ac+\frac{2}{3}a^2de\,x^3+a$

```
[In] int((e*x^2+d)^2*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c^2*e^2*x^13+2/11*c^2*d*e*x^11+1/9*(2*a*c*e^2+c^2*d^2)*x^9+4/7*a*c*d*e
*x^7+1/5*(a^2*e^2+2*a*c*d^2)*x^5+2/3*a^2*d*e*x^3+a^2*d^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = \frac{1}{13}\,c^2e^2x^{13} + \frac{2}{11}\,c^2dex^{11} + \frac{4}{7}\,acdex^7 + \frac{1}{9}\,\left(c^2d^2 + 2\,ace^2\right)x^9 + \frac{2}{3}\,a^2dex^3 + \frac{1}{5}\,\left(2\,acd^2 + a^2e^2\right)x^5 + a^2d^2x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^2*d^2 + 2*
a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = a^2d^2x + \frac{2a^2dex^3}{3} + \frac{4acdex^7}{7} + \frac{2c^2dex^{11}}{11} + \frac{c^2e^2x^{13}}{13} + x^9 \cdot \left(\frac{2ace^2}{9} + \frac{c^2d^2}{9}\right) + x^5\left(\frac{a^2e^2}{5} + \frac{2acd^2}{5}\right)$$

```
[In] integrate((e*x**2+d)**2*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**2*x + 2*a**2*d*e*x**3/3 + 4*a*c*d*e*x**7/7 + 2*c**2*d*e*x**11/11 +
c**2*e**2*x**13/13 + x**9*(2*a*c*e**2/9 + c**2*d**2/9) + x**5*(a**2*e**2/5
+ 2*a*c*d**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = \frac{1}{13}\,c^2e^2x^{13} + \frac{2}{11}\,c^2dex^{11} + \frac{4}{7}\,acdex^7 + \frac{1}{9}\,\left(c^2d^2 + 2\,ace^2\right)x^9 + \frac{2}{3}\,a^2dex^3 + \frac{1}{5}\,\left(2\,acd^2 + a^2e^2\right)x^5 + a^2d^2x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^2*d^2 + 2*
a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = \frac{1}{13}c^2e^2x^{13} + \frac{2}{11}c^2dex^{11} + \frac{1}{9}c^2d^2x^9 + \frac{2}{9}ace^2x^9 + \frac{4}{7}acdex^7 + \frac{2}{5}acd^2x^5 + \frac{1}{5}a^2e^2x^5 + \frac{2}{3}a^2dex^3 + a^2d^2x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 1/9*c^2*d^2*x^9 + 2/9*a*c*e^2*x^9 +
 4/7*a*c*d*e*x^7 + 2/5*a*c*d^2*x^5 + 1/5*a^2*e^2*x^5 + 2/3*a^2*d*e*x^3 + a^
2*d^2*x
```

Mupad [B] (verification not implemented)

Time = 14.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = x^5\left(\frac{a^2e^2}{5} + \frac{2\,c\,a\,d^2}{5}\right) + x^9\left(\frac{c^2d^2}{9} + \frac{2\,a\,c\,e^2}{9}\right) + a^2d^2x + \frac{c^2e^2x^{13}}{13} + \frac{2\,a^2\,d\,e\,x^3}{3} + \frac{2\,c^2\,d\,e\,x^{11}}{11} + \frac{4\,a\,c\,d\,e\,x^7}{7}$$

```
[In] int((a + c*x^4)^2*(d + e*x^2)^2,x)
```

```
[Out] x^5*((a^2*e^2)/5 + (2*a*c*d^2)/5) + x^9*((c^2*d^2)/9 + (2*a*c*e^2)/9) + a^2
*d^2*x + (c^2*e^2*x^13)/13 + (2*a^2*d*e*x^3)/3 + (2*c^2*d*e*x^11)/11 + (4*a
*c*d*e*x^7)/7
```

3.130 $\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e*
x^11
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1168}

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

```
[In] Int[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9
 + (c^2*e*x^11)/11
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(a^2d+a^2ex^2+2acdx^4+2acex^6+c^2dx^8+c^2ex^{10}\right)\,dx\\&=a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

```
[In] Integrate[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9
 + (c^2*e*x^11)/11
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
default	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
norman	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
risch	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
parallelrisch	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51

```
[In] int((e*x^2+d)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e*
x^11
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=\frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\,c^2dx^9+\frac{2}{7}\,acex^7+\frac{2}{5}\,acdx^5+\frac{1}{3}\,a^2ex^3+a^2dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*e
*x^3 + a^2*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right) \left(a+cx^4\right)^2 \, dx = a^2dx + \frac{a^2ex^3}{3} + \frac{2acdx^5}{5} + \frac{2acex^7}{7} + \frac{c^2dx^9}{9} + \frac{c^2ex^{11}}{11}$$

```
[In] integrate((e*x**2+d)*(c*x**4+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**3/3 + 2*a*c*d*x**5/5 + 2*a*c*e*x**7/7 + c**2*d*x**9/9
+ c**2*e*x**11/11
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right) \left(a+cx^4\right)^2 \, dx = \frac{1}{11}\,c^2ex^{11} + \frac{1}{9}\,c^2dx^9 + \frac{2}{7}\,acex^7 + \frac{2}{5}\,acdx^5 + \frac{1}{3}\,a^2ex^3 + a^2dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*e
*x^3 + a^2*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right) \left(a+cx^4\right)^2 \, dx = \frac{1}{11}\,c^2ex^{11} + \frac{1}{9}\,c^2dx^9 + \frac{2}{7}\,acex^7 + \frac{2}{5}\,acdx^5 + \frac{1}{3}\,a^2ex^3 + a^2dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*e
*x^3 + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^2\,dx=\frac{e\,a^2\,x^3}{3}+d\,a^2\,x+\frac{2\,e\,a\,c\,x^7}{7}+\frac{2\,d\,a\,c\,x^5}{5}+\frac{e\,c^2\,x^{11}}{11}+\frac{d\,c^2\,x^9}{9}$$

```
[In] int((a + c*x^4)^2*(d + e*x^2),x)
```

```
[Out] (a^2*e*x^3)/3 + (c^2*d*x^9)/9 + (c^2*e*x^11)/11 + a^2*d*x + (2*a*c*d*x^5)/5
 + (2*a*c*e*x^7)/7
```

3.131 $\int (a+cx^4)^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \left(a+cx^4\right)^2 \, dx = a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {200}

$$\int \left(a+cx^4\right)^2 \, dx = a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

```
[In] Int[(a + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2+2acx^4+c^2x^8\right) \, dx \\ &= a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(a+cx^4\right)^2\,dx=a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

```
[In] Integrate[(a + c*x^4)^2,x]
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$a^2x+\frac{2}{5}x^5ac+\frac{1}{9}c^2x^9$	22
default	$a^2x+\frac{2}{5}x^5ac+\frac{1}{9}c^2x^9$	22
norman	$a^2x+\frac{2}{5}x^5ac+\frac{1}{9}c^2x^9$	22
risch	$a^2x+\frac{2}{5}x^5ac+\frac{1}{9}c^2x^9$	22
parallelrisch	$a^2x+\frac{2}{5}x^5ac+\frac{1}{9}c^2x^9$	22

```
[In] int((c*x^4+a)^2,x,method=_RETURNVERBOSE)
[Out] a^2*x+2/5*x^5*a*c+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{5}\,acx^5+a^2x$$

```
[In] integrate((c*x^4+a)^2,x, algorithm="fricas")
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(a+cx^4\right)^2 \, dx = a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

```
[In] integrate((c*x**4+a)**2,x)
[Out] a**2*x + 2*a*c*x**5/5 + c**2*x**9/9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2 \, dx = \frac{1}{9}\,c^2x^9 + \frac{2}{5}\,acx^5 + a^2x$$

```
[In] integrate((c*x^4+a)^2,x, algorithm="maxima")
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2 \, dx = \frac{1}{9}\,c^2x^9 + \frac{2}{5}\,acx^5 + a^2x$$

```
[In] integrate((c*x^4+a)^2,x, algorithm="giac")
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2 \, dx = a^2\,x + \frac{2\,a\,c\,x^5}{5} + \frac{c^2\,x^9}{9}$$

```
[In] int((a + c*x^4)^2,x)
[Out] a^2*x + (c^2*x^9)/9 + (2*a*c*x^5)/5
```

3.132 $\int \frac{(a+cx^4)^2}{d+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = -\frac{cd(cd^2+2ae^2)\,x}{e^4}+\frac{c(cd^2+2ae^2)\,x^3}{3e^3} -\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{(cd^2+ae^2)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

```
[Out] -c*d*(2*a*e^2+c*d^2)*x/e^4+1/3*c*(2*a*e^2+c*d^2)*x^3/e^3-1/5*c^2*d*x^5/e^2+
1/7*c^2*x^7/e+(a*e^2+c*d^2)^2*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1168, 211}

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \frac{(ae^2+cd^2)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}-\frac{cdx(2ae^2+cd^2)}{e^4} +\frac{cx^3(2ae^2+cd^2)}{3e^3}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}$$

```
[In] Int[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] -((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^3)/(3*e^3) - (c^2
*d*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(Sqrt[d]*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{cd(cd^2+2ae^2)}{e^4}+\frac{c(cd^2+2ae^2)\,x^2}{e^3}-\frac{c^2dx^4}{e^2}+\frac{c^2x^6}{e} + \frac{c^2d^4+2acd^2e^2+a^2e^4}{e^4\,(d+ex^2)}\right) dx \\
&= -\frac{cd(cd^2+2ae^2)\,x}{e^4}+\frac{c(cd^2+2ae^2)\,x^3}{3e^3}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{(cd^2+ae^2)^2\int\frac{1}{d+ex^2}\,dx}{e^4} \\
&= -\frac{cd(cd^2+2ae^2)\,x}{e^4}+\frac{c(cd^2+2ae^2)\,x^3}{3e^3}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{(cd^2+ae^2)^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \frac{cx(70ae^2(-3d+ex^2)+c(-105d^3+35d^2ex^2-21de^2x^4+15e^3x^6))}{105e^4} + \frac{(cd^2+ae^2)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] (c*x*(70*a*e^2*(-3*d + e*x^2) + c*(-105*d^3 + 35*d^2*e*x^2 - 21*d*e^2*x^4 +
 15*e^3*x^6)))/(105*e^4) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(Sqrt[d]*e^(9/2))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c\left(-\frac{c\,x^{7} e^{3}}{7}+\frac{c d\,x^{5} e^{2}}{5}-\frac{\left(2 a\,e^{2}+c\,d^{2}\right) x^{3} e}{3}+d\left(2 a\,e^{2}+c\,d^{2}\right) x\right)}{e^{4}}+\frac{\left(a^{2} e^{4}+2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \arctan\left(\frac{e x}{\sqrt{e d}}\right)}{e^{4} \sqrt{e d}}$
risch	$\frac{c^{2} x^{7}}{7 e}-\frac{c^{2} d\,x^{5}}{5 e^{2}}+\frac{2 c a\,x^{3}}{3 e}+\frac{c^{2} d^{2} x^{3}}{3 e^{3}}-\frac{2 c a d x}{e^{2}}-\frac{c^{2} d^{3} x}{e^{4}}-\frac{\ln\left(e x+\sqrt{-e d}\right) a^{2}}{2 \sqrt{-e d}}-\frac{\ln\left(e x+\sqrt{-e d}\right) a c\,d^{2}}{e^{2} \sqrt{-e d}}-\frac{\ln\left(e x+\sqrt{-e d}\right) c^{2} d^{4}}{2 e^{4} \sqrt{-e d}}+$

```
[In] int((c*x^4+a)^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -c/e^4*(-1/7*c*x^7*e^3+1/5*c*d*x^5*e^2-1/3*(2*a*e^2+c*d^2)*x^3*e+d*(2*a*e^2
+c*d^2)*x)+(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^4/(e*d)^(1/2)*arctan(e*x/(e*d)
^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.48

$$\int \frac{\left(a+cx^4\right)^2}{d+ex^2}\,dx$$

$$= \Bigg[\frac{30\,c^2de^4x^7-42\,c^2d^2e^3x^5+70\,(c^2d^3e^2+2\,acde^4)x^3-105\,(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{-de}\log\Bigg(\frac{ex^2-2\,\sqrt{-de}x-}{ex^2+d}}{210\,de^5}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^2*d*e^4*x^7 - 42*c^2*d^2*e^3*x^5 + 70*(c^2*d^3*e^2 + 2*a*c*d*e
^4)*x^3 - 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-d*e)*log((e*x^2 - 2
*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5
), 1/105*(15*c^2*d*e^4*x^7 - 21*c^2*d^2*e^3*x^5 + 35*(c^2*d^3*e^2 + 2*a*c*d
*e^4)*x^3 + 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)*arctan(sqrt(d
*e)*x/d) - 105*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(100) = 200$.

Time = 0.23 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.19

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = -\frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + x^3\cdot\left(\frac{2ac}{3e}+\frac{c^2d^2}{3e^3}\right) + x\left(-\frac{2acd}{e^2}-\frac{c^2d^3}{e^4}\right) - \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}(ae^2+cd^2)^2}{a^2e^4+2acd^2e^2+c^2d^4}+x\right)}{2} + \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2\log\left(\frac{de^4\sqrt{-\frac{1}{de^9}}(ae^2+cd^2)^2}{a^2e^4+2acd^2e^2+c^2d^4}+x\right)}{2}$$

```
[In] integrate((c*x**4+a)**2/(e*x**2+d),x)
```

```
[Out] -c**2*d*x**5/(5*e**2) + c**2*x**7/(7*e) + x**3*(2*a*c/(3*e) + c**2*d**2/(3*
e**3)) + x*(-2*a*c*d/e**2 - c**2*d**3/e**4) - sqrt(-1/(d*e**9))*(a*e**2 + c
*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2
*a*c*d**2*e**2 + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2
*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2*a*c*d**2*
e**2 + c**2*d**4) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \frac{(c^2d^4+2\,acd^2e^2+a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^4} + \frac{15\,c^2e^6x^7-21\,c^2de^5x^5+35\,c^2d^2e^4x^3+70\,ace^6x^3-105\,c^2d^3e^3x-210\,acde^5x}{105\,e^7}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) +
 1/105*(15*c^2*e^6*x^7 - 21*c^2*d*e^5*x^5 + 35*c^2*d^2*e^4*x^3 + 70*a*c*e^6
*x^3 - 105*c^2*d^3*e^3*x - 210*a*c*d*e^5*x)/e^7
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.31

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = x^3\left(\frac{c^2\,d^2}{3\,e^3}+\frac{2\,a\,c}{3\,e}\right)+\frac{c^2\,x^7}{7\,e}-\frac{c^2\,d\,x^5}{5\,e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,\left(c\,d^2+a\,e^2\right)^2}{\sqrt{d}\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)^2}{\sqrt{d}\,e^{9/2}} - \frac{d\,x\,\left(\frac{c^2\,d^2}{e^3}+\frac{2\,a\,c}{e}\right)}{e}$$

```
[In] int((a + c*x^4)^2/(d + e*x^2),x)
```

```
[Out] x^3*((c^2*d^2)/(3*e^3) + (2*a*c)/(3*e)) + (c^2*x^7)/(7*e) - (c^2*d*x^5)/(5*
e^2) + (atan((e^(1/2)*x*(a*e^2 + c*d^2)^2)/(d^(1/2)*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2)))*(a*e^2 + c*d^2)^2)/(d^(1/2)*e^(9/2)) - (d*x*((c^2*d^2)/e^3 +
 (2*a*c)/e))/e
```

3.133 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx=\frac{c\left(3cd^2+2ae^2\right)x}{e^4}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}+\frac{\left(cd^2+ae^2\right)^2x}{2de^4\left(d+ex^2\right)}-\frac{\left(7cd^2-ae^2\right)\left(cd^2+ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

```
[Out] c*(2*a*e^2+3*c*d^2)*x/e^4-2/3*c^2*d*x^3/e^3+1/5*c^2*x^5/e^2+1/2*(a*e^2+c*d^
2)^2*x/d/e^4/(e*x^2+d)-1/2*(-a*e^2+7*c*d^2)*(a*e^2+c*d^2)*arctan(x*e^(1/2)/
d^(1/2))/d^(3/2)/e^(9/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1172, 1824, 211}

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx=-\frac{\left(7cd^2-ae^2\right)\left(ae^2+cd^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}+\frac{x\left(ae^2+cd^2\right)^2}{2de^4\left(d+ex^2\right)}+\frac{cx\left(2ae^2+3cd^2\right)}{e^4}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) +
 ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c*d^2 - a*e^2)*(c*d^2 +
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{\int \frac{-a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{2cd\left(cd^2+2ae^2\right)x^2}{e^3}+\frac{2c^2d^2x^4}{e^2}-\frac{2c^2dx^6}{e}}{d+ex^2}\,dx}{2d}\\
&= \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{\int \left(-\frac{2cd(3cd^2+2ae^2)}{e^4}+\frac{4c^2d^2x^2}{e^3}-\frac{2c^2dx^4}{e^2}+\frac{7c^2d^4+6acd^2e^2-a^2e^4}{e^4(d+ex^2)}\right)dx}{2d}\\
&= \frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{((7cd^2-ae^2)(cd^2+ae^2))\int \frac{1}{d+ex^2}\,dx}{2de^4}\\
&= \frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{(7cd^2-ae^2)(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\begin{aligned}
\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx &= \frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)}\\
&\quad - \frac{(7c^2d^4+6acd^2e^2-a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}
\end{aligned}$$

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^2,x]
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) +
 ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 6*a*c*d^2*e^2
- a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

method	result
default	$\frac{c\left(\frac{1}{5}c\,x^5e^2-\frac{2}{3}dc\,x^3e+2a\,e^2x+3c\,d^2x\right)}{e^4}+\frac{\frac{\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a^2e^4-6ac\,d^2e^2-7c^2d^4\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^4}$
risch	$\frac{c^2x^5}{5e^2}-\frac{2c^2d\,x^3}{3e^3}+\frac{2cax}{e^2}+\frac{3c^2d^2x}{e^4}+\frac{(a^2e^4+2ac\,d^2e^2+c^2d^4)x}{2d\,e^4(e\,x^2+d)}-\frac{\ln(ex+\sqrt{-ed})a^2}{4\sqrt{-ed}\,d}+\frac{3d\ln(ex+\sqrt{-ed})ac}{2e^2\sqrt{-ed}}+\frac{7d^3\ln(ex+\sqrt{-}}{4e^4\sqrt{-e}}$

```
[In] int((c*x^4+a)^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c/e^4*(1/5*c*x^5*e^2-2/3*d*c*x^3*e+2*a*e^2*x+3*c*d^2*x)+1/e^4*(1/2*(a^2*e^4
+2*a*c*d^2*e^2+c^2*d^4)/d*x/(e*x^2+d)+1/2*(a^2*e^4-6*a*c*d^2*e^2-7*c^2*d^4)
/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.01

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx = \left[\frac{12\,c^2d^2e^4x^7-28\,c^2d^3e^3x^5+20\,(7\,c^2d^4e^2+6\,acd^2e^4)x^3+15\,(7\,c^2d^5+6\,acd^3e^2-a^2de^4+(7\,c^2d^4e+6\,ac}{60\,(d^2e^6x^2+d^3e^5)}\right.$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^2*d^2*e^4*x^7 - 28*c^2*d^3*e^3*x^5 + 20*(7*c^2*d^4*e^2 + 6*a*c*
d^2*e^4)*x^3 + 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*d^4*e + 6
*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e
*x^2 + d)) + 30*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x^2 +
 d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 14*c^2*d^3*e^3*x^5 + 10*(7*c^2*d^4*e^2
 + 6*a*c*d^2*e^4)*x^3 - 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*
d^4*e + 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 15*
(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x^2 + d^3*e^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(122) = 244$.

Time = 0.41 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.40

$$
\begin{aligned}
\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx
&= -\frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + x\left(\frac{2ac}{e^2} + \frac{3c^2d^2}{e^4}\right) + \frac{x(a^2e^4 + 2acd^2e^2 + c^2d^4)}{2d^2e^4 + 2de^5x^2} \\
&\quad - \frac{\sqrt{-\frac{1}{d^3e^9}}\,(ae^2 - 7cd^2)\,(ae^2 + cd^2)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}(ae^2-7cd^2)(ae^2+cd^2)}{a^2e^4-6acd^2e^2-7c^2d^4} + x\right)}{4} \\
&\quad + \frac{\sqrt{-\frac{1}{d^3e^9}}\,(ae^2 - 7cd^2)\,(ae^2 + cd^2)\log\left(\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}(ae^2-7cd^2)(ae^2+cd^2)}{a^2e^4-6acd^2e^2-7c^2d^4} + x\right)}{4}
\end{aligned}
$$

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**2,x)
```

```
[Out] -2*c**2*d*x**3/(3*e**3) + c**2*x**5/(5*e**2) + x*(2*a*c/e**2 + 3*c**2*d**2/
e**4) + x*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(2*d**2*e**4 + 2*d*e**5
*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(-d*
*2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e*
*4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e**9))*(a*e**2 -
 7*c*d**2)*(a*e**2 + c*d**2)*log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7
*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)
/4
```

Maxima [F(-2)]

Exception generated.

$$
\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}
$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the ’assume’ command before evaluation *may* h
elp (example of legal syntax is ’assume(e>0)’, see ‘assume?‘ for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = -\frac{(7\,c^2d^4+6\,acd^2e^2-a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^4}+\frac{c^2d^4x+2\,acd^2e^2x+a^2e^4x}{2\,(ex^2+d)de^4} + \frac{3\,c^2e^8x^5-10\,c^2de^7x^3+45\,c^2d^2e^6x+30\,ace^8x}{15\,e^{10}}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)
*d*e^4) + 1/2*(c^2*d^4*x + 2*a*c*d^2*e^2*x + a^2*e^4*x)/((e*x^2 + d)*d*e^4)
 + 1/15*(3*c^2*e^8*x^5 - 10*c^2*d*e^7*x^3 + 45*c^2*d^2*e^6*x + 30*a*c*e^8*x
)/e^10
```

Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.40

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = x\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,a\,c}{e^2}\right)+\frac{c^2\,x^5}{5\,e^2}-\frac{2\,c^2\,d\,x^3}{3\,e^3}+\frac{x\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)}{2\,d\,\left(e^5\,x^2+d\,e^4\right)} - \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,\left(c\,d^2+a\,e^2\right)\,\left(a\,e^2-7\,c\,d^2\right)}{\sqrt{d}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2+7\,c^2\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)\,\left(a\,e^2-7\,c\,d^2\right)}{2\,d^{3/2}\,e^{9/2}}$$

```
[In] int((a + c*x^4)^2/(d + e*x^2)^2,x)
```

```
[Out] x*((3*c^2*d^2)/e^4 + (2*a*c)/e^2) + (c^2*x^5)/(5*e^2) - (2*c^2*d*x^3)/(3*e^
3) + (x*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(2*d*(d*e^4 + e^5*x^2)) - (ata
n((e^(1/2)*x*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(d^(1/2)*(7*c^2*d^4 - a^2*e
^4 + 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(2*d^(3/2)*e^(9/2)
)
```

3.134 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = -\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}+\frac{\left(cd^2+ae^2\right)^2x}{4de^4\left(d+ex^2\right)^2}+\frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)} + \frac{\left(35c^2d^4+6acd^2e^2+3a^2e^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}$$

```
[Out] -3*c^2*d*x/e^4+1/3*c^2*x^3/e^3+1/4*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^2+1/8*
(3*a^2-13*c^2*d^4/e^4-10*a*c*d^2/e^2)*x/d^2/(e*x^2+d)+1/8*(3*a^2*e^4+6*a*c*
d^2*e^2+35*c^2*d^4)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(9/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1172, 1828, 1167, 211}

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = \frac{\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}} + \frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{8d^2\left(d+ex^2\right)}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}$$

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (-3*c^2*d*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + a*e^2)^2*x)/(4*d*e^4*(d +
e*x^2)^2) + ((3*a^2 - (13*c^2*d^4)/e^4 - (10*a*c*d^2)/e^2)*x)/(8*d^2*(d + e
```

```
*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(8*d^(5/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\int \frac{-3a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{4cd\left(cd^2+2ae^2\right)x^2}{e^3}+\frac{4c^2d^2x^4}{e^2}-\frac{4c^2dx^6}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}\\
&= \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)} + \frac{\int \frac{3a^2+\frac{11c^2d^4}{e^4}+\frac{6acd^2}{e^2}-\frac{16c^2d^3x^2}{e^3}+\frac{8c^2d^2x^4}{e^2}}{d+ex^2}\,dx}{8d^2}\\
&= \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)} + \frac{\int \left(-\frac{24c^2d^3}{e^4}+\frac{8c^2d^2x^2}{e^3}+\frac{35c^2d^4+6acd^2e^2+3a^2e^4}{e^4\left(d+ex^2\right)}\right)dx}{8d^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2+ae^2)^2\,x}{4de^4\,(d+ex^2)^2} + \frac{\left(3a^2 - \frac{13c^2d^4}{e^4} - \frac{10acd^2}{e^2}\right)x}{8d^2\,(d+ex^2)} \\
&\quad + \frac{(35c^2d^4 + 6acd^2e^2 + 3a^2e^4)\int \frac{1}{d+ex^2}\,dx}{8d^2e^4} \\
&= -\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2+ae^2)^2\,x}{4de^4\,(d+ex^2)^2} + \frac{\left(3a^2 - \frac{13c^2d^4}{e^4} - \frac{10acd^2}{e^2}\right)x}{8d^2\,(d+ex^2)} \\
&\quad + \frac{(35c^2d^4 + 6acd^2e^2 + 3a^2e^4)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

$$
\begin{aligned}
&\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx \\
&= \frac{x(3a^2e^4(5d+3ex^2) - 6acd^2e^2(3d+5ex^2) - c^2d^2(105d^3 + 175d^2ex^2 + 56de^2x^4 - 8e^3x^6))}{24d^2e^4\,(d+ex^2)^2} \\
&\quad + \frac{(35c^2d^4 + 6acd^2e^2 + 3a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}
\end{aligned}
$$

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (x*(3*a^2*e^4*(5*d + 3*e*x^2) - 6*a*c*d^2*e^2*(3*d + 5*e*x^2) - c^2*d^2*(10
5*d^3 + 175*d^2*e*x^2 + 56*d*e^2*x^4 - 8*e^3*x^6)))/(24*d^2*e^4*(d + e*x^2)
^2) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(8*d^(5/2)*e^(9/2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.98

method	result
default	$-\frac{c^2\left(-\frac{1}{3}e\,x^3+3dx\right)}{e^4}+\frac{\frac{\frac{e\left(3a^2e^4-10ac\,d^2e^2-13c^2d^4\right)x^3}{8d^2}+\frac{\left(5a^2e^4-6ac\,d^2e^2-11c^2d^4\right)x}{8d}}{\left(e\,x^2+d\right)^2}+\frac{\left(3a^2e^4+6ac\,d^2e^2+35c^2d^4\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8d^2\sqrt{ed}}}{e^4}$
risch	$\frac{c^2x^3}{3e^3}-\frac{3c^2dx}{e^4}+\frac{\frac{e\left(3a^2e^4-10ac\,d^2e^2-13c^2d^4\right)x^3}{8d^2}+\frac{\left(5a^2e^4-6ac\,d^2e^2-11c^2d^4\right)x}{8d}}{e^4\left(e\,x^2+d\right)^2}-\frac{3\ln\left(ex+\sqrt{-ed}\right)a^2}{16\sqrt{-ed}\,d^2}-\frac{3\ln\left(ex+\sqrt{-ed}\right)ac}{8e^2\sqrt{-ed}}-35d^2$

```
[In] int((c*x^4+a)^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c^2/e^4*(-1/3*e*x^3+3*d*x)+1/e^4*((1/8*e*(3*a^2*e^4-10*a*c*d^2*e^2-13*c^2*
d^4)/d^2*x^3+1/8*(5*a^2*e^4-6*a*c*d^2*e^2-11*c^2*d^4)/d*x)/(e*x^2+d)^2+1/8*
(3*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)/d^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)
))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.33

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx$$

$$= \left[\frac{16\,c^2d^3e^4x^7 - 112\,c^2d^4e^3x^5 - 2\,(175\,c^2d^5e^2 + 30\,acd^3e^4 - 9\,a^2de^6)x^3 - 3\,(35\,c^2d^6 + 6\,acd^4e^2 + 3\,a^2d^2e^4 +}{}\right.$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^2*d^3*e^4*x^7 - 112*c^2*d^4*e^3*x^5 - 2*(175*c^2*d^5*e^2 + 30*a
*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 - 3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e
^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*e^6)*x^4 + 2*(35*c^2*d^5*e + 6
*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d
)/(e*x^2 + d)) - 6*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(d^3*e
^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 56*c^2*d^4*e^3
*x^5 - (175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 + 3*(35*c^2*d^6
 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*
e^6)*x^4 + 2*(35*c^2*d^5*e + 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqrt(d*e)*ar
ctan(sqrt(d*e)*x/d) - 3*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(
d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.66

$$\begin{aligned}
\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = {} & -\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} \\
& - \frac{\sqrt{-\frac{1}{d^5e^9}}\cdot\left(3a^2e^4 + 6acd^2e^2 + 35c^2d^4\right)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} \\
& + \frac{\sqrt{-\frac{1}{d^5e^9}}\cdot\left(3a^2e^4 + 6acd^2e^2 + 35c^2d^4\right)\log\left(d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} \\
& + \frac{x^3\cdot\left(3a^2e^5 - 10acd^2e^3 - 13c^2d^4e\right) + x\left(5a^2de^4 - 6acd^3e^2 - 11c^2d^5\right)}{8d^4e^4 + 16d^3e^5x^2 + 8d^2e^6x^4}
\end{aligned}$$

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**3,x)
```

```
[Out] -3*c**2*d*x/e**4 + c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a**2*e**4 +
 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/1
6 + sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log
(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 - 10*a*c*d**2*
e**3 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 6*a*c*d**3*e**2 - 11*c**2*d**5)
)/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = \frac{\left(35\,c^2d^4+6\,acd^2e^2+3\,a^2e^4\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^4} - \frac{13\,c^2d^4ex^3+10\,acd^2e^3x^3-3\,a^2e^5x^3+11\,c^2d^5x+6\,acd^3e^2x-5\,a^2de^4x}{8\,\left(ex^2+d\right)^2d^2e^4} + \frac{c^2e^6x^3-9\,c^2de^5x}{3\,e^9}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*
e)*d^2*e^4) - 1/8*(13*c^2*d^4*e*x^3 + 10*a*c*d^2*e^3*x^3 - 3*a^2*e^5*x^3 +
11*c^2*d^5*x + 6*a*c*d^3*e^2*x - 5*a^2*d*e^4*x)/((e*x^2 + d)^2*d^2*e^4) + 1
/3*(c^2*e^6*x^3 - 9*c^2*d*e^5*x)/e^9
```

Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx=\frac{c^2\,x^3}{3\,e^3}-\frac{\frac{x^3\,\left(-3\,a^2\,e^5+10\,a\,c\,d^2\,e^3+13\,c^2\,d^4\,e\right)}{8\,d^2}+\frac{x\,\left(-5\,a^2\,e^4+6\,a\,c\,d^2\,e^2+11\,c^2\,d^4\right)}{8\,d}}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4\right)}{8\,d^{5/2}\,e^{9/2}}-\frac{3\,c^2\,d\,x}{e^4}$$

```
[In] int((a + c*x^4)^2/(d + e*x^2)^3,x)
```

```
[Out] (c^2*x^3)/(3*e^3) - ((x^3*(13*c^2*d^4*e - 3*a^2*e^5 + 10*a*c*d^2*e^3))/(8*d
^2) + (x*(11*c^2*d^4 - 5*a^2*e^4 + 6*a*c*d^2*e^2))/(8*d))/(d^2*e^4 + e^6*x^
4 + 2*d*e^5*x^2) + (atan((e^(1/2)*x)/d^(1/2))*(3*a^2*e^4 + 35*c^2*d^4 + 6*a
*c*d^2*e^2))/(8*d^(5/2)*e^(9/2)) - (3*c^2*d*x)/e^4
```

3.135 $\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 184

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{c^2x}{e^4} + \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{(35c^2d^4-2acd^2e^2-5a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

```
[Out] c^2*x/e^4+1/6*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^3+1/24*(5*a^2-19*c^2*d^4/e^
4-14*a*c*d^2/e^2)*x/d^2/(e*x^2+d)^2+1/16*(5*a^2+29*c^2*d^4/e^4+2*a*c*d^2/e^
2)*x/d^3/(e*x^2+d)-1/16*(-5*a^2*e^4-2*a*c*d^2*e^2+35*c^2*d^4)*arctan(x*e^(1
/2)/d^(1/2))/d^(7/2)/e^(9/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1172, 1828, 1171, 396, 211}

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = -\frac{(-5a^2e^4-2acd^2e^2+35c^2d^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}} + \frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{24d^2(d+ex^2)^2} + \frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{16d^3(d+ex^2)} + \frac{x(ae^2+cd^2)^2}{6de^4(d+ex^2)^3} + \frac{c^2x}{e^4}$$

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) + ((5*a^2 - (19
*c^2*d^4)/e^4 - (14*a*c*d^2)/e^2)*x)/(24*d^2*(d + e*x^2)^2) + ((5*a^2 + (29
*c^2*d^4)/e^4 + (2*a*c*d^2)/e^2)*x)/(16*d^3*(d + e*x^2)) - ((35*c^2*d^4 - 2
*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(cd^2+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\int \frac{-5a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{6cd\left(cd^2+2ae^2\right)x^2}{e^3}+\frac{6c^2d^2x^4}{e^2}-\frac{6c^2dx^6}{e}}{\left(d+ex^2\right)^3}\,dx}{6d}$$

$$= \frac{\left(cd^2+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2\left(d+ex^2\right)^2} + \frac{\int \frac{3\left(5a^2+\frac{5c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)-\frac{48c^2d^3x^2}{e^3}+\frac{24c^2d^2x^4}{e^2}}{\left(d+ex^2\right)^2}\,dx}{24d^2}$$

$$= \frac{\left(cd^2+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2\left(d+ex^2\right)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3\left(d+ex^2\right)} - \frac{\int \frac{-3\left(5a^2-\frac{19c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)-\frac{48c^2d^3x^2}{e^3}}{d+ex^2}\,dx}{48d^3}$$

$$= \frac{c^2x}{e^4} + \frac{\left(cd^2+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2\left(d+ex^2\right)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3\left(d+ex^2\right)} - \frac{\left(35c^2d^4-2acd^2e^2-5a^2e^4\right)\int \frac{1}{d+ex^2}\,dx}{16d^3e^4}$$

$$= \frac{c^2x}{e^4} + \frac{\left(cd^2+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2\left(d+ex^2\right)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3\left(d+ex^2\right)} - \frac{\left(35c^2d^4-2acd^2e^2-5a^2e^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^4}\,dx = \frac{x(-2acd^2e^2(3d^2+8dex^2-3e^2x^4)+a^2e^4(33d^2+40dex^2+15e^2x^4)+c^2d^3(105d^3+280d^2ex^2+231de^2x^4+}{48d^3e^4\left(d+ex^2\right)^3} - \frac{\left(35c^2d^4-2acd^2e^2-5a^2e^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (x*(-2*a*c*d^2*e^2*(3*d^2 + 8*d*e*x^2 - 3*e^2*x^4) + a^2*e^4*(33*d^2 + 40*d
*e*x^2 + 15*e^2*x^4) + c^2*d^3*(105*d^3 + 280*d^2*e*x^2 + 231*d*e^2*x^4 + 4
8*e^3*x^6)))/(48*d^3*e^4*(d + e*x^2)^3) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*
a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

method	result
default	$\frac{c^{2} x}{e^{4}}+\frac{\frac{\frac{e^{2}\left(5 a^{2} e^{4}+2 a c\,d^{2} e^{2}+29 c^{2} d^{4}\right) x^{5}}{16 d^{3}}+\frac{e\left(5 a^{2} e^{4}-2 a c\,d^{2} e^{2}+17 c^{2} d^{4}\right) x^{3}}{6 d^{2}}+\frac{\left(11 a^{2} e^{4}-2 a c\,d^{2} e^{2}+19 c^{2} d^{4}\right) x}{16 d}}{\left(e\,x^{2}+d\right)^{3}}+\frac{\left(5 a^{2} e^{4}+2 a c\,d^{2} e^{2}-35 c^{2} d^{4}\right) \arctan}{16 d^{3} \sqrt{e d}}}{e^{4}}$
risch	$\frac{c^{2} x}{e^{4}}+\frac{\frac{e^{2}\left(5 a^{2} e^{4}+2 a c\,d^{2} e^{2}+29 c^{2} d^{4}\right) x^{5}}{16 d^{3}}+\frac{e\left(5 a^{2} e^{4}-2 a c\,d^{2} e^{2}+17 c^{2} d^{4}\right) x^{3}}{6 d^{2}}+\frac{\left(11 a^{2} e^{4}-2 a c\,d^{2} e^{2}+19 c^{2} d^{4}\right) x}{16 d}}{e^{4}\left(e\,x^{2}+d\right)^{3}}-\frac{5 \ln\left(e x+\sqrt{-e d}\right) a^{2}}{32 \sqrt{-e d}\, d^{3}}-\frac{\ln(ex}{16e}$

```
[In] int((c*x^4+a)^2/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*x/e^4+1/e^4*((1/16*e^2*(5*a^2*e^4+2*a*c*d^2*e^2+29*c^2*d^4)/d^3*x^5+1/6
*e*(5*a^2*e^4-2*a*c*d^2*e^2+17*c^2*d^4)/d^2*x^3+1/16*(11*a^2*e^4-2*a*c*d^2*
e^2+19*c^2*d^4)/d*x)/(e*x^2+d)^3+1/16*(5*a^2*e^4+2*a*c*d^2*e^2-35*c^2*d^4)/
d^3/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.60

$$\int \frac{\left(a+c x^4\right)^2}{\left(d+e x^2\right)^4}\,dx$$

$$=\left[\frac{96\,c^2 d^4 e^4 x^7+6\,(77\,c^2 d^5 e^3+2\,acd^3 e^5+5\,a^2 de^7)x^5+16\,(35\,c^2 d^6 e^2-2\,acd^4 e^4+5\,a^2 d^2 e^6)x^3+3\,(35\,c^2 d^7}{}\right.$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7
)*x^5 + 16*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 + 3*(35*c^2
*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5*
a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35
*c^2*d^6*e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*
sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d
^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5), 1/48*(4
8*c^2*d^4*e^4*x^7 + 3*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7)*x^5 +
8*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 - 3*(35*c^2*d^7 - 2*
a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5*a^2*e^7)*
x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35*c^2*d^6*
e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 3
*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^
7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.59

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{c^2x}{e^4} - \frac{\sqrt{-\frac{1}{d^7e^9}}\cdot(5a^2e^4+2acd^2e^2-35c^2d^4)\log\left(-d^4e^4\sqrt{-\frac{1}{d^7e^9}}+x\right)}{32}$$
$$+\frac{\sqrt{-\frac{1}{d^7e^9}}\cdot(5a^2e^4+2acd^2e^2-35c^2d^4)\log\left(d^4e^4\sqrt{-\frac{1}{d^7e^9}}+x\right)}{32}$$
$$+\frac{x^5\cdot(15a^2e^6+6acd^2e^4+87c^2d^4e^2)+x^3\cdot(40a^2de^5-16acd^3e^3+136c^2d^5e)+x(33a^2d^2e^4-6acd^4e^2+57}{48d^6e^4+144d^5e^5x^2+144d^4e^6x^4+48d^3e^7x^6}$$

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**4,x)
```

```
[Out] c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2
*d**4)*log(-d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))*(
5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)*log(d**4*e**4*sqrt(-1/(d**7*e
**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*c*d**2*e**4 + 87*c**2*d**4*e**2)
+ x**3*(40*a**2*d*e**5 - 16*a*c*d**3*e**3 + 136*c**2*d**5*e) + x*(33*a**2*d
**2*e**4 - 6*a*c*d**4*e**2 + 57*c**2*d**6))/(48*d**6*e**4 + 144*d**5*e**5*x
**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{c^2x}{e^4} - \frac{(35\,c^2d^4 - 2\,acd^2e^2 - 5\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^4}$$
$$+ \frac{87\,c^2d^4e^2x^5 + 6\,acd^2e^4x^5 + 15\,a^2e^6x^5 + 136\,c^2d^5ex^3 - 16\,acd^3e^3x^3 + 40\,a^2de^5x^3 + 57\,c^2d^6x - 6\,acd^4e^2x}{48\,(ex^2+d)^3d^3e^4}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x/e^4 - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arctan(e*x/sqrt(d
*e))/(sqrt(d*e)*d^3*e^4) + 1/48*(87*c^2*d^4*e^2*x^5 + 6*a*c*d^2*e^4*x^5 + 1
5*a^2*e^6*x^5 + 136*c^2*d^5*e*x^3 - 16*a*c*d^3*e^3*x^3 + 40*a^2*d*e^5*x^3 +
 57*c^2*d^6*x - 6*a*c*d^4*e^2*x + 33*a^2*d^2*e^4*x)/((e*x^2 + d)^3*d^3*e^4)
```

Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx$$
$$= \frac{\frac{x^3\,(5\,a^2\,e^5-2\,a\,c\,d^2\,e^3+17\,c^2\,d^4\,e)}{6\,d^2} + \frac{x\,(11\,a^2\,e^4-2\,a\,c\,d^2\,e^2+19\,c^2\,d^4)}{16\,d} + \frac{x^5\,(5\,a^2\,e^6+2\,a\,c\,d^2\,e^4+29\,c^2\,d^4\,e^2)}{16\,d^3}}{d^3\,e^4+3\,d^2\,e^5\,x^2+3\,d\,e^6\,x^4+e^7\,x^6}$$
$$+ \frac{c^2\,x}{e^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(5\,a^2\,e^4+2\,a\,c\,d^2\,e^2-35\,c^2\,d^4)}{16\,d^{7/2}\,e^{9/2}}$$

```
[In] int((a + c*x^4)^2/(d + e*x^2)^4,x)
```

```
[Out] ((x^3*(5*a^2*e^5 + 17*c^2*d^4*e - 2*a*c*d^2*e^3))/(6*d^2) + (x*(11*a^2*e^4
+ 19*c^2*d^4 - 2*a*c*d^2*e^2))/(16*d) + (x^5*(5*a^2*e^6 + 29*c^2*d^4*e^2 +
2*a*c*d^2*e^4))/(16*d^3))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x^2)
 + (c^2*x)/e^4 + (atan((e^(1/2)*x)/d^(1/2))*(5*a^2*e^4 - 35*c^2*d^4 + 2*a*c
*d^2*e^2))/(16*d^(7/2)*e^(9/2))
```

3.136 $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 223

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^5}\,dx &= \frac{\left(cd^2+ae^2\right)^2x}{8de^4\left(d+ex^2\right)^4}+\frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2\left(d+ex^2\right)^3}\\ &\quad+\frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3\left(d+ex^2\right)^2}-\frac{\left(93c^2d^4-6acd^2e^2-35a^2e^4\right)x}{128d^4e^4\left(d+ex^2\right)}\\ &\quad+\frac{\left(35c^2d^4+6acd^2e^2+35a^2e^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}\end{aligned}$$

```
[Out] 1/8*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^4+1/48*(7*a^2-25*c^2*d^4/e^4-18*a*c*d
^2/e^2)*x/d^2/(e*x^2+d)^3+1/192*(35*a^2+163*c^2*d^4/e^4+6*a*c*d^2/e^2)*x/d^
3/(e*x^2+d)^2-1/128*(-35*a^2*e^4-6*a*c*d^2*e^2+93*c^2*d^4)*x/d^4/e^4/(e*x^2
+d)+1/128*(35*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)*arctan(x*e^(1/2)/d^(1/2))/d
^(9/2)/e^(9/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

$= \{1172, 1828, 1171, 393, 211\}$

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{(35a^2e^4+6acd^2e^2+35c^2d^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}} - \frac{x(-35a^2e^4-6acd^2e^2+93c^2d^4)}{128d^4e^4(d+ex^2)} + \frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{48d^2(d+ex^2)^3} + \frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{192d^3(d+ex^2)^2} + \frac{x(ae^2+cd^2)^2}{8de^4(d+ex^2)^4}$$

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) + ((7*a^2 - (25*c^2*d^4)/e^4
- (18*a*c*d^2)/e^2)*x)/(48*d^2*(d + e*x^2)^3) + ((35*a^2 + (163*c^2*d^4)/e^
4 + (6*a*c*d^2)/e^2)*x)/(192*d^3*(d + e*x^2)^2) - ((93*c^2*d^4 - 6*a*c*d^2*
e^2 - 35*a^2*e^4)*x)/(128*d^4*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e
^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
```

```
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(cd^2+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\int \frac{-7a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{8cd\left(cd^2+2ae^2\right)x^2}{e^3}+\frac{8c^2d^2x^4}{e^2}-\frac{8c^2dx^6}{e}}{\left(d+ex^2\right)^4}\,dx}{8d}\\
&= \frac{\left(cd^2+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2\left(d+ex^2\right)^3} + \frac{\int \frac{35a^2+\frac{19c^2d^4}{e^4}+\frac{6acd^2}{e^2}-\frac{96c^2d^3x^2}{e^3}+\frac{48c^2d^2x^4}{e^2}}{\left(d+ex^2\right)^3}\,dx}{48d^2}\\
&= \frac{\left(cd^2+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2\left(d+ex^2\right)^3}\\
&\quad + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3\left(d+ex^2\right)^2} - \frac{\int \frac{-3\left(35a^2-\frac{29c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)-\frac{192c^2d^3x^2}{e^3}}{\left(d+ex^2\right)^2}\,dx}{192d^3}\\
&= \frac{\left(cd^2+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2\left(d+ex^2\right)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3\left(d+ex^2\right)^2}\\
&\quad - \frac{\left(93c^2d^4-6acd^2e^2-35a^2e^4\right)x}{128d^4e^4\left(d+ex^2\right)} + \frac{\left(35c^2d^4+6acd^2e^2+35a^2e^4\right)\int \frac{1}{d+ex^2}\,dx}{128d^4e^4}\\
&= \frac{\left(cd^2+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2\left(d+ex^2\right)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3\left(d+ex^2\right)^2}\\
&\quad - \frac{\left(93c^2d^4-6acd^2e^2-35a^2e^4\right)x}{128d^4e^4\left(d+ex^2\right)} + \frac{\left(35c^2d^4+6acd^2e^2+35a^2e^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.90

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$=\frac{\frac{\sqrt{d}\sqrt{e}x(-6acd^2e^2(3d^3+11d^2ex^2-11de^2x^4-3e^3x^6)+a^2e^4(279d^3+511d^2ex^2+385de^2x^4+105e^3x^6)-c^2d^4(105d^3+385d^2ex^2+511de^2x^4+279e^3x}{(d+ex^2)^4}}{384d^{9/2}e^{9/2}}$$

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((Sqrt[d]*Sqrt[e]*x*(-6*a*c*d^2*e^2*(3*d^3 + 11*d^2*e*x^2 - 11*d*e^2*x^4 -
3*e^3*x^6) + a^2*e^4*(279*d^3 + 511*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6
) - c^2*d^4*(105*d^3 + 385*d^2*e*x^2 + 511*d*e^2*x^4 + 279*e^3*x^6)))/(d +
e*x^2)^4 + 3*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(384*d^(9/2)*e^(9/2))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{(35a^2e^4+6ac\,d^2e^2-93c^2d^4)x^7}{128d^4e}+\frac{(385a^2e^4+66ac\,d^2e^2-511c^2d^4)x^5}{384d^3e^2}+\frac{(511a^2e^4-66ac\,d^2e^2-385c^2d^4)x^3}{384d^2e^3}+\frac{(93a^2e^4-6ac\,d^2e^2-35c^2d^4)x}{128e^4d}}{(e\,x^2+d)^4}+$
risch	$\frac{\frac{(35a^2e^4+6ac\,d^2e^2-93c^2d^4)x^7}{128d^4e}+\frac{(385a^2e^4+66ac\,d^2e^2-511c^2d^4)x^5}{384d^3e^2}+\frac{(511a^2e^4-66ac\,d^2e^2-385c^2d^4)x^3}{384d^2e^3}+\frac{(93a^2e^4-6ac\,d^2e^2-35c^2d^4)x}{128e^4d}}{(e\,x^2+d)^4}-$

```
[In] int((c*x^4+a)^2/(e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (1/128*(35*a^2*e^4+6*a*c*d^2*e^2-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+6
6*a*c*d^2*e^2-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4-66*a*c*d^2*e^2-38
5*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-6*a*c*d^2*e^2-35*c^2*d^4)/e^4/d*x)
/(e*x^2+d)^4+1/128*(35*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)/d^4/e^4/(e*d)^(1/2
)*arctan(e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 806, normalized size of antiderivative = 3.61

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx$$

$$= \Bigg[-\frac{6\,(93\,c^2d^5e^4-6\,acd^3e^6-35\,a^2de^8)x^7+2\,(511\,c^2d^6e^3-66\,acd^4e^5-385\,a^2d^2e^7)x^5+2\,(385\,c^2d^7e^2+66}{}$$

$$-\frac{3\,(93\,c^2d^5e^4-6\,acd^3e^6-35\,a^2de^8)x^7+(511\,c^2d^6e^3-66\,acd^4e^5-385\,a^2d^2e^7)x^5+(385\,c^2d^7e^2+66\,acd^5}{}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(6*(93*c^2*d^5*e^4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + 2*(511*c^2
*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*d^2*e^7)*x^5 + 2*(385*c^2*d^7*e^2 + 66*
a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 + 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2
*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^
5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e
^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5
)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2
*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 +
6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(93*c^2*d^5*e^4 - 6*a*c
*d^3*e^6 - 35*a^2*d*e^8)*x^7 + (511*c^2*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*
d^2*e^7)*x^5 + (385*c^2*d^7*e^2 + 66*a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 - 3
*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2
*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*
x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d
^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d)
 + 3*(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^
6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5)]
```

Sympy [A] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = -\frac{\sqrt{-\frac{1}{d^9e^9}}\cdot(35a^2e^4+6acd^2e^2+35c^2d^4)\log\left(-d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256}$$

$$+\frac{\sqrt{-\frac{1}{d^9e^9}}\cdot(35a^2e^4+6acd^2e^2+35c^2d^4)\log\left(d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256}$$

$$+\frac{x^7\cdot(105a^2e^7+18acd^2e^5-279c^2d^4e^3)+x^5\cdot(385a^2de^6+66acd^3e^4-511c^2d^5e^2)+x^3\cdot(511a^2d^2e^5-66}{384d^8e^4+1536d^7e^5x^2+2304d^6e^6x^4+1536d^5e^7x^6+}$$

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**5,x)
```

```
[Out] -sqrt(-1/(d**9*e**9))*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-
d**5*e**4*sqrt(-1/(d**9*e**9)) + x)/256 + sqrt(-1/(d**9*e**9))*(35*a**2*e**
4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(d**5*e**4*sqrt(-1/(d**9*e**9)) + x)
/256 + (x**7*(105*a**2*e**7 + 18*a*c*d**2*e**5 - 279*c**2*d**4*e**3) + x**5
*(385*a**2*d*e**6 + 66*a*c*d**3*e**4 - 511*c**2*d**5*e**2) + x**3*(511*a**2
*d**2*e**5 - 66*a*c*d**4*e**3 - 385*c**2*d**6*e) + x*(279*a**2*d**3*e**4 -
18*a*c*d**5*e**2 - 105*c**2*d**7))/(384*d**8*e**4 + 1536*d**7*e**5*x**2 + 2
304*d**6*e**6*x**4 + 1536*d**5*e**7*x**6 + 384*d**4*e**8*x**8)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.97

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{(35\,c^2d^4+6\,acd^2e^2+35\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\,\sqrt{de}d^4e^4} - \frac{279\,c^2d^4e^3x^7-18\,acd^2e^5x^7-105\,a^2e^7x^7+511\,c^2d^5e^2x^5-66\,acd^3e^4x^5-385\,a^2de^6x^5+385\,c^2d^6ex^3+}{384\,(ex^2+d)^4d^4e^4}$$

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt
(d*e)*d^4*e^4) - 1/384*(279*c^2*d^4*e^3*x^7 - 18*a*c*d^2*e^5*x^7 - 105*a^2*
e^7*x^7 + 511*c^2*d^5*e^2*x^5 - 66*a*c*d^3*e^4*x^5 - 385*a^2*d*e^6*x^5 + 38
5*c^2*d^6*e*x^3 + 66*a*c*d^4*e^3*x^3 - 511*a^2*d^2*e^5*x^3 + 105*c^2*d^7*x
+ 18*a*c*d^5*e^2*x - 279*a^2*d^3*e^4*x)/((e*x^2 + d)^4*d^4*e^4)
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4)}{128\,d^{9/2}\,e^{9/2}}$$
$$-\frac{\frac{x\,(-93\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4)}{128\,d\,e^4}-\frac{x^7\,(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2-93\,c^2\,d^4)}{128\,d^4\,e}+\frac{x^3\,(-511\,a^2\,e^4+66\,a\,c\,d^2\,e^2+385\,c^2\,d^4)}{384\,d^2\,e^3}-\frac{x^5\,(385\,a^2\,e^4+66\,a\,c\,d}{384\,d^3\,e}}{d^4+4\,d^3\,e\,x^2+6\,d^2\,e^2\,x^4+4\,d\,e^3\,x^6+e^4\,x^8}$$

```
[In] int((a + c*x^4)^2/(d + e*x^2)^5,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(35*a^2*e^4 + 35*c^2*d^4 + 6*a*c*d^2*e^2))/(128*
d^(9/2)*e^(9/2)) - ((x*(35*c^2*d^4 - 93*a^2*e^4 + 6*a*c*d^2*e^2))/(128*d*e^
4) - (x^7*(35*a^2*e^4 - 93*c^2*d^4 + 6*a*c*d^2*e^2))/(128*d^4*e) + (x^3*(38
5*c^2*d^4 - 511*a^2*e^4 + 66*a*c*d^2*e^2))/(384*d^2*e^3) - (x^5*(385*a^2*e^
4 - 511*c^2*d^4 + 66*a*c*d^2*e^2))/(384*d^3*e^2))/(d^4 + e^4*x^8 + 4*d^3*e*
x^2 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4)
```

3.137 $\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 437

$$\begin{aligned}
&\int \frac{(d+ex^2)^4}{a+cx^4}\,dx \\
&= \frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c} \\
&\quad-\frac{\left(c^2d^4-6acd^2e^2+a^2e^4+4\sqrt{a}\sqrt{c}de(cd^2-ae^2)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad+\frac{\left(c^2d^4-6acd^2e^2+a^2e^4+4\sqrt{a}\sqrt{c}de(cd^2-ae^2)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad-\frac{\left(c^2d^4-6acd^2e^2+a^2e^4-4\sqrt{a}\sqrt{c}de(cd^2-ae^2)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad+\frac{\left(c^2d^4-6acd^2e^2+a^2e^4-4\sqrt{a}\sqrt{c}de(cd^2-ae^2)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}
\end{aligned}$$

```
[Out] e^2*(-a*e^2+6*c*d^2)*x/c^2+4/3*d*e^3*x^3/c+1/5*e^4*x^5/c-1/8*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4-4*d*e*(
-a*e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4-4*d*e*(-a*
e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x
*2^(1/2)/a^(1/4))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4+4*d*e*(-a*e^2+c*d^2)*a^(1/
2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4+4*d*e*(-a*e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/
4)/c^(9/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1185, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}
&\int \frac{(d+ex^2)^4}{a+cx^4}\,dx \\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de(cd^2-ae^2)+c^2d^4\right)}{2\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de(cd^2-ae^2)+c^2d^4\right)}{2\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de(cd^2-ae^2)+c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de(cd^2-ae^2)+c^2d^4\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad+\frac{e^2x(6cd^2-ae^2)}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}
\end{aligned}$$

```
[In] Int[(d + e*x^2)^4/(a + c*x^4),x]
```

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(5*c) - ((c
^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) + ((c^2
*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((c^2*d
^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/
4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 -
a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{e^2(6cd^2-ae^2)}{c^2}+\frac{4de^3x^2}{c}+\frac{e^4x^4}{c}\right.\\ &\qquad \left.+\frac{c^2d^4-6acd^2e^2+a^2e^4+4cde(cd^2-ae^2)\,x^2}{c^2\,(a+cx^4)}\right)dx\\ &= \frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}+\frac{\int \frac{c^2d^4-6acd^2e^2+a^2e^4+4cde(cd^2-ae^2)x^2}{a+cx^4}\,dx}{c^2}\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c} \\
&\quad - \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c^2} \\
&\quad + \frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c^2} \\
&= \frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c} \\
&\quad + \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}c^{7/4}} \\
&\quad + \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}c^{7/4}} \\
&\quad + \frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c^2} \\
&\quad + \frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c^2} \\
&= \frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c} \\
&\quad + \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}} \\
&\quad - \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}} \\
&\quad + \frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}} \\
&\quad - \frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c} \\
&\quad - \frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}} \\
&\quad + \frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}} \\
&\quad + \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}} \\
&\quad - \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$$

$$= \frac{-120a^{3/4}\sqrt[4]{c}e^2(-6cd^2+ae^2)\,x+160a^{3/4}c^{5/4}de^3x^3+24a^{3/4}c^{5/4}e^4x^5-30\sqrt{2}\big(c^2d^4+4\sqrt{a}c^{3/2}d^3e-6acd^2e}{}$$

```
[In] Integrate[(d + e*x^2)^4/(a + c*x^4),x]
```

```
[Out] (-120*a^(3/4)*c^(1/4)*e^2*(-6*c*d^2 + a*e^2)*x + 160*a^(3/4)*c^(5/4)*d*e^3*
x^3 + 24*a^(3/4)*c^(5/4)*e^4*x^5 - 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*
d^3*e - 6*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] + 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*d^3*e - 6
*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)] - 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*c*d^2*
e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2] + 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*
c*d^2*e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(120*a^(3/4)*c^(9/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.27

method	result
risch	$\frac{e^{4} x^{5}}{5 c}+\frac{4 d\, e^{3} x^{3}}{3 c}-\frac{e^{4} a x}{c^{2}}+\frac{6 e^{2} d^{2} x}{c}+\frac{\sum_{\_R =\operatorname{RootOf}\left(c\,\_Z^{4}+a \right)}\frac{\left(4 d c e \left(-a\,e^{2}+c\,d^{2}\right) \_R^{2}+a^{2} e^{4}-6 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \ln \left(x -\_R \right)}{\_R^{3}}}{4 c^{3}}$
default	$-\frac{e^{2} \left(-\frac{1}{5} c\,x^{5} e^{2}-\frac{4}{3} d c\,x^{3} e +a\,e^{2} x -6 c\,d^{2} x \right)}{c^{2}}+\frac{\frac{\left(a^{2} e^{4}-6 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \right.}{8 a}}{}$

```
[In] int((e*x^2+d)^4/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e^4*x^5/c+4/3*d*e^3*x^3/c-e^4/c^2*a*x+6*e^2/c*d^2*x+1/4/c^3*sum((4*d*c*
e*(-a*e^2+c*d^2)*_R^2+a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)/_R^3*ln(x-_R),_R=RootO
f(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2878 vs. 2(352) = 704.

Time = 11.20 (sec) , antiderivative size = 2878, normalized size of antiderivative = 6.59

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*c*e^4*x^5 + 80*c*d*e^3*x^3 + 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c^2*
d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7
*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8
*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a
^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a^2*c^
6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^6*e^1
0 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^8*d^12 - 3
4*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^
4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e - a^4*c^7
*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a
^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^
2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^7*e -
 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16
- 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a
```

```
^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^
2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a
*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16 - 56*
a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^
4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^1
4 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a
^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^
6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x - (a*c^8*d^1
2 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a
^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e - a^
4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3
976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a
^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^
7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*
d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6
470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7
*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) + 15*c^2*sqrt(-(8*c^3*d^7*e -
 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-(c^8*d^16
- 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a
^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^
2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 -
 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c
^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^
8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 +
239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*d^3*e
 - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^
4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 +
924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c
^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-
(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^
6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 5
6*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*c^3*d^
7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-(c^8*
d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6
470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7
*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*
e^2 - 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*
a^5*c^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x -
(a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e
^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*
d^3*e - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^
12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^
10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(
-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*s
qrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^
```

```
10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^1
2 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) + 60*(6*c*d^2*e^2 -
 a*e^4)*x)/c^2
```

Sympy [A] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = x\left(-\frac{ae^4}{c^2}+\frac{6d^2e^2}{c}\right)$$
$$+\operatorname{RootSum}\left(256t^4a^3c^9+t^2\left(-256a^5c^5de^7+1792a^4c^6d^3e^5-1792a^3c^7d^5e^3+256a^2c^8d^7e\right)+a^8e^{16}+8a^7cd^2\right.$$
$$+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}$$

```
[In] integrate((e*x**2+d)**4/(c*x**4+a),x)
```

```
[Out] x*(-a*e**4/c**2 + 6*d**2*e**2/c) + RootSum(256*_t**4*a**3*c**9 + _t**2*(-25
6*a**5*c**5*d*e**7 + 1792*a**4*c**6*d**3*e**5 - 1792*a**3*c**7*d**5*e**3 +
256*a**2*c**8*d**7*e) + a**8*e**16 + 8*a**7*c*d**2*e**14 + 28*a**6*c**2*d**
4*e**12 + 56*a**5*c**3*d**6*e**10 + 70*a**4*c**4*d**8*e**8 + 56*a**3*c**5*d
**10*e**6 + 28*a**2*c**6*d**12*e**4 + 8*a*c**7*d**14*e**2 + c**8*d**16, Lam
bda(_t, _t*log(x + (256*_t**3*a**4*c**7*d*e**3 - 256*_t**3*a**3*c**8*d**3*e
 + 4*_t*a**7*c**2*e**12 - 264*_t*a**6*c**3*d**2*e**10 + 1980*_t*a**5*c**4*d
**4*e**8 - 3696*_t*a**4*c**5*d**6*e**6 + 1980*_t*a**3*c**6*d**8*e**4 - 264*
_t*a**2*c**7*d**10*e**2 + 4*_t*a*c**8*d**12)/(a**8*e**16 - 24*a**7*c*d**2*e
**14 - 36*a**6*c**2*d**4*e**12 + 88*a**5*c**3*d**6*e**10 + 198*a**4*c**4*d*
*8*e**8 + 88*a**3*c**5*d**10*e**6 - 36*a**2*c**6*d**12*e**4 - 24*a*c**7*d**
14*e**2 + c**8*d**16)))) + 4*d*e**3*x**3/(3*c) + e**4*x**5/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \frac{3\,ce^4x^5+20\,cde^3x^3+15\,(6\,cd^2e^2-ae^4)x}{15\,c^2}$$
$$+\frac{\frac{2\sqrt{2}\left(c^{\frac{5}{2}}d^4+4\sqrt{a}c^2d^3e-6\,ac^{\frac{3}{2}}d^2e^2-4\,a^{\frac{3}{2}}cde^3+a^2\sqrt{c}e^4\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(c^{\frac{5}{2}}d^4+4\sqrt{a}c^2d^3e-6\,ac^{\frac{3}{2}}d^2e^2-4\,a^{\frac{3}{2}}cde^3+\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{}}}}{}$$

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c*e^4*x^5 + 20*c*d*e^3*x^3 + 15*(6*c*d^2*e^2 - a*e^4)*x)/c^2 + 1/8*
(2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 - 4*a^(
3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a
^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(
c)) + 2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 -
4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*
sqrt(c)) + sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2
 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c
^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^
2*d^3*e - 6*a*c^(3/2)*d^2*e^2 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sq
rt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.17

$$\int \frac{\left(d+ex^2\right)^4}{a+cx^4}\,dx$$

$$= \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}c^3d^4-6\left(ac^3\right)^{\frac14}ac^2d^2e^2+\left(ac^3\right)^{\frac14}a^2ce^4+4\left(ac^3\right)^{\frac34}cd^3e-4\left(ac^3\right)^{\frac34}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right.}{2\left(\frac{a}{c}\right)^{\frac14}}\right.}{4\,ac^4}$$

$$+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}c^3d^4-6\left(ac^3\right)^{\frac14}ac^2d^2e^2+\left(ac^3\right)^{\frac14}a^2ce^4+4\left(ac^3\right)^{\frac34}cd^3e-4\left(ac^3\right)^{\frac34}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right.\right.}{2\left(\frac{a}{c}\right)^{\frac14}}\right.}{4\,ac^4}$$

$$+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}c^3d^4-6\left(ac^3\right)^{\frac14}ac^2d^2e^2+\left(ac^3\right)^{\frac14}a^2ce^4-4\left(ac^3\right)^{\frac34}cd^3e+4\left(ac^3\right)^{\frac34}ade^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}\right.}{8\,ac^4}$$

$$-\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac14}c^3d^4-6\left(ac^3\right)^{\frac14}ac^2d^2e^2+\left(ac^3\right)^{\frac14}a^2ce^4-4\left(ac^3\right)^{\frac34}cd^3e+4\left(ac^3\right)^{\frac34}ade^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}\right.}{8\,ac^4}$$

$$+\frac{3\,c^4e^4x^5+20\,c^4de^3x^3+90\,c^4d^2e^2x-15\,ac^3e^4x}{15\,c^5}$$

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3
)^(1/4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*c*d^3*e - 4*(a*c^3)^(3/4)*a*d*e^3)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/4*sqrt(
2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3)^(1/4)*a
^2*c*e^4 + 4*(a*c^3)^(3/4)*c*d^3*e - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^
3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3)^(1/4)*a^2*c*e^4
```

```
- 4*(a*c^3)^(3/4)*c*d^3*e + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt(2)*x*(a
/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*
c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*c*d^3*
e + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4) + 1/15*(3*c^4*e^4*x^5 + 20*c^4*d*e^3*x^3 + 90*c^4*d^2*e^2*x - 15*a*c
^3*e^4*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 4022, normalized size of antiderivative = 9.20

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^4/(a + c*x^4),x)
```

```
[Out] atan((((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a
^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*
((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8
*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2
*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*
(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^
4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5
*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^
2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))
^(1/2)*1i + ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6
+ 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2
*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7
*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d
^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^
4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2
) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c
^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^
3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3
*c^9))^(1/2)*1i)/(((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^
2*e^6 + 70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c
^5*d^2*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c
^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a
*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*
c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9
)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56
*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) -
 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(
16*a^3*c^9))^(1/2) - ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c
*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^
```

```
2*c^5*d^2*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^
2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 2
8*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a
^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*
c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 +
 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2
) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2)
)/(16*a^3*c^9))^(1/2) + (8*(a^5*d*e^11 - c^5*d^11*e - 3*a*c^4*d^9*e^3 + 3*a
^4*c*d^3*e^9 - 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7))/c^3))*((a^4*e^8*(-a^
3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7
 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1
/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/
2))/(16*a^3*c^9))^(1/2)*2i - x*((a*e^4)/c^2 - (6*d^2*e^2)/c) + atan((((4*x*
(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c^2*d^4*e
^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(-(a^4*e^8*(
-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*
e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)
^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^
(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^
3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*
a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^
3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2)*1i
+ ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c
^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(-(a
^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^
5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-
a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a
^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*
d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e
^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*
e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(
1/2)*1i)/(((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 +
70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e
^2)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*
e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^
6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4
*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)^(1/2
) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c
^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^
3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3
*c^9))^(1/2) - ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e
^6 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*
d^2*e^2)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8
*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c
^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^
```

```
2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)
^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*
a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) -
28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(1
6*a^3*c^9))^(1/2) + (8*(a^5*d*e^11 - c^5*d^11*e - 3*a*c^4*d^9*e^3 + 3*a^4*c
*d^3*e^9 - 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7))/c^3))*(-(a^4*e^8*(-a^3*c
^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 -
56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2)
 - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))
/(16*a^3*c^9))^(1/2)*2i + (e^4*x^5)/(5*c) + (4*d*e^3*x^3)/(3*c)
```

3.138 $\int \frac{\left(d+ex^2\right)^3}{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 370

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^3}{a+cx^4}\,dx &= \frac{3de^2x}{c}+\frac{e^3x^3}{3c} \\
&\quad - \frac{\left(\sqrt{c}d(cd^2-3ae^2)+\sqrt{a}e(3cd^2-ae^2)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{7/4}} \\
&\quad + \frac{\left(\sqrt{c}d(cd^2-3ae^2)+\sqrt{a}e(3cd^2-ae^2)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{7/4}} \\
&\quad - \frac{\left(\sqrt{c}d(cd^2-3ae^2)-\sqrt{a}e(3cd^2-ae^2)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}} \\
&\quad + \frac{\left(\sqrt{c}d(cd^2-3ae^2)-\sqrt{a}e(3cd^2-ae^2)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}
\end{aligned}
$$

```
[Out] 3*d*e^2*x/c+1/3*e^3*x^3/c-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7
/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*(-a*e
^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7/4)*2^(1/2)+1/4
*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e
^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a
^(1/4))*(e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(
7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1185, 1182, 1176, 631, 210, 1179, 642}

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$$
$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d(cd^2-3ae^2)+\sqrt{a}e(3cd^2-ae^2)\right)}{2\sqrt{2}a^{3/4}c^{7/4}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d(cd^2-3ae^2)+\sqrt{a}e(3cd^2-ae^2)\right)}{2\sqrt{2}a^{3/4}c^{7/4}}$$
$$-\frac{\left(\sqrt{c}d(cd^2-3ae^2)-\sqrt{a}e(3cd^2-ae^2)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}$$
$$+\frac{\left(\sqrt{c}d(cd^2-3ae^2)-\sqrt{a}e(3cd^2-ae^2)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}+\frac{3de^2x}{c}+\frac{e^3x^3}{3c}$$

```
[In] Int[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e
*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e^2)
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4)) - ((
Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqr
t[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3de^2}{c} + \frac{e^3x^2}{c} + \frac{cd^3 - 3ade^2 + e(3cd^2 - ae^2)\,x^2}{c\,(a + cx^4)}\right) dx \\
&= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\int \frac{cd^3 - 3ade^2 + e(3cd^2 - ae^2)x^2}{a + cx^4}\,dx}{c} \\
&= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} - \frac{\left(3cd^2e - ae^3 - \frac{\sqrt{c}\,d(cd^2 - 3ae^2)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4}\,dx}{2c^2} \\
&\quad + \frac{\left(3cd^2e - ae^3 + \frac{\sqrt{c}\,d(cd^2 - 3ae^2)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4}\,dx}{2c^2}
\end{aligned}$$

$$=\frac{3de^2x}{c}+\frac{e^3x^3}{3c}+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$
$$+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$
$$+\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4c^2}$$
$$+\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4c^2}$$

$$=\frac{3de^2x}{c}+\frac{e^3x^3}{3c}+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$
$$-\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$
$$+\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}}$$
$$-\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}}$$

$$=\frac{3de^2x}{c}+\frac{e^3x^3}{3c}-\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}}$$
$$+\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}}$$
$$+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$
$$-\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d(cd^2-3ae^2)}{\sqrt{a}}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$$
$$=\frac{72a^{3/4}c^{3/4}de^2x+8a^{3/4}c^{3/4}e^3x^3+6\sqrt{2}\left(-c^{3/2}d^3-3\sqrt{a}cd^2e+3a\sqrt{c}de^2+a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+}{}$$

```
[In] Integrate[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (72*a^(3/4)*c^(3/4)*d*e^2*x + 8*a^(3/4)*c^(3/4)*e^3*x^3 + 6*Sqrt[2]*(-(c^(3
/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 -
(Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e -
 3*a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 3*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e
^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*(c^(
3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^(3/4)*c^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.22

method	result
risch	$\frac{e^3x^3}{3c}+\frac{3d\,e^2x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+a\right)}\frac{\left(e\left(-a\,e^2+3c\,d^2\right)\_R^2-3d\,e^2a+d^3c\right)\ln\left(x-\_R\right)}{\_R^3}}{4c^2}$
default	$\frac{e^2\left(\frac{1}{3}e\,x^3+3dx\right)}{c}+\frac{\frac{\left(-3d\,e^2a+d^3c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-a\,e^3+3c\,d^2e\right)\sqrt{}}{}}{c}$

```
[In] int((e*x^2+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^3*x^3/c+3*d*e^2*x/c+1/4/c^2*sum((e*(-a*e^2+3*c*d^2)*_R^2-3*d*e^2*a+d^
3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. 2(287) = 574.

Time = 2.39 (sec) , antiderivative size = 2133, normalized size of antiderivative = 5.76

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*e^3*x^3 + 36*d*e^2*x - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*
a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4
 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12
)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*
e^4 + 27*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^9 - 1
8*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e
^8 + (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 +
255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*
d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^
2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 -
 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/
(a^3*c^7)))/(a*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e
^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*
a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*
c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 2
7*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x - (a*c^6*d^9 - 18*a^2*c
^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 + (3
*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2
*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^1
0 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5
 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^
3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^
7)))/(a*c^3))) - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*
c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3
*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/
(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 27*a^4*c
^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^9 - 18*a^2*c^5*d^7*
e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 - (3*a^3*c^
6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^
8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6
*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^
3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d
^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a
*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^3*sqr
t(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^
```

```
6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3)
)*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 27*a^4*c^2*d^4*
e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x - (a*c^6*d^9 - 18*a^2*c^5*d^7*e^2 + 6
0*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 - (3*a^3*c^6*d^2*e
 - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 -
 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/
(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^3*sqrt(
-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6
+ 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3)))
)/c
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$$
$$=\text{RootSum}\left(256t^4a^3c^7+t^2\cdot\left(192a^4c^4de^5-640a^3c^5d^3e^3+192a^2c^6d^5e\right)+a^6e^{12}+6a^5cd^2e^{10}+15a^4c^2d^4e^8+\right.$$
$$+\frac{3de^2x}{c}+\frac{e^3x^3}{3c}$$

```
[In] integrate((e*x**2+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**7 + _t**2*(192*a**4*c**4*d*e**5 - 640*a**3*c**5*d
**3*e**3 + 192*a**2*c**6*d**5*e) + a**6*e**12 + 6*a**5*c*d**2*e**10 + 15*a*
*4*c**2*d**4*e**8 + 20*a**3*c**3*d**6*e**6 + 15*a**2*c**4*d**8*e**4 + 6*a*c
**5*d**10*e**2 + c**6*d**12, Lambda(_t, _t*log(x + (-64*_t**3*a**4*c**5*e**
3 + 192*_t**3*a**3*c**6*d**2*e - 36*_t*a**5*c**2*d*e**8 + 336*_t*a**4*c**3*
d**3*e**6 - 504*_t*a**3*c**4*d**5*e**4 + 144*_t*a**2*c**5*d**7*e**2 - 4*_t*
a*c**6*d**9)/(a**6*e**12 - 12*a**5*c*d**2*e**10 - 27*a**4*c**2*d**4*e**8 +
27*a**2*c**4*d**8*e**4 + 12*a*c**5*d**10*e**2 - c**6*d**12))) + 3*d*e**2*x
/c + e**3*x**3/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx=\frac{e^3x^3+9\,de^2x}{3\,c}$$
$$+\frac{\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\sqrt{a}cd^2e-3\,a\sqrt{c}de^2-a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\sqrt{a}cd^2e-3\,a\sqrt{c}de^2-a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\right.}{2\sqrt{\sqrt{}}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/3*(e^3*x^3 + 9*d*e^2*x)/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2
*e - 3*a*sqrt(c)*d*e^2 - a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqr
t(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))
*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2
- a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/s
qrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(c^
(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c
^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*log(sqrt(
c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.11

$$
\begin{aligned}
\int \frac{(d+ex^2)^3}{a+cx^4}\,dx &= \frac{c^2e^3x^3+9\,c^2de^2x}{3\,c^3} \\
&+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2+3\,(ac^3)^{\frac{3}{4}}\,cd^2e-(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} \\
&+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2+3\,(ac^3)^{\frac{3}{4}}\,cd^2e-(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} \\
&+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2-3\,(ac^3)^{\frac{3}{4}}\,cd^2e+(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^4} \\
&- \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3-3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2-3\,(ac^3)^{\frac{3}{4}}\,cd^2e+(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^4}
\end{aligned}
$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/3*(c^2*e^3*x^3 + 9*c^2*d*e^2*x)/c^3 + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3
- 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e
^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1
/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)
^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*
c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*log
(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1
/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^
3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 2712, normalized size of antiderivative = 7.33

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^3/(a + c*x^4),x)
```

```
[Out] (e^3*x^3)/(3*c) - atan((a^3*e^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^3
*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)^(
1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*e^2
*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 +
 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*c^7)^
(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(-a^3*c
^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-a^3*c^7
)^(1/2))/(a*c^2)) - (c^3*d^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^3*e^
3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)^(1/2
))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*e^2*(-
a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 12
0*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*c^7)^(1/
2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(-a^3*c^7)
^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-a^3*c^7)^(
1/2))/(a*c^2)) + (a*c^2*d^4*e^2*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^3
*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)^(
1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*e^2
*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2
 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*c^7
)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(-a^3
*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-a^3*c
^7)^(1/2))/(a*c^2)) - (a^2*c*d^2*e^4*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (
5*d^3*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c
^7)^(1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^
4*e^2*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9
)/c^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^
3*c^7)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*
(-a^3*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-
a^3*c^7)^(1/2))/(a*c^2)))*(-(c^3*d^6*(-a^3*c^7)^(1/2) - a^3*e^6*(-a^3*c^7)^
(1/2) + 6*a^2*c^6*d^5*e + 6*a^4*c^4*d*e^5 - 20*a^3*c^5*d^3*e^3 - 15*a*c^2*d
^4*e^2*(-a^3*c^7)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a^3*c^7))^
(1/2)*2i - atan((a^3*e^6*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2))/(1
6*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/2))/(
16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(-a^3*
c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*a^
2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^(1/2))/
```

```
(a^2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c^7)^(1/
2))/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7)^(1/2)
)/(a*c^2)) - (c^3*d^6*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2))/(16*c
^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/2))/(16*
a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(-a^3*c^7
)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*a^2*d
^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^(1/2))/(a^
2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c^7)^(1/2))
/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7)^(1/2))/(
a*c^2)) + (a*c^2*d^4*e^2*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2))/(1
6*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/2))/(
16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(-a^3*
c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*
a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^(1/2)
)/(a^2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c^7)^(
1/2))/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7)^(1/
2))/(a*c^2)) - (a^2*c*d^2*e^4*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2
))/(16*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/
2))/(16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(
-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 +
 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^
(1/2))/(a^2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c
^7)^(1/2))/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7
)^(1/2))/(a*c^2)))*(-(a^3*e^6*(-a^3*c^7)^(1/2) - c^3*d^6*(-a^3*c^7)^(1/2) +
 6*a^2*c^6*d^5*e + 6*a^4*c^4*d*e^5 - 20*a^3*c^5*d^3*e^3 + 15*a*c^2*d^4*e^2*
(-a^3*c^7)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a^3*c^7))^(1/2)*2
i + (3*d*e^2*x)/c
```

3.139 $\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 297

$$\begin{aligned}\int \frac{(d+ex^2)^2}{a+cx^4}\,dx &= \frac{e^2x}{c}-\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}\\ &\quad+\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}\\ &\quad-\frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}\\ &\quad+\frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}\end{aligned}$$

```
[Out] e^2*x/c-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c*d^2-a*e^2
-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(c*d^2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5
/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^2-a*e^2+2*d*e*a^(
1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(c*d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1185, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{(d+ex^2)^2}{a+cx^4}\,dx &= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}c^{5/4}} \\ &+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}c^{5/4}} \\ &-\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}} \\ &+\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{e^2x}{c}\end{aligned}$$

```
[In] Int[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 + 2*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 - 2*
Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{c} + \frac{cd^2 - ae^2 + 2cdex^2}{c\,(a + cx^4)}\right) dx \\
&= \frac{e^2x}{c} + \frac{\int \frac{cd^2 - ae^2 + 2cdex^2}{a + cx^4}\,dx}{c} \\
&= \frac{e^2x}{c} + \frac{\left(cd^2 - 2\sqrt{a}\sqrt{c}de - ae^2\right)\int \frac{\sqrt{a}\sqrt{c} - cx^2}{a + cx^4}\,dx}{2\sqrt{a}c^{3/2}} + \frac{\left(cd^2 + 2\sqrt{a}\sqrt{c}de - ae^2\right)\int \frac{\sqrt{a}\sqrt{c} + cx^2}{a + cx^4}\,dx}{2\sqrt{a}c^{3/2}}
\end{aligned}$$

$$=\frac{e^2x}{c}-\frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}c^{5/4}}$$
$$-\frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}c^{5/4}}$$
$$+\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}c^{3/2}}$$
$$+\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}c^{3/2}}$$

$$=\frac{e^2x}{c}-\frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}$$
$$+\frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}$$
$$+\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}$$
$$-\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}$$

$$=\frac{e^2x}{c}-\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}$$
$$+\frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}$$
$$-\frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}$$
$$+\frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$$

$$= \frac{8a^{3/4}\sqrt[4]{c}e^2x - 2\sqrt{2}(cd^2+2\sqrt{a}\sqrt{c}de-ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(cd^2+2\sqrt{a}\sqrt{c}de-ae^2)\arctan(}{}$$

```
[In] Integrate[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (8*a^(3/4)*c^(1/4)*e^2*x - 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sq
rt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(c*d
^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2] + Sqrt[2]*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(3/4)*c^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.19

method	result
risch	$\frac{e^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+a)}\frac{(2\_R^2cde-ae^2+cd^2)\ln(x-\_R)}{\_R^3}}{4c^2}$
default	$\frac{e^2x}{c}+\frac{\frac{(-ae^2+cd^2)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right.}{}}{c}$

```
[In] int((e*x^2+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*x/c+1/4/c^2*sum((2*_R^2*c*d*e-a*e^2+c*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4
*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1480 vs. 2(216) = 432.

Time = 0.71 (sec) , antiderivative size = 1480, normalized size of antiderivative = 4.98

$$\int \frac{\left(d+ex^2\right)^2}{a+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*e^2*x + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a
*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))
/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2
*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^
4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*
e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3
+ a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d
^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 +
a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2
*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^
2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e
^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^
3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sq
rt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) + c*sqrt
(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^
8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x + (
a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*a^3*c^4*d
*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^
6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^
8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^
3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8
- 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*
c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3
*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^
4 - a^4*c*e^6 - 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^
2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*
d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a
^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))))/c
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$$
$$= \operatorname{RootSum}\left(256t^4a^3c^5 + t^2\left(-128a^3c^3de^3 + 128a^2c^4d^3e\right) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d^8,\right.$$
$$+ \frac{e^2x}{c}$$

```
[In] integrate((e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**5 + _t**2*(-128*a**3*c**3*d*e**3 + 128*a**2*c**4*
d**3*e) + a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 + 4*a*c**3
*d**6*e**2 + c**4*d**8, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c**4*d*e - 4
*_t*a**4*c*e**6 + 60*_t*a**3*c**2*d**2*e**4 - 60*_t*a**2*c**3*d**4*e**2 + 4
*_t*a*c**4*d**6)/(a**4*e**8 - 4*a**3*c*d**2*e**6 - 10*a**2*c**2*d**4*e**4 -
 4*a*c**3*d**6*e**2 + c**4*d**8)))) + e**2*x/c
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx = \frac{e^2x}{c}$$
$$+ \frac{\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}cde-a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}cde-a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}\left(\right.}{}}{8c}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] e^2*x/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^2 + 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*ar
ctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^2 + 2*s
qrt(a)*c*d*e - a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(
1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)
) + sqrt(2)*(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(c)*x^2
 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(3/2
)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$$
$$= \frac{e^2x}{c} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^2d^2 - (ac^3)^{\frac{1}{4}}\,ace^2 + 2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}$$
$$+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^2d^2 - (ac^3)^{\frac{1}{4}}\,ace^2 + 2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}$$
$$+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^2d^2 - (ac^3)^{\frac{1}{4}}\,ace^2 - 2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$
$$- \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^2d^2 - (ac^3)^{\frac{1}{4}}\,ace^2 - 2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] e^2*x/c + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a
*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*
(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/
4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 -
2*(a*c^3)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) -
 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/
4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 1479, normalized size of antiderivative = 4.98

$$
\begin{aligned}
\int \frac{(d+ex^2)^2}{a+cx^4}\,dx &= \frac{e^2\,x}{c} \\
&-2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^4\,x\,\sqrt{\frac{d\,e^3}{4\,c^2}-\frac{d^3\,e}{4\,a\,c}+\frac{d^4\,\sqrt{-a^3\,c^5}}{16\,a^3\,c^3}+\frac{e^4\,\sqrt{-a^3\,c^5}}{16\,a\,c^5}-\frac{3\,d^2\,e^2\,\sqrt{-a^3\,c^5}}{8\,a^2\,c^4}}}{4\,a^2\,d\,e^5-\frac{2\,d^6\,\sqrt{-a^3\,c^5}}{a^2}+4\,c^2\,d^5\,e+\frac{2\,a\,e^6\,\sqrt{-a^3\,c^5}}{c^3}-24\,a\,c\,d^3\,e^3-\frac{14\,d^2\,e^4\,\sqrt{-a^3\,c^5}}{c^2}+\frac{14\,d^4\,e^2\,\sqrt{-a^3\,c^5}}{a\,c}}\right. \\
&+\frac{8\,a^2\,c\,e^4\,x\,\sqrt{\frac{d\,e^3}{4\,c^2}-\frac{d^3\,e}{4\,a\,c}+\frac{d^4\,\sqrt{-a^3\,c^5}}{16\,a^3\,c^3}+\frac{e^4\,\sqrt{-a^3\,c^5}}{16\,a\,c^5}-\frac{3\,d^2\,e^2\,\sqrt{-a^3\,c^5}}{8\,a^2\,c^4}}}{4\,a^2\,d\,e^5-\frac{2\,d^6\,\sqrt{-a^3\,c^5}}{a^2}+4\,c^2\,d^5\,e+\frac{2\,a\,e^6\,\sqrt{-a^3\,c^5}}{c^3}-24\,a\,c\,d^3\,e^3-\frac{14\,d^2\,e^4\,\sqrt{-a^3\,c^5}}{c^2}+\frac{14\,d^4\,e^2\,\sqrt{-a^3\,c^5}}{a\,c}} \\
&\left.-\frac{48\,a\,c^2\,d^2\,e^2\,x\,\sqrt{\frac{d\,e^3}{4\,c^2}-\frac{d^3\,e}{4\,a\,c}+\frac{d^4\,\sqrt{-a^3\,c^5}}{16\,a^3\,c^3}+\frac{e^4\,\sqrt{-a^3\,c^5}}{16\,a\,c^5}-\frac{3\,d^2\,e^2\,\sqrt{-a^3\,c^5}}{8\,a^2\,c^4}}}{4\,a^2\,d\,e^5-\frac{2\,d^6\,\sqrt{-a^3\,c^5}}{a^2}+4\,c^2\,d^5\,e+\frac{2\,a\,e^6\,\sqrt{-a^3\,c^5}}{c^3}-24\,a\,c\,d^3\,e^3-\frac{14\,d^2\,e^4\,\sqrt{-a^3\,c^5}}{c^2}+\frac{14\,d^4\,e^2\,\sqrt{-a^3\,c^5}}{a\,c}}\right)\sqrt{\frac{a^2\,e^4\,\sqrt{}}{}} \\
&-2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^4\,x\,\sqrt{\frac{d\,e^3}{4\,c^2}-\frac{d^3\,e}{4\,a\,c}-\frac{d^4\,\sqrt{-a^3\,c^5}}{16\,a^3\,c^3}-\frac{e^4\,\sqrt{-a^3\,c^5}}{16\,a\,c^5}+\frac{3\,d^2\,e^2\,\sqrt{-a^3\,c^5}}{8\,a^2\,c^4}}}{\frac{2\,d^6\,\sqrt{-a^3\,c^5}}{a^2}+4\,a^2\,d\,e^5+4\,c^2\,d^5\,e-\frac{2\,a\,e^6\,\sqrt{-a^3\,c^5}}{c^3}-24\,a\,c\,d^3\,e^3+\frac{14\,d^2\,e^4\,\sqrt{-a^3\,c^5}}{c^2}-\frac{14\,d^4\,e^2\,\sqrt{-a^3\,c^5}}{a\,c}}\right. \\
&+\frac{8\,a^2\,c\,e^4\,x\,\sqrt{\frac{d\,e^3}{4\,c^2}-\frac{d^3\,e}{4\,a\,c}-\frac{d^4\,\sqrt{-a^3\,c^5}}{16\,a^3\,c^3}-\frac{e^4\,\sqrt{-a^3\,c^5}}{16\,a\,c^5}+\frac{3\,d^2\,e^2\,\sqrt{-a^3\,c^5}}{8\,a^2\,c^4}}}{\frac{2\,d^6\,\sqrt{-a^3\,c^5}}{a^2}+4\,a^2\,d\,e^5+4\,c^2\,d^5\,e-\frac{2\,a\,e^6\,\sqrt{-a^3\,c^5}}{c^3}-24\,a\,c\,d^3\,e^3+\frac{14\,d^2\,e^4\,\sqrt{-a^3\,c^5}}{c^2}-\frac{14\,d^4\,e^2\,\sqrt{-a^3\,c^5}}{a\,c}} \\
&\left.-\frac{48\,a\,c^2\,d^2\,e^2\,x\,\sqrt{\frac{d\,e^3}{4\,c^2}-\frac{d^3\,e}{4\,a\,c}-\frac{d^4\,\sqrt{-a^3\,c^5}}{16\,a^3\,c^3}-\frac{e^4\,\sqrt{-a^3\,c^5}}{16\,a\,c^5}+\frac{3\,d^2\,e^2\,\sqrt{-a^3\,c^5}}{8\,a^2\,c^4}}}{\frac{2\,d^6\,\sqrt{-a^3\,c^5}}{a^2}+4\,a^2\,d\,e^5+4\,c^2\,d^5\,e-\frac{2\,a\,e^6\,\sqrt{-a^3\,c^5}}{c^3}-24\,a\,c\,d^3\,e^3+\frac{14\,d^2\,e^4\,\sqrt{-a^3\,c^5}}{c^2}-\frac{14\,d^4\,e^2\,\sqrt{-a^3\,c^5}}{a\,c}}\right)\sqrt{-\frac{a^2\,e^4}{}}
\end{aligned}
$$

```
[In] int((d + e*x^2)^2/(a + c*x^4),x)
```

```
[Out] (e^2*x)/c - 2*atanh((8*c^3*d^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*
(-a^3*c^5)^(1/2))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2
*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)
^(1/2))/a^2 + 4*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3
 - (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c))
 + (8*a^2*c*e^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5)^(1/2
))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a^3*c^5)
^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4
*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14*d^2*e^4*
(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)) - (48*a*c^2*d^
2*e^2*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5)^(1/2))/(16*a^3
*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8
*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4*c^2*d^5*e
 + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14*d^2*e^4*(-a^3*c^5)
^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)))*((a^2*e^4*(-a^3*c^5)^(1
/2) + c^2*d^4*(-a^3*c^5)^(1/2) - 4*a^2*c^4*d^3*e + 4*a^3*c^3*d*e^3 - 6*a*c*
d^2*e^2*(-a^3*c^5)^(1/2))/(16*a^3*c^5))^(1/2) - 2*atanh((8*c^3*d^4*x*((d*e^
```

```
3)/(4*c^2) - (d^3*e)/(4*a*c) - (d^4*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (e^4*(
-a^3*c^5)^(1/2))/(16*a*c^5) + (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/
2))/((2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4*a^2*d*e^5 + 4*c^2*d^5*e - (2*a*e^6*(-
a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 + (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 -
(14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)) + (8*a^2*c*e^4*x*((d*e^3)/(4*c^2) - (d
^3*e)/(4*a*c) - (d^4*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (e^4*(-a^3*c^5)^(1/2)
)/(16*a*c^5) + (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*d^6*(-a
^3*c^5)^(1/2))/a^2 + 4*a^2*d*e^5 + 4*c^2*d^5*e - (2*a*e^6*(-a^3*c^5)^(1/2))
/c^3 - 24*a*c*d^3*e^3 + (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 - (14*d^4*e^2*(-a
^3*c^5)^(1/2))/(a*c)) - (48*a*c^2*d^2*e^2*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a
*c) - (d^4*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (e^4*(-a^3*c^5)^(1/2))/(16*a*c^
5) + (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*d^6*(-a^3*c^5)^(1
/2))/a^2 + 4*a^2*d*e^5 + 4*c^2*d^5*e - (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*
a*c*d^3*e^3 + (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 - (14*d^4*e^2*(-a^3*c^5)^(1
/2))/(a*c)))*(-(a^2*e^4*(-a^3*c^5)^(1/2) + c^2*d^4*(-a^3*c^5)^(1/2) + 4*a^2
*c^4*d^3*e - 4*a^3*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^5)^(1/2))/(16*a^3*c^5)
)^(1/2)
```

3.140 $\int \frac{d+ex^2}{a+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 247

$$\begin{aligned}\int \frac{d+ex^2}{a+cx^4}\,dx = &-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}\\ &-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}\\ &+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}\end{aligned}$$

```
[Out] -1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/
2))/a^(3/4)/c^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^
(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arcta
n(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2
)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used

$= \{1182, 1176, 631, 210, 1179, 642\}$

$$\int \frac{d+ex^2}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$

```
[In] Int[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] -1/2*((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(Sqr
t[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/
4)) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c}\\
&= \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c}\\
&\quad -\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}c^{3/4}}\\
&= -\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}\\
&\quad +\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}\\
&\quad +\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}\\
&\quad -\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}
\end{aligned}
$$

$$= -\frac{(\sqrt{c}d+\sqrt{a}e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{(\sqrt{c}d+\sqrt{a}e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$
$$-\frac{(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$
$$+\frac{(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^2}{a+cx^4}\,dx$$
$$=\frac{-2(\sqrt{c}d+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2(\sqrt{c}d+\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-(\sqrt{c}d-\sqrt{a}e)\left(\log\left(\sqrt{a}-\right.\right.}{4\sqrt{2}a^{3/4}c^{3/4}}$$

```
[In] Integrate[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d -
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(4*Sqrt[2]*a^(3/4)*c^(3
/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^2e+d\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$
default	$\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\right.}{8c\left(\frac{a}{c}\right.}$

```
[In] int((e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c*sum((_R^2*e+d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(166) = 332$.

Time = 0.30 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.11

$$\begin{aligned}\int \frac{d+ex^2}{a+cx^4}\,dx = {}& -\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\Bigg(-\left(c^2d^4-a^2e^4\right)x \\ &+\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\Bigg) \\ &+\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\Bigg(-\left(c^2d^4-a^2e^4\right)x \\ &-\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\Bigg) \\ &+\frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\log\Bigg(-\left(c^2d^4-a^2e^4\right)x \\ &+\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\Bigg) \\ &-\frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\log\Bigg(-\left(c^2d^4-a^2e^4\right)x \\ &-\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\Bigg)\end{aligned}$$

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*
e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^
2*e^2 + a^2*e^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^
```

```
2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt(-(a
*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))*log
(-(c^2*d^4 - a^2*e^4)*x - (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e
^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c
*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt((a*c*sqrt(-(c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^2*d^4 - a
^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))
 - a*c^2*d^3 + a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*
e^4)/(a^3*c^3)) - 2*d*e)/(a*c))) - 1/4*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x - (a^
3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - a*c^2*d^3 +
a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))
 - 2*d*e)/(a*c)))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int \frac{d+ex^2}{a+cx^4}\,dx$$
$$= \operatorname{RootSum}\left(256t^4a^3c^3+64t^2a^2c^2de+a^2e^4+2acd^2e^2+c^2d^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3c^2e+12ta^2cde^2-4ta}{a^2e^4-c^2d^4}\right.\right.\right.$$

```
[In] integrate((e*x**2+d)/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 64*_t**2*a**2*c**2*d*e + a**2*e**4 + 2*a*c*d*
*2*e**2 + c**2*d**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*c**2*e + 12*_t*a*
*2*c*d*e**2 - 4*_t*a*c**2*d**3)/(a**2*e**4 - c**2*d**4))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{d+ex^2}{a+cx^4}\,dx = {} & \frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} \\ & + \frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} \\ & + \frac{\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}} \\ & - \frac{\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}\end{aligned}$$

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + 1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)
*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(c))*sqrt(c)) + 1/8*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 + s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c
)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^
(3/4)*c^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int \frac{d+ex^2}{a+cx^4}\,dx = {} & \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} \\
& + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} \\
& + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} \\
& - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}
\end{aligned}
$$

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c
^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(
x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/
4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.43

$$
\begin{aligned}
\int \frac{d+ex^2}{a+cx^4}\,dx &= -2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right. \\
&\quad \left. -\frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right)\sqrt{-\frac{c\,d^2\,\sqrt{-a^3\,c^3}-a\,e^2\,\sqrt{-a^3\,c^3}+2\,a^2\,c^2\,d\,e}{16\,a^3\,c^3}} \\
&\quad -2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}-\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}+\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right. \\
&\quad \left. -\frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}-\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}+\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right)\sqrt{-\frac{a\,e^2\,\sqrt{-a^3\,c^3}-c\,d^2\,\sqrt{-a^3\,c^3}+2\,a^2\,c^2\,d\,e}{16\,a^3\,c^3}}
\end{aligned}
$$

```
[In] int((d + e*x^2)/(a + c*x^4),x)
```

```
[Out] - 2*atanh((8*c^3*d^2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c^
3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 + (
2*c*d^3*(-a^3*c^3)^(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2*e^
2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c^3)^(1/2))/(16*a^3*c
^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 + (2*c*d^3*(-a^3*c^3)^
(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a))*(-(c*d^2*(-a^3*c^3)^(1/2) - a*e
^2*(-a^3*c^3)^(1/2) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2) - 2*atanh((8*c^3*d
^2*x*((d^2*(-a^3*c^3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-a^3*c^3)
^(1/2))/(16*a^2*c^3))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(-a^3*c^3)
^(1/2))/a^2 + (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2*e^2*x*((d^2*(-a^3*c^
3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3
))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(-a^3*c^3)^(1/2))/a^2 + (2*d*
e^2*(-a^3*c^3)^(1/2))/a))*(-(a*e^2*(-a^3*c^3)^(1/2) - c*d^2*(-a^3*c^3)^(1/2
) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2)
```

3.141 $\int \frac{1}{a+cx^4}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 185

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[Out] 1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arctan
(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(3/4)*c^(1/4)) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqr
```

```
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))
 + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4
)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$
$$- \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.72

$$\int \frac{1}{a+cx^4}\,dx$$
$$= \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.15

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}}{4c}$	27
default	$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

```
[In] int(1/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.61

$$\begin{aligned}\int\frac{1}{a+cx^4}\,dx=&\ \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)+\frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)\\&-\frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)\\&-\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)\end{aligned}$$

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c))^(1/4) + x) + 1/4*I*(-1/(a^3*c))^(
1/4)*log(I*a*(-1/(a^3*c))^(1/4) + x) - 1/4*I*(-1/(a^3*c))^(1/4)*log(-I*a*(-
1/(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) +
x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.11

$$\int \frac{1}{a+cx^4}\,dx = \operatorname{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

```
[In] integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}$$

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.18

$$\int \frac{1}{a+cx^4}\,dx=-\frac{\operatorname{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,c^{1/4}}$$

```
[In] int(1/(a + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3/
4)*c^(1/4))
```

3.142 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 336

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = {} & \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ & + \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ & - \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ & + \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}\end{aligned}$$

```
[Out] 1/4*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(
3/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*
(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*c^(1/4)*ln(-a^(1/4
)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e
^2+c*d^2)*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)+e^(3/2)*arctan(x*e
^(1/2)/d^(1/2))/(a*e^2+c*d^2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1185, 211, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = &-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}\\ &+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}\\ &-\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}\\ &+\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}\\ &+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)}\end{aligned}$$

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)dx\\
&= \frac{c\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2}+\frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}\\
&= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\,(cd^2+ae^2)}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\,(cd^2+ae^2)}
\end{aligned}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\,(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\,(cd^2+ae^2)}$$
$$- \frac{\left(\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)} - \frac{\left(\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$+ \frac{\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$+ \frac{\left(\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$- \frac{\left(\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$+ \frac{\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$- \frac{\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$
$$+ \frac{\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

$$\int \frac{1}{(d+ex^2)\,(a+cx^4)}\,dx$$
$$= \frac{8a^{3/4}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{c}\sqrt{d}\Big(\left(-2\sqrt{c}d+2\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\Big(1-}{8a^{3/4}\sqrt{d}\,(cd^2+}$$

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.\right.}{8c\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{a\,e^{2}+c\,d^{2}}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{5}e^{4}+2a^{4}c\,d^{2}e^{2}+a^{3}c^{2}d^{4}\right)\_Z^{4}-4a^{2}cde\,\_Z^{2}+c\right)}\_R\ln\left(\left(\left(-2a^{5}e^{7}-2a^{4}c\,d^{2}e^{5}+2a^{3}c^{2}d^{4}e^{3}+2a^{2}c^{3}d^{6}e\right)\_R^{4}+(15a^{2}cd\right.\right.\right.}{4}$

```
[In] int(1/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] c/(a*e^2+c*d^2)*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)
+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(1/2
)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a
/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*
x-1)))+e^2/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. 2(247) = 494.

Time = 0.70 (sec) , antiderivative size = 4084, normalized size of antiderivative = 12.15

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a
```

```
^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*
e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*
d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2
+ a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^
4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d
^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^
4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
 a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^
2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^
5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (
a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a
^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^
2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e
^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4
- 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*
e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4
)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c
^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*
a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 -
 a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - 2*e*sqrt(-e/d)*log((e*x^2 + 2*d
*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*sqrt(e/d)*arctan
(x*sqrt(e/d)) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 -
 a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
```

```
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e +
 (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2
 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3
 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*
a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*
c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^
2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*
d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-
(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3
*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d
^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2
*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*
c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(
c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\,(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{1}{(d+ex^2)\,(a+cx^4)}\,dx = {} & \frac{e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2+ae^2)\sqrt{de}} \\ & + \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d-(ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ & + \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d-(ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ & + \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d+(ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ & - \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d+(ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}\end{aligned}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/2*((a*c^3)^(1/4)*
c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/
c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)*c^
2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)*c^2*
d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*
```

```
a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)
*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(
2)*a^2*c^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 15.59 (sec) , antiderivative size = 4802, normalized size of antiderivative = 14.29

$$\int \frac{1}{(d+ex^2)\,(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-
a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
 c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d
^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^
5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3
*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*
((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
 a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*
(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)
))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) +
2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4
*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(
16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512
*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^
6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32
*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c
)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2
) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^
(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5
*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a
^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5
 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*
c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*(
(a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
```

```
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((a
*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^
3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^
2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4
*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^
2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c
^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) +
20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2
) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*2i + atan(((((c*d^2*(-a^3*c)^(1/2) -
a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(2
56*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*
d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*
c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^
2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(
-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
)^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*1i - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(1
6*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d
^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1
/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d
^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 44
8*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e
^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*
e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*
x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^
4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i)/(((((c*d^2*(-a^3*c)^(1/2) - a
*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2
*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/
2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(25
6*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d
*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
- 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c
```

```
^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2
 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-
a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))
^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2) + (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a
^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(
-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*
d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1/2)
- a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*
d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e
^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^
3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))
*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4
+ a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(
(c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*2i - (log(16*a^2*e^2*(-d*e^3)^(3/2) + c^2*d^5*e^3*x - c^2*d^5*e*(-d*e
^3)^(1/2) + 16*a^2*d*e^7*x + a*c*d^2*(-d*e^3)^(3/2) + a*c*d^3*e^5*x)*(-d*e^
3)^(1/2))/(2*(c*d^3 + a*d*e^2)) + (log(c^2*d^5*e^3*x - 16*a^2*e^2*(-d*e^3)^
(3/2) + c^2*d^5*e*(-d*e^3)^(1/2) + 16*a^2*d*e^7*x + 4*a*c*d^2*(-d*e^3)^(3/2
) + a*c*d^3*e^5*x + 5*a*c*d^3*e^3*(-d*e^3)^(1/2))*(-d*e^3)^(1/2))/(2*c*d^3
+ 2*a*d*e^2)
```

3.143 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 453

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2\,(a+cx^4)}\,dx &= \frac{e^2x}{2d\,(cd^2+ae^2)\,(d+ex^2)} \\
&\quad + \frac{2c\sqrt{d}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\,(cd^2+ae^2)} \\
&\quad - \frac{c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)^2} \\
&\quad + \frac{c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)^2} \\
&\quad - \frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)^2} \\
&\quad + \frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)^2}
\end{aligned}$$

```
[Out] 1/2*e^2*x/d/(a*e^2+c*d^2)/(e*x^2+d)+1/2*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/d
^(3/2)/(a*e^2+c*d^2)+1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^
2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*c^(3/4)*
arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(
3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*c^(1/2))*(c*d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^
2*2^(1/2)+1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c*
d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+2*c*e^(3/2
)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(a*e^2+c*d^2)^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1185, 205, 211, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx = & -\frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2} \\ & +\frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2} \\ & -\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2} \\ & +\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2} \\ & +\frac{2c\sqrt{d}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(ae^2+cd^2)} \\ & +\frac{e^2x}{2d(d+ex^2)(ae^2+cd^2)}\end{aligned}$$

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a
*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 +
a*e^2)^2) + (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)
*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d
^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)^2}+\frac{2cde^2}{(cd^2+ae^2)^2(d+ex^2)}+\frac{c(cd^2-ae^2-2cdex^2)}{(cd^2+ae^2)^2(a+cx^4)}\right)dx\\
&= \frac{c\int \frac{cd^2-ae^2-2cdex^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2}+\frac{(2cde^2)\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2}+\frac{e^2\int \frac{1}{(d+ex^2)^2}\,dx}{cd^2+ae^2}\\
&= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)}+\frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}\\
&\quad+\frac{\left(\sqrt{c}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\sqrt{a}(cd^2+ae^2)^2}\\
&\quad+\frac{\left(\sqrt{c}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\sqrt{a}(cd^2+ae^2)^2}+\frac{e^2\int \frac{1}{d+ex^2}\,dx}{2d(cd^2+ae^2)}\\
&= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)}+\frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)}\\
&\quad+\frac{\left(\sqrt{c}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}(cd^2+ae^2)^2}\\
&\quad+\frac{\left(\sqrt{c}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}(cd^2+ae^2)^2}\\
&\quad-\frac{\left(c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\\
&\quad-\frac{\left(c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}
\end{aligned}
$$

$$= \frac{e^2x}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)}$$
$$- \frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{\left(c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$- \frac{\left(c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$= \frac{e^2x}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)}$$
$$- \frac{c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$- \frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.80

$$\int\frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx$$
$$= \frac{\frac{4e^2\left(cd^2+ae^2\right)x}{d(d+ex^2)} + \frac{4e^{3/2}\left(5cd^2+ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2\sqrt{2}c^{3/4}\left(-cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}} - \frac{2\sqrt{2}c^{3/4}\left(-cd^2+2\sqrt{a}\sqrt{c}de+a\right.}{a^{3/4}}}{8}$$

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] ((4*e^2*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (4*e^(3/2)*(5*c*d^2 + a*e^2)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt
```

```
[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) -
 (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*c^(3/4)*(-(c*d^2) - 2*Sqrt[
a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/a^(3/4) + (Sqrt[2]*c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4))/(8*(c*d^2 + a
*e^2)^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.68

method	result
default	$-\frac{c\left(\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\right.}{}\right.}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}$
risch	Expression too large to display

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -c/(a*e^2+c*d^2)^2*(1/8*(a*e^2-c*d^2)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/4*d*e/(a/
c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/
2)/(a/c)^(1/4)*x-1)))+e^2/(a*e^2+c*d^2)^2*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+
1/2*(a*e^2+5*c*d^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4193 vs. 2(352) = 704.

Time = 9.78 (sec) , antiderivative size = 8409, normalized size of antiderivative = 18.56

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.17

$$
\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx
&= \frac{e^2x}{2\left(cd^3+ade^2\right)\left(ex^2+d\right)} \\
&+ \frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2-2\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4+2\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)} \\
&+ \frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2-2\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4+2\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)} \\
&+ \frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\left(ac^3\right)^{\frac{3}{4}}de\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+2\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)} \\
&- \frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\left(ac^3\right)^{\frac{3}{4}}de\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+2\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)} \\
&+ \frac{\left(5\,cd^2e^2+ae^4\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4\right)\sqrt{de}}
\end{aligned}
$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*e^2*x/((c*d^3 + a*d*e^2)*(e*x^2 + d)) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a
*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2
+ sqrt(2)*a^3*c*e^4) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 -
 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4)
+ 1/4*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)
*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2
)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^2 - (a*c^
3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^
4) + 1/2*(5*c*d^2*e^2 + a*e^4)*arctan(e*x/sqrt(d*e))/((c^2*d^5 + 2*a*c*d^3*
e^2 + a^2*d*e^4)*sqrt(d*e))
```

Mupad [B] (verification not implemented)

Time = 16.80 (sec) , antiderivative size = 16369, normalized size of antiderivative = 36.13

$$\int \frac{1}{(d+ex^2)^2\,(a+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + c*x^4)*(d + e*x^2)^2),x)
```

```
[Out] (e^2*x)/(2*d*(d + e*x^2)*(a*e^2 + c*d^2)) - atan(((((((256*a^8*c^4*d*e^16 -
 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960
*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^
7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*
e^6 + 6*a^2*c^2*d^6*e^4)) + (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c
^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1
/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a
^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4
608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608
*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^1
0 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(
(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*
a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8
 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(32
*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*
e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))
/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^
6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d
^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3
*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)
 + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*
d^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2
 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*
d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-
a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11
*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^
2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*
(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*1i - (((((256*a^8*c^4*d*e^16 - 128*a*c
^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4*c^8
*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^5*d^
3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*
a^2*c^2*d^6*e^4)) - (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2
) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16
```

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*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d
^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*
c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6
*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^10 + a^4*
d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4
*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*
d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*
c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(32*a^6*c^5
*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 22
08*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^
10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*
((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4
*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8
 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (480*
a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*d^4*e^9
+ 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3
*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^
3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)
^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d
^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*
a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-
a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^
3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2
+ 6*a^5*c^2*d^4*e^4)))^(1/2)*1i)/((((((256*a^8*c^4*d*e^16 - 128*a*c^11*d^15
*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4*c^8*d^9*e^8
 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^5*d^3*e^14)/
(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*
d^6*e^4)) + (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^
2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^
8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4))
)^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*c^9*d^12
*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6*d^6*e^1
3 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^10 + a^4*d^2*e^8
+ 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c
^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 -
6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^
6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(32*a^6*c^5*d*e^14
- 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c
^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^10 + a^4
*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^
4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2
*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6
*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (480*a^2*c^8*
d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*d^4*e^9 + 96*a^4
```

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*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e
^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(
1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/
(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^
2*d^4*e^4)))^(1/2) + (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 +
 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^
4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)
^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2)
)/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*
c^2*d^4*e^4)))^(1/2) + (((((256*a^8*c^4*d*e^16 - 128*a*c^11*d^15*e^2 + 256*
a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4*c^8*d^9*e^8 + 10880*a^
5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^5*d^3*e^14)/(2*(c^4*d^1
0 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) -
 (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e
 - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4
*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*(51
2*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560
*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^
8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d
^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) +
 c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e
^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c
^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(32*a^6*c^5*d*e^14 - 48*a*c^10
*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8
+ 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 +
4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3
)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*
a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6
+ 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (480*a^2*c^8*d^6*e^7 - 2
00*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^
11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*
c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^
2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^
8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4))
)^(1/2) - (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*
d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a
^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*
a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*
e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4
)))^(1/2) + (5*c^8*d^3*e^6 + a*c^7*d*e^8)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3
*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2
) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^
2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^
4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*2i - (atan(((((x*(a^3*c^6*e^11 -
 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^
```

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2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (((240*a^2
*c^8*d^6*e^7 - 100*a*c^9*d^8*e^5 - 4*a^5*c^5*e^13 + 392*a^3*c^7*d^4*e^9 + 4
8*a^4*c^6*d^2*e^11)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4
*e^6 + 6*a^2*c^2*d^6*e^4) - (((x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 -
16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c
^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - ((a*e^2 + 5*c*d^2)*((128*a^8*c
^4*d*e^16 - 64*a*c^11*d^15*e^2 + 128*a^2*c^10*d^13*e^4 + 1728*a^3*c^9*d^11*
e^6 + 4480*a^4*c^8*d^9*e^8 + 5440*a^5*c^7*d^7*e^10 + 3456*a^6*c^6*d^5*e^12
+ 1088*a^7*c^5*d^3*e^14)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*
c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (x*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2)*(512
*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*
a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8
*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^
5*e^2)*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*
c^2*d^6*e^4)))*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)
))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^
5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*
a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2)*1i)/(4*(c^2*d^7 + a^2*d^3
*e^4 + 2*a*c*d^5*e^2)) + (((x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d^4
*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^
3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + (((240*a^2*c^8*d^6*e^7 - 100*a*c^9*d^8*e
^5 - 4*a^5*c^5*e^13 + 392*a^3*c^7*d^4*e^9 + 48*a^4*c^6*d^2*e^11)/(c^4*d^10
+ a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + ((
(x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^
9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*
e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*
c^2*d^6*e^4) + ((a*e^2 + 5*c*d^2)*((128*a^8*c^4*d*e^16 - 64*a*c^11*d^15*e^2
 + 128*a^2*c^10*d^13*e^4 + 1728*a^3*c^9*d^11*e^6 + 4480*a^4*c^8*d^9*e^8 + 5
440*a^5*c^7*d^7*e^10 + 3456*a^6*c^6*d^5*e^12 + 1088*a^7*c^5*d^3*e^14)/(c^4*
d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)
 + (x*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*
c^10*d^14*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^
7*d^8*e^11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^
2*e^17))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4*d^2*e^8
 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-d^3*e^3)^(1/2
))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3
)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d
^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^
2)*(-d^3*e^3)^(1/2)*1i)/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))/((5*c^
8*d^3*e^6 + a*c^7*d*e^8)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*
c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (((x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a
*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^
2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) - (((240*a^2*c^8*d^6*e^7 - 100*a*c
^9*d^8*e^5 - 4*a^5*c^5*e^13 + 392*a^3*c^7*d^4*e^9 + 48*a^4*c^6*d^2*e^11)/(c
```

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^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e
^4) - (((x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 102
4*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*
c^6*d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
+ 6*a^2*c^2*d^6*e^4) - ((a*e^2 + 5*c*d^2)*((128*a^8*c^4*d*e^16 - 64*a*c^11*
d^15*e^2 + 128*a^2*c^10*d^13*e^4 + 1728*a^3*c^9*d^11*e^6 + 4480*a^4*c^8*d^9
*e^8 + 5440*a^5*c^7*d^7*e^10 + 3456*a^6*c^6*d^5*e^12 + 1088*a^7*c^5*d^3*e^1
4)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*
d^6*e^4) - (x*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2)*(512*a^2*c^11*d^16*e^3 + 2
560*a^3*c^10*d^14*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 256
0*a^6*c^7*d^8*e^11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^
9*c^4*d^2*e^17))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4
*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-d^3*e
^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(
-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*
d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2
+ 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)) +
(((x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9)
)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d
^6*e^4) + (((240*a^2*c^8*d^6*e^7 - 100*a*c^9*d^8*e^5 - 4*a^5*c^5*e^13 + 392
*a^3*c^7*d^4*e^9 + 48*a^4*c^6*d^2*e^11)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d
^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + (((x*(32*a^6*c^5*d*e^14 - 4
8*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*
d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^10 + a^4*d^
2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + ((a*e^2 +
5*c*d^2)*((128*a^8*c^4*d*e^16 - 64*a*c^11*d^15*e^2 + 128*a^2*c^10*d^13*e^4
+ 1728*a^3*c^9*d^11*e^6 + 4480*a^4*c^8*d^9*e^8 + 5440*a^5*c^7*d^7*e^10 + 34
56*a^6*c^6*d^5*e^12 + 1088*a^7*c^5*d^3*e^14)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*
c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4) + (x*(a*e^2 + 5*c*d^2)*(
-d^3*e^3)^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*
c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6
*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(4*(c^2*d^7 + a^
2*d^3*e^4 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^
3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^2*d^3*
e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^7 + a^
2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(c^2*d^
7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2))/(4*(
c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2))))*(a*e^2 + 5*c*d^2)*(-d^3*e^3)^(1/2
)*1i)/(2*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)) - atan((((((256*a^8*c^4*
d*e^16 - 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^
6 + 8960*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 +
 2176*a^7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^
3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) + (x*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^
4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^
3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*
```

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e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^1
4*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^
11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))
/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^
6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*
d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^
3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2
) + (x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^
2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*
d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*
a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) -
4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^
7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e
^4)))^(1/2) + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 7
84*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*
c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-(a^2*e^4*(-a^3*c^3)^
(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*
c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(a^3*c^6*e^11 - 27*c^9*
d^6*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 +
4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^
3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6
*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*1i - (((((256*a^8*c^4*d*e
^16 - 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 +
 8960*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 21
76*a^7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c
*d^4*e^6 + 6*a^2*c^2*d^6*e^4)) - (x*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(
-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c
^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2
 + 6*a^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e
^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11
- 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c
^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e
^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3
*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c
^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) -
 (x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c
^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3
*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2
*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a
^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e
^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)
))^(1/2) + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*
a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3
```

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*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-(a^2*e^4*(-a^3*c^3)^(1/
2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d
^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(a^3*c^6*e^11 - 27*c^9*d^6
*e^5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a
*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^
(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*
c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*1i)/((((((256*a^8*c^4*d*e^16
 - 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 89
60*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*
a^7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^
4*e^6 + 6*a^2*c^2*d^6*e^4)) + (x*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^
3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)
^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5
+ 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4
608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*
d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)
)*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e
+ 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*
d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x
*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*
d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^
12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^
2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*
c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8
+ a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^
(1/2) + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3
*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^
8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-(a^2*e^4*(-a^3*c^3)^(1/2)
+ c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*
e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^
5 + 11*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^
3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/
2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d
^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (((((256*a^8*c^4*d*e^16 - 128
*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4
*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^
5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
+ 6*a^2*c^2*d^6*e^4)) - (x*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)
^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2)
)/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*
```

```
c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608
*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^
7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^10 +
 a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a
^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^
3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 +
4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(32*a
^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^
6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(
c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*
e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^
3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*
c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)
+ (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*d
^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2
+ 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*
d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-
a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11
*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c
^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2
*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3
*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (5*c^8*d^3*e^6 + a*c^7*d*e^8)/(c^4*
d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)
))*(-(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) - 4*a^2*c^3*d^3*e
 + 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4
*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*2i
```

3.144 $\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 363

$$\begin{aligned}\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = {} & -\frac{e^3x^3}{c(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\,(a+cx^4)} \\ & - \frac{3\big(\sqrt{c}d+\sqrt{a}e\big)\,(cd^2+ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}} \\ & + \frac{3\big(\sqrt{c}d+\sqrt{a}e\big)\,(cd^2+ae^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}} \\ & - \frac{3\big(\sqrt{c}d-\sqrt{a}e\big)\,(cd^2+ae^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}} \\ & + \frac{3\big(\sqrt{c}d-\sqrt{a}e\big)\,(cd^2+ae^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}\end{aligned}$$

```
[Out] -e^3*x^3/c/(c*x^4+a)+1/4*x*(d*(-3*a*e^2+c*d^2)+3*e*(a*e^2+c*d^2)*x^2)/a/c/(
c*x^4+a)-3/32*(a*e^2+c*d^2)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)+3/32*(a*e^2+c*d^2)*ln(a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)
/c^(7/4)*2^(1/2)+3/16*(a*e^2+c*d^2)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e
*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)+3/16*(a*e^2+c*d^2)*arctan(1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1221, 1872, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = &-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}{8\sqrt{2}a^{7/4}c^{7/4}}\\ &+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}{8\sqrt{2}a^{7/4}c^{7/4}}\\ &-\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}\\ &+\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}\\ &+\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4ac\,(a+cx^4)}-\frac{e^3x^3}{c\,(a+cx^4)}\end{aligned}$$

```
[In] Int[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

```
[Out] -((e^3*x^3)/(c*(a + c*x^4))) + (x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d^2 + a*e^2
)*x^2))/(4*a*c*(a + c*x^4)) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) + (3*(Sq
rt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) - (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4
)*c^(7/4)) + (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{e^3x^3}{c\left(a+cx^4\right)}-\frac{\int \frac{-cd^3-3e\left(cd^2+ae^2\right)x^2-3cde^2x^4}{\left(a+cx^4\right)^2}\,dx}{c} \\
&= -\frac{e^3x^3}{c\left(a+cx^4\right)}+\frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)}+\frac{\int \frac{3cd\left(cd^2+ae^2\right)+3ce\left(cd^2+ae^2\right)x^2}{a+cx^4}\,dx}{4ac^2} \\
&= -\frac{e^3x^3}{c\left(a+cx^4\right)}+\frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} \\
&\quad +\frac{\left(3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^{3/2}c^2} \\
&\quad +\frac{\left(3\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a^{3/2}c^2} \\
&= -\frac{e^3x^3}{c\left(a+cx^4\right)}+\frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} \\
&\quad -\frac{\left(3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}c^{7/4}} \\
&\quad -\frac{\left(3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}c^{7/4}} \\
&\quad +\frac{\left(3\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}c^2} \\
&\quad +\frac{\left(3\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^3x^3}{c\left(a+cx^4\right)}+\frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\left(a+cx^4\right)} \\
&\quad -\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}} \\
&\quad +\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}} \\
&\quad +\frac{\left(3\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}} \\
&\quad -\frac{\left(3\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}} \\
&= -\frac{e^3x^3}{c\left(a+cx^4\right)}+\frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\left(a+cx^4\right)} \\
&\quad -\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2+ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}} \\
&\quad +\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2+ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}} \\
&\quad -\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}} \\
&\quad +\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.02

$$\int\frac{\left(d+ex^2\right)^3}{\left(a+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{8a^{3/4}c^{3/4}\left(ae^2x\left(3d+ex^2\right)-cd^2x\left(d+3ex^2\right)\right)}{a+cx^4}-6\sqrt{2}\left(c^{3/2}d^3+\sqrt{a}cd^2e+a\sqrt{c}de^2+a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+6}{}$$

```
[In] Integrate[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(3/4)*(a*e^2*x*(3*d + e*x^2) - c*d^2*x*(d + 3*e*x^2)))/(a +
c*x^4) - 6*Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/
2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 +
Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4
```

```
)*x)/a^(1/4)] + 3*Sqrt[2]*(-(c^(3/2)*d^3) + Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e
^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] +
3*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.33

method	result
risch	$\frac{-\frac{e\left(a\,e^{2}-3c\,d^{2}\right)x^{3}}{4ac}-\frac{d\left(3a\,e^{2}-c\,d^{2}\right)x}{4ac}}{c\,x^{4}+a}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{4}+a\right)}\frac{\left(e\left(a\,e^{2}+c\,d^{2}\right)\_R^{2}+d\left(a\,e^{2}+c\,d^{2}\right)\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{16a\,c^{2}}$
default	$\frac{-\frac{e\left(a\,e^{2}-3c\,d^{2}\right)x^{3}}{4ac}-\frac{d\left(3a\,e^{2}-c\,d^{2}\right)x}{4ac}}{c\,x^{4}+a}+\frac{3\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right.\right.}{8a}\right.}{4ac}$

```
[In] int((e*x^2+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*e*(a*e^2-3*c*d^2)/a/c*x^3-1/4*d*(3*a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+3/16
/a/c^2*sum((e*(a*e^2+c*d^2)*_R^2+d*(a*e^2+c*d^2))/_R^3*ln(x-_R),_R=RootOf(_
Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. 2(280) = 560.

Time = 0.72 (sec) , antiderivative size = 2116, normalized size of antiderivative = 5.83

$$\int \frac{\left(d+ex^{2}\right)^{3}}{\left(a+cx^{4}\right)^{2}}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(3*c*d^2*e - a*e^3)*x^3 - 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*d^5*e
+ 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2
- a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10
+ a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2 + 2*
a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x + 27*(a
^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6*c^5*e*
sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 -
a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(-(2*c^2*d^5
```

```
*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e
^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^
10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2
*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^
10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^
2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^4*d^8*e
^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x
- 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6
*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6
*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(-(2*
c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5
*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c
*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) - 3*(a*c^2*x^4 + a^2*c)*sqrt(
-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^12 + 2*a
*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a
^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^
4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e
^10)*x + 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^
6 - a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*
c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sq
rt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^12 +
2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 +
2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c
)*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^1
2 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^
8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 +
 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8
- a^5*e^10)*x - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c
^2*d*e^6 - a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 -
 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^
7)))*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*
d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4
*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 4*(c*d^3 - 3*
a*d*e^2)*x)/(a*c^2*x^4 + a^2*c)
```

Sympy [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$$

$$= \operatorname{RootSum}\Bigg(65536t^4a^7c^7 + t^2\cdot\left(9216a^6c^4de^5 + 18432a^5c^5d^3e^3 + 9216a^4c^6d^5e\right) + 81a^6e^{12} + 486a^5cd^2e^{10} + 12$$

$$+ \frac{x^3(-ae^3 + 3cd^2e) + x(-3ade^2 + cd^3)}{4a^2c + 4ac^2x^4}$$

```
[In] integrate((e*x**2+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**7 + _t**2*(9216*a**6*c**4*d*e**5 + 18432*a**5*c
**5*d**3*e**3 + 9216*a**4*c**6*d**5*e) + 81*a**6*e**12 + 486*a**5*c*d**2*e*
*10 + 1215*a**4*c**2*d**4*e**8 + 1620*a**3*c**3*d**6*e**6 + 1215*a**2*c**4*
d**8*e**4 + 486*a*c**5*d**10*e**2 + 81*c**6*d**12, Lambda(_t, _t*log(x + (4
096*_t**3*a**6*c**5*e + 432*_t*a**5*c**2*d*e**6 + 720*_t*a**4*c**3*d**3*e**
4 + 144*_t*a**3*c**4*d**5*e**2 - 144*_t*a**2*c**5*d**7)/(27*a**5*e**10 + 81
*a**4*c*d**2*e**8 + 54*a**3*c**2*d**4*e**6 - 54*a**2*c**3*d**6*e**4 - 81*a*
c**4*d**8*e**2 - 27*c**5*d**10)))) + (x**3*(-a*e**3 + 3*c*d**2*e) + x*(-3*a
*d*e**2 + c*d**3))/(4*a**2*c + 4*a*c**2*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.80

$$\int \frac{\left(d+ex^2\right)^3}{\left(a+cx^4\right)^2}\,dx = \frac{(3\,cd^2e-ae^3)x^3+(cd^3-3\,ade^2)x}{4\,(ac^2x^4+a^2c)} + \frac{3\,(cd^2+ae^2)\left(\frac{2\,\sqrt{2}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(\sqrt{c}d-}{}\right.}{32\,ac}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((3*c*d^2*e - a*e^3)*x^3 + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*x^4 + a^2*c) +
 3/32*(c*d^2 + a*e^2)*(2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(
sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt
(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx &= \frac{3\,cd^2ex^3 - ae^3x^3 + cd^3x - 3\,ade^2x}{4\,(cx^4+a)ac} \\ &+ \frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 + (ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4} \\ &+ \frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 + (ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4} \\ &+ \frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 - (ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^4} \\ &- \frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 - (ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^4}\end{aligned}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*e*x^3 - a*e^3*x^3 + c*d^3*x - 3*a*d*e^2*x)/((c*x^4 + a)*a*c) +
 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^
(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*
c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/32*sqr
t(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*c*d
^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
^2*c^4) - 3/32*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 -
 (a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/
4) + sqrt(a/c))/(a^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 2560, normalized size of antiderivative = 7.05

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^3/(a + c*x^4)^2,x)
```

```
[Out] - ((d*x*(3*a*e^2 - c*d^2))/(4*a*c) + (e*x^3*(a*e^2 - 3*c*d^2))/(4*a*c))/(a
+ c*x^4) - 2*atanh((9*c^3*d^6*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (
9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2))/
(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a
^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*c*d^6*
e^3)/16 - (27*a^3*e^9)/(32*c^2) + (27*c^2*d^8*e)/(32*a) - (27*a^2*d^2*e^7)/
(16*c) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^5*c) - (27*d*e^8*(-a^7*c^7)^(1/2))
/(32*a*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^2*c^4) + (27*d^7*e^2*(-a^
7*c^7)^(1/2))/(16*a^4*c^2))) + (9*a*e^6*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^
4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^
7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/
2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*(
(27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c
^2*d^8*e)/(32*a^3) - (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) + (27*d*e^8*(-a^7
*c^7)^(1/2))/(32*a^3*c^5) + (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^4*c^4) - (2
7*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) + (9*c*d^2*e^4*x*((9*e^6*(-a^7*c
^7)^(1/2))/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2)
 - (9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*
e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6
*c^5))^(1/2))/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3
)/(16*a^2) - (27*c^2*d^8*e)/(32*a^3) - (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c)
 + (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^3*c^5) + (27*d^3*e^6*(-a^7*c^7)^(1/2))
/(16*a^4*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) - (9*c^2*d^4*e
^2*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d
^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(
128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*
c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a^2*e^9)/(32*c^2) + (27*a*d^2*e^7
)/(16*c) - (27*c*d^6*e^3)/(16*a) - (27*c^2*d^8*e)/(32*a^2) - (27*d^9*(-a^7*
c^7)^(1/2))/(32*a^6*c) + (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^2*c^5) + (27*d^3
*e^6*(-a^7*c^7)^(1/2))/(16*a^3*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^5
*c^2))))*(-(9*(c^3*d^6*(-a^7*c^7)^(1/2) - a^3*e^6*(-a^7*c^7)^(1/2) + 2*a^4*
c^6*d^5*e + 2*a^6*c^4*d*e^5 + 4*a^5*c^5*d^3*e^3 + a*c^2*d^4*e^2*(-a^7*c^7)^
(1/2) - a^2*c*d^2*e^4*(-a^7*c^7)^(1/2)))/(256*a^7*c^7))^(1/2) - 2*atanh((9*
c^3*d^6*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) -
 (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^(1/2)
)/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9*d^4*e^2*(
-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*c*d^6*e^3)/16 - (27*a^3*e^9)
/(32*c^2) + (27*c^2*d^8*e)/(32*a) - (27*a^2*d^2*e^7)/(16*c) - (27*d^9*(-a^7
*c^7)^(1/2))/(32*a^5*c) + (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a*c^5) + (27*d^3*
e^6*(-a^7*c^7)^(1/2))/(16*a^2*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^4*
c^2))) + (9*a*e^6*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(12
8*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*
c^7)^(1/2))/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9
*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a*e^9)/(32*c^2) +
(27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c^2*d^8*e)/(32*a^3) + (
```

```
27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) - (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^3*c
^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^4*c^4) + (27*d^7*e^2*(-a^7*c^7)^(
1/2))/(16*a^6*c^2))) + (9*c*d^2*e^4*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^
4) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^
3) - (9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/
(256*a^5*c^6) + (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*
a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c^2*d
^8*e)/(32*a^3) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) - (27*d*e^8*(-a^7*c^7
)^(1/2))/(32*a^3*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^4*c^4) + (27*d^
7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) - (9*c^2*d^4*e^2*x*((9*d^6*(-a^7*c^7
)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) -
 (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^2*e^
4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c
^5))^(1/2))/(2*((27*a^2*e^9)/(32*c^2) + (27*a*d^2*e^7)/(16*c) - (27*c*d^6*e
^3)/(16*a) - (27*c^2*d^8*e)/(32*a^2) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^6*c)
 - (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^2*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))
/(16*a^3*c^4) + (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^5*c^2))))*(-(9*(a^3*e^6
*(-a^7*c^7)^(1/2) - c^3*d^6*(-a^7*c^7)^(1/2) + 2*a^4*c^6*d^5*e + 2*a^6*c^4*
d*e^5 + 4*a^5*c^5*d^3*e^3 - a*c^2*d^4*e^2*(-a^7*c^7)^(1/2) + a^2*c*d^2*e^4*
(-a^7*c^7)^(1/2)))/(256*a^7*c^7))^(1/2)
```

3.145 $\int \frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 349

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx = & -\frac{e^2x}{3c\left(a+cx^4\right)}+\frac{x\left(3cd^2+ae^2+6cdex^2\right)}{12ac\left(a+cx^4\right)} \\
& -\frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} \\
& +\frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} \\
& -\frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}} \\
& +\frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}
\end{aligned}
$$

```
[Out] -1/3*e^2*x/c/(c*x^4+a)+1/12*x*(6*c*d*e*x^2+a*e^2+3*c*d^2)/a/c/(c*x^4+a)-1/3
2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2+a*e^2-2*d*e*a
^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(3*c*d^2+a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2
^(1/2)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2+a*e^2+2*d*e*a^(1/
2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4)
)*(3*c*d^2+a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1221, 1193, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = &-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)}{8\sqrt{2}a^{7/4}c^{5/4}}\\ &+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)}{8\sqrt{2}a^{7/4}c^{5/4}}\\ &-\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}\\ &+\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}\\ &+\frac{x(ae^2+3cd^2+6cdex^2)}{12ac\,(a+cx^4)}-\frac{e^2x}{3c\,(a+cx^4)}\end{aligned}$$

```
[In] Int[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

```
[Out] -1/3*(e^2*x)/(c*(a + c*x^4)) + (x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/(12*a*c*
(a + c*x^4)) - ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*c*d^2 + 2*Sqrt[a]
*Sqrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a
^(7/4)*c^(5/4)) - ((3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4)) + ((
3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\text{integral} = -\frac{e^2x}{3c\,(a+cx^4)} - \frac{\int \frac{-3cd^2-ae^2-6cdex^2}{(a+cx^4)^2}\,dx}{3c}$$

$$= -\frac{e^2x}{3c\left(a+cx^4\right)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\left(a+cx^4\right)} + \frac{\int \frac{3\left(3cd^2+ae^2\right)+6cdex^2}{a+cx^4}\,dx}{12ac}$$

$$= -\frac{e^2x}{3c\left(a+cx^4\right)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\left(a+cx^4\right)} + \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^{3/2}c^{3/2}} + \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a^{3/2}c^{3/2}}$$

$$= -\frac{e^2x}{3c\left(a+cx^4\right)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\left(a+cx^4\right)} - \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}c^{5/4}} - \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}c^{5/4}} + \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}c^{3/2}} + \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}c^{3/2}}$$

$$= -\frac{e^2x}{3c\left(a+cx^4\right)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\left(a+cx^4\right)} - \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}} + \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}} + \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} - \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}}$$

$$
\begin{aligned}
&= -\frac{e^2x}{3c\,(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\,(a+cx^4)} \\
&\quad - \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} \\
&\quad + \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} \\
&\quad - \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}} \\
&\quad + \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.85

$$
\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx
$$

$$
= \frac{-\frac{8a^{3/4}\sqrt[4]{c}(ae^2x-cdx(d+2ex^2))}{a+cx^4} - 2\sqrt{2}\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}\left(3cd^2+2\sqrt{a}\sqrt{cd}\right.}{}
$$

```
[In] Integrate[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(1/4)*(a*e^2*x - c*d*x*(d + 2*e*x^2)))/(a + c*x^4) - 2*Sqrt[
2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)] + 2*Sqrt[2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a
*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(3*c
*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\frac{ed\,x^3}{2a}-\frac{\left(a\,e^2-c\,d^2\right)x}{4ac}}{c\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(2ed\_R^2+\frac{a\,e^2+3c\,d^2}{c}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$\frac{\frac{ed\,x^3}{2a}-\frac{\left(a\,e^2-c\,d^2\right)x}{4ac}}{c\,x^4+a}+\frac{\frac{\left(a\,e^2+3c\,d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de\sqrt{2}\left(\ln\left(\frac{x^2-(}{x^2+(}\right.\right.}{}}{4ac}$

```
[In] int((e*x^2+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*e*d/a*x^3-1/4*(a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+1/16/a/c*sum((2*e*d*_R^2+
1/c*(a*e^2+3*c*d^2))/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. $2(264) = 528$.

Time = 0.72 (sec) , antiderivative size = 1596, normalized size of antiderivative = 4.57

$$\int \frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*c*d*e*x^3 + (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 +
36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5
)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2
 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt
(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a
^4*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^
4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2
*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e
^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36
*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))
 + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*sqrt(-
(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4
*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4
+ a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c
^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3
)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*
c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) -
12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38
*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt(-(81
```

```
*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^
8)/(a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a
^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d
^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a
^3*c^2))) + (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*
d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c
*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*sqrt(-(81*c^4
*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(
a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c
*e^6)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e
^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c
^2))) + 4*(c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c)
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$$

$$= \text{RootSum}\Bigg(65536t^4a^7c^5 + t^2\cdot\left(2048a^5c^3de^3 + 6144a^4c^4d^3e\right) + a^4e^8 + 20a^3cd^2e^6 + 118a^2c^2d^4e^4 + 180ac^3d$$

$$+ \frac{2cdex^3 + x(-ae^2 + cd^2)}{4a^2c + 4ac^2x^4}$$

```
[In] integrate((e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**5 + _t**2*(2048*a**5*c**3*d*e**3 + 6144*a**4*c*
*4*d**3*e) + a**4*e**8 + 20*a**3*c*d**2*e**6 + 118*a**2*c**2*d**4*e**4 + 18
0*a*c**3*d**6*e**2 + 81*c**4*d**8, Lambda(_t, _t*log(x + (-8192*_t**3*a**6*
c**4*d*e + 16*_t*a**5*c*e**6 - 48*_t*a**4*c**2*d**2*e**4 - 144*_t*a**3*c**3
*d**4*e**2 + 432*_t*a**2*c**4*d**6)/(a**4*e**8 + 12*a**3*c*d**2*e**6 + 38*a
**2*c**2*d**4*e**4 + 108*a*c**3*d**6*e**2 + 81*c**4*d**8)))) + (2*c*d*e*x**
3 + x*(-a*e**2 + c*d**2))/(4*a**2*c + 4*a*c**2*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = \frac{2\,cdex^3+(cd^2-ae^2)x}{4\,(ac^2x^4+a^2c)} + \frac{\frac{2\sqrt{2}\left(3\,c^{\frac{3}{2}}d^2+2\,\sqrt{a}cde+a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(3\,c^{\frac{3}{2}}d^2+2\,\sqrt{a}cde+a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \sqrt{2}(}{32\,ac}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*c*d*e*x^3 + (c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c) + 1/32*(2*sqrt(2)
*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sq
rt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a*sqr
t(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(s
qrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*c^(3/
2)*d^2 - 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^2 - 2*sqrt(a
)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt
(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx \\
&= \frac{2\,cdex^3+cd^2x-ae^2x}{4\,(cx^4+a)ac} \\
&+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2+2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} \\
&+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2+2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} \\
&+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} \\
&- \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}
\end{aligned}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*c*d*e*x^3 + c*d^2*x - a*e^2*x)/((c*x^4 + a)*a*c) + 1/16*sqrt(2)*(3*(
a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(
2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/3
2*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4
)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2
)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*l
og(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 1565, normalized size of antiderivative = 4.48

$$\int \frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^2/(a + c*x^4)^2,x)
```

```
[Out] 2*atanh((9*c^3*d^4*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (3*d^3*e)/(6
4*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) + (d
^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1/2)
)/(32*a^5) - (c*d^3*e^3)/8 - (a*d*e^5)/16 - (9*c^2*d^5*e)/(16*a) + (e^6*(-a
^7*c^5)^(1/2))/(32*a^2*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^3*c^2) + (
15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^4*c))) + (c*e^4*x*((9*d^4*(-a^7*c^5)^(1/
2))/(256*a^7*c^3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^
7*c^5)^(1/2))/(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/
2))/(2*((27*d^6*(-a^7*c^5)^(1/2))/(32*a^7) - (d*e^5)/(16*a) - (c*d^3*e^3)/(
8*a^2) - (9*c^2*d^5*e)/(16*a^3) + (e^6*(-a^7*c^5)^(1/2))/(32*a^4*c^3) + (5*
d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*
a^6*c))) + (c^2*d^2*e^2*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (3*d^3*
e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5)
 + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/((27*d^6*(-a^7*c^5)^(1/
2))/(32*a^6) - (d*e^5)/16 - (c*d^3*e^3)/(8*a) - (9*c^2*d^5*e)/(16*a^2) + (e
^6*(-a^7*c^5)^(1/2))/(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^4*c^
2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^5*c)))*((a^2*e^4*(-a^7*c^5)^(1/2)
+ 9*c^2*d^4*(-a^7*c^5)^(1/2) - 12*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 + 2*a*c*d
^2*e^2*(-a^7*c^5)^(1/2))/(256*a^7*c^5))^(1/2) - 2*atanh((9*c^3*d^4*x*(- (d*
e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-a^7*c^5)^(1/2))/(256*a^
7*c^3) - (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) - (d^2*e^2*(-a^7*c^5)^(1/2))/
(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1/2))/(32*a^5) + (c*d^3*e^3)/
8 + (a*d*e^5)/16 + (9*c^2*d^5*e)/(16*a) + (e^6*(-a^7*c^5)^(1/2))/(32*a^2*c^
3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^3*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/
```

```
2))/(32*a^4*c))) + (c*e^4*x*(- (d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c)
- (9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (e^4*(-a^7*c^5)^(1/2))/(256*a^5*
c^5) - (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c
^5)^(1/2))/(32*a^7) + (d*e^5)/(16*a) + (c*d^3*e^3)/(8*a^2) + (9*c^2*d^5*e)/
(16*a^3) + (e^6*(-a^7*c^5)^(1/2))/(32*a^4*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2
))/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^6*c))) + (c^2*d^2*e^2
*x*(- (d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-a^7*c^5)^(1/2)
)/(256*a^7*c^3) - (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) - (d^2*e^2*(-a^7*c^5
)^(1/2))/(128*a^6*c^4))^(1/2))/((d*e^5)/16 + (27*d^6*(-a^7*c^5)^(1/2))/(32*
a^6) + (c*d^3*e^3)/(8*a) + (9*c^2*d^5*e)/(16*a^2) + (e^6*(-a^7*c^5)^(1/2))/
(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^4*c^2) + (15*d^4*e^2*(-a^
7*c^5)^(1/2))/(32*a^5*c))))*(-(a^2*e^4*(-a^7*c^5)^(1/2) + 9*c^2*d^4*(-a^7*c^
5)^(1/2) + 12*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 + 2*a*c*d^2*e^2*(-a^7*c^5)^(1
/2))/(256*a^7*c^5))^(1/2) + ((d*e*x^3)/(2*a) - (x*(a*e^2 - c*d^2))/(4*a*c))
/(a + c*x^4)
```

3.146 $\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 275

$$\begin{aligned}\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = {} & \frac{x(d+ex^2)}{4a\left(a+cx^4\right)} - \frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} \\ & + \frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} \\ & - \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} \\ & + \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}\end{aligned}$$

```
[Out] 1/4*x*(e*x^2+d)/a/(c*x^4+a)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/c^(3
/4)*2^(1/2)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*d*c^(1/2
))/a^(7/4)/c^(3/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1
/2)+3*d*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1193, 1182, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = &-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{c}d\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\\ &+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+3\sqrt{c}d\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\\ &-\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}\\ &+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{x(d+ex^2)}{4a\,(a+cx^4)}\end{aligned}$$

```
[In] Int[(d + e*x^2)/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x^2))/(4*a*(a + c*x^4)) - ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d +
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3
/4)) - ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d - Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c
^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(d+ex^2)}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d-ex^2}{a+cx^4}\,dx}{4a} \\
&= \frac{x(d+ex^2)}{4a\left(a+cx^4\right)} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8ac} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8ac} \\
&= \frac{x(d+ex^2)}{4a\left(a+cx^4\right)} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} \\
&\quad - \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}c^{3/4}} - \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}c^{3/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(d+ex^2)}{4a(a+cx^4)} - \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} \\
&\quad + \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} \\
&\quad + \frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} \\
&\quad - \frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(d+ex^2)}{4a(a+cx^4)} - \frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} \\
&\quad + \frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} \\
&\quad - \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} \\
&\quad + \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.97

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx$$

$$= \frac{\frac{8ax(d+ex^2)}{a+cx^4} - \frac{2\sqrt{2}\sqrt[4]{a}\left(3\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt{2}\sqrt[4]{a}\left(3\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{\sqrt{2}\left(-3\sqrt[4]{a}\sqrt{c}d+a^{3/4}e\right)\mathrm{lo}}{c}}{32a^2}$$

```
[In] Integrate[(d + e*x^2)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x^2))/(a + c*x^4) - (2*Sqrt[2]*a^(1/4)*(3*Sqrt[c]*d + Sqrt[a
]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (2*Sqrt[2]*a^(1/4)*
(3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4)
+ (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d - a^(3/4
)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(32*a
^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\frac{e\,x^{3}}{4a}+\frac{dx}{4a}}{c\,x^{4}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(\_R^{2}e+3d\right)\ln\left(x-\_R\right)}{\_R^{3}}}{16ac}$
default	$d\left(\frac{x}{4a(c\,x^{4}+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^{2}}\right)+e\left(\frac{x^{3}}{4a(c\,x^{4}+a)}\right.$

```
[In] int((e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*e/a*x^3+1/4*d/a*x)/(c*x^4+a)+1/16/a/c*sum((_R^2*e+3*d)/_R^3*ln(x-_R),_
R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. 2(192) = 384.

Time = 0.29 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.17

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$$
$$= \frac{4\,ex^3-(acx^4+a^2)\sqrt{-\frac{a^3c\sqrt{-\frac{81\,c^2d^4-18\,acd^2e^2+a^2e^4}{a^7c^3}}+6\,de}{a^3c}}\log\left(-(81\,c^2d^4-a^2e^4)x+\left(a^6c^2e\sqrt{-\frac{81\,c^2d^4-18\,acd^2e^2+}{a^7c^3}}\right.\right.}{}$$

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*e*x^3 - (a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2
*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*x
+ (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 27*
a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^
2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (a*c*x^4 + a^2)*sqrt(-(a^3*c*s
qrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*l
og(-(81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^
2 + a^2*e^4)/(a^7*c^3)) + 27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt
(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (
a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a
^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*x + (a^6*c^2*e*sqrt(
```

```
-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 27*a^2*c^2*d^3 + 3*a^
3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c
^3)) - 6*d*e)/(a^3*c))) - (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 1
8*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^
2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^
3)) - 27*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*
c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))) + 4*d*x)/(a*c*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.49

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx$$
$$= \operatorname{RootSum}\Bigg(65536t^4a^7c^3 + 3072t^2a^4c^2de + a^2e^4 + 18acd^2e^2 + 81c^2d^4, \Bigg(t \mapsto t\log\Bigg(x + \frac{4096t^3a^6c^2e + 144ta}{a^2e^4 - 8}$$
$$+ \frac{dx+ex^3}{4a^2+4acx^4}$$

```
[In] integrate((e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 3072*_t**2*a**4*c**2*d*e + a**2*e**4 + 18*a
*c*d**2*e**2 + 81*c**2*d**4, Lambda(_t, _t*log(x + (4096*_t**3*a**6*c**2*e
+ 144*_t*a**3*c*d*e**2 - 432*_t*a**2*c**2*d**3)/(a**2*e**4 - 81*c**2*d**4))
)) + (d*x + e*x**3)/(4*a**2 + 4*a*c*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.92

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx = \frac{ex^3+dx}{4\left(acx^4+a^2\right)}$$
$$+ \frac{\frac{2\sqrt{2}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(3\sqrt{c}d-\sqrt{a}e)\log\Big(\sqrt{c}x^2+\sqrt{}}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{32\,a}$$

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e*x^3 + d*x)/(a*c*x^4 + a^2) + 1/32*(2*sqrt(2)*(3*sqrt(c)*d + sqrt(a)*
e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*sqrt(c)*d
+ sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqr
```

```
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*sq
rt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))
/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = {} & \frac{ex^3+dx}{4\,(cx^4+a)a} + \frac{\sqrt{2}\Big(3\,(ac^3)^{\frac14}\,c^2d+(ac^3)^{\frac34}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\,\left(\frac{a}{c}\right)^{\frac14}}\right)}{16\,a^2c^3} \\ & + \frac{\sqrt{2}\Big(3\,(ac^3)^{\frac14}\,c^2d+(ac^3)^{\frac34}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\,\left(\frac{a}{c}\right)^{\frac14}}\right)}{16\,a^2c^3} \\ & + \frac{\sqrt{2}\Big(3\,(ac^3)^{\frac14}\,c^2d-(ac^3)^{\frac34}\,e\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} \\ & - \frac{\sqrt{2}\Big(3\,(ac^3)^{\frac14}\,c^2d-(ac^3)^{\frac34}\,e\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}\end{aligned}$$

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(e*x^3 + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (
a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))
/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*sqrt(
2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4
) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3
/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 637, normalized size of antiderivative = 2.32

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = \frac{\frac{e\,x^3}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^4+a} - 2\,\mathrm{atanh}\left(\frac{c^2\,e^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}-\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}}}{2\,\left(\frac{c\,e^3}{32\,a}-\frac{9\,c^2\,d^2\,e}{32\,a^2}-\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^6}+\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^5}\right)} - \frac{9\,c^3\,d^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}-\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}}}{2\,\left(\frac{c\,e^3}{32}-\frac{9\,c^2\,d^2\,e}{32\,a}-\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^5}+\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^4}\right)}\right)\sqrt{-\frac{9\,c\,d^2\,\sqrt{-a^7\,c^3}-a\,e^2\,\sqrt{-a^7\,c^3}+6\,a^4\,c^2\,d\,e}{256\,a^7\,c^3}} - 2\,\mathrm{atanh}\left(\frac{c^2\,e^2\,x\,\sqrt{\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}-\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}}}{2\,\left(\frac{c\,e^3}{32\,a}-\frac{9\,c^2\,d^2\,e}{32\,a^2}+\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^6}-\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^5}\right)} - \frac{9\,c^3\,d^2\,x\,\sqrt{\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}-\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}}}{2\,\left(\frac{c\,e^3}{32}-\frac{9\,c^2\,d^2\,e}{32\,a}+\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^5}-\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^4}\right)}\right)\sqrt{-\frac{a\,e^2\,\sqrt{-a^7\,c^3}-9\,c\,d^2\,\sqrt{-a^7\,c^3}+6\,a^4\,c^2\,d\,e}{256\,a^7\,c^3}}$$

```
[In] int((d + e*x^2)/(a + c*x^4)^2,x)
```

```
[Out] ((e*x^3)/(4*a) + (d*x)/(4*a))/(a + c*x^4) - 2*atanh((c^2*e^2*x*((e^2*(-a^7*
c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d*e
)/(128*a^3*c))^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) - (27*c*d
^3*(-a^7*c^3)^(1/2))/(32*a^6) + (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) - (9*
c^3*d^2*x*((e^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1/2))/
(256*a^7*c^2) - (3*d*e)/(128*a^3*c))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d^2*e)/
(32*a) - (27*c*d^3*(-a^7*c^3)^(1/2))/(32*a^5) + (3*d*e^2*(-a^7*c^3)^(1/2))/
(32*a^4))))*(-(9*c*d^2*(-a^7*c^3)^(1/2) - a*e^2*(-a^7*c^3)^(1/2) + 6*a^4*c^
2*d*e)/(256*a^7*c^3))^(1/2) - 2*atanh((c^2*e^2*x*((9*d^2*(-a^7*c^3)^(1/2))/
(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3))
^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) + (27*c*d^3*(-a^7*c^3)^
(1/2))/(32*a^6) - (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) - (9*c^3*d^2*x*((9*
d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e^2*(-a^7*c^3)
^(1/2))/(256*a^6*c^3))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d^2*e)/(32*a) + (27*c
*d^3*(-a^7*c^3)^(1/2))/(32*a^5) - (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^4))))*(-
(a*e^2*(-a^7*c^3)^(1/2) - 9*c*d^2*(-a^7*c^3)^(1/2) + 6*a^4*c^2*d*e)/(256*a^
7*c^3))^(1/2)
```

3.147 $\int \frac{1}{\left(a+cx^4\right)^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 202

$$\int \frac{1}{\left(a+cx^4\right)^2} \, dx=\frac{x}{4a\left(a+cx^4\right)}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

```
[Out] 1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)
*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)-3
/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2^(1
/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {205, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{1}{\left(a+cx^4\right)^2} \, dx=-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{x}{4a\left(a+cx^4\right)}$$

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{4a\left(a+cx^4\right)}+\frac{3\int \frac{1}{a+cx^4}\,dx}{4a}\\
&= \frac{x}{4a\left(a+cx^4\right)}+\frac{3\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}}+\frac{3\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}}\\
&= \frac{x}{4a\left(a+cx^4\right)}+\frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}}+\frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}}\\
&\quad -\frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}\\
&= \frac{x}{4a\left(a+cx^4\right)}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}\\
&\quad +\frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}\\
&= \frac{x}{4a\left(a+cx^4\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}\\
&\quad -\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx = \frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x-\right.}{\sqrt[4]{c}}}{32a^{7/4}}$$

```
[In] Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{x}{4a\left(c\,x^{4}+a\right)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{16ac}$	46
default	$\frac{x}{4a\left(c\,x^{4}+a\right)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^{2}}$	118

```
[In] int(1/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/16/a/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+cx^4)^2}\,dx$$
$$= \frac{3\,(acx^4+a^2)(-\frac{1}{a^7c})^{\frac{1}{4}}\log\left(a^2(-\frac{1}{a^7c})^{\frac{1}{4}}+x\right)-3\,(-i\,acx^4-i\,a^2)(-\frac{1}{a^7c})^{\frac{1}{4}}\log\left(i\,a^2(-\frac{1}{a^7c})^{\frac{1}{4}}+x\right)-3\,(i\,acx}{16\,(acx^4+a}$$

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x)
- 3*(-I*a*c*x^4 - I*a^2)*(-1/(a^7*c))^(1/4)*log(I*a^2*(-1/(a^7*c))^(1/4) +
x) - 3*(I*a*c*x^4 + I*a^2)*(-1/(a^7*c))^(1/4)*log(-I*a^2*(-1/(a^7*c))^(1/4)
 + x) - 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) +
x) + 4*x)/(a*c*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a^2+4acx^4}+\operatorname{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(acx^4+a^2)}$$
$$+\frac{3\,\Bigg(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log(\sqrt{c}x^2-}{a^{\frac{3}{4}}}}{32\,a}$$

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c
))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int \frac{1}{(a+cx^4)^2}\,dx = {} & \frac{x}{4\,(cx^4+a)a} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} \\
& + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} \\
& + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} \\
& - \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}
\end{aligned}
$$

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx=\frac{x}{4\,a\,\left(c\,x^4+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,c^{1/4}}$$

```
[In] int(1/(a + c*x^4)^2,x)
```

```
[Out] x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4
)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

3.148 $\int \frac{1}{(d+ex^2)\,(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 689

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)\,(a+cx^4)^2}\,dx = {} & \frac{cx(d-ex^2)}{4a\,(cd^2+ae^2)\,(a+cx^4)} + \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} \\
& - \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)^2} \\
& - \frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)} \\
& + \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)^2} \\
& + \frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)} \\
& - \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)^2} \\
& - \frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\,(cd^2+ae^2)} \\
& + \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)^2} \\
& + \frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\,(cd^2+ae^2)}
\end{aligned}
$$

```
[Out] 1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/4*c^(1/4)*e^2*arctan(-1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/
2)+1/4*c^(1/4)*e^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2
))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^
(1/2)+1/8*c^(1/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*
a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^
(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1
/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/
2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2
)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+e^(7/2)*arctan(x*e^(1/2)/d^(1/
2))/(a*e^2+c*d^2)^2/d^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

$= \{1253, 211, 1193, 1182, 1176, 631, 210, 1179, 642\}$

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = &-\frac{\sqrt[4]{c}e^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}\\
&+\frac{\sqrt[4]{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}\\
&-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(3\sqrt{c}d-\sqrt{a}e\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}\\
&+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{c}d-\sqrt{a}e\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}\\
&-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}\\
&+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}\\
&-\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}\\
&+\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}\\
&+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)^2}+\frac{cx(d-ex^2)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}\end{aligned}$$

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1253

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{e^4}{\left(cd^2+ae^2\right)^2(d+ex^2)}+\frac{c(d-ex^2)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)^2}-\frac{ce^2(-d+ex^2)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right)dx\\
&= -\frac{\left(ce^2\right)\int \frac{-d+ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{e^4\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{c\int \frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2}\\
&= \frac{cx(d-ex^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}+\frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2}\\
&\quad+\frac{\left(e^2\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2}-\frac{c\int \frac{-3d+ex^2}{a+cx^4}\,dx}{4a\left(cd^2+ae^2\right)}\\
&= \frac{cx(d-ex^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2}+\frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)^2}\\
&\quad+\frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)^2}-\frac{\left(\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}\\
&\quad-\frac{\left(\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}\\
&\quad+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a\left(cd^2+ae^2\right)}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a\left(cd^2+ae^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{cx(d-ex^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} \\
&\quad -\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad +\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad +\frac{\left(\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad -\frac{\left(\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad +\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a\left(cd^2+ae^2\right)}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a\left(cd^2+ae^2\right)} \\
&\quad -\frac{\left(\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)} \\
&\quad -\frac{\left(\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\right)\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{cx(d-ex^2)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} \\
&\quad-\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad+\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad-\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad-\frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)} \\
&\quad+\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad+\frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)} \\
&\quad+\frac{\left(\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)} \\
&\quad-\frac{\left(\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{cx(d-ex^2)}{4a\left(cd^2+ae^2\right)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)} \\
&\quad + \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)} \\
&\quad - \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)} \\
&\quad + \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{\sqrt[4]{c}\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.62

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$$

$$= \frac{\frac{8c(cd^2+ae^2)x(d-ex^2)}{a(a+cx^4)} + \frac{32e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\sqrt[4]{c}(-3c^{3/2}d^3+\sqrt{a}cd^2e-7a\sqrt{c}de^2+5a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{2\sqrt{2}\sqrt[4]{c}(-3}{}}{}$$

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]
*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e
- 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
```

```
])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c
]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqr
t[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.48

method	result
default	$\frac{c\left(\frac{-\frac{e\left(a\,e^{2}+c\,d^{2}\right)x^{3}}{4a}+\frac{d\left(a\,e^{2}+c\,d^{2}\right)x}{4a}}{c\,x^{4}+a}+\frac{\frac{\left(7d\,e^{2}a+3d^{3}c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+}{4a}\right.}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}$
risch	Expression too large to display

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c/(a*e^2+c*d^2)^2*((-1/4*e*(a*e^2+c*d^2)/a*x^3+1/4*d*(a*e^2+c*d^2)/a*x)/(c*
x^4+a)+1/4/a*(1/8*(7*a*d*e^2+3*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-5*a*e
^3-c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))+e^4/(a*e^2+c*d^2)^2/(e*d)^(1/2)*arc
tan(e*x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. 2(518) = 1036.

Time = 12.22 (sec) , antiderivative size = 9892, normalized size of antiderivative = 14.36

$$\int \frac{1}{\left(d+ex^{2}\right)\left(a+cx^{4}\right)^{2}}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\,(a+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\,(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 621, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{e^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{de}}$$
$$+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-\left(ac^3\right)^{\frac{3}{4}}cd^2e-5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}$$
$$+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-\left(ac^3\right)^{\frac{3}{4}}cd^2e-5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}$$
$$+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+\left(ac^3\right)^{\frac{3}{4}}cd^2e+5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}$$
$$-\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+\left(ac^3\right)^{\frac{3}{4}}cd^2e+5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}$$
$$-\frac{cex^3-cdx}{4\left(cx^4+a\right)\left(acd^2+a^2e^2\right)}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)) +
 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)
*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(
1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(
2)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^
2 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^
3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3
)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^
2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3
*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*
c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*
a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/4*(c*e*x^3 - c*d*x)/((c*x^4 + a)
*(a*c*d^2 + a^2*e^2))
```

Mupad [B] (verification not implemented)

Time = 16.36 (sec) , antiderivative size = 17945, normalized size of antiderivative = 26.04

$$\int \frac{1}{(d+ex^2)\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^
4) - atan(((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c
^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8
*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c
^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^
(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a
^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*
c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*
c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a
^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 +
 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a
^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a
^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c
^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) +
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e
^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 +
21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8
*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(
128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^
2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
```

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*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4))^(1/2)*1i - (((((
65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 3
6864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 +
663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4
*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c
^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^
5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c
*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536
*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 3
27680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13
 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25
*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d
*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(
256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*
c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 +
7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 666
88*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c
^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^
5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) -
25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c
*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))
/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8
*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9
*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*
a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2
*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6
 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 8
1*c^7*d^4*e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d
^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16
 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e
^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*
e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*
e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/
2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*
a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(
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1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 +
 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3
- 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e
^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*
d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^
2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(
1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4
*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7
*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2
) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*
e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10
 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
+ 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*
a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*
e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(2
56*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^
2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c
^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*
e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)
^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d
^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a
^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1
/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2
*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*
c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(
1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 7
0*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)
^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d
^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8
*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*
c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6
*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a
^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*
c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^
8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)
))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d
^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^
7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^
8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e
^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^
5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2)
 + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c
*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^
```

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11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9
*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d
^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1
/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*
e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*
c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)
 - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 123
20*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(
a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4
*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*
d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/
2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10
*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*
c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532
*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(
-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*
a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*
e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^
4)))^(1/2)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i - atan(
((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e
^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e
^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x
*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e +
 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39
*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*
e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 -
 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e
^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^
4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d
^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/
2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a
^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1
/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 +
6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e
^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8
 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c
^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^
3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*
c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d
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^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*
a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432
*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*
c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c
*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(
1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70
*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612
*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) -
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((65536*a^11
*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c
^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9
*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a
^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-
a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*
d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*
c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*
a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(
-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*
a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*
e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^
4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c
^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7
*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c
*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(
1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70
*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^
(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13
+ 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33
296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5
*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6
*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 4
1*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^1
1*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*
e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 +
```

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 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*
(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i)/((125*a^2*c^5*e^12 + 81*c^7*d^4*
e^8 + 270*a*c^6*d^2*e^10)/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4
*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (((((65536*a^11*c^4*e^16 - 12288*a
^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 24576
0*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331
776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^
5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^
3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^
5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256
*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2
*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a
^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 32768
0*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))
/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*
c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a
^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7
*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 +
 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(11
52*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 2035
2*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*
a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^
3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-
a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a
*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e
^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4
)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*
e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^1
1)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^
6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6
*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a
^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8
 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(
1425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e
^9 + 2532*a^3*c^6*d^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
+ 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*
c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*
e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(2
56*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4)))^(1/2) + (((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 -
57344*a^5*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 +
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634880*a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^
14)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a
^6*c^2*d^4*e^4)) + (x*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2
) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^
2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^
4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(
65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 -
589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11
 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^
4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((25
*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a
^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^2*
c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^
2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 +
 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/
(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c
^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^
4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*
c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 +
4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*
c^10*d^11*e^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8
*d^7*e^7 + 21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 +
a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((
25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44
*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) - 39*a^
2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6
 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 +
 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d
^2*e^11))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
 + 6*a^6*c^2*d^4*e^4)))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1
/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*
e^2*(-a^7*c)^(1/2) - 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*
c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)
))*((25*a^3*e^6*(-a^7*c)^(1/2) - 9*c^3*d^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 - 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) -
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*2i + (atan(-(((((((4
5*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*e^5)/16
 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6*d^3*e^
11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6
*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 - 224*a^
5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*a^8*c^7
*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (x
```

```
*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7
*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*
a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(
512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*
d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*
d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*
c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*
d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a^8*e^8
+ a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*
(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/
(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*e^13 + 81*c^9*
d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11)
)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6
*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2) -
((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c^9*d^9*
e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*a^5*c^6
*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*
e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^14*e^2 -
 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8 + 2480*
a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(2*(a^8*
e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4
)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327
680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 +
327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e
^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*d^8 + 4
*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(
2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*c^11*d^13*e^2 - 490
24*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a
^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^
(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1425*a^4*c^5*e^13 +
81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^
2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*
e^2))/(((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16 + (305*a^2*c
^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9)/8 + (2081*
a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*a^4*c^11*d^1
4*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*c^8*d^8*e^8
+ 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5*d^2*e^14)/(
2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*
d^4*e^4)) - (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^
3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8
*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^
```

```
5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^8 + a^4*c^4*
d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(
1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1152*a^2*c^11*d^13*e^
2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 +
 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/
(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c
^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)))*(-
d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1425*a^4*c^5*
e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3
*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d
^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*e^4 + 2*a*c*
d^3*e^2) - ((125*a^2*c^5*e^12)/128 + (81*c^7*d^4*e^8)/128 + (135*a*c^6*d^2*
e^10)/64)/(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*
a^6*c^2*d^4*e^4) + ((((((45*a*c^10*d^11*e^3)/16 + (1277*a^6*c^5*d*e^13)/16
+ (305*a^2*c^9*d^9*e^5)/16 + (385*a^3*c^8*d^7*e^7)/8 + (657*a^4*c^7*d^5*e^9
)/8 + (2081*a^5*c^6*d^3*e^11)/16)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) - (((((256*a^11*c^4*e^16 - 48*
a^4*c^11*d^14*e^2 - 224*a^5*c^10*d^12*e^4 - 144*a^6*c^9*d^10*e^6 + 960*a^7*
c^8*d^8*e^8 + 2480*a^8*c^7*d^6*e^10 + 2592*a^9*c^6*d^4*e^12 + 1296*a^10*c^5
*d^2*e^14)/(2*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4)) + (x*(-d*e^7)^(1/2)*(65536*a^13*c^4*e^17 - 65536*a^6*
c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 327680
*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13 + 32
7680*a^12*c^5*d^2*e^15))/(512*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)*(a^8*e^
8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4))
)*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) + (x*(1152*a^2*
c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c
^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6
*d^3*e^12))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e
^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d
^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*e^2)) - (x*(1
425*a^4*c^5*e^13 + 81*c^9*d^8*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^
9 + 2532*a^3*c^6*d^2*e^11))/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 +
 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*(-d*e^7)^(1/2))/(c^2*d^5 + a^2*d*
e^4 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2)*1i)/(c^2*d^5 + a^2*d*e^4 + 2*a*c*d^3*
e^2)
```

3.149 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 864

$$
\begin{aligned}
&\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx \\
&= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}+\frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}+\frac{4c\sqrt{d}e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} \\
&+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)^2}-\frac{c^{3/4}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3} \\
&-\frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2} \\
&+\frac{c^{3/4}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3} \\
&+\frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2} \\
&-\frac{c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3} \\
&-\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2} \\
&+\frac{c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3} \\
&+\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}
\end{aligned}
$$

```
[Out] 1/2*e^4*x/d/(a*e^2+c*d^2)^2/(e*x^2+d)+1/4*c*x*(-2*c*d*e*x^2-a*e^2+c*d^2)/a/
(a*e^2+c*d^2)^2/(c*x^4+a)+1/2*e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*
e^2+c*d^2)^2+1/4*c^(3/4)*e^2*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-
a*e^2-4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+1/4*c^(3/4)*e^
2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-a*e^2-4*d*e*a^(1/2)*c^(1/2))
/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+1/16*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a
^(1/4))*(3*c*d^2-3*a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(
1/2)+1/16*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-3*a*e^2-2*d*
e*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/32*c^(3/4)*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2-3*a*e^2+2*d*e*a^(1/2)*c^(1
/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/32*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1
/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2-3*a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/(a
```

```
*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(3*c*d^2-a*e^2+4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3
*2^(1/2)+1/8*c^(3/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*
(3*c*d^2-a*e^2+4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+4*c*e
^(7/2)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(a*e^2+c*d^2)^3
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1253, 205, 211, 1193, 1182, 1176, 631, 210, 1179, 642}

$$
\begin{aligned}
&\int \frac{1}{(d+ex^2)^2\,(a+cx^4)^2}\,dx \\
&= \frac{xe^4}{2d\,(cd^2+ae^2)^2\,(ex^2+d)} + \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\,(cd^2+ae^2)^2} + \frac{4c\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{(cd^2+ae^2)^3} \\
&\quad - \frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^2}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)^3} \\
&\quad + \frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^2}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)^3} \\
&\quad - \frac{c^{3/4}\left(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^2}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)^3} \\
&\quad + \frac{c^{3/4}\left(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^2}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)^3} \\
&\quad - \frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)^2} \\
&\quad + \frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\,(cd^2+ae^2)^2} \\
&\quad - \frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\,(cd^2+ae^2)^2} \\
&\quad + \frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\,(cd^2+ae^2)^2} \\
&\quad + \frac{cx(cd^2-2cex^2d-ae^2)}{4a\,(cd^2+ae^2)^2\,(cx^4+a)}
\end{aligned}
$$

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2 - 2*c*d*e
*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[d]*e^(7/2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^
2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*
(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 + 4
*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 + 2*Sq
rt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 +
4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 + 2*S
qrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1253

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^4}{(cd^2+ae^2)^2(d+ex^2)^2} + \frac{4cde^4}{(cd^2+ae^2)^3(d+ex^2)} + \frac{c(cd^2-ae^2-2cdex^2)}{(cd^2+ae^2)^2(a+cx^4)^2} - \frac{ce^2(-3cd^2+ae^2+4cdex^2)}{(cd^2+ae^2)^3(a+cx^4)}\right)dx$$

$$=-\frac{(ce^2)\int \frac{-3cd^2+ae^2+4cdex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^3}+\frac{(4cde^4)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^3}+\frac{c\int \frac{cd^2-ae^2-2cdex^2}{\left(a+cx^4\right)^2}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{e^4\int \frac{1}{\left(d+ex^2\right)^2}\,dx}{\left(cd^2+ae^2\right)^2}$$

$$=\frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}+\frac{cx(cd^2-ae^2-2cdex^2)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}$$
$$+\frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3}+\frac{\left(\sqrt{c}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\sqrt{a}\left(cd^2+ae^2\right)^3}$$
$$+\frac{\left(\sqrt{c}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\sqrt{a}\left(cd^2+ae^2\right)^3}$$
$$-\frac{c\int \frac{-3\left(cd^2-ae^2\right)+2cdex^2}{a+cx^4}\,dx}{4a\left(cd^2+ae^2\right)^2}+\frac{e^4\int \frac{1}{d+ex^2}\,dx}{2d\left(cd^2+ae^2\right)^2}$$

$$=\frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}+\frac{cx(cd^2-ae^2-2cdex^2)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}+\frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3}$$
$$+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{\left(\sqrt{c}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\left(cd^2+ae^2\right)^3}$$
$$+\frac{\left(\sqrt{c}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\left(cd^2+ae^2\right)^3}$$
$$-\frac{\left(c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$
$$-\frac{\left(c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$
$$+\frac{\left(\sqrt{c}\left(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8a^{3/2}\left(cd^2+ae^2\right)^2}$$
$$+\frac{\left(\sqrt{c}\left(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^{3/2}\left(cd^2+ae^2\right)^2}$$

$$
\begin{aligned}
&= \frac{e^4 x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} \\
&\quad + \frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3} \\
&\quad + \frac{c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3} \\
&\quad + \frac{\left(c^{3/4}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3} \\
&\quad - \frac{\left(c^{3/4}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3} \\
&\quad + \frac{\left(\sqrt{c}\left(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{\left(\sqrt{c}\left(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{\left(c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2} \\
&\quad - \frac{\left(c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}
\end{aligned}
$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}+\frac{cx(cd^2-ae^2-2cdex^2)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}+\frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3}$$

$$+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)^2}-\frac{c^{3/4}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$

$$+\frac{c^{3/4}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$

$$-\frac{c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$

$$-\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}$$

$$+\frac{c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$

$$+\frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}$$

$$+\frac{\left(c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}$$

$$-\frac{\left(c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx(cd^2-ae^2-2cdex^2)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3}$$
$$+ \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)^2} - \frac{c^{3/4}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$
$$- \frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{c^{3/4}e^2\left(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$
$$+ \frac{c^{3/4}\left(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}$$
$$- \frac{c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$
$$- \frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}$$
$$+ \frac{c^{3/4}e^2\left(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$
$$+ \frac{c^{3/4}\left(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx$$

$$= \frac{\frac{16e^4(cd^2+ae^2)x}{d(d+ex^2)} + \frac{8c(cd^2+ae^2)x(-ae^2+cd(d-2ex^2))}{a(a+cx^4)} + \frac{16e^{7/2}(9cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2\sqrt{2}c^{3/4}\left(-3c^2d^4+2\sqrt{a}c^{3/2}d^3e-12acd^2e^2+18}{a^{7/4}}}{}$$

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] ((16*e^4*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (8*c*(c*d^2 + a*e^2)*x*(-(a*e
^2) + c*d*(d - 2*e*x^2)))/(a*(a + c*x^4)) + (16*e^(7/2)*(9*c*d^2 + a*e^2)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sq
rt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2*e^4
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(3/4)*(-3
```

```
*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*
e^3 + 7*a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (Sqrt[2
]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2
)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e +
 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^3)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.47

method	result
default	$-\frac{c\left(\frac{\frac{dce\left(a\,e^{2}+c\,d^{2}\right)x^{3}}{2a}+\frac{\left(a^{2}e^{4}-c^{2}d^{4}\right)x}{4a}}{c\,x^{4}+a}+\frac{\frac{\left(7a^{2}e^{4}-12ac\,d^{2}e^{2}-3c^{2}d^{4}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{8a}}{}\right.}{\left(a\,e^{2}+c\,d^{2}\right)^{3}}$
risch	Expression too large to display

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c/(a*e^2+c*d^2)^3*((1/2*d*c*e*(a*e^2+c*d^2)/a*x^3+1/4*(a^2*e^4-c^2*d^4)/a*
x)/(c*x^4+a)+1/4/a*(1/8*(7*a^2*e^4-12*a*c*d^2*e^2-3*c^2*d^4)*(a/c)^(1/4)/a*
2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1
/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^
(1/4)*x-1))+1/8*(18*a*c*d*e^3+2*c^2*d^3*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(
a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+
2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))+e^4/
(a*e^2+c*d^2)^3*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+9*c*d^2)/d/(e*d
)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7634 vs. 2(680) = 1360.

Time = 109.95 (sec) , antiderivative size = 15292, normalized size of antiderivative = 17.70

$$\int \frac{1}{\left(d+ex^{2}\right)^{2}\left(a+cx^{4}\right)^{2}}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d+ex^2)^2\,(a+cx^4)^2}\,dx$$

$$= \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^4+12\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e^2-7\,(ac^3)^{\frac{1}{4}}\,a^2ce^4-2\,(ac^3)^{\frac{3}{4}}\,cd^3e-18\,(ac^3)^{\frac{3}{4}}\,ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right.\right.}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{8\,\left(\sqrt{2}a^2c^4d^6+3\,\sqrt{2}a^3c^3d^4e^2+3\,\sqrt{2}a^4c^2d^2e^4+\sqrt{2}a^5ce^6\right)}$$

$$+\frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^4+12\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e^2-7\,(ac^3)^{\frac{1}{4}}\,a^2ce^4-2\,(ac^3)^{\frac{3}{4}}\,cd^3e-18\,(ac^3)^{\frac{3}{4}}\,ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{\right.}}{2\,\left(\frac{a}{c}\right)}\right.}{8\,\left(\sqrt{2}a^2c^4d^6+3\,\sqrt{2}a^3c^3d^4e^2+3\,\sqrt{2}a^4c^2d^2e^4+\sqrt{2}a^5ce^6\right)}$$

$$+\frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^4+12\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e^2-7\,(ac^3)^{\frac{1}{4}}\,a^2ce^4+2\,(ac^3)^{\frac{3}{4}}\,cd^3e+18\,(ac^3)^{\frac{3}{4}}\,ade^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)\right.}{16\,\left(\sqrt{2}a^2c^4d^6+3\,\sqrt{2}a^3c^3d^4e^2+3\,\sqrt{2}a^4c^2d^2e^4+\sqrt{2}a^5ce^6\right)}$$

$$-\frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^4+12\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e^2-7\,(ac^3)^{\frac{1}{4}}\,a^2ce^4+2\,(ac^3)^{\frac{3}{4}}\,cd^3e+18\,(ac^3)^{\frac{3}{4}}\,ade^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)\right.}{16\,\left(\sqrt{2}a^2c^4d^6+3\,\sqrt{2}a^3c^3d^4e^2+3\,\sqrt{2}a^4c^2d^2e^4+\sqrt{2}a^5ce^6\right)}$$

$$+\frac{(9\,cd^2e^4+ae^6)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,(c^3d^7+3\,ac^2d^5e^2+3\,a^2cd^3e^4+a^3de^6)\sqrt{de}}$$

$$-\frac{2\,c^2d^2e^2x^5-2\,ace^4x^5+c^2d^3ex^3+acde^3x^3-c^2d^4x+acd^2e^2x-2\,a^2e^4x}{4\,(ac^2d^5+2\,a^2cd^3e^2+a^3de^4)(cex^6+cdx^4+aex^2+ad)}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 7*(a*c^3)^(
1/4)*a^2*c*e^4 - 2*(a*c^3)^(3/4)*c*d^3*e - 18*(a*c^3)^(3/4)*a*d*e^3)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^6
+ 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6
) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 7*(a*c^
3)^(1/4)*a^2*c*e^4 - 2*(a*c^3)^(3/4)*c*d^3*e - 18*(a*c^3)^(3/4)*a*d*e^3)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*
d^6 + 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c
*e^6) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 7*
(a*c^3)^(1/4)*a^2*c*e^4 + 2*(a*c^3)^(3/4)*c*d^3*e + 18*(a*c^3)^(3/4)*a*d*e^
3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^6 + 3*sq
rt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) - 1/
16*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 7*(a*c^3)^(1
/4)*a^2*c*e^4 + 2*(a*c^3)^(3/4)*c*d^3*e + 18*(a*c^3)^(3/4)*a*d*e^3)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^6 + 3*sqrt(2)*a^3*
```

```
c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) + 1/2*(9*c*d^2
*e^4 + a*e^6)*arctan(e*x/sqrt(d*e))/((c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d
^3*e^4 + a^3*d*e^6)*sqrt(d*e)) - 1/4*(2*c^2*d^2*e^2*x^5 - 2*a*c*e^4*x^5 + c
^2*d^3*e*x^3 + a*c*d*e^3*x^3 - c^2*d^4*x + a*c*d^2*e^2*x - 2*a^2*e^4*x)/((a
*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4)*(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d)
)
```

Mupad [B] (verification not implemented)

Time = 17.35 (sec) , antiderivative size = 28923, normalized size of antiderivative = 33.48

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + c*x^4)^2*(d + e*x^2)^2),x)
```

```
[Out] ((x*(2*a^2*e^4 + c^2*d^4 - a*c*d^2*e^2))/(4*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*
d^2*e^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)) + (c*e^2*x^5*(a*e^2 - c*d^2))/(
2*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a*d + a*e*x^2 + c*d*x^4 + c*e*
x^6) + atan(((((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*
d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5
*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254
784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^
2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*
a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*
d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^
5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10
960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9
*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554
176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12
*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 +
56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c
^2*d^6*e^12)) - (x*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1
/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*
c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c
^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*
d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^1
0*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589
824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e
^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^
12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 -
 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*
e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d
^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 +
 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/
```

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2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 15
6*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2)
 - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2)
)/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 +
 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) -
(x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4
 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^1
3*e^10 + 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^
9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*
(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28
*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*
d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d
^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^
5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c
*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13
*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*
d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2))*(-(49*a^4*e^8
*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7
*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(
-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(
-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8
*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*
e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e
^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6
*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18
+ a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*
e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*
a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^
(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^
6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7
*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^
6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a
^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*1i - (((3584*a^10*c^5*e^21 +
 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 +
 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8
*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6
*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c
^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e
^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^2
4 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^
19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736
*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e
^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*
c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^
5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^1
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0*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) + (x*(-(49*a^4*e^8*(-a
^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2
*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7
*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7
*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5
*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)
))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*
c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2
752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*
d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*
a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2
*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a
^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d
^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12
*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*
e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2
) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6
*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^
6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*
c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*
a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12
+ 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5
*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^
11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*
e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(
-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7
*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c
^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c
^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*
e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a
^11*c^2*d^4*e^8)))^(1/2))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*
c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 4
04*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*
(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a
^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 +
 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(81*c^13*d^14*e^5
- 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636
*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575
*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14
+ 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c
^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(
-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c
^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a
^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a
^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c
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^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^
8)))^(1/2)*1i)/((((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^
13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*
a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1
254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12
*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 +
56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c
^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992
*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 +
 10960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*
c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5
554176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a
^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4
 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^1
0*c^2*d^6*e^12)) - (x*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)
^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a
^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^
7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c
^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*
a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 +
589824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^1
8*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512
*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^1
9 - 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d
^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^
7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^
8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^
(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 -
 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1
/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1
/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^
2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)
 - (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*
e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*
d^13*e^10 + 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056
*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(1
28*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 +
 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c
^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^
4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4
*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^
3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a
^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c
^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2))*(-(49*a^4*
e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*
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a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^
2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^
4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*
a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d
^4*e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^1
2*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*
d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^
18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^
14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 +
28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^
3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404
*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-
a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7
*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 2
0*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (((3584*a^10*c^5*e^21 +
 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 +
 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8
*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6
*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c
^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e
^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^2
4 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^
19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736
*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e
^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*
c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^
5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^1
0*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) + (x*(-(49*a^4*e^8*(-a
^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2
*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7
*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7
*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5
*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)
))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*
c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2
752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*
d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*
a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2
*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a
^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d
^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12
*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*
e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2
) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6
*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^
```

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6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*
c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*
a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12
+ 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5
*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^
11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*
e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(
-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7
*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c
^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c
^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*
e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a
^11*c^2*d^4*e^8)))^(1/2))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*
c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 4
04*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*
(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a
^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 +
 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(81*c^13*d^14*e^5
- 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636
*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575
*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14
+ 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c
^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(
-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c
^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a
^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a
^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c
^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^
8)))^(1/2) - (729*c^11*d^9*e^8 + 2916*a*c^10*d^7*e^10 + 2009*a^4*c^7*d*e^16
 - 2538*a^2*c^9*d^5*e^12 + 17764*a^3*c^8*d^3*e^14)/(256*(a^4*c^8*d^18 + a^1
2*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 +
 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*
c^2*d^6*e^12))))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2
) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^
3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3
)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^
12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*
c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*2i + atan(((((3584*a^10*c^5*e^21
 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7
 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d
^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c
^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5
*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10
*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e
^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*
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d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 254607
36*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9
*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^1
4*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*
a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d
^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (x*((49*a^4*e^8*(-
a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^
2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^
7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^
7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^
5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8
)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8
*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 +
2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8
*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824
*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^
2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*
a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*
d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12
*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*
e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2
) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6
*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^
6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*
c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*
a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12
+ 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5
*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^
11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*
e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(
(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*
e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^
3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^
2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e
^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^
11*c^2*d^4*e^8)))^(1/2))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^
3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404
*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-
a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7
*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 2
0*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 -
392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a
^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a
^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 +
8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4
```

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*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^
7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*
d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*
c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*
c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*
d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8))
)^(1/2)*1i - (((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*
d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5
*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254
784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^
2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*
a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*
d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^
5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10
960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9
*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554
176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12
*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 +
56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c
^2*d^6*e^12)) + (x*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/
2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c
^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^
3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d
^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10
*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 5898
24*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^
9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^1
2*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 -
2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e
^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^
16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 +
56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2)
 + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*
a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) -
 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/
(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 1
5*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x
*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 -
 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*
e^10 + 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*
c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a
^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a
^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^
8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*
(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e
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^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^
2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^
12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8
*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2))*((49*a^4*e^8*(-a
^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2
*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7
*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7
*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5
*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)
))^(1/2) + (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 +
 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^1
3 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^
12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4
+ 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10
*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2)
 + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3
*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)
^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^1
2 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c
^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*1i)/((((3584*a^10*c^5*e^21 + 1152
*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 2968
32*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13
 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*
e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^
16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 +
56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 2
4576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^
6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736*a^9*
c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 +
 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d
^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7
*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8
 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)) - (x*((49*a^4*e^8*(-a^7*c^3
)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7
 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^
(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^
(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*
e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/
2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d
^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512
*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e
^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c
^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16
+ 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5
*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^1
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2)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^
5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 6
8*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*
a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c
*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 +
 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^
21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^1
2*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^11*e^12 + 31554
56*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 -
 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^
4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 7
0*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12))*((49*a^4
*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252
*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e
^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e
^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6
*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*
d^4*e^8)))^(1/2))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2
) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^
3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3
)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^
12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*
c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7
*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10
*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*
d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c
^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e
^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^
(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 +
 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1
/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1
/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^
2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)
 + (((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 +
 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10
*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^
7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8
*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^
12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12))
 - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^5*c^14*d^2
1*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10960896*a^8
*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9*d^11*e^14
 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554176*a^13*c
^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16
+ 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5
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*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^1
2)) + (x*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^
4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5
 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) +
 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^
12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e
^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^1
4*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 589824
0*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12
*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^
15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(128
*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 2
8*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3
*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d
^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^
5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c
*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13
*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*
d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) + (x*(4096*a^1
2*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4
*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 221
9776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^
16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^1
8 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^1
4*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 2
8*a^10*c^2*d^6*e^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)
^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a
^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^
7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c
^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*
a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2))*((49*a^4*e^8*(-a^7*c^3)^(1
/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 1
56*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2
) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2
))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2
 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) +
 (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2
*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a
^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^1
6 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c
^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e
^12)))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) + 12*a^4*
c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*d^5*e^3 + 404*a^6*c^3*d^3*e^5 +
 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 3
0*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12
```

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*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6
 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (729*c^11*d^9*e^8 + 2916*a*c^10*d^7*e^10
+ 2009*a^4*c^7*d*e^16 - 2538*a^2*c^9*d^5*e^12 + 17764*a^3*c^8*d^3*e^14)/(25
6*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 +
28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^
3*d^8*e^10 + 28*a^10*c^2*d^6*e^12))))*((49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4
*d^8*(-a^7*c^3)^(1/2) + 12*a^4*c^5*d^7*e - 252*a^7*c^2*d*e^7 + 156*a^5*c^4*
d^5*e^3 + 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3
*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^
13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^
4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*2i + (atan((
(((x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^
2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*
a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^
16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*
c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*
e^12)) - (((7*a^10*c^5*e^21 + (9*a*c^14*d^18*e^3)/4 + (103*a^2*c^13*d^16*e^
5)/4 + (429*a^3*c^12*d^14*e^7)/4 + (2319*a^4*c^11*d^12*e^9)/4 + (10019*a^5*
c^10*d^10*e^11)/4 + (6009*a^6*c^9*d^8*e^13)/4 - (11105*a^7*c^8*d^6*e^15)/4
- (9803*a^8*c^7*d^4*e^17)/4 - 174*a^9*c^6*d^2*e^19)/(a^4*c^8*d^18 + a^12*d^
2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*
a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*
d^6*e^12) + ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*((x*(4096*a^12*c^5*d*e^22 -
 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6
 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*
d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^
10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e
^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7
*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6
*e^12)) - (((128*a^15*c^4*d*e^24 - 48*a^4*c^15*d^23*e^2 - 416*a^5*c^14*d^21
*e^4 - 688*a^6*c^13*d^19*e^6 + 3840*a^7*c^12*d^17*e^8 + 21408*a^8*c^11*d^15
*e^10 + 49728*a^9*c^10*d^13*e^12 + 67872*a^10*c^9*d^11*e^14 + 58752*a^11*c^
8*d^9*e^16 + 32400*a^12*c^7*d^7*e^18 + 10848*a^13*c^6*d^5*e^20 + 1936*a^14*
c^5*d^3*e^22)/(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7
*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8
 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12) - (x*(a*e^2 + 9*c*d^2)*(-d^3
*e^7)^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a
^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11
+ 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c
^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 5898
24*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(512*(c^3*d^9 + a^3*d^3*e^
6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^
11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*
e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(
a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*
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e^2 + 3*a^2*c*d^5*e^4))))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a
^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e
^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2
)*1i)/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)) + (((
x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c
^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5
*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16
+ 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5
*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^1
2)) + (((7*a^10*c^5*e^21 + (9*a*c^14*d^18*e^3)/4 + (103*a^2*c^13*d^16*e^5)/
4 + (429*a^3*c^12*d^14*e^7)/4 + (2319*a^4*c^11*d^12*e^9)/4 + (10019*a^5*c^1
0*d^10*e^11)/4 + (6009*a^6*c^9*d^8*e^13)/4 - (11105*a^7*c^8*d^6*e^15)/4 - (
9803*a^8*c^7*d^4*e^17)/4 - 174*a^9*c^6*d^2*e^19)/(a^4*c^8*d^18 + a^12*d^2*e
^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7
*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6
*e^12) - ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*((x*(4096*a^12*c^5*d*e^22 - 11
52*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c^13*d^17*e^6 -
140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 2219776*a^7*c^10*d^1
1*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16 + 362368*a^10*
c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16
 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^
5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^
12)) + (((128*a^15*c^4*d*e^24 - 48*a^4*c^15*d^23*e^2 - 416*a^5*c^14*d^21*e^
4 - 688*a^6*c^13*d^19*e^6 + 3840*a^7*c^12*d^17*e^8 + 21408*a^8*c^11*d^15*e^
10 + 49728*a^9*c^10*d^13*e^12 + 67872*a^10*c^9*d^11*e^14 + 58752*a^11*c^8*d
^9*e^16 + 32400*a^12*c^7*d^7*e^18 + 10848*a^13*c^6*d^5*e^20 + 1936*a^14*c^5
*d^3*e^22)/(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^
16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 +
56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12) + (x*(a*e^2 + 9*c*d^2)*(-d^3*e^
7)^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5 + 2293760*a^8*
c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^11*d^16*e^11 + 2
752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 5898240*a^13*c^8*
d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6*e^21 - 589824*
a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(512*(c^3*d^9 + a^3*d^3*e^6 +
 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*
c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6
 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(a*e
^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2
 + 3*a^2*c*d^5*e^4))))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*
c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6
+ 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*1
i)/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))/(((729*
c^11*d^9*e^8)/256 + (729*a*c^10*d^7*e^10)/64 + (2009*a^4*c^7*d*e^16)/256 -
(1269*a^2*c^9*d^5*e^12)/128 + (4441*a^3*c^8*d^3*e^14)/64)/(a^4*c^8*d^18 + a
^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4
```

```
 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^1
0*c^2*d^6*e^12) + (((x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d
^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^
9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*
d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*
d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10
 + 28*a^10*c^2*d^6*e^12)) - (((7*a^10*c^5*e^21 + (9*a*c^14*d^18*e^3)/4 + (10
3*a^2*c^13*d^16*e^5)/4 + (429*a^3*c^12*d^14*e^7)/4 + (2319*a^4*c^11*d^12*e^
9)/4 + (10019*a^5*c^10*d^10*e^11)/4 + (6009*a^6*c^9*d^8*e^13)/4 - (11105*a^
7*c^8*d^6*e^15)/4 - (9803*a^8*c^7*d^4*e^17)/4 - 174*a^9*c^6*d^2*e^19)/(a^4*
c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*
c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e
^10 + 28*a^10*c^2*d^6*e^12) + ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*((x*(4096
*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336
*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 +
 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^
7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8
*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6
*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10
 + 28*a^10*c^2*d^6*e^12)) - (((128*a^15*c^4*d*e^24 - 48*a^4*c^15*d^23*e^2 -
 416*a^5*c^14*d^21*e^4 - 688*a^6*c^13*d^19*e^6 + 3840*a^7*c^12*d^17*e^8 + 2
1408*a^8*c^11*d^15*e^10 + 49728*a^9*c^10*d^13*e^12 + 67872*a^10*c^9*d^11*e^
14 + 58752*a^11*c^8*d^9*e^16 + 32400*a^12*c^7*d^7*e^18 + 10848*a^13*c^6*d^5
*e^20 + 1936*a^14*c^5*d^3*e^22)/(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^
4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 7
0*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12) - (x*(a*e^
2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^
22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^
10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^1
5 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c
^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(512*(c^
3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)*(a^4*c^8*d^18 + a^
12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4
+ 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10
*c^2*d^6*e^12)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*
e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4))))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a
*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c
^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^
2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*
d^5*e^4)) - (((x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^
7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*
e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 +
 a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e
^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a
^10*c^2*d^6*e^12)) + (((7*a^10*c^5*e^21 + (9*a*c^14*d^18*e^3)/4 + (103*a^2*
```

```
c^13*d^16*e^5)/4 + (429*a^3*c^12*d^14*e^7)/4 + (2319*a^4*c^11*d^12*e^9)/4 +
 (10019*a^5*c^10*d^10*e^11)/4 + (6009*a^6*c^9*d^8*e^13)/4 - (11105*a^7*c^8*
d^6*e^15)/4 - (9803*a^8*c^7*d^4*e^17)/4 - 174*a^9*c^6*d^2*e^19)/(a^4*c^8*d^
18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^
14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 +
28*a^10*c^2*d^6*e^12) - ((a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*((x*(4096*a^12*
c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4 - 78336*a^4*c
^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^13*e^10 + 22197
76*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^9*c^8*d^7*e^16
 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*(a^4*c^8*d^18
+ a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*
e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*
a^10*c^2*d^6*e^12)) + (((128*a^15*c^4*d*e^24 - 48*a^4*c^15*d^23*e^2 - 416*a
^5*c^14*d^21*e^4 - 688*a^6*c^13*d^19*e^6 + 3840*a^7*c^12*d^17*e^8 + 21408*a
^8*c^11*d^15*e^10 + 49728*a^9*c^10*d^13*e^12 + 67872*a^10*c^9*d^11*e^14 + 5
8752*a^11*c^8*d^9*e^16 + 32400*a^12*c^7*d^7*e^18 + 10848*a^13*c^6*d^5*e^20
+ 1936*a^14*c^5*d^3*e^22)/(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14
 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*
c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12) + (x*(a*e^2 + 9*
c*d^2)*(-d^3*e^7)^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589824*a^7*c^14*d^22*e^5
 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e^9 + 5898240*a^10*c^1
1*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^12*c^9*d^12*e^15 - 58
98240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 - 2293760*a^15*c^6*d^6
*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*e^25))/(512*(c^3*d^9
+ a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)*(a^4*c^8*d^18 + a^12*d^2
*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a
^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d
^6*e^12)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 +
3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4))))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d
^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2))/(4*(c^3*d^9
 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4)))*(a*e^2 + 9*c*d^2)*(-d
^3*e^7)^(1/2))/(4*(c^3*d^9 + a^3*d^3*e^6 + 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^
4))))*(a*e^2 + 9*c*d^2)*(-d^3*e^7)^(1/2)*1i)/(2*(c^3*d^9 + a^3*d^3*e^6 + 3*
a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4))
```

3.150 $\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 388

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{4de(5cd^2-3ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{4\sqrt[4]{a}de(5cd^2-3ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} + \frac{\left(105c^2d^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2-252a^{3/2}\sqrt{c}de^3+25a^2e^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}$$

```
[Out] 1/21*e^2*(-5*a*e^2+42*c*d^2)*x*(c*x^4+a)^(1/2)/c^2+4/5*d*e^3*x^3*(c*x^4+a)^
(1/2)/c+1/7*e^4*x^5*(c*x^4+a)^(1/2)/c+4/5*d*e*(-3*a*e^2+5*c*d^2)*x*(c*x^4+a
)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-4/5*a^(1/4)*d*e*(-3*a*e^2+5*c*d^2)*(c
os(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*E
llipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2)
)*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)+1/210*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(105*c^2*d^4-210*a*
c*d^2*e^2+25*a^2*e^4+420*c^(3/2)*d^3*e*a^(1/2)-252*a^(3/2)*d*e^3*c^(1/2))*(
a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(9
/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1221, 1902, 1212, 226, 1210}

$$\begin{aligned}
\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx
&= \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(5(5a^2e^4-42acd^2e^2+21c^2d^4)+84\sqrt{a}\sqrt{c}de(5cd^2-3ae^2)\right)\operatorname{EllipticF}\Bigg(2\arctan}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}} \\
&\quad - \frac{4\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(5cd^2-3ae^2)\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} \\
&\quad + \frac{4dex\sqrt{a+cx^4}(5cd^2-3ae^2)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+cx^4}(42cd^2-5ae^2)}{21c^2} \\
&\quad + \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c}
\end{aligned}$$

```
[In] Int[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*(42*c*d^2 - 5*a*e^2)*x*Sqrt[a + c*x^4])/(21*c^2) + (4*d*e^3*x^3*Sqrt[a
 + c*x^4])/(5*c) + (e^4*x^5*Sqrt[a + c*x^4])/(7*c) + (4*d*e*(5*c*d^2 - 3*a*
e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (4*a^(1/4)*d*
e*(5*c*d^2 - 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sq
rt[a + c*x^4]) + ((84*Sqrt[a]*Sqrt[c]*d*e*(5*c*d^2 - 3*a*e^2) + 5*(21*c^2*d
^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
210*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{\int \frac{7cd^4+28cd^3ex^2+e^2(42cd^2-5ae^2)x^4+28cde^3x^6}{\sqrt{a+cx^4}}\,dx}{7c}\\
&= \frac{4de^3x^3\sqrt{a+cx^4}}{5c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{\int \frac{35c^2d^4+28cde(5cd^2-3ae^2)x^2+5ce^2(42cd^2-5ae^2)x^4}{\sqrt{a+cx^4}}\,dx}{35c^2}\\
&= \frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}\\
&\quad+\frac{\int \frac{5c(21c^2d^4-42acd^2e^2+5a^2e^4)+84c^2de(5cd^2-3ae^2)x^2}{\sqrt{a+cx^4}}\,dx}{105c^3}\\
&= \frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c}\\
&\quad+\frac{e^4x^5\sqrt{a+cx^4}}{7c}-\frac{\left(4\sqrt{a}de(5cd^2-3ae^2)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}}\\
&\quad+\frac{\left(105c^2d^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2-252a^{3/2}\sqrt{c}de^3+25a^2e^4\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{105c^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c}\\
&\quad+\frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{4de(5cd^2-3ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&\quad-\frac{4\sqrt[4]{a}de(5cd^2-3ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}\\
&\quad+\frac{\left(105c^2d^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2-252a^{3/2}\sqrt{c}de^3+25a^2e^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\bigg(}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.46

$$
\int\frac{\left(d+ex^2\right)^4}{\sqrt{a+cx^4}}\,dx
$$

$$
=\frac{5(21c^2d^4-42acd^2e^2+5a^2e^4)\,x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex\bigg(-e(a+cx^4)\left(25ae^2-\right.}{105c^2\sqrt{a+}}
$$

```
[In] Integrate[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

```
[Out] (5*(21*c^2*d^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x*(-(e*(a + c*x^4)*(25*a*e^2 - 3
*c*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4))) + 28*c*d*(5*c*d^2 - 3*a*e^2)*x^2*Sqr
t[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(105*c^2*
Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.74

method	result
elliptic	$\frac{e^{4} x^{5} \sqrt{c\,x^{4}+a}}{7 c}+\frac{4 d\,e^{3} x^{3} \sqrt{c\,x^{4}+a}}{5 c}+\frac{\left(6 e^{2} d^{2}-\frac{5 e^{4} a}{7 c}\right) x \sqrt{c\,x^{4}+a}}{3 c}+\frac{\left(d^{4}-\frac{\left(6 e^{2} d^{2}-\frac{5 e^{4} a}{7 c}\right) a}{3 c}\right) \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right.}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}$
risch	$-\frac{e^{2} x \left(-15 e^{2} x^{4} c-84 d e\,x^{2} c+25 a\,e^{2}-210 c\,d^{2}\right) \sqrt{c\,x^{4}+a}}{105 c^{2}}+\frac{\frac{25 a^{2} e^{4} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{105 c^{2} d^{4} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}}{\sqrt{}}}}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+}}}{}$
default	$\frac{d^{4} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+e^{4} \left(\frac{x^{5} \sqrt{c\,x^{4}+a}}{7 c}-\frac{5 a x \sqrt{c\,x^{4}+a}}{21 c^{2}}+\frac{5 a^{2} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{21 c^{2} \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}\right)$

```
[In] int((e*x^2+d)^4/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*e^4*x^5*(c*x^4+a)^(1/2)/c+4/5*d*e^3*x^3*(c*x^4+a)^(1/2)/c+1/3*(6*e^2*d^
2-5/7*e^4/c*a)/c*x*(c*x^4+a)^(1/2)+(d^4-1/3*(6*e^2*d^2-5/7*e^4/c*a)/c*a)/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2
)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*(4*
d^3*e-12/5*d*e^3/c*a)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I
))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{84\,(5\,acd^3e-3\,a^2de^3)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}\,E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+(105\,c^2d^4-420\,acd^3e-210\,acd^2e^2+252\,a^2d}{}$$

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(84*(5*a*c*d^3*e - 3*a^2*d*e^3)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arc
sin((-a/c)^(1/4)/x), -1) + (105*c^2*d^4 - 420*a*c*d^3*e - 210*a*c*d^2*e^2 +
 252*a^2*d*e^3 + 25*a^2*e^4)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c
)^(1/4)/x), -1) + (15*a*c*e^4*x^6 + 84*a*c*d*e^3*x^4 + 420*a*c*d^3*e - 252*
a^2*d*e^3 + 5*(42*a*c*d^2*e^2 - 5*a^2*e^4)*x^2)*sqrt(c*x^4 + a))/(a*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{d^4x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{d^3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3d^2e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{de^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \frac{e^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

```
[In] integrate((e*x**2+d)**4/(c*x**4+a)**(1/2),x)
```

```
[Out] d**4*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + d**3*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4
*exp_polar(I*pi)/a)/(sqrt(a)*gamma(7/4)) + 3*d**2*e**2*x**5*gamma(5/4)*hype
r((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + d*
e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(
sqrt(a)*gamma(11/4)) + e**4*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^4}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^4}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \int \frac{(e\,x^2+d)^4}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int((d + e*x^2)^4/(a + c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^4/(a + c*x^4)^(1/2), x)
```

3.151 $\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 326

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}+\frac{3e(5cd^2-ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{3\sqrt[4]{a}e(5cd^2-ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}$$

$$+\frac{\sqrt[4]{a}\left(15cd^2e-3ae^3+\frac{5\sqrt{c}d(cd^2-ae^2)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}$$

```
[Out] d*e^2*x*(c*x^4+a)^(1/2)/c+1/5*e^3*x^3*(c*x^4+a)^(1/2)/c+3/5*e*(-a*e^2+5*c*d
^2)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-3/5*a^(1/4)*e*(-a*e^2+5
*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a
^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x
^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/
2)+1/10*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*c^(1/2))*(15*c*d^2*e-3*a*e^3+5*d*(-a*e^2+c*d^2)*c^(1/2)/a^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1221, 1902, 1212, 226, 1210}

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = -\frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(5cd^2-ae^2)\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{5\sqrt{c}d(cd^2-ae^2)}{\sqrt{a}}-3ae^3+15cd^2e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}$$
$$+\frac{3ex\sqrt{a+cx^4}(5cd^2-ae^2)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

```
[In] Int[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (d*e^2*x*Sqrt[a + c*x^4])/c + (e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (3*e*(5*c*d
^2 - a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*a^(
1/4)*e*(5*c*d^2 - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)
*Sqrt[a + c*x^4]) + (a^(1/4)*(15*c*d^2*e - 3*a*e^3 + (5*Sqrt[c]*d*(c*d^2 -
a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a
 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^3x^3\sqrt{a+cx^4}}{5c}+\frac{\int \frac{5cd^3+3e(5cd^2-ae^2)x^2+15cde^2x^4}{\sqrt{a+cx^4}}\,dx}{5c}\\
&= \frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}+\frac{\int \frac{15cd(cd^2-ae^2)+9ce(5cd^2-ae^2)x^2}{\sqrt{a+cx^4}}\,dx}{15c^2}\\
&= \frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}-\frac{\left(3\sqrt{a}e(5cd^2-ae^2)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}}\\
&\quad+\frac{\left(5\sqrt{c}d(cd^2-ae^2)+3\sqrt{a}e(5cd^2-ae^2)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}}\\
&= \frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}+\frac{3e(5cd^2-ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&\quad-\frac{3\sqrt[4]{a}e(5cd^2-ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}\\
&\quad+\frac{\left(5\sqrt{c}d(cd^2-ae^2)+3\sqrt{a}e(5cd^2-ae^2)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10\sqrt[4]{a}c^{7/4}\sqrt{a+cx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.43

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{5d(cd^2-ae^2)\,x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex\left(e(5d+ex^2)\,(a+cx^4)+(5cd^2-ae^2)\right.}{5c\sqrt{a+cx^4}}$$

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (5*d*(c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
 -((c*x^4)/a)] + e*x*(e*(5*d + e*x^2)*(a + c*x^4) + (5*c*d^2 - a*e^2)*x^2*S
qrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(5*c*Sq
rt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{e^3x^3\sqrt{cx^4+a}}{5c}+\frac{de^2x\sqrt{cx^4+a}}{c}+\frac{\left(d^3-\frac{de^2a}{c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}}+\frac{i\left(3d^2e-\frac{3e^3a}{5c}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{}}$
risch	$\frac{e^2x(ex^2+5d)\sqrt{cx^4+a}}{5c}-\frac{-\frac{5d^3c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}}+\frac{5de^2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}}+\frac{i\left(3ae^3-15cd^2e\right)}{}}{5c}$
default	$\frac{d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}}+e^3\left(\frac{x^3\sqrt{cx^4+a}}{5c}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5c^{\frac{3}{2}}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}}\right.$

```
[In] int((e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e^3*x^3*(c*x^4+a)^(1/2)/c+d*e^2*x*(c*x^4+a)^(1/2)/c+(d^3-d*e^2/c*a)/(I/
a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)
*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*(3*d
^2*e-3/5*e^3/c*a)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{3\,(5\,acd^2e-a^2e^3)\sqrt{c}x\big(-\frac{a}{c}\big)^{\frac{3}{4}}\,E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+(5\,c^2d^3-15\,acd^2e-5\,acde^2+3\,a^2e^3)\sqrt{c}x\big(-\frac{a}{c}\big)^{\frac{3}{4}}}{5\,ac^2x}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(3*(5*a*c*d^2*e - a^2*e^3)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a
/c)^(1/4)/x), -1) + (5*c^2*d^3 - 15*a*c*d^2*e - 5*a*c*d*e^2 + 3*a^2*e^3)*sq
rt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + (a*c*e^3*x^4
+ 5*a*c*d*e^2*x^2 + 15*a*c*d^2*e - 3*a^2*e^3)*sqrt(c*x^4 + a))/(a*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.53

$$\begin{aligned}\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx &= \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}\\ &\quad+\frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\,\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\,\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}\end{aligned}$$

```
[In] integrate((e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)*hyp
er((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e
**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4
*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(e\,x^2+d)^3}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int((d + e*x^2)^3/(a + c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^3/(a + c*x^4)^(1/2), x)
```

3.152 $\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 264

$$\begin{aligned}
&\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+cx^4}}\,dx \\
&= \frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad -\frac{2\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\
&\quad +\frac{\left(3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}
\end{aligned}$$

```
[Out] 1/3*e^2*x*(c*x^4+a)^(1/2)/c+2*d*e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^
(1/2))-2*a^(1/4)*d*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/
(c*x^4+a)^(1/2)+1/6*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*(3*c*d^2-a*e^2+6*d*e*a^(1/2)*c^(1/2))*((c*x^4+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1221, 1212, 226, 1210}

$$\begin{aligned}
&\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx \\
&= \frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}} \\
&\quad - \frac{2\sqrt[4]{a}de(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\
&\quad + \frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\,(\sqrt{a}+\sqrt{c}x^2)} + \frac{e^2x\sqrt{a+cx^4}}{3c}
\end{aligned}$$

```
[In] Int[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(3*c) + (2*d*e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) - (2*a^(1/4)*d*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
c^(3/4)*Sqrt[a + c*x^4]) + ((3*c*d^2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{\int \frac{3cd^2-ae^2+6cdex^2}{\sqrt{a+cx^4}}\,dx}{3c}\\
&= \frac{e^2x\sqrt{a+cx^4}}{3c}-\frac{\left(2\sqrt{a}de\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}+\frac{\left(3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3c}\\
&= \frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&\quad -\frac{2\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}\\
&\quad +\frac{\left(3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.45

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{\left(3cd^2-ae^2\right)x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex\left(e\left(a+cx^4\right)+2cdx^2\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeon}\right.}{3c\sqrt{a+cx^4}}$$

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] ((3*c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -
((c*x^4)/a)] + e*x*(e*(a + c*x^4) + 2*c*d*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{e^2x\sqrt{c\,x^4+a}}{3c}+\frac{\left(d^2-\frac{a\,e^2}{3c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{2ied\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.\right.}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$
default	$\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+e^2\left(\frac{x\sqrt{c\,x^4+a}}{3c}-\frac{a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+\frac{2ied\sqrt{a}\,\sqrt{1-}}{}$
risch	$\frac{e^2x\sqrt{c\,x^4+a}}{3c}-\frac{\frac{a\,e^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{3c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{6id\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3c}$

```
[In] int((e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*x*(c*x^4+a)^(1/2)/c+(d^2-1/3*a/c*e^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2
)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+2*I*e*d*a^(1/2)/(I/a^(1/2)*c^(1/
2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(
c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(
x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.43

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{6\,a\sqrt{c}dex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+(3\,cd^2-6\,ade-ae^2)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+(}{3\,acx}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(6*a*sqrt(c)*d*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1)
+ (3*c*d^2 - 6*a*d*e - a*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/
c)^(1/4)/x), -1) + (a*e^2*x^2 + 6*a*d*e)*sqrt(c*x^4 + a))/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate((e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*ex
p_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/2,
5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx=\int \frac{{\left(e\,x^2+d\right)}^2}{\sqrt{c\,x^4+a}} \,d x$$

```
[In] int((d + e*x^2)^2/(a + c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^2/(a + c*x^4)^(1/2), x)
```

3.153 $\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*a
rctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d
*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+
a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1212, 226, 1210}

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = {} & \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}} \\ & -\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\ & +\frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}\end{aligned}$$

```
[In] Int[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sq
rt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sq
rt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\frac{\left(\sqrt{a}e\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}+\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx\\&=\frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}\\&\quad+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.34

$$\int\frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\frac{\sqrt{1+\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{a+cx^4}}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[a + c*x^4],x]
[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^4]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$	169
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$	169

```
[In] int((e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I
*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.39

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{a\sqrt{c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+(cd-ae)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+\sqrt{cx^4+a}ae}{acx}$$

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*sqrt(c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) + (c*d -
 a*e)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + sqrt(
c*x^4 + a)*a*e)/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_pol
ar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int((d + e*x^2)/(a + c*x^4)^(1/2),x)
[Out] int((d + e*x^2)/(a + c*x^4)^(1/2), x)
```

3.154 $\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 334

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx &= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2+ae^2}} \\
&+ \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}} \\
&- \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\sqrt{a+cx^4}}
\end{aligned}
$$

```
[Out] 1/2*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))*e^(1/2)/d
^(1/2)/(a*e^2+c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)-1/4*a^(3/4)*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^
(1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((
c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(c*x^4+a)^
(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1231, 226, 1721}

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}d\sqrt{a+cx^4}\,(cd^2-ae^2)} + \frac{\sqrt{e}\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{ae^2+cd^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

```
[In] Int[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])
/(2*Sqrt[d]*Sqrt[c*d^2 + a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) - (a^(3/4)*((S
qrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c
]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*S
qrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
```

```
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e} - \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e}\\ &= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2+ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}}\\ &\quad - \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.28

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{i\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d}, i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right), -1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}d\sqrt{a+cx^4}}$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[a +
c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	107
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	107

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I
*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.155 $\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 581

$$\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx &= -\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&+ \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt{e}\left(3cd^2+ae^2\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\left(cd^2+ae^2\right)^{3/2}} \\
&+ \frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\left(cd^2+ae^2\right)\sqrt{a+cx^4}} \\
&+ \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}} \\
&- \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(3cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\sqrt{a+cx^4}}
\end{aligned}$$

```
[Out] 1/4*(a*e^2+3*c*d^2)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^
(1/2))*e^(1/2)/d^(3/2)/(a*e^2+c*d^2)^(3/2)+1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e
^2+c*d^2)/(e*x^2+d)-1/2*e*x*c^(1/2)*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(a^(1/2
)+x^2*c^(1/2))+1/2*a^(1/4)*c^(1/4)*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2
)^(1/2)/d/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x
/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)-1/8
```

```
*(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1
/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4
)/d^2/(a*e^2+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1238, 1729, 1210, 1723, 226, 1721}

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = {} & \frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\sqrt{a+cx^4}\,(ae^2+cd^2)} \\
& - \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)(ae^2+3cd^2)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{c}d^2\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)(ae^2+cd^2)} \\
& + \frac{\sqrt{e}(ae^2+3cd^2)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\left(ae^2+cd^2\right)^{3/2}} \\
& + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)} \\
& + \frac{e^2x\sqrt{a+cx^4}}{2d\,(d+ex^2)\,(ae^2+cd^2)} - \frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)(ae^2+cd^2)}
\end{aligned}
$$

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -1/2*(Sqrt[c]*e*x*Sqrt[a + c*x^4])/(d*(c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^
2)) + (e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (Sqrt[e]*
(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a +
c*x^4])])/(4*d^(3/2)*(c*d^2 + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (c^(1
/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[
a]*e)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 + a*e^2)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/
4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)
*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
```

```
&& NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{-2cd^2-ae^2+2cdex^2+ce^2x^4}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{2d\left(cd^2+ae^2\right)} \\
&= \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{\sqrt{a}c^{3/2}de^2+ce\left(-2cd^2-ae^2\right)+\left(2c^2de^2-ce^2\left(cd-\sqrt{a}\sqrt{c}e\right)\right)x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{2cde\left(cd^2+ae^2\right)} \\
&\quad + \frac{\left(\sqrt{a}\sqrt{c}e\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2d\left(cd^2+ae^2\right)} \\
&= -\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} \\
&\quad + \frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\left(cd^2+ae^2\right)\sqrt{a+cx^4}} \\
&\quad + \frac{\sqrt{c}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{d\left(\sqrt{c}d-\sqrt{a}e\right)} - \frac{\left(\sqrt{a}e(3cd^2+ae^2)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{2d\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)} \\
&= -\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} \\
&\quad + \frac{\sqrt{e}(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\left(cd^2+ae^2\right)^{3/2}} \\
&\quad + \frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\left(cd^2+ae^2\right)\sqrt{a+cx^4}} \\
&\quad + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}} \\
&\quad - \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(3cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.59 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$$

$$= \frac{a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}de^2x + \sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cde^2x^5 - \sqrt{a}\sqrt{c}de(d+ex^2)\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) + \sqrt{c}d\left(i\sqrt{c}d+\sqrt{a}e\right.}{}$$

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e^2*x + Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*e^2*x^
5 - Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(I*Sqrt[c]*d + Sqrt[a]*e)*(d
 + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]
*x], -1] - (3*I)*c*d^3*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqr
t[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*d*e^2*Sqrt[1 + (
c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c
])/Sqrt[a]]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I
)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a
*e^3*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*Arc
Sinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d^2*(c
*d^2 + a*e^2)*(d + e*x^2)*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.96

method	result
default	$\frac{e^2x\sqrt{c\,x^4+a}}{2d(a\,e^2+c\,d^2)(e\,x^2+d)} - \frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} - \frac{i\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2d(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} + \frac{i\sqrt{c}\,e\sqrt{a}}{}$
elliptic	$\frac{e^2x\sqrt{c\,x^4+a}}{2d(a\,e^2+c\,d^2)(e\,x^2+d)} - \frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} - \frac{i\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2d(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} + \frac{i\sqrt{c}\,e\sqrt{a}}{}$

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2*c/(a*e^2+c*d^2)/(I/
a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)
*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I*
c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2)/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2/d^2
/(a*e^2+c*d^2)*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c
^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c
^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticP
i(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1
/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)^2}\,dx$$

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(e\,x^2+d)^2}\,dx$$

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2),x)
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

3.156 $\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 729

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx &= -\frac{3\sqrt{c}e(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2}\\
&+\frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}+\frac{3\sqrt{e}(5c^2d^4+2acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\left(cd^2+ae^2\right)^{5/2}}\\
&+\frac{3\sqrt[4]{a}\sqrt[4]{c}e(3cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}}\\
&+\frac{\sqrt[4]{c}\left(4cd^2-\sqrt{a}\sqrt{c}de+3ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\sqrt{a+cx^4}}\\
&-\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)\left(5c^2d^4+2acd^2e^2+a^2e^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\right.\right.}{32\sqrt[4]{a}\sqrt[4]{c}d^3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}}
\end{aligned}$$

```
[Out] 3/16*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)
/e^(1/2)/(c*x^4+a)^(1/2))*e^(1/2)/d^(5/2)/(a*e^2+c*d^2)^(5/2)+1/4*e^2*x*(c*
x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^2)*x*(c*x^4+a
)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-3/8*e*(a*e^2+3*c*d^2)*x*c^(1/2)*(c*x^
4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(a^(1/2)+x^2*c^(1/2))+3/8*a^(1/4)*c^(1/4)*e*
(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/d^2/(a*e^2+c*
d^2)^2/(c*x^4+a)^(1/2)-3/32*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(s
```

```
in(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c
^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^3/(a*e^2+c*d^2)^2/(-e*a^(1
/2)+d*c^(1/2))/(c*x^4+a)^(1/2)+1/8*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(4*c*d^2+3*a*e^2-d*e*a^(1/2)
*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d^2/(a*e^2+c*d^
2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1238, 1711, 1729, 1210, 1723, 226, 1721}

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx=\frac{3\sqrt{e}\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\left(ae^2+cd^2\right)^{5/2}}$$

$$-\frac{3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\right.\right.}{32\sqrt[4]{a}\sqrt[4]{c}d^3\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)^2}$$

$$+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}d^2\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)}$$

$$+\frac{3\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^2+3cd^2\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2\sqrt{a+cx^4}\left(ae^2+cd^2\right)^2}$$

$$+\frac{3e^2x\sqrt{a+cx^4}\left(ae^2+3cd^2\right)}{8d^2\left(d+ex^2\right)\left(ae^2+cd^2\right)^2}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}-\frac{3\sqrt{c}ex\sqrt{a+cx^4}\left(ae^2+3cd^2\right)}{8d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+cd^2\right)^2}$$

```
[In] Int[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (-3*Sqrt[c]*e*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2
*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d
 + e*x^2)^2) + (3*e^2*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 +
a*e^2)^2*(d + e*x^2)) + (3*Sqrt[e]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*Ar
cTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(16*d^(5/2
)*(c*d^2 + a*e^2)^(5/2)) + (3*a^(1/4)*c^(1/4)*e*(3*c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) +
(c^(1/4)*(4*c*d^2 - Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt
```

```
[a + c*x^4]) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*
e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c
*d^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1711

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), I
nt[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2
*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C
*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILt
Q[q, -1]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
```

```
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2}-\frac{\int\frac{-4cd^2-3ae^2+4cdex^2-ce^2x^4}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx}{4d\left(cd^2+ae^2\right)}\\
&= \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2}+\frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}\\
&\quad+\frac{\int\frac{8c^2d^4+5acd^2e^2+3a^2e^4-4cde\left(4cd^2+ae^2\right)x^2-3ce^2\left(3cd^2+ae^2\right)x^4}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{8d^2\left(cd^2+ae^2\right)^2}\\
&= \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2}+\frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}\\
&\quad+\frac{\int\frac{-3\sqrt{a}c^{3/2}de^2\left(3cd^2+ae^2\right)+ce\left(8c^2d^4+5acd^2e^2+3a^2e^4\right)+\left(3ce^2\left(cd-\sqrt{a}\sqrt{c}e\right)\left(3cd^2+ae^2\right)-4c^2de^2\left(4cd^2+ae^2\right)\right)x^2}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{8cd^2e\left(cd^2+ae^2\right)^2}\\
&\quad+\frac{\left(3\sqrt{a}\sqrt{c}e(3cd^2+ae^2)\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{8d^2\left(cd^2+ae^2\right)^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3\sqrt{c}e(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad + \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} + \frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} \\
&\quad + \frac{3\sqrt[4]{a}\sqrt[4]{c}e(3cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}} \\
&\quad + \frac{\left(\sqrt{c}\left(4cd^2-\sqrt{a}\sqrt{c}de+3ae^2\right)\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{4d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)} \\
&\quad - \frac{\left(3\sqrt{a}e\left(5c^2d^4+2acd^2e^2+a^2e^4\right)\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{8d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2} \\
&= -\frac{3\sqrt{c}e(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} \\
&\quad + \frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{3\sqrt{e}\left(5c^2d^4+2acd^2e^2+a^2e^4\right)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\left(cd^2+ae^2\right)^{5/2}} \\
&\quad + \frac{3\sqrt[4]{a}\sqrt[4]{c}e(3cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}} \\
&\quad + \frac{\sqrt[4]{c}\left(4cd^2-\sqrt{a}\sqrt{c}de+3ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\sqrt{a+cx^4}} \\
&\quad - \frac{3\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(5c^2d^4+2acd^2e^2+a^2e^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\Big(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\Big(}{32\sqrt[4]{c}d^3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.85 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.46

$$
\int\frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx
$$

$$
= \frac{\frac{de^2x\left(a+cx^4\right)\left(ae^2\left(5d+3ex^2\right)+cd^2\left(11d+9ex^2\right)\right)}{\left(d+ex^2\right)^2} + \sqrt{1+\frac{cx^4}{a}}\Big(-3\sqrt{a}\sqrt{c}de\left(3cd^2+ae^2\right)E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+i\Big(\sqrt{c}d\big(7c^{3/2}d^3-9i\sqrt{a}c}{8d^3\left(cd^2+ae^2\right.}
$$

```
[In] Integrate[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] ((d*e^2*x*(a + c*x^4)*(a*e^2*(5*d + 3*e*x^2) + c*d^2*(11*d + 9*e*x^2)))/(d
+ e*x^2)^2 + (Sqrt[1 + (c*x^4)/a]*(-3*Sqrt[a]*Sqrt[c]*d*e*(3*c*d^2 + a*e^2)
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(7*c^
(3/2)*d^3 - (9*I)*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - (3*I)*a^(3/2)*e^3)*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - 3*(5*c^2*d^4 + 2*a*c*
d^2*e^2 + a^2*e^4)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[
(I*Sqrt[c])/Sqrt[a]]*x], -1])))/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*(c*d^2 +
a*e^2)^2*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 1018, normalized size of antiderivative = 1.40

method	result	size
default	Expression too large to display	1018
elliptic	Expression too large to display	1018

```
[In] int(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^
2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^2/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*a*e
^2-7/8*c^2*d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*c^(1/2))^(1/2),I)-9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2
)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^
(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+9/8*I*c^(3/2
)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a
^(1/2)*c^(1/2))^(1/2),I)+3/8*I*c^(1/2)*e^3/d^2/(a*e^2+c*d^2)^2*a^(3/2)/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-3/8*I*c
^(1/2)*e^3/d^2/(a*e^2+c*d^2)^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8/d^3/(a*e^2+c*d^2)^2*e^4/(I/a^(1/2)*
c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2
)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a^2+3/4/(a*e^2+
c*d^2)^2*e^2/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/
2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/
2))^(1/2))*a*c+15/8*d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
```

```
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))
^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\left(d+ex^2\right)^3}\,dx$$

```
[In] integrate(1/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3 \sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(e\,x^2+d)^3}\,dx$$

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3),x)
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

3.157 $\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 213

$$\begin{aligned}\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx &= -\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}\\&\quad+\frac{3a^{3/4}e(5cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}\\&\quad+\frac{a^{3/4}\left(\frac{5\sqrt{c}d(cd^2+ae^2)}{\sqrt{a}}-3e(5cd^2+ae^2)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}\end{aligned}$$

```
[Out] -d*e^2*x*(-c*x^4+a)^(1/2)/c-1/5*e^3*x^3*(-c*x^4+a)^(1/2)/c+3/5*a^(3/4)*e*(a
*e^2+5*c*d^2)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(7/4)/(-c*
x^4+a)^(1/2)+1/5*a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-3*e*(a*e^2+5*c*d^
2)+5*d*(a*e^2+c*d^2)*c^(1/2)/a^(1/2))*(1-c*x^4/a)^(1/2)/c^(7/4)/(-c*x^4+a)^
(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1221, 1902, 1215, 230, 227, 1214, 1213, 435}

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$$
$$= \frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{5\sqrt{c}d(ae^2+cd^2)}{\sqrt{a}}-3e(ae^2+5cd^2)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}$$
$$+\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}(ae^2+5cd^2)\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}$$
$$-\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

```
[In] Int[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

```
[Out] -((d*e^2*x*Sqrt[a - c*x^4])/c) - (e^3*x^3*Sqrt[a - c*x^4])/(5*c) + (3*a^(3/
4)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((5*Sqrt[c]*d*(c*d^2 + a
*e^2))/Sqrt[a] - 3*e*(5*c*d^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
```

```
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^3x^3\sqrt{a-cx^4}}{5c}-\frac{\int \frac{-5cd^3-3e\left(5cd^2+ae^2\right)x^2-15cde^2x^4}{\sqrt{a-cx^4}}\,dx}{5c}\\
&= -\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}+\frac{\int \frac{15cd\left(cd^2+ae^2\right)+9ce\left(5cd^2+ae^2\right)x^2}{\sqrt{a-cx^4}}\,dx}{15c^2}\\
&= -\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}+\frac{\left(3\sqrt{a}e(5cd^2+ae^2)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{5c^{3/2}}\\
&\quad +\frac{\left(5\sqrt{c}d(cd^2+ae^2)-3\sqrt{a}e(5cd^2+ae^2)\right)\int \frac{1}{\sqrt{a-cx^4}}\,dx}{5c^{3/2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\left(3\sqrt{a}e(5cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{5c^{3/2}\sqrt{a-cx^4}} \\
&\quad + \frac{\left(\left(5\sqrt{c}d(cd^2+ae^2)-3\sqrt{a}e(5cd^2+ae^2)\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{5c^{3/2}\sqrt{a-cx^4}} \\
&= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} \\
&\quad + \frac{\sqrt[4]{a}\left(5\sqrt{c}d(cd^2+ae^2)-3\sqrt{a}e(5cd^2+ae^2)\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}} \\
&\quad + \frac{\left(3\sqrt{a}e(5cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{5c^{3/2}\sqrt{a-cx^4}} \\
&= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{3a^{3/4}e(5cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(5\sqrt{c}d(cd^2+ae^2)-3\sqrt{a}e(5cd^2+ae^2)\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66

$$
\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx
$$

$$
= \frac{5d(cd^2+ae^2)\,x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex\Bigg(e(5d+ex^2)\left(-a+cx^4\right)+\left(5cd^2+ae^2\right)x}{5c\sqrt{a-cx^4}}
$$

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

```
[Out] (5*d*(c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
 (c*x^4)/a] + e*x*(e*(5*d + e*x^2)*(-a + c*x^4) + (5*c*d^2 + a*e^2)*x^2*Sqr
t[1 - (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(5*c*Sqrt[a
- c*x^4])
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.04

method	result
elliptic	$-\frac{e^{3} x^{3} \sqrt{-c\,x^{4}+a}}{5c}-\frac{d\,e^{2} x \sqrt{-c\,x^{4}+a}}{c}+\frac{\left(d^{3}+\frac{d\,e^{2} a}{c}\right) \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{\left(3d^{2} e+\frac{3e^{3} a}{5c}\right) \sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}$
risch	$-\frac{e^{2} x \left(e\,x^{2}+5d\right) \sqrt{-c\,x^{4}+a}}{5c}+\frac{\frac{5d^{3} c \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{5d\,e^{2} a \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\left(3a\,e^{3}+15c\,d^{2} e\right) \sqrt{a}}{5c}$
default	$\frac{d^{3} \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+e^{3} \left(-\frac{x^{3} \sqrt{-c\,x^{4}+a}}{5c}-\frac{3a^{\frac{3}{2}} \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5c^{\frac{3}{2}} \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)$

```
[In] int((e*x^2+d)^3/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*e^3*x^3*(-c*x^4+a)^(1/2)/c-d*e^2*x*(-c*x^4+a)^(1/2)/c+(d^3+d*e^2/c*a)/
(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-(3
*d^2*e+3/5*e^3/c*a)*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(Ellipt
icF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{\left(d+ex^{2}\right)^{3}}{\sqrt{a-cx^{4}}}\,dx = -\frac{3\left(5\,acd^{2}e+a^{2}e^{3}\right)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-\left(5\,c^{2}d^{3}+15\,acd^{2}e+5\,acde^{2}+3\,a^{2}e^{3}\right)\sqrt{-c}x\left(\frac{a}{c}\right)}{5\,ac^{2}x}$$

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(5*a*c*d^2*e + a^2*e^3)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_e(arcsin((a
/c)^(1/4)/x), -1) - (5*c^2*d^3 + 15*a*c*d^2*e + 5*a*c*d*e^2 + 3*a^2*e^3)*sq
rt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + (a*c*e^3*x^4 +
 5*a*c*d*e^2*x^2 + 15*a*c*d^2*e + 3*a^2*e^3)*sqrt(-c*x^4 + a))/(a*c^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate((e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

```
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*
x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)
*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4
)) + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*p
i)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{-cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{-cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a-cx^4}}\,dx=\int \frac{{\left(e\,x^2+d\right)}^3}{\sqrt{a-c\,x^4}} \,dx$$

```
[In] int((d + e*x^2)^3/(a - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^3/(a - c*x^4)^(1/2), x)
```

3.158 $\int \frac{\left(d+ex^2\right)^2}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx=-\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}$$

```
[Out] -1/3*e^2*x*(-c*x^4+a)^(1/2)/c+2*a^(3/4)*d*e*EllipticE(c^(1/4)*x/a^(1/4),I)*
(1-c*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+a)^(1/2)+1/3*a^(1/4)*EllipticF(c^(1/4)*x/
a^(1/4),I)*(3*c*d^2+a*e^2-6*d*e*a^(1/2)*c^(1/2))*(1-c*x^4/a)^(1/2)/c^(5/4)/
(-c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1221, 1215, 230, 227, 1214, 1213, 435}

$$\begin{aligned}&\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx\\ &=\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}\\ &\quad+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}\\ &\quad-\frac{e^2x\sqrt{a-cx^4}}{3c}\end{aligned}$$

```
[In] Int[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

```
[Out] -1/3*(e^2*x*Sqrt[a - c*x^4])/c + (2*a^(3/4)*d*e*Sqrt[1 - (c*x^4)/a]*Ellipti
cE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(
3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcS
in[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(5/4)*Sqrt[a - c*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
```

```
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^2x\sqrt{a-cx^4}}{3c}-\frac{\int\frac{-3cd^2-ae^2-6cdex^2}{\sqrt{a-cx^4}}\,dx}{3c}\\
&= -\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{\left(2\sqrt{a}de\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{\sqrt{c}}-\frac{\left(-3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\int\frac{1}{\sqrt{a-cx^4}}\,dx}{3c}\\
&= -\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{\left(2\sqrt{a}de\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\sqrt{a-cx^4}}\\
&\quad-\frac{\left(\left(-3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{3c\sqrt{a-cx^4}}\\
&= -\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}\\
&\quad+\frac{\left(2\sqrt{a}de\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{\sqrt{c}\sqrt{a-cx^4}}
\end{aligned}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$
$$+ \frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$$
$$= \frac{(3cd^2+ae^2)\,x\sqrt{1-\frac{cx^4}{a}}\text{ Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex\left(-ae+cex^4+2cdx^2\sqrt{1-\frac{cx^4}{a}}\text{ Hypergeo}\right.}{3c\sqrt{a-cx^4}}$$

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

```
[Out] ((3*c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (
c*x^4)/a] + e*x*(-(a*e) + c*e*x^4 + 2*c*d*x^2*Sqrt[1 - (c*x^4)/a]*Hypergeom
etric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*c*Sqrt[a - c*x^4])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.15

method	result
elliptic	$-\frac{e^2x\sqrt{-cx^4+a}}{3c}+\frac{\left(d^2+\frac{ae^2}{3c}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}-\frac{2ed\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}\,\sqrt{c}}$
default	$\frac{d^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}+e^2\left(-\frac{x\sqrt{-cx^4+a}}{3c}+\frac{a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}\right)-\frac{2ed\sqrt{a}\,\sqrt{1-}}{}$
risch	$-\frac{e^2x\sqrt{-cx^4+a}}{3c}+\frac{\frac{a\,e^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}+\frac{3c\,d^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}-\frac{6d\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}}{3c}$

```
[In] int((e*x^2+d)^2/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*e^2*x*(-c*x^4+a)^(1/2)/c+(d^2+1/3*a/c*e^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(
1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(
```

```
1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-2*e*d*a^(1/2)/(1/a^(1/2)*c^(1
/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/
(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-Elliptic
E(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = -\frac{6\,a\sqrt{-c}dex\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-(3\,cd^2+6\,ade+ae^2)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+(a}{3\,acx}$$

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*sqrt(-c)*d*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1)
- (3*c*d^2 + 6*a*d*e + a*e^2)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c
)^(1/4)/x), -1) + (a*e^2*x^2 + 6*a*d*e)*sqrt(-c*x^4 + a))/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate((e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

```
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*
exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1
/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a-cx^4}}\,dx = \int \frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a-cx^4}}\,dx = \int \frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a-cx^4}}\,dx = \int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a-c\,x^4}}\,dx$$

```
[In] int((d + e*x^2)^2/(a - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^2/(a - c*x^4)^(1/2), x)
```

3.159 $\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

```
[Out] a^(3/4)*e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+
a)^(1/2)+a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-e+d*c^(1/2)/a^(1/2))*(1-c
*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1215, 230, 227, 1214, 1213, 435}

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

```
[In] Int[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c*
x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4
])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{\sqrt{c}} + \left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}\,dx$$

$$
=\frac{\left(\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\left(\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}
$$

$$
=\frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}+\frac{\left(\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}
$$

$$
=\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$
\int\frac{d+ex^2}{\sqrt{a-cx^4}}dx
=\frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{a-cx^4}}
$$

```
[In] Integrate[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[a - c*x^4])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}$	154
elliptic	$\frac{d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}$	154

```
[In] int((e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-
```

```
e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c
^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = -\frac{a\sqrt{-c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-(cd+ae)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)+\sqrt{-cx^4+a}ae}{acx}$$

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*sqrt(-c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - (c*d +
 a*e)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + sqrt(-
c*x^4 + a)*a*e)/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqr
t(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_p
olar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{a-c\,x^4}}\,dx$$

```
[In] int((d + e*x^2)/(a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(a - c*x^4)^(1/2), x)
```

3.160 $\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

```
[Out] a^(1/4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1
/2)/c^(1/4)/d/(-c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1233, 1232}

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

```
[In] Int[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{a-cx^4}}\\ &= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = -\frac{i\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d}, i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right), -1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d\sqrt{a-cx^4}}$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[a - c*x^
4])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, -\frac{e\sqrt{a}}{d\sqrt{c}}, \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}$	97
elliptic	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, -\frac{e\sqrt{a}}{d\sqrt{c}}, \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}$	97

```
[In] int(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),
-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.161 $\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 299

$$\begin{aligned}\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx &= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)}-\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\left(cd^2-ae^2\right)\sqrt{a-cx^4}}\\ &\quad-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2d\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}}\\ &\quad+\frac{\sqrt[4]{a}(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2\left(cd^2-ae^2\right)\sqrt{a-cx^4}}\end{aligned}$$

```
[Out] -1/2*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)-1/2*a^(3/4)*c^(1/4)*
e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d/(-a*e^2+c*d^2)/(-c*x^4
+a)^(1/2)+1/2*a^(1/4)*(-a*e^2+3*c*d^2)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1
/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/d^2/(-a*e^2+c*d^2)/(-c*x^4+a)^(1
/2)-1/2*a^(1/4)*c^(1/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d/
(e*a^(1/2)+d*c^(1/2))/(-c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {1238, 1731, 1215, 230, 227, 1214, 1213, 435, 1233, 1232}

$$\begin{aligned}&\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx\\&=-\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\,(cd^2-ae^2)}\\&\quad+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(3cd^2-ae^2)\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2\sqrt{a-cx^4}\,(cd^2-ae^2)}\\&\quad-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2d\sqrt{a-cx^4}\,(\sqrt{a}e+\sqrt{c}d)}-\frac{e^2x\sqrt{a-cx^4}}{2d\,(d+ex^2)\,(cd^2-ae^2)}\end{aligned}$$

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

```
[Out] -1/2*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)) - (a^(3/4)*c^(
1/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d
*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*Sqrt[1 - (c*x^4)/a]*El
lipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt
[a - c*x^4]) + (a^(1/4)*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-(
(Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/4)*d^2*
(c*d^2 - a*e^2)*Sqrt[a - c*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1731

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Di
st[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
```

`Q[c*d^2 - a*e^2, 0]`

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)}+\frac{\int \frac{2cd^2-ae^2-2cdex^2-ce^2x^4}{\left(d+ex^2\right)\sqrt{a-cx^4}}\,dx}{2d\left(cd^2-ae^2\right)}\\
&= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)}-\frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a-cx^4}}\,dx}{2de^2\left(cd^2-ae^2\right)}+\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(d+ex^2\right)\sqrt{a-cx^4}}\,dx}{2d\left(cd^2-ae^2\right)}\\
&= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)}-\frac{\sqrt{c}\int \frac{1}{\sqrt{a-cx^4}}\,dx}{2d\left(\sqrt{c}d+\sqrt{a}e\right)}-\frac{\left(\sqrt{a}\sqrt{c}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{2d\left(cd^2-ae^2\right)}\\
&\quad+\frac{\left(\left(3cd^2-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\left(d+ex^2\right)\sqrt{1-\frac{cx^4}{a}}}\,dx}{2d\left(cd^2-ae^2\right)\sqrt{a-cx^4}}\\
&= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)}\\
&\quad+\frac{\sqrt[4]{a}\left(3cd^2-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}d^2\left(cd^2-ae^2\right)\sqrt{a-cx^4}}\\
&\quad-\frac{\left(\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{2d\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}}-\frac{\left(\sqrt{a}\sqrt{c}e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{2d\left(cd^2-ae^2\right)\sqrt{a-cx^4}}\\
&= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)}-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}}\\
&\quad+\frac{\sqrt[4]{a}\left(3cd^2-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}d^2\left(cd^2-ae^2\right)\sqrt{a-cx^4}}\\
&\quad-\frac{\left(\sqrt{a}\sqrt{c}e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{2d\left(cd^2-ae^2\right)\sqrt{a-cx^4}}
\end{aligned}
$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)} - \frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\left(cd^2-ae^2\right)\sqrt{a-cx^4}}$$

$$- \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}}$$

$$+ \frac{\sqrt[4]{a}(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}d^2\left(cd^2-ae^2\right)\sqrt{a-cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.81 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx$$

$$= \frac{-a\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}de^2x+\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cde^2x^5+i\sqrt{a}\sqrt{c}de(d+ex^2)\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-i\sqrt{c}d(-}{}$$

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

```
[Out] (-(a*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^2*x) + Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*e^2*x
^5 + I*Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcS
inh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*Sqrt[c]*d*(-(Sqrt[c]*d) + Sqrt[a]*
e)*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[
a])]*x], -1] - (3*I)*c*d^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sq
rt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*d*e^2*Sqrt[1 -
(c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/
Sqrt[a])]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqr
t[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*e^3*
x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sq
rt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(2*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*(c*d^2 - a*
e^2)*(d + e*x^2)*Sqrt[a - c*x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(245) = 490$.

Time = 0.77 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.75

method	result
default	$\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{2\left(a\,e^{2}-c\,d^{2}\right) d \left(e\,x^{2}+d\right)}+\frac{c \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2\left(a\,e^{2}-c\,d^{2}\right) \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}-\frac{\sqrt{c}\,e \sqrt{a}\, \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2\left(a\,e^{2}-c\,d^{2}\right) d \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}+$
elliptic	$\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{2\left(a\,e^{2}-c\,d^{2}\right) d \left(e\,x^{2}+d\right)}+\frac{c \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2\left(a\,e^{2}-c\,d^{2}\right) \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}-\frac{\sqrt{c}\,e \sqrt{a}\, \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2\left(a\,e^{2}-c\,d^{2}\right) d \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}+$

```
[In] int(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*e^2-c*d^2)/(1
/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2
)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-1/2*
c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF
(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^
(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2/(a*
e^2-c*d^2)/d^2*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*
c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c
^(1/2))^(1/2))*a-3/2/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
Pi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1
/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^{2}\right)^{2}\sqrt{a-cx^{4}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2),x)
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

3.162 $\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 425

$$
\begin{aligned}
&\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx \\
&= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2} - \frac{3e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)} \\
&\quad - \frac{3a^{3/4}\sqrt[4]{c}e(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}} \\
&\quad - \frac{\sqrt[4]{a}\sqrt[4]{c}(7cd^2-2\sqrt{a}\sqrt{c}de-3ae^2)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8d^2\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2-ae^2\right)\sqrt{a-cx^4}} \\
&\quad + \frac{3\sqrt[4]{a}(5c^2d^4-2acd^2e^2+a^2e^4)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{c}d^3\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}
\end{aligned}
$$

```
[Out] -1/4*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^2-3/8*e^2*(-a*e^2+3*
c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)-3/8*a^(3/4)*c^(1/4
)*e*(-a*e^2+3*c*d^2)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d^2/(
-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)+3/8*a^(1/4)*(a^2*e^4-2*a*c*d^2*e^2+5*c^2*d
^4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/
c^(1/4)/d^3/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)-1/8*a^(1/4)*c^(1/4)*EllipticF
(c^(1/4)*x/a^(1/4),I)*(7*c*d^2-3*a*e^2-2*d*e*a^(1/2)*c^(1/2))*(1-c*x^4/a)^(
1/2)/d^2/(-a*e^2+c*d^2)/(e*a^(1/2)+d*c^(1/2))/(-c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1238, 1711, 1731, 1215, 230, 227, 1214, 1213, 435, 1233, 1232}

$$\begin{aligned}&\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx\\
&= -\frac{3a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}(3cd^2-ae^2)\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2\sqrt{a-cx^4}\,(cd^2-ae^2)^2}\\
&\quad+\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(a^2e^4-2acd^2e^2+5c^2d^4)\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{c}d^3\sqrt{a-cx^4}\,(cd^2-ae^2)^2}\\
&\quad-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8d^2\sqrt{a-cx^4}\,\left(\sqrt{a}e+\sqrt{c}d\right)(cd^2-ae^2)}\\
&\quad-\frac{3e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{8d^2\,(d+ex^2)\,(cd^2-ae^2)^2}-\frac{e^2x\sqrt{a-cx^4}}{4d\,(d+ex^2)^2\,(cd^2-ae^2)}\end{aligned}$$

```
[In] Int[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

```
[Out] -1/4*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)^2) - (3*e^2*(3*
c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(8*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)) - (
3*a^(3/4)*c^(1/4)*e*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (a^(
1/4)*c^(1/4)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/a
]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(Sqrt[c]*d + Sqrt[a]*e
)*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) + (3*a^(1/4)*(5*c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(8*c^(1/4)*d^3*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^
4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1711

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), I
nt[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2
*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C
*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILt
Q[q, -1]
```

Rule 1731

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Di
st[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^2 x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}+\frac{\int \frac{4cd^2-3ae^2-4cdex^2+ce^2x^4}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx}{4d\left(cd^2-ae^2\right)}\\
&= -\frac{e^2 x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)}\\
&\quad+\frac{\int \frac{8c^2d^4-5acd^2e^2+3a^2e^4-4cde\left(4cd^2-ae^2\right)x^2-3ce^2\left(3cd^2-ae^2\right)x^4}{\left(d+ex^2\right)\sqrt{a-cx^4}}\,dx}{8d^2\left(cd^2-ae^2\right)^2}\\
&= -\frac{e^2 x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)}\\
&\quad-\frac{\int \frac{-3cde^2\left(3cd^2-ae^2\right)+4cde^2\left(4cd^2-ae^2\right)+3ce^3\left(3cd^2-ae^2\right)x^2}{\sqrt{a-cx^4}}\,dx}{8d^2e^2\left(cd^2-ae^2\right)^2}\\
&\quad+\frac{\left(3\left(5c^2d^4-2acd^2e^2+a^2e^4\right)\right)\int \frac{1}{\left(d+ex^2\right)\sqrt{a-cx^4}}\,dx}{8d^2\left(cd^2-ae^2\right)^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2} - \frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)} \\
&\quad - \frac{\left(\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(7cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\right)\int\frac{1}{\sqrt{a-cx^4}}\,dx}{8d^2\left(cd^2-ae^2\right)^2} \\
&\quad - \frac{\left(3\sqrt{a}\sqrt{c}e(3cd^2-ae^2)\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{8d^2\left(cd^2-ae^2\right)^2} \\
&\quad + \frac{\left(3(5c^2d^4-2acd^2e^2+a^2e^4)\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{8d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2} - \frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)} \\
&\quad + \frac{3\sqrt[4]{a}(5c^2d^4-2acd^2e^2+a^2e^4)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{c}d^3\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}} \\
&\quad - \frac{\left(\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(7cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{8d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}} \\
&\quad - \frac{\left(3\sqrt{a}\sqrt{c}e(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{8d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2} - \frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)} \\
&\quad - \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(7cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}} \\
&\quad + \frac{3\sqrt[4]{a}(5c^2d^4-2acd^2e^2+a^2e^4)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{c}d^3\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}} \\
&\quad - \frac{\left(3\sqrt{a}\sqrt{c}e(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{8d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^2x\sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)} \\
&\quad-\frac{3a^{3/4}\sqrt[4]{c}e(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}} \\
&\quad-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(7cd^2-2\sqrt{a}\sqrt{c}de-3ae^2\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}} \\
&\quad+\frac{3\sqrt[4]{a}(5c^2d^4-2acd^2e^2+a^2e^4)\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{c}d^3\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.98 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx$$

$$=\frac{\frac{de^2x(a-cx^4)(ae^2(5d+3ex^2)-cd^2(11d+9ex^2))}{(d+ex^2)^2}-\frac{i\sqrt{1-\frac{cx^4}{a}}\Bigl(3\sqrt{a}\sqrt{c}de(-3cd^2+ae^2)E\Bigl(i\text{arcsinh}\Bigl(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\Bigr)\Big|-1\Bigr)+(-7c^2d^4+9\sqrt{a}c^{3/2}d^3}{}}{8d^3\left(cd^2-ae^2\right.}$$

```
[In] Integrate[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

```
[Out] ((d*e^2*x*(a - c*x^4)*(a*e^2*(5*d + 3*e*x^2) - c*d^2*(11*d + 9*e*x^2)))/(d
+ e*x^2)^2 - (I*Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*Sqrt[c]*d*e*(-3*c*d^2 + a*e^
2)*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-7*c^2*d^4 + 9*S
qrt[a]*c^(3/2)*d^3*e + a*c*d^2*e^2 - 3*a^(3/2)*Sqrt[c]*d*e^3)*EllipticF[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + 3*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt
[a])]*x], -1]))/Sqrt[-(Sqrt[c]/Sqrt[a])]])/(8*d^3*(c*d^2 - a*e^2)^2*Sqrt[a -
 c*x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(363) = 726$.

Time = 1.82 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.26

method	result
default	$\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{4(a\,e^{2}-c\,d^{2}) d (e\,x^{2}+d)^{2}}+\frac{3 e^{2} (a\,e^{2}-3 c\,d^{2}) x \sqrt{-c\,x^{4}+a}}{8 (a\,e^{2}-c\,d^{2})^{2} d^{2} (e\,x^{2}+d)}+\frac{c \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right) a\,e^{2}}{8 d (a\,e^{2}-c\,d^{2})^{2} \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{7 c^{2} d \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+}}{8 (a\,e^{2}-c\,d^{2})^{2} \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}$
elliptic	$\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{4(a\,e^{2}-c\,d^{2}) d (e\,x^{2}+d)^{2}}+\frac{3 e^{2} (a\,e^{2}-3 c\,d^{2}) x \sqrt{-c\,x^{4}+a}}{8 (a\,e^{2}-c\,d^{2})^{2} d^{2} (e\,x^{2}+d)}+\frac{c \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right) a\,e^{2}}{8 d (a\,e^{2}-c\,d^{2})^{2} \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{7 c^{2} d \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+}}{8 (a\,e^{2}-c\,d^{2})^{2} \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}$

```
[In] int(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2-3*c*d
^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/8*c/d/(a*e^2-c*d^2)^
2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*
a*e^2-7/8*c^2*d/(a*e^2-c*d^2)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x
*(1/a^(1/2)*c^(1/2))^(1/2),I)-3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^(3/2)/(
1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+9/8
*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
F(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^(3/2
)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-
9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8/(a*e^2-c*d^2)^2/d^3*e^4/(1/a^(1/2)*
c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^
(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a^2-3/4/(a*e^2-
c*d^2)^2*e^2/d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1
/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/
2))^(1/2))*a*c+15/8/(a*e^2-c*d^2)^2*d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))
^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{a-cx^4}\,\left(d+ex^2\right)^3}\,dx$$

```
[In] integrate(1/(e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^3}\,dx$$

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx=\int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^3}\,dx$$

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^3}\,dx$$

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3),x)
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

3.163 $\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 563

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx &= -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} \\
&\quad - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\,(cd^2-ae^2)^2\,(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)\,x\sqrt{a-cx^4}}{16d^3\,(cd^2-ae^2)^3\,(d+ex^2)} \\
&\quad - \frac{a^{3/4}\sqrt[4]{c}e(29c^2d^4-14acd^2e^2+5a^2e^4)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16d^3\,(cd^2-ae^2)^3\sqrt{a-cx^4}} \\
&\quad - \frac{\sqrt[4]{a}\sqrt[4]{c}(57c^2d^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+10a^{3/2}\sqrt{c}de^3+15a^2e^4)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{48d^3\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(\sqrt{c}d+\sqrt{a}e\right)^3\sqrt{a-cx^4}} \\
&\quad + \frac{\sqrt[4]{a}(35c^3d^6-7ac^2d^4e^2+17a^2cd^2e^4-5a^3e^6)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{16\sqrt[4]{c}d^4\,(cd^2-ae^2)^3\sqrt{a-cx^4}}
\end{aligned}
$$

```
[Out] -1/6*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^3-5/24*e^2*(-a*e^2+3
*c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)^2-1/16*e^2*(5*a^2
*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*x*(-c*x^4+a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/(e*
x^2+d)-1/16*a^(3/4)*c^(1/4)*e*(5*a^2*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*Ellipti
cE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(
1/2)+1/16*a^(1/4)*(-5*a^3*e^6+17*a^2*c*d^2*e^4-7*a*c^2*d^4*e^2+35*c^3*d^6)*
EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1
/4)/d^4/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(1/2)-1/48*a^(1/4)*c^(1/4)*EllipticF(c^
(1/4)*x/a^(1/4),I)*(57*c^2*d^4-32*a*c*d^2*e^2+15*a^2*e^4-30*c^(3/2)*d^3*e*a
^(1/2)+10*a^(3/2)*d*e^3*c^(1/2))*(1-c*x^4/a)^(1/2)/d^3/(-e*a^(1/2)+d*c^(1/2
))^2/(e*a^(1/2)+d*c^(1/2))^3/(-c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1238, 1711, 1731, 1215, 230, 227, 1214, 1213, 435, 1233, 1232}

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}(5a^2e^4-14acd^2e^2+29c^2d^4)}{16d^3(d+ex^2)(cd^2-ae^2)^3}$$

$$-\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}(5a^2e^4-14acd^2e^2+29c^2d^4)\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16d^3\sqrt{a-cx^4}\,(cd^2-ae^2)^3}$$

$$-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\left(10a^{3/2}\sqrt{c}de^3+15a^2e^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+57c^2d^4\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\right.}{48d^3\sqrt{a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(\sqrt{a}e+\sqrt{c}d\right)^3}$$

$$+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6)\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{16\sqrt[4]{c}d^4\sqrt{a-cx^4}\,(cd^2-ae^2)^3}$$

$$-\frac{5e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{24d^2(d+ex^2)^2(cd^2-ae^2)^2}-\frac{e^2x\sqrt{a-cx^4}}{6d(d+ex^2)^3(cd^2-ae^2)}$$

```
[In] Int[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

```
[Out] -1/6*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)^3) - (5*e^2*(3*
c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(24*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)^2)
- (e^2*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[a - c*x^4])/(16*d^3
*(c*d^2 - a*e^2)^3*(d + e*x^2)) - (a^(3/4)*c^(1/4)*e*(29*c^2*d^4 - 14*a*c*d
^2*e^2 + 5*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4
)], -1])/(16*d^3*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*(57*
c^2*d^4 - 30*Sqrt[a]*c^(3/2)*d^3*e - 32*a*c*d^2*e^2 + 10*a^(3/2)*Sqrt[c]*d*
e^3 + 15*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(48*d^3*(Sqrt[c]*d - Sqrt[a]*e)^2*(Sqrt[c]*d + Sqrt[a]*e)^3*Sqrt[a -
 c*x^4]) + (a^(1/4)*(35*c^3*d^6 - 7*a*c^2*d^4*e^2 + 17*a^2*c*d^2*e^4 - 5*a^
3*e^6)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c
^(1/4)*x)/a^(1/4)], -1])/(16*c^(1/4)*d^4*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
```

```
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1711

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), I
nt[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2
*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C
*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILt
Q[q, -1]
```

Rule 1731

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Di
st[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}+\frac{\int\frac{6cd^2-5ae^2-6cdex^2+3ce^2x^4}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx}{6d\left(cd^2-ae^2\right)}\\
&= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}\\
&\quad+\frac{\int\frac{24c^2d^4-29acd^2e^2+15a^2e^4-8cde\left(6cd^2-ae^2\right)x^2+5ce^2\left(3cd^2-ae^2\right)x^4}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx}{24d^2\left(cd^2-ae^2\right)^2}\\
&= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}\\
&\quad-\frac{e^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)x\sqrt{a-cx^4}}{16d^3\left(cd^2-ae^2\right)^3\left(d+ex^2\right)}\\
&\quad+\frac{\int\frac{48c^3d^6-19ac^2d^4e^2+46a^2cd^2e^4-15a^3e^6-4cde\left(36c^2d^4-11acd^2e^2+5a^2e^4\right)x^2-3ce^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)x^4}{\left(d+ex^2\right)\sqrt{a-cx^4}}\,dx}{48d^3\left(cd^2-ae^2\right)^3}
\end{aligned}
$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}$$

$$- \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)\,x\sqrt{a-cx^4}}{16d^3\left(cd^2-ae^2\right)^3\left(d+ex^2\right)}$$

$$- \frac{\int \frac{-3cde^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)+4cde^2\left(36c^2d^4-11acd^2e^2+5a^2e^4\right)+3ce^3\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)x^2}{\sqrt{a-cx^4}}\,dx}{48d^3e^2\left(cd^2-ae^2\right)^3}$$

$$+ \frac{(35c^3d^6-7ac^2d^4e^2+17a^2cd^2e^4-5a^3e^6)\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx}{16d^3\left(cd^2-ae^2\right)^3}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}$$

$$- \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)\,x\sqrt{a-cx^4}}{16d^3\left(cd^2-ae^2\right)^3\left(d+ex^2\right)}$$

$$- \frac{\left(\sqrt{a}\sqrt{c}e(29c^2d^4-14acd^2e^2+5a^2e^4)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{16d^3\left(cd^2-ae^2\right)^3}$$

$$- \frac{\left(\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(57c^2d^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+10a^{3/2}\sqrt{c}de^3+15a^2e^4\right)\right)\int \frac{1}{\sqrt{a-cx^4}}\,dx}{48d^3\left(cd^2-ae^2\right)^3}$$

$$+ \frac{\left((35c^3d^6-7ac^2d^4e^2+17a^2cd^2e^4-5a^3e^6)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{16d^3\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}$$

$$- \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)\,x\sqrt{a-cx^4}}{16d^3\left(cd^2-ae^2\right)^3\left(d+ex^2\right)}$$

$$+ \frac{\sqrt[4]{a}(35c^3d^6-7ac^2d^4e^2+17a^2cd^2e^4-5a^3e^6)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16\sqrt[4]{c}d^4\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

$$- \frac{\left(\sqrt{a}\sqrt{c}e(29c^2d^4-14acd^2e^2+5a^2e^4)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{16d^3\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

$$- \frac{\left(\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(57c^2d^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+10a^{3/2}\sqrt{c}de^3+15a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}\right)\int}{48d^3\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}$$

$$- \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)\,x\sqrt{a-cx^4}}{16d^3\left(cd^2-ae^2\right)^3\left(d+ex^2\right)}$$

$$- \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(57c^2d^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+10a^{3/2}\sqrt{c}de^3+15a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}F\left(\mathrm{s}\right.}{48d^3\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

$$+ \frac{\sqrt[4]{a}(35c^3d^6-7ac^2d^4e^2+17a^2cd^2e^4-5a^3e^6)\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16\sqrt[4]{c}d^4\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

$$- \frac{\left(\sqrt{a}\sqrt{c}e(29c^2d^4-14acd^2e^2+5a^2e^4)\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{16d^3\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}$$

$$- \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)\,x\sqrt{a-cx^4}}{16d^3\left(cd^2-ae^2\right)^3\left(d+ex^2\right)}$$

$$- \frac{a^{3/4}\sqrt[4]{c}e(29c^2d^4-14acd^2e^2+5a^2e^4)\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16d^3\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

$$- \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(57c^2d^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+10a^{3/2}\sqrt{c}de^3+15a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}F\left(\mathrm{s}\right.}{48d^3\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

$$+ \frac{\sqrt[4]{a}(35c^3d^6-7ac^2d^4e^2+17a^2cd^2e^4-5a^3e^6)\sqrt{1-\frac{cx^4}{a}}\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16\sqrt[4]{c}d^4\left(cd^2-ae^2\right)^3\sqrt{a-cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.38 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.81

$$\int\frac{1}{\left(d+ex^2\right)^4\sqrt{a-cx^4}}\,dx$$

$$= \frac{-\frac{de^2x\left(a-cx^4\right)\left(8\left(cd^3-ade^2\right)^2+10d\left(cd^2-ae^2\right)\left(3cd^2-ae^2\right)\left(d+ex^2\right)+3\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)\left(d+ex^2\right)^2\right)}{\left(cd^2-ae^2\right)^3\left(d+ex^2\right)^3} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}de(29c^2d^4-\right.}{}}{}$$

```
[In] Integrate[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

```
[Out] (-((d*e^2*x*(a - c*x^4)*(8*(c*d^3 - a*d*e^2)^2 + 10*d*(c*d^2 - a*e^2)*(3*c*
d^2 - a*e^2)*(d + e*x^2) + 3*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*(d +
 e*x^2)^2))/((c*d^2 - a*e^2)^3*(d + e*x^2)^3)) - (I*Sqrt[1 - (c*x^4)/a]*(3*
Sqrt[a]*Sqrt[c]*d*e*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*EllipticE[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*d*(57*c^(5/2)*d^5 - 87*Sq
rt[a]*c^2*d^4*e - 2*a*c^(3/2)*d^3*e^2 + 42*a^(3/2)*c*d^2*e^3 + 5*a^2*Sqrt[c
]*d*e^4 - 15*a^(5/2)*e^5)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x],
-1] + 3*(-35*c^3*d^6 + 7*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 + 5*a^3*e^6)*Elli
pticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -
1]))/(Sqrt[-(Sqrt[c]/Sqrt[a])]*(-(c*d^2) + a*e^2)^3))/(48*d^4*Sqrt[a - c*x^
4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1419 vs. 2(489) = 978.

Time = 2.47 (sec) , antiderivative size = 1420, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	1420
elliptic	Expression too large to display	1420

```
[In] int(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^3+5/24*e^2*(a*e^2-3*c*
d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+1/16*e^2*(5*a^2*e^4
-14*a*c*d^2*e^2+29*c^2*d^4)/(a*e^2-c*d^2)^3/d^3*x*(-c*x^4+a)^(1/2)/(e*x^2+d
)-35/16/(a*e^2-c*d^2)^3*d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1
/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/
a^(1/2)*c^(1/2))^(1/2))*c^3+19/16*c^3*d^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2
))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-
c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+5/16/(a*e^2-c*d^2)^
3/d^4*e^6/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^
(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(
1/2))*a^3+7/16/(a*e^2-c*d^2)^3*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
Pi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1
/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a*c^2+5/48*c/d^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*
c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*a^2*e^4-1/24*
c^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/
2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*
```

```
c^(1/2))^(1/2),I)*a*e^2-5/16*c^(1/2)*e^5/(a*e^2-c*d^2)^3/d^3*a^(5/2)/(1/a^(
1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^
2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+7/8*c^(3
/2)*e^3/(a*e^2-c*d^2)^3/d*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF
(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-29/16*c^(5/2)*e/(a*e^2-c*d^2)^3*d*a^(1/2)/(
1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+5/1
6*c^(1/2)*e^5/(a*e^2-c*d^2)^3/d^3*a^(5/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^
(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*E
llipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-7/8*c^(3/2)*e^3/(a*e^2-c*d^2)^3/d*a
^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2
),I)+29/16*c^(5/2)*e/(a*e^2-c*d^2)^3*d*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1
-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1
/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-17/16/(a*e^2-c*d^2)^3/d^2*e^4/
(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a
^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a^2*
c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^4}\,dx$$

```
[In] integrate(1/(e*x**2+d)**4/(-c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**4), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^4}\,dx$$

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^4}\,dx$$

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^4}\,dx$$

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4),x)
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4), x)
```

3.164 $\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{-a+cx^4}}+\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{-a+cx^4}}$$

```
[Out] a^(3/4)*e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(3/4)/(c*x^4-a
)^(1/2)+a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-e+d*c^(1/2)/a^(1/2))*(1-c*
x^4/a)^(1/2)/c^(3/4)/(c*x^4-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1215, 230, 227, 1214, 1213, 435}

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}$$

```
[In] Int[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c
*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[-a + c*x
^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{-a+cx^4}}\,dx}{\sqrt{c}} + \left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\sqrt{-a+cx^4}} + \frac{\left(\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}} + \frac{\left(\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{\sqrt{c}\sqrt{-a+cx^4}}$$

$$= \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx$$
$$= \frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{d\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{e\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}\,\sqrt{c}}$	160
elliptic	$\frac{d\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{e\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}\,\sqrt{c}}$	160

```
[In] int((e*x^2+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
```

```
+e*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(1/2)
*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \frac{a\sqrt{c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-(cd+ae)\sqrt{c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\sqrt{cx^4-a}ae}{acx}$$

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*sqrt(c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - (c*d + a
*e)*sqrt(c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + sqrt(c*x^
4 - a)*a*e)/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = -\frac{idx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}-\frac{iex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a)*gamma(5/4)
) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamm
a(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4-a}}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4-a}}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{c\,x^4-a}}\,dx$$

```
[In] int((d + e*x^2)/(c*x^4 - a)^(1/2),x)
[Out] int((d + e*x^2)/(c*x^4 - a)^(1/2), x)
```

3.165 $\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{-a+cx^4}}$$

```
[Out] a^(1/4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1
/2)/c^(1/4)/d/(c*x^4-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1233, 1232}

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{cx^4-a}}$$

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[-a + c*x^4])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}\\ &= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{-a+cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = -\frac{i\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d}, i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right), -1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d\sqrt{-a+cx^4}}$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[-a + c*x
^4])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

method	result	size
default	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}, \frac{e\sqrt{a}}{d\sqrt{c}}, \frac{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}$	99
elliptic	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}, \frac{e\sqrt{a}}{d\sqrt{c}}, \frac{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}$	99

```
[In] int(1/(e*x^2+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/2)
,e*a^(1/2)/d/c^(1/2),(1/a^(1/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 - a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{-a+cx^4}\,(d+ex^2)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a + c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4-a}\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)),x)
[Out] int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)), x)
```

3.166 $\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 54

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx=\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}$$

```
[Out] a^(3/4)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/(c*x^4-a)^
(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1214, 1213, 435}

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx=\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

```
[In] Int[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(1/4)*Sqrt[-a + c*x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}\\&=\frac{\left(\sqrt{a}\sqrt{1-\frac{cx^4}{a}}\right)\int\frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{\sqrt{-a+cx^4}}\\&=\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.59

$$\int\frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx=\frac{\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+\sqrt{c}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

```
[In] Integrate[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/
a] + Sqrt[c]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a +
 c*x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(42) = 84$.

Time = 0.62 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.93

method	result
default	$\frac{\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}$
elliptic	$\frac{\left(\sqrt{a}+x^{2}\sqrt{c}\right)\sqrt{-\left(-c\,x^{4}+a\right)c}\,\sqrt{-\left(-c\,x^{4}+a\right)a}\,\left(\frac{\sqrt{c}\,\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c^{2}x^{4}-a c}}+\frac{a\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{a c\,x}}\right.}{\sqrt{c\,x^{4}-a}\,\left(c\,x^{2}\sqrt{-\left(-c\,x^{4}+a\right)a}+a\sqrt{-\left(-c\,x^{4}+a\right)c}\right)}$

```
[In] int((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1
/2),I)+a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(
1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*(EllipticF(x*(-1/a^(1/2)*c^(
1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{a}+\sqrt{c}x^{2}}{\sqrt{-a+cx^{4}}}\,dx$$

$$= \frac{2\,acx\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-\sqrt{2}\left(\sqrt{2}acx\sqrt{\frac{a}{c}}+\sqrt{2}\sqrt{a}c^{\frac{3}{2}}x\sqrt{\frac{a}{c}}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+2\,\sqrt{}}{2\,acx}$$

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*c*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - sqrt(2)*(s
qrt(2)*a*c*x*sqrt(a/c) + sqrt(2)*sqrt(a)*c^(3/2)*x*sqrt(a/c))*(a/c)^(1/4)*e
lliptic_f(arcsin((a/c)^(1/4)/x), -1) + 2*sqrt(c*x^4 - a)*a*sqrt(c))/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = -\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{i\sqrt{c}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((a**(1/2)+x**2*c**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*gamma(5/4)) - I*sqrt
(c)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamma(7/
4))
```

Maxima [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}\,dx$$

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Giac [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}\,dx$$

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{c\,x^4-a}}\,dx$$

```
[In] int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2),x)
```

```
[Out] int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2), x)
```

3.167 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 52

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx=\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{-a+cx^4}}$$

```
[Out] EllipticE((c/a)^(1/4)*x,I)*(1-c*x^4/a)^(1/2)/(c/a)^(1/4)/(c*x^4-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1214, 1213, 435}

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx=\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4-a}}$$

```
[In] Int[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)^(1/4)*Sqr
t[-a + c*x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}} \\
&= \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{1+\sqrt{\frac{c}{a}}x^2}}{\sqrt{1-\sqrt{\frac{c}{a}}x^2}}\,dx}{\sqrt{-a+cx^4}} \\
&= \frac{\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{-a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$
\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+\sqrt{\frac{c}{a}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}
$$

```
[In] Integrate[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + Sqr
t[c/a]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4
])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(44) = 88.

Time = 1.85 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.17

method	result
default	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{\sqrt{\frac{c}{a}}\,\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}\,\sqrt{c}}$
elliptic	$\frac{\left(1+x^{2}\sqrt{\frac{c}{a}}\right)a\sqrt{-\frac{\left(-c\,x^{4}+a\right)c}{a}}\left(\frac{\sqrt{c}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{\frac{c^{2}x^{4}}{a}-c}}+\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}\right)}{c\,x^{2}\sqrt{c\,x^{4}-a}+a\sqrt{-\frac{\left(-c\,x^{4}+a\right)c}{a}}}$

```
[In] int((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
+(c/a)^(1/2)*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(
1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(
-1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(43) = 86.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.52

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$$

$$=\frac{2\,a\sqrt{c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}\sqrt{\frac{c}{a}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid-1)-\sqrt{2}\left(\sqrt{2}ax\sqrt{\frac{a}{c}}\sqrt{\frac{c}{a}}+\sqrt{2}cx\sqrt{\frac{a}{c}}\right)\sqrt{c}\left(\frac{a}{c}\right)^{\frac{1}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid-1)}{2\,acx}$$

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*sqrt(c)*x*(a/c)^(3/4)*sqrt(c/a)*elliptic_e(arcsin((a/c)^(1/4)/x),
-1) - sqrt(2)*(sqrt(2)*a*x*sqrt(a/c)*sqrt(c/a) + sqrt(2)*c*x*sqrt(a/c))*sqr
t(c)*(a/c)^(1/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + 2*sqrt(c*x^4 - a)*
a*sqrt(c/a))/(a*c*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

Time = 0.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = -\frac{ix^3\sqrt{\frac{c}{a}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((1+x**2*(c/a)**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x**3*sqrt(c/a)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)
*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a
)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Giac [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{x^2\,\sqrt{\frac{c}{a}}+1}{\sqrt{c\,x^4-a}}\,dx$$

```
[In] int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2),x)
[Out] int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2), x)
```

3.168 $\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 236

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx &= -\frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}}\\&\quad+\frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}}\end{aligned}$$

```
[Out] -e*x*(-c*x^4-a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a
)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*x^4-a)^(1/2)+1/2*a^(1/4)*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(
e+d*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*
x^4-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1212, 226, 1210}

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}} - \frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

```
[Out] -((e*x*Sqrt[-a - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1/4)*e*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[-a - c*x^4]) + (a^(1/4)
*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/
4)*Sqrt[-a - c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\left(\sqrt{a}e\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{-a-cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\sqrt{-a-cx^4}}\,dx \\ &= -\frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}} \\ &\quad + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{-a-cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34

$$\begin{aligned}&\int\frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx \\ &= \frac{\sqrt{1+\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{-a-cx^4}}\end{aligned}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[-a - c*x^4],x]
[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[-a - c*x^4
])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{d\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}}-\frac{ie\sqrt{a}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}\,\sqrt{c}}$	175
elliptic	$\frac{d\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}}-\frac{ie\sqrt{a}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}\,\sqrt{c}}$	175

```
[In] int((e*x^2+d)/(-c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*c^(1/2))^(1/2),I
)-I*e*a^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
-I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-I/a^(
1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.41

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx =$$
$$-\frac{a\sqrt{-c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+(cd-ae)\sqrt{-c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+\sqrt{-cx^4-}}{acx}$$

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*sqrt(-c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) + (c*d
 - a*e)*sqrt(-c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + sq
rt(-c*x^4 - a)*a*e)/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = -\frac{idx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{iex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*ex
p_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{-c\,x^4-a}}\,dx$$

```
[In] int((d + e*x^2)/(- a - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)/(- a - c*x^4)^(1/2), x)
```

3.169 $\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 347

$$\begin{aligned}\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx &= \frac{\sqrt{e}\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-ae^2}}\\&\quad+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a-cx^4}}\\&\quad-\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\sqrt{-a-cx^4}}\end{aligned}$$

```
[Out] 1/2*arctan(x*(-a*e^2-c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(-c*x^4-a)^(1/2))*e^(1/2)
/d^(1/2)/(-a*e^2-c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d*c^(1/2))/(-c*x^4-a)^(1/2)-1/4*a^(3/4)*(c
os(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*E
llipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/
e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^
2*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(-c*x^
4-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1231, 226, 1721}

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = -\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}d\sqrt{-a-cx^4}\,(cd^2-ae^2)} + \frac{\sqrt{e}\arctan\left(\frac{x\sqrt{-ae^2-cd^2}}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{-ae^2-cd^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

```
[In] Int[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a - c*x^4
])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a - c*x^4]) - (a^(
3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a
]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 -
a*e^2)*Sqrt[-a - c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
```

```
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c}\int \frac{1}{\sqrt{-a-cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e} - \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{-a-cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e}\\
&= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a-cx^4}}\\
&\quad - \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a-cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.28

$$
\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d}, i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right), -1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}d\sqrt{-a-cx^4}}
$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[-a -
 c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}, -\frac{i\sqrt{a}\,e}{\sqrt{c}\,d}, \frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}$	110
elliptic	$\frac{\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}, -\frac{i\sqrt{a}\,e}{\sqrt{c}\,d}, \frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}$	110

```
[In] int(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticPi(x*(-I/a^(1/2)*c^(1/2))^(1/2
),-I*a^(1/2)/c^(1/2)*e/d,(I/a^(1/2)*c^(1/2))^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/
2))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 - a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-a-cx^4}\,(d+ex^2)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a - c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-c\,x^4-a}\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)),x)
[Out] int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.170 $\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx=\frac{\text{EllipticPi}\left(-\frac{2b}{\sqrt{5}a},\arcsin\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

```
[Out] 1/10*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),I)*5^(3/4)/a*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1227, 551}

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx=\frac{\text{EllipticPi}\left(-\frac{2b}{\sqrt{5}a},\arcsin\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

```
[In] Int[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^
(1/4)*a)
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{5}\int \frac{1}{\sqrt{2\sqrt{5}-5x^2}\sqrt{2\sqrt{5}+5x^2}\,(a+bx^2)}\,dx\\ &= \frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}\end{aligned}$$

Mathematica [A] (verified)

Time = 10.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \frac{\text{EllipticPi}\left(-\frac{2b}{\sqrt{5}a},\arcsin\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^
(1/4)*a)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(32) = 64$.

Time = 1.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

method	result	size
default	$\frac{\sqrt{2}\,5^{\frac{3}{4}}\sqrt{1-\frac{x^{2}\sqrt{5}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{5}}{2}}\,\Pi\left(\frac{5^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b\sqrt{5}}{5a},\frac{\sqrt{-\frac{\sqrt{5}}{2}}\,\sqrt{2}\,5^{\frac{3}{4}}}{5}\right)}{5a\sqrt{-5x^{4}+4}}$	79
elliptic	$\frac{\sqrt{2}\,5^{\frac{3}{4}}\sqrt{1-\frac{x^{2}\sqrt{5}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{5}}{2}}\,\Pi\left(\frac{5^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b\sqrt{5}}{5a},\frac{\sqrt{-\frac{\sqrt{5}}{2}}\,\sqrt{2}\,5^{\frac{3}{4}}}{5}\right)}{5a\sqrt{-5x^{4}+4}}$	79

```
[In] int(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/a*2^(1/2)*5^(3/4)*(1-1/2*x^2*5^(1/2))^(1/2)*(1+1/2*x^2*5^(1/2))^(1/2)/(
-5*x^4+4)^(1/2)*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),1/5*(-1/2
*5^(1/2))^(1/2)*2^(1/2)*5^(3/4))
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 - 4*b*x^2 - 4*a), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{4-5x^4}\,(a+bx^2)}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(-5*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(4 - 5*x**4)*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{4-5\,x^4}}\,dx$$

```
[In] int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)), x)
```

3.171 $\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 310

$$
\begin{aligned}
\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx
&= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{5a^2+4b^2}\,x}{\sqrt{a}\sqrt{b}\sqrt{4+5x^4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}} \\
&\quad + \frac{\sqrt[4]{5}(\sqrt{5}a+2b)\,(2+\sqrt{5}x^2)\sqrt{\frac{4+5x^4}{\left(2+\sqrt{5}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\,(5a^2-4b^2)\sqrt{4+5x^4}} \\
&\quad - \frac{\left(\sqrt{5}a+2b\right)^2(2+\sqrt{5}x^2)\sqrt{\frac{4+5x^4}{\left(2+\sqrt{5}x^2\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{5}a-2b\right)^2}{8\sqrt{5}ab},2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt[4]{5}a\,(5a^2-4b^2)\sqrt{4+5x^4}}
\end{aligned}
$$

```
[Out] 1/2*arctan(x*(5*a^2+4*b^2)^(1/2)/a^(1/2)/b^(1/2)/(5*x^4+4)^(1/2))*b^(1/2)/a
^(1/2)/(5*a^2+4*b^2)^(1/2)+1/4*5^(1/4)*(cos(2*arctan(1/2*5^(1/4)*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*5^(1/4)*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*
5^(1/4)*x*2^(1/2))),1/2*2^(1/2))*(2*b+a*5^(1/2))*(2+x^2*5^(1/2))*((5*x^4+4)
/(2+x^2*5^(1/2))^2)^(1/2)/(5*a^2-4*b^2)*2^(1/2)/(5*x^4+4)^(1/2)-1/40*(cos(2
*arctan(1/2*5^(1/4)*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*5^(1/4)*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*5^(1/4)*x*2^(1/2))),-1/40*(-2*b+a*5^(1/2))^2
/a/b*5^(1/2),1/2*2^(1/2))*(2*b+a*5^(1/2))^2*(2+x^2*5^(1/2))*((5*x^4+4)/(2+x
^2*5^(1/2))^2)^(1/2)*5^(3/4)/a/(5*a^2-4*b^2)*2^(1/2)/(5*x^4+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1231, 226, 1721}

$$\begin{aligned}&\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx\\&=\frac{\sqrt{b}\arctan\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}}\\&\quad+\frac{\sqrt[4]{5}\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}\,(5a^2-4b^2)}\\&\quad-\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)^2\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{5}a-2b\right)^2}{8\sqrt{5}ab},2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt[4]{5}a\sqrt{5x^4+4}\,(5a^2-4b^2)}\end{aligned}$$

```
[In] Int[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[5*a^2 + 4*b^2]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + 5*x^4])])
/(2*Sqrt[a]*Sqrt[5*a^2 + 4*b^2]) + (5^(1/4)*(Sqrt[5]*a + 2*b)*(2 + Sqrt[5]*
x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticF[2*ArcTan[(5^(1/4)*x)/S
qrt[2]], 1/2])/(2*Sqrt[2]*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4]) - ((Sqrt[5]*a +
2*b)^2*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticPi[-
1/8*(Sqrt[5]*a - 2*b)^2/(Sqrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])
/(4*Sqrt[2]*5^(1/4)*a*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
```

```
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(2b\left(\sqrt{5}a+2b\right)\right)\int \frac{1+\frac{\sqrt{5}x^2}{2}}{(a+bx^2)\sqrt{4+5x^4}}\,dx}{5a^2-4b^2}+\frac{\left(5a+2\sqrt{5}b\right)\int \frac{1}{\sqrt{4+5x^4}}\,dx}{5a^2-4b^2}\\
&= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{5a^2+4b^2}x}{\sqrt{a}\sqrt{b}\sqrt{4+5x^4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}}+\frac{\sqrt[4]{5}\left(\sqrt{5}a+2b\right)\left(2+\sqrt{5}x^2\right)\sqrt{\frac{4+5x^4}{\left(2+\sqrt{5}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\left(5a^2-4b^2\right)\sqrt{4+5x^4}}\\
&\quad -\frac{\left(\sqrt{5}a+2b\right)^2\left(2+\sqrt{5}x^2\right)\sqrt{\frac{4+5x^4}{\left(2+\sqrt{5}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{5}a-2b\right)^2}{8\sqrt{5}ab};2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{4\sqrt{2}\sqrt[4]{5}a\left(5a^2-4b^2\right)\sqrt{4+5x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.16

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx=-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\text{EllipticPi}\left(-\frac{2ib}{\sqrt{5}a},i\text{arcsinh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{5}x\right),-1\right)}{\sqrt[4]{5}a}$$

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
[Out] ((-1/2 - I/2)*EllipticPi[((-2*I)*b)/(Sqrt[5]*a), I*ArcSinh[(1/2 + I/2)*5^(1
/4)*x], -1])/(5^(1/4)*a)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{5}\,x^2}{2}}\,\sqrt{1+\frac{i\sqrt{5}\,x^2}{2}}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{5}}\,x}{2},\frac{2i\sqrt{5}\,b}{5a},\frac{\sqrt{-\frac{i\sqrt{5}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{5}}}\right)}{a\sqrt{i\sqrt{5}}\,\sqrt{5x^4+4}}$	86
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{5}\,x^2}{2}}\,\sqrt{1+\frac{i\sqrt{5}\,x^2}{2}}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{5}}\,x}{2},\frac{2i\sqrt{5}\,b}{5a},\frac{\sqrt{-\frac{i\sqrt{5}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{5}}}\right)}{a\sqrt{i\sqrt{5}}\,\sqrt{5x^4+4}}$	86

```
[In] int(1/(b*x^2+a)/(5*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(1/2*I*5^(1/2))^(1/2)*(1-1/2*I*5^(1/2)*x^2)^(1/2)*(1+1/2*I*5^(1/2)*x^2)
^(1/2)/(5*x^4+4)^(1/2)*EllipticPi((1/2*I*5^(1/2))^(1/2)*x,2/5*I*5^(1/2)*b/a
,(-1/2*I*5^(1/2))^(1/2)/(1/2*I*5^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 + 4*b*x^2 + 4*a), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{5x^4+4}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(5*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(5*x**4 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{5\,x^4+4}}\,dx$$

```
[In] int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)), x)
```

3.172 $\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \frac{\text{EllipticPi}\left(-\frac{2b}{a\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

```
[Out] 1/2*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),I)/a/d^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1232}

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \frac{\text{EllipticPi}\left(-\frac{2b}{a\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

```
[In] Int[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*a*
d^(1/4))
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = -\frac{i\,\text{EllipticPi}\left(-\frac{2b}{a\sqrt{d}},i\text{arcsinh}\left(\frac{\sqrt{-\sqrt{d}}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}a\sqrt{-\sqrt{d}}}$$

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] ((-I)*EllipticPi[(-2*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[-Sqrt[d]]*x)/Sqrt[2]],
 -1])/(Sqrt[2]*a*Sqrt[-Sqrt[d]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 1.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{\sqrt{2}\sqrt{1-\frac{x^{2}\sqrt{d}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{d}}{2}}\,\Pi\left(\frac{d^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b}{a\sqrt{d}},\frac{\sqrt{-\frac{\sqrt{d}}{2}}\,\sqrt{2}}{d^{\frac{1}{4}}}\right)}{a\,d^{\frac{1}{4}}\sqrt{-d\,x^{4}+4}}$	78
elliptic	$\frac{\sqrt{2}\sqrt{1-\frac{x^{2}\sqrt{d}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{d}}{2}}\,\Pi\left(\frac{d^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b}{a\sqrt{d}},\frac{\sqrt{-\frac{\sqrt{d}}{2}}\,\sqrt{2}}{d^{\frac{1}{4}}}\right)}{a\,d^{\frac{1}{4}}\sqrt{-d\,x^{4}+4}}$	78

```
[In] int(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*2^(1/2)/d^(1/4)*(1-1/2*x^2*d^(1/2))^(1/2)*(1+1/2*x^2*d^(1/2))^(1/2)/(-d
*x^4+4)^(1/2)*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),(-1/2*d^(1/2)
)^(1/2)*2^(1/2)/d^(1/4))
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx=\int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-d*x^4 + 4)/(b*d*x^6 + a*d*x^4 - 4*b*x^2 - 4*a), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx=\int \frac{1}{(a+bx^2)\sqrt{-dx^4+4}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(-d*x**4+4)**(1/2),x)
[Out] Integral(1/((a + b*x**2)*sqrt(-d*x**4 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx=\int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx=\int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{4-d\,x^4}}\,dx$$

```
[In] int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)), x)
```

3.173 $\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 300

$$\begin{aligned}\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx &= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{4b^2+a^2d}\,x}{\sqrt{a}\sqrt{b}\sqrt{4+dx^4}}\right)}{2\sqrt{a}\sqrt{4b^2+a^2d}} \\ &\quad - \frac{\sqrt[4]{d}\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\left(2b-a\sqrt{d}\right)\sqrt{4+dx^4}} \\ &\quad + \frac{\left(2b+a\sqrt{d}\right)\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}a\left(2b-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{4+dx^4}}\end{aligned}$$

```
[Out] 1/2*arctan(x*(a^2*d+4*b^2)^(1/2)/a^(1/2)/b^(1/2)/(d*x^4+4)^(1/2))*b^(1/2)/a
^(1/2)/(a^2*d+4*b^2)^(1/2)-1/4*d^(1/4)*(cos(2*arctan(1/2*d^(1/4)*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*d^(1/4)*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*
d^(1/4)*x*2^(1/2))),1/2*2^(1/2))*(2+x^2*d^(1/2))*((d*x^4+4)/(2+x^2*d^(1/2))
^2)^(1/2)*2^(1/2)/(2*b-a*d^(1/2))/(d*x^4+4)^(1/2)+1/8*(cos(2*arctan(1/2*d^(
1/4)*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*d^(1/4)*x*2^(1/2)))*EllipticPi(s
in(2*arctan(1/2*d^(1/4)*x*2^(1/2))),-1/8*(2*b-a*d^(1/2))^2/a/b/d^(1/2),1/2*
2^(1/2))*(2*b+a*d^(1/2))*(2+x^2*d^(1/2))*((d*x^4+4)/(2+x^2*d^(1/2))^2)^(1/2
)/a/d^(1/4)*2^(1/2)/(2*b-a*d^(1/2))/(d*x^4+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1231, 226, 1721}

$$\begin{aligned}
&\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx \\
&= \frac{\sqrt{b}\arctan\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{dx^4+4}}\right)}{2\sqrt{a}\sqrt{a^2d+4b^2}} \\
&\quad - \frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)} \\
&\quad + \frac{\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\left(a\sqrt{d}+2b\right)\text{EllipticPi}\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}a\sqrt[4]{d}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}
\end{aligned}$$

```
[In] Int[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[4*b^2 + a^2*d]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + d*x^4])])
/(2*Sqrt[a]*Sqrt[4*b^2 + a^2*d]) - (d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x
^4)/(2 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(2*
Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt[4 + d*x^4]) + ((2*b + a*Sqrt[d])*(2 + Sqrt[d
]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*EllipticPi[-1/8*(2*b - a*Sqrt[
d])^2/(a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(4*Sqrt[2]*a*(2*b
 - a*Sqrt[d])*d^(1/4)*Sqrt[4 + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(2b)\int \frac{1+\frac{\sqrt{d}x^2}{2}}{(a+bx^2)\sqrt{4+dx^4}}\,dx}{2b-a\sqrt{d}} - \frac{\sqrt{d}\int \frac{1}{\sqrt{4+dx^4}}\,dx}{2b-a\sqrt{d}} \\
&= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{4b^2+a^2d}x}{\sqrt{a}\sqrt{b}\sqrt{4+dx^4}}\right)}{2\sqrt{a}\sqrt{4b^2+a^2d}} - \frac{\sqrt[4]{d}\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\left(2b-a\sqrt{d}\right)\sqrt{4+dx^4}} \\
&\quad + \frac{\left(2b+a\sqrt{d}\right)\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{4\sqrt{2}a\left(2b-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{4+dx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.22

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = -\frac{i\,\text{EllipticPi}\left(-\frac{2ib}{a\sqrt{d}}, i\text{arcsinh}\left(\frac{\sqrt{i\sqrt{d}}x}{\sqrt{2}}\right), -1\right)}{\sqrt{2}a\sqrt{i\sqrt{d}}}$$

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
[Out] ((-I)*EllipticPi[((-2*I)*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[I*Sqrt[d]]*x)/Sqrt
[2]], -1])/(Sqrt[2]*a*Sqrt[I*Sqrt[d]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{2}}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{d}}\,x}{2},\frac{2ib}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{d}}}\right)}{a\sqrt{i\sqrt{d}}\,\sqrt{d\,x^{4}+4}}$	86
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{2}}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{d}}\,x}{2},\frac{2ib}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{d}}}\right)}{a\sqrt{i\sqrt{d}}\,\sqrt{d\,x^{4}+4}}$	86

```
[In] int(1/(b*x^2+a)/(d*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(1/2*I*d^(1/2))^(1/2)*(1-1/2*I*d^(1/2)*x^2)^(1/2)*(1+1/2*I*d^(1/2)*x^2)
^(1/2)/(d*x^4+4)^(1/2)*EllipticPi((1/2*I*d^(1/2))^(1/2)*x,2*I/d^(1/2)*b/a,(
-1/2*I*d^(1/2))^(1/2)/(1/2*I*d^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx=\int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + 4)/(b*d*x^6 + a*d*x^4 + 4*b*x^2 + 4*a), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx=\int \frac{1}{(a+bx^2)\sqrt{dx^4+4}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(d*x**4 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^4+4}}\,dx$$

```
[In] int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)), x)
```

3.174 $\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \frac{a\sqrt{1-x^2}\sqrt{\frac{a(1+x^2)}{a+bx^2}}\,\text{EllipticPi}\left(\frac{b}{a+b},\arcsin\left(\frac{\sqrt{a+b}x}{\sqrt{a+bx^2}}\right),-\frac{a-b}{a+b}\right)}{\sqrt{a+b}\sqrt{1+x^2}\sqrt{\frac{a(1-x^2)}{a+bx^2}}}$$

```
[Out] a*EllipticPi(x*(a+b)^(1/2)/(b*x^2+a)^(1/2),b/(a+b),((-a+b)/(a+b))^(1/2))*(-
x^2+1)^(1/2)*(a*(x^2+1)/(b*x^2+a))^(1/2)/(a+b)^(1/2)/(x^2+1)^(1/2)/(a*(-x^2
+1)/(b*x^2+a))^(1/2)
```

Rubi [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
```

```
[Out] Defer[Int][Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx=\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
[Out] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\sqrt{-x^{4}+1}}dx$$

```
[In] int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
[Out] int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx=\int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 1)*sqrt(b*x^2 + a)/(x^4 - 1), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx=\int \frac{\sqrt{a+bx^2}}{\sqrt{-(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{1-x^4}}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

3.175 $\int (c+ex^2)^q (a+bx^4)^p \,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int (c+ex^2)^q (a+bx^4)^p \,dx = \text{Int}\left((c+ex^2)^q (a+bx^4)^p, x\right)$$

```
[Out] Unintegrable((e*x^2+c)^q*(b*x^4+a)^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+ex^2)^q (a+bx^4)^p \,dx = \int (c+ex^2)^q (a+bx^4)^p \,dx$$

```
[In] Int[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

```
[Out] Defer[Int][(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Rubi steps

$$\text{integral} = \int (c+ex^2)^q (a+bx^4)^p \,dx$$

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \,dx=\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \,dx$$

```
[In] Integrate[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

```
[Out] Integrate[(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(e\,x^{2}+c\right)^{q} \left(b\,x^{4}+a\right)^{p}dx$$

```
[In] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \,dx=\int { {\left(bx^{4} + a\right)}^{p} {\left(ex^{2} + c\right)}^{q}\,dx }$$

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+c)**q*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(bx^4+a\right)^p \left(ex^2+c\right)^q \, dx$$

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(bx^4+a\right)^p \left(ex^2+c\right)^q \, dx$$

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Mupad [N/A]

Not integrable

Time = 13.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(b\,x^4+a\right)^p \, \left(e\,x^2+c\right)^q \, dx$$

```
[In] int((a + b*x^4)^p*(c + e*x^2)^q,x)
```

```
[Out] int((a + b*x^4)^p*(c + e*x^2)^q, x)
```

3.176 $\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 204

$$\begin{aligned}
&\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx \\
&= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)} + c^3x\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) \\
&\quad - \frac{e\left(ae^2-bc^2(7+4p)\right)x^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)}{b(7+4p)} \\
&\quad + \frac{3}{5}ce^2x^5\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)
\end{aligned}$$

```
[Out] e^3*x^3*(b*x^4+a)^(p+1)/b/(7+4*p)+c^3*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/
4],-b*x^4/a)/((1+b*x^4/a)^p)-e*(a*e^2-b*c^2*(7+4*p))*x^3*(b*x^4+a)^p*hyperg
eom([3/4, -p],[7/4],-b*x^4/a)/b/(7+4*p)/((1+b*x^4/a)^p)+3/5*c*e^2*x^5*(b*x^
4+a)^p*hypergeom([5/4, -p],[9/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{1221, 1907, 252, 251, 372, 371\}$

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = c^3x\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + ex^3\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(c^2 - \frac{ae^2}{4bp+7b}\right) \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right) + \frac{3}{5}ce^2x^5\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right) + \frac{e^3x^3\left(a+bx^4\right)^{p+1}}{b(4p+7)}$$

```
[In] Int[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (e^3*x^3*(a + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (c^3*x*(a + b*x^4)^p*Hypergeo
metric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (e*(c^2 - (a*e^2
)/(7*b + 4*b*p))*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4
)/a)])/(1 + (b*x^4)/a)^p + (3*c*e^2*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4
, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
```

```
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)} \\
&\quad + \frac{\int (a+bx^4)^p \left(bc^3(7+4p) - 3e\left(ae^2 - bc^2(7+4p)\right)x^2 + 3bce^2(7+4p)x^4\right) dx}{b(7+4p)} \\
&= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)} \\
&\quad + \frac{\int \left(bc^3(7+4p)\left(a+bx^4\right)^p + 3e\left(-ae^2 + bc^2(7+4p)\right)x^2(a+bx^4)^p + 3bce^2(7+4p)x^4(a+bx^4)^p\right)\, d}{b(7+4p)} \\
&= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)} + c^3\int \left(a+bx^4\right)^p\, dx + \left(3ce^2\right)\int x^4\left(a+bx^4\right)^p\, dx \\
&\quad + \left(3e\left(c^2 - \frac{ae^2}{7b+4bp}\right)\right)\int x^2\left(a+bx^4\right)^p\, dx \\
&= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)} + \left(c^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p dx \\
&\quad + \left(3ce^2\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^4}{a}\right)^p dx \\
&\quad + \left(3e\left(c^2 - \frac{ae^2}{7b+4bp}\right)\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^4}{a}\right)^p dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + c^3x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)\\
&\quad + e\left(c^2-\frac{ae^2}{7b+4bp}\right)x^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)\\
&\quad + \frac{3}{5}ce^2x^5\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$
\begin{aligned}
&\int \left(c+ex^2\right)^3\left(a+bx^4\right)^p\,dx\\
&= \frac{1}{35}x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(35c^3\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)\right.\\
&\qquad + ex^2\left(35c^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)\right.\\
&\qquad \left.\left. + ex^2\left(21c\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)+5ex^2\,\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)\right)\right)\right)
\end{aligned}
$$

```
[In] Integrate[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(35*c^3*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*
x^2*(35*c^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + e*x^2*(21*c*Hyp
ergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*e*x^2*Hypergeometric2F1[7/4,
 -p, 11/4, -((b*x^4)/a)]))))/(35*(1 + (b*x^4)/a)^p)
```

Maple [F]

$$\int \left(e\,x^2+c\right)^3\left(b\,x^4+a\right)^p\,dx$$

```
[In] int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 60.59 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.82

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \frac{a^p c^3 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3a^p c^2 e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3a^p c e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{a^p e^3 x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{7}{4}, -p \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate((e*x**2+c)**3*(b*x**4+a)**p,x)
```

```
[Out] a**p*c**3*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4)) + 3*a**p*c**2*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + 3*a**p*c*e**2*x**5*gamma(5/4)*hyper(
(5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a**p*e**3*x**
7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(1
1/4))
```

Maxima [F]

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="maxima")
[Out] integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(b\,x^4+a\right)^p \, \left(e\,x^2+c\right)^3 \, dx$$

```
[In] int((a + b*x^4)^p*(c + e*x^2)^3,x)
[Out] int((a + b*x^4)^p*(c + e*x^2)^3, x)
```

3.177 $\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\begin{aligned}
&\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx \\
&= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} \\
&\quad - \frac{\left(ae^2-bc^2(5+4p)\right)x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)}{b(5+4p)} \\
&\quad + \frac{2}{3}cex^3\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)
\end{aligned}$$

```
[Out] e^2*x*(b*x^4+a)^(p+1)/b/(5+4*p)-(a*e^2-b*c^2*(5+4*p))*x*(b*x^4+a)^p*hyperge
om([1/4, -p],[5/4],-b*x^4/a)/b/(5+4*p)/((1+b*x^4/a)^p)+2/3*c*e*x^3*(b*x^4+a
)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

$= \{1221, 1218, 252, 251, 372, 371\}$

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx = x\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(c^2 - \frac{ae^2}{4bp+5b}\right) \text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -\frac{bx^4}{a}\right) + \frac{2}{3}cex^3\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4}, -p, \frac{7}{4}, -\frac{bx^4}{a}\right) + \frac{e^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

```
[In] Int[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b + 4*b*p))*
x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)
/a)^p + (2*c*e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/
a)])/(3*(1 + (b*x^4)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2+bc^2(5+4p)+2bce(5+4p)x^2\right)\left(a+bx^4\right)^p\,dx}{b(5+4p)} \\
&= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2\left(1-\frac{bc^2(5+4p)}{ae^2}\right)(a+bx^4)^p+2bce(5+4p)x^2(a+bx^4)^p\right)dx}{b(5+4p)} \\
&= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + (2ce)\int x^2\left(a+bx^4\right)^p\,dx - \left(-c^2+\frac{ae^2}{5b+4bp}\right)\int \left(a+bx^4\right)^p\,dx \\
&= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + \left(2ce\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^4}{a}\right)^p\,dx \\
&\quad - \left(\left(-c^2+\frac{ae^2}{5b+4bp}\right)\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p\,dx \\
&= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + \left(c^2-\frac{ae^2}{5b+4bp}\right)x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right) \\
&\quad + \frac{2}{3}cex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx = \frac{1}{15}x\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(15c^2 \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+ex^2\left(10c \,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+3ex^2 \,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)\right)\right)$$

```
[In] Integrate[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(15*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*
x^2*(10*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + 3*e*x^2*Hypergeom
etric2F1[5/4, -p, 9/4, -((b*x^4)/a)])))/(15*(1 + (b*x^4)/a)^p)
```

Maple [F]

$$\int \left(e\,x^2+c\right)^2 \left(b\,x^4+a\right)^p dx$$

```
[In] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^2 \left(bx^4+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 32.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.79

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx = \frac{a^p c^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, -p\\ \frac{5}{4}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p cex^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, -p\\ \frac{7}{4}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{a^p e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, -p\\ \frac{9}{4}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate((e*x**2+c)**2*(b*x**4+a)**p,x)
```

```
[Out] a**p*c**2*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4)) + a**p*c*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(2*gamma(7/4)) + a**p*e**2*x**5*gamma(5/4)*hyper((5/4, -p)
, (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^2 \left(bx^4+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^2 \left(bx^4+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^2\left(a+bx^4\right)^p\,dx=\int \left(b\,x^4+a\right)^p\,\left(e\,x^2+c\right)^2\,dx$$

```
[In] int((a + b*x^4)^p*(c + e*x^2)^2,x)
[Out] int((a + b*x^4)^p*(c + e*x^2)^2, x)
```

3.178 $\int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 96

$$\int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx = cx\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

```
[Out] c*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/3*e*x
^3*(b*x^4+a)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1218, 252, 251, 372, 371}

$$\int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx = cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

```
[In] Int[(c + e*x^2)*(a + b*x^4)^p,x]
```

```
[Out] (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x
^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a
)])/(3*(1 + (b*x^4)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(c\left(a+bx^4\right)^p+ex^2\left(a+bx^4\right)^p\right)\,dx\\
&= c\int \left(a+bx^4\right)^p\,dx+e\int x^2\left(a+bx^4\right)^p\,dx
\end{aligned}
$$

$$= \left(c\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int\left(1+\frac{bx^4}{a}\right)^p dx + \left(e\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^4}{a}\right)^p dx$$

$$= cx\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right) + \frac{1}{3}ex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int\left(c+ex^2\right)\left(a+bx^4\right)^p dx = \frac{1}{3}x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(3c\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+ex^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)\right)$$

```
[In] Integrate[(c + e*x^2)*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*x^2
*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]))/(3*(1 + (b*x^4)/a)^p)
```

Maple [F]

$$\int\left(e\,x^2+c\right)\left(b\,x^4+a\right)^p dx$$

```
[In] int((e*x^2+c)*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int \left(c+ex^2\right) \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)\left(bx^4+a\right)^p dx$$

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="fricas")
[Out] integral((e*x^2 + c)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int \left(c+ex^2\right) \left(a+bx^4\right)^p \, dx = \frac{a^p cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((e*x**2+c)*(b*x**4+a)**p,x)
[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(5/4)) + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int \left(c+ex^2\right) \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)\left(bx^4+a\right)^p dx$$

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="maxima")
[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(c+ex^2\right) \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)\left(bx^4+a\right)^p dx$$

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+ex^2) (a+bx^4)^p \, dx = \int (b\,x^4+a)^p \,(e\,x^2+c) \, dx$$

```
[In] int((a + b*x^4)^p*(c + e*x^2),x)
[Out] int((a + b*x^4)^p*(c + e*x^2), x)
```

3.179 $\int (a+bx^4)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

```
[Out] x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {252, 251}

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

```
[In] Int[(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4
)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int\left(1+\frac{bx^4}{a}\right)^p dx\\ &= x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int(a+bx^4)^p\,dx = x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

```
[In] Integrate[(a + b*x^4)^p,x]
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4
)/a)^p
```

Maple [F]

$$\int(b\,x^4+a)^p\,dx$$

```
[In] int((b*x^4+a)^p,x)
[Out] int((b*x^4+a)^p,x)
```

Fricas [F]

$$\int(a+bx^4)^p\,dx = \int(bx^4+a)^p\,dx$$

```
[In] integrate((b*x^4+a)^p,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^4\right)^p\,dx=\frac{a^p x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((b*x**4+a)**p,x)
```

```
[Out] a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gam
ma(5/4))
```

Maxima [F]

$$\int \left(a+bx^4\right)^p\,dx=\int \left(bx^4+a\right)^p\,dx$$

```
[In] integrate((b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^4\right)^p\,dx=\int \left(bx^4+a\right)^p\,dx$$

```
[In] integrate((b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(a+bx^4\right)^p\,dx=\frac{x\,\left(b\,x^4+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^p}$$

```
[In] int((a + b*x^4)^p,x)
```

```
[Out] (x*(a + b*x^4)^p*hypergeom([1/4, -p], 5/4, -(b*x^4)/a))/((b*x^4)/a + 1)^p
```

3.180 $\int \frac{(a+bx^4)^p}{c+ex^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{(a+bx^4)^p}{c+ex^2} \, dx = \frac{x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c} - \frac{ex^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

```
[Out] x*(b*x^4+a)^p*AppellF1(1/4,1,-p,5/4,e^2*x^4/c^2,-b*x^4/a)/c/((1+b*x^4/a)^p)
-1/3*e*x^3*(b*x^4+a)^p*AppellF1(3/4,1,-p,7/4,e^2*x^4/c^2,-b*x^4/a)/c^2/((1+
b*x^4/a)^p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1254, 441, 440, 525, 524}

$$\int \frac{(a+bx^4)^p}{c+ex^2} \, dx = \frac{x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c} - \frac{ex^3(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \operatorname{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

```
[In] Int[(a + b*x^4)^p/(c + e*x^2),x]
```

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/4, -((b*x
^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{c(a+bx^4)^p}{c^2-e^2x^4}+\frac{ex^2(a+bx^4)^p}{-c^2+e^2x^4}\right)dx\\ &= c\int \frac{(a+bx^4)^p}{c^2-e^2x^4}\,dx+e\int \frac{x^2(a+bx^4)^p}{-c^2+e^2x^4}\,dx\end{aligned}$$

$$= \left(c\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^4}{a}\right)^p}{c^2-e^2x^4}\,dx + \left(e\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int \frac{x^2\left(1+\frac{bx^4}{a}\right)^p}{-c^2+e^2x^4}\,dx$$

$$= \frac{x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c} - \frac{ex^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

Mathematica [F]

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \int \frac{(a+bx^4)^p}{c+ex^2}\,dx$$

```
[In] Integrate[(a + b*x^4)^p/(c + e*x^2),x]
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2), x]
```

Maple [F]

$$\int \frac{\left(b\,x^4+a\right)^p}{e\,x^2+c}dx$$

```
[In] int((b*x^4+a)^p/(e*x^2+c),x)
[Out] int((b*x^4+a)^p/(e*x^2+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \int \frac{(bx^4+a)^p}{ex^2+c}\,dx$$

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p/(e*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**4+a)**p/(e*x**2+c),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \int \frac{(bx^4+a)^p}{ex^2+c}\,dx$$

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \int \frac{(bx^4+a)^p}{ex^2+c}\,dx$$

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \int \frac{(b\,x^4+a)^p}{e\,x^2+c}\,dx$$

```
[In] int((a + b*x^4)^p/(c + e*x^2),x)
[Out] int((a + b*x^4)^p/(c + e*x^2), x)
```

3.181 $\int \frac{(a+bx^4)^p}{(c+ex^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2} \, dx = \frac{x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2} - \frac{2ex^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3} + \frac{e^2x^5(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{5c^4}$$

```
[Out] x*(b*x^4+a)^p*AppellF1(1/4,2,-p,5/4,e^2*x^4/c^2,-b*x^4/a)/c^2/((1+b*x^4/a)^
p)-2/3*e*x^3*(b*x^4+a)^p*AppellF1(3/4,2,-p,7/4,e^2*x^4/c^2,-b*x^4/a)/c^3/((
1+b*x^4/a)^p)+1/5*e^2*x^5*(b*x^4+a)^p*AppellF1(5/4,2,-p,9/4,e^2*x^4/c^2,-b*
x^4/a)/c^4/((1+b*x^4/a)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

= {1254, 441, 440, 525, 524}

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx = \frac{x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2} + \frac{e^2x^5(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{5c^4} - \frac{2ex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3}$$

```
[In] Int[(a + b*x^4)^p/(c + e*x^2)^2,x]
```

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 2, 7/4, -(
(b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^4)/a)^p) + (e^2*x^5*(a + b*x^4
)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 + (b*
x^4)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c^2(a+bx^4)^p}{(c^2-e^2x^4)^2}-\frac{2cex^2(a+bx^4)^p}{(c^2-e^2x^4)^2}+\frac{e^2x^4(a+bx^4)^p}{(-c^2+e^2x^4)^2}\right)dx\\
&= c^2\int \frac{(a+bx^4)^p}{(c^2-e^2x^4)^2}\,dx-(2ce)\int \frac{x^2(a+bx^4)^p}{(c^2-e^2x^4)^2}\,dx+e^2\int \frac{x^4(a+bx^4)^p}{(-c^2+e^2x^4)^2}\,dx\\
&= \left(c^2(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{(c^2-e^2x^4)^2}\,dx\\
&\quad-\left(2ce(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^4}{a}\right)^p}{(c^2-e^2x^4)^2}\,dx\\
&\quad+\left(e^2(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{x^4\left(1+\frac{bx^4}{a}\right)^p}{(-c^2+e^2x^4)^2}\,dx\\
&= \frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}\\
&\quad-\frac{2ex^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{3}{4};-p,2;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3}\\
&\quad+\frac{e^2x^5(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{5}{4};-p,2;\frac{9}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{5c^4}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx$$

```
[In] Integrate[(a + b*x^4)^p/(c + e*x^2)^2,x]
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2)^2, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+a\right)^{p}}{\left(e\,x^{2}+c\right)^{2}}dx$$

```
[In] int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

```
[Out] int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**4+a)**p/(e*x**2+c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx = \int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx = \int \frac{(b\,x^4+a)^p}{(e\,x^2+c)^2}\,dx$$

```
[In] int((a + b*x^4)^p/(c + e*x^2)^2,x)
```

```
[Out] int((a + b*x^4)^p/(c + e*x^2)^2, x)
```

3.182 $\int (1-x^2)^3 (1+bx^4)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\begin{aligned}\int (1-x^2)^3 (1+bx^4)^p \, dx = & -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) \\ & + \frac{(1-b(7+4p))x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)}{b(7+4p)} \\ & + \frac{3}{5}x^5\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right)\end{aligned}$$

```
[Out] -x^3*(b*x^4+1)^(p+1)/b/(7+4*p)+x*hypergeom([1/4, -p],[5/4],-b*x^4)+(1-b*(7+
4*p))*x^3*hypergeom([3/4, -p],[7/4],-b*x^4)/b/(7+4*p)+3/5*x^5*hypergeom([5/
4, -p],[9/4],-b*x^4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1221, 1907, 251, 371}

$$\begin{aligned}\int (1-x^2)^3 (1+bx^4)^p \, dx = & \, x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) \\ & + \frac{3}{5}x^5\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right) \\ & - x^3\left(1-\frac{1}{4bp+7b}\right)\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right) \\ & - \frac{x^3(bx^4+1)^{p+1}}{b(4p+7)}\end{aligned}$$

```
[In] Int[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] -((x^3*(1 + b*x^4)^(1 + p))/(b*(7 + 4*p))) + x*Hypergeometric2F1[1/4, -p, 5
/4, -(b*x^4)] - (1 - (7*b + 4*b*p)^(-1))*x^3*Hypergeometric2F1[3/4, -p, 7/4
, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
&\text{integral}\\
&= -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + \frac{\int (1+bx^4)^p\left(b(7+4p)+3(1-b(7+4p))x^2+3b(7+4p)x^4\right)\,dx}{b(7+4p)}\\
&= -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)}\\
&\quad + \frac{\int \left(b(7+4p)\left(1+bx^4\right)^p+3(1-b(7+4p))x^2(1+bx^4)^p+3b(7+4p)x^4(1+bx^4)^p\right)\,dx}{b(7+4p)}
\end{aligned}
$$

$$\begin{aligned}&=-\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)}+3\int x^4(1+bx^4)^p\,dx\\&\quad-\left(3\left(1-\frac{1}{7b+4bp}\right)\right)\int x^2(1+bx^4)^p\,dx+\int(1+bx^4)^p\,dx\\&=-\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)}+x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)\\&\quad-\left(1-\frac{1}{7b+4bp}\right)x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+\frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\begin{aligned}\int(1-x^2)^3(1+bx^4)^p\,dx&=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)\\&\quad-x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)\\&\quad+\frac{3}{5}x^5\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right)\\&\quad-\frac{1}{7}x^7\,\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-bx^4\right)\end{aligned}$$

```
[In] Integrate[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - x^3*Hypergeometric2F1[3/4, -p
, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5 - (x
^7*Hypergeometric2F1[7/4, -p, 11/4, -(b*x^4)])/7
```

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

method	result	size
meijerg	$-\frac{x^{7}{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-b\,x^{4}\right)}{7}+\frac{3x^{5}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-b\,x^{4}\right)}{5}-x^{3}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-b\,x^{4}\right)+x{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-b\,x^{4}\right)$	75

```
[In] int((-x^2+1)^3*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*x^7*hypergeom([7/4,-p],[11/4],-b*x^4)+3/5*x^5*hypergeom([5/4,-p],[9/4]
,-b*x^4)-x^3*hypergeom([3/4,-p],[7/4],-b*x^4)+x*hypergeom([1/4,-p],[5/4],-b
*x^4)
```

Fricas [F]

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p\,dx=\int -\left(x^2-1\right)^3\left(bx^4+1\right)^p\,dx$$

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^6 - 3*x^4 + 3*x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 50.67 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p\,dx=-\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{4},-p\\ \frac{11}{4}\end{matrix}\right|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{3x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\right|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{3x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\right|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\right|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((-x**2+1)**3*(b*x**4+1)**p,x)
```

```
[Out] -x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi))/(4*gamma
(11/4)) + 3*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)
)/(4*gamma(9/4)) - 3*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_po
lar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*ex
p_polar(I*pi))/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p\,dx=\int -\left(x^2-1\right)^3\left(bx^4+1\right)^p\,dx$$

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

Giac [F]

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = \int -\left(x^2-1\right)^3 \left(bx^4+1\right)^p \, dx$$

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="giac")
[Out] integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = -\int \left(x^2-1\right)^3 \left(b\,x^4+1\right)^p \, dx$$

```
[In] int(-(x^2 - 1)^3*(b*x^4 + 1)^p,x)
[Out] -int((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

3.183 $\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\begin{aligned}\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = {} & \frac{x\left(1+bx^4\right)^{1+p}}{b(5+4p)} \\ & - \frac{(1-b(5+4p))x \operatorname{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)}{b(5+4p)} \\ & - \frac{2}{3}x^3 \operatorname{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)\end{aligned}$$

```
[Out] x*(b*x^4+1)^(p+1)/b/(5+4*p)-(1-b*(5+4*p))*x*hypergeom([1/4, -p],[5/4],-b*x^
4)/b/(5+4*p)-2/3*x^3*hypergeom([3/4, -p],[7/4],-b*x^4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1221, 1218, 251, 371}

$$\begin{aligned}\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = {} & x\left(1-\frac{1}{4bp+5b}\right) \operatorname{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) \\ & - \frac{2}{3}x^3 \operatorname{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right) + \frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}\end{aligned}$$

```
[In] Int[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] (x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + (1 - (5*b + 4*b*p)^(-1))*x*Hypergeo
metric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4, -p, 7/4,
-(b*x^4)])/3
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \frac{\int (-1+b(5+4p)-2b(5+4p)x^2)(1+bx^4)^p\,dx}{b(5+4p)} \\
&= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \frac{\int \left((-1+b(5+4p))(1+bx^4)^p - 2b(5+4p)x^2(1+bx^4)^p\right)dx}{b(5+4p)} \\
&= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} - 2\int x^2(1+bx^4)^p\,dx + \left(1-\frac{1}{5b+4bp}\right)\int (1+bx^4)^p\,dx \\
&= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \left(1-\frac{1}{5b+4bp}\right)x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) - \frac{2}{3}x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right) + \frac{1}{5}x^5 \,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right)$$

```
[In] Integrate[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4,
 -p, 7/4, -(b*x^4)])/3 + (x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.65

method	result	size
meijerg	$\frac{x^5 {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-b\,x^4\right)}{5}-\frac{2x^3 {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-b\,x^4\right)}{3}+x {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-b\,x^4\right)$	56

```
[In] int((-x^2+1)^2*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*hypergeom([5/4,-p],[9/4],-b*x^4)-2/3*x^3*hypergeom([3/4,-p],[7/4],-
b*x^4)+x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Fricas [F]

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = \int (x^2-1)^2 (bx^4+1)^p \, dx$$

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4 - 2*x^2 + 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.98 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((-x**2+1)**2*(b*x**4+1)**p,x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi))/(4*gamma(9
/4)) - x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(2*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))
/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \int \left(x^2-1\right)^2 \left(bx^4+1\right)^p \, dx$$

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

Giac [F]

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \int \left(x^2-1\right)^2 \left(bx^4+1\right)^p \, dx$$

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx = \int \left(x^2-1\right)^2 \left(b\,x^4+1\right)^p \, dx$$

```
[In] int((x^2 - 1)^2*(b*x^4 + 1)^p,x)
[Out] int((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

3.184 $\int (1-x^2) (1+bx^4)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int (1-x^2) (1+bx^4)^p \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) - \frac{1}{3}x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)$$

```
[Out] x*hypergeom([1/4, -p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4, -p],[7/4],-b*x^
4)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1218, 251, 371}

$$\int (1-x^2) (1+bx^4)^p \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) - \frac{1}{3}x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)$$

```
[In] Int[(1 - x^2)*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\left(1+bx^4\right)^p - x^2\left(1+bx^4\right)^p\right)\,dx\\ &= \int \left(1+bx^4\right)^p\,dx - \int x^2\left(1+bx^4\right)^p\,dx\\ &= x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(1-x^2\right)\left(1+bx^4\right)^p\,dx &= x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)\\ &\quad - \frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)\end{aligned}$$

```
[In] Integrate[(1 - x^2)*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
meijerg	$x{}_{2}F_{1}\left(\frac{1}{4},-p;\frac{5}{4};-b\,x^{4}\right)-\frac{x^{3}{}_{2}F_{1}\left(\frac{3}{4},-p;\frac{7}{4};-b\,x^{4}\right)}{3}$	37

```
[In] int((-x^2+1)*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4,-p],[7/4],-b*x^4)
```

Fricas [F]

$$\int \left(1-x^{2}\right)\left(1+bx^{4}\right)^{p}\,dx=\int -\left(x^{2}-1\right)\left(bx^{4}+1\right)^{p}\,dx$$

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.87 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \left(1-x^{2}\right)\left(1+bx^{4}\right)^{p}\,dx=-\frac{x^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},-p\\\frac{7}{4}\end{matrix}\middle|bx^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|bx^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((-x**2+1)*(b*x**4+1)**p,x)
```

```
[Out] -x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(4*gamma(
7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*ga
mma(5/4))
```

Maxima [F]

$$\int (1-x^2) (1+bx^4)^p \, dx = \int -(x^2-1) (bx^4+1)^p \, dx$$

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="maxima")
[Out] -integrate((x^2 - 1)*(b*x^4 + 1)^p, x)
```

Giac [F]

$$\int (1-x^2) (1+bx^4)^p \, dx = \int -(x^2-1) (bx^4+1)^p \, dx$$

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="giac")
[Out] integrate(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-x^2) (1+bx^4)^p \, dx = -\int (x^2-1) \, (b\,x^4+1)^p \, dx$$

```
[In] int(-(x^2 - 1)*(b*x^4 + 1)^p,x)
[Out] -int((x^2 - 1)*(b*x^4 + 1)^p, x)
```

3.185 $\int (1+bx^4)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int (1+bx^4)^p \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -bx^4\right)$$

```
[Out] x*hypergeom([1/4, -p],[5/4],-b*x^4)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {251}

$$\int (1+bx^4)^p \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -bx^4\right)$$

```
[In] Int[(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\text{integral} = x \, {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -bx^4\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(1+bx^4\right)^p\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)$$

```
[In] Integrate[(1 + b*x^4)^p,x]
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
meijerg	$x{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-b\,x^4\right)$	17

```
[In] int((b*x^4+1)^p,x,method=_RETURNVERBOSE)
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Fricas [F]

$$\int \left(1+bx^4\right)^p\,dx = \int \left(bx^4+1\right)^p\,dx$$

```
[In] integrate((b*x^4+1)^p,x, algorithm="fricas")
[Out] integral((b*x^4 + 1)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \left(1+bx^4\right)^p\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

```
[In] integrate((b*x**4+1)**p,x)
[Out] x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4)
)
```

Maxima [F]

$$\int \left(1+bx^4\right)^p \, dx = \int \left(bx^4+1\right)^p \, dx$$

```
[In] integrate((b*x^4+1)^p,x, algorithm="maxima")
[Out] integrate((b*x^4 + 1)^p, x)
```

Giac [F]

$$\int \left(1+bx^4\right)^p \, dx = \int \left(bx^4+1\right)^p \, dx$$

```
[In] integrate((b*x^4+1)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + 1)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \left(1+bx^4\right)^p \, dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -b\,x^4\right)$$

```
[In] int((b*x^4 + 1)^p,x)
[Out] x*hypergeom([1/4, -p], 5/4, -b*x^4)
```

3.186 $\int \frac{(1+bx^4)^p}{1-x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = x\,\text{AppellF1}\left(\frac{1}{4},1,-p,\frac{5}{4},x^4,-bx^4\right) + \frac{1}{3}x^3\,\text{AppellF1}\left(\frac{3}{4},1,-p,\frac{7}{4},x^4,-bx^4\right)$$

```
[Out] x*AppellF1(1/4,1,-p,5/4,x^4,-b*x^4)+1/3*x^3*AppellF1(3/4,1,-p,7/4,x^4,-b*x^
4)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1254, 440, 524}

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = x\,\text{AppellF1}\left(\frac{1}{4},1,-p,\frac{5}{4},x^4,-bx^4\right) + \frac{1}{3}x^3\,\text{AppellF1}\left(\frac{3}{4},1,-p,\frac{7}{4},x^4,-bx^4\right)$$

```
[In] Int[(1 + b*x^4)^p/(1 - x^2),x]
```

```
[Out] x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1, -p, 7/4,
 x^4, -(b*x^4)])/3
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(1+bx^4)^p}{1-x^4} - \frac{x^2(1+bx^4)^p}{-1+x^4}\right) dx\\ &= \int \frac{(1+bx^4)^p}{1-x^4}\,dx - \int \frac{x^2(1+bx^4)^p}{-1+x^4}\,dx\\ &= xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right) + \frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)\end{aligned}$$

Mathematica [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = \int \frac{(1+bx^4)^p}{1-x^2}\,dx$$

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2),x]
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2), x]
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+1\right)^{p}}{-x^{2}+1}dx$$

```
[In] int((b*x^4+1)^p/(-x^2+1),x)
```

```
[Out] int((b*x^4+1)^p/(-x^2+1),x)
```

Fricas [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = \int -\frac{(bx^4+1)^p}{x^2-1}\,dx$$

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**4+1)**p/(-x**2+1),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = \int -\frac{(bx^4+1)^p}{x^2-1}\,dx$$

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1), x)
```

Giac [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx=\int -\frac{(bx^4+1)^p}{x^2-1}\,dx$$

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="giac")
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx=-\int \frac{(b\,x^4+1)^p}{x^2-1}\,dx$$

```
[In] int(-(b*x^4 + 1)^p/(x^2 - 1),x)
[Out] -int((b*x^4 + 1)^p/(x^2 - 1), x)
```

3.187 $\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\begin{aligned}\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx &= x\operatorname{AppellF1}\left(\frac{1}{4},2,-p,\frac{5}{4},x^4,-bx^4\right)\\ &\quad+\frac{2}{3}x^3\operatorname{AppellF1}\left(\frac{3}{4},2,-p,\frac{7}{4},x^4,-bx^4\right)\\ &\quad+\frac{1}{5}x^5\operatorname{AppellF1}\left(\frac{5}{4},2,-p,\frac{9}{4},x^4,-bx^4\right)\end{aligned}$$

```
[Out] x*AppellF1(1/4,2,-p,5/4,x^4,-b*x^4)+2/3*x^3*AppellF1(3/4,2,-p,7/4,x^4,-b*x^
4)+1/5*x^5*AppellF1(5/4,2,-p,9/4,x^4,-b*x^4)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1254, 440, 524}

$$\begin{aligned}\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx &= x\operatorname{AppellF1}\left(\frac{1}{4},2,-p,\frac{5}{4},x^4,-bx^4\right)\\ &\quad+\frac{1}{5}x^5\operatorname{AppellF1}\left(\frac{5}{4},2,-p,\frac{9}{4},x^4,-bx^4\right)\\ &\quad+\frac{2}{3}x^3\operatorname{AppellF1}\left(\frac{3}{4},2,-p,\frac{7}{4},x^4,-bx^4\right)\end{aligned}$$

```
[In] Int[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

```
[Out] x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4, 2, -p, 7/
4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x^4, -(b*x^4)])/5
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{(1+bx^4)^p}{(-1+x^4)^2}+\frac{2x^2(1+bx^4)^p}{(-1+x^4)^2}+\frac{x^4(1+bx^4)^p}{(-1+x^4)^2}\right)dx\\ &= 2\int \frac{x^2(1+bx^4)^p}{(-1+x^4)^2}\,dx+\int \frac{(1+bx^4)^p}{(-1+x^4)^2}\,dx+\int \frac{x^4(1+bx^4)^p}{(-1+x^4)^2}\,dx\\ &= xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)\\ &\quad+\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)\end{aligned}$$

Mathematica [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx=\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx$$

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^2,x]
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^2, x]
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+1\right)^{p}}{\left(-x^{2}+1\right)^{2}}dx$$

```
[In] int((b*x^4+1)^p/(-x^2+1)^2,x)
[Out] int((b*x^4+1)^p/(-x^2+1)^2,x)
```

Fricas [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="fricas")
[Out] integral((b*x^4 + 1)^p/(x^4 - 2*x^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**4+1)**p/(-x**2+1)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

Giac [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(b\,x^4+1)^p}{(x^2-1)^2}\,dx$$

```
[In] int((b*x^4 + 1)^p/(x^2 - 1)^2,x)
```

```
[Out] int((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

3.188 $\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\begin{aligned}\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = {} & x\,\text{AppellF1}\left(\frac{1}{4},3,-p,\frac{5}{4},x^4,-bx^4\right) \\ & + x^3\,\text{AppellF1}\left(\frac{3}{4},3,-p,\frac{7}{4},x^4,-bx^4\right) \\ & + \frac{3}{5}x^5\,\text{AppellF1}\left(\frac{5}{4},3,-p,\frac{9}{4},x^4,-bx^4\right) \\ & + \frac{1}{7}x^7\,\text{AppellF1}\left(\frac{7}{4},3,-p,\frac{11}{4},x^4,-bx^4\right)\end{aligned}$$

```
[Out] x*AppellF1(1/4,3,-p,5/4,x^4,-b*x^4)+x^3*AppellF1(3/4,3,-p,7/4,x^4,-b*x^4)+3
/5*x^5*AppellF1(5/4,3,-p,9/4,x^4,-b*x^4)+1/7*x^7*AppellF1(7/4,3,-p,11/4,x^4
,-b*x^4)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

= {1254, 440, 524}

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = x\operatorname{AppellF1}\left(\frac{1}{4},3,-p,\frac{5}{4},x^4,-bx^4\right) + \frac{1}{7}x^7\operatorname{AppellF1}\left(\frac{7}{4},3,-p,\frac{11}{4},x^4,-bx^4\right) + \frac{3}{5}x^5\operatorname{AppellF1}\left(\frac{5}{4},3,-p,\frac{9}{4},x^4,-bx^4\right) + x^3\operatorname{AppellF1}\left(\frac{3}{4},3,-p,\frac{7}{4},x^4,-bx^4\right)$$

```
[In] Int[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

```
[Out] x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3, -p, 7/4,
x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4, -(b*x^4)])/5 + (x^7*
AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{(1+bx^4)^p}{(-1+x^4)^3} - \frac{3x^2(1+bx^4)^p}{(-1+x^4)^3} - \frac{3x^4(1+bx^4)^p}{(-1+x^4)^3} - \frac{x^6(1+bx^4)^p}{(-1+x^4)^3}\right) dx$$

$$= -\left(3\int \frac{x^2(1+bx^4)^p}{(-1+x^4)^3}\,dx\right) - 3\int \frac{x^4(1+bx^4)^p}{(-1+x^4)^3}\,dx - \int \frac{(1+bx^4)^p}{(-1+x^4)^3}\,dx - \int \frac{x^6(1+bx^4)^p}{(-1+x^4)^3}\,dx$$

$$
\begin{aligned}
&= xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right)\\
&\quad+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right)+\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right)
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx=\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx$$

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^3,x]
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^3, x]
```

Maple [F]

$$\int \frac{(b\,x^4+1)^p}{(-x^2+1)^3}dx$$

```
[In] int((b*x^4+1)^p/(-x^2+1)^3,x)
[Out] int((b*x^4+1)^p/(-x^2+1)^3,x)
```

Fricas [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx=\int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="fricas")
[Out] integral(-(b*x^4 + 1)^p/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**4+1)**p/(-x**2+1)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx=\int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="maxima")
[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

Giac [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx=\int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="giac")
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx=\int -\frac{(b\,x^4+1)^p}{(x^2-1)^3}\,dx$$

```
[In] int(-(b*x^4 + 1)^p/(x^2 - 1)^3,x)
[Out] int(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

3.189 $\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx=-7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

```
[Out] -7*d^2*x-4/3*d*e*x^3-1/5*e^2*x^5+8*d^(5/2)*arctanh(x*e^(1/2)/d^(1/2))/e^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1164, 398, 214}

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx=\frac{8d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

```
[In] Int[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d+ex^2)^3}{d-ex^2}\,dx\\
&= \int \left(-7d^2-4dex^2-e^2x^4+\frac{8d^3}{d-ex^2}\right)dx\\
&= -7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\left(8d^3\right)\int \frac{1}{d-ex^2}\,dx\\
&= -7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

```
[In] Integrate[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{e^{2} x^{5}}{5}-\frac{4 d e \,x^{3}}{3}-7 d^{2} x +\frac{8 d^{3} \operatorname{arctanh}\left(\frac{e x}{\sqrt{e d}}\right)}{\sqrt{e d}}$	42
risch	$-\frac{e^{2} x^{5}}{5}-\frac{4 d e \,x^{3}}{3}-7 d^{2} x -\frac{4 \sqrt{e d}\, d^{2} \ln\left(\sqrt{e d}\, x -d \right)}{e}+\frac{4 \sqrt{e d}\, d^{2} \ln\left(-\sqrt{e d}\, x -d \right)}{e}$	74

```
[In] int((e*x^2+d)^4/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*e^2*x^5-4/3*d*e*x^3-7*d^2*x+8*d^3/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = \left[-\frac{1}{5}\,e^{2}x^{5} - \frac{4}{3}\,dex^{3} + 4\,d^{2}\sqrt{\frac{d}{e}}\log\left(\frac{ex^{2} + 2\,ex\sqrt{\frac{d}{e}} + d}{ex^{2} - d}\right) - 7\,d^{2}x,\right.$$
$$\left. -\frac{1}{5}\,e^{2}x^{5} - \frac{4}{3}\,dex^{3} - 8\,d^{2}\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right) - 7\,d^{2}x\right]$$

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/5*e^2*x^5 - 4/3*d*e*x^3 + 4*d^2*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e)
+ d)/(e*x^2 - d)) - 7*d^2*x, -1/5*e^2*x^5 - 4/3*d*e*x^3 - 8*d^2*sqrt(-d/e)*
arctan(e*x*sqrt(-d/e)/d) - 7*d^2*x]
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7d^{2}x - \frac{4dex^{3}}{3} - \frac{e^{2}x^{5}}{5} - 4\sqrt{\frac{d^{5}}{e}}\log\left(x - \frac{\sqrt{\frac{d^{5}}{e}}}{d^{2}}\right) + 4\sqrt{\frac{d^{5}}{e}}\log\left(x + \frac{\sqrt{\frac{d^{5}}{e}}}{d^{2}}\right)$$

```
[In] integrate((e*x**2+d)**4/(-e**2*x**4+d**2),x)
```

```
[Out] -7*d**2*x - 4*d*e*x**3/3 - e**2*x**5/5 - 4*sqrt(d**5/e)*log(x - sqrt(d**5/e
)/d**2) + 4*sqrt(d**5/e)*log(x + sqrt(d**5/e)/d**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -\frac{8\,d^3\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}} - \frac{3\,e^7x^5 + 20\,de^6x^3 + 105\,d^2e^5x}{15\,e^5}$$

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] -8*d^3*arctan(e*x/sqrt(-d*e))/sqrt(-d*e) - 1/15*(3*e^7*x^5 + 20*d*e^6*x^3 +
 105*d^2*e^5*x)/e^5
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7\,d^2\,x - \frac{e^2\,x^5}{5} - \frac{4\,d\,e\,x^3}{3} - \frac{d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x\,1\mathrm{i}}{\sqrt{d}}\right)\,8\mathrm{i}}{\sqrt{e}}$$

```
[In] int((d + e*x^2)^4/(d^2 - e^2*x^4),x)
```

```
[Out] - 7*d^2*x - (e^2*x^5)/5 - (d^(5/2)*atan((e^(1/2)*x*1i)/d^(1/2))*8i)/e^(1/2)
 - (4*d*e*x^3)/3
```

3.190 $\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx=-3dx-\frac{ex^3}{3}+\frac{4d^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

```
[Out] -3*d*x-1/3*e*x^3+4*d^(3/2)*arctanh(x*e^(1/2)/d^(1/2))/e^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1164, 398, 214}

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx=\frac{4d^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-3dx-\frac{ex^3}{3}$$

```
[In] Int[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

```
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d+ex^2)^2}{d-ex^2}\,dx\\ &= \int \left(-3d-ex^2+\frac{4d^2}{d-ex^2}\right)dx\\ &= -3dx-\frac{ex^3}{3}+\left(4d^2\right)\int \frac{1}{d-ex^2}\,dx\\ &= -3dx-\frac{ex^3}{3}+\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -3dx-\frac{ex^3}{3}+\frac{4d^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

```
[In] Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{e\,x^3}{3}-3dx+\frac{4d^2\,\text{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}$	31
risch	$-\frac{e\,x^3}{3}-3dx+\frac{2\sqrt{ed}\,d\ln\left(\sqrt{ed}\,x+d\right)}{e}-\frac{2\sqrt{ed}\,d\ln\left(-\sqrt{ed}\,x+d\right)}{e}$	55

```
[In] int((e*x^2+d)^3/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*e*x^3-3*d*x+4*d^2/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.37

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \left[-\frac{1}{3}\,ex^3+2\,d\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-3\,dx, -\frac{1}{3}\,ex^3 - 4\,d\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-3\,dx\right]$$

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="fricas")
[Out] [-1/3*e*x^3 + 2*d*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d))
- 3*d*x, -1/3*e*x^3 - 4*d*sqrt(-d/e)*arctan(e*x*sqrt(-d/e)/d) - 3*d*x]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -3dx-\frac{ex^3}{3}-2\sqrt{\frac{d^3}{e}}\log\left(x-\frac{\sqrt{\frac{d^3}{e}}}{d}\right)+2\sqrt{\frac{d^3}{e}}\log\left(x+\frac{\sqrt{\frac{d^3}{e}}}{d}\right)$$

```
[In] integrate((e*x**2+d)**3/(-e**2*x**4+d**2),x)
[Out] -3*d*x - e*x**3/3 - 2*sqrt(d**3/e)*log(x - sqrt(d**3/e)/d) + 2*sqrt(d**3/e)
*log(x + sqrt(d**3/e)/d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -\frac{4\,d^2\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}} - \frac{e^4x^3+9\,de^3x}{3\,e^3}$$

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="giac")
[Out] -4*d^2*arctan(e*x/sqrt(-d*e))/sqrt(-d*e) - 1/3*(e^4*x^3 + 9*d*e^3*x)/e^3
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \frac{4\,d^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{e\,x^3}{3} - 3\,d\,x$$

```
[In] int((d + e*x^2)^3/(d^2 - e^2*x^4),x)
[Out] (4*d^(3/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - (e*x^3)/3 - 3*d*x
```

3.191 $\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = -x + \frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

```
[Out] -x+2*arctanh(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1164, 396, 214}

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = \frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

```
[In] Int[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

```
[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{d+ex^2}{d-ex^2}\,dx\\ &= -x+(2d)\int \frac{1}{d-ex^2}\,dx\\ &= -x+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx=-x+\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

```
[In] Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
default	$-x+\frac{2d\,\text{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}$	22
risch	$-x-\frac{\sqrt{ed}\,\ln\left(\sqrt{ed}\,x-d\right)}{e}+\frac{\sqrt{ed}\,\ln\left(-\sqrt{ed}\,x-d\right)}{e}$	49

```
[In] int((e*x^2+d)^2/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
[Out] -x+2*d/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = \left[\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-x,-2\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-x\right]$$

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d)) - x, -2*sqrt(-d/e
)*arctan(e*x*sqrt(-d/e)/d) - x]
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = -x-\sqrt{\frac{d}{e}}\log\left(x-\sqrt{\frac{d}{e}}\right)+\sqrt{\frac{d}{e}}\log\left(x+\sqrt{\frac{d}{e}}\right)$$

```
[In] integrate((e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

```
[Out] -x - sqrt(d/e)*log(x - sqrt(d/e)) + sqrt(d/e)*log(x + sqrt(d/e))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = -\frac{2\,d\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}} - x$$

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] -2*d*arctan(e*x/sqrt(-d*e))/sqrt(-d*e) - x
```

Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = \frac{2\sqrt{d}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

```
[In] int((d + e*x^2)^2/(d^2 - e^2*x^4),x)
```

```
[Out] (2*d^(1/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - x
```

3.192 $\int \frac{d+ex^2}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

```
[Out] arctanh(x*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 214}

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

```
[In] Int[(d + e*x^2)/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
```

```
&& EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{d-ex^2}\,dx \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

```
[In] Integrate[(d + e*x^2)/(d^2 - e^2*x^4),x]
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}$	16
risch	$\frac{\ln\left(ex+\sqrt{ed}\right)}{2\sqrt{ed}}-\frac{\ln\left(-ex+\sqrt{ed}\right)}{2\sqrt{ed}}$	37

```
[In] int((e*x^2+d)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
[Out] 1/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = \left[\frac{\sqrt{de}\log\left(\frac{ex^2+2\sqrt{de}x+d}{ex^2-d}\right)}{2\,de}, -\frac{\sqrt{-de}\arctan\left(\frac{\sqrt{-de}x}{d}\right)}{de}\right]$$

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d))/(d*e), -sqrt(-d
*e)*arctan(sqrt(-d*e)*x/d)/(d*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = -\frac{\sqrt{\frac{1}{de}}\log\left(-d\sqrt{\frac{1}{de}}+x\right)}{2} + \frac{\sqrt{\frac{1}{de}}\log\left(d\sqrt{\frac{1}{de}}+x\right)}{2}$$

```
[In] integrate((e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d*e))*log(-d*sqrt(1/(d*e)) + x)/2 + sqrt(1/(d*e))*log(d*sqrt(1/(d*
e)) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = -\frac{\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}}$$

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
[Out] -arctan(e*x/sqrt(-d*e))/sqrt(-d*e)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

```
[In] int((d + e*x^2)/(d^2 - e^2*x^4),x)
[Out] atanh((e^(1/2)*x)/d^(1/2))/(d^(1/2)*e^(1/2))
```

3.193 $\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx=\frac{x}{4d^2(d+ex^2)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

```
[Out] 1/4*x/d^2/(e*x^2+d)+1/2*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)+1/4*arcta
nh(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1164, 425, 536, 214, 211}

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx=\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}+\frac{x}{4d^2(d+ex^2)}$$

```
[In] Int[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(4*d^2*(d + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(5/2)*Sqrt[e]) + A
rcTanh[(Sqrt[e]*x)/Sqrt[d]]/(4*d^(5/2)*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(d-ex^2)\left(d+ex^2\right)^2}\,dx\\
&= \frac{x}{4d^2\left(d+ex^2\right)} - \frac{\int \frac{-3de+e^2x^2}{(d-ex^2)(d+ex^2)}\,dx}{4d^2e}\\
&= \frac{x}{4d^2\left(d+ex^2\right)} + \frac{\int \frac{1}{d-ex^2}\,dx}{4d^2} + \frac{\int \frac{1}{d+ex^2}\,dx}{2d^2}\\
&= \frac{x}{4d^2\left(d+ex^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{\frac{\sqrt{d}x}{d+ex^2}+\frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{4d^{5/2}}$$

```
[In] Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x)/(d + e*x^2) + (2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ArcTan
h[(Sqrt[e]*x)/Sqrt[d]]/Sqrt[e])/(4*d^(5/2))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{4d^{2}\sqrt{ed}}+\frac{\frac{x}{e\,x^{2}+d}+\frac{2\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}}{4d^{2}}$	54
risch	$\frac{x}{4d^{2}(e\,x^{2}+d)}-\frac{\ln\left(-ex-\sqrt{-ed}\right)}{4\sqrt{-ed}\,d^{2}}+\frac{\ln\left(ex-\sqrt{-ed}\right)}{4\sqrt{-ed}\,d^{2}}+\frac{\ln\left(ex+\sqrt{ed}\right)}{8\sqrt{ed}\,d^{2}}-\frac{\ln\left(-ex+\sqrt{ed}\right)}{8\sqrt{ed}\,d^{2}}$	107

```
[In] int(1/(e*x^2+d)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d^2/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))+1/4/d^2*(x/(e*x^2+d)+2/(e*d)^(
1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.62

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx$$
$$= \left[\frac{2\,dex+4\,(ex^2+d)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)+(ex^2+d)\sqrt{de}\log\left(\frac{ex^2+2\,\sqrt{de}x+d}{ex^2-d}\right)}{8\,(d^3e^2x^2+d^4e)}, \frac{dex-(ex^2+d)\sqrt{-de}\arctan}{}\right.$$

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*d*e*x + 4*(e*x^2 + d)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (e*x^2 + d)
*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^3*e^2*x^2 + d^4
*e), 1/4*(d*e*x - (e*x^2 + d)*sqrt(-d*e)*arctan(sqrt(-d*e)*x/d) - (e*x^2 +
d)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^2*x^2 +
 d^4*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(63) = 126$.

Time = 0.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.14

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{x}{4d^3+4d^2ex^2} - \frac{\sqrt{\frac{1}{d^5e}}\log\left(-\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} - \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} + \frac{\sqrt{\frac{1}{d^5e}}\log\left(\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} + \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} - \frac{\sqrt{-\frac{1}{d^5e}}\log\left(-\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} - \frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^5e}}\log\left(\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} + \frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5} + x\right)}{4}$$

```
[In] integrate(1/(e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] x/(4*d**3 + 4*d**2*e*x**2) - sqrt(1/(d**5*e))*log(-d**8*e*(1/(d**5*e))**(3/
2)/10 - 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 + sqrt(1/(d**5*e))*log(d**8*e*(1/
(d**5*e))**(3/2)/10 + 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 - sqrt(-1/(d**5*e))
*log(-4*d**8*e*(-1/(d**5*e))**(3/2)/5 - 9*d**3*sqrt(-1/(d**5*e))/5 + x)/4 +
 sqrt(-1/(d**5*e))*log(4*d**8*e*(-1/(d**5*e))**(3/2)/5 + 9*d**3*sqrt(-1/(d*
*5*e))/5 + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^2} - \frac{\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{4\sqrt{-de}d^2} + \frac{x}{4(ex^2+d)d^2}$$

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2) - 1/4*arctan(e*x/sqrt(-d*e))/(sqr
t(-d*e)*d^2) + 1/4*x/((e*x^2 + d)*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{x}{4\,d^2\,(e\,x^2+d)} + \frac{\operatorname{atanh}\left(\frac{x\sqrt{d^5\,e}}{d^3}\right)\,\sqrt{d^5\,e}}{4\,d^5\,e} - \frac{\operatorname{atanh}\left(\frac{x\sqrt{-d^5\,e}}{d^3}\right)\,\sqrt{-d^5\,e}}{2\,d^5\,e}$$

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)),x)
```

```
[Out] x/(4*d^2*(d + e*x^2)) + (atanh((x*(d^5*e)^(1/2))/d^3)*(d^5*e)^(1/2))/(4*d^5
*e) - (atanh((x*(-d^5*e)^(1/2))/d^3)*(-d^5*e)^(1/2))/(2*d^5*e)
```

3.194 $\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \frac{x}{8d^2(d+ex^2)^2}+\frac{5x}{16d^3(d+ex^2)}+\frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

```
[Out] 1/8*x/d^2/(e*x^2+d)^2+5/16*x/d^3/(e*x^2+d)+7/16*arctan(x*e^(1/2)/d^(1/2))/d
^(7/2)/e^(1/2)+1/8*arctanh(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1164, 425, 541, 536, 214, 211}

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}+\frac{5x}{16d^3(d+ex^2)}+\frac{x}{8d^2(d+ex^2)^2}$$

```
[In] Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(8*d^2*(d + e*x^2)^2) + (5*x)/(16*d^3*(d + e*x^2)) + (7*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(16*d^(7/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(8*d^(7/2)*
Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(d-ex^2)\,(d+ex^2)^3}\,dx$$

$$
\begin{aligned}
&= \frac{x}{8d^2\left(d+ex^2\right)^2} - \frac{\int \frac{-7de+3e^2x^2}{(d-ex^2)(d+ex^2)^2}\,dx}{8d^2e} \\
&= \frac{x}{8d^2\left(d+ex^2\right)^2} + \frac{5x}{16d^3\left(d+ex^2\right)} + \frac{\int \frac{18d^2e^2-10de^3x^2}{(d-ex^2)(d+ex^2)}\,dx}{32d^4e^2} \\
&= \frac{x}{8d^2\left(d+ex^2\right)^2} + \frac{5x}{16d^3\left(d+ex^2\right)} + \frac{\int \frac{1}{d-ex^2}\,dx}{8d^3} + \frac{7\int \frac{1}{d+ex^2}\,dx}{16d^3} \\
&= \frac{x}{8d^2\left(d+ex^2\right)^2} + \frac{5x}{16d^3\left(d+ex^2\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$
\int \frac{1}{\left(d+ex^2\right)^2\left(d^2-e^2x^4\right)}\,dx = \frac{\frac{\sqrt{d}x(7d+5ex^2)}{(d+ex^2)^2} + \frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{16d^{7/2}}
$$

```
[In] Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x*(7*d + 5*e*x^2))/(d + e*x^2)^2 + (7*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/Sqrt[e] + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(16*d^(7/2))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{8d^3\sqrt{ed}} + \frac{\frac{\frac{5}{2}e\,x^3+\frac{7}{2}dx}{\left(e\,x^2+d\right)^2} + \frac{7\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2\sqrt{ed}}}{8d^3}$	64
risch	$\frac{\frac{5e\,x^3}{16d^3}+\frac{7x}{16d^2}}{\left(e\,x^2+d\right)^2} - \frac{7\ln\left(-ex-\sqrt{-ed}\right)}{32\sqrt{-ed}\,d^3} + \frac{7\ln\left(ex-\sqrt{-ed}\right)}{32\sqrt{-ed}\,d^3} + \frac{\ln\left(ex+\sqrt{ed}\right)}{16\sqrt{ed}\,d^3} - \frac{\ln\left(-ex+\sqrt{ed}\right)}{16\sqrt{ed}\,d^3}$	118

```
[In] int(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d^3/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))+1/8/d^3*((5/2*e*x^3+7/2*d*x)/(
e*x^2+d)^2+7/2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.12

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \Bigg[\frac{5\,de^2x^3 + 7\,d^2ex + 7\,(e^2x^4+2\,dex^2+d^2)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right) + (e^2x^4+2\,dex^2+d^2)\sqrt{de}\log\Bigg(\frac{ex^2+2\sqrt{de}x+d}{ex^2-d}}{16\,(d^4e^3x^4+2\,d^5e^2x^2+d^6e)}$$

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/16*(5*d*e^2*x^3 + 7*d^2*e*x + 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*ar
ctan(sqrt(d*e)*x/d) + (e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*log((e*x^2 + 2*
sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e), 1/32*
(10*d*e^2*x^3 + 14*d^2*e*x - 4*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*arcta
n(sqrt(-d*e)*x/d) - 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*log((e*x^2 - 2
*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(82) = 164$.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.89

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = &-\frac{\sqrt{\frac{1}{d^7e}}\log\left(-\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371} - \frac{351d^4\sqrt{\frac{1}{d^7e}}}{371} + x\right)}{16} \\ &+ \frac{\sqrt{\frac{1}{d^7e}}\log\left(\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371} + \frac{351d^4\sqrt{\frac{1}{d^7e}}}{371} + x\right)}{16} \\ &- \frac{7\sqrt{-\frac{1}{d^7e}}\log\left(-\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106} - \frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106} + x\right)}{32} \\ &+ \frac{7\sqrt{-\frac{1}{d^7e}}\log\left(\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106} + \frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106} + x\right)}{32} \\ &- \frac{-7dx - 5ex^3}{16d^5 + 32d^4ex^2 + 16d^3e^2x^4}\end{aligned}$$

```
[In] integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d**7*e))*log(-20*d**11*e*(1/(d**7*e))**(3/2)/371 - 351*d**4*sqrt(1
/(d**7*e))/371 + x)/16 + sqrt(1/(d**7*e))*log(20*d**11*e*(1/(d**7*e))**(3/2
)/371 + 351*d**4*sqrt(1/(d**7*e))/371 + x)/16 - 7*sqrt(-1/(d**7*e))*log(-24
5*d**11*e*(-1/(d**7*e))**(3/2)/106 - 351*d**4*sqrt(-1/(d**7*e))/106 + x)/32
 + 7*sqrt(-1/(d**7*e))*log(245*d**11*e*(-1/(d**7*e))**(3/2)/106 + 351*d**4*
sqrt(-1/(d**7*e))/106 + x)/32 - (-7*d*x - 5*e*x**3)/(16*d**5 + 32*d**4*e*x*
*2 + 16*d**3*e**2*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)}\,dx = \frac{7\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de}d^3} - \frac{\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{8\sqrt{-de}d^3} + \frac{5ex^3+7dx}{16\,(ex^2+d)^2d^3}$$

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 7/16*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3) - 1/8*arctan(e*x/sqrt(-d*e))/(sq
rt(-d*e)*d^3) + 1/16*(5*e*x^3 + 7*d*x)/((e*x^2 + d)^2*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex^2)^2\,(d^2-e^2x^4)}\,dx = \frac{\frac{7\,x}{16\,d^2}+\frac{5\,e\,x^3}{16\,d^3}}{d^2+2\,d\,e\,x^2+e^2\,x^4} + \frac{\mathrm{atanh}\left(\frac{x\,\sqrt{d^7\,e}}{d^4}\right)\,\sqrt{d^7\,e}}{8\,d^7\,e} - \frac{7\,\mathrm{atanh}\left(\frac{x\,\sqrt{-d^7\,e}}{d^4}\right)\,\sqrt{-d^7\,e}}{16\,d^7\,e}$$

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^2),x)
```

```
[Out] ((7*x)/(16*d^2) + (5*e*x^3)/(16*d^3))/(d^2 + e^2*x^4 + 2*d*e*x^2) + (atanh(
(x*(d^7*e)^(1/2))/d^4)*(d^7*e)^(1/2))/(8*d^7*e) - (7*atanh((x*(-d^7*e)^(1/2
))/d^4)*(-d^7*e)^(1/2))/(16*d^7*e)
```

3.195 $\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 62

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

```
[Out] -arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(1/2)+arctanh(x*2^(1/2)*e^(1/2)/(e*x^
2+d)^(1/2))*2^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1164, 399, 223, 212, 385, 214}

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4} \, dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

```
[In] Int[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sqrt{d+ex^2}}{d-ex^2}\,dx\\
&= (2d)\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}\,dx-\int \frac{1}{\sqrt{d+ex^2}}\,dx\\
&= (2d)\text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)-\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)+\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{\sqrt{e}}$$

```
[In] Integrate[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)] + Log
[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/Sqrt[e]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{\sqrt{2}\ \text{arctanh}\left(\frac{\sqrt{e\,x^2+d}\,\sqrt{2}}{2x\sqrt{e}}\right)-\text{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)}{\sqrt{e}}$	50
default	Expression too large to display	1356

```
[In] int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] (2^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2))-arctanh((e*x^2+d)^(
1/2)/x/e^(1/2)))/e^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.21

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \left[\frac{\sqrt{2}\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+d^2+\frac{4\,\sqrt{2}\left(3\,e^2x^3+dex\right)\sqrt{ex^2+d}}{\sqrt{e}}}{e^2x^4-2\,dex^2+d^2}\right)+2\,\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{}\right.}{4\,e}\right.$$

$$\left.-\frac{\sqrt{2}e\sqrt{-\frac{1}{e}}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-\frac{1}{e}}}{4\,(ex^3+dx)}\right)-2\,\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,e}\right]$$

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + d^2 + 4*sqrt(2)*(3*e^2
*x^3 + d*e*x)*sqrt(e*x^2 + d)/sqrt(e))/(e^2*x^4 - 2*d*e*x^2 + d^2)) + 2*sqr
t(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/e, -1/2*(sqrt(2)*e*sq
rt(-1/e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-1/e)/(e*x^3
 + d*x)) - 2*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e]
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = -\int \frac{\sqrt{d+ex^2}}{-d+ex^2}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(sqrt(d + e*x**2)/(-d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \int -\frac{(ex^2+d)^{\frac{3}{2}}}{e^2x^4-d^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 + d)^(3/2)/(e^2*x^4 - d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(46) = 92$.

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \frac{\sqrt{2}d\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{2\sqrt{e}|d|} + \frac{\log\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2\right)}{2\sqrt{e}}$$

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*d*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d)
- 6*d)/abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(sq
rt(e)*abs(d)) + 1/2*log((sqrt(e)*x - sqrt(e*x^2 + d))^2)/sqrt(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{d^2-e^2\,x^4}\,dx$$

```
[In] int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x)
[Out] int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4), x)
```

3.196 $\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d*2^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1164, 385, 214}

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

```
[In] Int[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}\,dx\\ &= \text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \frac{\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)}{\sqrt{2}d\sqrt{e}}$$

```
[In] Integrate[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
[Out] ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)]/(Sqrt[2]*d*Sqr
t[e])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}\,\sqrt{2}}{2x\sqrt{e}}\right)}{2d\sqrt{e}}$
default	$-\frac{e\left(\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^2 e+2\sqrt{ed}\left(x-\frac{\sqrt{ed}}{e}\right)+2d}+\frac{\sqrt{ed}\,\ln\left(\frac{\sqrt{ed}+e\left(x-\frac{\sqrt{ed}}{e}\right)}{\sqrt{e}}+\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^2 e+2\sqrt{ed}\left(x-\frac{\sqrt{ed}}{e}\right)+2d}\right)}{\sqrt{e}}-\sqrt{d}\,\sqrt{2}\,\ln\left(\frac{4d}{}\right.\right.}{2\left(\sqrt{ed}-\sqrt{-ed}\right)\left(\sqrt{ed}+\sqrt{-ed}\right)\sqrt{ed}}$

```
[In] int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*2^(1/2)/e^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.63

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \left[\frac{\sqrt{2}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}(3\,ex^3+dx)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)}{8\,d\sqrt{e}},\right.$$
$$\left.-\frac{\sqrt{2}\sqrt{-e}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4\,(e^2x^3+dex)}\right)}{4\,de}\right]$$

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(2)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(
e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2))/(d*sqrt(e)), -1/4*sq
rt(2)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e
^2*x^3 + d*e*x))/(d*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = -\int \frac{1}{-d\sqrt{d+ex^2}+ex^2\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d*sqrt(d + e*x**2) + e*x**2*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \int -\frac{\sqrt{ex^2+d}}{e^2x^4-d^2}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x^2 + d)/(e^2*x^4 - d^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(29) = 58$.

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6\,d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6\,d\right|}\right)}{4\sqrt{e}|d|}$$

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) -
6*d)/abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(sqrt
(e)*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}}{d^2-e^2\,x^4}\,dx$$

```
[In] int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4), x)
```

3.197 $\int \frac{1}{\sqrt{d+ex^2}(d^2-e^2x^4)} dx$

Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx=\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

```
[Out] 1/4*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d^2*2^(1/2)/e^(1/2)+1/2*x/d^
2/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1164, 390, 385, 214}

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}+\frac{x}{2d^2\sqrt{d+ex^2}}$$

```
[In] Int[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(2
*Sqrt[2]*d^2*Sqrt[e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(d-ex^2)\,(d+ex^2)^{3/2}}\,dx\\ &= \frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}\,dx}{2d}\\ &= \frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2d}\\ &= \frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx=\frac{\frac{2x}{\sqrt{d+ex^2}}+\frac{\sqrt{2}\text{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)}{\sqrt{e}}}{4d^2}$$

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
[Out] ((2*x)/Sqrt[d + e*x^2] + (Sqrt[2]*ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e
*x^2])/(Sqrt[2]*d)])/Sqrt[e])/(4*d^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,\sqrt{2}}{2x\sqrt{e}}\right)\sqrt{e\,x^{2}+d}+2x\sqrt{e}}{4\sqrt{e\,x^{2}+d}\,\sqrt{e}\,d^{2}}$
default	$\frac{e\sqrt{2}\,\ln\left(\frac{4d+2\sqrt{ed}\,\left(x-\frac{\sqrt{ed}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^{2}e+2\sqrt{ed}\,\left(x-\frac{\sqrt{ed}}{e}\right)+2d}}{x-\frac{\sqrt{ed}}{e}}\right)}{4\left(\sqrt{ed}-\sqrt{-ed}\right)\left(\sqrt{ed}+\sqrt{-ed}\right)\sqrt{ed}\,\sqrt{d}}-\frac{e\sqrt{2}\,\ln\left(\frac{4d-2\sqrt{ed}\,\left(x+\frac{\sqrt{ed}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{\left(x+\frac{\sqrt{ed}}{e}\right.}}{x+\frac{\sqrt{ed}}{e}}\right.}{4\left(\sqrt{ed}-\sqrt{-ed}\right)\left(\sqrt{ed}+\sqrt{-ed}\right.}$

```
[In] int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2))*(e*x^2+d)^(1/2)
+2*x*e^(1/2))/(e*x^2+d)^(1/2)/e^(1/2)/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.43

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = \left[\frac{\sqrt{2}(ex^2+d)\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}(3\,ex^3+dx)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)+8\,\sqrt{ex^2+d}ex}{16\,(d^2e^2x^2+d^3e)}, -\frac{\sqrt{2}(ex^2+d)\sqrt{-e}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4\,(e^2x^3+dex)}\right)-4\,\sqrt{ex^2+d}ex}{8\,(d^2e^2x^2+d^3e)}\right]$$

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(2)*(e*x^2 + d)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)
*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)
) + 8*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e), -1/8*(sqrt(2)*(e*x^2 + d)
*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x^
3 + d*e*x)) - 4*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = -\int \frac{1}{-d^2\sqrt{d+ex^2}+e^2x^4\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
[Out] -Integral(1/(-d**2*sqrt(d + e*x**2) + e**2*x**4*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = \int -\frac{1}{(e^2x^4-d^2)\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
[Out] -integrate(1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(45) = 90$.

Time $= 0.33$ (sec) , antiderivative size $= 101$, normalized size of antiderivative $= 1.66$

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{8d\sqrt{e}|d|} + \frac{x}{2\sqrt{ex^2+d}\,d^2}$$

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
[Out] 1/8*sqrt(2)*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) -
6*d)/abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(d*sq
rt(e)*abs(d)) + 1/2*x/(sqrt(e*x^2 + d)*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = \int \frac{1}{(d^2-e^2\,x^4)\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)),x)
[Out] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)), x)
```

3.198 $\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx=\frac{x}{6d^2(d+ex^2)^{3/2}}+\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

```
[Out] 1/6*x/d^2/(e*x^2+d)^(3/2)+1/8*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d^
3*2^(1/2)/e^(1/2)+7/12*x/d^3/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1164, 425, 541, 12, 385, 214}

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}+\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

```
[In] Int[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(6*d^2*(d + e*x^2)^(3/2)) + (7*x)/(12*d^3*Sqrt[d + e*x^2]) + ArcTanh[(Sqr
t[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*Sqrt[2]*d^3*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(d-ex^2)\left(d+ex^2\right)^{5/2}}\,dx\\ &= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}}-\frac{\int \frac{-5de+2e^2x^2}{(d-ex^2)(d+ex^2)^{3/2}}\,dx}{6d^2e}\\ &= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}}+\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\int \frac{3d^2e^2}{(d-ex^2)\sqrt{d+ex^2}}\,dx}{12d^4e^2}\end{aligned}$$

$$=\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}+\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\int\frac{1}{(d-ex^2)\sqrt{d+ex^2}}\,dx}{4d^2}$$

$$=\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}+\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\operatorname{Subst}\left(\int\frac{1}{d-2dex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{4d^2}$$

$$=\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}+\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int\frac{1}{\left(d+ex^2\right)^{3/2}\left(d^2-e^2x^4\right)}\,dx=\frac{\frac{2(9dx+7ex^3)}{(d+ex^2)^{3/2}}+\frac{3\sqrt{2}\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)}{\sqrt{e}}}{24d^3}$$

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((2*(9*d*x + 7*e*x^3))/(d + e*x^2)^(3/2) + (3*Sqrt[2]*ArcTanh[(d - e*x^2 +
Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)])/Sqrt[e])/(24*d^3)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{14e^{\frac{3}{2}}x^3+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}\,\sqrt{2}}{2x\sqrt{e}}\right)\left(e\,x^2+d\right)^{\frac{3}{2}}+18\sqrt{e}\,dx}{24\sqrt{e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}d^3}$
default	$-\frac{e\left(\frac{1}{2d\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^2e+2\sqrt{ed}\,\left(x-\frac{\sqrt{ed}}{e}\right)+2d}}-\frac{\sqrt{ed}\,\left(2e\left(x-\frac{\sqrt{ed}}{e}\right)+2\sqrt{ed}\right)}{4d^2e\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^2e+2\sqrt{ed}\,\left(x-\frac{\sqrt{ed}}{e}\right)+2d}}-\frac{\sqrt{2}\,\ln\left(\frac{4d+2\sqrt{ed}\,\left(x-\frac{\sqrt{ed}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{\left(x-\right.}}{x-\frac{\sqrt{e}}{e}}\right.}{4d^{\frac{3}{2}}}\right.}{2\left(\sqrt{ed}-\sqrt{-ed}\right)\left(\sqrt{ed}+\sqrt{-ed}\right)\sqrt{ed}}$

```
[In] int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(14*e^(3/2)*x^3+3*2^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2
))*(e*x^2+d)^(3/2)+18*e^(1/2)*d*x)/e^(1/2)/(e*x^2+d)^(3/2)/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.49

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \left[\frac{3\sqrt{2}(e^2x^4+2\,dex^2+d^2)\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}(3\,ex^3+dx)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)}{96\,(d^3e^3x^4+2\,d^4e^2x^2+d^5e)}\right.$$

$$\left.-\frac{3\sqrt{2}(e^2x^4+2\,dex^2+d^2)\sqrt{-e}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4\,(e^2x^3+dex)}\right)-4\,(7\,e^2x^3+9\,dex)\sqrt{ex^2+d}}{48\,(d^3e^3x^4+2\,d^4e^2x^2+d^5e)}\right]$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e)*log((17*e^2*x^4 + 14*d
*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4
- 2*d*e*x^2 + d^2)) + 8*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4
 + 2*d^4*e^2*x^2 + d^5*e), -1/48*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqr
t(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x^3 +
d*e*x)) - 4*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4 + 2*d^4*e^2
*x^2 + d^5*e)]
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = -\int \frac{1}{-d^3\sqrt{d+ex^2}-d^2ex^2\sqrt{d+ex^2}+de^2x^4\sqrt{d+ex^2}+e^3x^6\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d**3*sqrt(d + e*x**2) - d**2*e*x**2*sqrt(d + e*x**2) + d*e**2
*x**4*sqrt(d + e*x**2) + e**3*x**6*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \int -\frac{1}{(e^2x^4-d^2)(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((e^2*x^4 - d^2)*(e*x^2 + d)^(3/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(d^2-e^2x^4\right)}\,dx = \frac{x\left(\frac{7\,ex^2}{d^3}+\frac{9}{d^2}\right)}{12\,\left(ex^2+d\right)^{\frac{3}{2}}} + \frac{\sqrt{2}\log\left(\frac{\left|2\,\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\,\sqrt{2}|d|-6\,d\right|}{\left|2\,\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\,\sqrt{2}|d|-6\,d\right|}\right)}{16\,d^2\sqrt{e}|d|}$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/12*x*(7*e*x^2/d^3 + 9/d^2)/(e*x^2 + d)^(3/2) + 1/16*sqrt(2)*log(abs(2*(sq
rt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) - 6*d)/abs(2*(sqrt(e)*x - s
qrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(d^2*sqrt(e)*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(d^2-e^2x^4\right)}\,dx = \int \frac{1}{\left(d^2-e^2\,x^4\right)\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)),x)
```

```
[Out] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)), x)
```

3.199 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 153

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/4*x*(-b*x^2+a)*(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)-9/8*a*x*(-b*x^2+a)*(
b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/8*a^2*arctan(x*b^(1/2)/(-b*x^2+a)^(1
/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1166, 427, 396, 223, 209}

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}}$$

```
[In] Int[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-9*a*x*(a - b*x^2)*Sqrt[a + b*x^2])/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)*(a + b*x^2)^(3/2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
```

```
t[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2
*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{\left(a+bx^2\right)^2}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\ &= -\frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}}-\frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{-5a^2b-9ab^2x^2}{\sqrt{a-bx^2}}\,dx}{4b\sqrt{a^2-b^2x^4}}\end{aligned}$$

$$= -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}}\,dx}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = -\frac{(11ax+2bx^3)\sqrt{a^2-b^2x^4}}{8\sqrt{a+bx^2}} + \frac{19ia^2\log\left(-2i\sqrt{b}x + \frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

```
[In] Integrate[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/8*((11*a*x + 2*b*x^3)*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2] + (((19*I)/8)
*a^2*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[
b]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, \left(-2 b^{\frac{3}{2}} x^{3} \sqrt{-b\,x^{2}+a}-11 a x \sqrt{-b\,x^{2}+a}\, \sqrt{b}+19 \arctan\left(\frac{\sqrt{b}\, x}{\sqrt{-b\,x^{2}+a}}\right) a^{2}\right)}{8 \sqrt{b\,x^{2}+a}\, \sqrt{-b\,x^{2}+a}\, \sqrt{b}}$	96
risch	$-\frac{x \left(2 b\,x^{2}+11 a \right) \sqrt{-b\,x^{2}+a}\, \sqrt{\frac{-b^{2} x^{4}+a^{2}}{b\,x^{2}+a}}\, \sqrt{b\,x^{2}+a}}{8 \sqrt{-b^{2} x^{4}+a^{2}}}+\frac{19 a^{2} \arctan\left(\frac{\sqrt{b}\, x}{\sqrt{-b\,x^{2}+a}}\right) \sqrt{\frac{-b^{2} x^{4}+a^{2}}{b\,x^{2}+a}}\, \sqrt{b\,x^{2}+a}}{8 \sqrt{b}\, \sqrt{-b^{2} x^{4}+a^{2}}}$	143

```
[In] int((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-b^2*x^4+a^2)^(1/2)*(-2*b^(3/2)*x^3*(-b*x^2+a)^(1/2)-11*a*x*(-b*x^2+a)
^(1/2)*b^(1/2)+19*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*a^2)/(b*x^2+a)^(1/2)/(
-b*x^2+a)^(1/2)/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{19\,(a^2bx^2+a^3)\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{bx^2+a}\right)+2\,\sqrt{-b^2x^4+a^2}(2}{16\,(b^2x^2+ab)}\right.$$

$$\left.-\frac{19\,(a^2bx^2+a^3)\sqrt{b}\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)+\sqrt{-b^2x^4+a^2}(2\,b^2x^3+11\,abx)\sqrt{bx^2+a}}{8\,(b^2x^2+ab)}\right]$$

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(19*(a^2*b*x^2 + a^3)*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b
^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a)) + 2*sqrt(-b^2*
x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(b*x^2 + a))/(b^2*x^2 + a*b), -1/8*(1
9*(a^2*b*x^2 + a^3)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqr
t(b)/(b^2*x^3 + a*b*x)) + sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(
b*x^2 + a))/(b^2*x^2 + a*b)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

```
[In] integrate((b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

```
[In] int((a + b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a + b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.200 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 110

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx=-\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}+\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/2*x*(-b*x^2+a)*(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/2*a*arctan(x*b^(1/
2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1166, 396, 223, 209}

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx=\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/2*(x*(a - b*x^2)*Sqrt[a + b*x^2])/Sqrt[a^2 - b^2*x^4] + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(2*Sqrt[b]*Sqrt[a
^2 - b^2*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{a+bx^2}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}}\,dx}{2\sqrt{a^2-b^2x^4}}\\
&= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{x}{\sqrt{a-bx^2}}\right)}{2\sqrt{a^2-b^2x^4}}\\
&= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}+\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$
\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = -\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a+bx^2}}+\frac{3ia\log\left(-2i\sqrt{b}x+\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}
$$

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/2*(x*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2] + (((3*I)/2)*a*Log[(-2*I)*Sqrt
[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, \left(-x \sqrt{b}\, \sqrt{-b \,x^{2}+a}+3 \arctan \left(\frac{\sqrt{b}\, x}{\sqrt{-b \,x^{2}+a}}\right) a \right)}{2 \sqrt{b \,x^{2}+a}\, \sqrt{-b \,x^{2}+a}\, \sqrt{b}}$	75
risch	$-\frac{x \sqrt{-b \,x^{2}+a}\, \sqrt{\frac{-b^{2} x^{4}+a^{2}}{b \,x^{2}+a}}\, \sqrt{b \,x^{2}+a}}{2 \sqrt{-b^{2} x^{4}+a^{2}}}+\frac{3 a \arctan \left(\frac{\sqrt{b}\, x}{\sqrt{-b \,x^{2}+a}}\right) \sqrt{\frac{-b^{2} x^{4}+a^{2}}{b \,x^{2}+a}}\, \sqrt{b \,x^{2}+a}}{2 \sqrt{b}\, \sqrt{-b^{2} x^{4}+a^{2}}}$	131

```
[In] int((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b^2*x^4+a^2)^(1/2)*(-x*b^(1/2)*(-b*x^2+a)^(1/2)+3*arctan(b^(1/2)*x/(-
b*x^2+a)^(1/2))*a)/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.03

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx+3\,(abx^2+a^2)\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-}}{bx^2+a}\right.}{4\,(b^2x^2+ab)}\right.$$

$$\left.-\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx+3\,(abx^2+a^2)\sqrt{b}\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)}{2\,(b^2x^2+ab)}\right]$$

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(
-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt
(-b)*x - a^2)/(b*x^2 + a)))/(b^2*x^2 + a*b), -1/2*(sqrt(-b^2*x^4 + a^2)*sqr
t(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sq
rt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(b^2*x^2 + a*b)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{-(-a+bx^2)\,(a+bx^2)}}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral((a + b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

```
[In] int((a + b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a + b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.201 $\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)
/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1166, 223, 209}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt
[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{1+bx^2}\,dx,x,\frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int\frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{i\log\left(-2i\sqrt{b}x+\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
[Out] (I*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{-b^2x^4+a^2}\,\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+a}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{-b\,x^2+a}\,\sqrt{b}}$	54

```
[In] int((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)/(-b*x^2+a)^(1/2)/b^(1/2)*arctan(b^(1
/2)*x/(-b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{bx^2+a}\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)}{\sqrt{b}}\right]$$

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^
2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a))/b, -arctan(sqrt(-b^2*x^4 + a^2)*sqrt(
b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x))/sqrt(b)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx=\int \frac{\sqrt{bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.202 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 78

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(
1/2)/a*2^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1166, 385, 211}

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]
])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}(a+bx^2)}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a+2abx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a^2-b^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[2]*
a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(63) = 126.

Time = 0.59 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.19

method	result
default	$\frac{\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{b}\left(\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^{2}+a}+\sqrt{-ab}\,x+a\right)}{bx+\sqrt{-ab}}\right)\sqrt{b}-\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^{2}+a}-\sqrt{-ab}\,x+a\right)}{bx-\sqrt{-ab}}\right)\sqrt{b}+2\arctan\left(\right.\right.}{2\sqrt{b\,x^{2}+a}\,\sqrt{-b\,x^{2}+a}\,\left(-\sqrt{-ab}+\sqrt{ab}\right)\left(\sqrt{-ab}+\sqrt{ab}\right)\sqrt{-ab}}$

```
[In] int(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(
-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(1/
2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(
1/2)))*b^(1/2)+2*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2))
)^(1/2))*(-a*b)^(1/2)-2*(-a*b)^(1/2)*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2)))/(b
*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/(-(-a*b)^(1/2)+(a*b)^(1/2))/((-a*b)^(1/2)+(a
*b)^(1/2))/(-a*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.95

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{\sqrt{2}\sqrt{-b}\log\left(-\frac{3\,b^{2}x^{4}+2\,abx^{2}-2\,\sqrt{2}\sqrt{-b^{2}x^{4}+a^{2}}\sqrt{bx^{2}+a}\sqrt{-b}x-a^{2}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)}{4\,ab},\right.$$
$$\left.-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-b^{2}x^{4}+a^{2}}\sqrt{bx^{2}+a}\sqrt{b}}{2\,(b^{2}x^{3}+abx)}\right)}{2\,a\sqrt{b}}\right]$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^
4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*
b), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sq
rt(b)/(b^2*x^3 + a*b*x))/(a*sqrt(b))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\sqrt{a+bx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\sqrt{b\,x^2+a}}\,dx$$

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(1/2)), x)
```

3.203 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/4*x*(-b*x^2+a)/a^2/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/8*arctan(x*2^(1
/2)*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^2*2^(1/2)/
b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1166, 390, 385, 211}

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{3\sqrt{a+bx^2}\sqrt{a-bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}} + \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}$$

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(4*a^2*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(4*Sqrt[
2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}(a+bx^2)^2}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}(a+bx^2)}\,dx}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a+2abx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a-bx^2}+3\sqrt{2}\left(a+bx^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)\right)}{8a^2\sqrt{b}\sqrt{a-bx^2}\left(a+bx^2\right)^{3/2}}$$

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2] + 3*Sqrt[2]*(a + b*x^2)*A
rcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(8*a^2*Sqrt[b]*Sqrt[a - b*x^2]
*(a + b*x^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(101) = 202.

Time = 0.23 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.90

method	result
default	$-\frac{\sqrt{-b^2x^4+a^2}\,b^{\frac{5}{2}}\left(3\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^2+a}-\sqrt{-ab}\,x+a\right)}{bx-\sqrt{-ab}}\right)\sqrt{2}\,b^{\frac{3}{2}}\,x^2\sqrt{a}-3\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^2+a}+\sqrt{-ab}\,x+a\right)}{bx+\sqrt{-ab}}\right)\sqrt{2}\,b^{\frac{3}{2}}\,x^2\sqrt{a}+3\ln\left(\right.\right.}{}$

```
[In] int(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/
2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*b^(3/2)*x^2*a^(1/2)-3*ln(2
*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*
2^(1/2)*b^(3/2)*x^2*a^(1/2)+3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*
b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*a^(3/2)*b^(1/2)-3*ln(2*b*(2^(1/2)
*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^(1/2)*a^(
3/2)*b^(1/2)+4*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*b*x^2*(-a*b)^(1/2)-4*arct
an(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2))*b*x^2*(-a*b)
^(1/2)-4*(-b*x^2+a)^(1/2)*b^(1/2)*(-a*b)^(1/2)*x+4*arctan(b^(1/2)*x/(-b*x^2
+a)^(1/2))*a*(-a*b)^(1/2)-4*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(
a*b)^(1/2)))^(1/2))*a*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/(-(-a*
b)^(1/2)+(a*b)^(1/2))^2/((-a*b)^(1/2)+(a*b)^(1/2))^2/(b*x+(-a*b)^(1/2))/(b*
x-(-a*b)^(1/2))/(-a*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.38

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx=\left[\frac{4\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\,bx-3\sqrt{2}(b^2x^4+2\,abx^2+a^2)\sqrt{-b}\log\left(-\frac{3\,b^2x^4+}{}\right.}{16\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}\right.$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*
a*b*x^2 + a^2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x
^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(
a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^
2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*arctan(1/2*sqrt(
2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(a^2*b^
3*x^4 + 2*a^3*b^2*x^2 + a^4*b)]
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{-(-a+bx^2)\,(a+bx^2)}\,(a+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{a^2-b^2\,x^4}\,{(b\,x^2+a)}^{3/2}}\,dx$$

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(3/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(3/2)), x)
```

3.204 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 168

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/8*x*(-b*x^2+a)/a^2/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)+9/32*x*(-b*x^2+a)
/a^3/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/64*arctan(x*2^(1/2)*b^(1/2)/(-
b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^3*2^(1/2)/b^(1/2)/(-b^2*
x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1166, 425, 541, 12, 385, 211}

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}$$

```
[In] Int[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(8*a^2*(a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a - b
*x^2))/(32*a^3*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
```

```
qrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(32*Sqrt[2]*a^3
*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}(a+bx^2)^3}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a-bx^2)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} - \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{-7ab+2b^2x^2}{\sqrt{a-bx^2}(a+bx^2)^2}\,dx}{8a^2b\sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a-bx^2)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{19a^2b^2}{\sqrt{a-bx^2}(a+bx^2)}\,dx}{32a^4b^2\sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a-bx^2)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}(a+bx^2)}\,dx}{32a^2\sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a-bx^2)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a+2abx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{32a^2\sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a-bx^2)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a-bx^2}(13a+9bx^2)+19\sqrt{2}(a+bx^2)^2\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right.\right.}{64a^3\sqrt{b}\sqrt{a-bx^2}\left(a+bx^2\right)^{5/2}}$$

```
[In] Integrate[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2]*(13*a + 9*b*x^2) + 19*Sqr
t[2]*(a + b*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(64*a^3*Sq
rt[b]*Sqrt[a - b*x^2]*(a + b*x^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(138) = 276$.

Time = 0.23 (sec) , antiderivative size = 711, normalized size of antiderivative = 4.23

method	result
default	$-\frac{\sqrt{-b^{2}x^{4}+a^{2}}\, b^{\frac{9}{2}} \left(19\sqrt{2}\, \ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^{2}+a}-\sqrt{-ab}\,x+a\right)}{bx-\sqrt{-ab}}\right) b^{\frac{5}{2}} x^{4}\sqrt{a}-19\sqrt{2}\, \ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^{2}+a}+\sqrt{-ab}\,x+a\right)}{bx+\sqrt{-ab}}\right) b^{\frac{5}{2}} x^{4}\sqrt{a}+38\sqrt{2}\right.}{}$

```
[In] int(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*
x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*b^(5/2)*x^4*a^(1/2)-19*2
^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*
b)^(1/2)))*b^(5/2)*x^4*a^(1/2)+38*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a
)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*a^(3/2)*b^(3/2)*x^2-38*2^(1/2
)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1
/2)))*a^(3/2)*b^(3/2)*x^2+16*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*b^2*x^4*(-a
*b)^(1/2)-16*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1
/2))*b^2*x^4*(-a*b)^(1/2)-36*b^(3/2)*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x^3+19*2
^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*
b)^(1/2)))*a^(5/2)*b^(1/2)-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1
/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*a^(5/2)*b^(1/2)+32*arctan(b^(1/2)
*x/(-b*x^2+a)^(1/2))*a*b*x^2*(-a*b)^(1/2)-32*arctan(b^(1/2)*x/(1/b*(-b*x+(a
*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2))*a*b*x^2*(-a*b)^(1/2)-52*b^(1/2)*(-a*b)
^(1/2)*a*(-b*x^2+a)^(1/2)*x+16*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*a^2*(-a*b
)^(1/2)-16*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2
))*a^2*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/(-(-a*b)^(1/2)+(a*b)^
(1/2))^3/((-a*b)^(1/2)+(a*b)^(1/2))^3/(b*x+(-a*b)^(1/2))^2/(b*x-(-a*b)^(1/2
))^2/(-a*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.17

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{19\,\sqrt{2}(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-b}\log\left(-\frac{3\,b^2x^4+2\,abx^2-2\,\sqrt{2}\sqrt{-b^2x^4+a}}{b^2x^4+2\,abx^2+}\right.}{128\,(a^3b^4x^6+3\,a^4b^3x^4+3}\right.$$

$$-\frac{19\,\sqrt{2}(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{2\,(b^2x^3+abx)}\right)-2\,\sqrt{-b^2x^4+a^2}(9\,b^2x^3+13\,abx)\sqrt{}}{64\,(a^3b^4x^6+3\,a^4b^3x^4+3\,a^5b^2x^2+a^6b)}$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
[Out] [-1/128*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-b)*lo
g(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*
sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-b^2*x^4 + a^2)*(9*
b^2*x^3 + 13*a*b*x)*sqrt(b*x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b
^2*x^2 + a^6*b), -1/64*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3)*sqrt(b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)
/(b^2*x^3 + a*b*x)) - 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 + 13*a*b*x)*sqrt(b*
x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 + a^6*b)]
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(5/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(5/2)), x)
```

3.205 $\int \frac{\left(a-bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 152

$$\begin{aligned}\int \frac{\left(a-bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx =& -\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}}\\ &-\frac{x\left(a-bx^2\right)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}\end{aligned}$$

```
[Out] -1/4*x*(-b*x^2+a)^(3/2)*(b*x^2+a)/(-b^2*x^4+a^2)^(1/2)-9/8*a*x*(b*x^2+a)*(-
b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1166, 427, 396, 223, 212}

$$\begin{aligned}\int \frac{\left(a-bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx =& \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}\\ &-\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}}\end{aligned}$$

```
[In] Int[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-9*a*x*Sqrt[a - b*x^2]*(a + b*x^2))/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)^(3/2)*(a + b*x^2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
```

```
t[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^
2*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{\left(a-bx^2\right)^2}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}} \\
&= -\frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}}+\frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{5a^2b-9ab^2x^2}{\sqrt{a+bx^2}}\,dx}{4b\sqrt{a^2-b^2x^4}}
\end{aligned}
$$

$$= -\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{1}{8}\left(\frac{x(-11a+2bx^2)\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}} - \frac{19a^2\log(-a+bx^2)}{\sqrt{b}} + \frac{19a^2\log\left(abx-b^2x^3+\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}\right)}{\sqrt{b}}\right)$$

```
[In] Integrate[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] ((x*(-11*a + 2*b*x^2)*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2] - (19*a^2*Log[-a
 + b*x^2])/Sqrt[b] + (19*a^2*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*
Sqrt[a^2 - b^2*x^4]])/Sqrt[b])/8
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

method	result	siz
default	$\frac{\sqrt{-b^2x^4+a^2}\left(2b^{\frac{3}{2}}x^3\sqrt{b\,x^2+a}-11ax\sqrt{b}\,\sqrt{b\,x^2+a}+19\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)a^2\right)}{8\sqrt{-b\,x^2+a}\,\sqrt{b\,x^2+a}\,\sqrt{b}}$	94
risch	$\frac{x\left(-2b\,x^2+11a\right)\sqrt{b\,x^2+a}\,\sqrt{\frac{\left(-b\,x^2+a\right)\left(-b^2x^4+a^2\right)}{\left(b\,x^2-a\right)^2}}\,\left(b\,x^2-a\right)}{8\sqrt{-b\,x^2+a}\,\sqrt{-b^2x^4+a^2}}-\frac{19a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\sqrt{\frac{\left(-b\,x^2+a\right)\left(-b^2x^4+a^2\right)}{\left(b\,x^2-a\right)^2}}\,\left(b\,x^2-a\right)}{8\sqrt{b}\,\sqrt{-b\,x^2+a}\,\sqrt{-b^2x^4+a^2}}$	18

```
[In] int((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-b^2*x^4+a^2)^(1/2)*(2*b^(3/2)*x^3*(b*x^2+a)^(1/2)-11*a*x*b^(1/2)*(b*x
^2+a)^(1/2)+19*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*a^2)/(-b*x^2+a)^(1/2)/(b*x^2+a
)^(1/2)/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.74

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \Bigg[\frac{19\,(a^2bx^2-a^3)\sqrt{b}\log\left(\frac{2\,b^2x^4-abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{bx^2-a}\right)-2\,\sqrt{-b^2x^4+a^2}(2\,b^2x^3 -}{16\,(b^2x^2-ab)}$$

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(19*(a^2*b*x^2 - a^3)*sqrt(b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*
x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b*x^2 - a)) - 2*sqrt(-b^2*x^4
 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(-b*x^2 + a))/(b^2*x^2 - a*b), 1/8*(19*(
a^2*b*x^2 - a^3)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt
(-b)/(b^2*x^3 - a*b*x)) - sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(
-b*x^2 + a))/(b^2*x^2 - a*b)]
```

Sympy [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(a-bx^2)^{\frac{5}{2}}}{\sqrt{-(-a+bx^2)\,(a+bx^2)}}\,dx$$

```
[In] integrate((-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a - b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx=\int \frac{(-bx^2+a)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx=\int \frac{(a-b\,x^2)^{5/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

```
[In] int((a - b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a - b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.206 $\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 109

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} \, dx = -\frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}} + \frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/2*x*(b*x^2+a)*(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/2*a*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1166, 396, 223, 212}

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}} \, dx = \frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}} - \frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

```
[In] Int[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/2*(x*Sqrt[a - b*x^2]*(a + b*x^2))/Sqrt[a^2 - b^2*x^4] + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]*Sqrt[
a^2 - b^2*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{a-bx^2}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}}+\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$
\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{1}{2}\left(-\frac{x\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}}-\frac{3a\log(-a+bx^2)}{\sqrt{b}}\right.
\left.+\frac{3a\log\left(abx-b^2x^3+\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}\right)}{\sqrt{b}}\right)
$$

```
[In] Integrate[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-((x*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2]) - (3*a*Log[-a + b*x^2])/Sqrt[b]
 + (3*a*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]])
/Sqrt[b])/2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, \left(-x \sqrt{b}\, \sqrt{b\,x^{2}+a}+3 \ln\left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right) a\right)}{2 \sqrt{-b\,x^{2}+a}\, \sqrt{b\,x^{2}+a}\, \sqrt{b}}$	74
risch	$\frac{x \sqrt{b\,x^{2}+a}\, \sqrt{\frac{\left(-b\,x^{2}+a\right) \left(-b^{2} x^{4}+a^{2}\right)}{\left(b\,x^{2}-a\right)^{2}}}\, \left(b\,x^{2}-a\right)}{2 \sqrt{-b\,x^{2}+a}\, \sqrt{-b^{2} x^{4}+a^{2}}}-\frac{3 a \ln\left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right) \sqrt{\frac{\left(-b\,x^{2}+a\right) \left(-b^{2} x^{4}+a^{2}\right)}{\left(b\,x^{2}-a\right)^{2}}}\, \left(b\,x^{2}-a\right)}{2 \sqrt{b}\, \sqrt{-b\,x^{2}+a}\, \sqrt{-b^{2} x^{4}+a^{2}}}$	170

```
[In] int((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b^2*x^4+a^2)^(1/2)*(-x*b^(1/2)*(b*x^2+a)^(1/2)+3*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))*a)/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.17

$$\int \frac{\left(a-b x^{2}\right)^{3/2}}{\sqrt{a^{2}-b^{2} x^{4}}}\,dx=\Bigg[\frac{2\,\sqrt{-b^{2} x^{4}+a^{2}}\sqrt{-b x^{2}+a}\,b x+3\,\left(a b x^{2}-a^{2}\right)\sqrt{b}\log\Bigg(\frac{2\,b^{2} x^{4}-a b x^{2}-2\,\sqrt{-b^{2} x^{4}+a^{2}}\sqrt{-b x^{2}+a}\sqrt{b} x-}{b x^{2}-a}}{4\,\left(b^{2} x^{2}-a b\right)}$$

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(
b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(
b)*x - a^2)/(b*x^2 - a)))/(b^2*x^2 - a*b), 1/2*(sqrt(-b^2*x^4 + a^2)*sqrt(-
b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqr
t(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(b^2*x^2 - a*b)]
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx=\int \frac{(a-bx^2)^{\frac{3}{2}}}{\sqrt{-(-a+bx^2)\,(a+bx^2)}}\,dx$$

```
[In] integrate((-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral((a - b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx=\int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx=\int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx=\int \frac{(a-b\,x^2)^{3/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

```
[In] int((a - b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a - b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.207 $\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 64

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)
/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1166, 223, 212}

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqr
t[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{-\log\left(-a+bx^2\right)+\log\left(abx-b^2x^3+\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}\right)}{\sqrt{b}}$$

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
[Out] (-Log[-a + b*x^2] + Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2
- b^2*x^4]])/Sqrt[b]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\sqrt{-b^2x^4+a^2}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{-b\,x^2+a}\,\sqrt{b\,x^2+a}\,\sqrt{b}}$	54

```
[In] int((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)/(b*x^2+a)^(1/2)*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[\frac{\log\left(\frac{2\,b^2x^4-abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{bx^2-a}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{b^2x^3-abx}\right)}{b}\right]$$

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqr
t(b)*x - a^2)/(b*x^2 - a))/sqrt(b), sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sq
rt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x))/b]
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

```
[In] integrate((-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

```
[In] int((a - b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a - b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.208 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 77

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(
1/2)/a*2^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1166, 385, 214}

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a-2abx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$
\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a^2-b^2x^4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}
$$

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[2]
*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(62) = 124.

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.32

method	result
default	$\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{b}\,\bigg(\sqrt{a}\,\sqrt{2}\,\ln\bigg(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b\,x^2+a}-x\sqrt{ab}+a\right)}{bx+\sqrt{ab}}\bigg)\sqrt{b}-\sqrt{a}\,\sqrt{2}\,\ln\bigg(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b\,x^2+a}+x\sqrt{ab}+a\right)}{bx-\sqrt{ab}}\bigg)\sqrt{b}+2\sqrt{ab}\,\ln\bigg(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}+}{\sqrt{b}}}{2\sqrt{-b\,x^2+a}\,\sqrt{b\,x^2+a}\,\sqrt{ab}\,\left(\sqrt{-ab}-\sqrt{ab}\right)\left(\sqrt{-ab}+\sqrt{ab}\right)}$

```
[In] int(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(
b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(1/2)*
ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))
*b^(1/2)+2*(a*b)^(1/2)*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))-2*ln((b^(1
/2)*(-1/b*(-b*x+(-a*b)^(1/2))*(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*(a*b)
^(1/2))/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/(a*b)^(1/2)/((-a*b)^(1/2)-(a*b)^(1
/2))/((-a*b)^(1/2)+(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.01

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{3\,b^2x^4-2\,abx^2-2\,\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{b^2x^4-2\,abx^2+a^2}\right)}{4\,a\sqrt{b}},\ \frac{\sqrt{2}\sqrt{-b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{2\,(b^2x^3-abx)}\right)}{2\,ab}\right]$$

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*s
qrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2))/(a*sqrt(b)),
1/2*sqrt(2)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 +
a)*sqrt(-b)/(b^2*x^3 - a*b*x))/(a*b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\sqrt{a-bx^2}}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a - b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\sqrt{a-b\,x^2}}\,dx$$

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(1/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(1/2)), x)
```

3.209 $\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 124

$$\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx=\frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/4*x*(b*x^2+a)/a^2/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/8*arctanh(x*2^(
1/2)*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^2*2^(1/2)/
b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1166, 390, 385, 214}

$$\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx=\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}+\frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}$$

```
[In] Int[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(4*a^2*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*Sqrt
[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)^2\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)\sqrt{a+bx^2}}\,dx}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a-2abx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a+bx^2}+3\sqrt{2}(a-bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)}{8a^2\sqrt{b}\,(a-bx^2)^{3/2}\sqrt{a+bx^2}}$$

```
[In] Integrate[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a + b*x^2] + 3*Sqrt[2]*(a - b*x^2)*A
rcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(8*a^2*Sqrt[b]*(a - b*x^2)^(3
/2)*Sqrt[a + b*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(100) = 200.

Time = 0.23 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.02

method	result
default	$-\frac{\sqrt{-b^2x^4+a^2}\, b^{\frac{5}{2}} \left(-3\sqrt{2}\, \ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b\,x^2+a}+x\sqrt{ab}+a\right)}{bx-\sqrt{ab}}\right) b^{\frac{3}{2}} x^2 \sqrt{a}+3\sqrt{2}\, \ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b\,x^2+a}-x\sqrt{ab}+a\right)}{bx+\sqrt{ab}}\right) b^{\frac{3}{2}} x^2 \sqrt{a}+3\sqrt{2}\, \ln\left(\frac{2b(}{}\right.\right.}{}$

```
[In] int(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(-3*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^
2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*b^(3/2)*x^2*a^(1/2)+3*2^(1/2
)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)
))*b^(3/2)*x^2*a^(1/2)+3*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+x*
(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*a^(3/2)*b^(1/2)-3*2^(1/2)*ln(2*b*(2^(1/2)
*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*a^(3/2)*b^(1/2
)+4*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*b*x^2*(a*b)^(1/2)-4*ln((b^(1/
2)*(-1/b*(-b*x+(-a*b)^(1/2))*(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*b*x^2*
(a*b)^(1/2)+4*(b*x^2+a)^(1/2)*b^(1/2)*(a*b)^(1/2)*x-4*ln((b^(1/2)*(b*x^2+a)
^(1/2)+b*x)/b^(1/2))*a*(a*b)^(1/2)+4*ln((b^(1/2)*(-1/b*(-b*x+(-a*b)^(1/2))*
(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a*(a*b)^(1/2))/(-b*x^2+a)^(1/2)/(b*
x^2+a)^(1/2)/(-(-a*b)^(1/2)+(a*b)^(1/2))^2/((-a*b)^(1/2)+(a*b)^(1/2))^2/(b*
x-(a*b)^(1/2))/(b*x+(a*b)^(1/2))/(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.44

$$\int \frac{1}{\left(a-bx^{2}\right)^{3/2}\sqrt{a^{2}-b^{2}x^{4}}}\,dx=\left[\frac{4\sqrt{-b^{2}x^{4}+a^{2}}\sqrt{-bx^{2}+a}\,bx+3\sqrt{2}\left(b^{2}x^{4}-2\,abx^{2}+a^{2}\right)\sqrt{b}\log\left(-\frac{3\,b^{2}x^{4}-}{}\right.}{16\left(a^{2}b^{3}x^{4}-2\,a^{3}b^{2}x^{2}+a^{4}b\right)}\right.$$

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2
*a*b*x^2 + a^2)*sqrt(b)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x
^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)))/(
a^2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x
^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(1/2*sqr
t(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(a^
2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b)]
```

Sympy [F]

$$\int \frac{1}{\left(a-bx^{2}\right)^{3/2}\sqrt{a^{2}-b^{2}x^{4}}}\,dx=\int \frac{1}{\sqrt{-\left(-a+bx^{2}\right)\left(a+bx^{2}\right)}\left(a-bx^{2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a-bx^{2}\right)^{3/2}\sqrt{a^{2}-b^{2}x^{4}}}\,dx=\int \frac{1}{\sqrt{-b^{2}x^{4}+a^{2}}\left(-bx^{2}+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{-b^2x^4+a^2}(-bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,{\left(a-b\,x^2\right)}^{3/2}}\,dx$$

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(3/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(3/2)), x)
```

3.210 $\int \frac{1}{\left(a-bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 167

$$\int \frac{1}{\left(a-bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/8*x*(b*x^2+a)/a^2/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)+9/32*x*(b*x^2+a)/
a^3/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/64*arctanh(x*2^(1/2)*b^(1/2)/(
b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^3*2^(1/2)/b^(1/2)/(-b^2*
x^4+a^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1166, 425, 541, 12, 385, 214}

$$\int \frac{1}{\left(a-bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}+\frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}$$

```
[In] Int[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(8*a^2*(a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a + b
*x^2))/(32*a^3*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
```

```
qrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32*Sqrt[2]*a^
3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)^3\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a+bx^2)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{7ab+2b^2x^2}{(a-bx^2)^2\sqrt{a+bx^2}}\,dx}{8a^2b\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a+bx^2)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\\
&\quad+\frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{19a^2b^2}{(a-bx^2)\sqrt{a+bx^2}}\,dx}{32a^4b^2\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a+bx^2)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\\
&\quad+\frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)\sqrt{a+bx^2}}\,dx}{32a^2\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a+bx^2)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\\
&\quad+\frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a-2abx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{32a^2\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a+bx^2)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\\
&\quad+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a-bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a^2-b^2x^4}\Big(2\sqrt{b}x(13a-9bx^2)\sqrt{a+bx^2}+19\sqrt{2}(a-bx^2)^2\,\text{arctanh}\Big(\frac{\sqrt{2}\sqrt{b}}{\sqrt{a+b}}}{64a^3\sqrt{b}\left(a-bx^2\right)^{5/2}\sqrt{a+bx^2}}$$

```
[In] Integrate[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*(13*a - 9*b*x^2)*Sqrt[a + b*x^2] + 19*Sqr
t[2]*(a - b*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(64*a^3*S
qrt[b]*(a - b*x^2)^(5/2)*Sqrt[a + b*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(137) = 274$.

Time = 0.24 (sec) , antiderivative size = 728, normalized size of antiderivative = 4.36

method	result
default	$-\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, b^{\frac{9}{2}} \left(19 \sqrt{2}\, \ln\left(\frac{2 b \left(\sqrt{2}\, \sqrt{a}\, \sqrt{b\, x^{2}+a}+x \sqrt{a b}+a\right)}{b x -\sqrt{a b}}\right) b^{\frac{5}{2}} x^{4} \sqrt{a}-19 \sqrt{2}\, \ln\left(\frac{2 b \left(\sqrt{2}\, \sqrt{a}\, \sqrt{b\, x^{2}+a}-x \sqrt{a b}+a\right)}{b x +\sqrt{a b}}\right) b^{\frac{5}{2}} x^{4} \sqrt{a}-38 \sqrt{2}\, \ln\left(\right.\right.}{}$

```
[In] int(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x
^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*b^(5/2)*x^4*a^(1/2)-19*2^(1
/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/
2)))*b^(5/2)*x^4*a^(1/2)-38*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)
+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*a^(3/2)*b^(3/2)*x^2+38*2^(1/2)*ln(2*b*
(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*a^(3/2
)*b^(3/2)*x^2+16*ln((b^(1/2)*(-1/b*(-b*x+(-a*b)^(1/2))*(b*x+(-a*b)^(1/2)))^
(1/2)+b*x)/b^(1/2))*b^2*x^4*(a*b)^(1/2)-16*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)
/b^(1/2))*b^2*x^4*(a*b)^(1/2)-36*b^(3/2)*(a*b)^(1/2)*(b*x^2+a)^(1/2)*x^3+19
*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b
)^(1/2)))*a^(5/2)*b^(1/2)-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2
)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*a^(5/2)*b^(1/2)-32*ln((b^(1/2)*(-1/b*
(-b*x+(-a*b)^(1/2))*(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a*b*x^2*(a*b)^(
1/2)+32*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a*b*x^2*(a*b)^(1/2)+52*b^
(1/2)*(a*b)^(1/2)*a*(b*x^2+a)^(1/2)*x+16*ln((b^(1/2)*(-1/b*(-b*x+(-a*b)^(1/
2))*(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a^2*(a*b)^(1/2)-16*ln((b^(1/2)*
(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a^2*(a*b)^(1/2))/(-b*x^2+a)^(1/2)/(b*x^2+a)^(
1/2)/((-a*b)^(1/2)-(a*b)^(1/2))^3/((-a*b)^(1/2)+(a*b)^(1/2))^3/(b*x+(a*b)^(
1/2))^2/(b*x-(a*b)^(1/2))^2/(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.25

$$\int \frac{1}{\left(a-bx^2\right)^{5/2} \sqrt{a^2-b^2x^4}}\,dx = \left[\frac{19\,\sqrt{2}(b^3x^6-3\,ab^2x^4+3\,a^2bx^2-a^3)\sqrt{b}\log\left(-\frac{3\,b^2x^4-2\,abx^2-2\,\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-}}{b^2x^4-2\,abx^2+a^2}\right.}{128\,(a^3b^4x^6-3\,a^4b^3x^4+3\,a}\right.$$

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)*sqrt(b)*log(
-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*s
```

```
qrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)) + 4*sqrt(-b^2*x^4 + a^2)*(9*b^
2*x^3 - 13*a*b*x)*sqrt(-b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^
2*x^2 - a^6*b), 1/64*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3
)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b
)/(b^2*x^3 - a*b*x)) + 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 - 13*a*b*x)*sqrt(-
b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 - a^6*b)]
```

Sympy [F]

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a-bx^2\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(-bx^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(-bx^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\left(a-b\,x^2\right)^{5/2}}\,dx$$

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(5/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(5/2)), x)
```

3.211 $\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\operatorname{arcsinh}(x)}{\sqrt{-1+x^4}}$$

```
[Out] arcsinh(x)*(x^2-1)^(1/2)*(x^2+1)^(1/2)/(x^4-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1166, 221}

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx=\frac{\sqrt{x^2-1}\sqrt{x^2+1}\operatorname{arcsinh}(x)}{\sqrt{x^4-1}}$$

```
[In] Int[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{\sqrt{-1+x^4}}\\
&= \frac{\sqrt{-1+x^2}\sqrt{1+x^2}\sinh^{-1}(x)}{\sqrt{-1+x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$
\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = -\log\left(1-x^2\right)+\log\left(-x+x^3+\sqrt{-1+x^2}\sqrt{-1+x^4}\right)
$$

```
[In] Integrate[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
[Out] -Log[1 - x^2] + Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{x^4-1}\,\operatorname{arcsinh}(x)}{\sqrt{x^2-1}\,\sqrt{x^2+1}}$	25

```
[In] int((x^2-1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(24) = 48.

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.43

$$
\begin{aligned}
\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx &= \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)\\
&\quad-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)
\end{aligned}
$$

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x))
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{(x-1)(x+1)}}{\sqrt{(x-1)(x+1)(x^2+1)}}\,dx$$

```
[In] integrate((x**2-1)**(1/2)/(x**4-1)**(1/2),x)
[Out] Integral(sqrt((x - 1)*(x + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

```
[In] int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2),x)
[Out] int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

3.212 $\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = -\frac{\sqrt{-1+x^4}\arcsin(x)}{\sqrt{1-x^4}}$$

```
[Out] -arcsin(x)*(x^4-1)^(1/2)/(-x^4+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1166, 223, 212}

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}$$

```
[In] Int[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^4}} \\
&= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}} \\
&= \frac{\sqrt{-1+x^2}\sqrt{1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = -\log\left(1+x^2\right) + \log\left(x + x^3 + \sqrt{1+x^2}\sqrt{-1+x^4}\right)$$

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
[Out] -Log[1 + x^2] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{\sqrt{x^4-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}\,\sqrt{x^2-1}}$	33

```
[In] int((x^2+1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(x^2+1)^(1/2)*(x^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(20) = 40$.

Time $= 0.24$ (sec) , antiderivative size $= 65$, normalized size of antiderivative $= 2.71$

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right) - \frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)$$

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x))
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{(x-1)(x+1)(x^2+1)}}\,dx$$

```
[In] integrate((x**2+1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

```
[In] int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

3.213 $\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 73

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=-\frac{\sqrt{-1+x^4}\arcsin(x)}{\sqrt{1-x^2}\sqrt{1+x^2}}+\frac{\sqrt{-1+x^2}\sqrt{-1+x^4}\operatorname{arcsinh}(x)}{(1-x^2)\sqrt{1+x^2}}$$

```
[Out] -arcsin(x)*(x^4-1)^(1/2)/(-x^2+1)^(1/2)/(x^2+1)^(1/2)+arcsinh(x)*(x^2-1)^(1
/2)*(x^4-1)^(1/2)/(-x^2+1)/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6874, 1166, 221, 223, 212}

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\frac{\sqrt{x^2-1}\sqrt{x^2+1}\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}\operatorname{arcsinh}(x)}{\sqrt{x^4-1}}$$

```
[In] Int[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] -((Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]) + (Sqrt[-1 + x^
2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}+\frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\right) dx \\
&= -\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\, dx+\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\, dx \\
&= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\, dx}{\sqrt{-1+x^4}}-\frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1+x^2}}\, dx}{\sqrt{-1+x^4}} \\
&= -\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\sinh^{-1}(x)}{\sqrt{-1+x^4}}+\frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\, dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}} \\
&= -\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\sinh^{-1}(x)}{\sqrt{-1+x^4}}+\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \log\left(1-x^2\right)-\log\left(1+x^2\right) - \log\left(-x+x^3+\sqrt{-1+x^2}\sqrt{-1+x^4}\right) + \log\left(x+x^3+\sqrt{1+x^2}\sqrt{-1+x^4}\right)$$

```
[In] Integrate[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] Log[1 - x^2] - Log[1 + x^2] - Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
 + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{x^4-1}\,\operatorname{arcsinh}(x)}{\sqrt{x^2-1}\,\sqrt{x^2+1}}+\frac{\sqrt{x^4-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}\,\sqrt{x^2-1}}$	59

```
[In] int((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)+1/(x^2+1)^(1/2)*(x^
4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(53) = 106$.

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.88

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right) - \frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right) - \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right) + \frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)$$

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log((x^3 + sqrt(x^4 - 1
)*sqrt(x^2 - 1) - x)/(x^3 - x)) + 1/2*log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 -
1) - x)/(x^3 - x))
```

Sympy [F]

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{-\sqrt{x^2-1}+\sqrt{x^2+1}}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

```
[In] integrate((-(x**2-1)**(1/2)+(x**2+1)**(1/2))/(x**4-1)**(1/2),x)
```

```
[Out] Integral((-sqrt(x**2 - 1) + sqrt(x**2 + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)
), x)
```

Maxima [F]

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int -\frac{\sqrt{x^2-1}-\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

```
[In] int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2),x)
[Out] int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2), x)
```

3.214 $\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} \, dx$

Optimal result

Integrand size = 39, antiderivative size = 121

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} \, dx = \frac{(7c^2d^2-5bcde+b^2e^2)\,x}{c^3} + \frac{e(4cd-be)x^3}{3c^2} + \frac{e^2x^5}{5c} - \frac{(2cd-be)^3 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}$$

```
[Out] (b^2*e^2-5*b*c*d*e+7*c^2*d^2)*x/c^3+1/3*e*(-b*e+4*c*d)*x^3/c^2+1/5*e^2*x^5/
c-(-b*e+2*c*d)^3*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(7/2)/e^(1/2
)/(-b*e+c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1163, 398, 214}

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} \, dx = -\frac{(2cd-be)^3 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}} + \frac{x(b^2e^2-5bcde+7c^2d^2)}{c^3} + \frac{ex^3(4cd-be)}{3c^2} + \frac{e^2x^5}{5c}$$

```
[In] Int[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)/(3*c^2) +
 (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d -
b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(d+ex^2\right)^3}{\frac{-cd^2+bde}{d}+cex^2}\,dx\\
&= \int \left(\frac{7c^2d^2-5bcde+b^2e^2}{c^3}+\frac{e(4cd-be)x^2}{c^2}+\frac{e^2x^4}{c}+\frac{8c^3d^3-12bc^2d^2e+6b^2cde^2-b^3e^3}{c^3\left(-cd+be+cex^2\right)}\right)dx\\
&= \frac{\left(7c^2d^2-5bcde+b^2e^2\right)x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}+\frac{(2cd-be)^3\int\frac{1}{-cd+be+cex^2}\,dx}{c^3}\\
&= \frac{\left(7c^2d^2-5bcde+b^2e^2\right)x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex^2\right)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = -\frac{\left(-7c^2d^2+5bcde-b^2e^2\right)x}{c^3}-\frac{e(-4cd+be)x^3}{3c^2}+\frac{e^2x^5}{5c}-\frac{(-2cd+be)^3\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{c^{7/2}\sqrt{e}\sqrt{-cd+be}}$$

```
[In] Integrate[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((-7*c^2*d^2 + 5*b*c*d*e - b^2*e^2)*x)/c^3) - (e*(-4*c*d + b*e)*x^3)/(3*c
^2) + (e^2*x^5)/(5*c) - ((-2*c*d + b*e)^3*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-
(c*d) + b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11

method	result
default	$\frac{\frac{1}{5}e^{2}x^{5}c^{2}-\frac{1}{3}bc\,e^{2}x^{3}+\frac{4}{3}c^{2}de\,x^{3}+b^{2}e^{2}x-5bcdex+7c^{2}d^{2}x}{c^{3}}+\frac{\left(-b^{3}e^{3}+6b^{2}cd\,e^{2}-12b\,c^{2}d^{2}e+8c^{3}d^{3}\right)\arctan\left(\frac{xce}{\sqrt{\left(be-cd\right)ec}}\right)}{c^{3}\sqrt{\left(be-cd\right)ec}}$
risch	$\frac{e^{2}x^{5}}{5c}-\frac{b\,e^{2}x^{3}}{3c^{2}}+\frac{4de\,x^{3}}{3c}+\frac{b^{2}e^{2}x}{c^{3}}-\frac{5bdex}{c^{2}}+\frac{7d^{2}x}{c}-\frac{\ln\left(xce-\sqrt{-\left(be-cd\right)ec}\right)b^{3}e^{3}}{2c^{3}\sqrt{-\left(be-cd\right)ec}}+\frac{3\ln\left(xce-\sqrt{-\left(be-cd\right)ec}\right)b^{2}d\,e^{2}}{c^{2}\sqrt{-\left(be-cd\right)ec}}-$

```
[In] int((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/5*e^2*x^5*c^2-1/3*b*c*e^2*x^3+4/3*c^2*d*e*x^3+b^2*e^2*x-5*b*c*d*e*
x+7*c^2*d^2*x)+(-b^3*e^3+6*b^2*c*d*e^2-12*b*c^2*d^2*e+8*c^3*d^3)/c^3/((b*e-
c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(105) = 210.

Time = 0.26 (sec) , antiderivative size = 446, normalized size of antiderivative = 3.69

$$\int \frac{\left(d+ex^2\right)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$$
$$=\left[\frac{6\left(c^4de^3-bc^3e^4\right)x^5+10\left(4\,c^4d^2e^2-5\,bc^3de^3+b^2c^2e^4\right)x^3-15\left(8\,c^3d^3-12\,bc^2d^2e+6\,b^2cde^2-b^3e^3\right)\sqrt{c^2}}{30\left(c^5de-bc^4e^2\right)}\right.$$

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/30*(6*(c^4*d*e^3 - b*c^3*e^4)*x^5 + 10*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 +
b^2*c^2*e^4)*x^3 - 15*(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3
)*sqrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e + 2*sqrt(c^2*d*e - b*c*e
^2)*x)/(c*e*x^2 - c*d + b*e)) + 30*(7*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*
c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2), 1/15*(3*(c^4*d*e^3 - b*c^3
*e^4)*x^5 + 5*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 + b^2*c^2*e^4)*x^3 - 15*(8*c^3
*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*sqrt(-c^2*d*e + b*c*e^2)*a
rctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) + 15*(7*c^4*d^3*e - 12*b*c^3
*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(110) = 220$.

Time = 0.45 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.85

$$\begin{aligned}
&\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx \\
&= x^3\left(-\frac{be^2}{3c^2}+\frac{4de}{3c}\right)+x\left(\frac{b^2e^2}{c^3}-\frac{5bde}{c^2}+\frac{7d^2}{c}\right) \\
&\quad+\frac{\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3\log\left(x+\frac{-bc^3e\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3+c^4d\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3}{b^3e^3-6b^2cde^2+12bc^2d^2e-8c^3d^3}\right)}{2} \\
&\quad-\frac{\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3\log\left(x+\frac{bc^3e\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3-c^4d\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3}{b^3e^3-6b^2cde^2+12bc^2d^2e-8c^3d^3}\right)}{2} \\
&\quad+\frac{e^2x^5}{5c}
\end{aligned}$$

```
[In] integrate((e*x**2+d)**4/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x**3*(-b*e**2/(3*c**2) + 4*d*e/(3*c)) + x*(b**2*e**2/c**3 - 5*b*d*e/c**2 +
7*d**2/c) + sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log(x + (-b*c**3
*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 + c**4*d*sqrt(-1/(c**7*e*
(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 + 12*b*c**2*d*
*2*e - 8*c**3*d**3))/2 - sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log
(x + (b*c**3*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 - c**4*d*sqrt
(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 +
12*b*c**2*d**2*e - 8*c**3*d**3))/2 + e**2*x**5/(5*c)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{(8\,c^3d^3-12\,bc^2d^2e+6\,b^2cde^2-b^3e^3)\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}c^3} + \frac{3\,c^4e^7x^5+20\,c^4de^6x^3-5\,bc^3e^7x^3+105\,c^4d^2e^5x-75\,bc^3de^6x+15\,b^2c^2e^7x}{15\,c^5e^5}$$

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] (8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*arctan(c*e*x/sqrt(-c
^2*d*e + b*c*e^2))/(sqrt(-c^2*d*e + b*c*e^2)*c^3) + 1/15*(3*c^4*e^7*x^5 + 2
0*c^4*d*e^6*x^3 - 5*b*c^3*e^7*x^3 + 105*c^4*d^2*e^5*x - 75*b*c^3*d*e^6*x +
15*b^2*c^2*e^7*x)/(c^5*e^5)
```

Mupad [B] (verification not implemented)

Time = 7.96 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = x\left(\frac{3\,d^2}{c}+\frac{\left(\frac{e\,(b\,e-c\,d)}{c^2}-\frac{3\,d\,e}{c}\right)\,(b\,e-c\,d)}{c\,e}\right)-x^3\left(\frac{e\,(b\,e-c\,d)}{3\,c^2}-\frac{d\,e}{c}\right) + \frac{e^2\,x^5}{5\,c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x\,(b\,e-2\,c\,d)^3}{\sqrt{b\,e^2-c\,d\,e}\,(b^3\,e^3-6\,b^2\,c\,d\,e^2+12\,b\,c^2\,d^2\,e-8\,c^3\,d^3)}\right)\,(b\,e-2\,c\,d)^3}{c^{7/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

```
[In] int((d + e*x^2)^4/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] x*((3*d^2)/c + (((e*(b*e - c*d))/c^2 - (3*d*e)/c)*(b*e - c*d))/(c*e)) - x^3
*((e*(b*e - c*d))/(3*c^2) - (d*e)/c) + (e^2*x^5)/(5*c) - (atan((c^(1/2)*e*x
*(b*e - 2*c*d)^3)/((b*e^2 - c*d*e)^(1/2)*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^
2*e - 6*b^2*c*d*e^2)))*(b*e - 2*c*d)^3)/(c^(7/2)*(b*e^2 - c*d*e)^(1/2))
```

3.215 $\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 86

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}-\frac{(2cd-be)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}$$

```
[Out] (-b*e+3*c*d)*x/c^2+1/3*e*x^3/c-(-b*e+2*c*d)^2*arctanh(x*c^(1/2)*e^(1/2)/(-b
*e+c*d)^(1/2))/c^(5/2)/e^(1/2)/(-b*e+c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1163, 398, 214}

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{(2cd-be)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}+\frac{x(3cd-be)}{c^2}+\frac{ex^3}{3c}$$

```
[In] Int[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[(Sqrt[c]*S
qrt[e]*x)/Sqrt[c*d - b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d+ex^2)^2}{\frac{-cd^2+bde}{d}+cex^2}\,dx\\ &= \int \left(\frac{3cd-be}{c^2}+\frac{ex^2}{c}+\frac{4c^2d^2-4bcde+b^2e^2}{c^2(-cd+be+cex^2)}\right)dx\\ &= \frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}+\frac{(2cd-be)^2\int\frac{1}{-cd+be+cex^2}\,dx}{c^2}\\ &= \frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = -\frac{(-3cd+be)x}{c^2}+\frac{ex^3}{3c}+\frac{(-2cd+be)^2\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{c^{5/2}\sqrt{e}\sqrt{-cd+be}}$$

```
[In] Integrate[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
[Out] -(((-3*c*d + b*e)*x)/c^2) + (e*x^3)/(3*c) + ((-2*c*d + b*e)^2*ArcTan[(Sqrt[
c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

method	result
default	$-\frac{-\frac{1}{3}c\,x^3e+bex-3cdx}{c^2}+\frac{\left(b^2e^2-4bcde+4c^2d^2\right)\arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{c^2\sqrt{(be-cd)ec}}$
risch	$\frac{e\,x^3}{3c}-\frac{bex}{c^2}+\frac{3dx}{c}-\frac{\ln\left(xce+\sqrt{-(be-cd)ec}\right)b^2e^2}{2c^2\sqrt{-(be-cd)ec}}+\frac{2\ln\left(xce+\sqrt{-(be-cd)ec}\right)bde}{c\sqrt{-(be-cd)ec}}-\frac{2\ln\left(xce+\sqrt{-(be-cd)ec}\right)d^2}{\sqrt{-(be-cd)ec}}+\frac{\ln(-xce}{2c}$

```
[In] int((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/3*c*x^3*e+b*e*x-3*c*d*x)+(b^2*e^2-4*b*c*d*e+4*c^2*d^2)/c^2/((b*e
-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.62

$$\int \frac{\left(d+ex^2\right)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$$

$$=\left[\frac{2\left(c^3de^2-bc^2e^3\right)x^3+3\left(4\,c^2d^2-4\,bcde+b^2e^2\right)\sqrt{c^2de-bce^2}\log\left(\frac{cex^2+cd-be-2\,\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)+6\left(3\,c^3d^2e-4\right.}{6\left(c^4de-bc^3e^2\right)}\right.$$

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/6*(2*(c^3*d*e^2 - b*c^2*e^3)*x^3 + 3*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*s
qrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^2)
*x)/(c*e*x^2 - c*d + b*e)) + 6*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*x)
/(c^4*d*e - b*c^3*e^2), 1/3*((c^3*d*e^2 - b*c^2*e^3)*x^3 - 3*(4*c^2*d^2 - 4
*b*c*d*e + b^2*e^2)*sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^
2)*x/(c*d - b*e)) + 3*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*x)/(c^4*d*e
 - b*c^3*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(75) = 150.

Time = 0.35 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.20

$$
\begin{aligned}
&\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx \\
&= x\left(-\frac{be}{c^2}+\frac{3d}{c}\right) \\
&\quad - \frac{\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-2cd)^2\log\left(x+\frac{-bc^2e\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2+c^3d\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2} \\
&\quad + \frac{\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-2cd)^2\log\left(x+\frac{bc^2e\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2-c^3d\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2}+\frac{ex^3}{3c}
\end{aligned}
$$

```
[In] integrate((e*x**2+d)**3/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x*(-b*e/c**2 + 3*d/c) - sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(
x + (-b*c**2*e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 + c**3*d*sqrt
(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2
*d**2))/2 + sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(x + (b*c**2*
e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 - c**3*d*sqrt(-1/(c**5*e*(
b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + e
*x**3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$
\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: ValueError}
$$

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the ’assume’ command before evaluation *may* h
elp (example of legal syntax is ’assume(e*(b*e-c*d)>0)’, see ‘assume?‘ for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{(4\,c^2d^2-4\,bcde+b^2e^2)\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}\,c^2} + \frac{c^2e^4x^3+9\,c^2de^3x-3\,bce^4x}{3\,c^3e^3}$$

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] (4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/(s
qrt(-c^2*d*e + b*c*e^2)*c^2) + 1/3*(c^2*e^4*x^3 + 9*c^2*d*e^3*x - 3*b*c*e^4
*x)/(c^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = x\left(\frac{2\,d}{c}-\frac{b\,e-c\,d}{c^2}\right)+\frac{e\,x^3}{3\,c} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x\,(b\,e-2\,c\,d)^2}{\sqrt{b\,e^2-c\,d\,e}\,(b^2\,e^2-4\,b\,c\,d\,e+4\,c^2\,d^2)}\right)\,(b\,e-2\,c\,d)^2}{c^{5/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

```
[In] int((d + e*x^2)^3/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] x*((2*d)/c - (b*e - c*d)/c^2) + (e*x^3)/(3*c) + (atan((c^(1/2)*e*x*(b*e - 2
*c*d)^2)/((b*e^2 - c*d*e)^(1/2)*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)))*(b*e -
2*c*d)^2)/(c^(5/2)*(b*e^2 - c*d*e)^(1/2))
```

3.216 $\int \frac{\left(d+e x^2\right)^2}{-c d^2+b d e+b e^2 x^2+c e^2 x^4} \, dx$

Optimal result

Integrand size = 39, antiderivative size = 64

$$\int \frac{\left(d+e x^2\right)^2}{-c d^2+b d e+b e^2 x^2+c e^2 x^4} \, dx=\frac{x}{c}-\frac{(2 c d-b e) \text{arctanh}\left(\frac{\sqrt{c} \sqrt{e} x}{\sqrt{c d-b e}}\right)}{c^{3/2} \sqrt{e} \sqrt{c d-b e}}$$

```
[Out] x/c-(-b*e+2*c*d)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(3/2)/e^(1/2
)/(-b*e+c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1163, 396, 214}

$$\int \frac{\left(d+e x^2\right)^2}{-c d^2+b d e+b e^2 x^2+c e^2 x^4} \, dx=\frac{x}{c}-\frac{(2 c d-b e) \text{arctanh}\left(\frac{\sqrt{c} \sqrt{e} x}{\sqrt{c d-b e}}\right)}{c^{3/2} \sqrt{e} \sqrt{c d-b e}}$$

```
[In] Int[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^(3/2)
*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{d+ex^2}{\frac{-cd^2+bde}{d}+cex^2}\,dx\\ &= \frac{x}{c}-\frac{\left(-cde+\frac{e(-cd^2+bde)}{d}\right)\int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx}{ce}\\ &= \frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{x}{c}-\frac{(-2cd+be)\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{c^{3/2}\sqrt{e}\sqrt{-cd+be}}$$

```
[In] Integrate[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
[Out] x/c - ((-2*c*d + b*e)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(3
/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

method	result
default	$\frac{x}{c}+\frac{(-be+2cd)\arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{c\sqrt{(be-cd)ec}}$
risch	$\frac{x}{c}-\frac{\ln\left(xce-\sqrt{-(be-cd)ec}\right)be}{2c\sqrt{-(be-cd)ec}}+\frac{\ln\left(xce-\sqrt{-(be-cd)ec}\right)d}{\sqrt{-(be-cd)ec}}+\frac{\ln\left(-xce-\sqrt{-(be-cd)ec}\right)be}{2c\sqrt{-(be-cd)ec}}-\frac{\ln\left(-xce-\sqrt{-(be-cd)ec}\right)d}{\sqrt{-(be-cd)ec}}$

```
[In] int((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+(-b*e+2*c*d)/c/((b*e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.28

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \left[-\frac{\sqrt{c^2de-bce^2}(2\,cd-be)\log\left(\frac{cex^2+cd-be+2\,\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)-2\,(c^2de-bce^2)x}{2\,(c^3de-bc^2e^2)}, -\frac{\sqrt{-c^2de+bce^2}(2\,cd-be)\arctan\left(-\frac{\sqrt{-c^2de+bce^2}x}{cd-be}\right)-(c^2de-bce^2)x}{c^3de-bc^2e^2}\right]$$

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(sqrt(c^2*d*e - b*c*e^2)*(2*c*d - b*e)*log((c*e*x^2 + c*d - b*e + 2*s
qrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d + b*e)) - 2*(c^2*d*e - b*c*e^2)*x)
/(c^3*d*e - b*c^2*e^2), -(sqrt(-c^2*d*e + b*c*e^2)*(2*c*d - b*e)*arctan(-sq
rt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) - (c^2*d*e - b*c*e^2)*x)/(c^3*d*e - b
*c^2*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(54) = 108.

Time = 0.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.31

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)\log\left(x+\frac{-bce\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)+c^2d\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)}{be-2cd}\right)}{2} - \frac{\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)\log\left(x+\frac{bce\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)-c^2d\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)}{be-2cd}\right)}{2}+\frac{x}{c}$$

```
[In] integrate((e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (-b*c*e*sqrt(-1/(c**3*e
*(b*e - c*d)))*(b*e - 2*c*d) + c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e -
2*c*d))/(b*e - 2*c*d))/2 - sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(
x + (b*c*e*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d) - c**2*d*sqrt(-1/(c*
*3*e*(b*e - c*d)))*(b*e - 2*c*d))/(b*e - 2*c*d))/2 + x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(b*e-c*d)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{(2\,cd-be)\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}c}+\frac{x}{c}$$

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] (2*c*d - b*e)*arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/(sqrt(-c^2*d*e + b*c*e
^2)*c) + x/c
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{x}{c}-\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,e\,x}{\sqrt{b\,e^2-c\,d\,e}}\right)\,\left(b\,e-2\,c\,d\right)}{c^{3/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

```
[In] int((d + e*x^2)^2/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] x/c - (atan((c^(1/2)*e*x)/(b*e^2 - c*d*e)^(1/2))*(b*e - 2*c*d))/(c^(3/2)*(b
*e^2 - c*d*e)^(1/2))
```

3.217 $\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 49

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

```
[Out] -arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(1/2)/e^(1/2)/(-b*e+c*d)^(1/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1163, 214}

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

```
[In] Int[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[c*d -
b*e]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
```

```
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{-cd+be}}$$

```
[In] Integrate[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
[Out] ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[-(c*d)
 + b*e])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{\sqrt{(be-cd)ec}}$	33
risch	$-\frac{\ln\left(xce+\sqrt{-(be-cd)ec}\right)}{2\sqrt{-(be-cd)ec}}+\frac{\ln\left(-xce+\sqrt{-(be-cd)ec}\right)}{2\sqrt{-(be-cd)ec}}$	75

```
[In] int((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
[Out] 1/((b*e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.73

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \left[\frac{\log\left(\frac{cex^2+cd-be-2\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)}{2\sqrt{c^2de-bce^2}},\right.$$
$$\left. -\frac{\sqrt{-c^2de+bce^2}\arctan\left(-\frac{\sqrt{-c^2de+bce^2}x}{cd-be}\right)}{c^2de-bce^2}\right]$$

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas"
)
```

```
[Out] [1/2*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d
 + b*e))/sqrt(c^2*d*e - b*c*e^2), -sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c
^2*d*e + b*c*e^2)*x/(c*d - b*e))/(c^2*d*e - b*c*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(44) = 88$.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.53

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(-be\sqrt{-\frac{1}{ce(be-cd)}}+cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}$$
$$+\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(be\sqrt{-\frac{1}{ce(be-cd)}}-cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}$$

```
[In] integrate((e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -sqrt(-1/(c*e*(b*e - c*d)))*log(-b*e*sqrt(-1/(c*e*(b*e - c*d))) + c*d*sqrt(
-1/(c*e*(b*e - c*d))) + x)/2 + sqrt(-1/(c*e*(b*e - c*d)))*log(b*e*sqrt(-1/(
c*e*(b*e - c*d))) - c*d*sqrt(-1/(c*e*(b*e - c*d))) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(b*e-c*d)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}}$$

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")
```

```
[Out] arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/sqrt(-c^2*d*e + b*c*e^2)
```

Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{\mathrm{atan}\left(\frac{c\,e\,x}{\sqrt{b\,c\,e^2-c^2\,d\,e}}\right)}{\sqrt{b\,c\,e^2-c^2\,d\,e}}$$

```
[In] int((d + e*x^2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] atan((c*e*x)/(b*c*e^2 - c^2*d*e)^(1/2))/(b*c*e^2 - c^2*d*e)^(1/2)
```

3.218 $\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 136

$$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = -\frac{x}{2d(2cd-be)(d+ex^2)} - \frac{(4cd-be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2}$$

```
[Out] -1/2*x/d/(-b*e+2*c*d)/(e*x^2+d)-1/2*(-b*e+4*c*d)*arctan(x*e^(1/2)/d^(1/2))/
d^(3/2)/(-b*e+2*c*d)^2/e^(1/2)-c^(3/2)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)
^(1/2))/(-b*e+2*c*d)^2/e^(1/2)/(-b*e+c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {1163, 425, 536, 211, 214}

$$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = -\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(4cd-be)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2} - \frac{x}{2d(d+ex^2)(2cd-be)}$$

```
[In] Int[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -1/2*x/(d*(2*c*d - b*e)*(d + e*x^2)) - ((4*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqr
t[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{\left(d+ex^2\right)^2\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx \\
&= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)}+\frac{\int \frac{e(3cd-be)-ce^2x^2}{\left(d+ex^2\right)\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2de(2cd-be)}
\end{aligned}
$$

$$= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)} + \frac{c^2\int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx}{(2cd-be)^2} - \frac{(4cd-be)\int \frac{1}{d+ex^2}\,dx}{2d(2cd-be)^2}$$

$$= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)} - \frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int \frac{1}{(d+ex^2)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = -\frac{x}{2d(2cd-be)\left(d+ex^2\right)} + \frac{(-4cd+be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} + \frac{c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{\sqrt{e}(-2cd+be)^2\sqrt{-cd+be}}$$

```
[In] Integrate[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -1/2*x/(d*(2*c*d - b*e)*(d + e*x^2)) + ((-4*c*d + b*e)*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) + (c^(3/2)*ArcTan[(Sqrt[c]*Sqr
t[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*(-2*c*d + b*e)^2*Sqrt[-(c*d) + b*e])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^2\arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{(be-2cd)^2\sqrt{(be-cd)ec}}+\frac{\frac{(be-2cd)x}{2d\left(e\,x^2+d\right)}+\frac{(be-4cd)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{(be-2cd)^2}$
risch	$\frac{x}{2d(be-2cd)(e\,x^2+d)}+\frac{\sqrt{-(be-cd)ec}\,c\ln\left(\left(-\sqrt{-(be-cd)ec}\,b^4e^4+10\sqrt{-(be-cd)ec}\,b^3cd\,e^3-37\sqrt{-(be-cd)ec}\,b^2c^2d^2e^2+48\sqrt{-(be-}\right.\right.}{2}$

```
[In] int(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

```
[Out] c^2/(b*e-2*c*d)^2/((b*e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2))
+1/(b*e-2*c*d)^2*(1/2*(b*e-2*c*d)/d*x/(e*x^2+d)+1/2*(b*e-4*c*d)/d/(e*d)^(1/
2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 895, normalized size of antiderivative = 6.58

$$\int \frac{1}{(d+ex^2)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$$

$$= \left[\frac{2\left(cd^2e^2x^2+cd^3e\right)\sqrt{\frac{c}{cde-be^2}}\log\left(\frac{cex^2-2\left(cde-be^2\right)x\sqrt{\frac{c}{cde-be^2}}+cd-be}{cex^2-cd+be}\right)+(4\,cd^2-bde+(4\,cde-be^2)x^2)\sqrt{-de}}{4\left(4\,c^2d^5e-4\,bcd^4e^2+b^2d^3e^3+(4\,c^2d^4e^2-4\,bcd^3e^3+b^2d^2e^4)x^2\right.}\right.$$

$$-\frac{(4\,cd^2-bde+(4\,cde-be^2)x^2)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)-\left(cd^2e^2x^2+cd^3e\right)\sqrt{\frac{c}{cde-be^2}}\log\left(\frac{cex^2-2\left(cde-be^2\right)x\sqrt{\frac{c}{cd}}}{cex^2-cd+b}\right.}{2\left(4\,c^2d^5e-4\,bcd^4e^2+b^2d^3e^3+(4\,c^2d^4e^2-4\,bcd^3e^3+b^2d^2e^4)x^2\right)}$$

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/4*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 - 2*
(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e
)) + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sq
rt(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*
b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x
^2), -1/2*((4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(d*e)*arctan(sqrt(
d*e)*x/d) - (c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2
 - 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d +
 b*e)) + (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^
3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2), 1/4*(4*(c*d^2*e^2*x
^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2)))
 + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt
(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*
c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2
), 1/2*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqr
t(-c/(c*d*e - b*e^2))) - (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(d*e
)*arctan(sqrt(d*e)*x/d) - (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4
*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(b*e-c*d)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex^2)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \frac{c^2\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{\left(4\,c^2d^2-4\,bcde+b^2e^2\right)\sqrt{-c^2de+bce^2}} - \frac{(4\,cd-be)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\left(4\,c^2d^3-4\,bcd^2e+b^2de^2\right)\sqrt{de}} - \frac{x}{2\left(2\,cd^2-bde\right)\left(ex^2+d\right)}$$

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] c^2*arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/((4*c^2*d^2 - 4*b*c*d*e + b^2*e^
2)*sqrt(-c^2*d*e + b*c*e^2)) - 1/2*(4*c*d - b*e)*arctan(e*x/sqrt(d*e))/((4*
c^2*d^3 - 4*b*c*d^2*e + b^2*d*e^2)*sqrt(d*e)) - 1/2*x/((2*c*d^2 - b*d*e)*(e
*x^2 + d))
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 3901, normalized size of antiderivative = 28.68

$$\int \frac{1}{(d+ex^2)\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx=\text{Too large to display}$$

```
[In] int(1/((d + e*x^2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)

[Out] - x/(2*(d + e*x^2)*(2*c*d^2 - b*d*e)) - (atan((((((96*c^7*d^6*e^6 - 224*b*
c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 +
 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*
c^2*d^4*e)) - (x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^
5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^
12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*
b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 -
4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) - (x*(b^2*c^3*e^8 + 20*c^5*
d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^
3*e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2
*c*d*e^3) - (((((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 20
8*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5
 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e)) + (x*(-c^3*e*(b*e - c*d
))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 -
128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 -
4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-
c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b
^2*c*d*e^3)) + (x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2
*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^3*e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4
 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3))/((((((96*c^7*d^6*e^6 - 2
24*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*
e^9 + 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 -
12*b*c^2*d^4*e)) - (x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b
^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d
^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e
 + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e
^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) - (x*(b^2*c^3*e^8 + 20
*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))
*(-c^3*e*(b*e - c*d))^(1/2))/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b
^2*c*d*e^3) - ((b*c^4*e^6)/2 - 2*c^5*d*e^5)/(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^
2*c*d^3*e^2 - 12*b*c^2*d^4*e) + (((((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2
*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2
*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e)) + (
x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384
*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*
d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 -
```

```
 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 - 4*c^3*d^3*e + 8
*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) + (x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c
^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^3*e*(b*e - c*d)
)^(1/2))/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*
e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c
*d*e^3) - (atan(((((x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(2*(4
*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) - ((-d^3*e)^(1/2)*((96*c^7*d^6*e^6 -
 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^
3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 1
2*b*c^2*d^4*e) - (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)*(256*b*c^6*d^6*e^8 - 512*b
^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d
^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(4*c^2*d^5*e + b^2*d^3
*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2*d^5*e + b^2*d^3*e^3 - 4*b*
c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d)*1i)/(4*(4*c^2*d^5*e + b^2*d^3*e^3
 - 4*b*c*d^4*e^2)) + (((x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(
2*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) + ((-d^3*e)^(1/2)*((96*c^7*d^6*e
^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^
4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2
 - 12*b*c^2*d^4*e) + (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)*(256*b*c^6*d^6*e^8 - 5
12*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c
^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(4*c^2*d^5*e + b^2
*d^3*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2*d^5*e + b^2*d^3*e^3 -
4*b*c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d)*1i)/(4*(4*c^2*d^5*e + b^2*d^3
*e^3 - 4*b*c*d^4*e^2)))/(((b*c^4*e^6)/2 - 2*c^5*d*e^5)/(8*c^3*d^5 - b^3*d^2
*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e) + (((x*(b^2*c^3*e^8 + 20*c^5*d^2*e
^6 - 8*b*c^4*d*e^7))/(2*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) - ((-d^3*e
)^(1/2)*((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c
^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^2
*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e) - (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)*
(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c
^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3
*e)*(4*c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2*
d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d))/(4*(4*
c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)) - (((x*(b^2*c^3*e^8 + 20*c^5*d^2*
e^6 - 8*b*c^4*d*e^7))/(2*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) + ((-d^3*
e)^(1/2)*((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*
c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^
2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e) + (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)
*(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*
c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^
3*e)*(4*c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2
*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d))/(4*(4
*c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2))))*(-d^3*e)^(1/2)*(b*e - 4*c*d)*1
i)/(2*(4*c^2*d^5*e + b^2*d^3*e^3 - 4*b*c*d^4*e^2))
```

3.219 $\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 187

$$\begin{aligned}&\int \frac{1}{(d+ex^2)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx\\&=-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)}\\&\quad-\frac{\left(28c^2d^2-16bcde+3b^2e^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3}\end{aligned}$$

```
[Out] -1/4*x/d/(-b*e+2*c*d)/(e*x^2+d)^2-1/8*(-3*b*e+10*c*d)*x/d^2/(-b*e+2*c*d)^2/
(e*x^2+d)-1/8*(3*b^2*e^2-16*b*c*d*e+28*c^2*d^2)*arctan(x*e^(1/2)/d^(1/2))/d
^(5/2)/(-b*e+2*c*d)^3/e^(1/2)-c^(5/2)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^
(1/2))/(-b*e+2*c*d)^3/e^(1/2)/(-b*e+c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1163, 425, 541, 536, 211, 214}

$$\begin{aligned}&\int \frac{1}{(d+ex^2)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx\\&=-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3b^2e^2-16bcde+28c^2d^2\right)}{8d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3}\\&\quad-\frac{x(10cd-3be)}{8d^2\left(d+ex^2\right)(2cd-be)^2}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}\end{aligned}$$

```
[In] Int[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -1/4*x/(d*(2*c*d - b*e)*(d + e*x^2)^2) - ((10*c*d - 3*b*e)*x)/(8*d^2*(2*c*d
 - b*e)^2*(d + e*x^2)) - ((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]*(2*c*d - b*e)^3) - (c^(5/2)*ArcTanh[(S
qrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^
3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
```

```
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(d+ex^2)^3\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx \\
&= -\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}+\frac{\int \frac{e(7cd-3be)-3ce^2x^2}{(d+ex^2)^2\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{4de(2cd-be)} \\
&= -\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)} \\
&\quad +\frac{\int \frac{e^2\left(18c^2d^2-13bcde+3b^2e^2\right)-ce^3(10cd-3be)x^2}{(d+ex^2)\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{8d^2e^2(2cd-be)^2} \\
&= -\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)} \\
&\quad +\frac{c^3\int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx}{(2cd-be)^3}-\frac{\left(28c^2d^2-16bcde+3b^2e^2\right)\int \frac{1}{d+ex^2}\,dx}{8d^2(2cd-be)^3} \\
&= -\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)} \\
&\quad -\frac{\left(28c^2d^2-16bcde+3b^2e^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int \frac{1}{\left(d+ex^2\right)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx \\
&= \frac{1}{8}\left(-\frac{\left(28c^2d^2-16bcde+3b^2e^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}(2cd-be)^3}\right. \\
&\qquad \left. -\frac{\frac{(-2cd+be)x\left(-be\left(5d+3ex^2\right)+2cd\left(7d+5ex^2\right)\right)}{d^2\left(d+ex^2\right)^2}+\frac{8c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{\sqrt{e}\sqrt{-cd+be}}}{(-2cd+be)^3}\right)
\end{aligned}
$$

```
[In] Integrate[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] (-((((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*Sqrt[e]*(2*c*d - b*e)^3)) - (((-2*c*d + b*e)*x*(-(b*e*(5*d + 3*e*x^2))
 + 2*c*d*(7*d + 5*e*x^2)))/(d^2*(d + e*x^2)^2) + (8*c^(5/2)*ArcTan[(Sqrt[c]
*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*Sqrt[-(c*d) + b*e]))/(-2*c*d + b*
e)^3)/8
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{c^{3} \arctan\left(\frac{x c e}{\sqrt{(b e-c d) e c}}\right)}{(b e-2 c d)^{3} \sqrt{(b e-c d) e c}}+\frac{\frac{\frac{e\left(3 b^{2} e^{2}-16 b c d e+20 c^{2} d^{2}\right) x^{3}}{8 d^{2}}+\frac{\left(5 b^{2} e^{2}-24 b c d e+28 c^{2} d^{2}\right) x}{8 d}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(3 b^{2} e^{2}-16 b c d e+28 c^{2} d^{2}\right) \arctan\left(\frac{e x}{\sqrt{e d}}\right)}{8 d^{2} \sqrt{e d}}}{(b e-2 c d)^{3}}$	174
risch	Expression too large to display	1012

```
[In] int(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE
)
```

```
[Out] -c^3/(b*e-2*c*d)^3/((b*e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2)
)+1/(b*e-2*c*d)^3*((1/8*e*(3*b^2*e^2-16*b*c*d*e+20*c^2*d^2)/d^2*x^3+1/8*(5*
b^2*e^2-24*b*c*d*e+28*c^2*d^2)/d*x)/(e*x^2+d)^2+1/8*(3*b^2*e^2-16*b*c*d*e+2
8*c^2*d^2)/d^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(161) = 322$.

Time = 0.85 (sec) , antiderivative size = 1765, normalized size of antiderivative = 9.44

$$\int \frac{1}{(d+ex^2)^2\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fri
cas")
```

```
[Out] [-1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 + 8*(c^2*d^3*
e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x
^2 + 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*
d + b*e)) - (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 -
16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*
e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(28*
c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*
e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6
*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6
```

```
*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3
+ 3*b^2*d*e^4)*x^3 + (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d
^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 +
 3*b^2*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 4*(c^2*d^3*e^3*x^4 + 2
*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 + 2*(c*d
*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e)) +
 (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2
*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^
4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^
3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*
d^2*e^3 + 3*b^2*d*e^4)*x^3 - 16*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*
d^5*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2))) - (28*
c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3
*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(-
d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(28*c^2*d^4*e - 24*b
*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^
6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 -
 b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 -
 b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x
^3 - 8*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(-c/(c*d*e - b
*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2))) + (28*c^2*d^4 - 16*b*c*d^3*e +
3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2
*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d)
 + (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c
^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*
e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*
e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 (-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e*(b*e-c*d)>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.26

$$\begin{aligned}&\int \frac{1}{(d+ex^2)^2 (-cd^2+bde+be^2x^2+ce^2x^4)}\,dx\\&=\frac{c^3\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{(8\,c^3d^3-12\,bc^2d^2e+6\,b^2cde^2-b^3e^3)\sqrt{-c^2de+bce^2}}\\&\quad-\frac{(28\,c^2d^2-16\,bcde+3\,b^2e^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,(8\,c^3d^5-12\,bc^2d^4e+6\,b^2cd^3e^2-b^3d^2e^3)\sqrt{de}}\\&\quad-\frac{10\,cdex^3-3\,be^2x^3+14\,cd^2x-5\,bdex}{8\,(4\,c^2d^4-4\,bcd^3e+b^2d^2e^2)(ex^2+d)^2}\end{aligned}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="gia
c")
```

```
[Out] c^3*arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/((8*c^3*d^3 - 12*b*c^2*d^2*e + 6
*b^2*c*d*e^2 - b^3*e^3)*sqrt(-c^2*d*e + b*c*e^2)) - 1/8*(28*c^2*d^2 - 16*b*
c*d*e + 3*b^2*e^2)*arctan(e*x/sqrt(d*e))/((8*c^3*d^5 - 12*b*c^2*d^4*e + 6*b
^2*c*d^3*e^2 - b^3*d^2*e^3)*sqrt(d*e)) - 1/8*(10*c*d*e*x^3 - 3*b*e^2*x^3 +
14*c*d^2*x - 5*b*d*e*x)/((4*c^2*d^4 - 4*b*c*d^3*e + b^2*d^2*e^2)*(e*x^2 + d
)^2)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 6267, normalized size of antiderivative = 33.51

$$\int \frac{1}{(d+ex^2)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x^2)^2*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

```
[Out] ((x*(5*b*e - 14*c*d))/(8*d*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)) + (e*x^3*(3*b
*e - 10*c*d))/(8*d^2*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)))/(d^2 + e^2*x^4 + 2
*d*e*x^2) - (atan(((((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e
^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^4*d^4*e^4
 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^10*d^
10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 +
 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*
b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10 + b^6*d^4*e^6
 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^
2*d^6*e^4 - 192*b*c^5*d^9*e)) - (x*(-c^5*e*(b*e - c*d))^(1/2)*(16384*b*c^8*
d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*b^4*c^5*d
^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7*c^2*d^4*
e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^
2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*
d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^4*e^5 + 8*c^4*
d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b
*e - c*d))^(1/2)*1i)/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2
*d^2*e^3 - 7*b^3*c*d*e^4) + (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*
c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^
4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) + (((57
6*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*
d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^
12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10 + b^
6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 +
60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) + (x*(-c^5*e*(b*e - c*d))^(1/2)*(163
84*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*
b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7
*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c
^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18
*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^4*e^5
 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(
-c^5*e*(b*e - c*d))^(1/2)*1i)/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 1
8*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4))/(((9*b^3*c^5*e^8)/32 - (35*c^8*d^3*e^5)
/4 + (61*b*c^7*d^2*e^6)/8 - (39*b^2*c^6*d*e^7)/16)/(64*c^6*d^10 + b^6*d^4*e
^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*
c^2*d^6*e^4 - 192*b*c^5*d^9*e) + (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 8
```

```
96*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8
 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) -
(((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3
*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d
^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10
 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e
^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) - (x*(-c^5*e*(b*e - c*d))^(1/2)
*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 4
0960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 25
6*b^7*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*
b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2
 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^
4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4
)))*(-c^5*e*(b*e - c*d))^(1/2))/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 +
 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4) - (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e
^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c
^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*
e)) + (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 32
88*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6
*c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^
6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3
*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) + (x*(-c^5*e*(b*e - c*d))
^(1/2)*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^
10 - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^1
3 + 256*b^7*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3
 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d
^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/
(2*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c
*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3
*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2)*1i)
/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d
*e^4) - (atan(((((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 -
 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(32*(16*c^4*d^8 + b^4*d^4*e^4 - 8
*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^10*d^10*e
^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 194
0*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7*
c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(64*c^6*d^10 + b^6*d^4*e^6 - 12*b
^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e
^4 - 192*b*c^5*d^9*e) - (x*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*
d*e)*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10
 - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13
+ 256*b^7*c^2*d^4*e^14))/(512*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2
 + 6*b^2*c*d^6*e^3)*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^
2*d^6*e^2 - 32*b*c^3*d^7*e)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b
*c*d*e))/(16*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^
```

```
3)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e)*1i)/(16*(8*c^3*d^
8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)) + (((x*(9*b^4*c^3*
e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5
*d^2*e^8))/(32*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6
*e^2 - 32*b*c^3*d^7*e)) + (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*
b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^
5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^
2*e^14)/2)/(64*c^6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*
e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e) + (x*(-d^
5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e)*(16384*b*c^8*d^10*e^8 - 49
152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*b^4*c^5*d^7*e^11 + 153
60*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7*c^2*d^4*e^14))/(512*(
8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)*(16*c^4*d^8
 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)))*(
-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e))/(16*(8*c^3*d^8*e - b^3
*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)))*(-d^5*e)^(1/2)*(3*b^2*e^2
+ 28*c^2*d^2 - 16*b*c*d*e)*1i)/(16*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^
7*e^2 + 6*b^2*c*d^6*e^3)))/(((9*b^3*c^5*e^8)/32 - (35*c^8*d^3*e^5)/4 + (61*
b*c^7*d^2*e^6)/8 - (39*b^2*c^6*d*e^7)/16)/(64*c^6*d^10 + b^6*d^4*e^6 - 12*b
^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e
^4 - 192*b*c^5*d^9*e) + (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*
d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(32*(16*c^4*d^8 + b^4*d^
4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^
10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*
e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 -
 (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(64*c^6*d^10 + b^6*d^4*e
^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*
c^2*d^6*e^4 - 192*b*c^5*d^9*e) - (x*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2
- 16*b*c*d*e)*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6
*d^8*e^10 - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*
d^5*e^13 + 256*b^7*c^2*d^4*e^14))/(512*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^
2*d^7*e^2 + 6*b^2*c*d^6*e^3)*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 +
24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d
^2 - 16*b*c*d*e))/(16*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2
*c*d^6*e^3)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e))/(16*(8*
c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)) - (((x*(9*b^
4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b
^2*c^5*d^2*e^8))/(32*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c
^2*d^6*e^2 - 32*b*c^3*d^7*e)) + (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 +
 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*
b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*
c^2*d^2*e^14)/2)/(64*c^6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^
4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e) + (
x*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e)*(16384*b*c^8*d^10*e^
8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*b^4*c^5*d^7*e^11
```

```
 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7*c^2*d^4*e^14))/
(512*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)*(16*c
^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*
e)))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*b*c*d*e))/(16*(8*c^3*d^8*e
 - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3)))*(-d^5*e)^(1/2)*(3*b^
2*e^2 + 28*c^2*d^2 - 16*b*c*d*e))/(16*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2
*d^7*e^2 + 6*b^2*c*d^6*e^3))))*(-d^5*e)^(1/2)*(3*b^2*e^2 + 28*c^2*d^2 - 16*
b*c*d*e)*1i)/(8*(8*c^3*d^8*e - b^3*d^5*e^4 - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6
*e^3))
```

3.220 $\int \frac{\left(d+ex^2\right)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 139

$$\int \frac{\left(d+ex^2\right)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{x\sqrt{d+ex^2}}{2c}+\frac{(5cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}-\frac{(2cd-be)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}$$

```
[Out] 1/2*(-2*b*e+5*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^2/e^(1/2)-(-b*e+2*c
*d)^(3/2)*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(
1/2))/c^2/e^(1/2)/(-b*e+c*d)^(1/2)+1/2*x*(e*x^2+d)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1163, 427, 537, 223, 212, 385, 214}

$$\int \frac{\left(d+ex^2\right)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{(2cd-be)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}+\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(5cd-2be)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

```
[In] Int[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) + ((5*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e
*x^2]])/(2*c^2*Sqrt[e]) - ((2*c*d - b*e)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[2*c*d
- b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c^2*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(d+ex^2\right)^{3/2}}{\frac{-cd^2+bde}{d}+cex^2}\,dx \\
&= \frac{x\sqrt{d+ex^2}}{2c} + \frac{\int \frac{de(3cd-be)+e^2(5cd-2be)x^2}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2ce} \\
&= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c^2} + \frac{(2cd-be)^2\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{c^2} \\
&= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} \\
&\quad + \frac{(2cd-be)^2\mathrm{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c^2} \\
&= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$
\int \frac{\left(d+ex^2\right)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = -\frac{-cx\sqrt{d+ex^2}+\frac{2(2cd-be)\sqrt{2c^2d^2-3bcde+b^2e^2}\,\mathrm{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{\sqrt{e}(cd-be)}+\frac{(5cd-2be)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{\sqrt{e}}}{2c^2}
$$

```
[In] Integrate[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -1/2*(-(c*x*Sqrt[d + e*x^2]) + (2*(2*c*d - b*e)*Sqrt[2*c^2*d^2 - 3*b*c*d*e
+ b^2*e^2]*ArcTanh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2]))/Sqr
t[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]])/(Sqrt[e]*(c*d - b*e)) + ((5*c*d - 2*b*
e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/Sqrt[e])/c^2
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{-\frac{2(be-2cd)^{2}\operatorname{arctanh}\left(\frac{(be-cd)\sqrt{e\,x^{2}+d}}{x\sqrt{e(be-2cd)(be-cd)}}\right)}{\sqrt{e(be-2cd)(be-cd)}}-\frac{\sqrt{e\,x^{2}+d}\,cx\sqrt{e}-2\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)be+5\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)cd}{\sqrt{e}}}{2c^{2}}$
risch	$\frac{x\sqrt{e\,x^{2}+d}}{2c}-\frac{\frac{(2be-5cd)\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{c\sqrt{e}}-\frac{\left(b^{2}e^{2}-4bcde+4c^{2}d^{2}\right)\ln\left(\frac{-\frac{2(be-2cd)}{c}-\frac{2\sqrt{-(be-cd)ec}\left(x+\frac{\sqrt{-(be-cd)ec}}{ec}\right)}{c}+2\sqrt{-\frac{be-}{c}}}{x}\right.}{\sqrt{-(be-cd)ec}\,c\sqrt{-}}}{}$
default	Expression too large to display

```
[In] int((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2/c^2*(-2*(b*e-2*c*d)^2/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)*arctanh((b*e-c*d
)*(e*x^2+d)^(1/2)/x/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2))-((e*x^2+d)^(1/2)*c*x*e
^(1/2)-2*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))*b*e+5*arctanh((e*x^2+d)^(1/2)/x
/e^(1/2))*c*d)/e^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 1079, normalized size of antiderivative = 7.76

$$\int \frac{\left(d+ex^{2}\right)^{5/2}}{-cd^{2}+bde+be^{2}x^{2}+ce^{2}x^{4}}\,dx=\left[\frac{2\sqrt{ex^{2}+d}cex-(5cd-2be)\sqrt{e}\log\left(-2ex^{2}+2\sqrt{ex^{2}+d}\sqrt{e}x-d\right)-}{}\right.$$

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(2*sqrt(e*x^2 + d)*c*e*x - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*e
- b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b
*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*
x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c
*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)
))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2
)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x - 2*(5*c*d - 2*b*e)*sqrt(-e)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*
```

```
e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24
*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3
)*x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b
*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^
2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x
^2)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x + 2*(2*c*d*e - b*e^2)*sqrt(-(2*
c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)
*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*e - b*e^
2)*x^3 + (2*c*d^2 - b*d*e)*x)) - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d))/(c^2*e), 1/2*(sqrt(e*x^2 + d)*c*e*x - (5*c*d
 - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*c*d*e - b*e^2)*s
qrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e -
2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*
e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)))/(c^2*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d)**(5/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\int \frac{(ex^2+d)^{\frac{5}{2}}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate((e*x^2 + d)^(5/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(e\,x^2+d)^{5/2}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

```
[In] int((d + e*x^2)^(5/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] int((d + e*x^2)^(5/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

3.221 $\int \frac{\left(d+ex^2\right)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 108

$$\int \frac{\left(d+ex^2\right)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)-arctanh(x*e^(1/2)*(-b*e+2*c*d)
^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))*(-b*e+2*c*d)^(1/2)/c/e^(1/2)/(-b*e
+c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1163, 399, 223, 212, 385, 214}

$$\int \frac{\left(d+ex^2\right)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\operatorname{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

```
[In] Int[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d - b*e]*ArcTa
nh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c*Sqr
t[e]*Sqrt[c*d - b*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{d+ex^2}}{\frac{-cd^2+bde}{d}+cex^2}\,dx \\
&= \frac{\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} - \frac{\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{ce} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} \\
&\quad - \frac{\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)\text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{ce}
\end{aligned}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx =$$
$$-\frac{\sqrt{2c^2d^2-3bcde+b^2e^2}\operatorname{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)+(cd-be)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{c\sqrt{e}(cd-be)}$$

```
[In] Integrate[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -((Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]*ArcTanh[(-(b*e) + c*(d - e*x^2 + S
qrt[e]*x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]] + (c*d -
b*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(c*Sqrt[e]*(c*d - b*e)))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$-\frac{\frac{(be-2cd)\operatorname{arctanh}\left(\frac{(be-cd)\sqrt{e\,x^2+d}}{x\sqrt{e(be-2cd)(be-cd)}}\right)}{\sqrt{e(be-2cd)(be-cd)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)}{\sqrt{e}}}{c}$	100
default	Expression too large to display	2436

```
[In] int((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/c*((b*e-2*c*d)*arctanh((b*e-c*d)*(e*x^2+d)^(1/2)/x/(e*(b*e-2*c*d)*(b*e-c
*d))^(1/2))/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)-1/e^(1/2)*arctanh((e*x^2+d)^(1/
2)/x/e^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 940, normalized size of antiderivative = 8.70

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \left[\frac{e\sqrt{\frac{2\,cd-be}{cde-be^2}}\log\left(\frac{c^2d^4-2\,bcd^3e+b^2d^2e^2+(17\,c^2d^2e^2-24\,bcde^3+8\,b^2e^4)x^4+2\,(7\,c^2d^3e-11\,bc}{c^2e^2x^4+c^2}\right.}{}\right.$$

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^
2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*
e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2
*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*
sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b
^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) + 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^
2 + d)*sqrt(e)*x - d))/(c*e), 1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*lo
g((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8
*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*
c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b
^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*
x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) - 4*sqrt(
-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e), 1/2*(e*sqrt(-(2*c*d - b*e)/(
c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e
*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*e - b*e^2)*x^3 + (2*
c*d^2 - b*d*e)*x)) + sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d
))/(c*e), 1/2*(e*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b
*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e
- b*e^2))/((2*c*d*e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)) - 2*sqrt(-e)*arcta
n(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e)]
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{\sqrt{d+ex^2}}{be-cd+cex^2}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(b*e - c*d + c*e*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

```
[In] int((d + e*x^2)^(3/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

3.222 $\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 76

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

```
[Out] -arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/e^(
1/2)/(-b*e+c*d)^(1/2)/(-b*e+2*c*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {1163, 385, 214}

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

```
[In] Int[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])]/
(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e(-cd^2+bde)}{d}\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}\,x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = -\frac{\operatorname{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{\sqrt{e}\sqrt{2c^2d^2-3bcde+b^2e^2}}$$

```
[In] Integrate[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*d
^2 - 3*b*c*d*e + b^2*e^2]]/(Sqrt[e]*Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{(be-cd)\sqrt{e\,x^{2}+d}}{x\sqrt{e(be-2cd)(be-cd)}}\right)}{\sqrt{e(be-2cd)(be-cd)}}$	64
default	Expression too large to display	1425

```
[In] int((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)*arctanh((b*e-c*d)*(e*x^2+d)^(1/2)/x/(e*(b
*e-2*c*d)*(b*e-c*d))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(62) = 124.

Time = 0.30 (sec) , antiderivative size = 432, normalized size of antiderivative = 5.68

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$$

$$= \Bigg[\frac{\log\bigg(\frac{c^{2}d^{4}-2\,bcd^{3}e+b^{2}d^{2}e^{2}+\left(17\,c^{2}d^{2}e^{2}-24\,bcde^{3}+8\,b^{2}e^{4}\right)x^{4}+2\,\left(7\,c^{2}d^{3}e-11\,bcd^{2}e^{2}+4\,b^{2}de^{3}\right)x^{2}-4\,\sqrt{2\,c^{2}d^{2}e-3\,bcde^{2}+b^{2}e^{3}}\,\left(\left(3\,cde-2\,be^{2}\right)x^{3}+}{c^{2}e^{2}x^{4}+c^{2}d^{2}-2\,bcde+b^{2}e^{2}-2\,\left(c^{2}de-bce^{2}\right)x^{2}}}{4\,\sqrt{2\,c^{2}d^{2}e-3\,bcde^{2}+b^{2}e^{3}}}$$

$$-\frac{\sqrt{-2\,c^{2}d^{2}e+3\,bcde^{2}-b^{2}e^{3}}\,\arctan\left(-\frac{\sqrt{-2\,c^{2}d^{2}e+3\,bcde^{2}-b^{2}e^{3}}\left(cd^{2}-bde+\left(3\,cde-2\,be^{2}\right)x^{2}\right)\sqrt{ex^{2}+d}}{2\,\left(\left(2\,c^{2}d^{2}e^{2}-3\,bcde^{3}+b^{2}e^{4}\right)x^{3}+\left(2\,c^{2}d^{3}e-3\,bcd^{2}e^{2}+b^{2}de^{3}\right)x\right)}\right)}{2\,\left(2\,c^{2}d^{2}e-3\,bcde^{2}+b^{2}e^{3}\right)}\Bigg]$$

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*
e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 -
 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 + (c*
d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e
^2 - 2*(c^2*d*e - b*c*e^2)*x^2))/sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3),
 -1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(-1/2*sqrt(-2*c^2*d^
2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt
(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2*d^3*e - 3
*b*c*d^2*e^2 + b^2*d*e^3)*x))/(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\int \frac{1}{\sqrt{d+ex^2}\,(be-cd+cex^2)}\,dx$$

```
[In] integrate((e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x**2)*(b*e - c*d + c*e*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\int \frac{\sqrt{ex^2+d}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{\arctan\left(\frac{\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2c-3\,cd+2\,be}{2\,\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)}{\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}\sqrt{e}}$$

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] -arctan(1/2*((sqrt(e)*x - sqrt(e*x^2 + d))^2*c - 3*c*d + 2*b*e)/sqrt(-2*c^2
*d^2 + 3*b*c*d*e - b^2*e^2))/(sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2)*sqrt(e
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\int \frac{\sqrt{e\,x^2+d}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

```
[In] int((d + e*x^2)^(1/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
[Out] int((d + e*x^2)^(1/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

3.223 $\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 106

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} - \frac{c\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-be}\,x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\,(2cd-be)^{3/2}}$$

```
[Out] -c*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/(
-b*e+2*c*d)^(3/2)/e^(1/2)/(-b*e+c*d)^(1/2)-x/d/(-b*e+2*c*d)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {1163, 390, 385, 214}

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = -\frac{c\,\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\,(2cd-be)^{3/2}} - \frac{x}{d\sqrt{d+ex^2}\,(2cd-be)}$$

```
[In] Int[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b
*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d
- b*e)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\left(d+ex^2\right)^{3/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx \\
&= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}}+\frac{c\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2cd-be} \\
&= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}}+\frac{c\text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2cd-be} \\
&= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}\,x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx =$$
$$-\frac{\sqrt{e}(2c^2d^2-3bcde+b^2e^2)\,x+cd\sqrt{2c^2d^2-3bcde+b^2e^2}\sqrt{d+ex^2}\operatorname{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{d\sqrt{e}(cd-be)(-2cd+be)^2\sqrt{d+ex^2}}$$

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -((Sqrt[e]*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*x + c*d*Sqrt[2*c^2*d^2 - 3*b*c
*d*e + b^2*e^2]*Sqrt[d + e*x^2]*ArcTanh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*
Sqrt[d + e*x^2]))/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]])/(d*Sqrt[e]*(c*d -
 b*e)*(-2*c*d + b*e)^2*Sqrt[d + e*x^2]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$\frac{-cd\operatorname{arctanh}\left(\frac{(be-cd)\sqrt{e\,x^2+d}}{x\sqrt{e(be-2cd)(be-cd)}}\right)\sqrt{e\,x^2+d}+x\sqrt{e(be-2cd)(be-cd)}}{(be-2cd)\sqrt{e\,x^2+d}\,\sqrt{e(be-2cd)(be-cd)}\,d}$
default	$-\frac{c\sqrt{\left(x+\frac{\sqrt{-ed}}{e}\right)^2 e-2\sqrt{-ed}\left(x+\frac{\sqrt{-ed}}{e}\right)}}{2d\left(\sqrt{-ed}\,c+\sqrt{-(be-cd)ec}\right)\left(-\sqrt{-ed}\,c+\sqrt{-(be-cd)ec}\right)\left(x+\frac{\sqrt{-ed}}{e}\right)}-\frac{c\sqrt{\left(x-\frac{\sqrt{-ed}}{e}\right)^2 e+2\sqrt{-ed}\left(x-\right.}}{2d\left(\sqrt{-ed}\,c+\sqrt{-(be-cd)ec}\right)\left(-\sqrt{-ed}\,c+\sqrt{-(\right.}$

```
[In] int(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVER
BOSE)
```

```
[Out] (-c*d*arctanh((b*e-c*d)*(e*x^2+d)^(1/2)/x/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2))*
(e*x^2+d)^(1/2)+x*(e*(b*e-2*c*d)*(b*e-c*d))^(1/2))/(b*e-2*c*d)/(e*x^2+d)^(1
/2)/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(90) = 180$.

Time = 0.34 (sec) , antiderivative size = 701, normalized size of antiderivative = 6.61

$$\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$$
$$= \left[-\frac{4\left(2\,c^2d^2e-3\,bcde^2+b^2e^3\right)\sqrt{ex^2+d}x+\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\left(cdex^2+cd^2\right)\log\left(\frac{c^2d^4-2\,bcd^3e+b^2d^2e^2+}{}\right.}{4\left(4\,c^3d^5e-8\,bc^2d^4e^2+5\,b^2cd^3e^3-b^3d^2e^4+(\right.}\right.$$
$$-\frac{2\left(2\,c^2d^2e-3\,bcde^2+b^2e^3\right)\sqrt{ex^2+d}x+\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\left(cdex^2+cd^2\right)\arctan\left(-\frac{\sqrt{-2\,c^2d^2e+3\,b}}{2\left(\left(2\,c^2d^2e^2-\right.\right.}\right.}{2\left(4\,c^3d^5e-8\,bc^2d^4e^2+5\,b^2cd^3e^3-b^3d^2e^4+\left(4\,c^3d^4e^2-8\,bc^2d^3e^3+5\,b^2cd^2e^4-\right.\right.}$$

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"fricas")
```

```
[Out] [-1/4*(4*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(2*c
^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*(c*d*e*x^2 + c*d^2)*log((c^2*d^4 - 2*b*c*
d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(
7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 + 4*sqrt(2*c^2*d^2*e - 3*b*
c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2
 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)
*x^2)))/(4*c^3*d^5*e - 8*b*c^2*d^4*e^2 + 5*b^2*c*d^3*e^3 - b^3*d^2*e^4 + (4
*c^3*d^4*e^2 - 8*b*c^2*d^3*e^3 + 5*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2), -1/2*(2
*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(-2*c^2*d^2*
e + 3*b*c*d*e^2 - b^2*e^3)*(c*d*e*x^2 + c*d^2)*arctan(-1/2*sqrt(-2*c^2*d^2*
e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e
*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2*d^3*e - 3*b
*c*d^2*e^2 + b^2*d*e^3)*x)))/(4*c^3*d^5*e - 8*b*c^2*d^4*e^2 + 5*b^2*c*d^3*e
^3 - b^3*d^2*e^4 + (4*c^3*d^4*e^2 - 8*b*c^2*d^3*e^3 + 5*b^2*c*d^2*e^4 - b^3
*d*e^5)*x^2)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \int \frac{1}{\left(d+ex^2\right)^{\frac{3}{2}}\left(be-cd+cex^2\right)}\,dx$$

```
[In] integrate(1/(e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/((d + e*x**2)**(3/2)*(b*e - c*d + c*e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{(ce^2x^4+be^2x^2-cd^2+bde)\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*sqrt(e*x^2 + d)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \frac{c\sqrt{e}\arctan\left(-\frac{\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2c-3\,cd+2\,be}{2\,\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)}{\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}(2\,cde-be^2)} - \frac{x}{(2\,cd^2-bde)\sqrt{ex^2+d}}$$

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"giac")
```

```
[Out] c*sqrt(e)*arctan(-1/2*((sqrt(e)*x - sqrt(e*x^2 + d))^2*c - 3*c*d + 2*b*e)/s
qrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2))/(sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e
^2)*(2*c*d*e - b*e^2)) - x/((2*c*d^2 - b*d*e)*sqrt(e*x^2 + d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2\right)}\,dx$$

```
[In] int(1/((d + e*x^2)^(1/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

```
[Out] int(1/((d + e*x^2)^(1/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)), x)
```

3.224 $\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 149

$$\int \frac{1}{(d+ex^2)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx=-\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}}-\frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}}-\frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}}$$

```
[Out] -1/3*x/d/(-b*e+2*c*d)/(e*x^2+d)^(3/2)-c^2*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1
/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/(-b*e+2*c*d)^(5/2)/e^(1/2)/(-b*e+c*d)
^(1/2)-1/3*(-2*b*e+7*c*d)*x/d^2/(-b*e+2*c*d)^2/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1163, 425, 541, 12, 385, 214}

$$\int \frac{1}{(d+ex^2)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx=-\frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}}-\frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}(2cd-be)^2}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

```
[In] Int[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -1/3*x/(d*(2*c*d - b*e)*(d + e*x^2)^(3/2)) - ((7*c*d - 2*b*e)*x)/(3*d^2*(2*
c*d - b*e)^2*Sqrt[d + e*x^2]) - (c^2*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/
```

```
(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^
(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{\left(d+ex^2\right)^{5/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx \\
&= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}}+\frac{\int \frac{e(5cd-2be)-2ce^2x^2}{\left(d+ex^2\right)^{3/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{3de(2cd-be)} \\
&= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}}-\frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}}+\frac{\int \frac{3c^2d^2e^2}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{3d^2e^2(2cd-be)^2} \\
&= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}}-\frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}}+\frac{c^2\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{(2cd-be)^2} \\
&= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}}-\frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} \\
&\quad +\frac{c^2\text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{(2cd-be)^2} \\
&= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}}-\frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.17

$$
\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = -\frac{\frac{(-2cd+be)x\left(-be\left(3d+2ex^2\right)+cd\left(9d+7ex^2\right)\right)}{d^2\left(d+ex^2\right)^{3/2}}+\frac{3c^2\sqrt{2c^2d^2-3bcde+b^2e^2}\,\text{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{\sqrt{e}(-cd+be)}}{3(-2cd+be)^3}
$$

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),
x]
```

```
[Out] -1/3*((((-2*c*d + b*e)*x*(-(b*e*(3*d + 2*e*x^2)) + c*d*(9*d + 7*e*x^2)))/(d^
2*(d + e*x^2)^(3/2)) + (3*c^2*Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]*ArcTanh
[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*d^2 - 3*b*
c*d*e + b^2*e^2]])/(Sqrt[e]*(-(c*d) + b*e)))/(-2*c*d + b*e)^3
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02

method	result	size
pseudoelliptic	$\frac{c^{2} d^{2} \operatorname{arctanh}\left(\frac{(b e-c d) \sqrt{e\,x^{2}+d}}{x \sqrt{e(b e-2 c d)(b e-c d)}}\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}+x\left(\frac{2 b\,e^{2} x^{2}}{3}+d\left(-\frac{7 c\,x^{2}}{3}+b\right) e-3 c\,d^{2}\right) \sqrt{e(b e-2 c d)(b e-c d)}}{\sqrt{e(b e-2 c d)(b e-c d)}\,\left(e\,x^{2}+d\right)^{\frac{3}{2}}(b e-2 c d)^{2} d^{2}}$	152
default	Expression too large to display	1551

```
[In] int(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVER
BOSE)
```

```
[Out] (c^2*d^2*arctanh((b*e-c*d)*(e*x^2+d)^(1/2)/x/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2
))*(e*x^2+d)^(3/2)+x*(2/3*b*e^2*x^2+d*(-7/3*c*x^2+b)*e-3*c*d^2)*(e*(b*e-2*c
*d)*(b*e-c*d))^(1/2))/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)/(e*x^2+d)^(3/2)/(b*e-
2*c*d)^2/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(127) = 254.

Time = 0.51 (sec) , antiderivative size = 1063, normalized size of antiderivative = 7.13

$$\int \frac{1}{(d+ex^2)^{3/2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \Bigg[\frac{3\,(c^2d^2e^2x^4+2\,c^2d^3ex^2+c^2d^4)\sqrt{2\,c^2d^2e-3\,bcde^2+b}}{12}$$

$$-\frac{3\,(c^2d^2e^2x^4+2\,c^2d^3ex^2+c^2d^4)\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\arctan\left(-\frac{\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\,(cd^2-bde+(3\,cde-2\,be}{2\,((2\,c^2d^2e^2-3\,bcde^3+b^2e^4)x^3+(2\,c^2d^3e-3\,bcd^2}\right.}{6\,(8\,c^4d^8e-20\,bc^3d^7e^2+18\,b^2c^2d^6e^3-7\,b^3cd^5e^4+b^4d^4e^5+(8\,c^4d^6e^3-20\,bc^3d^5e^4+}$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(2*c^2*d^2*e - 3
*b*c*d*e^2 + b^2*e^3)*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^
2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4
*b^2*d*e^3)*x^2 - 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2
*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 -
2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) - 4*((14*c^3*d^3*e^2 - 25
*b*c^2*d^2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b*c^
2*d^3*e^2 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^4*d^8*e -
 20*b*c^3*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^4*e^5 + (8
*c^4*d^6*e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^
4*d^2*e^7)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 -
 7*b^3*c*d^4*e^5 + b^4*d^3*e^6)*x^2), -1/6*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*
```

```
e*x^2 + c^2*d^4)*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(-1/2*sqr
t(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2
)*x^2)*sqrt(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^
2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)) + 2*((14*c^3*d^3*e^2 - 25*b*c^2*d^
2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b*c^2*d^3*e^2
 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^4*d^8*e - 20*b*c^3
*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^4*e^5 + (8*c^4*d^6*
e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^4*d^2*e^7
)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 - 7*b^3*c*
d^4*e^5 + b^4*d^3*e^6)*x^2)]
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^{\frac{5}{2}}\left(be-cd+cex^2\right)}\,dx$$

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
[Out] Integral(1/((d + e*x**2)**(5/2)*(b*e - c*d + c*e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx=\int \frac{1}{\left(ce^2x^4+be^2x^2-cd^2+bde\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"maxima")
[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(127) = 254.

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.30

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx=
-\frac{c^2\sqrt{e}\arctan\left(\frac{\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2c-3\,cd+2\,be}{2\,\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)}{\left(4\,c^2d^2e-4\,bcde^2+b^2e^3\right)\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}
-\frac{\left(\frac{\left(28\,c^3d^3e^2-36\,bc^2d^2e^3+15\,b^2cde^4-2\,b^3e^5\right)x^2}{16\,c^4d^6e-32\,bc^3d^5e^2+24\,b^2c^2d^4e^3-8\,b^3cd^3e^4+b^4d^2e^5}+\frac{3\,\left(12\,c^3d^4e-16\,bc^2d^3e^2+7\,b^2cd^2e^3-b^3de^4\right)}{16\,c^4d^6e-32\,bc^3d^5e^2+24\,b^2c^2d^4e^3-8\,b^3cd^3e^4+b^4d^2e^5}\right)x}{3\,\left(ex^2+d\right)^{\frac{3}{2}}}$$

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"giac")
```

```
[Out] -c^2*sqrt(e)*arctan(1/2*((sqrt(e)*x - sqrt(e*x^2 + d))^2*c - 3*c*d + 2*b*e)
/sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2))/((4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*
e^3)*sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2)) - 1/3*((28*c^3*d^3*e^2 - 36*b*
c^2*d^2*e^3 + 15*b^2*c*d*e^4 - 2*b^3*e^5)*x^2/(16*c^4*d^6*e - 32*b*c^3*d^5*
e^2 + 24*b^2*c^2*d^4*e^3 - 8*b^3*c*d^3*e^4 + b^4*d^2*e^5) + 3*(12*c^3*d^4*e
 - 16*b*c^2*d^3*e^2 + 7*b^2*c*d^2*e^3 - b^3*d*e^4)/(16*c^4*d^6*e - 32*b*c^3
*d^5*e^2 + 24*b^2*c^2*d^4*e^3 - 8*b^3*c*d^3*e^4 + b^4*d^2*e^5))*x/(e*x^2 +
d)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx=\int \frac{1}{\left(e\,x^2+d\right)^{3/2}\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2\right)}\,dx$$

```
[In] int(1/((d + e*x^2)^(3/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

```
[Out] int(1/((d + e*x^2)^(3/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)), x)
```

3.225 $\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 183

$$\begin{aligned}\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx &= \frac{26x\sqrt{1+x^2+x^4}}{45\left(1+x^2\right)}+\frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4} \\ &\quad +\frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2} \\ &\quad -\frac{26(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\middle|\frac{1}{4}\right)}{45\sqrt{1+x^2+x^4}} \\ &\quad +\frac{7(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{15\sqrt{1+x^2+x^4}}\end{aligned}$$

```
[Out] 1/3*x*(x^4+x^2+1)^(3/2)+1/9*x^3*(x^4+x^2+1)^(3/2)+26/45*x*(x^4+x^2+1)^(1/2)
/(x^2+1)+2/45*x*(6*x^2+7)*(x^4+x^2+1)^(1/2)-26/45*(x^2+1)*(cos(2*arctan(x))
^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^
2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+7/15*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos
(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)
/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

= {1220, 1693, 1190, 1211, 1117, 1209}

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx = \frac{7(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}} - \frac{26(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{45\sqrt{x^4+x^2+1}} + \frac{1}{3}\left(x^4+x^2+1\right)^{3/2}x + \frac{2}{45}\left(6x^2+7\right)\sqrt{x^4+x^2+1}x + \frac{26\sqrt{x^4+x^2+1}x}{45\left(x^2+1\right)} + \frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$

```
[In] Int[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (26*x*Sqrt[1 + x^2 + x^4])/(45*(1 + x^2)) + (2*x*(7 + 6*x^2)*Sqrt[1 + x^2 +
 x^4])/45 + (x*(1 + x^2 + x^4)^(3/2))/3 + (x^3*(1 + x^2 + x^4)^(3/2))/9 - (
26*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])
/(45*Sqrt[1 + x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*
EllipticF[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}\int\sqrt{1+x^2+x^4}\left(9+24x^2+21x^4\right)\,dx\\
&= \frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}+\frac{1}{63}\int\left(42+84x^2\right)\sqrt{1+x^2+x^4}\,dx\\
&= \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}\\
&\quad+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}+\frac{1}{945}\int\frac{336+546x^2}{\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}\\
&\quad+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}-\frac{26}{45}\int\frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{14}{15}\int\frac{1}{\sqrt{1+x^2+x^4}}\,dx
\end{aligned}$$

$$= \frac{26x\sqrt{1+x^2+x^4}}{45\left(1+x^2\right)} + \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{3}x\left(1+x^2+x^4\right)^{3/2} + \frac{1}{9}x^3\left(1+x^2 + x^4\right)^{3/2} - \frac{26(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{45\sqrt{1+x^2+x^4}} + \frac{7(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{15\sqrt{1+x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

$$\int \left(1+x^2\right)^3\sqrt{1+x^2+x^4}\,dx$$

$$= \frac{x(29+61x^2+81x^4+57x^6+25x^8+5x^{10})+26\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\text{arcsinh}\left((-1)^{5/6}x\right.\right.}{45\sqrt{1+}}$$

```
[In] Integrate[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(29 + 61*x^2 + 81*x^4 + 57*x^6 + 25*x^8 + 5*x^10) + 26*(-1)^(1/3)*Sqrt[1
 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] + 2*(-1)^(5/6)*(9*I + 4*Sqrt[3])*Sqrt[1 + (-1)^(1/3)*x^2]*S
qrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(45
*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.28

method	result
risch	$\frac{x\left(5x^6+20x^4+32x^2+29\right)\sqrt{x^4+x^2+1}}{45}+\frac{32\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{45\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-104\sqrt{1-\left(-\frac{1}{2}\right.}$
default	$\frac{29x\sqrt{x^4+x^2+1}}{45}+\frac{32\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{45\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-104\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}\right.}$
elliptic	$\frac{29x\sqrt{x^4+x^2+1}}{45}+\frac{32\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{45\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-104\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}\right.}$

```
[In] int((x^2+1)^3*(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45*x*(5*x^6+20*x^4+32*x^2+29)*(x^4+x^2+1)^(1/2)+32/45/(-2+2*I*3^(1/2))^(1
/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(
x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2)
)^(1/2))-104/45/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(
1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(Elliptic
F(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*
(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.72

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx$$
$$= \frac{13\sqrt{2}(\sqrt{-3}x-x)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}) - \sqrt{2}(5\sqrt{-3}x-21x)\sqrt{\sqrt{-3}-1}F(\mathrm{ar}}{90x}$$

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/90*(13*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*
sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(5*sqrt(-3)*x
- 21*x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)
/x), 1/2*sqrt(-3) - 1/2) + 2*(5*x^8 + 20*x^6 + 32*x^4 + 29*x^2 + 26)*sqrt(x
^4 + x^2 + 1))/x
```

Sympy [F]

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx = \int \sqrt{(x^2-x+1)(x^2+x+1)}\left(x^2+1\right)^3\,dx$$

```
[In] integrate((x**2+1)**3*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3, x)
```

Maxima [F]

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\, dx = \int \sqrt{x^4+x^2+1}\left(x^2+1\right)^3 dx$$

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

Giac [F]

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\, dx = \int \sqrt{x^4+x^2+1}\left(x^2+1\right)^3 dx$$

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\, dx = \int \left(x^2+1\right)^3 \sqrt{x^4+x^2+1}\, dx$$

```
[In] int((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2),x)
[Out] int((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2), x)
```

3.226 $\int (1+x^2)^2 \sqrt{1+x^2+x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 164

$$\begin{aligned}\int (1+x^2)^2 \sqrt{1+x^2+x^4}\,dx &= \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{2}{21}x(4+3x^2)\sqrt{1+x^2+x^4} \\ &\quad + \frac{1}{7}x(1+x^2+x^4)^{3/2} \\ &\quad - \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} \\ &\quad + \frac{4(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{7\sqrt{1+x^2+x^4}}\end{aligned}$$

```
[Out] 1/7*x*(x^4+x^2+1)^(3/2)+2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+2/21*x*(3*x^2+4)*(x
^4+x^2+1)^(1/2)-2/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Ell
ipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2
)+4/7*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*a
rctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{1220, 1190, 1211, 1117, 1209\}$

$$\begin{aligned}\int \left(1+x^2\right)^2\sqrt{1+x^2+x^4}\,dx = {} & \frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\mathrm{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{7\sqrt{x^4+x^2+1}} \\ & - \frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}} \\ & + \frac{1}{7}x\left(x^4+x^2+1\right)^{3/2} \\ & + \frac{2}{21}x\left(3x^2+4\right)\sqrt{x^4+x^2+1} + \frac{2x\sqrt{x^4+x^2+1}}{3\left(x^2+1\right)}\end{aligned}$$

```
[In] Int[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*x*(4 + 3*x^2)*Sqrt[1 + x^2 + x
^4])/21 + (x*(1 + x^2 + x^4)^(3/2))/7 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (4*(1 +
x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(7*Sqrt
[1 + x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{7}\int\left(6+10x^2\right)\sqrt{1+x^2+x^4}\,dx\\
&= \frac{2}{21}x\left(4+3x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{7}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{105}\int\frac{50+70x^2}{\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{2}{21}x\left(4+3x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{7}x\left(1+x^2+x^4\right)^{3/2}\\
&\quad-\frac{2}{3}\int\frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{8}{7}\int\frac{1}{\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{2}{21}x\left(4+3x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{7}x\left(1+x^2+x^4\right)^{3/2}\\
&\quad-\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}+\frac{4(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{7\sqrt{1+x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.61 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int\left(1+x^2\right)^2\sqrt{1+x^2+x^4}\,dx$$

$$=\frac{x(11+20x^2+23x^4+12x^6+3x^8)+14\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}}{21\sqrt{1+x^2+}}$$

```
[In] Integrate[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 20*x^2 + 23*x^4 + 12*x^6 + 3*x^8) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1
/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(
2/3)] + 2*(-1)^(1/3)*(-7 + 5*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(21*Sqrt[1
+ x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.40

method	result
risch	$\frac{x(3x^{4}+9x^{2}+11)\sqrt{x^{4}+x^{2}+1}}{21}+\frac{20\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{21\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}}{}$
default	$\frac{11x\sqrt{x^{4}+x^{2}+1}}{21}+\frac{20\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{21\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\right.}}{}$
elliptic	$\frac{11x\sqrt{x^{4}+x^{2}+1}}{21}+\frac{20\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{21\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\right.}}{}$

```
[In] int((x^2+1)^2*(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*x*(3*x^4+9*x^2+11)*(x^4+x^2+1)^(1/2)+20/21/(-2+2*I*3^(1/2))^(1/2)*(1-(
-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+
1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))
-8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2
*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2
+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(
1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76

$$\int \left(1+x^{2}\right)^{2}\sqrt{1+x^{2}+x^{4}}\,dx$$
$$=\frac{7\sqrt{2}\left(\sqrt{-3}x-x\right)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})-2\sqrt{2}\left(\sqrt{-3}x-6x\right)\sqrt{\sqrt{-3}-1}F(\mathrm{arc}}{42x}$$

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/42*(7*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*s
qrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - 2*sqrt(2)*(sqrt(-3)*x -
 6*x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x
), 1/2*sqrt(-3) - 1/2) + 2*(3*x^6 + 9*x^4 + 11*x^2 + 14)*sqrt(x^4 + x^2 + 1
))/x
```

Sympy [F]

$$\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\, dx = \int \sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(x^2+1\right)^2\, dx$$

```
[In] integrate((x**2+1)**2*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2, x)
```

Maxima [F]

$$\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\, dx = \int \sqrt{x^4+x^2+1}\left(x^2+1\right)^2\, dx$$

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

Giac [F]

$$\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\, dx = \int \sqrt{x^4+x^2+1}\left(x^2+1\right)^2\, dx$$

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\, dx = \int \left(x^2+1\right)^2 \sqrt{x^4+x^2+1}\, dx$$

```
[In] int((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2), x)
```

3.227 $\int \left(1+x^2\right) \sqrt{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 145

$$\int \left(1+x^2\right) \sqrt{1+x^2+x^4} \, dx = \frac{3x\sqrt{1+x^2+x^4}}{5\left(1+x^2\right)}+\frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4} - \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}}$$

```
[Out] 3/5*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/5*x*(x^2+2)*(x^4+x^2+1)^(1/2)-3/5*(x^2+1)
*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2
)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3/5*(x^2+1)*(cos(2*arctan
(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)
/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1190, 1211, 1117, 1209}

$$\int \left(1+x^2\right) \sqrt{1+x^2+x^4} \, dx = \frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}} - \frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}} + \frac{1}{5}\left(x^2+2\right)\sqrt{x^4+x^2+1}x+\frac{3\sqrt{x^4+x^2+1}x}{5\left(x^2+1\right)}$$

```
[In] Int[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (3*x*Sqrt[1 + x^2 + x^4])/(5*(1 + x^2)) + (x*(2 + x^2)*Sqrt[1 + x^2 + x^4])
/5 - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/4])/(5*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{15}\int\frac{9+9x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4}-\frac{3}{5}\int\frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{6}{5}\int\frac{1}{\sqrt{1+x^2+x^4}}\,dx\end{aligned}$$

$$= \frac{3x\sqrt{1+x^2+x^4}}{5(1+x^2)} + \frac{1}{5}x(2+x^2)\sqrt{1+x^2+x^4} - \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.16

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx$$

$$= \frac{2x+3x^3+3x^5+x^7+3\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+\frac{3}{2}\sqrt{2+(1}}{5\sqrt{1+x^2+x^4}}$$

```
[In] Integrate[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + 3*x^5 + x^7 + 3*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
 (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (3*Sqrt[2
 + (1 - I*Sqrt[3])*x^2]*Sqrt[2 + (1 + I*Sqrt[3])*x^2]*EllipticF[ArcSin[(x +
 I*Sqrt[3]*x)/2], (I/2)*(I + Sqrt[3])])/2)/(5*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.54

method	result
risch	$\frac{x(x^2+2)\sqrt{x^4+x^2+1}}{5}+\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{5\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}-\frac{12\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-(}{}$
default	$\frac{2x\sqrt{x^4+x^2+1}}{5}+\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{5\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}-\frac{12\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-(-\frac{1}{2}-}{5}$
elliptic	$\frac{2x\sqrt{x^4+x^2+1}}{5}+\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{5\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}-\frac{12\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-(-\frac{1}{2}-}{5}$

```
[In] int((x^2+1)*(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x*(x^2+2)*(x^4+x^2+1)^(1/2)+6/5/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3
^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ell
ipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-12/5/(-2+2*
```

```
I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))
*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2
))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2)
,1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx = \frac{3\sqrt{2}(\sqrt{-3}x-x)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}) + 6\sqrt{2}x\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right.}{20x}$$

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(3*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*s
qrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) + 6*sqrt(2)*x*sqrt(sqrt(-
3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) -
 1/2) + 4*(x^4 + 2*x^2 + 3)*sqrt(x^4 + x^2 + 1))/x
```

Sympy [F]

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx = \int \sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)\,dx$$

```
[In] integrate((x**2+1)*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1), x)
```

Maxima [F]

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx = \int \sqrt{x^4+x^2+1}(x^2+1)\,dx$$

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

Giac [F]

$$\int \left(1+x^2\right) \sqrt{1+x^2+x^4}\, dx = \int \sqrt{x^4+x^2+1}\left(x^2+1\right) dx$$

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+x^2\right) \sqrt{1+x^2+x^4}\, dx = \int \left(x^2+1\right) \sqrt{x^4+x^2+1}\, dx$$

```
[In] int((x^2 + 1)*(x^2 + x^4 + 1)^(1/2),x)
[Out] int((x^2 + 1)*(x^2 + x^4 + 1)^(1/2), x)
```

3.228 $\int \frac{\sqrt{1+x^2+x^4}}{1+x^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\begin{aligned}\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx &= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} + \frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &\quad - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}\\ &\quad + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}\end{aligned}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))+x*(x^4+x^2+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2*
arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+
x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3/4*(x^2+1)*(cos(2*arctan(x))^2)^
(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)
^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1222, 1153, 1117, 1209, 1224, 1712, 209}

$$\begin{aligned}\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx &= \frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)\\ &\quad + \frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}\\ &\quad - \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{x^2+1}\end{aligned}$$

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - ((1 +
 x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1
 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*
ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
```

```
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2}{\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx \\
&\quad + \int \frac{1}{\sqrt{1+x^2+x^4}}\,dx - \int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} \\
&\quad + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&\quad - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.90 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx$$

$$= \frac{\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)-\text{EllipticF}\left(i\text{arcsinh}\left((-1)^{5/6}x\right),(-1\right.\right.}{\sqrt{1+x^2+x^4}}$$

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] ((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1
)^(2/3)] + (-1)^(1/3)*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)]))/Sqrt[1 + x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.14

method	result
default	$-\frac{4\sqrt{1+\frac{x^{2}}{2}-\frac{i x^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{i x^{2}\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}\,\left(1+i\sqrt{3}\right)}+\frac{4\sqrt{1+\frac{x^{2}}{2}-\frac{i x^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{i x^{2}\sqrt{3}}{2}}\,E\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}\,\left(1+i\sqrt{3}\right)}$
elliptic	$-\frac{4\sqrt{1+\frac{x^{2}}{2}-\frac{i x^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{i x^{2}\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}\,\left(1+i\sqrt{3}\right)}+\frac{4\sqrt{1+\frac{x^{2}}{2}-\frac{i x^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{i x^{2}\sqrt{3}}{2}}\,E\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}\,\left(1+i\sqrt{3}\right)}$

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/
2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*x*(-2+
2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/(-2+2*I*3^(1/2))^(1/2)*(1+
1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x
^2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*
I*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2)
)^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-
1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/
2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx$$
$$=\frac{2\sqrt{2}\left(\sqrt{-3}x-x\right)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2\,x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})-\sqrt{2}\left(\sqrt{-3}x-3\,x\right)\sqrt{\sqrt{-3}-1}F(\mathrm{arcsi}}{8\,x}$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sq
rt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(sqrt(-3)*x - 3*
x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x),
1/2*sqrt(-3) - 1/2) + 4*x*arctan(x/sqrt(x^4 + x^2 + 1)) + 8*sqrt(x^4 + x^2
+ 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{\sqrt{(x^2-x+1)\,(x^2+x+1)}}{x^2+1}\,dx$$

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1),x)
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx=\int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1),x)
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1), x)
```

3.229 $\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^2} \, dx=\frac{\left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}} E\left(2 \arctan (x)\left|\frac{1}{4}\right.\right)}{2 \sqrt{1+x^2+x^4}}$$

```
[Out] 1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arc
tan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1239}

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^2} \, dx=\frac{\left(x^2+1\right) \sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}} E\left(2 \arctan (x)\left|\frac{1}{4}\right.\right)}{2 \sqrt{x^4+x^2+1}}$$

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(
2*Sqrt[1 + x^2 + x^4])
```

Rule 1239

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> With[{q = Rt[e/d, 2]}, Simp[c*(d + e*x^2)*(Sqrt[(e^2*(a + b*x^2 + c
*x^4))/(c*(d + e*x^2)^2)]/(2*d*e^2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*
ArcTan[q*x], (2*c*d - b*e)/(4*c*d)], x]] /; FreeQ[{a, b, c, d, e}, x] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

```
&& PosQ[e/d]
```

Rubi steps

$$\text{integral} = \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx$$
$$= \frac{\frac{x+x^3+x^5}{1+x^2} + (-1)^{2/3}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left((-1)^{5/6}x\right),(-1)^{2/3}\right) + \sqrt[3]{-1}\sqrt{1+\sqrt[3]{}}}{2\sqrt{1+x^2}}$$

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) + (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*
Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/
(2*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 224, normalized size of antiderivative = 4.57

method	result
default	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{\sqrt{-}}$
risch	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{\sqrt{-}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{\sqrt{-}}$

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(
1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ellip
ticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3^(
1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)
^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*
(-2+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx = -\frac{2\sqrt{2}\sqrt{-3}(x^2+1)\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})+\sqrt{2}(x^2-\sqrt{-3}(x^2+1)+1)}{8\,(x^2+1)}$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(2)*sqrt(-3)*(x^2 + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2
*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sqrt(2)*(x^2 - sqrt(-
3)*(x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqrt(s
qrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 4*sqrt(x^4 + x^2 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{(x^2-x+1)\,(x^2+x+1)}}{(x^2+1)^2}\,dx$$

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**2,x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^2}\,dx$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^2}\,dx$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^2}\,dx$$

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^2,x)
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^2, x)
```

3.230 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2} + \frac{1}{4}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

```
[Out] 1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/4*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {1242, 1237, 1710, 1607, 1726, 1209, 1714, 1117, 1712, 209, 12, 1331, 1224}

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \frac{1}{4}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{4\,(x^2+1)^2}$$

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/
(4*Sqrt[1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1714

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
```

```
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

integral

$$= \int \left(\frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}} - \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\right) dx$$

$$= \int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx - \int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} - \frac{1}{4}\int \frac{-3+2x^2-x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{-1+2x^2+x^4}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{8}\int \frac{-10x^2-6x^4}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx - \frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{2x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{8}\int \frac{x^2(-10-6x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx + \int \frac{x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$

$$\begin{aligned}
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&\quad - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} \\
&\quad + \frac{1}{8}\int\frac{-6-10x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int\frac{1}{\sqrt{1+x^2+x^4}}\,dx \\
&\quad - \frac{1}{2}\int\frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx + \frac{3}{4}\int\frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} \\
&\quad + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} + \frac{1}{4}\int\frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx \\
&\quad - \frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) - \int\frac{1}{\sqrt{1+x^2+x^4}}\,dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} \\
&\quad + \frac{1}{4}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)
\end{aligned}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.89

$$\int\frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^3}\,dx$$

$$= \frac{\frac{x(2+x^2)(1+x^2+x^4)}{(1+x^2)^2} + \sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(-E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right) + \text{EllipticF}\left(i\text{arc}\right.\right.}{4\surd}$$

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] ((x*(2 + x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)
*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2
/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) + 2*(-1)^(2/3)*Sqrt[
1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSi
nh[(-1)^(5/6)*x], (-1)^(2/3)])/(4*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.70

method	result
risch	$\frac{\sqrt{x^4+x^2+1}\,x\left(x^2+2\right)}{4\left(x^2+1\right)^2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-E\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$
default	$\frac{x\sqrt{x^4+x^2+1}}{4\left(x^2+1\right)^2}+\frac{x\sqrt{x^4+x^2+1}}{4x^2+4}+\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+i}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{4\left(x^2+1\right)^2}+\frac{x\sqrt{x^4+x^2+1}}{4x^2+4}+\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+i}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{}}$

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(x^4+x^2+1)^(1/2)*x*(x^2+2)/(x^2+1)^2+1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2
+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(
1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1
/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/
2)))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+
1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3
^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/
2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^3}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{-3}\left(x^4+2x^2+1\right)\sqrt{\sqrt{-3}-1}\,F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)+\sqrt{2}\left(x^4+2x^2-\sqrt{-}\right.}{}$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*sqrt(2)*sqrt(-3)*(x^4 + 2*x^2 + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(a
rcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sqrt(2)*(x^4
 + 2*x^2 - sqrt(-3)*(x^4 + 2*x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(ar
```

```
csin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 4*(x^4 + 2*x^
2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1)) - 4*sqrt(x^4 + x^2 + 1)*(x^3 + 2*x))/(
x^4 + 2*x^2 + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \int \frac{\sqrt{(x^2-x+1)\,(x^2+x+1)}}{(x^2+1)^3}\,dx$$

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**3,x)
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^3,x)
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^3, x)
```

3.231 $\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 166

$$\begin{aligned}\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4} \, dx = {} & \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3}+\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2}+\frac{1}{4}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \\ & +\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{3\sqrt{1+x^2+x^4}} \\ & -\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{8\sqrt{1+x^2+x^4}}\end{aligned}$$

```
[Out] 1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^3+1/6*x*(x^
4+x^2+1)^(1/2)/(x^2+1)^2+1/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arcta
n(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^
2+1)^(1/2)-1/8*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Elliptic
F(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules

used = {1242, 1237, 1710, 1600, 1211, 1117, 1209, 1607, 1726, 1714, 1712, 209, 12, 1331}

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx = \frac{1}{4}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\mathrm{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}} + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{6\left(x^2+1\right)^2} + \frac{\sqrt{x^4+x^2+1}x}{6\left(x^2+1\right)^3}$$

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x
^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/
(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) - ((1 + x
^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(8*Sqrt[
1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1714

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{(1+x^2)^4\sqrt{1+x^2+x^4}} - \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\right) dx$$

$$=\int \frac{1}{\left(1+x^2\right)^4 \sqrt{1+x^2+x^4}} \, dx-\int \frac{1}{\left(1+x^2\right)^3 \sqrt{1+x^2+x^4}} \, dx+\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}} \, dx$$

$$=\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3}-\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}+\frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)}-\frac{1}{6}\int \frac{-5+2x^2-3x^4}{\left(1+x^2\right)^3 \sqrt{1+x^2+x^4}} \, dx+\frac{1}{4}\int \frac{-3+2x^2-x^4}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}} \, dx-\frac{1}{2}\int \frac{-1+2x^2+x^4}{\left(1+x^2\right) \sqrt{1+x^2+x^4}} \, dx$$

$$=\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3}+\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2}-\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)}+\frac{1}{24}\int \frac{10-8x^2+10x^4}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}} \, dx-\frac{1}{8}\int \frac{-10x^2-6x^4}{\left(1+x^2\right) \sqrt{1+x^2+x^4}} \, dx+\frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}} \, dx-\frac{1}{2}\int \frac{2x^2}{\left(1+x^2\right) \sqrt{1+x^2+x^4}} \, dx$$

$$=\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3}+\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2}-\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}-\frac{1}{48}\int \frac{8+36x^2+28x^4}{\left(1+x^2\right) \sqrt{1+x^2+x^4}} \, dx-\frac{1}{8}\int \frac{x^2\left(-10-6x^2\right)}{\left(1+x^2\right) \sqrt{1+x^2+x^4}} \, dx-\int \frac{x^2}{\left(1+x^2\right) \sqrt{1+x^2+x^4}} \, dx$$

$$=\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3}+\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2}-\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}-\frac{1}{48}\int \frac{8+28x^2}{\sqrt{1+x^2+x^4}} \, dx-\frac{1}{8}\int \frac{-6-10x^2}{\left(1+x^2\right) \sqrt{1+x^2+x^4}} \, dx-\frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}} \, dx+\frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right) \sqrt{1+x^2+x^4}} \, dx-\frac{3}{4}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}} \, dx$$

$$=\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3}+\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2}+\frac{7x\sqrt{1+x^2+x^4}}{12\left(1+x^2\right)}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}-\frac{1}{4}\int \frac{1-x^2}{\left(1+x^2\right) \sqrt{1+x^2+x^4}} \, dx+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2} \, dx,x,\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{7}{12}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}} \, dx-\frac{3}{4}\int \frac{1}{\sqrt{1+x^2+x^4}} \, dx+\int \frac{1}{\sqrt{1+x^2+x^4}} \, dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{8\sqrt{1+x^2+x^4}} - \frac{1}{4}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{8\sqrt{1+x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.45

$$\int\frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx$$

$$= \frac{\frac{x(1+x^2+x^4)(4+5x^2+2x^4)}{(1+x^2)^3} - 2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right) - \text{EllipticF}\left(\right.\right.}{}$$

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

```
[Out] ((x*(1 + x^2 + x^4)*(4 + 5*x^2 + 2*x^4))/(1 + x^2)^3 - 2*(-1)^(1/3)*Sqrt[1
+ (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - (-1)^(2
/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(
-1)^(5/6)*x], (-1)^(2/3)] + 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]
)/(6*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.05

method	result
risch	$\frac{\sqrt{x^4+x^2+1}\,x\left(2x^4+5x^2+4\right)}{6\left(x^2+1\right)^3}-\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}$
default	$\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^3}+\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^2}+\frac{x\sqrt{x^4+x^2+1}}{3x^2+3}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+4\sqrt{1+\frac{x^2}{2}-}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^3}+\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^2}+\frac{x\sqrt{x^4+x^2+1}}{3x^2+3}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+4\sqrt{1+\frac{x^2}{2}-}$

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(x^4+x^2+1)^(1/2)*x*(2*x^4+5*x^2+4)/(x^2+1)^3-1/3/(-2+2*I*3^(1/2))^(1/2
)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^
4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^
(1/2))+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1
/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2
*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2
*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+1/2/(-1/2+1/2*I*3^(1/2))^(1/
2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/
(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^
(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx = -\frac{4\,\sqrt{2}\left(x^6+3\,x^4+3\,x^2-\sqrt{-3}\left(x^6+3\,x^4+3\,x^2+1\right)+1\right)\sqrt{\sqrt{-3}-1}E\left(\arcsin\left(\frac{1}{2}\,\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\right.}{}$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(4*sqrt(2)*(x^6 + 3*x^4 + 3*x^2 - sqrt(-3)*(x^6 + 3*x^4 + 3*x^2 + 1)
+ 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1))
, 1/2*sqrt(-3) - 1/2) - sqrt(2)*(3*x^6 + 9*x^4 + 9*x^2 - 5*sqrt(-3)*(x^6 +
3*x^4 + 3*x^2 + 1) + 3)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*x*
sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 12*(x^6 + 3*x^4 + 3*x^2 + 1)*arc
tan(x/sqrt(x^4 + x^2 + 1)) - 8*(2*x^5 + 5*x^3 + 4*x)*sqrt(x^4 + x^2 + 1))/(
x^6 + 3*x^4 + 3*x^2 + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx=\int \frac{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}{\left(x^2+1\right)^4}\,dx$$

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**4,x)
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx=\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^4}\,dx$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx=\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^4}\,dx$$

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx=\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^4}\,dx$$

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^4,x)
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^4, x)
```

3.232 $\int \frac{\left(1+x^2\right)^3}{\sqrt{1+x^2+x^4}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 159

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}} \, dx = \frac{11}{15} x \sqrt{1+x^2+x^4} + \frac{1}{5} x^3 \sqrt{1+x^2+x^4} + \frac{14 x \sqrt{1+x^2+x^4}}{15 \left(1+x^2\right)} - \frac{14 (1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} E\left(2 \arctan(x) \middle| \frac{1}{4}\right)}{15 \sqrt{1+x^2+x^4}} + \frac{3 (1+x^2) \sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}} \operatorname{EllipticF}\left(2 \arctan(x), \frac{1}{4}\right)}{5 \sqrt{1+x^2+x^4}}$$

```
[Out] 11/15*x*(x^4+x^2+1)^(1/2)+1/5*x^3*(x^4+x^2+1)^(1/2)+14/15*x*(x^4+x^2+1)^(1/
2)/(x^2+1)-14/15*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Ellipt
icE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3
/5*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arct
an(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{1220, 1693, 1211, 1117, 1209\}$

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}} - \frac{14(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}} + \frac{14\sqrt{x^4+x^2+1}x}{15\,(x^2+1)} + \frac{11}{15}\sqrt{x^4+x^2+1}x + \frac{1}{5}\sqrt{x^4+x^2+1}x^3$$

```
[In] Int[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (11*x*Sqrt[1 + x^2 + x^4])/15 + (x^3*Sqrt[1 + x^2 + x^4])/5 + (14*x*Sqrt[1
+ x^2 + x^4])/(15*(1 + x^2)) - (14*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sq
rt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^
2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
```

```
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{1}{5}\int\frac{5+12x^2+11x^4}{\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{1}{15}\int\frac{4+14x^2}{\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}-\frac{14}{15}\int\frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{6}{5}\int\frac{1}{\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{14x\sqrt{1+x^2+x^4}}{15\,(1+x^2)}\\
&\quad-\frac{14(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{15\sqrt{1+x^2+x^4}}+\frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int\frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx$$

$$=\frac{x(11+14x^2+14x^4+3x^6)+14\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+2}{15\sqrt{1+x^2+x^4}}$$

```
[In] Integrate[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 14*x^2 + 14*x^4 + 3*x^6) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*
Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2
*(-1)^(1/3)*(-7 + 2*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3
)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(15*Sqrt[1 + x^2 + x
^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.42

method	result
risch	$\frac{x(3x^2+11)\sqrt{x^4+x^2+1}}{15}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{15\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{56\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-(}}{}$
default	$\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{15\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x^3\sqrt{x^4+x^2+1}}{5}+\frac{11x\sqrt{x^4+x^2+1}}{15}-\frac{56\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}{}$
elliptic	$\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{15\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x^3\sqrt{x^4+x^2+1}}{5}+\frac{11x\sqrt{x^4+x^2+1}}{15}-\frac{56\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}{}$

```
[In] int((x^2+1)^3/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*x*(3*x^2+11)*(x^4+x^2+1)^(1/2)+8/15/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/
2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2
)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-56/15/
(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^
(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*
3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))
^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

$$\int \frac{\left(1+x^2\right)^3}{\sqrt{1+x^2+x^4}}\,dx$$
$$=\frac{7\sqrt{2}\left(\sqrt{-3}x-x\right)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2\,x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})-\sqrt{2}\left(5\sqrt{-3}x-9\,x\right)\sqrt{\sqrt{-3}-1}F(\mathrm{arcs}}{30\,x}$$

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(7*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*s
qrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(5*sqrt(-3)*x -
 9*x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x
), 1/2*sqrt(-3) - 1/2) + 2*(3*x^4 + 11*x^2 + 14)*sqrt(x^4 + x^2 + 1))/x
```

Sympy [F]

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^3}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

```
[In] integrate((x**2+1)**3/(x**4+x**2+1)**(1/2),x)
[Out] Integral((x**2 + 1)**3/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Maxima [F]

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}}\,dx$$

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

Giac [F]

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}}\,dx$$

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}}\,dx$$

```
[In] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(1/2),x)
[Out] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(1/2), x)
```

3.233 $\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}} \, dx = \frac{1}{3}x\sqrt{1+x^2+x^4} + \frac{4x\sqrt{1+x^2+x^4}}{3(1+x^2)} - \frac{4(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

```
[Out] 1/3*x*(x^4+x^2+1)^(1/2)+4/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-4/3*(x^2+1)*(cos(2*
arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+
x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+(x^2+1)*(cos(2*arctan(x))^2)^(1/2
)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^
(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{1220, 1211, 1117, 1209\}$

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} - \frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}} + \frac{4\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{1}{3}\sqrt{x^4+x^2+1}x$$

```
[In] Int[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/3 + (4*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) - (4*(1
 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*S
qrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Elliptic
F[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
```

```
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{1}{3}\int\frac{2+4x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{1+x^2+x^4}-\frac{4}{3}\int\frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+2\int\frac{1}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{4x\sqrt{1+x^2+x^4}}{3\,(1+x^2)}-\frac{4(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}\\ &\quad+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int\frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$=\frac{x+x^3+x^5+4\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+2\sqrt[3]{-1}\left(-2+\sqrt[3]{-1}\right)}{3\sqrt{1+x^2+x^4}}$$

```
[In] Integrate[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x + x^3 + x^5 + 4*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*
x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(1/3)*(-2 + (-
1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*Arc
Sinh[(-1)^(5/6)*x], (-1)^(2/3)])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.59

method	result
default	$\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{3}-$
risch	$\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{3}-$
elliptic	$\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{3}-$

```
[In] int((x^2+1)^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*
I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/3*x*(x^4+x^2+1)^(1/2)-16/3/(-2+2*I*3^(1/2
))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1
/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2)
,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2
+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int \frac{\left(1+x^2\right)^2}{\sqrt{1+x^2+x^4}}\,dx$$
$$=\frac{2\sqrt{2}\left(\sqrt{-3}x-x\right)\sqrt{\sqrt{-3}-1}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-\sqrt{2}\left(\sqrt{-3}x-3x\right)\sqrt{\sqrt{-3}-1}\,F(\mathrm{arcsi}}{6x}$$

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sq
rt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(sqrt(-3)*x - 3*
x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x),
1/2*sqrt(-3) - 1/2) + 2*sqrt(x^4 + x^2 + 1)*(x^2 + 4))/x
```

Sympy [F]

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^2}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

```
[In] integrate((x**2+1)**2/(x**4+x**2+1)**(1/2),x)
[Out] Integral((x**2 + 1)**2/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Maxima [F]

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^2}{\sqrt{x^4+x^2+1}}\,dx$$

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

Giac [F]

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^2}{\sqrt{x^4+x^2+1}}\,dx$$

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^2}{\sqrt{x^4+x^2+1}}\,dx$$

```
[In] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(1/2),x)
[Out] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(1/2), x)
```

3.234 $\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

```
[Out] x*(x^4+x^2+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan
(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2
+1)^(1/2)+(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin
(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1211, 1117, 1209}

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} - \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{x^2+1}$$

```
[In] Int[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^
2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + ((1 + x^2)*Sqrt[(1
 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx-\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{x\sqrt{1+x^2+x^4}}{1+x^2}-\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,(E(i\operatorname{arcsinh}((-1)^{5/6}x)\,|(-1)^{2/3})+(-1+\sqrt[3]{-1})\ \text{EllipticF}\ (i\operatorname{arcsinh}((}{\sqrt{1+x^2+x^4}}$$

```
[In] Integrate[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1 + (-1)^(1/3))*EllipticF[I*ArcSinh[
(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}-\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,\left(F\left(\frac{x\sqrt{-2+2}}{2}\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}-\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,\left(F\left(\frac{x\sqrt{-2+2}}{2}\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x}}$

```
[In] int((x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2))-4/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/
2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^
(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-
EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{2}\left(\sqrt{-3}x-x\right)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2\,x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})+2\sqrt{2}x\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2\,x}\right.}{4\,x}$$

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sqrt
(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) + 2*sqrt(2)*x*sqrt(sqrt(-3)
- 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/
2) + 4*sqrt(x^4 + x^2 + 1))/x
```

Sympy [F]

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{(x^2-x+1)\,(x^2+x+1)}}\,dx$$

```
[In] integrate((x**2+1)/(x**4+x**2+1)**(1/2),x)
[Out] Integral((x**2 + 1)/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Maxima [F]

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Giac [F]

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

```
[In] int((x^2 + 1)/(x^2 + x^4 + 1)^(1/2),x)
[Out] int((x^2 + 1)/(x^2 + x^4 + 1)^(1/2), x)
```

3.235 $\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))+1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(
2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/
(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1224, 1117, 1712, 209}

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

```
[In] Int[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx+\frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{1+x^2+x^4}}\right)\\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticPi}\left(\sqrt[3]{-1},i\text{arcsinh}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)}{\sqrt{1+x^2+x^4}}$$

```
[In] Integrate[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-
1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/Sqrt[1 + x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.51

method	result	size
default	$\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\Pi\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}$	104
elliptic	$\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\Pi\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}$	104

```
[In] int(1/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2
+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))
^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(
1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\frac{1}{8}\sqrt{2}(\sqrt{-3}+1)\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}) + \frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*(sqrt(-3) + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2
)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + 1/2*arctan(x/sqrt(x^4 + x^2
+ 1))
```

Sympy [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)}\,dx$$

```
[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}\,dx$$

```
[In] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
[Out] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

3.236 $\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 118

$$\begin{aligned}\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx &= \frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &\quad+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}\\ &\quad-\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}\end{aligned}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))+1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(
2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/
(x^4+x^2+1)^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*E
llipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1
/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used

$= \{1237, 1726, 1209, 12, 1331, 1117, 1712, 209\}$

$$\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} + \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

```
[In] Int[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[
(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 +
 x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{-1+2x^2+x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx - \frac{1}{2}\int \frac{2x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \int \frac{x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx
\end{aligned}$$

$$= \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\big|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\big|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\big|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\big|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.92

$$\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x+x^3+x^5}{1+x^2} - (-1)^{2/3}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left((-1)^{5/6}x\right),(-1)^{2/3}\right) + \sqrt[3]{-1}$$

```
[In] Integrate[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*
Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) +
 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[
(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(2*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.78

method	result
risch	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}}{\sqrt{-2}}$
default	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right.}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right.}$

```
[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(
1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ellip
ticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3^(
1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)
^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*
(-2+2*I*3^(1/2))^(1/2)))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*
3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Ellipt
icPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/
2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.19

$$\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = -\frac{\sqrt{2}\left(x^2-\sqrt{-3}(x^2+1)+1\right)\sqrt{\sqrt{-3}-1}\,E\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-\sqrt{2}\left(x^2-\sqrt{-3}(x^2\right.}{}$$

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*(x^2 - sqrt(-3)*(x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(
arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(x^
2 - sqrt(-3)*(x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2
)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 4*(x^2 + 1)*arctan(x/sqrt(x^
4 + x^2 + 1)) - 4*sqrt(x^4 + x^2 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(x^2+1\right)^2}\,dx$$

```
[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^2}\,dx$$

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^2}\,dx$$

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\left(x^2+1\right)^2 \sqrt{x^4+x^2+1}}\,dx$$

```
[In] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)),x)
[Out] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)), x)
```

3.237 $\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\begin{aligned}\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}} \, dx = {} & \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{1}{4}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \\ & + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} \\ & - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}\end{aligned}$$

```
[Out] 1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+3/4*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)-1/2*(x^2+1)*(cos(2*arct
an(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+
1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

= {1237, 1710, 1607, 1726, 1209, 1714, 1117, 1712, 209}

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = \frac{1}{4}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}} + \frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

```
[In] Int[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4]
)/(4*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
```

```
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1714

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
```

```
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{1}{4}\int \frac{-3+2x^2-x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{-10x^2-6x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{x^2(-10-6x^2)}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{-6-10x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx + \frac{3}{4}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} \\
&\quad + \frac{1}{4}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx - \int \frac{1}{\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} \\
&\quad - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&\quad + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.65

$$\int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{\frac{x(4+3x^2)(1+x^2+x^4)}{(1+x^2)^2} - 3\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right) - \text{EllipticF}\left(ia\right.\right.}{}$$

```
[In] Integrate[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x*(4 + 3*x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 - 3*(-1)^(1/3)*Sqrt[1 + (-1)^(
1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)
^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)*Sq
rt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5
/6)*x], (-1)^(2/3)] + 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(
2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(4*S
qrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.37

method	result
risch	$\frac{\sqrt{x^4+x^2+1}\,x(3x^2+4)}{4(x^2+1)^2}-\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+3\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}$
default	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{3x\sqrt{x^4+x^2+1}}{4(x^2+1)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+3\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{3x\sqrt{x^4+x^2+1}}{4(x^2+1)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+3\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}$

```
[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(x^4+x^2+1)^(1/2)*x*(3*x^2+4)/(x^2+1)^2-1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1
/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)
^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+3
/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3
^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I
*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2)
)^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*
x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1
)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1
/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = -\frac{3\sqrt{2}\left(x^4+2x^2-\sqrt{-3}(x^4+2x^2+1)+1\right)\sqrt{\sqrt{-3}-1}E\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-2}{}$$

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*sqrt(2)*(x^4 + 2*x^2 - sqrt(-3)*(x^4 + 2*x^2 + 1) + 1)*sqrt(sqrt(-
3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) -
 1/2) - 2*sqrt(2)*(2*x^4 + 4*x^2 - sqrt(-3)*(x^4 + 2*x^2 + 1) + 2)*sqrt(sqr
t(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3
) - 1/2) - 4*(x^4 + 2*x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1)) - 4*sqrt(x^4 +
 x^2 + 1)*(3*x^3 + 4*x))/(x^4 + 2*x^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)^3}\,dx$$

```
[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)^3}\,dx$$

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

Giac [F]

$$\int \frac{1}{\left(1+x^2\right)^3 \sqrt{1+x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^3}\,dx$$

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+x^2\right)^3 \sqrt{1+x^2+x^4}}\,dx=\int \frac{1}{\left(x^2+1\right)^3 \sqrt{x^4+x^2+1}}\,dx$$

```
[In] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2)), x)
```

3.238 $\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} \, dx = -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} - \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

```
[Out] -1/3*x*(-x^2+1)/(x^4+x^2+1)^(1/2)+2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-2/3*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+(x^2+1)*(cos(2*arctan(x
))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(
x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1219, 1211, 1117, 1209}

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} \, dx = \frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}} - \frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}} + \frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)} - \frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}}$$

```
[In] Int[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] -1/3*(x*(1 - x^2))/Sqrt[1 + x^2 + x^4] + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
x^2)) - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x
], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3}\int \frac{4+2x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} - \frac{2}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx + 2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx$$

$$= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{2x\sqrt{1+x^2+x^4}}{3\,(1+x^2)} - \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx = \frac{-x+x^3+2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+}{3\sqrt{1+x}}$$

```
[In] Integrate[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (-x + x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*
EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(5/6)*Sqrt[3 + 3*(-
1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (
-1)^(2/3)])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.55

method	result
risch	$\frac{x(x^2-1)}{3\sqrt{x^4+x^2+1}} + \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}} - \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{3\sqrt{}}$
elliptic	$-\frac{2\left(\frac{1}{6}x-\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}} + \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}} - \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{}}{2}\right.}}{3}$
default	$-\frac{4\left(-\frac{1}{6}x+\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}} + \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}} - \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\right.}}{}$

```
[In] int((x^2+1)^3/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(x^2-1)/(x^4+x^2+1)^(1/2)+8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3
^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ell
```

```
ipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/(-2+2*I
*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*
x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2)
)^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),
1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(x^5+x^3-\sqrt{-3}(x^5+x^3+x)+x\right)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2\,x}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}) - \sqrt{2}\left(3\,x^5+3\,x^3+\right.}{6\,(x^5+}$$

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(2)*(x^5 + x^3 - sqrt(-3)*(x^5 + x^3 + x) + x)*sqrt(sqrt(-3) - 1)
*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) -
 sqrt(2)*(3*x^5 + 3*x^3 + sqrt(-3)*(x^5 + x^3 + x) + 3*x)*sqrt(sqrt(-3) - 1
)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2)
- 2*(3*x^4 + x^2 + 2)*sqrt(x^4 + x^2 + 1))/(x^5 + x^3 + x)
```

Sympy [F]

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^3}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)**3/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^3}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^3}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^3}{(x^4+x^2+1)^{3/2}}\,dx$$

```
[In] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(3/2),x)
[Out] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(3/2), x)
```

3.239 $\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} \, dx = \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} - \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\, E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

```
[Out] 1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)-2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+2/3*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1219, 1209}

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} \, dx = \frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\, E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}} - \frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{(2x^2+1)\,x}{3\sqrt{x^4+x^2+1}}$$

```
[In] Int[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) - (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
 x^2)) + (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[
x], 1/4])/(3*Sqrt[1 + x^2 + x^4])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}}+\frac{1}{3}\int\frac{2-2x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}}-\frac{2x\sqrt{1+x^2+x^4}}{3\,(1+x^2)}+\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.61

$$\int\frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx=\frac{x+2x^3-2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-}{3\sqrt{1}}$$

```
[In] Integrate[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x + 2*x^3 - 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]
*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - I*Sqrt[2 + (1 + I*Sqrt[3]
)*x^2]*Sqrt[6 + (3 - (3*I)*Sqrt[3])*x^2]*EllipticF[ArcSin[(x + I*Sqrt[3]*x)
/2], (I/2)*(I + Sqrt[3])]])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.30

method	result
risch	$\frac{x\left(2x^2+1\right)}{3\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x}}{3\sqrt{-}}$
elliptic	$-\frac{2\left(-\frac{1}{3}x^3-\frac{1}{6}x\right)}{\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{}}{2}\right.}}{3}$
default	$-\frac{2\left(-\frac{1}{6}x+\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{}}{2}\right.}}{3}$

```
[In] int((x^2+1)^2/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I
*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*E
llipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+8/3/(-2+2
*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2)
)*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/
2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.34

$$\int \frac{\left(1+x^2\right)^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{-3}\left(x^4+x^2+1\right)\sqrt{\sqrt{-3}-1}F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)+\sqrt{2}\left(x^4+x^2-\sqrt{-3}(x^4+\right.}{6\left(x^4+x^2+\right.}$$

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(2)*sqrt(-3)*(x^4 + x^2 + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(arcs
in(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sqrt(2)*(x^4 +
x^2 - sqrt(-3)*(x^4 + x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/
2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 2*sqrt(x^4 + x^2 + 1
)*(2*x^3 + x))/(x^4 + x^2 + 1)
```

Sympy [F]

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^2}{((x^2-x+1)\,(x^2+x+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x**2+1)**2/(x**4+x**2+1)**(3/2),x)
[Out] Integral((x**2 + 1)**2/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^2}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^2}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^2}{(x^4+x^2+1)^{3/2}}\,dx$$

```
[In] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(3/2),x)
[Out] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(3/2), x)
```

3.240 $\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 96

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx=\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}}-\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{3\sqrt{1+x^2+x^4}}$$

```
[Out] 1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)-1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/3*(x^2+1)
*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2
)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1192, 1209}

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx=\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{3\sqrt{x^4+x^2+1}}-\frac{\sqrt{x^4+x^2+1}x}{3\left(x^2+1\right)}+\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}$$

```
[In] Int[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2
)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/
4])/(3*Sqrt[1 + x^2 + x^4])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx \\ &= \frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} - \frac{x\sqrt{1+x^2+x^4}}{3\,(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.67

$$\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx = \frac{2x+x^3-\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\,\text{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)-\frac{1}{2}}{3\sqrt{1}}$$

```
[In] Integrate[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (2*x + x^3 - (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*E
llipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - (I/2)*Sqrt[2 + (1 + I*Sqrt[
3])*x^2]*Sqrt[6 + (3 - (3*I)*Sqrt[3])*x^2]*EllipticF[ArcSin[(x + I*Sqrt[3]*
x)/2], (I/2)*(I + Sqrt[3])]])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.32

method	result
risch	$\frac{x(x^2+2)}{3\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x}{3\sqrt{-}}$
elliptic	$-\frac{2\left(-\frac{1}{6}x^3-\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{}}{2}}{3}$
default	$-\frac{2\left(-\frac{1}{6}x+\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{}}{2}}{3}$

```
[In] int((x^2+1)/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3
^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ell
ipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/3/(-2+2*I
*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*
x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2)
)^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),
1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.36

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{-3}(x^4+x^2+1)\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})+\sqrt{2}(x^4+x^2-\sqrt{-3}(x^4+}{12\,(x^4+x^2+}$$

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(2)*sqrt(-3)*(x^4 + x^2 + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(arc
sin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sqrt(2)*(x^4 +
 x^2 - sqrt(-3)*(x^4 + x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1
/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 4*sqrt(x^4 + x^2 +
1)*(x^3 + 2*x))/(x^4 + x^2 + 1)
```

Sympy [F]

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx=\int \frac{x^2+1}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x**2+1)/(x**4+x**2+1)**(3/2),x)
[Out] Integral((x**2 + 1)/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx=\int \frac{x^2+1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx=\int \frac{x^2+1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx=\int \frac{x^2+1}{\left(x^4+x^2+1\right)^{3/2}}\,dx$$

```
[In] int((x^2 + 1)/(x^2 + x^4 + 1)^(3/2),x)
[Out] int((x^2 + 1)/(x^2 + x^4 + 1)^(3/2), x)
```

3.241 $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 166

$$\begin{aligned}\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx = &-\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)}\\ &+\frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)-\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}\\ &+\frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}\end{aligned}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)+2/3*x*(x^
4+x^2+1)^(1/2)/(x^2+1)-2/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(
x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+
1)^(1/2)+3/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(
sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used

$= \{1235, 1133, 1211, 1117, 1209, 1224, 1712, 209\}$

$$\begin{aligned}\int \frac{1}{(1+x^2)\left(1+x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) \\ &+ \frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} \\ &- \frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}} + \frac{2\sqrt{x^4+x^2+1}x}{3\left(x^2+1\right)} - \frac{\left(2x^2+1\right)x}{3\sqrt{x^4+x^2+1}}\end{aligned}$$

```
[In] Int[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/3*(x*(1 + 2*x^2))/Sqrt[1 + x^2 + x^4] + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1
+ x^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^
4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (3*(
1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*
Sqrt[1 + x^2 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
```

```
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{x^2}{(1+x^2+x^4)^{3/2}}\,dx + \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx \\
&= -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3}\int \frac{1+2x^2}{\sqrt{1+x^2+x^4}}\,dx \\
&\quad + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} \\
&\quad + \frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&\quad - \frac{2}{3}\int\frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx + \int\frac{1}{\sqrt{1+x^2+x^4}}\,dx \\
&= -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&\quad - \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.23

$$\int\frac{1}{(1+x^2)\left(1+x^2+x^4\right)^{3/2}}\,dx = \frac{-x-2x^3+2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)|\right.}{}$$

```
[In] Integrate[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-x - 2*x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2
]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*(-2 + (-1)^(1
/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[
(-1)^(5/6)*x], (-1)^(2/3)] + 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
 (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)
])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{x\left(2x^{2}+1\right)}{3\sqrt{x^{4}+x^{2}+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{3\sqrt{}}$
default	$-\frac{2\left(\frac{1}{3}x^{3}+\frac{1}{6}x\right)}{\sqrt{x^{4}+x^{2}+1}}+\frac{2\sqrt{1+\frac{x^{2}}{2}-\frac{ix^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{ix^{2}\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}-\frac{8\sqrt{1+\frac{x^{2}}{2}-\frac{ix^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{ix^{2}\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2}}{2}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}\left(1+\right.}$
elliptic	$-\frac{2\left(\frac{1}{3}x^{3}+\frac{1}{6}x\right)}{\sqrt{x^{4}+x^{2}+1}}+\frac{2\sqrt{1+\frac{x^{2}}{2}-\frac{ix^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{ix^{2}\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}-\frac{8\sqrt{1+\frac{x^{2}}{2}-\frac{ix^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{ix^{2}\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2}}{2}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}\left(1+\right.}$

```
[In] int(1/(x^2+1)/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*
I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*
EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/(-2+
2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2
))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1
/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/
2),1/2*(-2+2*I*3^(1/2))^(1/2)))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2
*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)
*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*
I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int \frac{1}{(1+x^2)\left(1+x^2+x^4\right)^{3/2}}\,dx = \frac{4\sqrt{2}\left(x^4+x^2-\sqrt{-3}(x^4+x^2+1)+1\right)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-}}\right.}{}$$

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(4*sqrt(2)*(x^4 + x^2 - sqrt(-3)*(x^4 + x^2 + 1) + 1)*sqrt(sqrt(-3) -
1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2)
 - sqrt(2)*(9*x^4 + 9*x^2 + sqrt(-3)*(x^4 + x^2 + 1) + 9)*sqrt(sqrt(-3) - 1
)*elliptic_f(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2)
+ 12*(x^4 + x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1)) - 8*sqrt(x^4 + x^2 + 1)*
(2*x^3 + x))/(x^4 + x^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x^2)\left(1+x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left((x^2-x+1)\left(x^2+x+1\right)\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

```
[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(3/2),x)
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)\left(1+x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x^2)\left(1+x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)\left(1+x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2+1\right)\,\left(x^4+x^2+1\right)^{3/2}}\,dx$$

```
[In] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(3/2)),x)
[Out] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(3/2)), x)
```

3.242 $\int \frac{1}{\left(1+x^2\right)^2 \left(1+x^2+x^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{1}{\left(1+x^2\right)^2 \left(1+x^2+x^4\right)^{3/2}} \, dx = -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, E\left(2\arctan(x)\middle|\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}$$

```
[Out] arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)+1/3*x*(x^4+x^2+
1)^(1/2)/(x^2+1)+1/6*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*El
lipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/
2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {1242, 1192, 1209, 1237, 1726, 12, 1331, 1117, 1712, 209, 1224}

$$\int \frac{1}{\left(1+x^2\right)^2 \left(1+x^2+x^4\right)^{3/2}} \, dx = \arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) + \frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\, E\left(2\arctan(x)\middle|\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}} + \frac{\sqrt{x^4+x^2+1}x}{3\left(x^2+1\right)} - \frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}$$

```
[In] Int[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/3*(x*(2 + x^2))/Sqrt[1 + x^2 + x^4] + (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^
2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 +
x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(6*Sqrt[1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{-1-x^2}{(1+x^2+x^4)^{3/2}} + \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}} \right) dx$$

$$=\int \frac{-1-x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx+\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}}\,dx+\int \frac{1}{\left(1+x^2\right) \sqrt{1+x^2+x^4}}\,dx$$

$$=-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)}+\frac{1}{3}\int \frac{-1+x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx$$
$$+\frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx-\frac{1}{2}\int \frac{-1+2x^2+x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$=-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}+\frac{5x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$
$$+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}+\frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx$$
$$-\frac{1}{2}\int \frac{2x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

$$=-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$
$$+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}$$
$$+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}-\int \frac{x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$=-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$
$$+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$
$$-\frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx+\frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$=-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$
$$+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

$$=-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$
$$+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.51

$$\int \frac{1}{(1+x^2)^2\,(1+x^2+x^4)^{3/2}}\,dx = \frac{-2x(1+x^2)\,(2+x^2)+3x(1+x^2+x^4)-\sqrt[3]{-1}(1+x^2)\,\sqrt{1+\sqrt[3]{-1}x^2}\,\sqrt{1}{}$$

```
[In] Integrate[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-2*x*(1 + x^2)*(2 + x^2) + 3*x*(1 + x^2 + x^4) - (-1)^(1/3)*(1 + x^2)*Sqrt
[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/
6)*x], (-1)^(2/3)] + (-1 + 5*(-1)^(1/3))*EllipticF[I*ArcSinh[(-1)^(5/6)*x],
 (-1)^(2/3)] - 12*(-1)^(1/3)*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x]
, (-1)^(2/3)]))/(6*(1 + x^2)*Sqrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.05

method	result
risch	$\frac{x(x^4-3x^2-1)}{6(x^2+1)\sqrt{x^4+x^2+1}}-\frac{5\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}\right.}{}$
default	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}-\frac{5\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}-\frac{5\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{}}$

```
[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(x^4-3*x^2-1)/(x^2+1)/(x^4+x^2+1)^(1/2)-5/3/(-2+2*I*3^(1/2))^(1/2)*(1
-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^
2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2
))+2/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1
/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(
-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3
^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+
1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x
```

```
^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2))
,(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int \frac{1}{(1+x^2)^2\,(1+x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{-3}(x^6+2\,x^4+2\,x^2+1)\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})+\sqrt{2}(x^6+2\,x^4+}{}$$

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*sqrt(-3)*(x^6 + 2*x^4 + 2*x^2 + 1)*sqrt(sqrt(-3) - 1)*elli
ptic_f(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sqrt
(2)*(x^6 + 2*x^4 + 2*x^2 - sqrt(-3)*(x^6 + 2*x^4 + 2*x^2 + 1) + 1)*sqrt(sqr
t(-3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3
) - 1/2) - 24*(x^6 + 2*x^4 + 2*x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1)) - 4*(
x^5 - 3*x^3 - x)*sqrt(x^4 + x^2 + 1))/(x^6 + 2*x^4 + 2*x^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x^2)^2\,(1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{\left((x^2-x+1)\,(x^2+x+1)\right)^{\frac{3}{2}}\,(x^2+1)^2}\,dx$$

```
[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**2), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)^2\,(1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}}(x^2+1)^2}\,dx$$

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(1+x^2\right)^2 \left(1+x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^2}\,dx$$

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+x^2\right)^2 \left(1+x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(x^2+1\right)^2 \left(x^4+x^2+1\right)^{3/2}}\,dx$$

```
[In] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(3/2)),x)
[Out] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(3/2)), x)
```

3.243 $\int \frac{1}{\left(1+x^2\right)^3\left(1+x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 190

$$\begin{aligned}\int \frac{1}{(1+x^2)^3\,(1+x^2+x^4)^{3/2}}\,dx = & -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} \\ & +\frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2}-\frac{x\sqrt{1+x^2+x^4}}{3\,(1+x^2)} \\ & +\frac{3}{4}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{19(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{12\sqrt{1+x^2+x^4}} \\ & -\frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}\end{aligned}$$

```
[Out] 3/4*arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(-x^2+1)/(x^4+x^2+1)^(1/2)+1/4*x*(x^4
+x^2+1)^(1/2)/(x^2+1)^2-1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+19/12*(x^2+1)*(cos(
2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^
4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)-5/4*(x^2+1)*(cos(2*arctan(x))^2
)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+
1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {1242, 1106, 1211, 1117, 1209, 1237, 1710, 1607, 1726, 1714, 1712, 209, 12, 1331}

$$\begin{aligned}\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}}\,dx = {} & \frac{3}{4}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) \\ & - \frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} \\ & + \frac{19(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{12\sqrt{x^4+x^2+1}} \\ & - \frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2} - \frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}}\end{aligned}$$

```
[In] Int[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/3*(x*(1 - x^2))/Sqrt[1 + x^2 + x^4] + (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^
2)^2) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (3*ArcTan[x/Sqrt[1 + x^2 +
x^4]])/4 + (19*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcT
an[x], 1/4])/(12*Sqrt[1 + x^2 + x^4]) - (5*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1714

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{\left(1+x^2+x^4\right)^{3/2}}+\frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}+\frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\right)dx\\
&= -\int \frac{1}{\left(1+x^2+x^4\right)^{3/2}}\,dx+\int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx+\int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx\\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}\\
&\quad+\frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)}-\frac{1}{4}\int \frac{-3+2x^2-x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx\\
&\quad-\frac{1}{3}\int \frac{2+x^2}{\sqrt{1+x^2+x^4}}\,dx-\frac{1}{2}\int \frac{-1+2x^2+x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx\\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}+\frac{5x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)}\\
&\quad+\frac{1}{8}\int \frac{-10x^2-6x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx+\frac{1}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx\\
&\quad+\frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx-\frac{1}{2}\int \frac{2x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx-\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx\\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}+\frac{5x\sqrt{1+x^2+x^4}}{12\left(1+x^2\right)}\\
&\quad+\frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}-\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}\\
&\quad+\frac{1}{8}\int \frac{x^2(-10-6x^2)}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx-\int \frac{x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx\\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}+\frac{5x\sqrt{1+x^2+x^4}}{12\left(1+x^2\right)}\\
&\quad+\frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}-\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}\\
&\quad+\frac{1}{8}\int \frac{-6-10x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx-\frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx\\
&\quad+\frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx+\frac{3}{4}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} \\
&\quad + \frac{19(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{12\sqrt{1+x^2+x^4}} \\
&\quad - \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx \\
&\quad + \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) - \int \frac{1}{\sqrt{1+x^2+x^4}}\,dx \\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} \\
&\quad + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{19(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{12\sqrt{1+x^2+x^4}} \\
&\quad - \frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&\quad + \frac{19(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{12\sqrt{1+x^2+x^4}} - \frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(1+x^2\right)^3\left(1+x^2+x^4\right)^{3/2}}\,dx = \frac{4x(-1+x^2)\left(1+x^2\right)^2 + 3x(1+x^2+x^4) + 15x(1+x^2)\left(1+x^2+x^4\right) -}{}$$

```
[In] Integrate[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (4*x*(-1 + x^2)*(1 + x^2)^2 + 3*x*(1 + x^2 + x^4) + 15*x*(1 + x^2)*(1 + x^2
 + x^4) - (-1)^(1/3)*(1 + x^2)^2*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/
3)*x^2]*(19*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-9 + (10*I)*S
qrt[3])*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 18*(-1)^(1/3)*Elli
pticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/(12*(1 + x^2)^2*S
qrt[1 + x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.82

method	result
risch	$\frac{x\left(19x^{6}+37x^{4}+29x^{2}+14\right)}{12\left(x^{2}+1\right)^{2}\sqrt{x^{4}+x^{2}+1}}-\frac{10\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}+19\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}$
default	$\frac{x\sqrt{x^{4}+x^{2}+1}}{4\left(x^{2}+1\right)^{2}}+\frac{5x\sqrt{x^{4}+x^{2}+1}}{4\left(x^{2}+1\right)}-\frac{2\left(\frac{1}{6}x-\frac{1}{6}x^{3}\right)}{\sqrt{x^{4}+x^{2}+1}}-\frac{10\sqrt{1+\frac{x^{2}}{2}-\frac{ix^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{ix^{2}\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}+19\sqrt{1+}$
elliptic	$\frac{x\sqrt{x^{4}+x^{2}+1}}{4\left(x^{2}+1\right)^{2}}+\frac{5x\sqrt{x^{4}+x^{2}+1}}{4\left(x^{2}+1\right)}-\frac{2\left(\frac{1}{6}x-\frac{1}{6}x^{3}\right)}{\sqrt{x^{4}+x^{2}+1}}-\frac{10\sqrt{1+\frac{x^{2}}{2}-\frac{ix^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{ix^{2}\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}+19\sqrt{1+}$

```
[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x*(19*x^6+37*x^4+29*x^2+14)/(x^2+1)^2/(x^4+x^2+1)^(1/2)-10/3/(-2+2*I*3
^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^
2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2
*I*3^(1/2))^(1/2))+19/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)
^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(
EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-Elliptic
E(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+3/2/(-1/2+1/2*I
*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^
(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(
-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(1+x^{2}\right)^{3}\left(1+x^{2}+x^{4}\right)^{3/2}}\,dx =$$

$$-\frac{19\sqrt{2}\left(x^{8}+3x^{6}+4x^{4}+3x^{2}-\sqrt{-3}\left(x^{8}+3x^{6}+4x^{4}+3x^{2}+1\right)+1\right)\sqrt{\sqrt{-3}-1}\,E\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\right)}{}$$

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/48*(19*sqrt(2)*(x^8 + 3*x^6 + 4*x^4 + 3*x^2 - sqrt(-3)*(x^8 + 3*x^6 + 4*
x^4 + 3*x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sq
```

```
rt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 2*sqrt(2)*(15*x^8 + 45*x^6 + 60*x^
4 + 45*x^2 - 4*sqrt(-3)*(x^8 + 3*x^6 + 4*x^4 + 3*x^2 + 1) + 15)*sqrt(sqrt(-
3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) -
 1/2) - 36*(x^8 + 3*x^6 + 4*x^4 + 3*x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1))
- 4*(19*x^7 + 37*x^5 + 29*x^3 + 14*x)*sqrt(x^4 + x^2 + 1))/(x^8 + 3*x^6 + 4
*x^4 + 3*x^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}} (x^2+1)^3}\,dx$$

```
[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(3/2),x)
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}} (x^2+1)^3}\,dx$$

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

Giac [F]

$$\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}} (x^2+1)^3}\,dx$$

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)^3\,(1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^2+1)^3\,(x^4+x^2+1)^{3/2}}\,dx$$

```
[In] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(3/2)),x)
[Out] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(3/2)), x)
```

3.244 $\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\begin{aligned}\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right) \, dx = {} & ad^4x+\frac{1}{3}d^3(bd+4ae)x^3+\frac{1}{5}d^2\left(cd^2+4bde+6ae^2\right)x^5 \\ & +\frac{2}{7}de\left(2cd^2+e(3bd+2ae)\right)x^7 \\ & +\frac{1}{9}e^2\left(6cd^2+e(4bd+ae)\right)x^9 \\ & +\frac{1}{11}e^3(4cd+be)x^{11}+\frac{1}{13}ce^4x^{13}\end{aligned}$$

```
[Out] a*d^4*x+1/3*d^3*(4*a*e+b*d)*x^3+1/5*d^2*(6*a*e^2+4*b*d*e+c*d^2)*x^5+2/7*d*e
*(2*c*d^2+e*(2*a*e+3*b*d))*x^7+1/9*e^2*(6*c*d^2+e*(a*e+4*b*d))*x^9+1/11*e^3
*(b*e+4*c*d)*x^11+1/13*c*e^4*x^13
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1167}

$$\begin{aligned}\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right) \, dx = {} & \frac{1}{9}e^2x^9\left(e(ae+4bd)+6cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right) \\ & +\frac{2}{7}dex^7\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd) \\ & +ad^4x+\frac{1}{11}e^3x^{11}(be+4cd)+\frac{1}{13}ce^4x^{13}\end{aligned}$$

```
[In] Int[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)
/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2*(6*c*d^2 + e*(4*b*d
 + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad^4+d^3(bd+4ae)x^2+d^2\left(cd^2+4bde+6ae^2\right)x^4+2de\left(2cd^2+e(3bd+2ae)\right)x^6\right.\\&\qquad\left.+e^2\left(6cd^2+e(4bd+ae)\right)x^8+e^3(4cd+be)x^{10}+ce^4x^{12}\right)\,dx\\&=ad^4x+\frac{1}{3}d^3(bd+4ae)x^3+\frac{1}{5}d^2\left(cd^2+4bde+6ae^2\right)x^5+\frac{2}{7}de\left(2cd^2+e(3bd+2ae)\right)x^7\\&\quad+\frac{1}{9}e^2\left(6cd^2+e(4bd+ae)\right)x^9+\frac{1}{11}e^3(4cd+be)x^{11}+\frac{1}{13}ce^4x^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(d+ex^2\right)^4\left(a+bx^2+cx^4\right)\,dx &= ad^4x+\frac{1}{3}d^3(bd+4ae)x^3+\frac{1}{5}d^2\left(cd^2+4bde+6ae^2\right)x^5\\&\quad+\frac{2}{7}de\left(2cd^2+3bde+2ae^2\right)x^7+\frac{1}{9}e^2\left(6cd^2+4bde+ae^2\right)x^9\\&\quad+\frac{1}{11}e^3(4cd+be)x^{11}+\frac{1}{13}ce^4x^{13}\end{aligned}$$

```
[In] Integrate[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)
/5 + (2*d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + 4*b*d*e
+ a*e^2)*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

method	result
norman	$\frac{c\,e^{4} x^{13}}{13}+\left(\frac{1}{11} e^{4} b +\frac{4}{11} d\,e^{3} c \right) x^{11}+\left(\frac{1}{9} e^{4} a +\frac{4}{9} d\,e^{3} b +\frac{2}{3} e^{2} d^{2} c \right) x^{9}+\left(\frac{4}{7} d\,e^{3} a +\frac{6}{7} e^{2} d^{2} b +\frac{4}{7} d^{3} e c \right) x^{7}$
default	$\frac{c\,e^{4} x^{13}}{13}+\frac{\left(e^{4} b +4 d\,e^{3} c \right) x^{11}}{11}+\frac{\left(e^{4} a +4 d\,e^{3} b +6 e^{2} d^{2} c \right) x^{9}}{9}+\frac{\left(4 d\,e^{3} a +6 e^{2} d^{2} b +4 d^{3} e c \right) x^{7}}{7}+\frac{\left(6 e^{2} d^{2} a +4 d^{3} e b +d^{4} c \right) x^{5}}{5}+$
gosper	$\frac{1}{13} c\,e^{4} x^{13}+\frac{1}{11} x^{11} e^{4} b +\frac{4}{11} c d\,e^{3} x^{11}+\frac{1}{9} x^{9} e^{4} a +\frac{4}{9} x^{9} d\,e^{3} b +\frac{2}{3} x^{9} e^{2} d^{2} c +\frac{4}{7} x^{7} d\,e^{3} a +\frac{6}{7} x^{7} e^{2} d^{2} b +$
risch	$\frac{1}{13} c\,e^{4} x^{13}+\frac{1}{11} x^{11} e^{4} b +\frac{4}{11} c d\,e^{3} x^{11}+\frac{1}{9} x^{9} e^{4} a +\frac{4}{9} x^{9} d\,e^{3} b +\frac{2}{3} x^{9} e^{2} d^{2} c +\frac{4}{7} x^{7} d\,e^{3} a +\frac{6}{7} x^{7} e^{2} d^{2} b +$
parallelrisch	$\frac{1}{13} c\,e^{4} x^{13}+\frac{1}{11} x^{11} e^{4} b +\frac{4}{11} c d\,e^{3} x^{11}+\frac{1}{9} x^{9} e^{4} a +\frac{4}{9} x^{9} d\,e^{3} b +\frac{2}{3} x^{9} e^{2} d^{2} c +\frac{4}{7} x^{7} d\,e^{3} a +\frac{6}{7} x^{7} e^{2} d^{2} b +$

```
[In] int((e*x^2+d)^4*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c*e^4*x^13+(1/11*e^4*b+4/11*d*e^3*c)*x^11+(1/9*e^4*a+4/9*d*e^3*b+2/3*e
^2*d^2*c)*x^9+(4/7*d*e^3*a+6/7*e^2*d^2*b+4/7*d^3*e*c)*x^7+(6/5*e^2*d^2*a+4/
5*d^3*e*b+1/5*d^4*c)*x^5+(4/3*d^3*e*a+1/3*d^4*b)*x^3+a*d^4*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right)\,dx &= \frac{1}{13}\,ce^4x^{13}+\frac{1}{11}\,\left(4\,cde^3+be^4\right)x^{11}\\ &\quad+\frac{1}{9}\,\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^9\\ &\quad+\frac{2}{7}\,\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^7+ad^4x\\ &\quad+\frac{1}{5}\,\left(cd^4+4\,bd^3e+6\,ad^2e^2\right)x^5+\frac{1}{3}\,\left(bd^4+4\,ad^3e\right)x^3\end{aligned}$$

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/13*c*e^4*x^13 + 1/11*(4*c*d*e^3 + b*e^4)*x^11 + 1/9*(6*c*d^2*e^2 + 4*b*d*
e^3 + a*e^4)*x^9 + 2/7*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^7 + a*d^4*x
+ 1/5*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^5 + 1/3*(b*d^4 + 4*a*d^3*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right)\,dx = ad^4x+\frac{ce^4x^{13}}{13}+x^{11}\left(\frac{be^4}{11}+\frac{4cde^3}{11}\right)+x^9\left(\frac{ae^4}{9}+\frac{4bde^3}{9}+\frac{2cd^2e^2}{3}\right)+x^7\cdot\left(\frac{4ade^3}{7}+\frac{6bd^2e^2}{7}+\frac{4cd^3e}{7}\right)+x^5\cdot\left(\frac{6ad^2e^2}{5}+\frac{4bd^3e}{5}+\frac{cd^4}{5}\right)+x^3\cdot\left(\frac{4ad^3e}{3}+\frac{bd^4}{3}\right)$$

```
[In] integrate((e*x**2+d)**4*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**13/13 + x**11*(b*e**4/11 + 4*c*d*e**3/11) + x**9*(a*e*
*4/9 + 4*b*d*e**3/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 6*b*d**2*e**2
/7 + 4*c*d**3*e/7) + x**5*(6*a*d**2*e**2/5 + 4*b*d**3*e/5 + c*d**4/5) + x**
3*(4*a*d**3*e/3 + b*d**4/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right)\,dx = \frac{1}{13}\,ce^4x^{13}+\frac{1}{11}\,\left(4\,cde^3+be^4\right)x^{11}+\frac{1}{9}\,\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^9+\frac{2}{7}\,\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^7+ad^4x+\frac{1}{5}\,\left(cd^4+4\,bd^3e+6\,ad^2e^2\right)x^5+\frac{1}{3}\,\left(bd^4+4\,ad^3e\right)x^3$$

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^4*x^13 + 1/11*(4*c*d*e^3 + b*e^4)*x^11 + 1/9*(6*c*d^2*e^2 + 4*b*d*
e^3 + a*e^4)*x^9 + 2/7*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^7 + a*d^4*x
+ 1/5*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^5 + 1/3*(b*d^4 + 4*a*d^3*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10

$$\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right)\,dx = \frac{1}{13}ce^4x^{13}+\frac{4}{11}cde^3x^{11}+\frac{1}{11}be^4x^{11}+\frac{2}{3}cd^2e^2x^9+\frac{4}{9}bde^3x^9+\frac{1}{9}ae^4x^9+\frac{4}{7}cd^3ex^7+\frac{6}{7}bd^2e^2x^7+\frac{4}{7}ade^3x^7+\frac{1}{5}cd^4x^5+\frac{4}{5}bd^3ex^5+\frac{6}{5}ad^2e^2x^5+\frac{1}{3}bd^4x^3+\frac{4}{3}ad^3ex^3+ad^4x$$

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/11*b*e^4*x^11 + 2/3*c*d^2*e^2*x^9 +
 4/9*b*d*e^3*x^9 + 1/9*a*e^4*x^9 + 4/7*c*d^3*e*x^7 + 6/7*b*d^2*e^2*x^7 + 4/
7*a*d*e^3*x^7 + 1/5*c*d^4*x^5 + 4/5*b*d^3*e*x^5 + 6/5*a*d^2*e^2*x^5 + 1/3*b
*d^4*x^3 + 4/3*a*d^3*e*x^3 + a*d^4*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97

$$\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right)\,dx = x^3\left(\frac{b\,d^4}{3}+\frac{4\,a\,e\,d^3}{3}\right)+x^{11}\left(\frac{b\,e^4}{11}+\frac{4\,c\,d\,e^3}{11}\right)+x^5\left(\frac{c\,d^4}{5}+\frac{4\,b\,d^3\,e}{5}+\frac{6\,a\,d^2\,e^2}{5}\right)+x^9\left(\frac{2\,c\,d^2\,e^2}{3}+\frac{4\,b\,d\,e^3}{9}+\frac{a\,e^4}{9}\right)+\frac{c\,e^4\,x^{13}}{13}+a\,d^4\,x+\frac{2\,d\,e\,x^7\,\left(2\,c\,d^2+3\,b\,d\,e+2\,a\,e^2\right)}{7}$$

```
[In] int((d + e*x^2)^4*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((b*d^4)/3 + (4*a*d^3*e)/3) + x^11*((b*e^4)/11 + (4*c*d*e^3)/11) + x^5*
((c*d^4)/5 + (6*a*d^2*e^2)/5 + (4*b*d^3*e)/5) + x^9*((a*e^4)/9 + (2*c*d^2*e
^2)/3 + (4*b*d*e^3)/9) + (c*e^4*x^13)/13 + a*d^4*x + (2*d*e*x^7*(2*a*e^2 +
2*c*d^2 + 3*b*d*e))/7
```

3.245 $\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) dx = ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3e(bd+ae)\right)x^5 +\frac{1}{7}e\left(3cd^2+e(3bd+ae)\right)x^7+\frac{1}{9}e^2(3cd+be)x^9+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x+1/3*d^2*(3*a*e+b*d)*x^3+1/5*d*(c*d^2+3*e*(a*e+b*d))*x^5+1/7*e*(3*c*
d^2+e*(a*e+3*b*d))*x^7+1/9*e^2*(b*e+3*c*d)*x^9+1/11*c*e^3*x^11
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1167}

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) dx = \frac{1}{7}ex^7\left(e(ae+3bd)+3cd^2\right)+\frac{1}{5}dx^5\left(3e(ae+bd)+cd^2\right) +\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

```
[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^5)/5 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e^3
*x^11)/11
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad^3+d^2(bd+3ae)x^2+d\left(cd^2+3e(bd+ae)\right)x^4+e\left(3cd^2+e(3bd+ae)\right)x^6 \right. \\ &\qquad \left. +e^2(3cd+be)x^8+ce^3x^{10}\right)\,dx \\ &= ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3e(bd+ae)\right)x^5 \\ &\quad +\frac{1}{7}e\left(3cd^2+e(3bd+ae)\right)x^7+\frac{1}{9}e^2(3cd+be)x^9+\frac{1}{11}ce^3x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \left(d+ex^2\right)^3\left(a+bx^2+cx^4\right)\,dx = ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3bde+3ae^2\right)x^5 +\frac{1}{7}e\left(3cd^2+3bde+ae^2\right)x^7+\frac{1}{9}e^2(3cd+be)x^9+\frac{1}{11}ce^3x^{11}$$

```
[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^5)/5
 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e
^3*x^11)/11
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

method	result
norman	$\frac{c\,e^3x^{11}}{11}+\left(\frac{1}{9}e^3b+\frac{1}{3}d\,e^2c\right)x^9+\left(\frac{1}{7}a\,e^3+\frac{3}{7}d\,e^2b+\frac{3}{7}c\,d^2e\right)x^7+\left(\frac{3}{5}d\,e^2a+\frac{3}{5}d^2eb+\frac{1}{5}d^3c\right)x^5+(d$
default	$\frac{c\,e^3x^{11}}{11}+\frac{\left(e^3b+3d\,e^2c\right)x^9}{9}+\frac{\left(a\,e^3+3d\,e^2b+3c\,d^2e\right)x^7}{7}+\frac{\left(3d\,e^2a+3d^2eb+d^3c\right)x^5}{5}+\frac{\left(3d^2ea+d^3b\right)x^3}{3}+a\,d^3x$
gosper	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{9}x^9e^3b+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7d\,e^2b+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{3}{5}x^5d^2eb+\frac{1}{5}x^5d^3$
risch	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{9}x^9e^3b+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7d\,e^2b+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{3}{5}x^5d^2eb+\frac{1}{5}x^5d^3$
parallelrisch	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{9}x^9e^3b+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7d\,e^2b+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{3}{5}x^5d^2eb+\frac{1}{5}x^5d^3$

```
[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c*e^3*x^11+(1/9*e^3*b+1/3*d*e^2*c)*x^9+(1/7*a*e^3+3/7*d*e^2*b+3/7*c*d^
2*e)*x^7+(3/5*d*e^2*a+3/5*d^2*e*b+1/5*d^3*c)*x^5+(d^2*e*a+1/3*d^3*b)*x^3+a*
d^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) \, dx = \frac{1}{11} ce^3x^{11} + \frac{1}{9} \left(3\,cde^2 + be^3\right)x^9 + \frac{1}{7} \left(3\,cd^2e + 3\,bde^2 + ae^3\right)x^7 + \frac{1}{5} \left(cd^3 + 3\,bd^2e + 3\,ade^2\right)x^5 + ad^3x + \frac{1}{3} \left(bd^3 + 3\,ad^2e\right)x^3$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/11*c*e^3*x^11 + 1/9*(3*c*d*e^2 + b*e^3)*x^9 + 1/7*(3*c*d^2*e + 3*b*d*e^2
+ a*e^3)*x^7 + 1/5*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^5 + a*d^3*x + 1/3*(b*d
^3 + 3*a*d^2*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) \, dx = ad^3x + \frac{ce^3x^{11}}{11} + x^9 \left(\frac{be^3}{9} + \frac{cde^2}{3}\right) + x^7 \left(\frac{ae^3}{7} + \frac{3bde^2}{7} + \frac{3cd^2e}{7}\right) + x^5 \cdot \left(\frac{3ade^2}{5} + \frac{3bd^2e}{5} + \frac{cd^3}{5}\right) + x^3 \left(ad^2e + \frac{bd^3}{3}\right)$$

```
[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**11/11 + x**9*(b*e**3/9 + c*d*e**2/3) + x**7*(a*e**3/7
+ 3*b*d*e**2/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + 3*b*d**2*e/5 + c*d**3
/5) + x**3*(a*d**2*e + b*d**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)\,dx = \frac{1}{11}\,ce^3x^{11}+\frac{1}{9}\left(3\,cde^2+be^3\right)x^9+\frac{1}{7}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^7 + \frac{1}{5}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^5+ad^3x+\frac{1}{3}\left(bd^3+3\,ad^2e\right)x^3$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*e^3*x^11 + 1/9*(3*c*d*e^2 + b*e^3)*x^9 + 1/7*(3*c*d^2*e + 3*b*d*e^2
+ a*e^3)*x^7 + 1/5*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^5 + a*d^3*x + 1/3*(b*d
^3 + 3*a*d^2*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)\,dx = \frac{1}{11}\,ce^3x^{11}+\frac{1}{3}\,cde^2x^9+\frac{1}{9}\,be^3x^9+\frac{3}{7}\,cd^2ex^7 + \frac{3}{7}\,bde^2x^7+\frac{1}{7}\,ae^3x^7+\frac{1}{5}\,cd^3x^5+\frac{3}{5}\,bd^2ex^5 + \frac{3}{5}\,ade^2x^5+\frac{1}{3}\,bd^3x^3+ad^2ex^3+ad^3x$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/9*b*e^3*x^9 + 3/7*c*d^2*e*x^7 + 3/7*b
*d*e^2*x^7 + 1/7*a*e^3*x^7 + 1/5*c*d^3*x^5 + 3/5*b*d^2*e*x^5 + 3/5*a*d*e^2*
x^5 + 1/3*b*d^3*x^3 + a*d^2*e*x^3 + a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)\,dx = x^3\left(\frac{b\,d^3}{3}+a\,e\,d^2\right)+x^9\left(\frac{b\,e^3}{9}+\frac{c\,d\,e^2}{3}\right) + x^5\left(\frac{c\,d^3}{5}+\frac{3\,b\,d^2\,e}{5}+\frac{3\,a\,d\,e^2}{5}\right) + x^7\left(\frac{3\,c\,d^2\,e}{7}+\frac{3\,b\,d\,e^2}{7}+\frac{a\,e^3}{7}\right)+\frac{c\,e^3\,x^{11}}{11}+a\,d^3\,x$$

```
[In] int((d + e*x^2)^3*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((b*d^3)/3 + a*d^2*e) + x^9*((b*e^3)/9 + (c*d*e^2)/3) + x^5*((c*d^3)/5
+ (3*a*d*e^2)/5 + (3*b*d^2*e)/5) + x^7*((a*e^3)/7 + (3*b*d*e^2)/7 + (3*c*d^
2*e)/7) + (c*e^3*x^11)/11 + a*d^3*x
```

3.246 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx = ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5 +\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x+1/3*d*(2*a*e+b*d)*x^3+1/5*(c*d^2+e*(a*e+2*b*d))*x^5+1/7*e*(b*e+2*c*
d)*x^7+1/9*c*e^2*x^9
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1167}

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx = \frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd) +ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\text{integral} = \int \left(ad^2 + d(bd+2ae)x^2 + \left(cd^2+e(2bd+ae)\right)x^4 + e(2cd+be)x^6 + ce^2x^8\right)\,dx$$

$$= ad^2x + \frac{1}{3}d(bd+2ae)x^3 + \frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5 + \frac{1}{7}e(2cd+be)x^7 + \frac{1}{9}ce^2x^9$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx = ad^2x + \frac{1}{3}d(bd+2ae)x^3 + \frac{1}{5}\left(cd^2+2bde+ae^2\right)x^5 + \frac{1}{7}e(2cd+be)x^7 + \frac{1}{9}ce^2x^9$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	siz
default	$\frac{c\,e^2x^9}{9}+\frac{(b\,e^2+2dce)x^7}{7}+\frac{(a\,e^2+2bde+c\,d^2)x^5}{5}+\frac{(2eda+b\,d^2)x^3}{3}+a\,d^2x$	70
norman	$\frac{c\,e^2x^9}{9}+\left(\frac{1}{7}b\,e^2+\frac{2}{7}dce\right)x^7+\left(\frac{1}{5}a\,e^2+\frac{2}{5}bde+\frac{1}{5}c\,d^2\right)x^5+\left(\frac{2}{3}eda+\frac{1}{3}b\,d^2\right)x^3+a\,d^2x$	71
gosper	$\frac{1}{9}c\,e^2x^9+\frac{1}{7}x^7b\,e^2+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{2}{5}x^5bde+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+\frac{1}{3}x^3b\,d^2+a\,d^2x$	77
risch	$\frac{1}{9}c\,e^2x^9+\frac{1}{7}x^7b\,e^2+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{2}{5}x^5bde+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+\frac{1}{3}x^3b\,d^2+a\,d^2x$	77
parallelrisch	$\frac{1}{9}c\,e^2x^9+\frac{1}{7}x^7b\,e^2+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{2}{5}x^5bde+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+\frac{1}{3}x^3b\,d^2+a\,d^2x$	77

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2*
a*d*e+b*d^2)*x^3+a*d^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\,dx = \frac{1}{9}\,ce^2x^9+\frac{1}{7}\left(2\,cde+be^2\right)x^7+\frac{1}{5}\left(cd^2+2\,bde+ae^2\right)x^5 + ad^2x+\frac{1}{3}\left(bd^2+2\,ade\right)x^3$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\,dx = ad^2x+\frac{ce^2x^9}{9}+x^7\left(\frac{be^2}{7}+\frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5}+\frac{2bde}{5}+\frac{cd^2}{5}\right)+x^3\cdot\left(\frac{2ade}{3}+\frac{bd^2}{3}\right)$$

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2
*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\,dx = \frac{1}{9}\,ce^2x^9+\frac{1}{7}\left(2\,cde+be^2\right)x^7+\frac{1}{5}\left(cd^2+2\,bde+ae^2\right)x^5 + ad^2x+\frac{1}{3}\left(bd^2+2\,ade\right)x^3$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{9}ce^2x^9+\frac{2}{7}cdex^7+\frac{1}{7}be^2x^7+\frac{1}{5}cd^2x^5+\frac{2}{5}bdex^5+\frac{1}{5}ae^2x^5+\frac{1}{3}bd^2x^3+\frac{2}{3}adex^3+ad^2x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 1/7*b*e^2*x^7 + 1/5*c*d^2*x^5 + 2/5*b*d*e*x
^5 + 1/5*a*e^2*x^5 + 1/3*b*d^2*x^3 + 2/3*a*d*e*x^3 + a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 7.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=x^5\left(\frac{c\,d^2}{5}+\frac{2\,b\,d\,e}{5}+\frac{a\,e^2}{5}\right)+x^3\left(\frac{b\,d^2}{3}+\frac{2\,a\,e\,d}{3}\right)+x^7\left(\frac{b\,e^2}{7}+\frac{2\,c\,d\,e}{7}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x$$

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3) +
 x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x
```

3.247 $\int (d+ex^2)(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int (d+ex^2)\left(a+bx^2+cx^4\right)\,dx=adx+\frac{1}{3}(bd+ae)x^3+\frac{1}{5}(cd+be)x^5+\frac{1}{7}cex^7$$

```
[Out] a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1167}

$$\int (d+ex^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

```
[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(ad+(bd+ae)x^2+(cd+be)x^4+cex^6\right)\,dx\\&=adx+\frac{1}{3}(bd+ae)x^3+\frac{1}{5}(cd+be)x^5+\frac{1}{7}cex^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (d+ex^2) (a+bx^2+cx^4) \, dx = adx + \frac{1}{3}(bd+ae)x^3 + \frac{1}{5}(cd+be)x^5 + \frac{1}{7}cex^7$$

```
[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
default	$adx+\frac{(ae+bd)x^3}{3}+\frac{(be+cd)x^5}{5}+\frac{ce\,x^7}{7}$	37
norman	$\frac{ce\,x^7}{7}+\left(\frac{be}{5}+\frac{cd}{5}\right)x^5+\left(\frac{ae}{3}+\frac{bd}{3}\right)x^3+adx$	39
gosper	$\frac{1}{7}ce\,x^7+\frac{1}{5}x^5be+\frac{1}{5}cd\,x^5+\frac{1}{3}ae\,x^3+\frac{1}{3}x^3bd+adx$	41
risch	$\frac{1}{7}ce\,x^7+\frac{1}{5}x^5be+\frac{1}{5}cd\,x^5+\frac{1}{3}ae\,x^3+\frac{1}{3}x^3bd+adx$	41
parallelrisch	$\frac{1}{7}ce\,x^7+\frac{1}{5}x^5be+\frac{1}{5}cd\,x^5+\frac{1}{3}ae\,x^3+\frac{1}{3}x^3bd+adx$	41

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (d+ex^2) (a+bx^2+cx^4) \, dx = \frac{1}{7}cex^7 + \frac{1}{5}(cd+be)x^5 + \frac{1}{3}(bd+ae)x^3 + adx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/7*c*e*x^7 + 1/5*(c*d + b*e)*x^5 + 1/3*(b*d + a*e)*x^3 + a*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)\,dx=adx+\frac{cex^7}{7}+x^5\left(\frac{be}{5}+\frac{cd}{5}\right)+x^3\left(\frac{ae}{3}+\frac{bd}{3}\right)$$

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a),x)
[Out] a*d*x + c*e*x**7/7 + x**5*(b*e/5 + c*d/5) + x**3*(a*e/3 + b*d/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}\,cex^7+\frac{1}{5}\,(cd+be)x^5+\frac{1}{3}\,(bd+ae)x^3+adx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/7*c*e*x^7 + 1/5*(c*d + b*e)*x^5 + 1/3*(b*d + a*e)*x^3 + a*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}\,cex^7+\frac{1}{5}\,cdx^5+\frac{1}{5}\,bex^5+\frac{1}{3}\,bdx^3+\frac{1}{3}\,aex^3+adx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/5*b*e*x^5 + 1/3*b*d*x^3 + 1/3*a*e*x^3 + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)\,dx=\frac{c\,e\,x^7}{7}+\left(\frac{b\,e}{5}+\frac{c\,d}{5}\right)\,x^5+\left(\frac{a\,e}{3}+\frac{b\,d}{3}\right)\,x^3+a\,d\,x$$

```
[In] int((d + e*x^2)*(a + b*x^2 + c*x^4),x)
[Out] x^3*((a*e)/3 + (b*d)/3) + x^5*((b*e)/5 + (c*d)/5) + a*d*x + (c*e*x^7)/7
```

3.248 $\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx=-\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

```
[Out] -(-b*e+c*d)*x/e^2+1/3*c*x^3/e+(a*e^2-b*d*e+c*d^2)*arctan(x*e^(1/2)/d^(1/2))
/e^(5/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1167, 211}

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx=\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(ae^2-bde+cd^2)}{\sqrt{d}e^{5/2}}-\frac{x(cd-be)}{e^2}+\frac{cx^3}{3e}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{cd-be}{e^2}+\frac{cx^2}{e}+\frac{cd^2-bde+ae^2}{e^2(d+ex^2)}\right) dx\\
&= -\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\int \frac{1}{d+ex^2}\,dx}{e^2}\\
&= -\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \frac{(-cd+be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

```
[Out] ((-(c*d) + b*e)*x)/e^2 + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result
default	$\frac{\frac{1}{3}c\,x^3e+bex-cdx}{e^2}+\frac{(a\,e^2-bde+c\,d^2)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{e^2\sqrt{ed}}$
risch	$\frac{c\,x^3}{3e}+\frac{bx}{e}-\frac{cdx}{e^2}-\frac{\ln(ex+\sqrt{-ed})a}{2\sqrt{-ed}}+\frac{\ln(ex+\sqrt{-ed})bd}{2e\sqrt{-ed}}-\frac{\ln(ex+\sqrt{-ed})c\,d^2}{2e^2\sqrt{-ed}}+\frac{\ln(-ex+\sqrt{-ed})a}{2\sqrt{-ed}}-\frac{\ln(-ex+\sqrt{-ed})bd}{2e\sqrt{-ed}}+$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/3*c*x^3*e+b*e*x-c*d*x)+(a*e^2-b*d*e+c*d^2)/e^2/(e*d)^(1/2)*arctan(
e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.41

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \left[\frac{2\,cde^2x^3 - 3\,(cd^2-bde+ae^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right) - 6\,(cd^2e-bde^2)x}{6\,de^3}, \frac{cde^2x^3+3\,(cd^2-bde+ae}{}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*d*e^2*x^3 - 3*(c*d^2 - b*d*e + a*e^2)*sqrt(-d*e)*log((e*x^2 - 2*s
qrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(c*d^2*e - b*d*e^2)*x)/(d*e^3), 1/3*(c*d*
e^2*x^3 + 3*(c*d^2 - b*d*e + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*
d^2*e - b*d*e^2)*x)/(d*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \frac{cx^3}{3e} + x\left(\frac{b}{e} - \frac{cd}{e^2}\right) - \frac{\sqrt{-\frac{1}{de^5}}\,(ae^2-bde+cd^2)\log\left(-de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}\,(ae^2-bde+cd^2)\log\left(de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d),x)
```

```
[Out] c*x**3/(3*e) + x*(b/e - c*d/e**2) - sqrt(-1/(d*e**5))*(a*e**2 - b*d*e + c*d
**2)*log(-d*e**2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 - b*d
*e + c*d**2)*log(d*e**2*sqrt(-1/(d*e**5)) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \frac{(cd^2-bde+ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^2} + \frac{ce^2x^3-3\,cdex+3\,be^2x}{3\,e^3}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) + 1/3*(c*e^2*
x^3 - 3*c*d*e*x + 3*b*e^2*x)/e^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = x\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)+\frac{c\,x^3}{3\,e}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\sqrt{d}\,e^{5/2}}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2),x)
```

```
[Out] x*(b/e - (c*d)/e^2) + (c*x^3)/(3*e) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c
*d^2 - b*d*e))/(d^(1/2)*e^(5/2))
```

3.249 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx=\frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1171, 396, 211}

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx=-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3cd^2-e(ae+bd))}{2d^{3/2}e^{5/2}}+\frac{x(ae^2-bde+cd^2)}{2de^2\,(d+ex^2)}+\frac{cx}{e^2}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)}-\frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\ &= \frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\int\frac{1}{d+ex^2}\,dx}{2de^2}\\ &= \frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)}-\frac{(3cd^2-bde-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
default	$\frac{cx}{e^2}+\frac{\frac{\left(a\,e^2-bde+c\,d^2\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a\,e^2+bde-3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^2}$
risch	$\frac{cx}{e^2}+\frac{(a\,e^2-bde+c\,d^2)x}{2d\,e^2(e\,x^2+d)}-\frac{\ln\left(ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}-\frac{\ln\left(ex+\sqrt{-ed}\right)b}{4e\sqrt{-ed}}+\frac{3d\ln\left(ex+\sqrt{-ed}\right)c}{4e^2\sqrt{-ed}}+\frac{\ln\left(-ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}+\frac{\ln\left(-ex+\sqrt{-ed}\right)}{4e\sqrt{-ed}}$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*d
^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx$$

$$=\left[\frac{4\,cd^2e^2x^3+(3\,cd^3-bd^2e-ade^2+(3\,cd^2e-bde^2-ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)+2\,(3\,cd^3e-bd^2}{4\,(d^2e^4x^2+d^3e^3)}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(75) = 150.

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{cx}{e^2} + \frac{x(ae^2-bde+cd^2)}{2d^2e^2+2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2+bde-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2+bde-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{cx}{e^2} - \frac{\left(3\,cd^2-bde-ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)de^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*
e^2) + 1/2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx=\frac{c\,x}{e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2+b\,d\,e+a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}}+\frac{x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.250 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx=\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d(d+ex^2)^2}-\frac{(5cd^2-e(bd+3ae))\,x}{8d^2e^2(d+ex^2)}+\frac{(3cd^2+e(bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[Out] 1/4*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^2-1/8*(5*c*d^2-e*(3*a*e+b*d))*x/d^2/
e^2/(e*x^2+d)+1/8*(3*c*d^2+e*(3*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)
/e^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1171, 393, 211}

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx=\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(3ae+bd)+3cd^2)}{8d^{5/2}e^{5/2}}-\frac{x(5cd^2-e(3ae+bd))}{8d^2e^2(d+ex^2)}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(4*d*e^2*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d +
3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{4de^2\,(d+ex^2)^2}-\frac{\int \frac{-3a+\frac{d(cd-be)}{e^2}-\frac{4cdx^2}{e}}{(d+ex^2)^2}\,dx}{4d}\\
&= \frac{(cd^2-bde+ae^2)\,x}{4de^2\,(d+ex^2)^2}-\frac{(5cd^2-e(bd+3ae))\,x}{8d^2e^2\,(d+ex^2)}-\frac{\left(-\frac{4cd^2}{e}+e\left(-3a+\frac{d(cd-be)}{e^2}\right)\right)\int\frac{1}{d+ex^2}\,dx}{8d^2e}\\
&= \frac{(cd^2-bde+ae^2)\,x}{4de^2\,(d+ex^2)^2}-\frac{(5cd^2-e(bd+3ae))\,x}{8d^2e^2\,(d+ex^2)}+\frac{(3cd^2+e(bd+3ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\begin{aligned}
\int\frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx &= \frac{x(-cd^2(3d+5ex^2)+e(bd(-d+ex^2)+ae(5d+3ex^2)))}{8d^2e^2\,(d+ex^2)^2}\\
&\quad+\frac{(3cd^2+e(bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}
\end{aligned}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))
/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(3a\,e^2+bde-5c\,d^2\right)x^3}{8d^2e}+\frac{\left(5a\,e^2-bde-3c\,d^2\right)x}{8d\,e^2}}{\left(e\,x^2+d\right)^2}+\frac{\left(3a\,e^2+bde+3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8e^2d^2\sqrt{ed}}$
risch	$\frac{\frac{\left(3a\,e^2+bde-5c\,d^2\right)x^3}{8d^2e}+\frac{\left(5a\,e^2-bde-3c\,d^2\right)x}{8d\,e^2}}{\left(e\,x^2+d\right)^2}-\frac{3\ln\left(ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^2}-\frac{\ln\left(ex+\sqrt{-ed}\right)b}{16\sqrt{-ed}\,ed}-\frac{3\ln\left(ex+\sqrt{-ed}\right)c}{16\sqrt{-ed}\,e^2}+\frac{3\ln\left(-ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^2}+$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/d/e^2*x)
/(e*x^2+d)^2+1/8*(3*a*e^2+b*d*e+3*c*d^2)/e^2/d^2/(e*d)^(1/2)*arctan(e*x/(e*
d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.40

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx$$

$$= \Bigg[-\frac{2\,(5\,cd^3e^2-bd^2e^3-3\,ade^4)x^3+(3\,cd^4+bd^3e+3\,ad^2e^2+(3\,cd^2e^2+bde^3+3\,ae^4)x^4+2\,(3\,cd^3e+bd^2e}{16\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}$$

$$-\frac{(5\,cd^3e^2-bd^2e^3-3\,ade^4)x^3-(3\,cd^4+bd^3e+3\,ad^2e^2+(3\,cd^2e^2+bde^3+3\,ae^4)x^4+2\,(3\,cd^3e+bd^2e^2+}{8\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e +
3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*
e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 +
d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x
^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*d^4 +
 b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3
```

```
*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4
*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.70

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16} + \frac{x^3\cdot(3ae^3+bde^2-5cd^2e)+x(5ade^2-bd^2e-3cd^3)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/
(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*l
og(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*
c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e*
*3*x**2 + 8*d**2*e**4*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^2} - \frac{5\,cd^2ex^3-bde^2x^3-3\,ae^3x^3+3\,cd^3x+bd^2ex-5\,ade^2x}{8\left(ex^2+d\right)^2d^2e^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2) -
 1/8*(5*c*d^2*e*x^3 - b*d*e^2*x^3 - 3*a*e^3*x^3 + 3*c*d^3*x + b*d^2*e*x - 5
*a*d*e^2*x)/((e*x^2 + d)^2*d^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}} - \frac{\frac{x\,\left(3\,c\,d^2+b\,d\,e-5\,a\,e^2\right)}{8\,d\,e^2}-\frac{x^3\,\left(-5\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^2\,e}}{d^2+2\,d\,e\,x^2+e^2\,x^4}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2))
 - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 + b
*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)
```

3.251 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\,(d+ex^2)^3} - \frac{(7cd^2-e(bd+5ae))\,x}{24d^2e^2\,(d+ex^2)^2} + \frac{(cd^2+e(bd+5ae))\,x}{16d^3e^2\,(d+ex^2)} + \frac{(cd^2+e(bd+5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

```
[Out] 1/6*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^3-1/24*(7*c*d^2-e*(5*a*e+b*d))*x/d^2
/e^2/(e*x^2+d)^2+1/16*(c*d^2+e*(5*a*e+b*d))*x/d^3/e^2/(e*x^2+d)+1/16*(c*d^2
+e*(5*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(5/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1171, 393, 205, 211}

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e(5ae+bd)+cd^2)}{16d^{7/2}e^{5/2}} - \frac{x(7cd^2-e(5ae+bd))}{24d^2e^2\,(d+ex^2)^2} + \frac{x(ae^2-bde+cd^2)}{6de^2\,(d+ex^2)^3} + \frac{x(e(5ae+bd)+cd^2)}{16d^3e^2\,(d+ex^2)}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^2)^3) - ((7*c*d^2 - e*(b*d +
5*a*e))*x)/(24*d^2*e^2*(d + e*x^2)^2) + ((c*d^2 + e*(b*d + 5*a*e))*x)/(16*d
```

```
^3*e^2*(d + e*x^2)) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(16*d^(7/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^2)^3}-\frac{\int \frac{-5a+\frac{d(cd-be)}{e^2}-\frac{6cdx^2}{e}}{(d+ex^2)^3}\,dx}{6d}\\
&= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^2)^3}-\frac{(7cd^2-e(bd+5ae))\,x}{24d^2e^2\,(d+ex^2)^2}+\frac{(cd^2+e(bd+5ae))\int \frac{1}{(d+ex^2)^2}\,dx}{8d^2e^2}\\
&= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^2)^3}-\frac{(7cd^2-e(bd+5ae))\,x}{24d^2e^2\,(d+ex^2)^2}\\
&\quad+\frac{(cd^2+e(bd+5ae))\,x}{16d^3e^2\,(d+ex^2)}+\frac{(cd^2+e(bd+5ae))\int \frac{1}{d+ex^2}\,dx}{16d^3e^2}
\end{aligned}
$$

$$= \frac{(cd^2 - bde + ae^2)\,x}{6de^2\,(d+ex^2)^3} - \frac{(7cd^2 - e(bd+5ae))\,x}{24d^2e^2\,(d+ex^2)^2} + \frac{(cd^2 + e(bd+5ae))\,x}{16d^3e^2\,(d+ex^2)} + \frac{(cd^2 + e(bd+5ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \frac{x(cd^2(-3d^2-8dex^2+3e^2x^4)+e(bd(-3d^2+8dex^2+3e^2x^4)+ae(33d^2+40dex^2+15e^2x^4)))}{48d^3e^2\,(d+ex^2)^3} + \frac{(cd^2+e(bd+5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + e*(b*d*(-3*d^2 + 8*d*e*x^2 + 3
*e^2*x^4) + a*e*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4))))/(48*d^3*e^2*(d + e*x^
2)^3) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)
*e^(5/2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{(5a\,e^2+bde+c\,d^2)x^5}{16d^3}+\frac{(5a\,e^2+bde-c\,d^2)x^3}{6d^2e}+\frac{(11a\,e^2-bde-c\,d^2)x}{16d\,e^2}}{(e\,x^2+d)^3}+\frac{(5a\,e^2+bde+c\,d^2)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{16d^3e^2\sqrt{ed}}$
risch	$\frac{\frac{(5a\,e^2+bde+c\,d^2)x^5}{16d^3}+\frac{(5a\,e^2+bde-c\,d^2)x^3}{6d^2e}+\frac{(11a\,e^2-bde-c\,d^2)x}{16d\,e^2}}{(e\,x^2+d)^3}-\frac{5\ln(ex+\sqrt{-ed})a}{32\sqrt{-ed}\,d^3}-\frac{\ln(ex+\sqrt{-ed})b}{32\sqrt{-ed}\,e\,d^2}-\frac{\ln(ex+\sqrt{-ed})c}{32\sqrt{-ed}\,e^2d}+5\ln$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*a*e^2+b*d*e+c*d^2)/d^3*x^5+1/6*(5*a*e^2+b*d*e-c*d^2)/d^2/e*x^3+1/1
6*(11*a*e^2-b*d*e-c*d^2)/d/e^2*x)/(e*x^2+d)^3+1/16*(5*a*e^2+b*d*e+c*d^2)/d^
3/e^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.53

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx$$
$$= \left[\frac{6\,(cd^3e^3+bd^2e^4+5\,ade^5)x^5-16\,(cd^4e^2-bd^3e^3-5\,ad^2e^4)x^3-3\,((cd^2e^3+bde^4+5\,ae^5)x^6+cd^5+bd^4e}{96\,(d^4e^6}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - b*d^3*e^
3 - 5*a*d^2*e^4)*x^3 - 3*((c*d^2*e^3 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d
^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e
 + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d
)/(e*x^2 + d)) - 6*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6 + 3
*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + b*d^2*e^4 + 5
*a*d*e^5)*x^5 - 8*(c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3
 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 +
b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*sqr
t(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x)/(d
^4*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.61

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = -\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+bde+cd^2)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}$$
$$+\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+bde+cd^2)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}$$
$$+\frac{x^5\cdot(15ae^4+3bde^3+3cd^2e^2)+x^3\cdot(40ade^3+8bd^2e^2-8cd^3e)+x(33ad^2e^2-3bd^3e-3cd^4)}{48d^6e^2+144d^5e^3x^2+144d^4e^4x^4+48d^3e^5x^6}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(-d**4*e**2*sqrt(-1/(d
**7*e**5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(d
**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*b*d*e**3 + 3*c
*d**2*e**2) + x**3*(40*a*d*e**3 + 8*b*d**2*e**2 - 8*c*d**3*e) + x*(33*a*d**
2*e**2 - 3*b*d**3*e - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 + 144*d
**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \frac{(cd^2+bde+5\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^2} + \frac{3\,cd^2e^2x^5+3\,bde^3x^5+15\,ae^4x^5-8\,cd^3ex^3+8\,bd^2e^2x^3+40\,ade^3x^3-3\,cd^4x-3\,bd^3ex+33\,ad^2e^2x}{48\,(ex^2+d)^3d^3e^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + b*d*e + 5*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e^2) +
1/48*(3*c*d^2*e^2*x^5 + 3*b*d*e^3*x^5 + 15*a*e^4*x^5 - 8*c*d^3*e*x^3 + 8*b*
d^2*e^2*x^3 + 40*a*d*e^3*x^3 - 3*c*d^4*x - 3*b*d^3*e*x + 33*a*d^2*e^2*x)/((
e*x^2 + d)^3*d^3*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \frac{\frac{x^5\,(c\,d^2+b\,d\,e+5\,a\,e^2)}{16\,d^3}-\frac{x\,(c\,d^2+b\,d\,e-11\,a\,e^2)}{16\,d\,e^2}+\frac{x^3\,(-c\,d^2+b\,d\,e+5\,a\,e^2)}{6\,d^2\,e}}{d^3+3\,d^2\,e\,x^2+3\,d\,e^2\,x^4+e^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2+b\,d\,e+5\,a\,e^2)}{16\,d^{7/2}\,e^{5/2}}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^4,x)
```

```
[Out] ((x^5*(5*a*e^2 + c*d^2 + b*d*e))/(16*d^3) - (x*(c*d^2 - 11*a*e^2 + b*d*e))/
(16*d*e^2) + (x^3*(5*a*e^2 - c*d^2 + b*d*e))/(6*d^2*e))/(d^3 + e^3*x^6 + 3*
d^2*e*x^2 + 3*d*e^2*x^4) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2 + b*
d*e))/(16*d^(7/2)*e^(5/2))
```

3.252 $\int (d+ex^2)^3 (a+bx^2+cx^4)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 223

$$
\begin{aligned}
\int (d+ex^2)^3 (a+bx^2+cx^4)^2 \, dx = {} & a^2d^3x + \frac{1}{3}ad^2(2bd+3ae)x^3 \\
& + \frac{1}{5}d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^5 \\
& + \frac{1}{7}\left(2bcd^3+3b^2d^2e+6acd^2e+6abde^2+a^2e^3\right)x^7 \\
& + \frac{1}{9}\left(c^2d^3+6cde(bd+ae)+be^2(3bd+2ae)\right)x^9 \\
& + \frac{1}{11}e\left(3c^2d^2+b^2e^2+2ce(3bd+ae)\right)x^{11} \\
& + \frac{1}{13}ce^2(3cd+2be)x^{13} + \frac{1}{15}c^2e^3x^{15}
\end{aligned}
$$

```
[Out] a^2*d^3*x+1/3*a*d^2*(3*a*e+2*b*d)*x^3+1/5*d*(b^2*d^2+6*a*b*d*e+a*(3*a*e^2+2
*c*d^2))*x^5+1/7*(a^2*e^3+6*a*b*d*e^2+6*a*c*d^2*e+3*b^2*d^2*e+2*b*c*d^3)*x^
7+1/9*(c^2*d^3+6*c*d*e*(a*e+b*d)+b*e^2*(2*a*e+3*b*d))*x^9+1/11*e*(3*c^2*d^2
+b^2*e^2+2*c*e*(a*e+3*b*d))*x^11+1/13*c*e^2*(2*b*e+3*c*d)*x^13+1/15*c^2*e^3
*x^15
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1167}

$$\begin{aligned}\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx &= \frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)\\&\quad+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)\\&\quad+\frac{1}{5}dx^5\left(6abde+a\left(3ae^2+2cd^2\right)+b^2d^2\right)\\&\quad+\frac{1}{9}x^9\left(6cde(ae+bd)+be^2(2ae+3bd)+c^2d^3\right)\\&\quad+\frac{1}{3}ad^2x^3(3ae+2bd)+\frac{1}{13}ce^2x^{13}(2be+3cd)+\frac{1}{15}c^2e^3x^{15}\end{aligned}$$

```
[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 +
(c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \Big(a^2d^3+ad^2(2bd+3ae)x^2+d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^4\\&\quad+\left(2bcd^3+3b^2d^2e+6acd^2e+6abde^2+a^2e^3\right)x^6\\&\quad+\left(c^2d^3+6cde(bd+ae)+be^2(3bd+2ae)\right)x^8\\&\quad+e\left(3c^2d^2+b^2e^2+2ce(3bd+ae)\right)x^{10}+ce^2(3cd+2be)x^{12}+c^2e^3x^{14}\Big)\,dx\\&=a^2d^3x+\frac{1}{3}ad^2(2bd+3ae)x^3+\frac{1}{5}d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^5\\&\quad+\frac{1}{7}\left(2bcd^3+3b^2d^2e+6acd^2e+6abde^2+a^2e^3\right)x^7\\&\quad+\frac{1}{9}\left(c^2d^3+6cde(bd+ae)+be^2(3bd+2ae)\right)x^9\\&\quad+\frac{1}{11}e\left(3c^2d^2+b^2e^2+2ce(3bd+ae)\right)x^{11}+\frac{1}{13}ce^2(3cd+2be)x^{13}+\frac{1}{15}c^2e^3x^{15}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+bx^2+cx^4\right)^2\,dx = {} & a^2d^3x+\frac{1}{3}ad^2(2bd+3ae)x^3\\ & +\frac{1}{5}d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^5\\ & +\frac{1}{7}\left(2bcd^3+3b^2d^2e+6acd^2e+6abde^2+a^2e^3\right)x^7\\ & +\frac{1}{9}\left(c^2d^3+6cde(bd+ae)+be^2(3bd+2ae)\right)x^9\\ & +\frac{1}{11}e\left(3c^2d^2+b^2e^2+2ce(3bd+ae)\right)x^{11}\\ & +\frac{1}{13}ce^2(3cd+2be)x^{13}+\frac{1}{15}c^2e^3x^{15}\end{aligned}$$

```
[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 +
(c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.98

method	result
default	$\frac{c^2e^3x^{15}}{15}+\frac{\left(2e^3bc+3d\,e^2c^2\right)x^{13}}{13}+\frac{\left(3d^2e\,c^2+6d\,e^2bc+e^3\left(2ac+b^2\right)\right)x^{11}}{11}+\frac{\left(c^2d^3+6d^2ebc+3d\,e^2\left(2ac+b^2\right)+2e^3ab\right)x^9}{9}+$
norman	$a^2d^3x+\left(d^2e\,a^2+\frac{2}{3}a\,d^3b\right)x^3+\left(\frac{3}{5}d\,e^2a^2+\frac{6}{5}d^2eab+\frac{2}{5}d^3ac+\frac{1}{5}b^2d^3\right)x^5+\left(\frac{1}{7}e^3a^2+\frac{6}{7}abd\,e^2+\right.$
gosper	$a^2d^3x+a^2d^2e\,x^3+\frac{2}{3}x^3a\,d^3b+\frac{3}{5}x^5d\,e^2a^2+\frac{6}{5}x^5d^2eab+\frac{2}{5}x^5d^3ac+\frac{1}{5}x^5b^2d^3+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7a$
risch	$a^2d^3x+a^2d^2e\,x^3+\frac{2}{3}x^3a\,d^3b+\frac{3}{5}x^5d\,e^2a^2+\frac{6}{5}x^5d^2eab+\frac{2}{5}x^5d^3ac+\frac{1}{5}x^5b^2d^3+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7a$
parallelrisch	$a^2d^3x+a^2d^2e\,x^3+\frac{2}{3}x^3a\,d^3b+\frac{3}{5}x^5d\,e^2a^2+\frac{6}{5}x^5d^2eab+\frac{2}{5}x^5d^3ac+\frac{1}{5}x^5b^2d^3+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7a$

```
[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*c^2*e^3*x^15+1/13*(2*b*c*e^3+3*c^2*d*e^2)*x^13+1/11*(3*d^2*e*c^2+6*d*e
^2*b*c+e^3*(2*a*c+b^2))*x^11+1/9*(c^2*d^3+6*d^2*e*b*c+3*d*e^2*(2*a*c+b^2)+2
*e^3*a*b)*x^9+1/7*(2*b*c*d^3+3*d^2*e*(2*a*c+b^2)+6*a*b*d*e^2+e^3*a^2)*x^7+1
/5*(d^3*(2*a*c+b^2)+6*d^2*e*a*b+3*d*e^2*a^2)*x^5+1/3*(3*a^2*d^2*e+2*a*b*d^3
)*x^3+a^2*d^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx = {} & \frac{1}{15}\, c^2e^3x^{15} + \frac{1}{13}\left(3\, c^2de^2 + 2\, bce^3\right)x^{13} \\ & + \frac{1}{11}\left(3\, c^2d^2e + 6\, bcde^2 + \left(b^2+2\, ac\right)e^3\right)x^{11} \\ & + \frac{1}{9}\left(c^2d^3 + 6\, bcd^2e + 2\, abe^3 + 3\left(b^2+2\, ac\right)de^2\right)x^9 \\ & + \frac{1}{7}\left(2\, bcd^3 + 6\, abde^2 + a^2e^3 + 3\left(b^2+2\, ac\right)d^2e\right)x^7 \\ & + a^2d^3x + \frac{1}{5}\left(6\, abd^2e + 3\, a^2de^2 + \left(b^2+2\, ac\right)d^3\right)x^5 \\ & + \frac{1}{3}\left(2\, abd^3 + 3\, a^2d^2e\right)x^3\end{aligned}$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*c^2*e^3*x^15 + 1/13*(3*c^2*d*e^2 + 2*b*c*e^3)*x^13 + 1/11*(3*c^2*d^2*e
 + 6*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^11 + 1/9*(c^2*d^3 + 6*b*c*d^2*e + 2*a
*b*e^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^9 + 1/7*(2*b*c*d^3 + 6*a*b*d*e^2 + a^2*e^
3 + 3*(b^2 + 2*a*c)*d^2*e)*x^7 + a^2*d^3*x + 1/5*(6*a*b*d^2*e + 3*a^2*d*e^2
 + (b^2 + 2*a*c)*d^3)*x^5 + 1/3*(2*a*b*d^3 + 3*a^2*d^2*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx = {} & a^2d^3x + \frac{c^2e^3x^{15}}{15} + x^{13} \cdot \left(\frac{2bce^3}{13} + \frac{3c^2de^2}{13}\right) + x^{11} \\ & \cdot \left(\frac{2ace^3}{11} + \frac{b^2e^3}{11} + \frac{6bcde^2}{11} + \frac{3c^2d^2e}{11}\right) + x^9 \\ & \cdot \left(\frac{2abe^3}{9} + \frac{2acde^2}{3} + \frac{b^2de^2}{3} + \frac{2bcd^2e}{3} + \frac{c^2d^3}{9}\right) \\ & + x^7\left(\frac{a^2e^3}{7} + \frac{6abde^2}{7} + \frac{6acd^2e}{7} + \frac{3b^2d^2e}{7} + \frac{2bcd^3}{7}\right) \\ & + x^5 \cdot \left(\frac{3a^2de^2}{5} + \frac{6abd^2e}{5} + \frac{2acd^3}{5} + \frac{b^2d^3}{5}\right) \\ & + x^3\left(a^2d^2e + \frac{2abd^3}{3}\right)\end{aligned}$$

```
[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**3*x + c**2*e**3*x**15/15 + x**13*(2*b*c*e**3/13 + 3*c**2*d*e**2/13)
 + x**11*(2*a*c*e**3/11 + b**2*e**3/11 + 6*b*c*d*e**2/11 + 3*c**2*d**2*e/11
) + x**9*(2*a*b*e**3/9 + 2*a*c*d*e**2/3 + b**2*d*e**2/3 + 2*b*c*d**2*e/3 +
c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*b*d*e**2/7 + 6*a*c*d**2*e/7 + 3*b**2
*d**2*e/7 + 2*b*c*d**3/7) + x**5*(3*a**2*d*e**2/5 + 6*a*b*d**2*e/5 + 2*a*c*
d**3/5 + b**2*d**3/5) + x**3*(a**2*d**2*e + 2*a*b*d**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+bx^2+cx^4\right)^2\,dx &= \frac{1}{15}\,c^2e^3x^{15}+\frac{1}{13}\left(3\,c^2de^2+2\,bce^3\right)x^{13}\\&\quad+\frac{1}{11}\left(3\,c^2d^2e+6\,bcde^2+\left(b^2+2\,ac\right)e^3\right)x^{11}\\&\quad+\frac{1}{9}\left(c^2d^3+6\,bcd^2e+2\,abe^3+3\left(b^2+2\,ac\right)de^2\right)x^9\\&\quad+\frac{1}{7}\left(2\,bcd^3+6\,abde^2+a^2e^3+3\left(b^2+2\,ac\right)d^2e\right)x^7\\&\quad+a^2d^3x+\frac{1}{5}\left(6\,abd^2e+3\,a^2de^2+\left(b^2+2\,ac\right)d^3\right)x^5\\&\quad+\frac{1}{3}\left(2\,abd^3+3\,a^2d^2e\right)x^3\end{aligned}$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*c^2*e^3*x^15 + 1/13*(3*c^2*d*e^2 + 2*b*c*e^3)*x^13 + 1/11*(3*c^2*d^2*e
 + 6*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^11 + 1/9*(c^2*d^3 + 6*b*c*d^2*e + 2*a
*b*e^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^9 + 1/7*(2*b*c*d^3 + 6*a*b*d*e^2 + a^2*e^
3 + 3*(b^2 + 2*a*c)*d^2*e)*x^7 + a^2*d^3*x + 1/5*(6*a*b*d^2*e + 3*a^2*d*e^2
 + (b^2 + 2*a*c)*d^3)*x^5 + 1/3*(2*a*b*d^3 + 3*a^2*d^2*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.17

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{15}c^2e^3x^{15} + \frac{3}{13}c^2de^2x^{13} + \frac{2}{13}bce^3x^{13} + \frac{3}{11}c^2d^2ex^{11} + \frac{6}{11}bcde^2x^{11} + \frac{1}{11}b^2e^3x^{11} + \frac{2}{11}ace^3x^{11} + \frac{1}{9}c^2d^3x^9 + \frac{2}{3}bcd^2ex^9 + \frac{1}{3}b^2de^2x^9 + \frac{2}{3}acde^2x^9 + \frac{2}{9}abe^3x^9 + \frac{2}{7}bcd^3x^7 + \frac{3}{7}b^2d^2ex^7 + \frac{6}{7}acd^2ex^7 + \frac{6}{7}abde^2x^7 + \frac{1}{7}a^2e^3x^7 + \frac{1}{5}b^2d^3x^5 + \frac{2}{5}acd^3x^5 + \frac{6}{5}abd^2ex^5 + \frac{3}{5}a^2de^2x^5 + \frac{2}{3}abd^3x^3 + a^2d^2ex^3 + a^2d^3x$$

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 2/13*b*c*e^3*x^13 + 3/11*c^2*d^2*
e*x^11 + 6/11*b*c*d*e^2*x^11 + 1/11*b^2*e^3*x^11 + 2/11*a*c*e^3*x^11 + 1/9*
c^2*d^3*x^9 + 2/3*b*c*d^2*e*x^9 + 1/3*b^2*d*e^2*x^9 + 2/3*a*c*d*e^2*x^9 + 2
/9*a*b*e^3*x^9 + 2/7*b*c*d^3*x^7 + 3/7*b^2*d^2*e*x^7 + 6/7*a*c*d^2*e*x^7 +
6/7*a*b*d*e^2*x^7 + 1/7*a^2*e^3*x^7 + 1/5*b^2*d^3*x^5 + 2/5*a*c*d^3*x^5 + 6
/5*a*b*d^2*e*x^5 + 3/5*a^2*d*e^2*x^5 + 2/3*a*b*d^3*x^3 + a^2*d^2*e*x^3 + a^
2*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.99

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx = x^7 \left(\frac{a^2\,e^3}{7} + \frac{6\,a\,b\,d\,e^2}{7} + \frac{6\,c\,a\,d^2\,e}{7} + \frac{3\,b^2\,d^2\,e}{7} + \frac{2\,c\,b\,d^3}{7}\right) + x^9 \left(\frac{b^2\,d\,e^2}{3} + \frac{2\,b\,c\,d^2\,e}{3} + \frac{2\,a\,b\,e^3}{9} + \frac{c^2\,d^3}{9} + \frac{2\,a\,c\,d\,e^2}{3}\right) + x^5 \left(\frac{3\,a^2\,d\,e^2}{5} + \frac{6\,a\,b\,d^2\,e}{5} + \frac{2\,c\,a\,d^3}{5} + \frac{b^2\,d^3}{5}\right) + x^{11} \left(\frac{b^2\,e^3}{11} + \frac{6\,b\,c\,d\,e^2}{11} + \frac{3\,c^2\,d^2\,e}{11} + \frac{2\,a\,c\,e^3}{11}\right) + a^2\,d^3\,x + \frac{c^2\,e^3\,x^{15}}{15} + \frac{a\,d^2\,x^3\,\left(3\,a\,e+2\,b\,d\right)}{3} + \frac{c\,e^2\,x^{13}\,\left(2\,b\,e+3\,c\,d\right)}{13}$$

```
[In] int((d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^7*((a^2*e^3)/7 + (3*b^2*d^2*e)/7 + (2*b*c*d^3)/7 + (6*a*b*d*e^2)/7 + (6*a
*c*d^2*e)/7) + x^9*((c^2*d^3)/9 + (b^2*d*e^2)/3 + (2*a*b*e^3)/9 + (2*a*c*d*
```

```
e^2)/3 + (2*b*c*d^2*e)/3) + x^5*((b^2*d^3)/5 + (3*a^2*d*e^2)/5 + (2*a*c*d^3
)/5 + (6*a*b*d^2*e)/5) + x^11*((b^2*e^3)/11 + (3*c^2*d^2*e)/11 + (2*a*c*e^3
)/11 + (6*b*c*d*e^2)/11) + a^2*d^3*x + (c^2*e^3*x^15)/15 + (a*d^2*x^3*(3*a*
e + 2*b*d))/3 + (c*e^2*x^13*(2*b*e + 3*c*d))/13
```

3.253 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx = {} & a^2d^2x+\frac{2}{3}ad(bd+ae)x^3 \\ & +\frac{1}{5}\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^5 \\ & +\frac{2}{7}\left(bcd^2+b^2de+2acde+abe^2\right)x^7 \\ & +\frac{1}{9}\left(c^2d^2+b^2e^2+2ce(2bd+ae)\right)x^9 \\ & +\frac{2}{11}ce(cd+be)x^{11}+\frac{1}{13}c^2e^2x^{13}\end{aligned}$$

```
[Out] a^2*d^2*x+2/3*a*d*(a*e+b*d)*x^3+1/5*(b^2*d^2+4*a*b*d*e+a*(a*e^2+2*c*d^2))*x
^5+2/7*(a*b*e^2+2*a*c*d*e+b^2*d*e+b*c*d^2)*x^7+1/9*(c^2*d^2+b^2*e^2+2*c*e*(
a*e+2*b*d))*x^9+2/11*c*e*(b*e+c*d)*x^11+1/13*c^2*e^2*x^13
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1167}

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx = {} & a^2d^2x+\frac{1}{9}x^9\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right) \\ & +\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right) \\ & +\frac{1}{5}x^5\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right) \\ & +\frac{2}{3}adx^3(ae+bd)+\frac{2}{11}cex^{11}(be+cd)+\frac{1}{13}c^2e^2x^{13}\end{aligned}$$

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2
+ a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((
c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)/9 + (2*c*e*(c*d + b*e)*x^11)/
11 + (c^2*e^2*x^13)/13
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2d^2+2ad(bd+ae)x^2+\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^4\right.\\ &\quad +2\left(bcd^2+b^2de+2acde+abe^2\right)x^6+\left(c^2d^2+b^2e^2+2ce(2bd+ae)\right)x^8\\ &\quad \left.+2ce(cd+be)x^{10}+c^2e^2x^{12}\right)\,dx\\ &= a^2d^2x+\frac{2}{3}ad(bd+ae)x^3+\frac{1}{5}\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^5\\ &\quad +\frac{2}{7}\left(bcd^2+b^2de+2acde+abe^2\right)x^7\\ &\quad +\frac{1}{9}\left(c^2d^2+b^2e^2+2ce(2bd+ae)\right)x^9+\frac{2}{11}ce(cd+be)x^{11}+\frac{1}{13}c^2e^2x^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2\,dx &= a^2d^2x+\frac{2}{3}ad(bd+ae)x^3+\frac{1}{5}\left(b^2d^2+2acd^2+4abde+a^2e^2\right)x^5\\ &\quad +\frac{2}{7}\left(bcd^2+b^2de+2acde+abe^2\right)x^7\\ &\quad +\frac{1}{9}\left(c^2d^2+4bcde+b^2e^2+2ace^2\right)x^9\\ &\quad +\frac{2}{11}ce(cd+be)x^{11}+\frac{1}{13}c^2e^2x^{13}\end{aligned}$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 2*a*c*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((
c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^9)/9 + (2*c*e*(c*d + b*e)*x^11
)/11 + (c^2*e^2*x^13)/13
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00

method	result
default	$\frac{c^2e^2x^{13}}{13}+\frac{(2bc\,e^2+2ed\,c^2)x^{11}}{11}+\frac{(c^2d^2+4bcde+e^2(2ac+b^2))x^9}{9}+\frac{(2bc\,d^2+2ed(2ac+b^2)+2ab\,e^2)x^7}{7}+\frac{(d^2(2ac+b^2)+4}{5}$
norman	$\frac{c^2e^2x^{13}}{13}+\big(\frac{2}{11}bc\,e^2+\frac{2}{11}ed\,c^2\big)\,x^{11}+\big(\frac{2}{9}e^2ac+\frac{1}{9}b^2e^2+\frac{4}{9}bcde+\frac{1}{9}c^2d^2\big)\,x^9+\big(\frac{2}{7}ab\,e^2+\frac{4}{7}acde+$
gosper	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}x^{11}bc\,e^2+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9b^2e^2+\frac{4}{9}x^9bcde+\frac{1}{9}x^9c^2d^2+\frac{2}{7}x^7ab\,e^2+$
risch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}x^{11}bc\,e^2+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9b^2e^2+\frac{4}{9}x^9bcde+\frac{1}{9}x^9c^2d^2+\frac{2}{7}x^7ab\,e^2+$
parallelrisch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}x^{11}bc\,e^2+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9b^2e^2+\frac{4}{9}x^9bcde+\frac{1}{9}x^9c^2d^2+\frac{2}{7}x^7ab\,e^2+$

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c^2*e^2*x^13+1/11*(2*b*c*e^2+2*c^2*d*e)*x^11+1/9*(c^2*d^2+4*b*c*d*e+e^
2*(2*a*c+b^2))*x^9+1/7*(2*b*c*d^2+2*e*d*(2*a*c+b^2)+2*a*b*e^2)*x^7+1/5*(d^2
*(2*a*c+b^2)+4*a*b*d*e+e^2*a^2)*x^5+1/3*(2*a^2*d*e+2*a*b*d^2)*x^3+a^2*d^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2\,dx &= \frac{1}{13}\,c^2e^2x^{13}+\frac{2}{11}\,\left(c^2de+bce^2\right)x^{11}\\ &\quad+\frac{1}{9}\,\left(c^2d^2+4\,bcde+\left(b^2+2\,ac\right)e^2\right)x^9\\ &\quad+\frac{2}{7}\,\left(bcd^2+abe^2+\left(b^2+2\,ac\right)de\right)x^7\\ &\quad+\frac{1}{5}\,\left(4\,abde+a^2e^2+\left(b^2+2\,ac\right)d^2\right)x^5\\ &\quad+a^2d^2x+\frac{2}{3}\,\left(abd^2+a^2de\right)x^3\end{aligned}$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*(c^2*d*e + b*c*e^2)*x^11 + 1/9*(c^2*d^2 + 4*b*c*d*
e + (b^2 + 2*a*c)*e^2)*x^9 + 2/7*(b*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)*d*e)*x^
7 + 1/5*(4*a*b*d*e + a^2*e^2 + (b^2 + 2*a*c)*d^2)*x^5 + a^2*d^2*x + 2/3*(a*
b*d^2 + a^2*d*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx = {} & a^2d^2x+\frac{c^2e^2x^{13}}{13}+x^{11}\cdot\left(\frac{2bce^2}{11}+\frac{2c^2de}{11}\right)\\ & +x^9\cdot\left(\frac{2ace^2}{9}+\frac{b^2e^2}{9}+\frac{4bcde}{9}+\frac{c^2d^2}{9}\right)\\ & +x^7\cdot\left(\frac{2abe^2}{7}+\frac{4acde}{7}+\frac{2b^2de}{7}+\frac{2bcd^2}{7}\right)\\ & +x^5\left(\frac{a^2e^2}{5}+\frac{4abde}{5}+\frac{2acd^2}{5}+\frac{b^2d^2}{5}\right)\\ & +x^3\cdot\left(\frac{2a^2de}{3}+\frac{2abd^2}{3}\right)\end{aligned}$$

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**2*x + c**2*e**2*x**13/13 + x**11*(2*b*c*e**2/11 + 2*c**2*d*e/11) +
x**9*(2*a*c*e**2/9 + b**2*e**2/9 + 4*b*c*d*e/9 + c**2*d**2/9) + x**7*(2*a*b
*e**2/7 + 4*a*c*d*e/7 + 2*b**2*d*e/7 + 2*b*c*d**2/7) + x**5*(a**2*e**2/5 +
4*a*b*d*e/5 + 2*a*c*d**2/5 + b**2*d**2/5) + x**3*(2*a**2*d*e/3 + 2*a*b*d**2
/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx = {} & \frac{1}{13}c^2e^2x^{13}+\frac{2}{11}\left(c^2de+bce^2\right)x^{11}\\ & +\frac{1}{9}\left(c^2d^2+4\,bcde+\left(b^2+2\,ac\right)e^2\right)x^9\\ & +\frac{2}{7}\left(bcd^2+abe^2+\left(b^2+2\,ac\right)de\right)x^7\\ & +\frac{1}{5}\left(4\,abde+a^2e^2+\left(b^2+2\,ac\right)d^2\right)x^5\\ & +a^2d^2x+\frac{2}{3}\left(abd^2+a^2de\right)x^3\end{aligned}$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*(c^2*d*e + b*c*e^2)*x^11 + 1/9*(c^2*d^2 + 4*b*c*d*
e + (b^2 + 2*a*c)*e^2)*x^9 + 2/7*(b*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)*d*e)*x^
7 + 1/5*(4*a*b*d*e + a^2*e^2 + (b^2 + 2*a*c)*d^2)*x^5 + a^2*d^2*x + 2/3*(a*
b*d^2 + a^2*d*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.17

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{13}\,c^2e^2x^{13} + \frac{2}{11}\,c^2dex^{11} + \frac{2}{11}\,bce^2x^{11} + \frac{1}{9}\,c^2d^2x^9 + \frac{4}{9}\,bcdex^9 + \frac{1}{9}\,b^2e^2x^9 + \frac{2}{9}\,ace^2x^9 + \frac{2}{7}\,bcd^2x^7 + \frac{2}{7}\,b^2dex^7 + \frac{4}{7}\,acdex^7 + \frac{2}{7}\,abe^2x^7 + \frac{1}{5}\,b^2d^2x^5 + \frac{2}{5}\,acd^2x^5 + \frac{4}{5}\,abdex^5 + \frac{1}{5}\,a^2e^2x^5 + \frac{2}{3}\,abd^2x^3 + \frac{2}{3}\,a^2dex^3 + a^2d^2x$$

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 2/11*b*c*e^2*x^11 + 1/9*c^2*d^2*x^9
 + 4/9*b*c*d*e*x^9 + 1/9*b^2*e^2*x^9 + 2/9*a*c*e^2*x^9 + 2/7*b*c*d^2*x^7 +
2/7*b^2*d*e*x^7 + 4/7*a*c*d*e*x^7 + 2/7*a*b*e^2*x^7 + 1/5*b^2*d^2*x^5 + 2/5
*a*c*d^2*x^5 + 4/5*a*b*d*e*x^5 + 1/5*a^2*e^2*x^5 + 2/3*a*b*d^2*x^3 + 2/3*a^
2*d*e*x^3 + a^2*d^2*x
```

Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx = x^5\left(\frac{a^2\,e^2}{5}+\frac{4\,a\,b\,d\,e}{5}+\frac{2\,c\,a\,d^2}{5}+\frac{b^2\,d^2}{5}\right) + x^9\left(\frac{b^2\,e^2}{9}+\frac{4\,b\,c\,d\,e}{9}+\frac{c^2\,d^2}{9}+\frac{2\,a\,c\,e^2}{9}\right) + x^7\left(\frac{2\,b^2\,d\,e}{7}+\frac{2\,c\,b\,d^2}{7}+\frac{2\,a\,b\,e^2}{7}+\frac{4\,a\,c\,d\,e}{7}\right) + a^2\,d^2\,x + \frac{c^2\,e^2\,x^{13}}{13} + \frac{2\,a\,d\,x^3\,(a\,e+b\,d)}{3} + \frac{2\,c\,e\,x^{11}\,(b\,e+c\,d)}{11}$$

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((a^2*e^2)/5 + (b^2*d^2)/5 + (2*a*c*d^2)/5 + (4*a*b*d*e)/5) + x^9*((b^2
*e^2)/9 + (c^2*d^2)/9 + (2*a*c*e^2)/9 + (4*b*c*d*e)/9) + x^7*((2*a*b*e^2)/7
 + (2*b*c*d^2)/7 + (2*b^2*d*e)/7 + (4*a*c*d*e)/7) + a^2*d^2*x + (c^2*e^2*x^
13)/13 + (2*a*d*x^3*(a*e + b*d))/3 + (2*c*e*x^11*(b*e + c*d))/11
```

3.254 $\int (d+ex^2)(a+bx^2+cx^4)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 96

$$\int (d+ex^2)(a+bx^2+cx^4)^2 \, dx = a^2dx+\frac{1}{3}a(2bd+ae)x^3+\frac{1}{5}(b^2d+2acd+2abe)x^5 +\frac{1}{7}(2bcd+b^2e+2ace)x^7+\frac{1}{9}c(cd+2be)x^9+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x+1/3*a*(a*e+2*b*d)*x^3+1/5*(2*a*b*e+2*a*c*d+b^2*d)*x^5+1/7*(2*a*c*e+
b^2*e+2*b*c*d)*x^7+1/9*c*(2*b*e+c*d)*x^9+1/11*c^2*e*x^11
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1167}

$$\int (d+ex^2)(a+bx^2+cx^4)^2 \, dx = a^2dx+\frac{1}{7}x^7(2ace+b^2e+2bcd)+\frac{1}{5}x^5(2abe+2acd+b^2d) +\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

```
[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^11)
/11
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2d+a(2bd+ae)x^2+\left(b^2d+2acd+2abe\right)x^4+\left(2bcd+b^2e+2ace\right)x^6 \right. \\
&\qquad \left. +c(cd+2be)x^8+c^2ex^{10}\right)\,dx \\
&= a^2dx+\frac{1}{3}a(2bd+ae)x^3+\frac{1}{5}\left(b^2d+2acd+2abe\right)x^5 \\
&\quad +\frac{1}{7}\left(2bcd+b^2e+2ace\right)x^7+\frac{1}{9}c(cd+2be)x^9+\frac{1}{11}c^2ex^{11}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= a^2dx+\frac{1}{3}a(2bd+ae)x^3+\frac{1}{5}\left(b^2d+2acd+2abe\right)x^5 \\
&\quad +\frac{1}{7}\left(2bcd+b^2e+2ace\right)x^7+\frac{1}{9}c(cd+2be)x^9+\frac{1}{11}c^2ex^{11}
\end{aligned}
$$

```
[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^11)
/11
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

method	result
default	$\frac{c^{2} e\,x^{11}}{11}+\frac{\left(2ebc+c^{2} d\right) x^{9}}{9}+\frac{\left(2bcd+e\left(2ac+b^{2}\right)\right) x^{7}}{7}+\frac{\left(d\left(2ac+b^{2}\right)+2abe\right) x^{5}}{5}+\frac{\left(e\,a^{2}+2dab\right) x^{3}}{3}+a^{2} d x$
norman	$\frac{c^{2} e\,x^{11}}{11}+\left(\frac{2}{9} ebc+\frac{1}{9} c^{2} d\right) x^{9}+\left(\frac{2}{7} ace+\frac{1}{7} b^{2} e+\frac{2}{7} bcd\right) x^{7}+\left(\frac{2}{5} abe+\frac{2}{5} acd+\frac{1}{5} b^{2} d\right) x^{5}+(\frac{1}{3} e\,a^{2}+$
gosper	$\frac{1}{11} c^{2} e\,x^{11}+\frac{2}{9} x^{9} ebc+\frac{1}{9} c^{2} d\,x^{9}+\frac{2}{7} ace\,x^{7}+\frac{1}{7} x^{7} b^{2} e+\frac{2}{7} x^{7} bcd+\frac{2}{5} x^{5} abe+\frac{2}{5} acd\,x^{5}+\frac{1}{5} x^{5} b^{2} d+$
risch	$\frac{1}{11} c^{2} e\,x^{11}+\frac{2}{9} x^{9} ebc+\frac{1}{9} c^{2} d\,x^{9}+\frac{2}{7} ace\,x^{7}+\frac{1}{7} x^{7} b^{2} e+\frac{2}{7} x^{7} bcd+\frac{2}{5} x^{5} abe+\frac{2}{5} acd\,x^{5}+\frac{1}{5} x^{5} b^{2} d+$
parallelrisch	$\frac{1}{11} c^{2} e\,x^{11}+\frac{2}{9} x^{9} ebc+\frac{1}{9} c^{2} d\,x^{9}+\frac{2}{7} ace\,x^{7}+\frac{1}{7} x^{7} b^{2} e+\frac{2}{7} x^{7} bcd+\frac{2}{5} x^{5} abe+\frac{2}{5} acd\,x^{5}+\frac{1}{5} x^{5} b^{2} d+$

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^2*e*x^11+1/9*(2*b*c*e+c^2*d)*x^9+1/7*(2*b*c*d+e*(2*a*c+b^2))*x^7+1/5
*(d*(2*a*c+b^2)+2*a*b*e)*x^5+1/3*(a^2*e+2*a*b*d)*x^3+a^2*d*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (d+ex^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\,\left(c^2d+2\,bce\right)x^9+\frac{1}{7}\,\left(2\,bcd+\left(b^2+2\,ac\right)e\right)x^7 +\frac{1}{5}\,\left(2\,abe+\left(b^2+2\,ac\right)d\right)x^5+a^2dx+\frac{1}{3}\,\left(2\,abd+a^2e\right)x^3$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*(c^2*d + 2*b*c*e)*x^9 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*
e)*x^7 + 1/5*(2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + a^2*d*x + 1/3*(2*a*b*d + a^2
*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int (d+ex^2)\left(a+bx^2+cx^4\right)^2\,dx = a^2dx+\frac{c^2ex^{11}}{11}+x^9\cdot\left(\frac{2bce}{9}+\frac{c^2d}{9}\right)+x^7\cdot\left(\frac{2ace}{7}+\frac{b^2e}{7} +\frac{2bcd}{7}\right)+x^5\cdot\left(\frac{2abe}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)+x^3\left(\frac{a^2e}{3}+\frac{2abd}{3}\right)$$

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + c**2*e*x**11/11 + x**9*(2*b*c*e/9 + c**2*d/9) + x**7*(2*a*c*e/7
+ b**2*e/7 + 2*b*c*d/7) + x**5*(2*a*b*e/5 + 2*a*c*d/5 + b**2*d/5) + x**3*(a
**2*e/3 + 2*a*b*d/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (d+ex^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\,\left(c^2d+2\,bce\right)x^9+\frac{1}{7}\,\left(2\,bcd+\left(b^2+2\,ac\right)e\right)x^7 +\frac{1}{5}\,\left(2\,abe+\left(b^2+2\,ac\right)d\right)x^5+a^2dx+\frac{1}{3}\,\left(2\,abd+a^2e\right)x^3$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*(c^2*d + 2*b*c*e)*x^9 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*
e)*x^7 + 1/5*(2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + a^2*d*x + 1/3*(2*a*b*d + a^2
*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int (d+ex^2)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\,c^2dx^9+\frac{2}{9}\,bcex^9+\frac{2}{7}\,bcdx^7+\frac{1}{7}\,b^2ex^7+\frac{2}{7}\,acex^7+\frac{1}{5}\,b^2dx^5+\frac{2}{5}\,acdx^5+\frac{2}{5}\,abex^5+\frac{2}{3}\,abdx^3+\frac{1}{3}\,a^2ex^3+a^2dx$$

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/9*b*c*e*x^9 + 2/7*b*c*d*x^7 + 1/7*b^2*e
*x^7 + 2/7*a*c*e*x^7 + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*e*x^5 + 2/3*
a*b*d*x^3 + 1/3*a^2*e*x^3 + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (d+ex^2)\left(a+bx^2+cx^4\right)^2\,dx=x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,e\,b}{5}+\frac{2\,a\,c\,d}{5}\right)+x^7\left(\frac{e\,b^2}{7}+\frac{2\,c\,d\,b}{7}+\frac{2\,a\,c\,e}{7}\right)+x^3\left(\frac{e\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^9\left(\frac{d\,c^2}{9}+\frac{2\,b\,e\,c}{9}\right)+\frac{c^2\,e\,x^{11}}{11}+a^2\,d\,x$$

```
[In] int((d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((b^2*d)/5 + (2*a*b*e)/5 + (2*a*c*d)/5) + x^7*((b^2*e)/7 + (2*a*c*e)/7
+ (2*b*c*d)/7) + x^3*((a^2*e)/3 + (2*a*b*d)/3) + x^9*((c^2*d)/9 + (2*b*c*e)
/9) + (c^2*e*x^11)/11 + a^2*d*x
```

3.255 $\int \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \left(a+bx^2+cx^4\right)^2 \, dx=a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1104}

$$\int \left(a+bx^2+cx^4\right)^2 \, dx=a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[In] Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right) dx\\&=a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2+cx^4\right)^2\,dx=a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$a^{2}x+\frac{2ab\,x^{3}}{3}+\frac{\left(2ac+b^{2}\right)x^{5}}{5}+\frac{2bc\,x^{7}}{7}+\frac{c^{2}x^{9}}{9}$	42
norman	$\frac{c^{2}x^{9}}{9}+\frac{2bc\,x^{7}}{7}+\left(\frac{2ac}{5}+\frac{b^{2}}{5}\right)x^{5}+\frac{2ab\,x^{3}}{3}+a^{2}x$	43
gosper	$\frac{1}{9}c^{2}x^{9}+\frac{2}{7}bc\,x^{7}+\frac{2}{5}x^{5}ac+\frac{1}{5}b^{2}x^{5}+\frac{2}{3}ab\,x^{3}+a^{2}x$	44
risch	$\frac{1}{9}c^{2}x^{9}+\frac{2}{7}bc\,x^{7}+\frac{2}{5}x^{5}ac+\frac{1}{5}b^{2}x^{5}+\frac{2}{3}ab\,x^{3}+a^{2}x$	44
parallelrisch	$\frac{1}{9}c^{2}x^{9}+\frac{2}{7}bc\,x^{7}+\frac{2}{5}x^{5}ac+\frac{1}{5}b^{2}x^{5}+\frac{2}{3}ab\,x^{3}+a^{2}x$	44

```
[In] int((c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2+cx^4\right)^2\,dx=a^2x+\frac{2abx^3}{3}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}+x^5\cdot\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

```
[In] integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5+a^2x+\frac{2}{15}\,\left(3\,cx^5+5\,bx^3\right)a$$

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)*
a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5+\frac{2}{5}\,acx^5+\frac{2}{3}\,abx^3+a^2x$$

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2+cx^4\right)^2 \, dx = a^2 \, x + x^5 \left(\frac{b^2}{5} + \frac{2 \, a \, c}{5}\right) + \frac{c^2 \, x^9}{9} + \frac{2 \, a \, b \, x^3}{3} + \frac{2 \, b \, c \, x^7}{7}$$

```
[In] int((a + b*x^2 + c*x^4)^2,x)
```

```
[Out] a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7)
/7
```

3.256 $\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 143

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx = -\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^3}{3e^3} - \frac{c(cd-2be)x^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{\left(cd^2-bde+ae^2\right)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

```
[Out] -(-b*e+c*d)*(c*d^2-e*(-2*a*e+b*d))*x/e^4+1/3*(c^2*d^2+b^2*e^2-2*c*e*(-a*e+b
*d))*x^3/e^3-1/5*c*(-2*b*e+c*d)*x^5/e^2+1/7*c^2*x^7/e+(a*e^2-b*d*e+c*d^2)^2
*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1167, 211}

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx = \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}e^{9/2}} + \frac{x^3\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{3e^3} - \frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}-\frac{cx^5(cd-2be)}{5e^2}+\frac{c^2x^7}{7e}$$

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(5*e^2) + (c^2*x^7)/(
```

```
7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(
9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}\right. \\
&\quad \left. -\frac{c(cd-2be)x^4}{e^2}+\frac{c^2x^6}{e}+\frac{c^2d^4-2bcd^3e+b^2d^2e^2+2acd^2e^2-2abde^3+a^2e^4}{e^4\left(d+ex^2\right)}\right)dx \\
&= -\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^3}{3e^3} \\
&\quad -\frac{c(cd-2be)x^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{\left(cd^2-bde+ae^2\right)^2\int\frac{1}{d+ex^2}\,dx}{e^4} \\
&= -\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^3}{3e^3} \\
&\quad -\frac{c(cd-2be)x^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{\left(cd^2-bde+ae^2\right)^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{d+ex^2}\,dx=\frac{(-cd+be)\left(cd^2-bde+2ae^2\right)x}{e^4}+\frac{\left(c^2d^2-2bcde+b^2e^2+2ace^2\right)x^3}{3e^3}+\frac{c(-cd+2be)x^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{\left(cd^2-bde+ae^2\right)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] ((-(c*d) + b*e)*(c*d^2 - b*d*e + 2*a*e^2)*x)/e^4 + ((c^2*d^2 - 2*b*c*d*e +
b^2*e^2 + 2*a*c*e^2)*x^3)/(3*e^3) + (c*(-(c*d) + 2*b*e)*x^5)/(5*e^2) + (c^2
*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[d]*e^(9/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

method	result
default	$\frac{\frac{c^2x^7e^3}{7}+\frac{\left((be-cd)ce^2+e^3bc\right)x^5}{5}+\frac{\left((be-cd)be^2+ec\left(2ae^2-bde+cd^2\right)\right)x^3}{3}+(be-cd)\left(2ae^2-bde+cd^2\right)x}{e^4}+\frac{(a^2e^4-2abde^3+2acd^2e^2+b^2d^2e}{e^4\surd}$
risch	$\frac{c^2x^7}{7e}+\frac{2x^5bc}{5e}-\frac{c^2dx^5}{5e^2}+\frac{x^3b^2}{3e}-\frac{2x^3dbc}{3e^2}+\frac{2cax^3}{3e}+\frac{c^2d^2x^3}{3e^3}+\frac{2abx}{e}-\frac{2cadx}{e^2}-\frac{b^2dx}{e^2}+\frac{2bcd^2x}{e^3}-\frac{c^2d^3x}{e^4}-\frac{\ln(ex+}{2\surd}$

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/7*c^2*x^7*e^3+1/5*((b*e-c*d)*c*e^2+e^3*b*c)*x^5+1/3*((b*e-c*d)*b*e
^2+e*c*(2*a*e^2-b*d*e+c*d^2))*x^3+(b*e-c*d)*(2*a*e^2-b*d*e+c*d^2)*x)+(a^2*e
^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/e^4/(e*d)^(1/
2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.84

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{d+ex^2}\,dx$$
$$=\left[\frac{30\,c^2de^4x^7-42\,(c^2d^2e^3-2\,bcde^4)x^5+70\,(c^2d^3e^2-2\,bcd^2e^3+(b^2+2\,ac)de^4)x^3-105\,(c^2d^4-2\,bcd^3e-}{}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^2*d*e^4*x^7 - 42*(c^2*d^2*e^3 - 2*b*c*d*e^4)*x^5 + 70*(c^2*d^3
*e^2 - 2*b*c*d^2*e^3 + (b^2 + 2*a*c)*d*e^4)*x^3 - 105*(c^2*d^4 - 2*b*c*d^3*
e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*sqrt(-d*e)*log((e*x^2 -
2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e - 2*b*c*d^3*e^2 - 2*a*b*d
*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)/(d*e^5), 1/105*(15*c^2*d*e^4*x^7 - 21*(c^2
*d^2*e^3 - 2*b*c*d*e^4)*x^5 + 35*(c^2*d^3*e^2 - 2*b*c*d^2*e^3 + (b^2 + 2*a*
c)*d*e^4)*x^3 + 105*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 +
 2*a*c)*d^2*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 105*(c^2*d^4*e - 2*b*c*d
^3*e^2 - 2*a*b*d*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)/(d*e^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(133) = 266$.

Time = 0.49 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.59

$$\begin{aligned}
&\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx \\
&= \frac{c^2x^7}{7e} + x^5\cdot\left(\frac{2bc}{5e} - \frac{c^2d}{5e^2}\right) + x^3\cdot\left(\frac{2ac}{3e} + \frac{b^2}{3e} - \frac{2bcd}{3e^2} + \frac{c^2d^2}{3e^3}\right) \\
&\quad + x\left(\frac{2ab}{e} - \frac{2acd}{e^2} - \frac{b^2d}{e^2} + \frac{2bcd^2}{e^3} - \frac{c^3d^3}{e^4}\right) \\
&\quad - \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2-bde+cd^2)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}\,(ae^2-bde+cd^2)^2}{a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4}+x\right)}{2} \\
&\quad + \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2-bde+cd^2)^2\log\left(\frac{de^4\sqrt{-\frac{1}{de^9}}\,(ae^2-bde+cd^2)^2}{a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4}+x\right)}{2}
\end{aligned}$$

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d),x)
```

```
[Out] c**2*x**7/(7*e) + x**5*(2*b*c/(5*e) - c**2*d/(5*e**2)) + x**3*(2*a*c/(3*e)
+ b**2/(3*e) - 2*b*c*d/(3*e**2) + c**2*d**2/(3*e**3)) + x*(2*a*b/e - 2*a*c*
d/e**2 - b**2*d/e**2 + 2*b*c*d**2/e**3 - c**2*d**3/e**4) - sqrt(-1/(d*e**9)
)*(a*e**2 - b*d*e + c*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*
e + c*d**2)**2/(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2
 - 2*b*c*d**3*e + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c
*d**2)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**2/(a**2*e
**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2
*d**4) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx = \frac{(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^4} + \frac{15\,c^2e^6x^7-21\,c^2de^5x^5+42\,bce^6x^5+35\,c^2d^2e^4x^3-70\,bcde^5x^3+35\,b^2e^6x^3+70\,ace^6x^3-105\,c^2d^3e^3x+2}{105\,e^7}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^
4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) + 1/105*(15*c^2*e^6*x^7 - 21*c^2*d
*e^5*x^5 + 42*b*c*e^6*x^5 + 35*c^2*d^2*e^4*x^3 - 70*b*c*d*e^5*x^3 + 35*b^2*
e^6*x^3 + 70*a*c*e^6*x^3 - 105*c^2*d^3*e^3*x + 210*b*c*d^2*e^4*x - 105*b^2*
d*e^5*x - 210*a*c*d*e^5*x + 210*a*b*e^6*x)/e^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx$$

$$= x^3\left(\frac{b^2+2\,a\,c}{3\,e}+\frac{d\left(\frac{c^2\,d}{e^2}-\frac{2\,b\,c}{e}\right)}{3\,e}\right) - x\left(\frac{d\left(\frac{b^2+2\,a\,c}{e}+\frac{d\left(\frac{c^2\,d}{e^2}-\frac{2\,b\,c}{e}\right)}{e}\right)}{e}-\frac{2\,a\,b}{e}\right) - x^5\left(\frac{c^2\,d}{5\,e^2}-\frac{2\,b\,c}{5\,e}\right)+\frac{c^2\,x^7}{7\,e} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2}{\sqrt{d}\,\left(a^2\,e^4-2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2-2\,b\,c\,d^3\,e+c^2\,d^4\right)}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)^2}{\sqrt{d}\,e^{9/2}}$$

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2),x)
```

```
[Out] x^3*((2*a*c + b^2)/(3*e) + (d*((c^2*d)/e^2 - (2*b*c)/e))/(3*e)) - x*((d*((2
*a*c + b^2)/e + (d*((c^2*d)/e^2 - (2*b*c)/e))/e))/e - (2*a*b)/e) - x^5*((c^
2*d)/(5*e^2) - (2*b*c)/(5*e)) + (c^2*x^7)/(7*e) + (atan((e^(1/2)*x*(a*e^2 +
 c*d^2 - b*d*e)^2)/(d^(1/2)*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3
- 2*b*c*d^3*e + 2*a*c*d^2*e^2)))*(a*e^2 + c*d^2 - b*d*e)^2)/(d^(1/2)*e^(9/2
))
```

3.257 $\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx = \frac{\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{\left(cd^2-bde+ae^2\right)^2x}{2de^4\left(d+ex^2\right)} - \frac{\left(cd^2-bde+ae^2\right)\left(7cd^2-e(3bd+ae)\right)\arctan\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

```
[Out] (3*c^2*d^2+b^2*e^2-2*c*e*(-a*e+2*b*d))*x/e^4-2/3*c*(-b*e+c*d)*x^3/e^3+1/5*c
^2*x^5/e^2+1/2*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)-1/2*(a*e^2-b*d*e+c*d
^2)*(7*c*d^2-e*(a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(9/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1171, 1824, 211}

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)\left(7cd^2-e(ae+3bd)\right)}{2d^{3/2}e^{9/2}} + \frac{x\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4} + \frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)} - \frac{2cx^3(cd-be)}{3e^3} + \frac{c^2x^5}{5e^2}$$

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 - (2*c*(c*d - b*e)*x^3)
/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 - b*d*e + a*e^2)^2*x)/(2*d*e^4*(d +
e*x^2)) - ((c*d^2 - b*d*e + a*e^2)*(7*c*d^2 - e*(3*b*d + a*e))*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-a^2e^2\right)}{e^4}-\frac{2d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{2cd(cd-2be)x^4}{e^2}-\frac{2c^2dx^6}{e}}{d+ex^2}\,dx}{2d}\\
&= \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\int \left(-\frac{2d\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)}{e^4}+\frac{4cd(cd-be)x^2}{e^3}-\frac{2c^2dx^4}{e^2}+\frac{7c^2d^4-10bcd^3e+3b^2d^2e^2+6acd^2e^2-2abde^3-a^2e^4}{e^4\left(d+ex^2\right)}\right)dx}{2d}\\
&= \frac{\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)}\\
&\quad - \frac{\left(7c^2d^4-10bcd^3e+3b^2d^2e^2+6acd^2e^2-2abde^3-a^2e^4\right)\int \frac{1}{d+ex^2}\,dx}{2de^4}
\end{aligned}$$

$$= \frac{(3c^2d^2+b^2e^2-2ce(2bd-ae))\,x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2-bde+ae^2)^2\,x}{2de^4\,(d+ex^2)} - \frac{(7cd^2-3bde-ae^2)\,(cd^2-bde+ae^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = \frac{(3c^2d^2+b^2e^2+2ce(-2bd+ae))\,x}{e^4} + \frac{2c(-cd+be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+e(-bd+ae))^2\,x}{2de^4\,(d+ex^2)} - \frac{(7c^2d^4+2cd^2e(-5bd+3ae)-e^2(-3b^2d^2+2abde+a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 + 2*c*e*(-2*b*d + a*e))*x)/e^4 + (2*c*(-(c*d) + b*e)*
x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(2*d*e^
4*(d + e*x^2)) - ((7*c^2*d^4 + 2*c*d^2*e*(-5*b*d + 3*a*e) - e^2*(-3*b^2*d^2
 + 2*a*b*d*e + a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.28

method	result
default	$\frac{\frac{1}{5}e^2x^5c^2+\frac{2}{3}bc\,e^2x^3-\frac{2}{3}c^2de\,x^3+2e^2acx+b^2e^2x-4bcdex+3c^2d^2x}{e^4} + \frac{\frac{\left(a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-2bc\,d^3e+c^2d^4\right)x}{2d\left(e\,x^2+d\right)}+}{e^4}$
risch	$\frac{c^2x^5}{5e^2}+\frac{2bc\,x^3}{3e^2}-\frac{2c^2d\,x^3}{3e^3}+\frac{2cax}{e^2}+\frac{b^2x}{e^2}-\frac{4bcdx}{e^3}+\frac{3c^2d^2x}{e^4}+\frac{(a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-2bc\,d^3e+c^2d^4)x}{2d\,e^4(e\,x^2+d)}-$

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/5*e^2*x^5*c^2+2/3*b*c*e^2*x^3-2/3*c^2*d*e*x^3+2*e^2*a*c*x+b^2*e^2*
x-4*b*c*d*e*x+3*c^2*d^2*x)+1/e^4*(1/2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^
2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/d*x/(e*x^2+d)+1/2*(a^2*e^4+2*a*b*d*e^3-6*a*c
*d^2*e^2-3*b^2*d^2*e^2+10*b*c*d^3*e-7*c^2*d^4)/d/(e*d)^(1/2)*arctan(e*x/(e*
d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.61

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx = \left[\frac{12\,c^2d^2e^4x^7 - 4\,(7\,c^2d^3e^3 - 10\,bcd^2e^4)x^5 + 20\,(7\,c^2d^4e^2 - 10\,bcd^3e^3 + 3\,(b^2+2\,ac)d^2e^4)x^3 + 15\,(7\,c^2d^5 - 1}{}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^2*d^2*e^4*x^7 - 4*(7*c^2*d^3*e^3 - 10*b*c*d^2*e^4)*x^5 + 20*(7*
c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^3 + 15*(7*c^2*d^5
 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4 + 3*(b^2 + 2*a*c)*d^3*e^2 + (7*
c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^
3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 30*(7*c^
2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*
e^3)*x)/(d^2*e^6*x^2 + d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 2*(7*c^2*d^3*e^3
 - 10*b*c*d^2*e^4)*x^5 + 10*(7*c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^2 + 2*a*
c)*d^2*e^4)*x^3 - 15*(7*c^2*d^5 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4
+ 3*(b^2 + 2*a*c)*d^3*e^2 + (7*c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a
^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 15
*(7*c^2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c
)*d^3*e^3)*x)/(d^2*e^6*x^2 + d^3*e^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(156) = 312$.

Time = 1.17 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.92

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx = \frac{c^2x^5}{5e^2} + x^3\cdot\left(\frac{2bc}{3e^2} - \frac{2c^2d}{3e^3}\right) + x\left(\frac{2ac}{e^2} + \frac{b^2}{e^2} - \frac{4bcd}{e^3} + \frac{3c^2d^2}{e^4}\right)$$
$$+ \frac{x(a^2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4)}{2d^2e^4 + 2de^5x^2}$$
$$- \frac{\sqrt{-\frac{1}{d^3e^9}}\left(ae^2 - bde + cd^2\right)\left(ae^2 + 3bde - 7cd^2\right)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-bde+cd^2\right)\left(ae^2+3bde-7cd^2\right)}{a^2e^4+2abde^3-6acd^2e^2-3b^2d^2e^2+10bcd^3e-7c^2d^4} + x\right)}{4}$$
$$+ \frac{\sqrt{-\frac{1}{d^3e^9}}\left(ae^2 - bde + cd^2\right)\left(ae^2 + 3bde - 7cd^2\right)\log\left(\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-bde+cd^2\right)\left(ae^2+3bde-7cd^2\right)}{a^2e^4+2abde^3-6acd^2e^2-3b^2d^2e^2+10bcd^3e-7c^2d^4} + x\right)}{4}$$

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**2,x)
```

```
[Out] c**2*x**5/(5*e**2) + x**3*(2*b*c/(3*e**2) - 2*c**2*d/(3*e**3)) + x*(2*a*c/e
**2 + b**2/e**2 - 4*b*c*d/e**3 + 3*c**2*d**2/e**4) + x*(a**2*e**4 - 2*a*b*d
*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(2*d**
2*e**4 + 2*d*e**5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a
*e**2 + 3*b*d*e - 7*c*d**2)*log(-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b
*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*
a*c*d**2*e**2 - 3*b**2*d**2*e**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4 + sq
rt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)*
log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*
d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e
**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx &= -\frac{(7\,c^2d^4-10\,bcd^3e+3\,b^2d^2e^2+6\,acd^2e^2-2\,abde^3-a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^4}\\ &\quad+\frac{c^2d^4x-2\,bcd^3ex+b^2d^2e^2x+2\,acd^2e^2x-2\,abde^3x+a^2e^4x}{2\,(ex^2+d)de^4}\\ &\quad+\frac{3\,c^2e^8x^5-10\,c^2de^7x^3+10\,bce^8x^3+45\,c^2d^2e^6x-60\,bcde^7x+15\,b^2e^8x+30\,ace^8x}{15\,e^{10}}\end{aligned}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*c^2*d^4 - 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 - 2*a*b*d*e^
3 - a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^4) + 1/2*(c^2*d^4*x - 2*b
```

```
*c*d^3*e*x + b^2*d^2*e^2*x + 2*a*c*d^2*e^2*x - 2*a*b*d*e^3*x + a^2*e^4*x)/(
(e*x^2 + d)*d*e^4) + 1/15*(3*c^2*e^8*x^5 - 10*c^2*d*e^7*x^3 + 10*b*c*e^8*x^
3 + 45*c^2*d^2*e^6*x - 60*b*c*d*e^7*x + 15*b^2*e^8*x + 30*a*c*e^8*x)/e^10
```

Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = x\left(\frac{b^2+2\,a\,c}{e^2}+\frac{2\,d\left(\frac{2\,c^2\,d}{e^3}-\frac{2\,b\,c}{e^2}\right)}{e}-\frac{c^2\,d^2}{e^4}\right)-x^3\left(\frac{2\,c^2\,d}{3\,e^3}-\frac{2\,b\,c}{3\,e^2}\right)+\frac{c^2\,x^5}{5\,e^2}+\frac{x\,(a^2\,e^4-2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2-2\,b\,c\,d^3\,e+c^2\,d^4)}{2\,d\,(e^5\,x^2+d\,e^4)}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,(c\,d^2-b\,d\,e+a\,e^2)\,(-7\,c\,d^2+3\,b\,d\,e+a\,e^2)}{\sqrt{d}\,(a^2\,e^4+2\,a\,b\,d\,e^3-6\,a\,c\,d^2\,e^2-3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-7\,c^2\,d^4)}\right)\,(c\,d^2-b\,d\,e+a\,e^2)\,(-7\,c\,d^2+3\,b\,d\,e+a\,e^2)}{2\,d^{3/2}\,e^{9/2}}$$

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x)
```

```
[Out] x*((2*a*c + b^2)/e^2 + (2*d*((2*c^2*d)/e^3 - (2*b*c)/e^2))/e - (c^2*d^2)/e^
4) - x^3*((2*c^2*d)/(3*e^3) - (2*b*c)/(3*e^2)) + (c^2*x^5)/(5*e^2) + (x*(a^
2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2))
/(2*d*(d*e^4 + e^5*x^2)) + (atan((e^(1/2)*x*(a*e^2 + c*d^2 - b*d*e)*(a*e^2
- 7*c*d^2 + 3*b*d*e))/(d^(1/2)*(a^2*e^4 - 7*c^2*d^4 - 3*b^2*d^2*e^2 + 2*a*b
*d*e^3 + 10*b*c*d^3*e - 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2 - b*d*e)*(a*e^2 - 7
*c*d^2 + 3*b*d*e))/(2*d^(3/2)*e^(9/2))
```

3.258 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx &= -\frac{c(3cd-2be)x}{e^4}+\frac{c^2x^3}{3e^3}+\frac{(cd^2-bde+ae^2)^2\,x}{4de^4\,(d+ex^2)^2}\\ &\quad-\frac{(13cd^2-5bde-3ae^2)\,(cd^2-bde+ae^2)\,x}{8d^2e^4\,(d+ex^2)}\\ &\quad+\frac{(35c^2d^4-6cd^2e(5bd-ae)+e^2(3b^2d^2+2abde+3a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}\end{aligned}$$

```
[Out] -c*(-2*b*e+3*c*d)*x/e^4+1/3*c^2*x^3/e^3+1/4*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(
e*x^2+d)^2-1/8*(-3*a*e^2-5*b*d*e+13*c*d^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e
*x^2+d)+1/8*(35*c^2*d^4-6*c*d^2*e*(-a*e+5*b*d)+e^2*(3*a^2*e^2+2*a*b*d*e+3*b
^2*d^2))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(9/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{1171, 1828, 1167, 211\}$

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$$
$$= \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2(3a^2e^2+2abde+3b^2d^2)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}}$$
$$-\frac{x(-3ae^2-5bde+13cd^2)\left(ae^2-bde+cd^2\right)}{8d^2e^4\left(d+ex^2\right)}$$
$$+\frac{x(ae^2-bde+cd^2)^2}{4de^4\left(d+ex^2\right)^2}-\frac{cx(3cd-2be)}{e^4}+\frac{c^2x^3}{3e^3}$$

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^3)/(3*e^3) + ((c*d^2 - b*d*e + a*e^2)
^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 5*b*d*e - 3*a*e^2)*(c*d^2 - b*
d*e + a*e^2)*x)/(8*d^2*e^4*(d + e*x^2)) + ((35*c^2*d^4 - 6*c*d^2*e*(5*b*d -
 a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(8*d^(5/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
```

```
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\int \frac{\frac{\left(cd^2-bde-ae^2\right)\left(cd^2-bde+3ae^2\right)}{e^4}-\frac{4d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{4cd(cd-2be)x^4}{e^2}-\frac{4c^2dx^6}{e}}{\left(d+ex^2\right)^2}\,dx}{4d} \\
&= \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} \\
&\quad + \frac{\int \frac{\frac{11c^2d^4-2cd^2e(7bd-3ae)+e^2\left(3b^2d^2+2abde+3a^2e^2\right)}{e^4}-\frac{16cd^2(cd-be)x^2}{e^3}+\frac{8c^2d^2x^4}{e^2}}{d+ex^2}\,dx}{8d^2} \\
&= \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} \\
&\quad + \frac{\int \left(-\frac{8cd^2(3cd-2be)}{e^4}+\frac{8c^2d^2x^2}{e^3}+\frac{35c^2d^4-30bcd^3e+3b^2d^2e^2+6acd^2e^2+2abde^3+3a^2e^4}{e^4\left(d+ex^2\right)}\right)dx}{8d^2} \\
&= -\frac{c(3cd-2be)x}{e^4}+\frac{c^2x^3}{3e^3}+\frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} \\
&\quad - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} \\
&\quad + \frac{\left(35c^2d^4-6cd^2e(5bd-ae)+e^2\left(3b^2d^2+2abde+3a^2e^2\right)\right)\int \frac{1}{d+ex^2}\,dx}{8d^2e^4} \\
&= -\frac{c(3cd-2be)x}{e^4}+\frac{c^2x^3}{3e^3}+\frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} \\
&\quad - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} \\
&\quad + \frac{\left(35c^2d^4-6cd^2e(5bd-ae)+e^2\left(3b^2d^2+2abde+3a^2e^2\right)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx &= \frac{c(-3cd+2be)x}{e^4}+\frac{c^2x^3}{3e^3}+\frac{(cd^2+e(-bd+ae))^2\,x}{4de^4\,(d+ex^2)^2}\\ &\quad -\frac{(13c^2d^4-2cd^2e(9bd-5ae)+e^2(5b^2d^2-2abde-3a^2e^2))\,x}{8d^2e^4\,(d+ex^2)}\\ &\quad +\frac{(35c^2d^4+6cd^2e(-5bd+ae)+e^2(3b^2d^2+2abde+3a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}\end{aligned}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (c*(-3*c*d + 2*b*e)*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + e*(-(b*d) + a*e)
)^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c^2*d^4 - 2*c*d^2*e*(9*b*d - 5*a*e) +
 e^2*(5*b^2*d^2 - 2*a*b*d*e - 3*a^2*e^2))*x)/(8*d^2*e^4*(d + e*x^2)) + ((35
*c^2*d^4 + 6*c*d^2*e*(-5*b*d + a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^
2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.18

method	result
default	$\frac{c\left(\frac{1}{3}cx^3e+2bex-3cdx\right)}{e^4}+\frac{\frac{\frac{e\left(3a^2e^4+2abde^3-10acd^2e^2-5b^2d^2e^2+18bcd^3e-13c^2d^4\right)x^3}{8d^2}+\frac{\left(5a^2e^4-2abde^3-6acd^2e^2-3b^2d^2e^2+14bcd^3e-11c^2\right.}{8d}}{\left(ex^2+d\right)^2}}{e^4}$
risch	$\frac{c^2x^3}{3e^3}+\frac{2cbx}{e^3}-\frac{3c^2dx}{e^4}+\frac{\frac{e\left(3a^2e^4+2abde^3-10acd^2e^2-5b^2d^2e^2+18bcd^3e-13c^2d^4\right)x^3}{8d^2}+\frac{\left(5a^2e^4-2abde^3-6acd^2e^2-3b^2d^2e^2+14bcd^3e-11c\right.}{8d}}{e^4\left(ex^2+d\right)^2}$

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c/e^4*(1/3*c*x^3*e+2*b*e*x-3*c*d*x)+1/e^4*((1/8*e*(3*a^2*e^4+2*a*b*d*e^3-10
*a*c*d^2*e^2-5*b^2*d^2*e^2+18*b*c*d^3*e-13*c^2*d^4)/d^2*x^3+1/8*(5*a^2*e^4-
2*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2+14*b*c*d^3*e-11*c^2*d^4)/d*x)/(e*x^
2+d)^2+1/8*(3*a^2*e^4+2*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2-30*b*c*d^3*e+
35*c^2*d^4)/d^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(185) = 370$.

Time = 0.26 (sec) , antiderivative size = 794, normalized size of antiderivative = 3.95

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx$$
$$= \left[\frac{16\,c^2d^3e^4x^7 - 16\,(7\,c^2d^4e^3 - 6\,bcd^3e^4)x^5 - 2\,(175\,c^2d^5e^2 - 150\,bcd^4e^3 - 6\,abd^2e^5 - 9\,a^2de^6 + 15\,(b^2 + 2\,a}{}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^2*d^3*e^4*x^7 - 16*(7*c^2*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - 2*(175
*c^2*d^5*e^2 - 150*b*c*d^4*e^3 - 6*a*b*d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 + 2*
a*c)*d^3*e^4)*x^3 - 3*(35*c^2*d^6 - 30*b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^
2*e^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*a*b*
d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^4 + 2*(35*c^2*d^5*e - 30*b*c
*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x^2)*sqrt
(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(35*c^2*d^6*e - 30
*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3*(b^2 + 2*a*c)*d^4*e^3)*x)/
(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 8*(7*c^2
*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - (175*c^2*d^5*e^2 - 150*b*c*d^4*e^3 - 6*a*b*
d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^4)*x^3 + 3*(35*c^2*d^6 - 30*
b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^2*e^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c
^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*a*b*d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2
*e^4)*x^4 + 2*(35*c^2*d^5*e - 30*b*c*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5
+ 3*(b^2 + 2*a*c)*d^3*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c^2
*d^6*e - 30*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3*(b^2 + 2*a*c)*d
^4*e^3)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

Sympy [A] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = \frac{c^2x^3}{3e^3} + x\left(\frac{2bc}{e^3} - \frac{3c^2d}{e^4}\right)$$
$$- \frac{\sqrt{-\frac{1}{d^5e^9}} \cdot (3a^2e^4 + 2abde^3 + 6acd^2e^2 + 3b^2d^2e^2 - 30bcd^3e + 35c^2d^4)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16}$$
$$+ \frac{\sqrt{-\frac{1}{d^5e^9}} \cdot (3a^2e^4 + 2abde^3 + 6acd^2e^2 + 3b^2d^2e^2 - 30bcd^3e + 35c^2d^4)\log\left(d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16}$$
$$+ \frac{x^3 \cdot (3a^2e^5 + 2abde^4 - 10acd^2e^3 - 5b^2d^2e^3 + 18bcd^3e^2 - 13c^2d^4e) + x(5a^2de^4 - 2abd^2e^3 - 6acd^3e^2 - 3}{8d^4e^4 + 16d^3e^5x^2 + 8d^2e^6x^4}$$

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**3,x)
```

```
[Out] c**2*x**3/(3*e**3) + x*(2*b*c/e**3 - 3*c**2*d/e**4) - sqrt(-1/(d**5*e**9))*
(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**2 - 30*b*c*d
**3*e + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + sqrt(-1
/(d**5*e**9))*(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e
**2 - 30*b*c*d**3*e + 35*c**2*d**4)*log(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)
/16 + (x**3*(3*a**2*e**5 + 2*a*b*d*e**4 - 10*a*c*d**2*e**3 - 5*b**2*d**2*e*
*3 + 18*b*c*d**3*e**2 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 2*a*b*d**2*e**
3 - 6*a*c*d**3*e**2 - 3*b**2*d**3*e**2 + 14*b*c*d**4*e - 11*c**2*d**5))/(8*
d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx &= \frac{\left(35\,c^2d^4-30\,bcd^3e+3\,b^2d^2e^2+6\,acd^2e^2+2\,abde^3+3\,a^2e^4\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^4}\\ &\quad-\frac{13\,c^2d^4ex^3-18\,bcd^3e^2x^3+5\,b^2d^2e^3x^3+10\,acd^2e^3x^3-2\,abde^4x^3-3\,a^2e^5x^3+11\,c^2d^5x-14\,bcd^4ex+3}{8\left(ex^2+d\right)^2d^2e^4}\\ &\quad+\frac{c^2e^6x^3-9\,c^2de^5x+6\,bce^6x}{3\,e^9}\end{aligned}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*c^2*d^4 - 30*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 2*a*b*d*e^
3 + 3*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^4) - 1/8*(13*c^2*d^4*
```

```
e*x^3 - 18*b*c*d^3*e^2*x^3 + 5*b^2*d^2*e^3*x^3 + 10*a*c*d^2*e^3*x^3 - 2*a*b
*d*e^4*x^3 - 3*a^2*e^5*x^3 + 11*c^2*d^5*x - 14*b*c*d^4*e*x + 3*b^2*d^3*e^2*
x + 6*a*c*d^3*e^2*x + 2*a*b*d^2*e^3*x - 5*a^2*d*e^4*x)/((e*x^2 + d)^2*d^2*e
^4) + 1/3*(c^2*e^6*x^3 - 9*c^2*d*e^5*x + 6*b*c*e^6*x)/e^9
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx = \frac{c^2\,x^3}{3\,e^3} - x\left(\frac{3\,c^2\,d}{e^4} - \frac{2\,b\,c}{e^3}\right) - \frac{\frac{x\,\left(-5\,a^2\,e^4+2\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2-14\,b\,c\,d^3\,e+11\,c^2\,d^4\right)}{8\,d} - \frac{x^3\,\left(3\,a^2\,e^5+2\,a\,b\,d\,e^4-10\,a\,c\,d^2\,e^3-5\,b^2\,d^2\,e^3+18\,b\,c\,d^3\,e^2-13\,c^2\,d^4\,e\right)}{8\,d^2}}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,a^2\,e^4+2\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2-30\,b\,c\,d^3\,e+35\,c^2\,d^4\right)}{8\,d^{5/2}\,e^{9/2}}$$

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x)
```

```
[Out] (c^2*x^3)/(3*e^3) - x*((3*c^2*d)/e^4 - (2*b*c)/e^3) - ((x*(11*c^2*d^4 - 5*a
^2*e^4 + 3*b^2*d^2*e^2 + 2*a*b*d*e^3 - 14*b*c*d^3*e + 6*a*c*d^2*e^2))/(8*d)
 - (x^3*(3*a^2*e^5 - 13*c^2*d^4*e - 5*b^2*d^2*e^3 + 2*a*b*d*e^4 - 10*a*c*d^
2*e^3 + 18*b*c*d^3*e^2))/(8*d^2))/(d^2*e^4 + e^6*x^4 + 2*d*e^5*x^2) + (atan
((e^(1/2)*x)/d^(1/2))*(3*a^2*e^4 + 35*c^2*d^4 + 3*b^2*d^2*e^2 + 2*a*b*d*e^3
 - 30*b*c*d^3*e + 6*a*c*d^2*e^2))/(8*d^(5/2)*e^(9/2))
```

3.259 $\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 250

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^4} \, dx &= \frac{c^2x}{e^4}+\frac{\left(cd^2-bde+ae^2\right)^2x}{6de^4\left(d+ex^2\right)^3}-\frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2}\\&\quad+\frac{\left(29c^2d^4-2cd^2e(11bd-ae)+e^2\left(b^2d^2+2abde+5a^2e^2\right)\right)x}{16d^3e^4\left(d+ex^2\right)}\\&\quad-\frac{\left(35c^2d^4-2cd^2e(5bd+ae)-e^2\left(b^2d^2+2abde+5a^2e^2\right)\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}\end{aligned}$$

```
[Out] c^2*x/e^4+1/6*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)^3-1/24*(-5*a*e^2-7*b*
d*e+19*c*d^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e*x^2+d)^2+1/16*(29*c^2*d^4-2*
c*d^2*e*(-a*e+11*b*d)+e^2*(5*a^2*e^2+2*a*b*d*e+b^2*d^2))*x/d^3/e^4/(e*x^2+d
)-1/16*(35*c^2*d^4-2*c*d^2*e*(a*e+5*b*d)-e^2*(5*a^2*e^2+2*a*b*d*e+b^2*d^2))
*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(9/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{1171, 1828, 396, 211\}$

$$\begin{aligned}&\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^4}\,dx\\ &= -\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(ae+5bd)+35c^2d^4\right)}{16d^{7/2}e^{9/2}}\\ &\quad+\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{16d^3e^4\left(d+ex^2\right)}\\ &\quad-\frac{x\left(-5ae^2-7bde+19cd^2\right)\left(ae^2-bde+cd^2\right)}{24d^2e^4\left(d+ex^2\right)^2}+\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}+\frac{c^2x}{e^4}\end{aligned}$$

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((19*
c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(24*d^2*e^4*(d + e*x^
2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*e) + e^2*(b^2*d^2 + 2*a*b*d*e
+ 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d
 + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(16*d^(7/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3}\\
&\quad-\frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-5a^2e^2\right)}{e^4}-\frac{6d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{6cd(cd-2be)x^4}{e^2}-\frac{6c^2dx^6}{e}}{\left(d+ex^2\right)^3}\,dx}{6d}\\
&= \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3}-\frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2}\\
&\quad+\frac{\int \frac{\frac{3\left(5c^2d^4-2cd^2e(3bd-ae)+e^2\left(b^2d^2+2abde+5a^2e^2\right)\right)}{e^4}-\frac{48cd^2(cd-be)x^2}{e^3}+\frac{24c^2d^2x^4}{e^2}}{\left(d+ex^2\right)^2}\,dx}{24d^2}\\
&= \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3}-\frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2}\\
&\quad+\frac{\left(29c^2d^4-2cd^2e(11bd-ae)+e^2(b^2d^2+2abde+5a^2e^2)\right)x}{16d^3e^4\left(d+ex^2\right)}\\
&\quad-\frac{\int \frac{\frac{3\left(19c^2d^4-2cd^2e(5bd+ae)-e^2\left(b^2d^2+2abde+5a^2e^2\right)\right)}{e^4}-\frac{48c^2d^3x^2}{e^3}}{d+ex^2}\,dx}{48d^3}\\
&= \frac{c^2x}{e^4}+\frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3}-\frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2}\\
&\quad+\frac{\left(29c^2d^4-2cd^2e(11bd-ae)+e^2(b^2d^2+2abde+5a^2e^2)\right)x}{16d^3e^4\left(d+ex^2\right)}\\
&\quad-\frac{\left(35c^2d^4-2cd^2e(5bd+ae)-e^2(b^2d^2+2abde+5a^2e^2)\right)\int \frac{1}{d+ex^2}\,dx}{16d^3e^4}\\
&= \frac{c^2x}{e^4}+\frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3}-\frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2}\\
&\quad+\frac{\left(29c^2d^4-2cd^2e(11bd-ae)+e^2(b^2d^2+2abde+5a^2e^2)\right)x}{16d^3e^4\left(d+ex^2\right)}\\
&\quad-\frac{\left(35c^2d^4-2cd^2e(5bd+ae)-e^2(b^2d^2+2abde+5a^2e^2)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx$$
$$=\frac{c^2x}{e^4}+\frac{(cd^2+e(-bd+ae))^2x}{6de^4(d+ex^2)^3}$$
$$-\frac{(19c^2d^4+2cd^2e(-13bd+7ae)+e^2(7b^2d^2-2abde-5a^2e^2))x}{24d^2e^4(d+ex^2)^2}$$
$$+\frac{(29c^2d^4+2cd^2e(-11bd+ae)+e^2(b^2d^2+2abde+5a^2e^2))x}{16d^3e^4(d+ex^2)}$$
$$-\frac{(35c^2d^4-2cd^2e(5bd+ae)-e^2(b^2d^2+2abde+5a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((
19*c^2*d^4 + 2*c*d^2*e*(-13*b*d + 7*a*e) + e^2*(7*b^2*d^2 - 2*a*b*d*e - 5*a
^2*e^2))*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 + 2*c*d^2*e*(-11*b*d
+ a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2))
 - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^
2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.14

method	result
default	$\frac{c^2x}{e^4}+\frac{\frac{\frac{e^2\left(5a^2e^4+2abde^3+2acd^2e^2+b^2d^2e^2-22bcd^3e+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4+2abde^3-2acd^2e^2-b^2d^2e^2-10bcd^3e+17c^2d^4\right)x^3}{6d^2}+\left(11a^2e^4-2\right.}{\left(ex^2+d\right)^3}}{e^4}$
risch	$\frac{c^2x}{e^4}+\frac{\frac{e^2\left(5a^2e^4+2abde^3+2acd^2e^2+b^2d^2e^2-22bcd^3e+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4+2abde^3-2acd^2e^2-b^2d^2e^2-10bcd^3e+17c^2d^4\right)x^3}{6d^2}+\left(11a^2e^4-2\right.}{e^4\left(ex^2+d\right)^3}$

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*x/e^4+1/e^4*((1/16*e^2*(5*a^2*e^4+2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2
-22*b*c*d^3*e+29*c^2*d^4)/d^3*x^5+1/6*e*(5*a^2*e^4+2*a*b*d*e^3-2*a*c*d^2*e^
2-b^2*d^2*e^2-10*b*c*d^3*e+17*c^2*d^4)/d^2*x^3+1/16*(11*a^2*e^4-2*a*b*d*e^3
-2*a*c*d^2*e^2-b^2*d^2*e^2-10*b*c*d^3*e+19*c^2*d^4)/d*x)/(e*x^2+d)^3+1/16*(
5*a^2*e^4+2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2+10*b*c*d^3*e-35*c^2*d^4)/d^
3/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(234) = 468.

Time = 0.27 (sec) , antiderivative size = 1016, normalized size of antiderivative = 4.06

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx$$
$$= \Bigg[\frac{96\,c^2d^4e^4x^7+6\,(77\,c^2d^5e^3-22\,bcd^4e^4+2\,abd^2e^6+5\,a^2de^7+(b^2+2\,ac)d^3e^5)x^5+16\,(35\,c^2d^6e^2-10\,bcd^5}{}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 - 22*b*c*d^4*e^4 + 2*a*b*d^2*
e^6 + 5*a^2*d*e^7 + (b^2 + 2*a*c)*d^3*e^5)*x^5 + 16*(35*c^2*d^6*e^2 - 10*b*
c*d^5*e^3 + 2*a*b*d^3*e^5 + 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 + 3*
(35*c^2*d^7 - 10*b*c*d^6*e - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*
d^5*e^2 + (35*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2
 + 2*a*c)*d^2*e^5)*x^6 + 3*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5
 - 5*a^2*d*e^6 - (b^2 + 2*a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*
e^2 - 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*sqrt(-d*e
)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 10*b*c*
d^6*e^2 - 2*a*b*d^4*e^4 + 11*a^2*d^3*e^5 - (b^2 + 2*a*c)*d^5*e^3)*x)/(d^4*e
^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5), 1/48*(48*c^2*d^4*e^4*x^7
 + 3*(77*c^2*d^5*e^3 - 22*b*c*d^4*e^4 + 2*a*b*d^2*e^6 + 5*a^2*d*e^7 + (b^2
+ 2*a*c)*d^3*e^5)*x^5 + 8*(35*c^2*d^6*e^2 - 10*b*c*d^5*e^3 + 2*a*b*d^3*e^5
+ 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 - 3*(35*c^2*d^7 - 10*b*c*d^6*e
 - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*d^5*e^2 + (35*c^2*d^4*e^3
- 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2 + 2*a*c)*d^2*e^5)*x^6 + 3
*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5 - 5*a^2*d*e^6 - (b^2 + 2*
a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*e^2 - 2*a*b*d^3*e^4 - 5*a^
2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 3
*(35*c^2*d^7*e - 10*b*c*d^6*e^2 - 2*a*b*d^4*e^4 + 11*a^2*d^3*e^5 - (b^2 + 2
*a*c)*d^5*e^3)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx$$

$$= \frac{c^2x}{e^4} - \frac{(35\,c^2d^4 - 10\,bcd^3e - b^2d^2e^2 - 2\,acd^2e^2 - 2\,abde^3 - 5\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^4}$$

$$+ \frac{87\,c^2d^4e^2x^5 - 66\,bcd^3e^3x^5 + 3\,b^2d^2e^4x^5 + 6\,acd^2e^4x^5 + 6\,abde^5x^5 + 15\,a^2e^6x^5 + 136\,c^2d^5ex^3 - 80\,bcd^4e^2}{}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x/e^4 - 1/16*(35*c^2*d^4 - 10*b*c*d^3*e - b^2*d^2*e^2 - 2*a*c*d^2*e^2 -
 2*a*b*d*e^3 - 5*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e^4) + 1/48*
(87*c^2*d^4*e^2*x^5 - 66*b*c*d^3*e^3*x^5 + 3*b^2*d^2*e^4*x^5 + 6*a*c*d^2*e^
4*x^5 + 6*a*b*d*e^5*x^5 + 15*a^2*e^6*x^5 + 136*c^2*d^5*e*x^3 - 80*b*c*d^4*e
^2*x^3 - 8*b^2*d^3*e^3*x^3 - 16*a*c*d^3*e^3*x^3 + 16*a*b*d^2*e^4*x^3 + 40*a
^2*d*e^5*x^3 + 57*c^2*d^6*x - 30*b*c*d^5*e*x - 3*b^2*d^4*e^2*x - 6*a*c*d^4*
e^2*x - 6*a*b*d^3*e^3*x + 33*a^2*d^2*e^4*x)/((e*x^2 + d)^3*d^3*e^4)
```

Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{\frac{x^5\,(5\,a^2\,e^6+2\,a\,b\,d\,e^5+2\,a\,c\,d^2\,e^4+b^2\,d^2\,e^4-22\,b\,c\,d^3\,e^3+29\,c^2\,d^4\,e^2)}{16\,d^3}-\frac{x\,(-11\,a^2\,e^4+2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-19\,c^2\,d^4)}{16\,d}+x^3\,(}{d^3\,e^4+3\,d^2\,e^5\,x^2+3\,d\,e^6\,x^4+e^7\,x^6} + \frac{c^2\,x}{e^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(5\,a^2\,e^4+2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-35\,c^2\,d^4)}{16\,d^{7/2}\,e^{9/2}}$$

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x)
```

```
[Out] ((x^5*(5*a^2*e^6 + b^2*d^2*e^4 + 29*c^2*d^4*e^2 + 2*a*b*d*e^5 + 2*a*c*d^2*e
^4 - 22*b*c*d^3*e^3))/(16*d^3) - (x*(b^2*d^2*e^2 - 19*c^2*d^4 - 11*a^2*e^4
+ 2*a*b*d*e^3 + 10*b*c*d^3*e + 2*a*c*d^2*e^2))/(16*d) + (x^3*(5*a^2*e^5 + 1
7*c^2*d^4*e - b^2*d^2*e^3 + 2*a*b*d*e^4 - 2*a*c*d^2*e^3 - 10*b*c*d^3*e^2))/
(6*d^2))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x^2) + (c^2*x)/e^4 +
(atan((e^(1/2)*x)/d^(1/2))*(5*a^2*e^4 - 35*c^2*d^4 + b^2*d^2*e^2 + 2*a*b*d*
e^3 + 10*b*c*d^3*e + 2*a*c*d^2*e^2))/(16*d^(7/2)*e^(9/2))
```

3.260 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 317

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx &= \frac{(cd^2-bde+ae^2)^2\,x}{8de^4\,(d+ex^2)^4}-\frac{(25cd^2-9bde-7ae^2)\,(cd^2-bde+ae^2)\,x}{48d^2e^4\,(d+ex^2)^3}\\&\quad+\frac{(163c^2d^4-2cd^2e(59bd-3ae)+e^2(3b^2d^2+10abde+35a^2e^2))\,x}{192d^3e^4\,(d+ex^2)^2}\\&\quad-\frac{(93c^2d^4-2cd^2e(5bd+3ae)-e^2(3b^2d^2+10abde+35a^2e^2))\,x}{128d^4e^4\,(d+ex^2)}\\&\quad+\frac{(35c^2d^4+2cd^2e(5bd+3ae)+e^2(3b^2d^2+10abde+35a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}\end{aligned}$$

```
[Out] 1/8*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)^4-1/48*(-7*a*e^2-9*b*d*e+25*c*d
^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e*x^2+d)^3+1/192*(163*c^2*d^4-2*c*d^2*e*
(-3*a*e+59*b*d)+e^2*(35*a^2*e^2+10*a*b*d*e+3*b^2*d^2))*x/d^3/e^4/(e*x^2+d)^
2-1/128*(93*c^2*d^4-2*c*d^2*e*(3*a*e+5*b*d)-e^2*(35*a^2*e^2+10*a*b*d*e+3*b^
2*d^2))*x/d^4/e^4/(e*x^2+d)+1/128*(35*c^2*d^4+2*c*d^2*e*(3*a*e+5*b*d)+e^2*(
35*a^2*e^2+10*a*b*d*e+3*b^2*d^2))*arctan(x*e^(1/2)/d^(1/2))/d^(9/2)/e^(9/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1171, 1828, 393, 211}

$$\begin{aligned}
&\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx \\
&= \frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(e^2(35a^2e^2+10abde+3b^2d^2)+2cd^2e(3ae+5bd)+35c^2d^4)}{128d^{9/2}e^{9/2}} \\
&\quad -\frac{x(-e^2(35a^2e^2+10abde+3b^2d^2)-2cd^2e(3ae+5bd)+93c^2d^4)}{128d^4e^4(d+ex^2)} \\
&\quad +\frac{x(e^2(35a^2e^2+10abde+3b^2d^2)-2cd^2e(59bd-3ae)+163c^2d^4)}{192d^3e^4(d+ex^2)^2} \\
&\quad +\frac{x(ae^2-bde+cd^2)^2}{8de^4(d+ex^2)^4}-\frac{x(-7ae^2-9bde+25cd^2)(ae^2-bde+cd^2)}{48d^2e^4(d+ex^2)^3}
\end{aligned}$$

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^2 - 9*b*d*
e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(48*d^2*e^4*(d + e*x^2)^3) + ((163*
c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2
*e^2))*x)/(192*d^3*e^4*(d + e*x^2)^2) - ((93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3
*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(128*d^4*e^4*(d + e*x
^2)) + ((35*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d
*e + 35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-7a^2e^2\right)}{e^4}-\frac{8d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{8cd(cd-2be)x^4}{e^2}-\frac{8c^2dx^6}{e}}{\left(d+ex^2\right)^4}\,dx}{8d}\\
&= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3}\\
&\quad + \frac{\int \frac{\frac{19c^2d^4-2cd^2e(11bd-3ae)+e^2\left(3b^2d^2+10abde+35a^2e^2\right)}{e^4}-\frac{96cd^2(cd-be)x^2}{e^3}+\frac{48c^2d^2x^4}{e^2}}{\left(d+ex^2\right)^3}\,dx}{48d^2}\\
&= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3}\\
&\quad + \frac{\left(163c^2d^4-2cd^2e(59bd-3ae)+e^2(3b^2d^2+10abde+35a^2e^2)\right)x}{192d^3e^4\left(d+ex^2\right)^2}\\
&\quad - \frac{\int \frac{\frac{3\left(29c^2d^4-2cd^2e(5bd+3ae)-e^2\left(3b^2d^2+10abde+35a^2e^2\right)\right)}{e^4}-\frac{192c^2d^3x^2}{e^3}}{\left(d+ex^2\right)^2}\,dx}{192d^3}\\
&= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3}\\
&\quad + \frac{\left(163c^2d^4-2cd^2e(59bd-3ae)+e^2(3b^2d^2+10abde+35a^2e^2)\right)x}{192d^3e^4\left(d+ex^2\right)^2}\\
&\quad - \frac{\left(93c^2d^4-2cd^2e(5bd+3ae)-e^2(3b^2d^2+10abde+35a^2e^2)\right)x}{128d^4e^4\left(d+ex^2\right)}\\
&\quad + \frac{\left(35c^2d^4+2cd^2e(5bd+3ae)+e^2(3b^2d^2+10abde+35a^2e^2)\right)\int \frac{1}{d+ex^2}\,dx}{128d^4e^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3} \\
&\quad + \frac{\left(163c^2d^4-2cd^2e(59bd-3ae)+e^2\left(3b^2d^2+10abde+35a^2e^2\right)\right)x}{192d^3e^4\left(d+ex^2\right)^2} \\
&\quad - \frac{\left(93c^2d^4-2cd^2e(5bd+3ae)-e^2\left(3b^2d^2+10abde+35a^2e^2\right)\right)x}{128d^4e^4\left(d+ex^2\right)} \\
&\quad + \frac{\left(35c^2d^4+2cd^2e(5bd+3ae)+e^2\left(3b^2d^2+10abde+35a^2e^2\right)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.09

$$
\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^5}\,dx
$$

$$
= \frac{\frac{48d^{7/2}\sqrt{e}\left(cd^2+e(-bd+ae)\right)^2x}{\left(d+ex^2\right)^4} - \frac{8d^{5/2}\sqrt{e}\left(25c^2d^4+2cd^2e(-17bd+9ae)+e^2\left(9b^2d^2-2abde-7a^2e^2\right)\right)x}{\left(d+ex^2\right)^3} + \frac{2d^{3/2}\sqrt{e}\left(163c^2d^4+2cd^2e(-59bd+3ae)\right.}{\left(d+ex^2\right)}}{}
$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((48*d^(7/2)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^2*x)/(d + e*x^2)^4 - (8*d^(
5/2)*Sqrt[e]*(25*c^2*d^4 + 2*c*d^2*e*(-17*b*d + 9*a*e) + e^2*(9*b^2*d^2 - 2
*a*b*d*e - 7*a^2*e^2))*x)/(d + e*x^2)^3 + (2*d^(3/2)*Sqrt[e]*(163*c^2*d^4 +
 2*c*d^2*e*(-59*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x
)/(d + e*x^2)^2 - (3*Sqrt[d]*Sqrt[e]*(93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e
) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(d + e*x^2) + 3*(35*c^2*d
^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(384*d^(9/2)*e^(9/2))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.09

method	result
default	$\frac{\frac{\left(35a^2e^4+10abd\,e^3+6ac\,d^2e^2+3b^2d^2e^2+10bc\,d^3e-93c^2d^4\right)x^7}{128d^4e}+\frac{\left(385a^2e^4+110abd\,e^3+66ac\,d^2e^2+33b^2d^2e^2-146bc\,d^3e-511c^2d^4\right)x^5}{384d^3e^2}+\frac{\left(511a^2e^4+\right.}{}}{\left(e\,x^2+d\right)^4}$
risch	$\frac{\frac{\left(35a^2e^4+10abd\,e^3+6ac\,d^2e^2+3b^2d^2e^2+10bc\,d^3e-93c^2d^4\right)x^7}{128d^4e}+\frac{\left(385a^2e^4+110abd\,e^3+66ac\,d^2e^2+33b^2d^2e^2-146bc\,d^3e-511c^2d^4\right)x^5}{384d^3e^2}+\frac{\left(511a^2e^4+\right.}{}}{\left(e\,x^2+d\right)^4}$

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2+10*b*c*d^3*e-93
*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+110*a*b*d*e^3+66*a*c*d^2*e^2+33*b^2*
d^2*e^2-146*b*c*d^3*e-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4+146*a*b*d
*e^3-66*a*c*d^2*e^2-33*b^2*d^2*e^2-110*b*c*d^3*e-385*c^2*d^4)/d^2/e^3*x^3+1
/128*(93*a^2*e^4-10*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2-10*b*c*d^3*e-35*c
^2*d^4)/e^4/d*x)/(e*x^2+d)^4+1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3
*b^2*d^2*e^2+10*b*c*d^3*e+35*c^2*d^4)/d^4/e^4/(e*d)^(1/2)*arctan(e*x/(e*d)^
(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(299) = 598$.

Time = 0.29 (sec) , antiderivative size = 1266, normalized size of antiderivative = 3.99

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(6*(93*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8
 - 3*(b^2 + 2*a*c)*d^3*e^6)*x^7 + 2*(511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 11
0*a*b*d^3*e^6 - 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + 2*(385*c^
2*d^7*e^2 + 110*b*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 +
 2*a*c)*d^5*e^4)*x^3 + 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a
^2*d^4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 1
0*a*b*d*e^7 + 35*a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3
 + 10*b*c*d^4*e^4 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5
)*x^6 + 6*(35*c^2*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^
6 + 3*(b^2 + 2*a*c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*
b*d^4*e^4 + 35*a^2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(-d*e)*log((
e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^8*e + 10*b*c*d^7*e^2
 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x
^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(9
3*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8 - 3*(b^2 + 2
*a*c)*d^3*e^6)*x^7 + (511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 110*a*b*d^3*e^6 -
 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + (385*c^2*d^7*e^2 + 110*b
*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 + 2*a*c)*d^5*e^4)*
x^3 - 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a^2*d^4*e^4 + 3*(b
^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 10*a*b*d*e^7 + 35*
a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3 + 10*b*c*d^4*e^4
 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5)*x^6 + 6*(35*c^2
*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^6 + 3*(b^2 + 2*a*
c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*b*d^4*e^4 + 35*a^
2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) +
```

```
 3*(35*c^2*d^8*e + 10*b*c*d^7*e^2 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^
2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8
*e^6*x^2 + d^9*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**5,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$= \frac{(35\,c^2d^4+10\,bcd^3e+3\,b^2d^2e^2+6\,acd^2e^2+10\,abde^3+35\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\,\sqrt{de}d^4e^4} - \frac{279\,c^2d^4e^3x^7-30\,bcd^3e^4x^7-9\,b^2d^2e^5x^7-18\,acd^2e^5x^7-30\,abde^6x^7-105\,a^2e^7x^7+511\,c^2d^5e^2x^5+146\,b}{}$$

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 10*a*b*d
*e^3 + 35*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^4*e^4) - 1/384*(279*c
^2*d^4*e^3*x^7 - 30*b*c*d^3*e^4*x^7 - 9*b^2*d^2*e^5*x^7 - 18*a*c*d^2*e^5*x^
7 - 30*a*b*d*e^6*x^7 - 105*a^2*e^7*x^7 + 511*c^2*d^5*e^2*x^5 + 146*b*c*d^4*
e^3*x^5 - 33*b^2*d^3*e^4*x^5 - 66*a*c*d^3*e^4*x^5 - 110*a*b*d^2*e^5*x^5 - 3
85*a^2*d*e^6*x^5 + 385*c^2*d^6*e*x^3 + 110*b*c*d^5*e^2*x^3 + 33*b^2*d^4*e^3
*x^3 + 66*a*c*d^4*e^3*x^3 - 146*a*b*d^3*e^4*x^3 - 511*a^2*d^2*e^5*x^3 + 105
*c^2*d^7*x + 30*b*c*d^6*e*x + 9*b^2*d^5*e^2*x + 18*a*c*d^5*e^2*x + 30*a*b*d
^4*e^3*x - 279*a^2*d^3*e^4*x)/((e*x^2 + d)^4*d^4*e^4)
```

Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(35\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e+35\,c^2\,d^4)}{128\,d^{9/2}\,e^{9/2}} - \frac{\frac{x\,(-93\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e+35\,c^2\,d^4)}{128\,d\,e^4} - \frac{x^7\,(35\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-93\,c^2\,d^4)}{128\,d^4\,e}}{d^4+4\,d^3\,e\,x^2+}$$

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(35*a^2*e^4 + 35*c^2*d^4 + 3*b^2*d^2*e^2 + 10*a*
b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^2*e^2))/(128*d^(9/2)*e^(9/2)) - ((x*(35*c^
2*d^4 - 93*a^2*e^4 + 3*b^2*d^2*e^2 + 10*a*b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^
2*e^2))/(128*d*e^4) - (x^7*(35*a^2*e^4 - 93*c^2*d^4 + 3*b^2*d^2*e^2 + 10*a*
b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^2*e^2))/(128*d^4*e) + (x^3*(385*c^2*d^4 -
511*a^2*e^4 + 33*b^2*d^2*e^2 - 146*a*b*d*e^3 + 110*b*c*d^3*e + 66*a*c*d^2*e
^2))/(384*d^2*e^3) - (x^5*(385*a^2*e^4 - 511*c^2*d^4 + 33*b^2*d^2*e^2 + 110
*a*b*d*e^3 - 146*b*c*d^3*e + 66*a*c*d^2*e^2))/(384*d^3*e^2))/(d^4 + e^4*x^8
 + 4*d^3*e*x^2 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4)
```

3.261 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2} \, dx=\frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-e(bd+ae)\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1171, 396, 211}

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2} \, dx=-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)}-\frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\
&= \frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\int \frac{1}{d+ex^2}\,dx}{2de^2}\\
&= \frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx=\frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)}-\frac{(3cd^2-bde-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
default	$\frac{cx}{e^{2}}+\frac{\frac{\left(a\,e^{2}-bde+c\,d^{2}\right)x}{2d\left(e\,x^{2}+d\right)}+\frac{\left(a\,e^{2}+bde-3c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^{2}}$
risch	$\frac{cx}{e^{2}}+\frac{\left(a\,e^{2}-bde+c\,d^{2}\right)x}{2d\,e^{2}\left(e\,x^{2}+d\right)}-\frac{\ln\left(ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}-\frac{\ln\left(ex+\sqrt{-ed}\right)b}{4e\sqrt{-ed}}+\frac{3d\ln\left(ex+\sqrt{-ed}\right)c}{4e^{2}\sqrt{-ed}}+\frac{\ln\left(-ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}+\frac{\ln\left(-ex+\sqrt{-ed}\right)b}{4e\sqrt{-ed}}$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*d
^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx$$

$$=\left[\frac{4\,cd^2e^2x^3+\left(3\,cd^3-bd^2e-ade^2+\left(3\,cd^2e-bde^2-ae^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right)+2\left(3\,cd^3e-bd^2e\right.}{4\left(d^2e^4x^2+d^3e^3\right)}\right.$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(75) = 150.

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{cx}{e^2}+\frac{x(ae^2-bde+cd^2)}{2d^2e^2+2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2+bde-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2+bde-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{cx}{e^2} - \frac{\left(3\,cd^2-bde-ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)de^2}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*
e^2) + 1/2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx=\frac{c\,x}{e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2+b\,d\,e+a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}}+\frac{x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.262 $\int \frac{a+x^2\left(b+cx^2\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx=\frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1828, 396, 211}

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx=-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3cd^2-e(ae+bd))}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\,(d+ex^2)}+\frac{cx}{e^2}$$

```
[In] Int[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\int\frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\
&= \frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-e(bd+ae)\right)\int\frac{1}{d+ex^2}\,dx}{2de^2}\\
&= \frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$
\int\frac{a+x^2(b+cx^2)}{\left(d+ex^2\right)^2}\,dx=\frac{cx}{e^2}+\frac{\left(cd^2-bde+ae^2\right)x}{2de^2\left(d+ex^2\right)}-\frac{\left(3cd^2-bde-ae^2\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}
$$

```
[In] Integrate[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
default	$\frac{cx}{e^2}+\frac{\frac{\left(a\,e^2-bde+c\,d^2\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a\,e^2+bde-3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^2}$
risch	$\frac{cx}{e^2}+\frac{(a\,e^2-bde+c\,d^2)x}{2d\,e^2(e\,x^2+d)}-\frac{\ln(ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}-\frac{\ln(ex+\sqrt{-ed})b}{4e\sqrt{-ed}}+\frac{3d\ln(ex+\sqrt{-ed})c}{4e^2\sqrt{-ed}}+\frac{\ln(-ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}+\frac{\ln(-ex+\sqrt{-ed})}{4e\sqrt{-ed}}$

```
[In] int((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*d
^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \frac{a+x^2(b+cx^2)}{\left(d+ex^2\right)^2}\,dx$$

$$=\left[\frac{4\,cd^2e^2x^3+(3\,cd^3-bd^2e-ade^2+(3\,cd^2e-bde^2-ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)+2\,(3\,cd^3e-bd^2}{4\,(d^2e^4x^2+d^3e^3)}\right.$$

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(75) = 150$.

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} + \frac{x(ae^2-bde+cd^2)}{2d^2e^2+2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

```
[In] integrate((a+x**2*(c*x**2+b))/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} - \frac{(3\,cd^2-bde-ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)de^2}$$

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*
e^2) + 1/2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx = \frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2+b\,d\,e+a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

```
[In] int((a + x^2*(b + c*x^2))/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.263 $\int \frac{\left(d+ex^2\right)^4}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 459

$$\int \frac{\left(d+ex^2\right)^4}{a+bx^2+cx^4} \, dx = \frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c}$$
$$+\frac{\left(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)+\frac{2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)}{\sqrt{b^2-4ac}}\right)\mathrm{a}}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)-\frac{2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)}{\sqrt{b^2-4ac}}\right)\mathrm{a}}{\sqrt{2}c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] e^2*(6*c^2*d^2+b^2*e^2-c*e*(a*e+4*b*d))*x/c^3+1/3*e^3*(-b*e+4*c*d)*x^3/c^2+
1/5*e^4*x^5/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e
*(-b*e+2*c*d)*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))+(2*c^4*d^4+b^4*e^4-4*b^2*
c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*
d^2))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e*(-b*e+2*c*d)*(2*c
^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))+(-2*c^4*d^4-b^4*e^4+4*b^2*c*e^3*(a*e+b*d)+4
*c^3*d^2*e*(3*a*e+b*d)-2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))/(-4*a*c+b^2
)^(1/2))/c^(7/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1184, 1180, 211}

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx$$

$$= \frac{\arctan\Big(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\Big)\Big(\frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}+e(2cd-be)(-2ce(ae+}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\arctan\Big(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\Big)\Big(e(2cd-be)(-2ce(ae+bd)+b^2e^2+2c^2d^2)-\frac{2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-}{\sqrt{b^2-4ac}}}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$+\frac{e^2x(-ce(ae+4bd)+b^2e^2+6c^2d^2)}{c^3}+\frac{e^3x^3(4cd-be)}{3c^2}+\frac{e^4x^5}{5c}$$

```
[In] Int[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2
*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*
d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2
*e^2 - 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e)
- 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))}{c^3}+\frac{e^3(4cd-be)x^2}{c^2}+\frac{e^4x^4}{c} + \frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace^3(4bd+ae)+e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)x^2}{c^3\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c} + \frac{\int \frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace^3(4bd+ae)+e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)x^2}{a+bx^2+cx^4}\,dx}{c^3}$$

$$= \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c} + \frac{\Big(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right) - \frac{2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+}{\sqrt{b^2-4ac}}}{2c^3} + \frac{\Big(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right) + \frac{2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+}{\sqrt{b^2-4ac}}}{2c^3}$$

$$= \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c} + \frac{\Big(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right) + \frac{2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+}{\sqrt{b^2-4ac}}}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\Big(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right) - \frac{2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+}{\sqrt{b^2-4ac}}}{\sqrt{2}c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c} + \frac{\left(2c^4d^4+b^3\left(b-\sqrt{b^2-4ac}\right)e^4+4c^3d^2e\left(-bd+\sqrt{b^2-4ac}d-3ae\right)+2bce^3\left(-2b^2d+2b\sqrt{b^2-4ac}d-2a\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-}} - \frac{\left(2c^4d^4+b^3\left(b+\sqrt{b^2-4ac}\right)e^4-4c^3d^2e\left(bd+\sqrt{b^2-4ac}d+3ae\right)-2bce^3\left(2b^2d+a\sqrt{b^2-4ac}e+2b\left(\sqrt{b^2}\right.\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4a}}$$

```
[In] Integrate[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((2*c^4*d^4 + b^3*(b - Sqrt[b^2 - 4*a*c])*
e^4 + 4*c^3*d^2*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*b*c*e^3*(-2*b^
2*d + 2*b*Sqrt[b^2 - 4*a*c]*d - 2*a*b*e + a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^
2*(3*b^2*d^2 - 3*b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*e*(-2*Sqrt[b^2 - 4*a
*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4*d^4
+ b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 4*c^3*d^2*e*(b*d + Sqrt[b^2 - 4*a*c]*d
+ 3*a*e) - 2*b*c*e^3*(2*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 2*b*(Sqrt[b^2 - 4*a
*c]*d + a*e)) + 2*c^2*e^2*(3*b^2*d^2 + a*e*(2*Sqrt[b^2 - 4*a*c]*d + a*e) +
3*b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.49

method	result
risch	$\frac{e^4x^5}{5c}-\frac{e^4b\,x^3}{3c^2}+\frac{4d\,e^3x^3}{3c}-\frac{e^4ax}{c^2}+\frac{e^4b^2x}{c^3}-\frac{4e^3bdx}{c^2}+\frac{6e^2d^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{(e(2abc\,e^3-4a\,c^2d\,e^2-b^3e^3-}{}}{}$
default	$-\frac{e^2(-\frac{1}{5}e^2x^5c^2+\frac{1}{3}bc\,e^2x^3-\frac{4}{3}c^2de\,x^3+e^2acx-b^2e^2x+4bcdex-6c^2d^2x)}{c^3}+\frac{\frac{(2abc\,e^4\sqrt{-4ac+b^2}-4a\,c^2d\,e^3\sqrt{-4ac+b^2}-b^3e^4\sqrt{-4ac+b^2}+}{}}{}$

```
[In] int((e*x^2+d)^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e^4*x^5/c-1/3*e^4/c^2*b*x^3+4/3*d*e^3*x^3/c-e^4/c^2*a*x+e^4/c^3*b^2*x-4
*e^3/c^2*b*d*x+6*e^2/c*d^2*x+1/2/c^3*sum((e*(2*a*b*c*e^3-4*a*c^2*d*e^2-b^3*
e^3+4*b^2*c*d*e^2-6*b*c^2*d^2*e+4*c^3*d^3)*_R^2+a^2*c*e^4-a*b^2*e^4+4*a*b*c
*d*e^3-6*a*c^2*d^2*e^2+c^3*d^4)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_
Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16218 vs. $2(421) = 842$.

Time = 270.80 (sec) , antiderivative size = 16218, normalized size of antiderivative = 35.33

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^4}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*e^4*x^5 + 5*(4*c^2*d*e^3 - b*c*e^4)*x^3 + 15*(6*c^2*d^2*e^2 - 4
*b*c*d*e^3 + (b^2 - a*c)*e^4)*x)/c^3 + integrate((c^3*d^4 - 6*a*c^2*d^2*e^2
 + 4*a*b*c*d*e^3 - (a*b^2 - a^2*c)*e^4 + (4*c^3*d^3*e - 6*b*c^2*d^2*e^2 + 4
*(b^2*c - a*c^2)*d*e^3 - (b^3 - 2*a*b*c)*e^4)*x^2)/(c*x^4 + b*x^2 + a), x)/
c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9306 vs. 2(421) = 842.

Time = 1.30 (sec) , antiderivative size = 9306, normalized size of antiderivative = 20.27

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(4*(2*b^4*c^5 - 16*a*b^2*c^6 + 32*a^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6)*c^2*d^3*e - 6*(2
*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d^2*e^2 + 4
*(2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*
(b^2 - 4*a*c)*a^2*c^5)*c^2*d*e^3 - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c
^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sq
```

```
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5
*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*c^2*e^4 + 2
*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c^6 - 2*b^4*c^6 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^7 + sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^2*c^7 + 16*a*b^2*c^7 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*c^8 - 32*a^2*c^8 + 2*(b^2 - 4*a*c)*b^2*c^6 - 8*(b^2 - 4*a*c)*a*c^7
)*d^4*abs(c) - 12*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*a*b^4*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*c^6 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 + 16*a^2*b^2*c^6 - 4*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^7 - 32*a^3*c^7 + 2*(b^2 - 4*a*c
)*a*b^2*c^5 - 8*(b^2 - 4*a*c)*a^2*c^6)*d^2*e^2*abs(c) + 8*(sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 2*a*
b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^3*c^5 + 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^6 - 32*a^3*b*c^6 + 2*(b^2 - 4*a*c)*a*b^3*c^4 - 8*(b^2 - 4*a*
c)*a^2*b*c^5)*d*e^3*abs(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^6*c^2 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 2*a*b^6*c^3 + 24*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 1
8*a^2*b^4*c^4 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 5*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^5 - 48*a^3*b^2*c^5 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*c^6 + 32*a^4*c^6 + 2*(b^2 - 4*a*c)*a*b^4*c^3 - 10*(b^2 - 4
*a*c)*a^2*b^2*c^4 + 8*(b^2 - 4*a*c)*a^3*c^5)*e^4*abs(c) + 2*(2*b^3*c^8 - 8*
a*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^6
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^7 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^7 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^8 - 2*(b^2 - 4*a*c)*
b*c^8)*d^4 - 4*(2*b^4*c^7 - 8*a*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^2*c^7 - 2*(b^2 - 4*a*c)*b^2*c^7)*d^3*e + 6*(2*b^5*c^6 - 12*a*b^
3*c^7 + 16*a^2*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^5*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
```

```
4*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c
^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^7 - 2*(b^2 - 4
*a*c)*b^3*c^6 + 4*(b^2 - 4*a*c)*a*b*c^7)*d^2*e^2 - 4*(2*b^6*c^5 - 14*a*b^4*
c^6 + 24*a^2*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^6*c^3 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
5*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^5 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 3*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*(b^
2 - 4*a*c)*b^4*c^5 + 6*(b^2 - 4*a*c)*a*b^2*c^6)*d*e^3 + (2*b^7*c^4 - 16*a*b
^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^5*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^6 - 2*(b^2 - 4*a*c)*b^5*c^4 + 8*(b^2 - 4*a*c)*a*b^3*c^5 - 4*(b^2 - 4*a*c)
*a^2*b*c^6)*e^4)*arctan(2*sqrt(1/2)*x/sqrt((b*c^5 + sqrt(b^2*c^10 - 4*a*c^1
1))/c^6))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*a^3*c^7 + 8*a^2*b*
c^7 + a*b^2*c^7 - 4*a^2*c^8)*c^2) - 1/8*(4*(2*b^4*c^5 - 16*a*b^2*c^6 + 32*a
^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3
+ 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(
b^2 - 4*a*c)*a*c^6)*c^2*d^3*e - 6*(2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b
^2 - 4*a*c)*a*b*c^5)*c^2*d^2*e^2 + 4*(2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2
*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
```

```
4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2
 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3
 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c
^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*d*e^3 - (2*b
^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*
(b^2 - 4*a*c)*a^2*b*c^4)*c^2*e^4 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 2*b^4*c^6 + 16*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^7 - 16*a*b^2*c^7 -
 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^8 + 32*a^2*c^8 - 2*(b^2 - 4*
a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7)*d^4*abs(c) + 12*(sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*a*b^4*
c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^6 - 16*a^2*b^2*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*c^7 + 32*a^3*c^7 - 2*(b^2 - 4*a*c)*a*b^2*c^5 + 8*(b^2 - 4*a*c)*a^2*c^6)*
d^2*e^2*abs(c) - 8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 -
 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^
2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*a^2*b*c^5)*d*e^3*abs(c) + 2*(sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c
^3 + 2*a*b^6*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 +
 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 18*a^2*b^4*c^4 - 16*sqrt(2)*sqrt(b*c - s
```

```
qrt(b^2 - 4*a*c)*c)*a^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b*c^5 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 48*a^3*b^2
*c^5 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 - 32*a^4*c^6 - 2*(
b^2 - 4*a*c)*a*b^4*c^3 + 10*(b^2 - 4*a*c)*a^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3
*c^5)*e^4*abs(c) + 2*(2*b^3*c^8 - 8*a*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b*c^8 - 2*(b^2 - 4*a*c)*b*c^8)*d^4 - 4*(2*b^4*c^7 - 8*a*b^2*c^8
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^7 - 2*(b^2 - 4*a*c)*b^2
*c^7)*d^3*e + 6*(2*b^5*c^6 - 12*a*b^3*c^7 + 16*a^2*b*c^8 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b*c^7 - 2*(b^2 - 4*a*c)*b^3*c^6 + 4*(b^2 - 4*a*c)*a*b*c^7
)*d^2*e^2 - 4*(2*b^6*c^5 - 14*a*b^4*c^6 + 24*a^2*b^2*c^7 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^3 + 7*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*(b^2 - 4*a*c)*b^4*c^5 + 6*(b^2 - 4*a*c)*a*
b^2*c^6)*d*e^3 + (2*b^7*c^4 - 16*a*b^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^5*c^4 + 8*(b^2
- 4*a*c)*a*b^3*c^5 - 4*(b^2 - 4*a*c)*a^2*b*c^6)*e^4)*arctan(2*sqrt(1/2)*x/s
qrt((b*c^5 - sqrt(b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^5 - 8*a^2*b^2*c^6 -
 2*a*b^3*c^6 + 16*a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*c^2) + 1/1
5*(3*c^4*e^4*x^5 + 20*c^4*d*e^3*x^3 - 5*b*c^3*e^4*x^3 + 90*c^4*d^2*e^2*x -
60*b*c^3*d*e^3*x + 15*b^2*c^2*e^4*x - 15*a*c^3*e^4*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 10.92 (sec) , antiderivative size = 29551, normalized size of antiderivative = 64.38

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^4/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*((b*((b*e^4)/c^2 - (4*d*e^3)/c))/c - (a*e^4)/c^2 + (6*d^2*e^2)/c) - x^3*(
(b*e^4)/(3*c^2) - (4*d*e^3)/(3*c)) + atan(((((16*a*c^8*d^4 + 16*a^3*c^6*e^4
 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^
2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 - (
2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c
- b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 2
8*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8
- 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d
^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*
c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^
5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^
2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2
)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7
*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a
*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c
^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d
^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*
e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2
*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2
)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*
d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2
))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b
^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*
a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8
+ a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*
d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^
2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b
^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a
^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c
^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*
(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70
*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*
b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*
```

```
c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) +
24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c
 - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4
*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^
2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e
^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) - (2*x*(b^8*e^8 + 2
*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b
^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2
 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2
*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*
d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 2
80*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*
a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8
 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) -
11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 +
 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(
1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*
c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*
e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d
^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^
4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2
) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^
3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*
d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e
^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6
+ 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*
b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)
^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^
3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1
/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7
*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a
^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8
*a^2*b^2*c^8)))^(1/2)*1i - (((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4
 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5
*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 + (2*x*(4*b^3*c^7 -
 16*a*b*c^8)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2)
- a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8
+ 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3
*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4
*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c
 - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*
a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 +
700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*
```

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a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c
*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3
 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112
*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3
*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d
^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(
4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*
a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c
 - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*
c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*(-(a*b^9
*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c
 - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e -
64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-
(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*
c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 33
6*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 -
 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e
^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*
a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^
3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e
^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 +
80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*
a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e
^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2)
 + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(
4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32
*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^
2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c
^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4
*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140
*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5
*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b
^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*
b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4
*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^
5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*
a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8
 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*
e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c
^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*
b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^
3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^
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3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c -
 b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7
*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 -
56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b
^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c
^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^
4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a
*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c -
 b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*
c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)
^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e
^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^
(1/2)*1i)/((((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e
^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*
b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(
a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4
*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7
*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e
^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4
*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2)
 + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*
e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*
d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*
(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c -
b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*
d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e
^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 -
 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^
3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^
(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2)
 - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c
 - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d
^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2)
- 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7
 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)
^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^
8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^
5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3
*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*
d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1
/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*
```

b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^
3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d
*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^
6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*
a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^
2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*
d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^
(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e
^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84
*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 -
 8*a^2*b^2*c^8)))^(1/2) - (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*
c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4
 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^
4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c
^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 +
56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3
*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*
c^3*d*e^7))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1
/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*
e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3
*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448
*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4
*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 +
448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^
5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) -
 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b
^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5
*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 -
 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264
*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c
^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4
*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) -
 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4
*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*
b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) - (2*(a^4*
b^3*e^12 - 4*c^7*d^11*e + b^7*d^4*e^8 - 4*a*b^6*d^3*e^9 - 4*a^3*b^4*d*e^11
- 12*a*c^6*d^9*e^3 + 4*a^5*c^2*d*e^11 + 22*b*c^6*d^10*e^2 - 8*b^6*c*d^5*e^7
 + 6*a^2*b^5*d^2*e^10 - 8*a^2*c^5*d^7*e^5 + 8*a^3*c^4*d^5*e^7 + 12*a^4*c^3*
d^3*e^9 - 52*b^2*c^5*d^9*e^3 + 69*b^3*c^4*d^8*e^4 - 56*b^4*c^3*d^7*e^5 + 28
*b^5*c^2*d^6*e^6 - 2*a^5*b*c*e^12 - 48*a^2*b^2*c^3*d^5*e^7 + 50*a^2*b^3*c^2
*d^4*e^8 + 8*a^3*b^2*c^2*d^3*e^9 + 54*a*b*c^5*d^8*e^4 + 26*a*b^5*c*d^4*e^8
+ 4*a^4*b^2*c*d*e^11 - 104*a*b^2*c^4*d^7*e^5 + 112*a*b^3*c^3*d^6*e^6 - 72*a
*b^4*c^2*d^5*e^7 + 28*a^2*b*c^4*d^6*e^6 - 28*a^2*b^4*c*d^3*e^9 - 20*a^3*b*c

```
^3*d^4*e^8 + 8*a^3*b^3*c*d^2*e^10 - 18*a^4*b*c^2*d^2*e^10))/c^5 + (((16*a*c
^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*
e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b
^2*c^6*d^2*e^2)/c^5 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(a*b^9*e^8 + b^3*c^7*
d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2)
 - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^
7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3
)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b
^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d
^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^
3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5
*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(
1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a
*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c
^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*
d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e
^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56
*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b
^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3
*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)
^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*
e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8
4*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7
 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*
c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 +
 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^
8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7
*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^
2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*
c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*
b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b
^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d
^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56
*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7
*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6
*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*
d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b
^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b
^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^
4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3
)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1
/2) + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2
*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^
```

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6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c
^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d
*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 +
 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a
*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*(-(
a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4
*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7
*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e
^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4
*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2)
 + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*
e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*
d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*
(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c -
b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*
d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e
^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 -
 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^
3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^
(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2)
 - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c
 - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2)))*(-(a*b^9*e^8 + b^3*c^7*d^8 +
 c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11
*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 4
2*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/
2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*
d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^
3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2
*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*
e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2)
- 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*
c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^
3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7
 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 +
304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*
c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3
)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*
e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2
) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2
*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a
^2*b^2*c^8)))^(1/2)*2i + atan(((((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7
```

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*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3
*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 - (2*x*(4*b^3*c
^7 - 16*a*b*c^8)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e
^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e
^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*
c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*
a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*
a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 4
48*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5
 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) -
28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^
4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*
e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 +
112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*
a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^
6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*
(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) -
28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*
a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b
^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*((c^7
*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*
c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e
+ 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*
(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*
b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) -
336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5
 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2
*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(
4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2
)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4
*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2
- 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 56
0*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d
*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/
2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(
-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) -
32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c -
b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3
*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) - (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a
^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 1
40*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d
^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a
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*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*
a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d
^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*
e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3
*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^
8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2
*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*
c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3
*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b
^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a
^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c
 - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^
7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 +
 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*
b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*
c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c
^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*
a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c
 - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b
*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2
)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*
e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))
^(1/2)*1i - (((16*a*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*
e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2
*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*((
c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4
*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7
*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e
^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4
*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2)
 - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*
e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*
d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*
(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c -
b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*
d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e
^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 +
 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^
3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^
(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2)
 - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c
 - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
```

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a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*((c^7*d^8*(-(4*a*c - b^2
)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) +
 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7
 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^
(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8
*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5
*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*
d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d
^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b
^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3
*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*
e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6
 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a
*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2
)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d
^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(
1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^
7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*
a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 -
8*a^2*b^2*c^8)))^(1/2) + (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c
^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4
- 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4
*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^
7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 5
6*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*
c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c
^3*d*e^7))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^
8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^
8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c
^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a
^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a
*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 44
8*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5
- 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 2
8*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4
*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e
^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 1
12*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a
^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6
*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(
-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 2
8*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a
*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^
```

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5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2)*1i)/((((16*a
*c^8*d^4 + 16*a^3*c^6*e^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^
5*e^4 - 96*a^2*c^7*d^2*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a
*b^2*c^6*d^2*e^2)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*((c^7*d^8*(-(4*a*c -
b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2
) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e
^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^
3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*
b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*
d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c
^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^
5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^
(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*
a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*
c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2
*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*
e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5
6*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c -
b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^
3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3
)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d
*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) +
84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7
 - 8*a^2*b^2*c^8)))^(1/2))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^
7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 -
 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^
8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7
*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^
2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*
c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*
b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b
^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d
^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56
*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7
*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6
*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*
d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b
^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b
^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^
4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3
)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1
/2) - (2*x*(b^8*e^8 + 2*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2
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*b^4*c^2*e^8 - 16*a^3*b^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^
6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c
^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d
*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 +
56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a
*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*((c
^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*
a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*
e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^
8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*
a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2)
- 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e
^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d
^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(
-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b
^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d
^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^
2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 +
560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3
*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(
1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6
*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2)
- 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c
- b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) - (2*(a^4*b^3*e^12 - 4*c^7*d^11
*e + b^7*d^4*e^8 - 4*a*b^6*d^3*e^9 - 4*a^3*b^4*d*e^11 - 12*a*c^6*d^9*e^3 +
4*a^5*c^2*d*e^11 + 22*b*c^6*d^10*e^2 - 8*b^6*c*d^5*e^7 + 6*a^2*b^5*d^2*e^10
- 8*a^2*c^5*d^7*e^5 + 8*a^3*c^4*d^5*e^7 + 12*a^4*c^3*d^3*e^9 - 52*b^2*c^5*
d^9*e^3 + 69*b^3*c^4*d^8*e^4 - 56*b^4*c^3*d^7*e^5 + 28*b^5*c^2*d^6*e^6 - 2*
a^5*b*c*e^12 - 48*a^2*b^2*c^3*d^5*e^7 + 50*a^2*b^3*c^2*d^4*e^8 + 8*a^3*b^2*
c^2*d^3*e^9 + 54*a*b*c^5*d^8*e^4 + 26*a*b^5*c*d^4*e^8 + 4*a^4*b^2*c*d*e^11
- 104*a*b^2*c^4*d^7*e^5 + 112*a*b^3*c^3*d^6*e^6 - 72*a*b^4*c^2*d^5*e^7 + 28
*a^2*b*c^4*d^6*e^6 - 28*a^2*b^4*c*d^3*e^9 - 20*a^3*b*c^3*d^4*e^8 + 8*a^3*b^
3*c*d^2*e^10 - 18*a^4*b*c^2*d^2*e^10))/c^5 + (((16*a*c^8*d^4 + 16*a^3*c^6*e
^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*
e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 +
(2*x*(4*b^3*c^7 - 16*a*b*c^8)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7
*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 -
28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8
+ 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*
d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2
*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c
^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b
^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^

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2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^
7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*
a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*
c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*
d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d
*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^
2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^
2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4
*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/
2))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b
^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*
a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8
 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*
d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^
2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b
^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a
^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c
^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*
(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70
*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*
b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*
c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) +
24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c
 - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4
*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^
2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e
^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) + (2*x*(b^8*e^8 + 2
*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b
^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2
 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2
*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*
d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 2
80*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*
a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*((c^7*d^8*(-(4*a*c - b^2)^
3)^(1/2) - b^3*c^7*d^8 - a*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 1
1*a^2*b^7*c*e^8 - 28*a^5*b*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 -
42*a^3*b^5*c^2*e^8 + 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1
/2) + 448*a^3*c^7*d^5*e^3 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c
*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e
^3 + 490*a^2*b^3*c^5*d^4*e^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^
```

```
2*e^6 + 1008*a^3*b^2*c^5*d^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4
*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2)
 + 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3
*c^6*d^6*e^2 + 56*a*b^4*c^5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d
^3*e^5 - 28*a*b^7*c^2*d^2*e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^
7 - 840*a^3*b*c^6*d^4*e^4 + 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 -
 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b
*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^
3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3
*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/
2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*
(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^
2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*
a^2*b^2*c^8)))^(1/2)))*((c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^7*d^8 - a
*b^9*e^8 - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) + 11*a^2*b^7*c*e^8 - 28*a^5*b
*c^4*e^8 - 64*a^2*c^8*d^7*e + 64*a^5*c^5*d*e^7 - 42*a^3*b^5*c^2*e^8 + 63*a^
4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) + 448*a^3*c^7*d^5*e^3
 - 448*a^4*c^6*d^3*e^5 + 4*a*b*c^8*d^8 + 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8
*(-(4*a*c - b^2)^3)^(1/2) - 336*a^2*b^2*c^6*d^5*e^3 + 490*a^2*b^3*c^5*d^4*e
^4 - 448*a^2*b^4*c^4*d^3*e^5 + 252*a^2*b^5*c^3*d^2*e^6 + 1008*a^3*b^2*c^5*d
^3*e^5 - 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1
/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^2*c^7*d^7*e + 5*
a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^3*c^6*d^6*e^2 + 56*a*b^4*c^
5*d^5*e^3 - 70*a*b^5*c^4*d^4*e^4 + 56*a*b^6*c^3*d^3*e^5 - 28*a*b^7*c^2*d^2*
e^6 + 112*a^2*b*c^7*d^6*e^2 - 80*a^2*b^6*c^2*d*e^7 - 840*a^3*b*c^6*d^4*e^4
+ 264*a^3*b^4*c^3*d*e^7 + 560*a^4*b*c^5*d^2*e^6 - 304*a^4*b^2*c^4*d*e^7 - 2
8*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^
2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^
4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1
/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5
*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) +
 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2)*2i +
(e^4*x^5)/(5*c)
```

3.264 $\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 316

$$\begin{aligned}\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx &= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c} \\ &\quad+\frac{\left(e(3c^2d^2+b^2e^2-ce(3bd+ae))+\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad+\frac{\left(e(3c^2d^2+b^2e^2-ce(3bd+ae))-\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

```
[Out] e^2*(-b*e+3*c*d)*x/c^2+1/3*e^3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*(e*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))+(-b*e+2*c*d)*(c
^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*(e*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))-(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-
c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1184, 1180, 211}

$$\begin{aligned}\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx &= \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2)+\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2)-\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\\ &\quad+\frac{e^2x(3cd-be)}{c^2}+\frac{e^3x^3}{3c}\end{aligned}$$

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^2*e^2 - c*
e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 -
 c*e*(3*b*d + a*e)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)
))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2(3cd-be)}{c^2}+\frac{e^3x^2}{c} + \frac{c^2d^3-3acde^2+abe^3+e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)x^2}{c^2\left(a+bx^2+cx^4\right)}\right)dx \\
&= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\int \frac{c^2d^3-3acde^2+abe^3+e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)x^2}{a+bx^2+cx^4}\,dx}{c^2} \\
&= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c} + \frac{\left(e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)-\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2} \\
&\quad + \frac{\left(e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2} \\
&= \frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c} + \frac{\left(e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{\left(e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)-\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.27

$$\int \frac{\left(d+ex^2\right)^3}{a+bx^2+cx^4}\,dx$$

$$= \frac{6\sqrt{c}e^2(3cd-be)x+2c^{3/2}e^3x^3+\frac{3\sqrt{2}\left(2c^3d^3+b^2\left(-b+\sqrt{b^2-4ac}\right)e^3+3c^2de\left(-bd+\sqrt{b^2-4ac}d-2ae\right)+ce^2\left(3b^2d-3b\sqrt{b^2-4ac}d+3abe-\right.\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*e^2*(3*c*d - b*e)*x + 2*c^(3/2)*e^3*x^3 + (3*Sqrt[2]*(2*c^3*d^3
+ b^2*(-b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*
d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^
2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(-2*c^3*d^3 + b^2
*(b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e
)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.41

method	result
risch	$\frac{e^{3} x^{3}}{3 c}-\frac{e^{3} b x}{c^{2}}+\frac{3 d \,e^{2} x}{c}+\frac{\sum_{\_R =\operatorname{RootOf}\left(c \,\_Z^{4}+\_Z^{2} b +a \right)}\frac{\left(e \left(-e^{2} a c +b^{2} e^{2}-3 b c d e +3 c^{2} d^{2}\right) \_R^{2}+e^{3} a b -3 a c d \,e^{2}+c^{2} d^{3}\right) \ln \left(x -\_R \right)}{2 c \,\_R^{3}+\_R b}}{2 c^{2}}$
default	$-\frac{e^{2} \left(-\frac{1}{3} c \,x^{3} e +b e x -3 c d x \right)}{c^{2}}+\frac{\frac{\left(-e^{3} a c \sqrt{-4 a c +b^{2}}+b^{2} e^{3} \sqrt{-4 a c +b^{2}}-3 d \,e^{2} b c \sqrt{-4 a c +b^{2}}+3 d^{2} e \,c^{2} \sqrt{-4 a c +b^{2}}-3 a b c \,e^{3}+6 a \,c^{2} d \,e^{2}+b^{3} e^{3}-3 b^{2} c d\right.}{2 c \sqrt{-4 a c +b^{2}}\, \sqrt{\left(b +\sqrt{-4 a c +b^{2}}\right) c}}}{}$

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^3*x^3/c-e^3/c^2*b*x+3*d*e^2*x/c+1/2/c^2*sum((e*(-a*c*e^2+b^2*e^2-3*b*
c*d*e+3*c^2*d^2)*_R^2+e^3*a*b-3*a*c*d*e^2+c^2*d^3)/(2*_R^3*c+_R*b)*ln(x-_R)
,_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9584 vs. 2(280) = 560.

Time = 44.92 (sec) , antiderivative size = 9584, normalized size of antiderivative = 30.33

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx=\int \frac{(ex^2+d)^3}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*e^3*x^3 + 3*(3*c*d*e^2 - b*e^3)*x)/c^2 - integrate(-(c^2*d^3 - 3*a*c
*d*e^2 + a*b*e^3 + (3*c^2*d^2*e - 3*b*c*d*e^2 + (b^2 - a*c)*e^3)*x^2)/(c*x^
4 + b*x^2 + a), x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6418 vs. 2(280) = 560.

Time = 1.14 (sec) , antiderivative size = 6418, normalized size of antiderivative = 20.31

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(3*(2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d^2*e - 3*(2
*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2*d*e^2 + (2*b^
6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
```

```
 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c
)*a^2*c^4)*c^2*e^3 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^3*c^5 - 2*b^4*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*c^6 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 + 16*a*b^2*c^6 - 4*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^7 - 32*a^2*c^7 + 2*(b^2 - 4*a*c)*b^2*c^5 -
 8*(b^2 - 4*a*c)*a*c^6)*d^3*abs(c) - 6*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 +
16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3
*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*d*e^2*abs(c) +
2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*
c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*e^3*abs(c) + 2*(2*b^3*c^7 - 8*a*b*c^8 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^7 - 2*(b^2 - 4*a*c)*b*c^7)*d^3 -
 3*(2*b^4*c^6 - 8*a*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d^2*e + 3*(2*b^5*c^5 - 12*a*b^3*c^6 + 16*a
^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^
3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^
5 + 4*(b^2 - 4*a*c)*a*b*c^6)*d*e^2 - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2
*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 +
7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(
```

```
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c
^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*e^3)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqr
t(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*
a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) - 1/8*(3*(2*b^4*c^4 - 1
6*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*
a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d^2*e - 3*(2*b^5*c^3 - 16*a*b^3*c
^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2
 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 -
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c
)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2*d*e^2 + (2*b^6*c^2 - 18*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 +
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)
*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2*e^3 -
2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3*c^5 + 2*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 8
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^2*c^6 - 16*a*b^2*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*c^7 + 32*a^2*c^7 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^
6)*d^3*abs(c) + 6*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sq
```

```
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c
)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*d*e^2*abs(c) - 2*(sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^
5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)
*a^2*b*c^4)*e^3*abs(c) + 2*(2*b^3*c^7 - 8*a*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b*c^7 - 2*(b^2 - 4*a*c)*b*c^7)*d^3 - 3*(2*b^4*c^6 - 8*a*b
^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 2*(b^2 - 4*a*
c)*b^2*c^6)*d^2*e + 3*(2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a
*b*c^6)*d*e^2 - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*
a*b^2*c^5)*e^3)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))
/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6
 + a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/3*(c^2*e^3*x^3 + 9*c^2*d*e^2*x - 3*b*c*e
^3*x)/c^3
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 17954, normalized size of antiderivative = 56.82

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^3/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3
- 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^6
*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6
*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2
)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^
2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a
^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*d
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/
2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*a
*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^
3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^
6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^
2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a
^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c
- b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(
1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*
a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*
d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) -
(2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^
2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b
^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4
*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))
/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4
*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2
*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c -
 b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 12
0*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a
*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3
```

```
)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^
4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 1
08*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c
^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)
^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2)
*1i - (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3
 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^6
*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6
*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2
)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^
2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a
^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*d
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/
2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*a
*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^
3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^
6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^
2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a
^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c
 - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(
1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*
a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*
d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) +
(2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^
2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b
^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4
*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))
/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4
*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2
*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c -
 b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 12
0*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a
*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3
)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^
4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 1
08*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c
```

```
^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)
^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2)
*1i)/((2*(3*c^5*d^8*e - a^4*c*e^9 + a^3*b^2*e^9 - b^5*d^3*e^6 + 3*a*b^4*d^2
*e^7 - 3*a^2*b^3*d*e^8 + 8*a*c^4*d^6*e^3 - 12*b*c^4*d^7*e^2 + 6*b^4*c*d^4*e
^5 + 6*a^2*c^3*d^4*e^5 + 19*b^2*c^3*d^6*e^3 - 15*b^3*c^2*d^5*e^4 - 24*a*b*c
^3*d^5*e^4 - 14*a*b^3*c*d^3*e^6 + 27*a*b^2*c^2*d^4*e^5 - 12*a^2*b*c^2*d^3*e
^6 + 9*a^2*b^2*c*d^2*e^7))/c^3 + (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*
c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 - (
2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20
*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 +
 a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6
 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15
*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*
c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^
3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 18
0*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d
*e^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 -
 8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 2
0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6
+ a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^
6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 1
5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2
*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e
^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1
80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
d*e^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1
/2) - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5
- 8*a^2*b^2*c^6)))^(1/2) - (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a
*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2
- 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e -
 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e
^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*
a*c - b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6
- 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e
^6 + a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6
*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4
- 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*
b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^
3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5
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+ 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)
^(1/2) - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c
^5 - 8*a^2*b^2*c^6)))^(1/2) + (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3
*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 + (2*x
*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b
^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^
4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^
3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 -
6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^
2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e
^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 +
 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a
^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^
5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*
a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c -
b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a
^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a
^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 -
 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a
^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*
e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3
+ 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*
a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e
^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8
*a^2*b^2*c^6)))^(1/2) + (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a*c^
5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 2
0*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - 6*
b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^5
- 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 2
0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6
+ a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^
6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 1
5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2
*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e
^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1
80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
d*e^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1
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/2) - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5
- 8*a^2*b^2*c^6)))^(1/2)))*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c -
b^2)^3)^(1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a
^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a
^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 -
 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a
^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*
e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3
+ 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*
a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e
^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8
*a^2*b^2*c^6)))^(1/2)*2i + atan(((((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*
c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 - (
2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c
 - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20
*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 -
 a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6
 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 15
*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2*
c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^
3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 18
0*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c^2*d
*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/
2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 -
 8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c
 - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 2
0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6
- a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^
6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 1
5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2
*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e
^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1
80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c^2*
d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1
/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5
- 8*a^2*b^2*c^6)))^(1/2) - (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a
*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2
- 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e -
 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e
^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*
a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6
```

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- 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e
^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6
*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4
+ 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*
b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^
3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5
+ 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c
^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)
^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c
^5 - 8*a^2*b^2*c^6)))^(1/2)*1i - (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*
c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 + (
2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c
- b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20
*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 -
 a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6
 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 15
*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2*
c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^
3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 18
0*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c^2*d
*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/
2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 -
 8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*a*c
 - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 2
0*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6
- a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^
6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 + 1
5*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*b^2
*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e
^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 1
80*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c^2*
d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1
/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5
- 8*a^2*b^2*c^6)))^(1/2) + (2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a
*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2
- 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e -
 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e
^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 + c^5*d^6*(-(4*
a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6
- 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e
^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6
*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4
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+ 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e + 3*a^2*
b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^
3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5
+ 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*c^4*d^4*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b*c
^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)
^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c
^5 - 8*a^2*b^2*c^6)))^(1/2)*1i)/((2*(3*c^5*d^8*e - a^4*c*e^9 + a^3*b^2*e^9
- b^5*d^3*e^6 + 3*a*b^4*d^2*e^7 - 3*a^2*b^3*d*e^8 + 8*a*c^4*d^6*e^3 - 12*b*
c^4*d^7*e^2 + 6*b^4*c*d^4*e^5 + 6*a^2*c^3*d^4*e^5 + 19*b^2*c^3*d^6*e^3 - 15
*b^3*c^2*d^5*e^4 - 24*a*b*c^3*d^5*e^4 - 14*a*b^3*c*d^3*e^6 + 27*a*b^2*c^2*d
^4*e^5 - 12*a^2*b*c^2*d^3*e^6 + 9*a^2*b^2*c*d^2*e^7))/c^3 + (((16*a*c^6*d^3
 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 +
12*a*b^2*c^4*d*e^2)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*
c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(
1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d
*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*
c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 1
05*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a
*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^
4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2
+ 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15
*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2
*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3
*c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^
(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*
d*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3
*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 -
105*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*
a*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d
^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2
 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 1
5*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^
2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) - (2*x*(b^6*e^6 + 2*c^6
*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^
2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*
b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c
^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 +
b^3*c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^
3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c
^4*d*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*
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a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3
 - 105*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) -
12*a*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^
4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*
e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5
 - 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2
*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) + (((16*a*c^6*d^3 -
4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3 - 48*a^2*c^5*d*e^2 + 12*
a*b^2*c^4*d*e^2)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(a*b^7*e^6 + b^3*c^5
*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2
) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^
5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5
*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*
a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^
2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e
^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 4
8*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a*
c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e^
4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^
5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/
2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e
^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^
5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105
*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b
^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*
e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 +
48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a
*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e
^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) + (2*x*(b^6*e^6 + 2*c^6*d^
6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 30*a^2*c^4*d^2*e
^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b^4*c^2*d^2*e^4 - 6*a*b^4
*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4*d^3*e^3 + 30*a*b^3*c^2*
d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))/c^3)*(-(a*b^7*e^6 + b^3
*c^5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^
(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*
d*e^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3
*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 -
105*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*
a*b^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d
```

```
^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2
 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 1
5*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^
2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2)))*(-(a*b^7*e^6 + b^3*c^
5*d^6 + c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) - a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/
2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e
^5 + 25*a^3*b^3*c^2*e^6 - a^3*c^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^
5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105
*a^2*b^3*c^3*d^2*e^4 + 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b
^2*c^5*d^5*e + 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*
e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 +
48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 - 15*a
*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^2*c^2*d^2*e
^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^3*c*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2)*2i - x*((b*e^3)/c^2 - (3*d
*e^2)/c) + (e^3*x^3)/(3*c)
```

3.265 $\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 238

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \frac{e^2x}{c} + \frac{\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] e^2*x/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e*(-b*e
+2*c*d)+(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*(e*(-b*e+2*c*d)+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*
a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1184, 1180, 211}

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{e^2x}{c}$$

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e) - (2*c^2*
d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{c}+\frac{cd^2-ae^2+e(2cd-be)x^2}{c\left(a+bx^2+cx^4\right)}\right)dx \\
&= \frac{e^2x}{c}+\frac{\int \frac{cd^2-ae^2+e(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{e^2x}{c}+\frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&\quad +\frac{\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} \\
&= \frac{e^2x}{c}+\frac{\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad +\frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.13

$$\int \frac{\left(d+ex^2\right)^2}{a+bx^2+cx^4}\,dx$$

$$= \frac{2\sqrt{c}e^2x+\frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{}\right.\right.}{\sqrt{b^2-4ac}\sqrt{b}}}{2c^{3/2}}$$

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*e^2*x + (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*
c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4
*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.32

method	result
risch	$\frac{e^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(e(-be+2cd)\_R^2-a\,e^2+c\,d^2\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{e^2x}{c}+\frac{\left(-b\,e^2\sqrt{-4ac+b^2}+2dce\sqrt{-4ac+b^2}+2e^2ac-b^2e^2+2bcde-2c^2d^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-b\,e^2\sqrt{-4ac+b^2}+\right.}{}$

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*x/c+1/2/c*sum((e*(-b*e+2*c*d)*_R^2-a*e^2+c*d^2)/(2*_R^3*c+_R*b)*ln(x-_R
),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4690 vs. 2(204) = 408.

Time = 4.22 (sec) , antiderivative size = 4690, normalized size of antiderivative = 19.71

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^2*x - sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2
*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3
 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2
*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^
2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5
 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7
- (a^3*b^2 - a^4*c)*e^8)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*
c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c -
 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 +
 (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 -
 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c
^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a
*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*
(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2
*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^
2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 -
4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*
c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3
*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c +
a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) + sqrt(
1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c -
2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((
c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(
a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8
*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^
2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*
d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 +
 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^
2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*
e^8)*x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4
*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28
*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^
```

```
2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*
a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^
6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^
4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c
^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7
)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a
^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6
*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b
^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a
^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b
^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^3*
d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (
a*b^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d
^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c
^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*
c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^
7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d
^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4
+ 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2
*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x + sqrt(1/2)*(
(b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2
- 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 -
4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6
 + ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*
b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^
5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^
2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4
 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4
 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b
^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*
e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)
*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2
)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7))
)/(a*b^2*c^3 - 4*a^2*c^4))) + sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e
+ 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e
^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d
^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a
^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^
4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*
a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 -
4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2
*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 +
a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*
d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e
^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3
```

```
*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 + ((a*b^3*c^4 - 4*a
^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^
4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3
*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^
3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4
*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6
*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4
- (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*
e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*
b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 -
 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2
*c^4))))/c
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^2}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^2)/(c*x^4 + b*x^2
 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4110 vs. $2(204) = 408$.

Time = 0.91 (sec) , antiderivative size = 4110, normalized size of antiderivative = 17.27

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] e^2*x/c + 1/8*(2*(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*d*e
- (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*e^2 + 2*(sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 -
2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^2*c^5 + 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*c^6 - 32*a^2*c^6 + 2*(b^2 - 4*a*c)*b^2*c^4 - 8*(b^2 - 4*a*c)*a*c^5)*d^2*ab
s(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c
^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*e^2*abs(c) + 2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d^2 - 2*
(2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*d*e + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^
6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(
```

```
b^2 - 4*a*c)*a*b*c^5)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 -
4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*
a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*(2*b^4*c^3 - 16*a*b^2*c^4
+ 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 +
 8*(b^2 - 4*a*c)*a*c^4)*c^2*d*e - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a
*c)*a*b*c^3)*c^2*e^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 4*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^4
 + 8*(b^2 - 4*a*c)*a*c^5)*d^2*abs(c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4
- 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a
^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*e^2*abs(c) +
2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^6 -
 2*(b^2 - 4*a*c)*b*c^6)*d^2 - 2*(2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*d*e + (2*b^5*
c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
```

```
c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5
 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*e^2)*arctan(2*sqrt(1/
2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4
 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 9600, normalized size of antiderivative = 40.34

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^2/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c
 - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*
a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4
+ 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e
- 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e +
6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2
*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e^4 + b^3
*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^
(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(
1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^
3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2
*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2
) - (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^
2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^
3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^
2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c
*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b
*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a
^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^
3 - 8*a^2*b^2*c^4)))^(1/2)*1i - (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^
3*d^2 + 4*a*b^2*c^2*e^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b
^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3
)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)
^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*
e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a
^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1
```

```
/2))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b
^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*
c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*
b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*
a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c
^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 1
2*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3
*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*
a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4
+ 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e
- 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e +
6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2
*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((2*(2*c^3*d^5*e -
 a^2*b*e^6 - b^3*d^2*e^4 + 4*a*c^2*d^3*e^3 - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*
e^3 + 2*a*b^2*d*e^5 + 2*a^2*c*d*e^5 - 6*a*b*c*d^2*e^4))/c + (((16*a*c^4*d^2
 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c - (2*x*(4*b^3*c^3 -
16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) -
 a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 +
a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 -
4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 -
 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b
^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-
(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e
^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3
*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e
 + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*
c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(b^4*e^4 + 2
*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c
*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*(-(a*b^5*e^4 +
 b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)
^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^
3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*
d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24
*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e
^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^
(1/2) + (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)
/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(
4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^
4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*
e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e
+ 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c
```

```
^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e^4 + b
^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3
)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)
^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*
e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a
^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1
/2) + (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*
c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*
e^3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*
b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2
*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a
*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24
*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*
c^3 - 8*a^2*b^2*c^4)))^(1/2)))*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*
c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 +
12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e -
32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*
a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*2i + atan(((((16*a*c^4*d
^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c - (2*x*(4*b^3*c^3
- 16*a*b*c^4)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4
- a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 +
 a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 +
 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2
+ 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*
b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b
^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e
^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3
*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e
 - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*
c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(b^4*e^4 + 2
*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c
*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*((c^3*d^4*(-(4
*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^
3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3
)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d
*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*
a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^
3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(
```

```
1/2)*1i - (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^
2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b
^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e
^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3
*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e
 - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*
c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*((c^3*d^4*(-(4*
a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3
)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)
^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*
e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a
^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1
/2) + (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*
c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*
e^3))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b
^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*
c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*
b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*
a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c
^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((2*(2*c^3*d^5*e - a^2*b*e^6 - b^3*d^2*e^4
+ 4*a*c^2*d^3*e^3 - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*e^3 + 2*a*b^2*d*e^5 + 2*a
^2*c*d*e^5 - 6*a*b*c*d^2*e^4))/c + (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2
*c^3*d^2 + 4*a*b^2*c^2*e^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((c^3*d^4*(-(
4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)
^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^
3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*
d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24
*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e
^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^
(1/2))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*
b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2
*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a
*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24
*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*
c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 -
12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^
3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*
c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4
- 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e
+ 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e -
6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2
```

```
*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (((16*a*c^4*d^2 - 16
*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c + (2*x*(4*b^3*c^3 - 16*a*
b*c^4)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2
*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*
e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*
c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^
2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3
 - 8*a^2*b^2*c^4)))^(1/2))/c)*((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*
d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12
*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32
*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*
b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(b^4*e^4 + 2*c^4*d^
4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 -
4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*((c^3*d^4*(-(4*a*c -
b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2
) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2)
 - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4 + 4*a*b^4*c*d*e^3 +
8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3*d^2*e^2 - 24*a^2*b^2
*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)))*
((c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^3*d^4 - a*b^5*e^4 - a*b^2*e^4*(-
(4*a*c - b^2)^3)^(1/2) + 7*a^2*b^3*c*e^4 - 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(
4*a*c - b^2)^3)^(1/2) - 32*a^2*c^4*d^3*e + 32*a^3*c^3*d*e^3 + 4*a*b*c^4*d^4
 + 4*a*b^4*c*d*e^3 + 8*a*b^2*c^3*d^3*e - 6*a*b^3*c^2*d^2*e^2 + 24*a^2*b*c^3
*d^2*e^2 - 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^
2*b^2*c^4)))^(1/2)*2i + (e^2*x)/c
```

3.266 $\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 174

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx=\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d)/
(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c+b^2
)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1180, 211}

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e -
(2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\ &\quad+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx \\ &= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \frac{\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2
 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt
[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(\_R^{2}e+d\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}\right)}{2}$
default	$4c\left(\frac{\left(e\sqrt{-4ac+b^{2}}+be-2cd\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(e\sqrt{-4ac+b^{2}}-be+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8c\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)$

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((_R^2*e+d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1525 vs. 2(140) = 280.

Time = 0.36 (sec) , antiderivative size = 1525, normalized size of antiderivative = 8.76

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*
sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x + sqr
t(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2
*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (
a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2
- 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a
*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 -
 b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a
*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c
^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))
*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4
 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^
2))) + 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^
```

```
2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))
/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*
x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c
- 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*
e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b
^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d^
2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^
2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d
^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c -
4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3
*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4
*a^2*c^2)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx=\int \frac{ex^2+d}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. 2(140) = 280.

Time = 0.82 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.06

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2
 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*
c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b
^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arcta
n(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*
a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 4109, normalized size of antiderivative = 23.61

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^
2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2
+ a*b^4*c)))^(1/2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*
c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*
d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^
2*b^2*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4
*b*c^2*d*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d
*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i +
((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*
c)))^(1/2) + 4*b^2*c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)
^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*
a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^
2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d
*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a
*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((x*(8*b
^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*
d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a
^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/
2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2
) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*
e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^
4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(
a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d
*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((x*(8*b^3*c^2 - 32
*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e
- 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 4*b^2*
c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d
^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^
2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2
```

```
) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 +
 a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*
a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*c^2*d^2*e + 2*a*c*e^3 - 2*b*
c*d*e^2))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e
- 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i - ata
n((((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/
2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c
*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b
^4*c)))^(1/2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b
^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 -
 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2
*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^
2*d*e))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e -
4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i + ((x*(
8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3
*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 1
6*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^
(1/2) + 4*b^2*c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(
1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b
*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a
*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*
(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((x*(8*b^3*c^
2 - 32*a*b*c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 +
 c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^
2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) -
4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b
^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 +
 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c))
)^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3
*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(
8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((x*(8*b^3*c^2 - 32*a*b*
c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a
*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 4*b^2*c^2*d
 - 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 +
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2
*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + x
*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 - a*e^
2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4
```

```
*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c
^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*c^2*d^2*e + 2*a*c*e^3 - 2*b*c*d*e
^2))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a
*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i
```

3.267 $\int \frac{1}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{a+bx^2+cx^4} dx = \frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*
a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1107, 211}

$$\int \frac{1}{a+bx^2+cx^4} dx = \frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \frac{\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-1),x]
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

```
[In] int(1/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*sum(1/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(114) = 228$.

Time $= 0.28$ (sec) , antiderivative size $= 613$, normalized size of antiderivative $= 4.09$

$$
\begin{aligned}
\int \frac{1}{a+bx^2+cx^4}\,dx = {} & -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx\right. \\
& \left.+\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) \\
& +\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx\right. \\
& \left.-\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) \\
& -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx\right. \\
& \left.+\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) \\
& +\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx\right. \\
& \left.-\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)
\end{aligned}
$$

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$
$$= \mathrm{RootSum}\left(t^4\cdot\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c, \left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab}{c}\right.\right.\right.$$

```
[In] integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Maxima [F]

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \int \frac{1}{cx^4+bx^2+a}\,dx$$

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. 2(114) = 228.

Time = 0.58 (sec) , antiderivative size = 1026, normalized size of antiderivative = 6.84

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$

$$= \frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^4 - 8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}ab^2c - 2\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^3c - 2\,b^4c + 16\,\sqrt{2}\right.}{}$$

$$+ \frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^4 - 8\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}ab^2c - 2\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^3c + 2\,b^4c + 16\,\sqrt{2}\right.}{}$$

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^
2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs
(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.09

$$\int \frac{1}{a+bx^2+cx^4}\,dx =$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right.$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-}\right.$$

```
[In] int(1/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*
a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
- 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2
) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(b^3 + (
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 +
 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i
)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

3.268 $\int \frac{1}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 254

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} \\ &\quad -\frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} \\ &\quad +\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}\end{aligned}$$

```
[Out] e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/d^(1/2)-1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*
c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

= {1184, 211, 1180}

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)} - \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx \\ &= \frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}\end{aligned}$$

$$
\begin{aligned}
&=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)}-\frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)}\\
&\quad-\frac{\left(c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)}\\
&=-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}\\
&\quad-\frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.08

$$
\begin{aligned}
\int\frac{1}{(d+ex^2)\left(a+bx^2+cx^4\right)}\,dx&=\frac{\sqrt{c}\left(-2cd+be+\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-cd^2+bde-ae^2\right)}\\
&\quad+\frac{\sqrt{c}\left(2cd-be+\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\left(-cd^2+bde-ae^2\right)}\\
&\quad+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)}
\end{aligned}
$$

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) +
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85

method	result
default	$\frac{4c\left(\frac{\left(be-2cd-e\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\operatorname{arctan}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(-e\sqrt{-4ac+b^{2}}-be+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{a\,e^{2}-bde+c\,d^{2}}+\frac{e^{2}\operatorname{arct}}{(a\,e^{2}-bd}$
risch	Expression too large to display

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)*c*(1/8*(b*e-2*c*d-e*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))-1/8*(-e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)))+e^2/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arctan(e*
x/(e*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7995 vs. 2(210) = 420.

Time = 14.27 (sec) , antiderivative size = 16013, normalized size of antiderivative = 63.04

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7664 vs. 2(210) = 420.

Time = 1.78 (sec) , antiderivative size = 7664, normalized size of antiderivative = 30.17

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] e^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + 1/8*(2*(2*b
^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^
4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^
2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e + 4*(2*b^5*c^
3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^
2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 - (2*b^6*c^2 + 4*a*b^
4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c
```

+ 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2
+ 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4
*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 + 2*(2*a*b^5*c^2 - 6*a^
2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^
2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d*e^4 - (2*a^2*b^4*c^2 -
8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
2*(b^2 - 4*a*c)*a^2*b^2*c^2)*e^5 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2*c^4 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 32*a^2*c^5 + 2*(b^2 - 4*
a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) - 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2
 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*
b*c^3)*d^2*e*abs(c*d^2 - b*d*e + a*e^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 2*b^6*c + 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 14*a*b^4*c^2
 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*
(b^2 - 4*a*c)*a^2*c^3)*d*e^2*abs(c*d^2 - b*d*e + a*e^2) - 2*(sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c
+ 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a

```
^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c
^2)*e^3*abs(c*d^2 - b*d*e + a*e^2) - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a
*c^3)*(c*d^2 - b*d*e + a*e^2)^2*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2
*d*e + a*b*e^2 + sqrt((b*c*d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*
e + a^2*e^2)*(c^2*d^2 - b*c*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2))
)/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*
b^2*c^4 - 4*a^2*c^5)*d^4*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8
*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a
^2*b*c^4)*d^3*e*abs(c*d^2 - b*d*e + a*e^2)*abs(c) + (a*b^6 - 6*a^2*b^4*c -
2*a*b^5*c + 4*a^2*b^3*c^2 + a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b
^2*c^3 - 8*a^3*c^4)*d^2*e^2*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a^2*b^5
- 8*a^3*b^3*c - 2*a^2*b^4*c + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 -
4*a^3*b*c^3)*d*e^3*abs(c*d^2 - b*d*e + a*e^2)*abs(c) + (a^3*b^4 - 8*a^4*b^2
*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*e^4*
abs(c*d^2 - b*d*e + a*e^2)*abs(c)) - 1/8*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(
2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^
4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2
 - 4*a*c)*a*b*c^4)*d^3*e^2 - (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
```

```
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4
*a*c)*a*b^2*c^3)*d^2*e^3 + 2*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b
^2 - 4*a*c)*a^2*b*c^3)*d*e^4 - (2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2
)*e^5 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^3*c^3 + 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*c^5 + 32*a^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a
*c)*a*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*
a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^
4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*e*abs(c*d^2 - b*
d*e + a*e^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 7*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^5*c + 2*b^6*c + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 14*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^3 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2
 - 4*a*c)*b^4*c + 6*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*d*e^
2*abs(c*d^2 - b*d*e + a*e^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c
^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2
*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*e^3*abs(c*d^2 - b*d*e +
 a*e^2) - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

```
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c
^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^
2)^2*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c*d^2 - b^2*d*e + a*b*e^2 - sqrt((b*c*
d^2 - b^2*d*e + a*b*e^2)^2 - 4*(a*c*d^2 - a*b*d*e + a^2*e^2)*(c^2*d^2 - b*c
*d*e + a*c*e^2)))/(c^2*d^2 - b*c*d*e + a*c*e^2)))/((a*b^4*c^2 - 8*a^2*b^2*c
^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^4*ab
s(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2
+ 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*d^3*e*abs(c*d^2 -
 b*d*e + a*e^2)*abs(c) + (a*b^6 - 6*a^2*b^4*c - 2*a*b^5*c + 4*a^2*b^3*c^2 +
 a*b^4*c^2 + 32*a^4*c^3 + 16*a^3*b*c^3 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^2
*abs(c*d^2 - b*d*e + a*e^2)*abs(c) - 2*(a^2*b^5 - 8*a^3*b^3*c - 2*a^2*b^4*c
 + 16*a^4*b*c^2 + 8*a^3*b^2*c^2 + a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^3*abs(c*d^
2 - b*d*e + a*e^2)*abs(c) + (a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c
^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*e^4*abs(c*d^2 - b*d*e + a*e^2)*
abs(c))
```

Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 23640, normalized size of antiderivative = 93.07

$$\int \frac{1}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2
 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3
*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2
 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3
*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*
b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d
^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2
*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*
b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 +
b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^
2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
```

```
 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^
3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^
2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*
c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) -
16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*
c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^
2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*
b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*
b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 +
512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^
5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6
+ 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e
^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 -
640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96
*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2
*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c
^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*
a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5
*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a
^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c
^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2
*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2
*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 +
 a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*
b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 1
6*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c
*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2
*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b
^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*
c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*
d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2
*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3
*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2
*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^
4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^
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2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*
d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^
4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16
*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*
a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 1
6*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4
*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2
*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*
d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 1
2*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*
a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*
e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*
d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a
^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5
*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*
e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
 - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16
*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*
d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b
*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e
 - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 -
 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*
d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 +
128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^
2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128
*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3
*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*
d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3
*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128
*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d
^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*
b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*
c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3
*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7
))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7
*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2
*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4
*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2
*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e -
 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*
c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 -
4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^
5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*
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d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d
^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12
*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a
^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e
^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d
^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^
2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d
^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a
*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^
2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*
a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d
^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 +
192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3
 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*
d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2
*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/
2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4
*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c
^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32
*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(
-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b
^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2
*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4
 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2
*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*
a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*
d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*
e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4
*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e
^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 2
88*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7
+ 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*
a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6
*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^
5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 1
6*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5
*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 6
4*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*
c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*
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e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5
- 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*
d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16
*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*
e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*
e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*
c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3
*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 -
 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*
b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*
b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^
3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*
a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((-
(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^
3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*
d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4
+ 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*
b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a
^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d
^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 -
112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^
6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d
^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d
^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a
*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^
2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*
a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d
^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2
 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8
*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a
^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
- 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*
(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d
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^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 1
28*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2
*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*
c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4
*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4
*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4
 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) -
64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d
^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96
*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^
4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7
- 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 12
8*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^
2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d
^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3
*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)
))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^
5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^
2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*
b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2
*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a
^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^
2*e^2)))^(1/2)))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a
*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*
d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b
^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a
*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3
 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i + atan((((-(b^5*e
^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^
2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e +
2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a
^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*
e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c
^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e -
 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*
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b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32
*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5
 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c
^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*
d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^
3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2
)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c
^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e
+ 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 +
16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d
^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5
*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 1
6*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 3
2*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4
*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192
*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c
^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^
2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d
^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*
e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8
- 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c
^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4
 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c
^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*
e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*
a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*
b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*
e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4
- 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 3
2*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e
^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/
2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16
*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^
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2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d
^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3
*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)
))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b
*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*
e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4
+ 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6
*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b
^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 +
 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 +
16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 -
 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e
^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^
5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c
*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
 + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 1
6*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5
*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*
b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*
e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*
d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 +
a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b
^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16
*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*
d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^
9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^
3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4
*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e
^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^
2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 -
672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*
a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*
b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^
8 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3
*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6
 - 240*a^2*b^2*c^4*d^2*e^6 + 256*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384
*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4
*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 +
 b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 +
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a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c
^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3
 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d
^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a
^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4
 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5
*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*
e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e
 + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16
*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*
d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b
*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e
 - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i)/(((-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^
2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 -
8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*
a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)
*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e
^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d
*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a
*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*
a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3
*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e
^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*
b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*
a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^
3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3
*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e
^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*
a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e
 + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b
^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c
^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 -
32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^
3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*
d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d
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^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e
^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 -
1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288
*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e
^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b
^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e
^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 3
2*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3
*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c
^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3
*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e +
12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16
*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2
*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c
*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*
a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^
3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5
) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a
*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*
d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b
^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a
*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3
 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - ((-(b^5*e^2 + b^3*
c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^
4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a
^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e
+ 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^
4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e
^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^
4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b
*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^
2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 -
8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*
a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)
*(256*a^4*c^4*e^8 + x*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)
```

```
^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e
- 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2
*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b
^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4
+ a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2
- 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*
d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^
3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^
3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*
e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*
a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 3
84*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*
b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5
*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c
^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 64*a*c^7*d^6*e^
2 + 16*a^2*b^4*c^2*e^8 - 128*a^3*b^2*c^3*e^8 + 128*a^2*c^6*d^4*e^4 + 448*a^
3*c^5*d^2*e^6 + 16*b^2*c^6*d^6*e^2 - 64*b^3*c^5*d^5*e^3 + 96*b^4*c^4*d^4*e^
4 - 64*b^5*c^3*d^3*e^5 + 16*b^6*c^2*d^2*e^6 - 240*a^2*b^2*c^4*d^2*e^6 + 256
*a*b*c^6*d^5*e^3 - 32*a*b^5*c^2*d*e^7 - 384*a^3*b*c^4*d*e^7 - 416*a*b^2*c^5
*d^4*e^4 + 288*a*b^3*c^4*d^3*e^5 - 32*a*b^4*c^3*d^2*e^6 - 128*a^2*b*c^5*d^3
*e^5 + 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b
^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b
^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3
*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a
^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) + 4*b^
3*c^3*e^6 + 4*c^6*d^3*e^3 - 4*b*c^5*d^2*e^4 - 4*b^2*c^4*d*e^5 - 16*a*b*c^4*
e^6 + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2
 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8
*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a
^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3
- 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2))
)*(-(b^5*e^2 + b^3*c^2*d^2 - b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - c^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*
a*b^3*c*e^2 + a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*
c^2*d*e + 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*
d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*
a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e -
32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c
^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*2i - (log(b^5*d*(-d*e^3)^(5/2) - b^
```

```
5*d^3*e^8*x + c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(-d
*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b^4
*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e^3
)^(3/2) + a*b^4*d^2*e^9*x + 2*a*c^4*d^6*e^5*x - 2*b*c^4*d^7*e^4*x + 2*b^4*c
*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2) +
 17*a^2*c^3*d^4*e^7*x + 16*a^3*c^2*d^2*e^9*x + b^2*c^3*d^6*e^5*x - b^3*c^2*
d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2) +
 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) - 12*a*b^2*c^2*d^4*e^7*x - 12*a^2*b*c^2*d^3
*e^8*x - 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) + 2*a*
b*c^3*d^5*e^6*x + 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(-d
*e^3)^(1/2))/(2*(c*d^3 + a*d*e^2 - b*d^2*e)) + (log(b^5*d*(-d*e^3)^(5/2) +
b^5*d^3*e^8*x - c^5*d^8*e^3*x + 2*a*c^4*d^5*(-d*e^3)^(3/2) - 16*a^3*c^2*e*(
-d*e^3)^(5/2) - c^5*d^8*e*(-d*e^3)^(1/2) + b^2*c^3*d^5*(-d*e^3)^(3/2) - a*b
^4*e*(-d*e^3)^(5/2) - 7*a*b^3*c*d*(-d*e^3)^(5/2) + 17*a^2*c^3*d^3*e^2*(-d*e
^3)^(3/2) - a*b^4*d^2*e^9*x - 2*a*c^4*d^6*e^5*x + 2*b*c^4*d^7*e^4*x - 2*b^4
*c*d^4*e^7*x + 12*a^2*b*c^2*d*(-d*e^3)^(5/2) + 8*a^2*b^2*c*e*(-d*e^3)^(5/2)
 - 17*a^2*c^3*d^4*e^7*x - 16*a^3*c^2*d^2*e^9*x - b^2*c^3*d^6*e^5*x + b^3*c^
2*d^5*e^6*x - b^3*c^2*d^4*e*(-d*e^3)^(3/2) + 2*b^4*c*d^3*e^2*(-d*e^3)^(3/2)
 + 2*b*c^4*d^7*e^2*(-d*e^3)^(1/2) + 12*a*b^2*c^2*d^4*e^7*x + 12*a^2*b*c^2*d
^3*e^8*x + 8*a^2*b^2*c*d^2*e^9*x - 12*a*b^2*c^2*d^3*e^2*(-d*e^3)^(3/2) - 2*
a*b*c^3*d^5*e^6*x - 7*a*b^3*c*d^3*e^8*x + 2*a*b*c^3*d^4*e*(-d*e^3)^(3/2))*(
-d*e^3)^(1/2))/(2*c*d^3 + 2*a*d*e^2 - 2*b*d^2*e)
```

3.269 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 429

$$
\begin{aligned}
&\int \frac{1}{(d+ex^2)^2\,(a+bx^2+cx^4)}\,dx \\
&= \frac{e^2x}{2d\,(cd^2-bde+ae^2)\,(d+ex^2)} \\
&\quad + \frac{\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\
&\quad - \frac{\sqrt{c}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\
&\quad + \frac{e^{3/2}(2cd-be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\,(cd^2-bde+ae^2)}
\end{aligned}
$$

```
[Out] 1/2*e^2*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)+1/2*e^(3/2)*arctan(x*e^(1/2)/d^(1
/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)+e^(3/2)*(-b*e+2*c*d)*arctan(x*e^(1/2)/d^(1
/2))/(a*e^2-b*d*e+c*d^2)^2/d^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*c*e*(b
*d+a*e+d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(-4*a*c+b^2)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*c^(1/2)*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d
+a*e-d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1184, 205, 211, 1180}

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx &= \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)^2}\\ &\quad-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)^2}\\ &\quad+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(2cd-be)}{\sqrt{d}\,(ae^2-bde+cd^2)^2}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(ae^2-bde+cd^2)}+\frac{e^2x}{2d(d+ex^2)(ae^2-bde+cd^2)}\end{aligned}$$

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c^2*d^2 + b
*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]*(2*
c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d
+ a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (
e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e
+ a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d
*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^2}{(cd^2-bde+ae^2)\,(d+ex^2)^2}-\frac{e^2(-2cd+be)}{(cd^2-bde+ae^2)^2\,(d+ex^2)}\right.\\
&\qquad\left.+\frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)x^2}{(cd^2-bde+ae^2)^2\,(a+bx^2+cx^4)}\right)dx\\
&= \frac{\int \frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{(cd^2-bde+ae^2)^2}+\frac{\left(e^2(2cd-be)\right)\int \frac{1}{d+ex^2}\,dx}{(cd^2-bde+ae^2)^2}+\frac{e^2\int \frac{1}{(d+ex^2)^2}\,dx}{cd^2-bde+ae^2}\\
&= \frac{e^2x}{2d\,(cd^2-bde+ae^2)\,(d+ex^2)}+\frac{e^{3/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2}+\frac{e^2\int \frac{1}{d+ex^2}\,dx}{2d\,(cd^2-bde+ae^2)}\\
&\quad+\frac{\left(c\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)^2}\\
&\quad-\frac{\left(c\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)^2}\\
&= \frac{e^2x}{2d\,(cd^2-bde+ae^2)\,(d+ex^2)}\\
&\quad+\frac{\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2}\\
&\quad-\frac{\sqrt{c}\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2}\\
&\quad+\frac{e^{3/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\,(cd^2-bde+ae^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx = \frac{\frac{e^2(cd^2+e(-bd+ae))x}{d(d+ex^2)} + \frac{\sqrt{2}\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}(-2c^2d^2+b(-b+\sqrt{b}}{}}{2\left(cd^2+e(-bd+ae)\right)^2}$$

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((e^2*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^2)) + (Sqrt[2]*Sqrt[c]*(2*c
^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d +
 a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-2*c^2*d^2 + b*(-b
 + Sqrt[b^2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]) + (e^(3/2)*(5*c*d^2 + e*(-3*b*d + a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/d^(3/2))/(2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.80

method	result
default	$\frac{4c\left(\frac{\left(b\,e^2\sqrt{-4ac+b^2}-2dce\sqrt{-4ac+b^2}+2e^2ac-b^2e^2+2bcde-2c^2d^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(b\,e^2\sqrt{-4ac+b^2}-2dce\sqrt{-4ac+b^2}-2e^2a}{8\sqrt{-4a}}\right.}{\left(a\,e^2-bde+c\,d^2\right)^2}$
risch	Expression too large to display

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)^2*c*(1/8*(b*e^2*(-4*a*c+b^2)^(1/2)-2*d*c*e*(-4*a*c+b^
2)^(1/2)+2*e^2*a*c-b^2*e^2+2*b*c*d*e-2*c^2*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))-1/8*(b*e^2*(-4*a*c+b^2)^(1/2)-2*d*c*e*(-4*a*c+b^2)^(1/2)-2*e^2*a
*c+b^2*e^2-2*b*c*d*e+2*c^2*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+e
^2/(a*e^2-b*d*e+c*d^2)^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2-
3*b*d*e+5*c*d^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2394 vs. $2(366) = 732$.

Time = 1.83 (sec) , antiderivative size = 2394, normalized size of antiderivative = 5.58

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*e^2*x/((c*d^3 - b*d^2*e + a*d*e^2)*(e*x^2 + d)) + 1/2*(sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 2*b^4*c^3
+ 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2
*c^4 + 16*a*b^2*c^4 + 2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^5 - 32*a^2*c^5 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b*c^4 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4 - 2*(b^2 - 4*a*c)*b
*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 +
2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4 + sqrt((b*c^2*d^4 - 2*b^2*c*d^3
*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4)^2 - 4*(a*c^
2*d^4 - 2*a*b*c*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a^2*b*d*e^3 + a
^3*e^4)*(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*
c*d*e^3 + a^2*c*e^4)))/(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d
^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b
^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^2*abs(c) - 2*(
a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^
3*c^3 - 4*a^2*b*c^4 + (a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 +
 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*sqrt(b^2 - 4*a*c))*d*e*abs(c) + (a*b^
6 - 10*a^2*b^4*c - 2*a*b^5*c + 32*a^3*b^2*c^2 + 12*a^2*b^3*c^2 + a*b^4*c^2
- 32*a^4*c^3 - 16*a^3*b*c^3 - 6*a^2*b^2*c^3 + 8*a^3*c^4 + (a*b^5 - 8*a^2*b^
3*c - 2*a*b^4*c + 16*a^3*b*c^2 + 8*a^2*b^2*c^2 + a*b^3*c^2 - 4*a^2*b*c^3)*s
qrt(b^2 - 4*a*c))*e^2*abs(c)) + 1/2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 16*a*b^2*c^4 +
2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 + 32*a^2*c^5 -
8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c
)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*arctan(2*sqrt(1/
2)*x/sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*
b^2*d*e^3 + a^2*b*e^4 - sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a
*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4)^2 - 4*(a*c^2*d^4 - 2*a*b*c*d^3*e
+ a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4)*(c^3*d^4 - 2*b
*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))
/(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3
 + a^2*c*e^4)))/(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 +
8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*d^2*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^
2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 +
```

```
(a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c
^3 - 4*a^2*c^4)*sqrt(b^2 - 4*a*c))*d*e*abs(c) + (a*b^6 - 10*a^2*b^4*c - 2*a
*b^5*c + 32*a^3*b^2*c^2 + 12*a^2*b^3*c^2 + a*b^4*c^2 - 32*a^4*c^3 - 16*a^3*
b*c^3 - 6*a^2*b^2*c^3 + 8*a^3*c^4 + (a*b^5 - 8*a^2*b^3*c - 2*a*b^4*c + 16*a
^3*b*c^2 + 8*a^2*b^2*c^2 + a*b^3*c^2 - 4*a^2*b*c^3)*sqrt(b^2 - 4*a*c))*e^2*
abs(c)) + 1/2*(5*c*d^2*e^2 - 3*b*d*e^3 + a*e^4)*arctan(e*x/sqrt(d*e))/((c^2
*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + 2*a*c*d^3*e^2 - 2*a*b*d^2*e^3 + a^2*d*e^
4)*sqrt(d*e))
```

Mupad [B] (verification not implemented)

Time = 11.41 (sec) , antiderivative size = 91169, normalized size of antiderivative = 212.52

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan(((((x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8
*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48
*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e
^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^
8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a
^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*
b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 -
 12*a^2*b*c*d^5*e^5)) - (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c
^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 +
 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c
^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^
9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*
a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11
+ 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b
^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^
5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5
*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10
+ a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d
^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*
d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2
*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - ((-d^3*e^3)^(1/2)*((x*(32*c^11*d^13*e^2
 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*
d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6
- 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336
*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c
^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*
e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6
```

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*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*
c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720
*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^1
3 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10
*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7
*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*
e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^
6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d
*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^
4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e
^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*
e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d
^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16
 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 1
0880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*
b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*
c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*
d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^1
1*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*
b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 40
0*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e
^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*
c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680
*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^
13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^
4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^
6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 36
48*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 -
 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 -
 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 +
 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e
^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2
*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 -
4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b
^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 -
4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^
5)) - (x*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16*e
^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9
 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 -
1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*
b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8
*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2
*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a
^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e
^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^
```

2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 1376
0*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8
*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^
2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4
224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*
e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4
*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 2150
4*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^
15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^
3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c
^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a
*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 57
6*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 +
320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e
^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*
d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(8*(c^2*d^7
+ a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)
*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7
+ 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 +
6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3
+ 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c
*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*
a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a
^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(
-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^
2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)
*(a*e^2 + 5*c*d^2 - 3*b*d*e)*1i)/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 -
2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)) + (((x*(54*c^9*d^6*e^5 - 2*a^
3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a
^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e
^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^
2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5
- 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6
*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e -
12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (((2*a^2*b
^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26
*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*
d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 -
464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^
2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4
*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^
2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^
5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*
e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e
^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*

```
b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c
*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*
c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5))
+ ((-d^3*e^3)^(1/2)*((x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d
^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28
*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*
c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^
10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8
+ 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*
a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 -
 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2
*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b
^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 74
0*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 78
52*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 81
6*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 72
16*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 56
96*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 3
36*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*
d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7
*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*
d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - ((
(128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^
3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c
^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^
14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e
^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32
*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7
 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4
*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3
*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 -
9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4
*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b
^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20
672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*
e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c
^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3
*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b
^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^
9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*
a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 1
6*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2
*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7
+ 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 +
 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3
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 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-d^3*e^3)^(1/2)*(a*e^2 +
 5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216
*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^
7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^1
0*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^
14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*
e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*
e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^
5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1
664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^1
2*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3
*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12
+ 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6
*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^
4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 -
 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d
^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b
^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 84
48*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^
17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^1
6*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^
7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*
c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2
*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11
520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^1
4 + 5376*a^8*b*c^4*d^3*e^16))/(8*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b
*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6
*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^
6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8
*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c
*d^5*e^5)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2
*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(a*
e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d
^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 -
 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^
4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*1i)/(
4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*
c*d^5*e^2)))/((5*c^8*d^3*e^6 - 3*b*c^7*d^2*e^7 + a*c^7*d*e^8)/(c^4*d^10 + a
^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6
*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*
d^6*e^4 - 12*a^2*b*c*d^5*e^5) - (((x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*
a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 +
 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*
d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/
```

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(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^
7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6
 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e
^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((2*a^2*b^6*c^2*e^13 - 20
0*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^1
3 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^
2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5
*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a
^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 +
464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^
4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b
^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b
*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*
c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*
a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2
*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a
*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - ((-d^3*e^3)^(1
/2)*((x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6
*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^1
5 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 2
88*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4
*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d
^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*
e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^
4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^
3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12
- 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^
2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8
*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*
e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8
*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4
*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d
*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*
b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2
*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c
^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (((128*a*c^11*d^15
*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 -
 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 21
76*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4
*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^
5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^
10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4
*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*
a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8
 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4
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*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^
4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12
- 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*
d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b
^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 10
24*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 +
8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 +
 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 +
 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^
9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*
e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4
*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^
2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e
^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^
6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d
*e)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^
7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 -
 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512
*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^
7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c
^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b
^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 -
 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*
d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^
3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^1
0 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^
2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280
*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^
13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^
5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^
5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14
+ 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d
^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2
*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b
^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 953
6*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1
856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4
 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^
9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c
^4*d^3*e^16))/(8*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b
*d^4*e^3 + 2*a*c*d^5*e^2)*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d
^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*
e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d
^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-d^
3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d
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^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2 -
3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4
*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c
^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^
5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^
3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)) + (((x*
(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a
^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3
*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2
*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*
e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*
e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*
d^5*e^5)) + (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14
*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7
*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 +
 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c
^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*
d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^
8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e
^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^
10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12
- 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^
8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a
^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*
b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 -
 12*a^2*b*c*d^5*e^5)) + ((-d^3*e^3)^(1/2)*((x*(32*c^11*d^13*e^2 + 48*a^6*b*
c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4
*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c
^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^1
1*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 -
 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^
9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*
a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12
 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5
*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4
*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*
a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*
a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a
*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a
^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*
a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 +
b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c
*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*
c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*
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a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c
^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7
*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^1
5*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5
 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 25
6*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 1216
0*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*
e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^
2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a
^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12
 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5
*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4
*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 +
 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^
3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9
*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*
c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c
^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b
*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*
a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*
a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*
e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3
 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*
e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-d
^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16*e^3 + 5120*a^
3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*
c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4
*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15
*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^
7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 +
8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d
^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^
2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 +
 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^
6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088
*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e
^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*
c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*
a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^1
3 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^
5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6
*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 -
 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2
 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^1
4*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*
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d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c
^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a
^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 2
0224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(8*(c^2*d^7 + a^2*d^3*e
^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)*(c^4*d^10 +
 a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^
8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d
^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*
c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d
*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3
+ 2*a*c*d^5*e^2)))*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 +
 b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1
/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 -
2*b*c*d^6*e - 2*a*b*d^4*e^3 + 2*a*c*d^5*e^2)))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*
c*d^2 - 3*b*d*e))/(4*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2
*a*b*d^4*e^3 + 2*a*c*d^5*e^2))))*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*
e)*1i)/(2*(c^2*d^7 + a^2*d^3*e^4 + b^2*d^5*e^2 - 2*b*c*d^6*e - 2*a*b*d^4*e^
3 + 2*a*c*d^5*e^2)) - atan(((((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a
^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e
^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b
^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^
4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 +
672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e
^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984
*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*
a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2
*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d
^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c
^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*
c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a
*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^
4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9
*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^
14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1
792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*
b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2
*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12
864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e
^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c
^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*
a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^1
0 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^
3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^
5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 -
 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*
```

e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^
11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d
^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^
12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c
^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b
*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^
6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e
^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^
8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*
c*d^5*e^5)) - (x*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4
+ c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e -
32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4
*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 +
4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*
(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^
3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) +
42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*
a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2
*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4
+ 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64
*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 14
4*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4
*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*
e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a
^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^
4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120
*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a
^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*
c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d
^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12
*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10
+ 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^
7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648
*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^1
2 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4
*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1
088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^1
0*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b
^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 64
00*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*
e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2
*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*
a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^1

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6 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*
e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*
d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c
^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^1
0*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 3712
0*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12
+ 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d
^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2
+ 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4
 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*
e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^
4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*
c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c
^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b
^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a
*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e
^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*
e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c
^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*
d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c
^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d
^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d
^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*
d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a
^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*
d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3
- 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(32*c^11*d^13*
e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^
10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e
^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 +
336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^
5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d
^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*
d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b
^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3
720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*
e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d
^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*
d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d
^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7
*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^
3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 +
 a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^
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8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d
^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*
c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*
c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 3
2*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 +
 a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 +
 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c
^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*
c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a
*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^
2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96
*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5
*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4
*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^
2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e
+ 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^
3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^
5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2))*((b^4*e^4*(-
(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)
^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3
*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*
c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24
*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1
/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^
2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5
*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4
 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*
e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2
*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3
*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^
4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*
d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*
a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5
*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d
^3*e^5)))^(1/2) - (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 -
118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4
*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^
3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a
^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6
```

*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*
d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^
4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*
a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a
^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9
*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2
*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^
3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a
c - b^2)^3)^(1/2))/(8(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b
^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*
c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a
^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c
^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c
^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*
c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e +
32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*
c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*
e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*1i - (((2*a^2*b
^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26
*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*
d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 -
 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^
2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4
*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^
2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^
5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*
e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e
^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*
b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c
*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*
c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5))
- (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 345
6*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a
^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^
9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^
11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9
- 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10
*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6
*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920
*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^1
0 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2
*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a
^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14

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- 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*
d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b
^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a
*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224
*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*
a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30
720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13
 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16
)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*
e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e
^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7
*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*((b^4*e^4*(-(4*a*c -
b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) +
20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e -
25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e
^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^
4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b
^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d
^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8
+ 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*
b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*
a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*
d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*
d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2
*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 -
 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2
*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^
5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))
^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12
*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^1
3 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 +
512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480
*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^1
0*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^
2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^
8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c
^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840
*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*
e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8
*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1
280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6
*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3
*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576
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*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^
14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^
4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*
b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*
a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 -
9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9
- 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*
e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7
*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*
b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5
 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 +
6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e -
 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*
(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^
3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c
^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^
5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) -
24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^
(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*
a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a
^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e
^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^
2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a
^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a
^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*
a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*
c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 +
6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b
^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3
*d^3*e^5)))^(1/2) - (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^
11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*
a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c
^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^1
0*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 +
 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a
^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 -
3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*
e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^
4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740
*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 785
2*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816
*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 721
6*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 569
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6*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 33
6*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d
^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*
e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d
^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^
4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c -
 b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4
*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2)
- 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2
*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1
/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^
2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2
 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d
^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8
*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*
b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6
- 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2
+ 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4
 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*
a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*
e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20
*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5
*b*c^3*d^3*e^5)))^(1/2))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 -
 b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5
*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*
e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*
c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3
 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3
 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*
d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8
 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*
d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*
e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*
e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2
*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*
b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7
*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 4
4*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(54*c^9*d^6*e^5 -
 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 -
 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*
d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4
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*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5
*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^
3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9
*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*
e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b
^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b
^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) -
6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2
) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 -
 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8
 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d
^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^
6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 -
44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 +
32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 +
 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*
b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^
7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a
^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b
*c^3*d^3*e^5)))^(1/2)*1i)/((5*c^8*d^3*e^6 - 3*b*c^7*d^2*e^7 + a*c^7*d*e^8)/
(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 +
 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 +
6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3
+ 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5) + (((2*a^2*b^6*c^2*e^13 - 200*a*
c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 +
480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^
8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8
 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b
^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*
a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*
c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c
^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7
*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*
d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*
b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2
*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c
^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15
*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 -
 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 21
76*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4
*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^
5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^
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10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4
*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*
a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8
 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4
*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^
4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12
- 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*
d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b
^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 10
24*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 +
8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 +
 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 +
 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^
9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*
e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4
*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^
2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e
^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^
6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*
c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 3
2*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 +
 a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 +
 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c
^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*
c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a
*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^
2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96
*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5
*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4
*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^
2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e
+ 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^
3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^
5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^1
1*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*
d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4
*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3
 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 +
3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*
b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 -
 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^
5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^
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2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^
8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6
*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 5376
0*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*
e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*
c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952
*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^1
5 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^
3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^
7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 2
56*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 -
 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*
e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9
*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^
9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^
6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*
(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 +
 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 +
6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3
+ 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(
1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*
c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^
3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*
b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e -
 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 +
72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2
*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*
c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7
 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d
^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 +
 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 -
 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5
+ 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b
*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2
+ 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a
^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) +
(x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*
d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2
048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^
5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*
d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^
9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 -
 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4
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*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2
*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648
*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^1
3 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7
+ 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10
+ 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7
+ 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11
 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14
))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3
*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*
e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^
7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2
)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*
a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*
a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2
+ 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d
^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*
c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*
e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a
*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 1
6*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5
*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4
*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6
*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5
*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^
3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64
*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^
6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 -
 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1
/2))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(
-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*
d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^
3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^
3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b
^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4
*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^
3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^
8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5
 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3
*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4
*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^
3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d
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^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6
*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4
*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6
- 192*a^5*b*c^3*d^3*e^5)))^(1/2) - (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22
*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9
+ 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7
*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))
/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e
^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^
6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*
e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^
3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^
3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^
2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 +
4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3
*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*
d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^
2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b
*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*
a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d
*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c
^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e
^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e
^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*
e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a
^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*
e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 1
92*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2
) + (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4
*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9
 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4
*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e
^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10
 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^
6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 207
2*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*
a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a
^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*
d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4
*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*
d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*
b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*
d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7
*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^
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2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2
240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^
9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^
2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9
 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^
5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b
^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 4
80*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6
*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6
*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 238
4*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^
14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^1
3*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*
d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d
^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8
*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*
b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*
b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5
 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6
*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e -
12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*((b^4*e
^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^
2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^
4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6
*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^
2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2)
 - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^
3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 -
72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8
 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^
4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6
*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 4
4*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 3
2*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 +
32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b
^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7
 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^
3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*
c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 921
6*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a
^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^
10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d
^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11
*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14
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*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b
^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 +
1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^
12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^
3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12
 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^
6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a
^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11
- 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*
d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*
b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8
448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e
^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^
16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c
^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8
*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^
2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 1
1520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^
14 + 5376*a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4
*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^
3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4
*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5
)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-
(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d
*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3
)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3
 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^
2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*
a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3
*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8
 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5
+ 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*
d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*
d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3
*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^
7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*
b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*
e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 -
 192*a^5*b*c^3*d^3*e^5)))^(1/2) - (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15
+ 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*
c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^
8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 2
68*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6
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*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4
*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c
^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^
2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11
 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*
d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8
*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5
*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*
d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^
2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5
*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e
^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e
^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d
^5*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*
d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3
*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c -
b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c
*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*
c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e
*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b
^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2
*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^
3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^
6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^
2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b
^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*
c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 6
4*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*
c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2
*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) -
 b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^
4 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*
e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*
d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*
b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2
*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^
8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a
^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2
 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^
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2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^
3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a
^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d
^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a
^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c
^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(54
*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*
b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^
8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*
c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4
 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 -
4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2
 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5
*e^5)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*c^4*d^4 - b^7*e^4 + c^4*d^
4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4 + 32*a^2*c^5*d^3*e - 32*a^3*c
^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^
2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d
*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c
- b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(
-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) + 42*a*b^3
*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c
*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*
e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*
c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*
c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2
*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^
3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*
a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*
d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e
^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)))*((b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) -
b^3*c^4*d^4 - b^7*e^4 + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) + 20*a^3*b*c^3*e^4
 + 32*a^2*c^5*d^3*e - 32*a^3*c^4*d*e^3 + 4*b^4*c^3*d^3*e - 25*a^2*b^3*c^2*e
^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*b^5*c^2*d^2*e^2 + 4*a*b*c^5*d
^4 + 9*a*b^5*c*e^4 + 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^2*c^4*d^3*e - 32*a*b
^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*
a*c - b^2)^3)^(1/2) + 42*a*b^3*c^3*d^2*e^2 - 72*a^2*b*c^4*d^2*e^2 + 72*a^2*
b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8
 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^
2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2
+ 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2
*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3
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*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^
5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^
7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^
2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^
4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*2i - atan
(((((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c
^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 +
 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c
^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^1
0 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 -
 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6
- 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*
a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*
b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3
*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^
6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e
^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^
8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*
c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^
13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e
^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2
+ 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 224
0*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*
c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*
b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 +
3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*
e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4
*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480
*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e
^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c
^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*
a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14
 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*
e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^
10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7
*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d
^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*
c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^
2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 -
4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a
^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12
*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*(-(b^7*e^
4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2
)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4
*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*
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b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2
*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2)
+ 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3
)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 7
2*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 +
 a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4
*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*
d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44
*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32
*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 3
2*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^
7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7
+ 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3
*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c
^3*d^3*e^5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216
*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^
7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^1
0*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^
14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*
e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*
e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^
5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1
664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^1
2*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3
*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12
+ 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6
*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^
4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 -
 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d
^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b
^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 84
48*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^
17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^1
6*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^
7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*
c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2
*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11
520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^1
4 + 5376*a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*
a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3
*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*
b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)
))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-
(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d
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*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3
)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3
 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^
2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*
a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3
*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8
 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5
+ 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*
d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*
d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3
*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^
7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*
b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*
e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 -
 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15
+ 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*
c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^
8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 2
68*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6
*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4
*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c
^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^
2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11
 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*
d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8
*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5
*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*
d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^
2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5
*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e
^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
- 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e
^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d
^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4
*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^
3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*
c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a
*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*
e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*
b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^
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2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d
^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a
^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b
^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*
b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5
*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 -
64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6
*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^
2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*
(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*
e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^
2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^
5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*
a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a
^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e
^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*
e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8
*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e
^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*
a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*
a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64
*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5
*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32
*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b
*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) - (x*(
54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^
2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*
e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*
b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e
^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6
- 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e
^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d
^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4
*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^
3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*
c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a
*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*
e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*
b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^
2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d
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^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a
^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b
^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*
b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5
*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 -
64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6
*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^
2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*1i - (((2*a^2*b^6*c^2*e^13 - 200*a*c
^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 4
80*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8
*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8
 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^
2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a
^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c
^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^
5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*
d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d
*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b
*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*
d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^
2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*
e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 -
8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 217
6*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*
c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5
*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^1
0 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*
c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a
^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8
+ 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*
d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4
*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 -
 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d
^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^
5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 102
4*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8
464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 +
1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 +
512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9
 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e
^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*
d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2
 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^
4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6
*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*
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c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 3
2*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 +
 a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 -
 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c
^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*
c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a
*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^
2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96
*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5
*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4
*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^
2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e
+ 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^
3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^
5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^1
1*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*
d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4
*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3
 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 +
3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*
b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 -
 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^
5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^
2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^
8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6
*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 5376
0*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*
e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*
c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952
*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^1
5 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^
3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^
7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 2
56*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 -
 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*
e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9
*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^
9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^
6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*
(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 +
 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 +
6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3
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+ 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4
*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b
^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a
*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e
+ 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 -
 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^
2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7
*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^
7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*
d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2
+ 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3
- 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5
 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*
b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2
 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*
a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) -
 (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5
*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 -
2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a
^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7
*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e
^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8
- 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^
4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^
2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 64
8*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^
13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7
 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10
 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7
 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^1
1 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^1
4))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^
3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4
*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d
^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^
4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 2
0*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 2
5*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^
2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4
*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^
3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^
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2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8
*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 +
 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b
^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a
^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d
^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d
^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*
d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 -
64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*
d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5
 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^
(1/2))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^
4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c
^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^
2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d
*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c
- b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(
-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3
*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c
*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*
e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*
c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*
c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2
*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^
3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*
a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*
d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e
^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 -
 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e
^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*
c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^
9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^
3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4
*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d
^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^
4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 2
0*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 2
5*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^
2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4
*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^
3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^
2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8
```

```
*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 +
 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b
^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a
^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d
^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d
^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*
d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 -
64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*
d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5
 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^
(1/2)*1i)/((5*c^8*d^3*e^6 - 3*b*c^7*d^2*e^7 + a*c^7*d*e^8)/(c^4*d^10 + a^4*
d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2
 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^
4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6
*e^4 - 12*a^2*b*c*d^5*e^5) + (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*
a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*
e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*
b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d
^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 +
 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*
e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 198
4*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160
*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^
2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*
d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*
c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2
*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*
a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c
^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^
9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e
^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 +
1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896
*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^
2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 1
2864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*
e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*
c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120
*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^
10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c
^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a
^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15
- 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14
*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d
^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*
d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d
```

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^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*
c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*
b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d
^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*
e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d
^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b
*c*d^5*e^5)) - (x*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/
2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e
 + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-
(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4
 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^
4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*
c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a
*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 -
8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a
^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^
4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 +
64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 +
144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 -
 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*
d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4
*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*
b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 51
20*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120
*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^
9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8
*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 - 4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^
12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^
10 + 8192*a^2*b^2*c^9*d^14*e^5 + 5056*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*
c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^11*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 36
48*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e
^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 27840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b
^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 +
 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d
^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4
*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 +
6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^
6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 - 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b
^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 32
0*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e
^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^1
7*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^
8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7
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*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b
^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37
120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^1
2 + 20224*a^7*b*c^5*d^5*e^14 + 5376*a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4
*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^
2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e
^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^
6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c -
 b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a
^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^
2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*
a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*
d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3
*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^
4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c
^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^
5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^
2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^
2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c
^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 3
2*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c
^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e
^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(32*c^11*d^
13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b
*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^
9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12
 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260
*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^
3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c
^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^
2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10
+ 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d
^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^
9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c
^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^
3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*
c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4
*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^1
0 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3
*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^
2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b
^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(
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4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4
 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e
^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d
^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b
^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*
b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8
 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^
2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2
+ 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2
*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3
*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^
5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^
7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^
2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^
4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2))*(-(b^7*e
^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^
2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^
4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6
*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^
2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2)
 + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^
3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 +
72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8
+ a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^
4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6
*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 4
4*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 3
2*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 +
32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b
^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7
 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^
3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*
c^3*d^3*e^5)))^(1/2) - (x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e
^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^
7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6
*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^1
0 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3
*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^
2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b
^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(
4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4
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 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e
^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d
^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b
^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*
b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8
 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^
2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2
 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2
*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3
*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^
5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^
7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^
2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^
4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (((2*a^
2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 +
 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c
^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^
7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8
*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*
b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7
*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6
*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d
^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^
3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4
*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^
3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4
*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5
)) - (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 -
3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 691
2*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3
*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6
*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e
^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d
^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*
b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17
920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*
e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*
c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 752
0*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^
14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c
^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^
6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 729
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6*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 +
224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 1
76*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 +
 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e
^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e
^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d
^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^
4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*
d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (x*(-(b^7*e^4 + b^3*c
^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2
) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*
e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d
^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^
2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) -
 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c
^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*
e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*
a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 +
 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*
c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*
c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3
*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e
^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6
*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^
3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^
5)))^(1/2)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*
d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6
*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^
2 + 512*b^4*c^9*d^16*e^3 - 1792*b^5*c^8*d^15*e^4 + 3584*b^6*c^7*d^14*e^5 -
4480*b^7*c^6*d^13*e^6 + 3584*b^8*c^5*d^12*e^7 - 1792*b^9*c^4*d^11*e^8 + 512
*b^10*c^3*d^10*e^9 - 64*b^11*c^2*d^9*e^10 + 8192*a^2*b^2*c^9*d^14*e^5 + 505
6*a^2*b^3*c^8*d^13*e^6 - 31104*a^2*b^4*c^7*d^12*e^7 + 40256*a^2*b^5*c^6*d^1
1*e^8 - 22784*a^2*b^6*c^5*d^10*e^9 + 3648*a^2*b^7*c^4*d^9*e^10 + 1664*a^2*b
^8*c^3*d^8*e^11 - 576*a^2*b^9*c^2*d^7*e^12 + 45312*a^3*b^2*c^8*d^12*e^7 - 2
7840*a^3*b^3*c^7*d^11*e^8 - 13760*a^3*b^4*c^6*d^10*e^9 + 27520*a^3*b^5*c^5*
d^9*e^10 - 12416*a^3*b^6*c^4*d^8*e^11 + 1088*a^3*b^7*c^3*d^7*e^12 + 320*a^3
*b^8*c^2*d^6*e^13 + 53760*a^4*b^2*c^7*d^10*e^9 - 30400*a^4*b^3*c^6*d^9*e^10
 + 1280*a^4*b^4*c^5*d^8*e^11 + 4224*a^4*b^5*c^4*d^7*e^12 - 1280*a^4*b^6*c^3
*d^6*e^13 + 320*a^4*b^7*c^2*d^5*e^14 + 6400*a^5*b^2*c^6*d^8*e^11 - 2624*a^5
*b^3*c^5*d^7*e^12 + 5952*a^5*b^4*c^4*d^6*e^13 - 2752*a^5*b^5*c^3*d^5*e^14 -
 576*a^5*b^6*c^2*d^4*e^15 - 21504*a^6*b^2*c^5*d^6*e^13 + 832*a^6*b^3*c^4*d^
5*e^14 + 4736*a^6*b^4*c^3*d^4*e^15 + 320*a^6*b^5*c^2*d^3*e^16 - 8448*a^7*b^
2*c^4*d^4*e^15 - 2624*a^7*b^3*c^3*d^3*e^16 - 64*a^7*b^4*c^2*d^2*e^17 + 512*

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a^8*b^2*c^3*d^2*e^17 + 256*a*b*c^11*d^17*e^2 - 2304*a*b^2*c^10*d^16*e^3 + 8
512*a*b^3*c^9*d^15*e^4 - 16704*a*b^4*c^8*d^14*e^5 + 18240*a*b^5*c^7*d^13*e^
6 - 9536*a*b^6*c^6*d^12*e^7 - 576*a*b^7*c^5*d^11*e^8 + 3648*a*b^8*c^4*d^10*
e^9 - 1856*a*b^9*c^3*d^9*e^10 + 320*a*b^10*c^2*d^8*e^11 - 5376*a^2*b*c^10*d
^15*e^4 - 25344*a^3*b*c^9*d^13*e^6 - 37120*a^4*b*c^8*d^11*e^8 - 11520*a^5*b
*c^7*d^9*e^10 + 20736*a^6*b*c^6*d^7*e^12 + 20224*a^7*b*c^5*d^5*e^14 + 5376*
a^8*b*c^4*d^3*e^16))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5
*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^
3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9
*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))*(-(b^7
*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*
b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) +
 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*
c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/
2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2
)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2
+ 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^
8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*
d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b
^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 -
 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 +
 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4
+ 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a
*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e
^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*
a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*
b*c^3*d^3*e^5)))^(1/2) - (x*(32*c^11*d^13*e^2 + 48*a^6*b*c^4*e^15 + 96*a*c^
10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*d^12*e^3 + 4*a^4*b^5*c^2*e^15
- 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6 - 4416*a^3*c^8*d^7*e^8 - 2528*
a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336*b^2*c^9*d^11*e^4 - 268*b^3*c^
8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c^6*d^8*e^7 - 1568*b^6*c^5*d^7*
e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*e^10 + 52*b^9*c^2*d^4*e^11 - 7
584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6*e^9 + 5936*a^2*b^4*c^5*d^5*e^
10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*c^3*d^3*e^12 + 104*a^2*b^7*c^2
*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720*a^3*b^3*c^5*d^4*e^11 + 1280*a
^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^13 - 4272*a^4*b^2*c^5*d^3*e^12
+ 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10*e^5 + 3632*a*b^2*c^8*d^9*e^6
- 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7*e^8 - 4936*a*b^5*c^5*d^6*e^9
+ 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*e^11 - 128*a*b^8*c^2*d^3*e^12
+ 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^6*e^9 - 32*a^3*b^6*c^2*d*e^14
+ 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d*e^14 + 752*a^5*b*c^5*d^2*e^13
 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*
b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c
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*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*
c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))
*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4
*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e
^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^
(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 +
 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)
^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*
c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d
^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3
*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 +
 a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 +
6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^
2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^
6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d
^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*
e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*
c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^
4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 1
92*a^5*b*c^3*d^3*e^5)))^(1/2))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c
 - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*
a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a
^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9
*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2
*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^
3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a
*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b
^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*
c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a
^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c
^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c
^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*
c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e +
32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*
c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*
e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2) + (x*(54*c^9*d^
6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8*d^5*e^6 + a^2*b^2*c^5
*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48*b^3*c^6*d^3*e^8 + 9*b
^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e^10 + 10*a^2*b*c^6*d*e
^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*
b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c
*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*
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c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)))
*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + c^4*d^4*(-(4
*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32*a^2*c^5*d^3*e + 32*a^3*c^4*d*e
^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^3)^
(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 - 9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 +
 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)
^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*
c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2) - 42*a*b^3*c^3*d
^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3
*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 +
 a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2*c^5*d^8 - 4*a^2*b^7*d^3*e^5 +
6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^
2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^
6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*c^2*d^4*e^4 + 144*a^4*b^2*c^3*d
^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*
e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e + 32*a^5*b^3*c*d*e^7 - 64*a^6*b*
c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^
4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5*e^3 - 44*a^4*b^4*c*d^2*e^6 - 1
92*a^5*b*c^3*d^3*e^5)))^(1/2)))*(-(b^7*e^4 + b^3*c^4*d^4 + b^4*e^4*(-(4*a*c
 - b^2)^3)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^4 - 32
*a^2*c^5*d^3*e + 32*a^3*c^4*d*e^3 - 4*b^4*c^3*d^3*e + 25*a^2*b^3*c^2*e^4 +
a^2*c^2*e^4*(-(4*a*c - b^2)^3)^(1/2) + 6*b^5*c^2*d^2*e^2 - 4*a*b*c^5*d^4 -
9*a*b^5*c*e^4 - 4*b^6*c*d*e^3 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^2*c^4*d^3*e + 32*a*b^4*c^
2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^3)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - 42*a*b^3*c^3*d^2*e^2 + 72*a^2*b*c^4*d^2*e^2 - 72*a^2*b^2*c
^3*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^6*d^8 + a^5*b^4*e^8 + 16*a^7*c^2*e^8 + a*
b^4*c^4*d^8 - 8*a^6*b^2*c*e^8 + a*b^8*d^4*e^4 - 4*a^4*b^5*d*e^7 - 8*a^2*b^2
*c^5*d^8 - 4*a^2*b^7*d^3*e^5 + 6*a^3*b^6*d^2*e^6 + 64*a^4*c^5*d^6*e^2 + 96*
a^5*c^4*d^4*e^4 + 64*a^6*c^3*d^2*e^6 - 44*a^2*b^4*c^3*d^6*e^2 + 20*a^2*b^5*
c^2*d^5*e^3 + 64*a^3*b^2*c^4*d^6*e^2 + 32*a^3*b^3*c^3*d^5*e^3 - 74*a^3*b^4*
c^2*d^4*e^4 + 144*a^4*b^2*c^3*d^4*e^4 + 32*a^4*b^3*c^2*d^3*e^5 + 64*a^5*b^2
*c^2*d^2*e^6 - 4*a*b^5*c^3*d^7*e - 4*a*b^7*c*d^5*e^3 - 64*a^3*b*c^5*d^7*e +
 32*a^5*b^3*c*d*e^7 - 64*a^6*b*c^2*d*e^7 + 6*a*b^6*c^2*d^6*e^2 + 32*a^2*b^3
*c^4*d^7*e + 4*a^2*b^6*c*d^4*e^4 + 20*a^3*b^5*c*d^3*e^5 - 192*a^4*b*c^4*d^5
*e^3 - 44*a^4*b^4*c*d^2*e^6 - 192*a^5*b*c^3*d^3*e^5)))^(1/2)*2i + (e^2*x)/(
2*d*(d + e*x^2)*(a*e^2 + c*d^2 - b*d*e))
```

3.270 $\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 563

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \frac{x\left(c\left(b^2d^3-2ad(cd^2-3ae^2)-\frac{abe(3cd^2+ae^2)}{c}\right)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,x^2\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(ab^3e^3+6ac\left(2cd+\sqrt{b^2-4ac}e\right)(cd^2+ae^2)-b^2\left(c^2d^3-3acde^2+a\sqrt{b^2-4ac}e^3\right)-bc\left(ae^2\left(3\sqrt{b^2-4a}\right.\right.\right.}{2\sqrt{2}ac^{3/2}\,(b^2-4ac)^{3/2}\,\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(ab^3e^3+6ac\left(2cd-\sqrt{b^2-4ac}e\right)(cd^2+ae^2)-b^2\left(c^2d^3-3acde^2-a\sqrt{b^2-4ac}e^3\right)+bc\left(cd^2\left(\sqrt{b^2-4ac}\right.\right.\right.}{2\sqrt{2}ac^{3/2}\,(b^2-4ac)^{3/2}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(c*(b^2*d^3-2*a*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)/c)-(a*b^2*e^
3+2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^2)/a/c/(-4*a*c+b^2)/(c*
x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(a*
b^3*e^3+6*a*c*(a*e^2+c*d^2)*(2*c*d+e*(-4*a*c+b^2)^(1/2))-b^2*(c^2*d^3-3*a*c
*d*e^2+a*e^3*(-4*a*c+b^2)^(1/2))-b*c*(c*d^2*(12*a*e+d*(-4*a*c+b^2)^(1/2))+a
*e^2*(8*a*e+3*d*(-4*a*c+b^2)^(1/2))))/a/c^(3/2)/(-4*a*c+b^2)^(3/2)*2^(1/2)/
(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*(a*b^3*e^3+6*a*c*(a*e^2+c*d^2)*(2*c*d-e*(-4*a*c+b^2)^(1/2))-b^
2*(c^2*d^3-3*a*c*d*e^2-a*e^3*(-4*a*c+b^2)^(1/2))+b*c*(c*d^2*(-12*a*e+d*(-4*
a*c+b^2)^(1/2))+a*e^2*(-8*a*e+3*d*(-4*a*c+b^2)^(1/2))))/a/c^(3/2)/(-4*a*c+b
^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1219, 1180, 211}

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx =$$

$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ab^3e^3-b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac(ae^2+cd^2)\left(e\sqrt{b^2-4ac}+2cd\right)\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(ab^3e^3-b^2\left(-ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac(ae^2+cd^2)\left(2cd-e\sqrt{b^2-4ac}\right.\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$+\frac{x\left(c\left(-\frac{abe(ae^2+3cd^2)}{c}-2ad(cd^2-3ae^2)+b^2d^3\right)-x^2(ab^2e^3-bcd(3ae^2+cd^2)+2ace(3cd^2-ae^2))\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d^3 - 2*a*d*(c*d^2 - 3*a*e^2) - (a*b*e*(3*c*d^2 + a*e^2))/c) - (
a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^2))/(2*a
*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((a*b^3*e^3 + 6*a*c*(2*c*d + Sqrt[b
^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 -
4*a*c]*e^3) - b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2
- 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(
c^2*d^3 - 3*a*c*d*e^2 - a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4
*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

integral

$$=\frac{x\left(c\left(b^2d^3-2ad(cd^2-3ae^2)-\frac{abe\left(3cd^2+ae^2\right)}{c}\right)-\left(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{-b^2d^3+6ad\left(cd^2+ae^2\right)-\frac{abe\left(3cd^2+ae^2\right)}{c}+\left(-\frac{ab^2e^3}{c}+6ae\left(cd^2+ae^2\right)-b\left(cd^3+3ade^2\right)\right)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$=\frac{x\left(c\left(b^2d^3-2ad(cd^2-3ae^2)-\frac{abe\left(3cd^2+ae^2\right)}{c}\right)-\left(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(ab^3e^3+6ac\left(2cd-\sqrt{b^2-4ac}e\right)\left(cd^2+ae^2\right)-b^2\left(c^2d^3-3acde^2-a\sqrt{b^2-4ac}e^3\right)+bc\big(cd^2\big(\sqrt{b}\right.}{4ac\left(b^2-4ac\right)^{3/2}}-\frac{\left(ab^3e^3+6ac\left(2cd+\sqrt{b^2-4ac}e\right)\left(cd^2+ae^2\right)-b^2\left(c^2d^3-3acde^2+a\sqrt{b^2-4ac}e^3\right)-bc\big(ae^2\big(3\sqrt{}\right.}{4ac\left(b^2-4ac\right)^{3/2}}$$

$$=\frac{x\left(c\left(b^2d^3-2ad(cd^2-3ae^2)-\frac{abe\left(3cd^2+ae^2\right)}{c}\right)-\left(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(ab^3e^3+6ac\left(2cd+\sqrt{b^2-4ac}e\right)\left(cd^2+ae^2\right)-b^2\left(c^2d^3-3acde^2+a\sqrt{b^2-4ac}e^3\right)-bc\big(ae^2\big(3\sqrt{}\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2}}}+\frac{\left(ab^3e^3+6ac\left(2cd-\sqrt{b^2-4ac}e\right)\left(cd^2+ae^2\right)-b^2\left(c^2d^3-3acde^2-a\sqrt{b^2-4ac}e^3\right)+bc\big(cd^2\big(\sqrt{b}\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2}}}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2\sqrt{c}x(b^2(cd^3-ae^3x^2)+b(-a^2e^3+c^2d^3x^2-3acde(d-ex^2))+2ac(ae^2(3d+ex^2)-cd^2(d+3ex^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-ab^3e^3-6ac\left(2cd+\sqrt{b^2-4ac}e\right)(cd^2+ae^2}{}}{}$$

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b^2*(c*d^3 - a*e^3*x^2) + b*(-(a^2*e^3) + c^2*d^3*x^2 - 3*a*
c*d*e*(d - e*x^2)) + 2*a*c*(a*e^2*(3*d + e*x^2) - c*d^2*(d + 3*e*x^2))))/((
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*e^3) - 6*a*c*(2*c*d +
 Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt
[b^2 - 4*a*c]*e^3) + b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sq
rt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a
*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(-(c^2
*d^3) + 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4*a
*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]))/(4*a*c^(3/2))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.57

method	result
risch	$\frac{-\frac{\left(2a^2c\,e^3-a\,b^2e^3+3abcd\,e^2-6a\,c^2d^2e+b\,c^2d^3\right)x^3}{2ac\left(4ac-b^2\right)}+\frac{\left(a^2b\,e^3-6a^2cd\,e^2+3abc\,d^2e+2a\,c^2d^3-b^2c\,d^3\right)x}{2c\left(4ac-b^2\right)a}}{c\,x^4+b\,x^2+a} + \frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)} \left(\frac{(6a^2c}{}\right.}{}$
default	$\frac{-\frac{\left(2a^2c\,e^3-a\,b^2e^3+3abcd\,e^2-6a\,c^2d^2e+b\,c^2d^3\right)x^3}{2ac\left(4ac-b^2\right)}+\frac{\left(a^2b\,e^3-6a^2cd\,e^2+3abc\,d^2e+2a\,c^2d^3-b^2c\,d^3\right)x}{2c\left(4ac-b^2\right)a}}{c\,x^4+b\,x^2+a} + \frac{\left(6a^2c\,e^3\sqrt{-4ac+b^2}-a\,b^2e^3\sqrt{-4ac+b^2}-\right.}{}$

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(2*a^2*c*e^3-a*b^2*e^3+3*a*b*c*d*e^2-6*a*c^2*d^2*e+b*c^2*d^3)/a/c/(4*
a*c-b^2)*x^3+1/2/c*(a^2*b*e^3-6*a^2*c*d*e^2+3*a*b*c*d^2*e+2*a*c^2*d^3-b^2*c
```

```
*d^3)/(4*a*c-b^2)/a*x)/(c*x^4+b*x^2+a)+1/4/a/c*sum(((6*a^2*c*e^3-a*b^2*e^3-
3*a*b*c*d*e^2+6*a*c^2*d^2*e-b*c^2*d^3)/(4*a*c-b^2)*_R^2-(a^2*b*e^3-6*a^2*c*
d*e^2+3*a*b*c*d^2*e-6*a*c^2*d^3+b^2*c*d^3)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(
x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12117 vs. 2(507) = 1014.

Time = 107.28 (sec) , antiderivative size = 12117, normalized size of antiderivative = 21.52

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(ex^2+d)^3}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*x^
3 - (3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3)*x)/
(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c
^2)*x^2) - 1/2*integrate(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 + (b^2
*c - 6*a*c^2)*d^3 + (b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 + (a*b^2 - 6
*a^2*c)*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8992 vs. 2(507) = 1014.

Time = 1.77 (sec) , antiderivative size = 8992, normalized size of antiderivative = 15.97

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c^2*d^3*x^3 - 6*a*c^2*d^2*e*x^3 + 3*a*b*c*d*e^2*x^3 - a*b^2*e^3*x^3
+ 2*a^2*c*e^3*x^3 + b^2*c*d^3*x - 2*a*c^2*d^3*x - 3*a*b*c*d^2*e*x + 6*a^2*c
*d*e^2*x - a^2*b*e^3*x)/((c*x^4 + b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/16*
((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 -
2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^3 - 6*(2*a*b^2*c^4 - 8*a^2
*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a
*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^2*e + 3*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(
a*b^2*c - 4*a^2*c^2)^2*d*e^2 + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 24*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*
(b^2 - 4*a*c)*a^2*c^3)*(a*b^2*c - 4*a^2*c^2)^2*e^3 + 2*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 2*a*b^
6*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 20*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^5 + 28*a^2*b^4*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*c^6 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 1
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 128*a^3*b^2*c^6 + 2
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 192*a^4*c^7 + 2*(b^2 -
4*a*c)*a*b^4*c^4 - 20*(b^2 - 4*a*c)*a^2*b^2*c^5 + 48*(b^2 - 4*a*c)*a^3*c^6)
```

```
*d^3*abs(a*b^2*c - 4*a^2*c^2) + 6*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*a^2*b^5*c^4 + 16*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3
*c^5 + 16*a^3*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6
 - 32*a^4*b*c^6 + 2*(b^2 - 4*a*c)*a^2*b^3*c^4 - 8*(b^2 - 4*a*c)*a^3*b*c^5)*
d^2*e*abs(a*b^2*c - 4*a^2*c^2) - 12*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2*a^3*b^4*c^4 + 16*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c
^5 + 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 3
2*a^5*c^6 + 2*(b^2 - 4*a*c)*a^3*b^2*c^4 - 8*(b^2 - 4*a*c)*a^4*c^5)*d*e^2*ab
s(a*b^2*c - 4*a^2*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5
*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 2*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*a^3*b^5*c^3 + 16*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 +
16*a^4*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 32*a
^5*b*c^5 + 2*(b^2 - 4*a*c)*a^3*b^3*c^3 - 8*(b^2 - 4*a*c)*a^4*b*c^4)*e^3*abs
(a*b^2*c - 4*a^2*c^2) + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 224*a^4*b^3*c^8 -
 384*a^5*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^7*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^5*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^6*c^5 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^3*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^4*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^6 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
5*b*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b^3*c^7 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*b^3*c^7 - 96*(b^
2 - 4*a*c)*a^4*b*c^8)*d^3 + 12*(2*a^3*b^6*c^6 - 16*a^4*b^4*c^7 + 32*a^5*b^2
*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^
4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5
- 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(
b^2 - 4*a*c)*a^3*b^4*c^6 + 8*(b^2 - 4*a*c)*a^4*b^2*c^7)*d^2*e - 3*(2*a^3*b^
7*c^5 - 8*a^4*b^5*c^6 - 32*a^5*b^3*c^7 + 128*a^6*b*c^8 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4
```

```
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 - 2*(b^2 - 4*a*c)*a^3*b^5*c^5 +
32*(b^2 - 4*a*c)*a^5*b*c^7)*d*e^2 - (2*a^3*b^8*c^4 - 32*a^4*b^6*c^5 + 160*a
^5*b^4*c^6 - 256*a^6*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^6*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^5*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^5*b^2*c^6 - 2*(b^2 - 4*a*c)*a^3*b^6*c^4 + 24*(b^2 - 4*a*c)*a^4
*b^4*c^5 - 64*(b^2 - 4*a*c)*a^5*b^2*c^6)*e^3)*arctan(2*sqrt(1/2)*x/sqrt((a*
b^3*c - 4*a^2*b*c^2 + sqrt((a*b^3*c - 4*a^2*b*c^2)^2 - 4*(a^2*b^2*c - 4*a^3
*c^2)*(a*b^2*c^2 - 4*a^2*c^3)))/(a*b^2*c^2 - 4*a^2*c^3)))/((a^3*b^6*c^3 - 1
2*a^4*b^4*c^4 - 2*a^3*b^5*c^4 + 48*a^5*b^2*c^5 + 16*a^4*b^3*c^5 + a^3*b^4*c
^5 - 64*a^6*c^6 - 32*a^5*b*c^6 - 8*a^4*b^2*c^6 + 16*a^5*c^7)*abs(a*b^2*c -
4*a^2*c^2)*abs(c)) - 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^
3 - 6*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^2*e + 3*(2*a*b^3*
c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^2*d*e^2 + (2*a*b^4*c^2 - 20*
a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^
```

```
2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a*b^2*c - 4*a^2*c^2)^2*e^
3 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 14*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^5*c^4 + 2*a*b^6*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^2*c^5 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 28*a^2*b^4*c^5 - 96*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b*c^6 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^6 + 128*a^3*b^2*c^6 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7
 - 192*a^4*c^7 - 2*(b^2 - 4*a*c)*a*b^4*c^4 + 20*(b^2 - 4*a*c)*a^2*b^2*c^5 -
 48*(b^2 - 4*a*c)*a^3*c^6)*d^3*abs(a*b^2*c - 4*a^2*c^2) - 6*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4
+ 2*a^2*b^5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 16*a^3*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^6 + 32*a^4*b*c^6 - 2*(b^2 - 4*a*c)*a^2*b^3*c^4 + 8
*(b^2 - 4*a*c)*a^3*b*c^5)*d^2*e*abs(a*b^2*c - 4*a^2*c^2) + 12*(sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^
4 + 2*a^3*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c^5 - 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*c^6 + 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^3*b^2*c^4 + 8*(b^2
- 4*a*c)*a^4*c^5)*d*e^2*abs(a*b^2*c - 4*a^2*c^2) - 2*(sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 2*a^3
*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 8*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^3*c^4 - 16*a^4*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^4*b*c^5 + 32*a^5*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^3 + 8*(b^2 -
 4*a*c)*a^4*b*c^4)*e^3*abs(a*b^2*c - 4*a^2*c^2) + (2*a^2*b^7*c^6 - 40*a^3*b
^5*c^7 + 224*a^4*b^3*c^8 - 384*a^5*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4
*a*c)*a^3*b^3*c^7 - 96*(b^2 - 4*a*c)*a^4*b*c^8)*d^3 + 12*(2*a^3*b^6*c^6 - 1
6*a^4*b^4*c^7 + 32*a^5*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
```

```
b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^5*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^4*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3*b^4*c^6 + 8*(b^2 - 4*a*c)*a^4*b^
2*c^7)*d^2*e - 3*(2*a^3*b^7*c^5 - 8*a^4*b^5*c^6 - 32*a^5*b^3*c^7 + 128*a^6*
b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^
4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4
 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 -
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 + 16
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 - 2*(b
^2 - 4*a*c)*a^3*b^5*c^5 + 32*(b^2 - 4*a*c)*a^5*b*c^7)*d*e^2 - (2*a^3*b^8*c^
4 - 32*a^4*b^6*c^5 + 160*a^5*b^4*c^6 - 256*a^6*b^2*c^7 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 2*(b^2 - 4*a*c)*a^3*b^6*
c^4 + 24*(b^2 - 4*a*c)*a^4*b^4*c^5 - 64*(b^2 - 4*a*c)*a^5*b^2*c^6)*e^3)*arc
tan(2*sqrt(1/2)*x/sqrt((a*b^3*c - 4*a^2*b*c^2 - sqrt((a*b^3*c - 4*a^2*b*c^2
)^2 - 4*(a^2*b^2*c - 4*a^3*c^2)*(a*b^2*c^2 - 4*a^2*c^3)))/(a*b^2*c^2 - 4*a^
2*c^3)))/((a^3*b^6*c^3 - 12*a^4*b^4*c^4 - 2*a^3*b^5*c^4 + 48*a^5*b^2*c^5 +
16*a^4*b^3*c^5 + a^3*b^4*c^5 - 64*a^6*c^6 - 32*a^5*b*c^6 - 8*a^4*b^2*c^6 +
16*a^5*c^7)*abs(a*b^2*c - 4*a^2*c^2)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 29030, normalized size of antiderivative = 51.56

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^3*(b*c^2*d^3 - a*b^2*e^3 + 2*a^2*c*e^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^
2))/(2*a*c*(4*a*c - b^2)) - (x*(2*a*c^2*d^3 + a^2*b*e^3 - b^2*c*d^3 - 6*a^2
```

```
*c*d*e^2 + 3*a*b*c*d^2*e))/(2*a*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - ata
n(((((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a^6*c
^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c^6*d^
3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3072*a
^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3*b^6*
c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5*b^2*
c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) -
 (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 -
 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e
^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c
^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^
6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b
^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) -
18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 8
8*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 -
768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e
^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*
d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6
*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4
*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b
^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*
b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)
^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*
b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*
b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*
a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)
))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 -
9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^
6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^
6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6
 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^
5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 1
8432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88
*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 7
68*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^
2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d
^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(
1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*
a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*
b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^
6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b
*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^
(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b
^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b
```

^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d^6 -
a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a^4*b^
2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e
^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e
^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b
*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3
+ a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11
*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b
^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216
*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^
6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a
^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*
(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 -
384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 +
3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^
4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*
d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*
d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/
2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*
a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^
5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6
*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c
^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1
280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i - (((6144
*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a^6*c^6*d*e^2 -
288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c^6*d^3 + 16*a^3
*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3072*a^5*b*c^6*d
^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3*b^6*c^3*d*e^2
+ 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5*b^2*c^5*d*e^2)
/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27*a
*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d
^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4
*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 -
288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^
2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6
- 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*
c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*
c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^
6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a
^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 -
13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*
a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*
c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d
^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e

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^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*
e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6
*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 -
 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 -
6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^
4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*
b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^
6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*
c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 -
288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2
*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6
- 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c
^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c
^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6
*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^
5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 1
3824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a
^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c
*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^
5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^
5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e
^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*
a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 -
24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6
144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^
6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6
- 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*a^
2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b
^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5
 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^4*
c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 384
0*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 +
 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d
^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 38
40*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2
*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c -
 b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^
7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b
^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*
a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 +
8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b
^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^
5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*
d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d
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*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(
-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(40
96*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^
6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i)/((5*a^4*b^4*e^9 +
216*a^6*c^2*e^9 + 5*b^3*c^5*d^9 - 66*a^5*b^2*c*e^9 + a*b^7*d^3*e^6 - 9*a^3*
b^5*d*e^8 + 216*a^2*c^6*d^8*e - 9*b^4*c^4*d^8*e + 3*a^2*b^6*d^2*e^7 + 864*a
^3*c^5*d^6*e^3 + 1296*a^4*c^4*d^4*e^5 + 864*a^5*c^3*d^2*e^7 + 3*b^5*c^3*d^7
*e^2 + b^6*c^2*d^6*e^3 - 36*a*b*c^6*d^9 + 624*a^2*b^2*c^4*d^6*e^3 - 6*a^2*b
^3*c^3*d^5*e^4 - 108*a^2*b^4*c^2*d^4*e^5 + 1020*a^3*b^2*c^3*d^4*e^5 + 128*a
^3*b^3*c^2*d^3*e^6 + 384*a^4*b^2*c^2*d^2*e^7 + 54*a*b^2*c^5*d^8*e + 6*a*b^6
*c*d^4*e^5 + 153*a^4*b^3*c*d*e^8 - 612*a^5*b*c^2*d*e^8 + 24*a*b^3*c^4*d^7*e
^2 - 46*a*b^4*c^3*d^6*e^3 - 3*a*b^5*c^2*d^5*e^4 - 720*a^2*b*c^5*d^7*e^2 - 3
*a^2*b^5*c*d^3*e^6 - 1944*a^3*b*c^4*d^5*e^4 - 90*a^3*b^4*c*d^2*e^7 - 1872*a
^4*b*c^3*d^3*e^6)/(4*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*
c^3)) + (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*
a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c
^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3
072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3
*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5
*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^
3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*
d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*
c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*
a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c
^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*
a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/
2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^
2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e
^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*
d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2
*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2
)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*
e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 38
4*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*
a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664
*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^
2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 +
 a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840
*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 +
 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2
*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d
^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c
^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a
^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^
7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a
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^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2
) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2
 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^
2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d
^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*
c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e
 - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384
*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a
^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*
a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2
)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 +
a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*
a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d
^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a
^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*
d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*
d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*
a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4
*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3
*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*
a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) -
 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b
^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) -
288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3
*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e
^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e
^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d
^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5
*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2
*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9
)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e -
 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 46
08*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 38
4*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*
a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^
5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (((61
44*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a^6*c^6*d*e^2
 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c^6*d^3 + 16*a
^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3072*a^5*b*c^6
*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3*b^6*c^3*d*e^
2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5*b^2*c^5*d*e^
2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27
*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4
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*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a
^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5
 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*
b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e
^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^
7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^
8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*
b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768
*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3
 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) +
9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^1
0*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6
*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d
*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^
2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) -
 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3
 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7
 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*
c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*
a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*
d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^
4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5
 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b
^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^
6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7
*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8
*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b
^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*
a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 -
 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9
*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10
*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*
d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*
e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2
*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) -
6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3
 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 -
 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*
e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^
6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*
a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a
*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e
^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^
4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3
840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6
```

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 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8
*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 -
3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c
^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c
 - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*
b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4
*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 1382
4*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4
+ 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a
*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*
c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^
7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4
*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e
*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(
4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*
b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)))*((27*a*b^9*c^4*d^6
 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c
- b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*
a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7
*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a
*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*
b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3
+ 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3
+ 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^
3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d
^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^
3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*
e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 10
8*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a
^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a
^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a
^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e
^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10
*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2
*c^8)))^(1/2)*2i - atan(((((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*
b*c^5*e^3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3
 - 5632*a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^
5*b^3*c^4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c
^4*d^2*e - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4
*d*e^2 - 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^
2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6
+ 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*
e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*
c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6
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 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^
7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*
a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a
^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*
a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 1
3824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e
^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) -
6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b
^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2
*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*
c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^
5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a
^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 -
 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*
b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 +
 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e
^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c
^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6
 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7
*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a
*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^
3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a
^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13
824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^
4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^
2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6
*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^
6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*
c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c
^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5
*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32
*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^
6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^5*c^3*e
^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3
*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 -
 102*a^2*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 -
 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*
b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d
*e^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b
^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^
2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c
 - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5
*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c -
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 b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*
c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*
a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*
a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 -
384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e
^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^
5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*
(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^
2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b
^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b
*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b
^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(
-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4
 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8
)))^(1/2)*1i - (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3
+ 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^
4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*
e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e -
 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4
608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4
*b^2*c^3)) + (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5
*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840
*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e
- 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^
4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6
+ 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)
^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4
*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^
5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b
^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*
a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*
c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c
^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5
*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e
 - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5
+ 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a
*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^
9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6
 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^
7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*
a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*
b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*
a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e -
 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4
*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 +
```

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 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^
9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*
d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5
*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^
3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a
^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^
3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*
e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e
 - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 +
 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*
c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9
*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6
 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^5*c^3*e^6 - 72*a^
2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6
 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b
^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b
^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^
5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*
(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^
6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2
) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^
(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 150
4*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(
1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 -
b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^
3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^
2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^
7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*
a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4
+ 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c -
 b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*
d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e
^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e
^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e
^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c -
b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5
*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*
1i)/((5*a^4*b^4*e^9 + 216*a^6*c^2*e^9 + 5*b^3*c^5*d^9 - 66*a^5*b^2*c*e^9 +
a*b^7*d^3*e^6 - 9*a^3*b^5*d*e^8 + 216*a^2*c^6*d^8*e - 9*b^4*c^4*d^8*e + 3*a
^2*b^6*d^2*e^7 + 864*a^3*c^5*d^6*e^3 + 1296*a^4*c^4*d^4*e^5 + 864*a^5*c^3*d
^2*e^7 + 3*b^5*c^3*d^7*e^2 + b^6*c^2*d^6*e^3 - 36*a*b*c^6*d^9 + 624*a^2*b^2
*c^4*d^6*e^3 - 6*a^2*b^3*c^3*d^5*e^4 - 108*a^2*b^4*c^2*d^4*e^5 + 1020*a^3*b
^2*c^3*d^4*e^5 + 128*a^3*b^3*c^2*d^3*e^6 + 384*a^4*b^2*c^2*d^2*e^7 + 54*a*b
^2*c^5*d^8*e + 6*a*b^6*c*d^4*e^5 + 153*a^4*b^3*c*d*e^8 - 612*a^5*b*c^2*d*e^
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8 + 24*a*b^3*c^4*d^7*e^2 - 46*a*b^4*c^3*d^6*e^3 - 3*a*b^5*c^2*d^5*e^4 - 720
*a^2*b*c^5*d^7*e^2 - 3*a^2*b^5*c*d^3*e^6 - 1944*a^3*b*c^4*d^5*e^4 - 90*a^3*
b^4*c*d^2*e^7 - 1872*a^4*b*c^3*d^3*e^6)/(4*(64*a^5*c^4 - a^2*b^6*c + 12*a^3
*b^4*c^2 - 48*a^4*b^2*c^3)) + (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024
*a^6*b*c^5*e^3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^
5*d^3 - 5632*a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 7
68*a^5*b^3*c^4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*
b^5*c^4*d^2*e - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^
4*c^4*d*e^2 - 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b
^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11
*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b
^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216
*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^
6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a
^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*
(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 -
384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 +
3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^
4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*
d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*
d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/
2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*
a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^
5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6
*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c
^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1
280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*
c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 +
 a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*
e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^
9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*
a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6
*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^
5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(
-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 3
84*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3
744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4
 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d
^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d
^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2
) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a
^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5
*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*
b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^
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3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 12
80*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^5*
c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 1
6*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*
e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*
e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6
*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*
c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^
6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c
 - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4
*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^
7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*
a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7
*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3
 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3
 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e
^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*
d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b
^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2
*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 1
08*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*
a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*
a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*
a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*
e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^1
0*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^
2*c^8)))^(1/2) + (((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^
3 + 6144*a^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*
a^4*b^2*c^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^
4*e^3 - 3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e
 - 96*a^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 -
 4608*a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a
^4*b^2*c^3)) + (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a
^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 38
40*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*
e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*
a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^
6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^
2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c
^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*
c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5
*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 819
2*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*
a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10
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*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d
^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5
*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^
5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4
*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*
a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c
^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*
b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c -
8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^
5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 384
0*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e
 - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a
^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6
 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2
)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^
4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c
^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*
b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192
*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a
*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*
c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^
5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*
e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5
 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*
a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a
^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^
6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^5*c^3*e^6 - 72*
a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e
^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2
*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3
*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*
e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(
2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^6 - b^11*c^3*
d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 + 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1
/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9
)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1
504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)
^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6
 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*
c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*
c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*
b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 374
4*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^
4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c
 - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^
```

```
4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d
*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4
*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d
*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a
^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2
)))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 +
 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e
^6 - 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c
^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^
6 + a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b
^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 - b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) -
18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 8
8*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 -
768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e
^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*
d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 + 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^
(1/2) - 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6
*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4
*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b
^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*
b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 - 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)
^(1/2) + 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*
b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*
b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*2i
```

3.271 $\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 386

$$
\begin{aligned}
\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx
&= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad + \frac{\left(bcd^2-4acde+abe^2+\frac{8abcde+b^2(cd^2-ae^2)-4ac(3cd^2+ae^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{\left(bcd^2-4acde+abe^2-\frac{8abcde+b^2(cd^2-ae^2)-4ac(3cd^2+ae^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

```
[Out] 1/2*x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^2
)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2))*(b*c*d^2-4*a*c*d*e+a*b*e^2+(8*a*b*c*d*e+b^2*(-a*e^2+c*d^2)
-4*a*c*(a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/
(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*(b*c*d^2-4*a*c*d*e+a*b*e^2+(-8*a*b*c*d*e-b^2*(-a*e^2+c*d^2)+4*
a*c*(a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1219, 1180, 211}

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd^2-ae^2)+8abcde-4ac(ae^2+3cd^2)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd^2-ae^2)+8abcde-4ac(ae^2+3cd^2)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{x(x^2(abe^2-4acde+bcd^2)-2abde-2a(cd^2-ae^2)+b^2d^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2
 - 4*a*c*d*e + a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^
2 + a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
```

```
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad -\frac{\int \frac{-b^2d^2-2abde+2a(3cd^2+ae^2)+(-bcd^2+4acde-abe^2)x^2}{a+bx^2+cx^4}\,dx}{2a\,(b^2-4ac)} \\
&= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad +\frac{\left(bcd^2-4acde+abe^2-\frac{8abcde+b^2(cd^2-ae^2)-4ac(3cd^2+ae^2)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\,(b^2-4ac)} \\
&\quad +\frac{\left(bcd^2-4acde+abe^2+\frac{8abcde+b^2(cd^2-ae^2)-4ac(3cd^2+ae^2)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\,(b^2-4ac)} \\
&= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad +\frac{\left(bcd^2-4acde+abe^2+\frac{8abcde+b^2(cd^2-ae^2)-4ac(3cd^2+ae^2)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad +\frac{\left(bcd^2-4acde+abe^2-\frac{8abcde+b^2(cd^2-ae^2)-4ac(3cd^2+ae^2)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2x(b^2d^2+2a^2e^2+bcd^2x^2+abe(-2d+ex^2)-2acd(d+2ex^2))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(b^2(cd^2-ae^2)-4ac\left(3cd^2+e\left(\sqrt{b^2-4ac}d+ae\right)\right)+b\left(a\sqrt{b^2-4ac}e^2+cd\left(\sqrt{b^2-4}\right.\right.\right.}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*(
d + 2*e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d^2 -
 a*e^2) - 4*a*c*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b*(a*Sqrt[b^2 -
 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d^2) + a*e^2) + b*(a*Sqrt[b^2 - 4*a*c]*
e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e)) + 4*a*c*(3*c*d^2 + e*(-(Sqrt[b^2 -
 4*a*c]*d) + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.58

method	result
risch	$\frac{-\frac{\left(ab\,e^{2}-4acde+bc\,d^{2}\right)x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2e^{2}a^{2}-2abde-2d^{2}ac+b^{2}d^{2}\right)x}{2a\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(-\frac{\left(ab\,e^{2}-4acde+bc\,d^{2}\right)\_R^{2}}{4ac-b^{2}}+\frac{2e^{2}a^{2}-2abde+}{4ac-}\right.}{2c\_R^{3}+\_R b}}{4a}$
default	$\frac{-\frac{\left(ab\,e^{2}-4acde+bc\,d^{2}\right)x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2e^{2}a^{2}-2abde-2d^{2}ac+b^{2}d^{2}\right)x}{2a\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(\frac{\left(-ab\,e^{2}\sqrt{-4ac+b^{2}}+4acde\sqrt{-4ac+b^{2}}-bc\,d^{2}\sqrt{-4ac+b^{2}}-4a^{2}c\,e^{2}-a\,b^{2}e^{2}+8a\right.}{8c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4a}\right.}}\right.}{}$

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a*(a*b*e^2-4*a*c*d*e+b*c*d^2)/(4*a*c-b^2)*x^3-1/2*(2*a^2*e^2-2*a*b*d*
e-2*a*c*d^2+b^2*d^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/a*sum((-(a*b*e^2-
4*a*c*d*e+b*c*d^2)/(4*a*c-b^2)*_R^2+(2*a^2*e^2-2*a*b*d*e+6*a*c*d^2-b^2*d^2)
/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7338 vs. 2(344) = 688.

Time = 12.70 (sec) , antiderivative size = 7338, normalized size of antiderivative = 19.01

$$\int \frac{\left(d+ex^2\right)^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(ex^2+d)^2}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 - (2*a*b*d*e - 2*a^2*e^2 - (b^2 -
2*a*c)*d^2)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*
a^2*b*c)*x^2) + 1/2*integrate((2*a*b*d*e - 2*a^2*e^2 + (b^2 - 6*a*c)*d^2 +
(b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2
*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6390 vs. 2(344) = 688.

Time = 1.38 (sec) , antiderivative size = 6390, normalized size of antiderivative = 16.55

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d^2*x^3 - 4*a*c*d*e*x^3 + a*b*e^2*x^3 + b^2*d^2*x - 2*a*c*d^2*x -
2*a*b*d*e*x + 2*a^2*e^2*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*(
(2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(
b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d^2 - 4*(2*a*b^2*c^3 - 8*a^2*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2
```

```
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*
b^2 - 4*a^2*c)^2*d*e + (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*e^2
+ 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^5*c^2 - 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 28*a^2*b^4*c^3 - 96*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^4 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4
 - 128*a^3*b^2*c^4 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 1
92*a^4*c^5 + 2*(b^2 - 4*a*c)*a*b^4*c^2 - 20*(b^2 - 4*a*c)*a^2*b^2*c^3 + 48*
(b^2 - 4*a*c)*a^3*c^4)*d^2*abs(a*b^2 - 4*a^2*c) + 4*(sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*a^2*b^
5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 8*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c^3 + 16*a^3*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^4 - 32*a^4*b*c^4 + 2*(b^2 - 4*a*c)*a^2*b^3*c^2 - 8*(b^2 - 4*
a*c)*a^3*b*c^3)*d*e*abs(a*b^2 - 4*a^2*c) - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^
2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*a^3*b^4*c^2 +
 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^3 + 16*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
c^4 - 32*a^5*c^4 + 2*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3)*e
^2*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b^3*c^5
 - 384*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^6*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
5*b*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b^3*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*b^3*c^4 - 96*(b^
2 - 4*a*c)*a^4*b*c^5)*d^2 + 8*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b^2*
c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 1
```

```
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*(b^2
 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*d*e - (2*a^3*b^7*c^2 -
 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 32*(b^2 - 4*a*c
)*a^5*b*c^4)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^
3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c -
 4*a^2*c^2)))/((a^3*b^6*c - 12*a^4*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3
 + 16*a^4*b^3*c^3 + a^3*b^4*c^3 - 64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4
 + 16*a^5*c^5)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^3 - 8*a*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(a*
b^2 - 4*a^2*c)^2*d^2 - 4*(2*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*d*e +
(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2
 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*e^2 - 2*(sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 2*a*b^6
*c^2 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^3 - 28*a^2*b^4*c^3 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^4*c^4 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 10
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 128*a^3*b^2*c^4 + 24
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 192*a^4*c^5 - 2*(b^2 - 4
*a*c)*a*b^4*c^2 + 20*(b^2 - 4*a*c)*a^2*b^2*c^3 - 48*(b^2 - 4*a*c)*a^3*c^4)*
d^2*abs(a*b^2 - 4*a^2*c) - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 2*a^2*b^5*c^2 + 16*sqrt(2)*sqrt
```

```
(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 -
16*a^3*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 32*a
^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^3*c^2 + 8*(b^2 - 4*a*c)*a^3*b*c^3)*d*e*abs
(a*b^2 - 4*a^2*c) + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c -
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 2*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 2*a^3*b^4*c^2 + 16*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
4*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 16*a^4*b^2*
c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 32*a^5*c^4 - 2*(b
^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - 4*a*c)*a^4*c^3)*e^2*abs(a*b^2 - 4*a^2*c)
 + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b^3*c^5 - 384*a^5*b*c^6 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c + 20*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 112*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 32*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 192*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 96*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 16*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 48*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)
*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*b^3*c^4 - 96*(b^2 - 4*a*c)*a^4*b*c^5)*d
^2 + 8*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*(b^2 - 4*a*c)*a^3*b^4*c^3 +
 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*d*e - (2*a^3*b^7*c^2 - 8*a^4*b^5*c^3 - 32*a^5
*b^3*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^5*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6
*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b
^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b
*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 32*(b^2 - 4*a*c)*a^5*b*c^4)*e^2)*arcta
n(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a
^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6
*c - 12*a^4*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3 + 16*a^4*b^3*c^3 + a^3
*b^4*c^3 - 64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4 + 16*a^5*c^5)*abs(a*b^
```

```
2 - 4*a^2*c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 11.08 (sec) , antiderivative size = 18785, normalized size of antiderivative = 48.67

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b
^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e
^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a
^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 -
64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11*c*d^4 + a^3*b^9*e
^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d
^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(
-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2
*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2
*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1
344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 7
2*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4
*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b
^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^1
2*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c
^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b
^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^1
1*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4
 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^
4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*
c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d
^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*
b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*
a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*
c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^
5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c
^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^4 + 8*
a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^
2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 8
0*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a
```

```
*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-
(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(
1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504
*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*
c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2
*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e
 - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 25
6*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656
*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3
*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c
^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c
^6)))^(1/2)*1i - (((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2
- 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^
3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5
*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*
(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11*c*d^4
+ a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*
a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 -
b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^
3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*
a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*
d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*
c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*
e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e
- 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^
7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 384
0*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2
+ 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*
c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*
b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 7
68*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e
+ 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^
4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e
 + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^
2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 3
84*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072
*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2
*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 128
0*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c
^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2
*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^
3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/
(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3
```

```
*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a
*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c
- b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3
*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 5
12*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*
b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^
8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4
*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d
*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) -
4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24
*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 614
4*a^8*b^2*c^6)))^(1/2)*1i)/((5*b^3*c^4*d^6 - 3*a^3*b^3*c*e^6 - 4*a^4*b*c^2*
e^6 + 144*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 - 6*b^4*c^3*d^5*e + 160*a^3*c^4*
d^3*e^3 + b^5*c^2*d^4*e^2 - 36*a*b*c^5*d^6 + 152*a^2*b^2*c^3*d^3*e^3 - 29*a
^2*b^3*c^2*d^2*e^4 + 36*a*b^2*c^4*d^5*e + a*b^5*c*d^2*e^4 + 2*a^2*b^4*c*d*e
^5 + 11*a*b^3*c^3*d^4*e^2 - 8*a*b^4*c^2*d^3*e^3 - 300*a^2*b*c^4*d^4*e^2 - 1
40*a^3*b*c^3*d^2*e^4 + 36*a^3*b^2*c^2*d*e^5)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2)) + (((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*
a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c
^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 -
2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^
3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*
(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c
^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^
7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 204
8*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3
*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 12
8*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2
 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^
3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*
b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^
2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6
*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 1
6*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^
2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)
^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2
)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*
a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4
*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*
a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*
a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9
*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^
2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d
^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c
```

- b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5
*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)
+ (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2
*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^
2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^
3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3
*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b
*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c
d^4(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 2
88*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b
^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e
^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e
^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 2
56*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768
*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a
^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7
*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e
^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a
^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^
4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 153
6*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
)) + (x*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27
*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2)
- 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3
*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840
*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^
3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4
*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2
+ 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3
072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*
a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 -
1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b
*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 1
6*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c -
b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a
*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2)
+ 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3
*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*
e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2
+ 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2
*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^
4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6
*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(

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-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 +
 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6))
)^(1/2) - (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d
^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^
3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 1
2*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d
^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 38
40*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4
 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d
*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 -
96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c
^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b
^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d
^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3
*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9
*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 +
3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)))*(-(b^11*c*d^4 + a^3*b^9*e^4
+ a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4
+ 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4
*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^
7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^
4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344
*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a
^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^
4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*
c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/
2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c
 - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5
- 6144*a^8*b^2*c^6)))^(1/2)*2i - ((x^3*(a*b*e^2 + b*c*d^2 - 4*a*c*d*e))/(2*
a*(4*a*c - b^2)) + (x*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e))/(2*a*(
4*a*c - b^2)))/(a + b*x^2 + c*x^4) + atan(((((6144*a^5*c^6*d^2 + 2048*a^6*c
^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 56
32*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^
2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e +
 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2)) - (x*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 +
 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/
2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*
d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3
840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c
*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*
c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e
^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3
+ 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 +
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 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3
 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^
5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*
e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4
*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/
2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a
^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^
3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e
^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e +
 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*
a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a
^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e
*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2
 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6
)))^(1/2) + (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4
*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*
c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 -
 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4
*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3
840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^
4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*
d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 +
 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*
c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*
b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*
d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^
3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^
9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 +
 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i - (((6144*a^5*c^6*d^2 + 20
48*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*
d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 153
6*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c
^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2)) + (x*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11
*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2
)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*
a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4
*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*
a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*
a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9
*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^
2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d
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^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5
*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*
(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(
a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) -
a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*
d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2
)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4
+ 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^
6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c
^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2
*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*
e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3
+ 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b
*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*
b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8
*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a
*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*
a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3
*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(
(a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2
*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*
b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*
a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c
^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*
a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 +
 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*
b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^
2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*
(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b
^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i)/((5*b^3*c^4*d^6 -
 3*a^3*b^3*c*e^6 - 4*a^4*b*c^2*e^6 + 144*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 -
 6*b^4*c^3*d^5*e + 160*a^3*c^4*d^3*e^3 + b^5*c^2*d^4*e^2 - 36*a*b*c^5*d^6 +
 152*a^2*b^2*c^3*d^3*e^3 - 29*a^2*b^3*c^2*d^2*e^4 + 36*a*b^2*c^4*d^5*e + a*
b^5*c*d^2*e^4 + 2*a^2*b^4*c*d*e^5 + 11*a*b^3*c^3*d^4*e^2 - 8*a*b^4*c^2*d^3*
e^3 - 300*a^2*b*c^4*d^4*e^2 - 140*a^3*b*c^3*d^2*e^4 + 36*a^3*b^2*c^2*d*e^5)
/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (((6144*a^5*c
^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a
^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b^4*c^
3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 38
4*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9
*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(
4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1
```

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/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*
a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c
^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*
e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e
 + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256
*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*
a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*
e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^
2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^
6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3
*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^
9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4
 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(
4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b
^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e
^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 134
4*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*
a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b
^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4
*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1
/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*
c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5
 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3
*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^
2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 1
6*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3
*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 2
7*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2)
 + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^
3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 384
0*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d
^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^
4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2
 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 +
3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2
*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 -
 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (((6144*
a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1
920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e^2 + 384*a^4*b
^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e
 - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^
3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^
4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^
```

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9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 +
1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*
b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3
*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d
^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e
- 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 +
6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c
*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^
10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b
^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^
4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c -
b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^
6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^
4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*
a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*
c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2
+ 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3
+ 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*
a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^
5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*
b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^
4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^
4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c
^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*
e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)))*((a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^
4 + 27*a*b^9*c^2*d^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^
(1/2) + 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c
^6*d^3*e - 2048*a^7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4
- 3840*a^4*b^3*c^5*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^1
0*c*d^3*e - 128*a^3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b
^3*c^4*d^2*e^2 + 24*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^
2*e^2 - 384*a^3*b^6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e
^3 + 3072*a^5*b^2*c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^
2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*
c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)))*((a
^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*e^4 - b^11*c*d^4 + 27*a*b^9*c^2*d
^4 + 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*
c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) - 6144*a^6*c^6*d^3*e - 2048*a^
7*c^5*d*e^3 - 288*a^2*b^7*c^3*d^4 + 1504*a^3*b^5*c^4*d^4 - 3840*a^4*b^3*c^5
*d^4 + 96*a^5*b^5*c^2*e^4 - 512*a^6*b^3*c^3*e^4 - 4*a*b^10*c*d^3*e - 128*a^
3*b^7*c^2*d^2*e^2 + 1344*a^4*b^5*c^3*d^2*e^2 - 5120*a^5*b^3*c^4*d^2*e^2 + 2
4*a^3*b^8*c*d*e^3 + 72*a^2*b^8*c^2*d^3*e + 2*a^2*b^9*c*d^2*e^2 - 384*a^3*b^
```

```
6*c^3*d^3*e + 256*a^4*b^4*c^4*d^3*e - 256*a^4*b^6*c^2*d*e^3 + 3072*a^5*b^2*
c^5*d^3*e + 768*a^5*b^4*c^3*d*e^3 + 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^
2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4
096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6
*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*2i
```

3.272 $\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 293

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bd-2ae+\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(bd-2ae-\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*d-
2*a*e+(4*a*b*e-12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(b*d-2*a*e+(-4*a*b*e+12*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2)
)/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{1192, 1180, 211\}$

$$\int \frac{d+ex^2}{(a+bx^2+cx^4)^2}\,dx = \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)}{2\sqrt{2}a\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)}{2\sqrt{2}a\,(b^2-4ac)\,\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(cx^2(bd-2ae)-abe-2acd+b^2d)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*a*c*d + 4*a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a
*e - (b^2*d - 12*a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-b^2d+6acd-abe-c(bd-2ae)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}\\
&= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\\
&\quad+\frac{\left(c\left(bd-2ae-\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)}\\
&\quad+\frac{\left(c\left(bd-2ae+\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)}\\
&= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\\
&\quad+\frac{\sqrt{c}\left(bd-2ae+\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad+\frac{\sqrt{c}\left(bd-2ae-\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.06

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$=\frac{\frac{2x\left(b^2d+b\left(-ae+cdx^2\right)-2ac\left(d+ex^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}d+4ae\right)-2a\left(6cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\sqrt{2}\sqrt{c}\left(-b^2d+1\right.}{4a}$$

```
[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2) - 2*a*c*(d + e*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d + 4*a
*e) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*e - 2
*a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\frac{c(2ae-bd)x^3}{2a\left(4ac-b^2\right)}+\frac{\left(abe+2acd-b^2d\right)x}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{c(2ae-bd)\_R^2}{4ac-b^2}-\frac{abe-6acd+b^2d}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{4a}$
default	$16c^2\left(\frac{\frac{\left(-d\sqrt{-4ac+b^2}+2ae-bd\right)\sqrt{-4ac+b^2}\,x}{16ac\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}+\frac{\left(12\sqrt{-4ac+b^2}\,acd-3\sqrt{-4ac+b^2}\,b^2d-8a^2ce-6a\,b^2e+28abcd-3b^3d\right)\left(\sqrt{-4ac+b^2}-2b\right)\sqrt{2}\,\mathrm{arctanh}}{16a\left(4ac+3b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)\sqrt{-4ac+b^2}}\right.$

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*c*(2*a*e-b*d)/a/(4*a*c-b^2)*x^3+1/2*(a*b*e+2*a*c*d-b^2*d)/a/(4*a*c-b^2
)*x)/(c*x^4+b*x^2+a)+1/4/a*sum((c*(2*a*e-b*d)/(4*a*c-b^2)*_R^2-(a*b*e-6*a*c
*d+b^2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4573 vs. 2(251) = 502.

Time = 1.98 (sec) , antiderivative size = 4573, normalized size of antiderivative = 15.61

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d - 2*a*c*e)*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b
^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b
*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*
c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3
*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2
*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 - (3*b^5*c - 65*
a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*d^2*e^2 -
 (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*c^2)*e^4)*x + 1/
2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^
```

```
4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4*b*c^3)*d^2*
e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2 + (a^3*b
^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b
^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*a^5*b^6*c + 128*
a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b
^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c
)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(
(b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2
)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2
*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*b^3*c +
60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 1
2*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3
 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2
 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 - (3*b^5
*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*d
^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*c^2)*e^4
)*x - 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3
+ 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4*b*c
^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2
+ (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b^7*c + 1
44*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*a^5*b^6*
c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4
- 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 -
 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24
*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c
+ 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2
*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*
b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2
*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 +
4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5
*b^2*c^2 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4
- (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^
2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*
```

```
c^2)*e^4)*x + 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*
b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112
*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3
)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 + ((a^3*b^9 - 20*a^4*b
^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*
a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4
 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a
^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^
2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*
a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^
3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c
- 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5
 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*
e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2
)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c
^4)*d^4 - (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 2
8*a^2*b^2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c
+ 4*a^4*c^2)*e^4)*x - 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 -
672*a^3*b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c
^2 - 112*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32
*a^5*c^3)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 + ((a^3*b^9 -
20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*
b^8 - 8*a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 +
 a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*
e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 -
6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*
a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b
^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^
2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - 2*(a*b*e - (
b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3
- 4*a^2*b*c)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{ex^2+d}{\left(cx^4+bx^2+a\right)^2}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*e)*x^3 - (a*b*e - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2
*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate(-(
a*b*e + (b*c*d - 2*a*c*e)*x^2 + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a
*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4425 vs. $2(251) = 502$.

Time = 1.32 (sec) , antiderivative size = 4425, normalized size of antiderivative = 15.10

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*e*x^3 + b^2*d*x - 2*a*c*d*x - a*b*e*x)/((c*x^4 + b*x
^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d - 2*
(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*
c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^
2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
```

```
4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2
 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(
b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c)
 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*b
^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*e*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 -
 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 -
4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d + 4*(2*a^3*b^6*c^2 - 16*
a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*e)*
arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 -
 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^
3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*
b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2
 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^
```

```
2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a
*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a
^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 - 4
*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c) - 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
^4*c + 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 +
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 32*a^4*b*c^3 - 2*(b^2 - 4*a*c)*a^2*b^3*c +
8*(b^2 - 4*a*c)*a^3*b*c^2)*e*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^3
*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*
a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d + 4*(2*a^3*b^6*c^2 - 16*a^4*b^4
*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5
*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4
*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*
c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*e)*arctan(
2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2
*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 -
 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2
 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^
```

```
2*c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 12350, normalized size of antiderivative = 42.15

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4
*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*
a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2)) - (x*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)
^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*
c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*
a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2
 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e
 + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b
*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*
c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 +
 a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840
*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1
504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b
^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6
*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1
536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4
096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7
*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 +
 b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40
*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^
2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(
1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*
c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2
+ 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2)
 + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*
c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6*d - 28
8*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*
e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5
*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11
```

```
*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c -
b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288
*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5
*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)
^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128
*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2
))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c
^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c
^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a
^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2
- 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e
+ 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*
d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*
d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 -
768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d
^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a
*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2
*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d
*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*
a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a
^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b
^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*
a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c
^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*
b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e
^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e
 - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*
b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*
c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4
 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5
*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*
d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*
a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*
c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^
9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*
```

```
a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6
*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5*d^
2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2
*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10
*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 -
96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*
a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2
*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2
 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (((614
4*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d +
 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d
- 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2
*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*
a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*
d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2
*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b
^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*
c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b
^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1
024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^
2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d
^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*
c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 -
 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 3
6*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*
c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6
- 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6
144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2
 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d
*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 +
a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*
a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 15
04*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^
3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*
c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 15
36*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 40
96*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*
b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (8*a^3*c^4*e^3 + 5*b^3*c^4*d^3 + 72*a
^2*c^5*d^2*e - 3*b^4*c^3*d^2*e + 6*a^2*b^2*c^3*e^3 - 36*a*b*c^5*d^3 + 18*a*
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b^2*c^4*d^2*e + 3*a*b^3*c^3*d*e^2 - 60*a^2*b*c^4*d*e^2)/(4*(a^2*b^6 - 64*a^
5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e
^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b
*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^
3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3
*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d
*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^
5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^
9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*a^5*c^6*d - 288*a^2*b^6*c
^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3
*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2
*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^2*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2
) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2
*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 5
12*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) -
3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3
*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*
b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 +
3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c
^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d
^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*
b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^
2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6
*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8
*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x
*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*
b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d
^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*
e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*
b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9
*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 1
92*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2)*1i - (((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 563
2*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e
+ 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2)) + (x*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2
- b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^
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6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3
840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c
*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c
*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2
*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*
b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^
2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3
840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2
 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^
5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*
a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e
 + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^
2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e -
 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^
7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^
2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^
4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^5*c^6*d -
288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^
2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c
^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^2
*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c -
 b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 28
8*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^
5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2
)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 12
8*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/
2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a
^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*
c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^1
1*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c
^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a
^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2
 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e
 - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*
d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
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+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*
d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9
*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 7
68*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^
2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*
b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*
b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a
^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^
8*b^2*c^5)))^(1/2) + (((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c
^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*
b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^
2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^
2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c
^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 +
27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36
*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c
^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3
- 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))*((a^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^2*b^
7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^2*e^
2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^4*b^
4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*
a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3
*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a
^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c
 - b^2)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e -
288*a^2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*
b^5*c^2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e +
128*a^4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (8*a^3*c^4*e^
3 + 5*b^3*c^4*d^3 + 72*a^2*c^5*d^2*e - 3*b^4*c^3*d^2*e + 6*a^2*b^2*c^3*e^3
- 36*a*b*c^5*d^3 + 18*a*b^2*c^4*d^2*e + 3*a*b^3*c^3*d*e^2 - 60*a^2*b*c^4*d*
e^2)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*((a^2*e^2
```

```
*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*e^2 - b^11*d^2 + b^2*d^2*(-(4*a*c - b^2
)^9)^(1/2) + 3840*a^5*b*c^5*d^2 + 768*a^6*b*c^4*e^2 - 2*a*b^10*d*e - 288*a^
2*b^7*c^2*d^2 + 1504*a^3*b^5*c^3*d^2 - 3840*a^4*b^3*c^4*d^2 + 96*a^4*b^5*c^
2*e^2 - 512*a^5*b^3*c^3*e^2 + 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)
^(1/2) - 3072*a^6*c^5*d*e + 36*a^2*b^8*c*d*e - 192*a^3*b^6*c^2*d*e + 128*a^
4*b^4*c^3*d*e + 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/
(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b
^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i + ((x*(a*b*e - b^2
*d + 2*a*c*d))/(2*a*(4*a*c - b^2)) + (c*x^3*(2*a*e - b*d))/(2*a*(4*a*c - b^
2)))/(a + b*x^2 + c*x^4)
```

3.273 $\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 252

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

$= \{1106, 1180, 211\}$

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x(-2ac+b^2+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[In] Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{x(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{b^2-2ac-2(b^2-4ac)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\,(b^2-4ac)}$$

$$
\begin{aligned}
&= \frac{x(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\,(b^2-4ac)^{3/2}} \\
&\quad + \frac{\left(c\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\,(b^2-4ac)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\,\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\,\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.96

$$
\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{2x(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}
$$

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{b c\,x^{3}}{2a\left(4ac-b^{2}\right)}+\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(-\frac{bc\_R^{2}}{4ac-b^{2}}+\frac{6ac-b^{2}}{4ac-b^{2}}\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}}{4a}$
default	$16c^{2}\left(-\frac{-\frac{\left(-b\sqrt{-4ac+b^{2}}+4ac-b^{2}\right)x}{16a\,c^{2}\left(x^{2}+\frac{b}{2c}-\frac{\sqrt{-4ac+b^{2}}}{2c}\right)}-\frac{\left(b^{2}-12ac+b\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{16ac\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}}{4\sqrt{-4ac+b^{2}}\,(4ac-b^{2})}-\frac{\frac{\left(b\sqrt{-4ac+b^{2}}+4ac-b^{2}\right)x}{16a\,c^{2}\left(x^{2}+\frac{\sqrt{-4ac+b^{2}}}{2c}+\frac{b}{2c}\right)}+}{}\right.$

```
[In] int(1/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a*b*c/(4*a*c-b^2)*x^3+1/2*(2*a*c-b^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a
)+1/4/a*sum((-b*c/(4*a*c-b^2)*_R^2+(6*a*c-b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*b)
*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. 2(206) = 412.

Time = 0.41 (sec) , antiderivative size = 2309, normalized size of antiderivative = 9.16

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
```

```
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \int \frac{1}{(cx^4+bx^2+a)^2}\,dx$$

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. 2(206) = 412.

Time = 0.61 (sec) , antiderivative size = 2682, normalized size of antiderivative = 10.64

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) - 1
/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^
3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c
)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b
^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sq
```

```
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c
)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sq
rt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2
 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*
a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 6
4*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)
*abs(c)) + 1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5
*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7
+ 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b
*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*
(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))
*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2
- 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a
^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3
*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^
2 - 4*a^2*c)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 6404, normalized size of antiderivative = 25.41

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 +
 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 38
40*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*
b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
- b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 384
0*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2
 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 409
6*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b
^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a
^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-
(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1
504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c
^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^
5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^
5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11
```

```
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16
*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 24
0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(
1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4
- 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^
8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27
*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 2
4*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*a^5*c^3 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2)*2i + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*
b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3
- 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2
*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5
*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*
```

```
(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c
^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^
3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*
(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*
c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^
3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*
a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*
b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*
a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 +
 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b
^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3
840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^
2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^
2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^
7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*
c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*
(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7
*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c
^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((
((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a
^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x
*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2
- 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^
9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 -
1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b
*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 1
6*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5
*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c
 + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^
10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2
*c^5)))^(1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3
*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2)) + (x*(-(b^11 - b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
```

```
(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4)
)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))
/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 - b^2*(-(4*a*c - b^2)^9)
^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3
*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^
9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))))*(-(b^11 - b^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^
3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a
^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5)))^(1/2)*2i
```

3.274 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 660

$$\begin{aligned}
\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx
&= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)x^2)}{2a(b^2-4ac)(cd^2-bde+ae^2)(a+bx^2+cx^4)} \\
&\quad - \frac{\sqrt{c}e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\
&\quad + \frac{\sqrt{c}\left(bcd-b^2e+2ace+\frac{b^2cd-12ac^2d-b^3e+8abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} \\
&\quad - \frac{\sqrt{c}e^2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\
&\quad + \frac{\sqrt{c}\left(bcd-b^2e+2ace-\frac{b^2cd-12ac^2d-b^3e+8abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} \\
&\quad + \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2}
\end{aligned}$$

```
[Out] 1/2*x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^2)/a/(-4
*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)+e^(7/2)*arctan(x*e^(1/2)/d^(1
/2))/(a*e^2-b*d*e+c*d^2)^2/d^(1/2)-1/2*e^2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*
d*e+c*d^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2*e+2*a*c*e+(8*a*b*c*e-1
2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*
```

```
d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*e^2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(
a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1
/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2*e+2*a*c*e+(-8*
a*b*c*e+12*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/(a*e^2
-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1252, 211, 1192, 1180}

$$\begin{aligned}
&\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx \\
&= -\frac{\sqrt{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)^2}-\frac{\sqrt{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)^2} \\
&\quad+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)}{2\sqrt{2}a\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)} \\
&\quad+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)}{2\sqrt{2}a\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)} \\
&\quad+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)^2} \\
&\quad+\frac{x(cx^2(2ace+b^2(-e)+bcd)+3abce-2ac^2d+b^3(-e)+b^2cd)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)\,(ae^2-bde+cd^2)}
\end{aligned}$$

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x
^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^2 + c*x^4)) - (Sqr
t[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*a*c^2
*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b
^2*e + 2*a*c*e - (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
```

```
b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{e^4}{(cd^2-bde+ae^2)^2 (d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)^2} - \frac{e^2(-cd+be+cex^2)}{(cd^2-bde+ae^2)^2 (a+bx^2+cx^4)} \right) dx$$

$$= -\frac{e^2 \int \frac{-cd+be+cex^2}{a+bx^2+cx^4}\,dx}{(cd^2-bde+ae^2)^2} + \frac{e^4 \int \frac{1}{d+ex^2}\,dx}{(cd^2-bde+ae^2)^2} + \frac{\int \frac{cd-be-cex^2}{(a+bx^2+cx^4)^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^2)}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,(a+bx^2+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2} - \frac{\int \frac{-b^2cd+6ac^2d+b^3e-5abce-c(bcd-b^2e+2ace)x^2}{a+bx^2+cx^4}\,dx}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)} - \frac{\left(ce^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)^2} - \frac{\left(ce^2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)^2}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^2)}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,(a+bx^2+cx^4)} - \frac{\sqrt{c}e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} - \frac{\sqrt{c}e^2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2} + \frac{\left(c\left(bcd-b^2e+2ace-\frac{b^2cd-12ac^2d-b^3e+8abce}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\,(b^2-4ac)\,(cd^2-bde+ae^2)} + \frac{\left(c\left(bcd-b^2e+2ace+\frac{b^2cd-12ac^2d-b^3e+8abce}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\,(b^2-4ac)\,(cd^2-bde+ae^2)}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^2)}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,(a+bx^2+cx^4)} - \frac{\sqrt{c}e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} + \frac{\sqrt{c}\left(bcd-b^2e+2ace+\frac{b^2cd-12ac^2d-b^3e+8abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}e^2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} + \frac{\sqrt{c}\left(bcd-b^2e+2ace-\frac{b^2cd-12ac^2d-b^3e+8abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.07

$$\int \frac{1}{(d+ex^2)\,(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2(cd^2+e(-bd+ae))x(b^3e-bc(3ae+cdx^2)+2ac^2(d-ex^2)+b^2c(-d+ex^2))}{a(-b^2+4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\Big(b^4de^2+2ac\Big(-6c^2d^3+5a\sqrt{b^2-4ac}e^3+cde\Big(\sqrt{b^2-4ac}d-14ae\Big)}{}}{}$$

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*(c*d^2 + e*(-(b*d) + a*e))*x*(b^3*e - b*c*(3*a*e + c*d*x^2) + 2*a*c^2*(
d - e*x^2) + b^2*c*(-d + e*x^2)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) +
(Sqrt[2]*Sqrt[c]*(b^4*d*e^2 + 2*a*c*(-6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3
 + c*d*e*(Sqrt[b^2 - 4*a*c]*d - 14*a*e)) + b^3*e*(-2*c*d^2 + e*(Sqrt[b^2 -
4*a*c]*d - 3*a*e)) + b^2*(c^2*d^3 - 3*a*Sqrt[b^2 - 4*a*c]*e^3 - c*d*e*(2*Sq
rt[b^2 - 4*a*c]*d + 3*a*e)) + b*c*(a*e^2*(-(Sqrt[b^2 - 4*a*c]*d) + 16*a*e)
+ c*d^2*(Sqrt[b^2 - 4*a*c]*d + 20*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*Sqrt[c]*(-(b^4*d*e^2) - b^2*(c^2*d^3 + 3*a*Sqrt[b^2 - 4*a*c]*e^3
 + c*d*e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + b^3*e*(2*c*d^2 + e*(Sqrt[b^2 -
4*a*c]*d + 3*a*e)) + 2*a*c*(6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(
Sqrt[b^2 - 4*a*c]*d + 14*a*e)) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 20*a*e)
- a*e^2*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (4*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d])/(4*(c*d^2 + e*(-(b*d) +
a*e))^2)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\frac{\frac{c\left(2a^2c\,e^3-a\,b^2e^3-abcd\,e^2+2a\,c^2d^2e+b^3d\,e^2-2b^2c\,d^2e+b\,c^2d^3\right)x^3}{2a\left(4ac-b^2\right)}+\frac{(3a^2b\,e^3c-2a^2c^2d\,e^2-a\,b^3e^3-2a\,b^2cd\,e^2+5ab\,c^2d^2e-2a\,c^3d^3+b^4d\,e^2-2b^3}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}}{}$
risch	Expression too large to display

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*e^2-b*d*e+c*d^2)^2*((1/2*c*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*c^2
*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/a/(4*a*c-b^2)*x^3+1/2*(3*a^2*b*c*
e^3-2*a^2*c^2*d*e^2-a*b^3*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^3*d^3+b
^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/a/(4
*a*c-b^2)*c*(1/8*(10*a^2*c*e^3*(-4*a*c+b^2)^(1/2)-3*a*b^2*e^3*(-4*a*c+b^2)^
(1/2)-a*b*c*d*e^2*(-4*a*c+b^2)^(1/2)+2*a*c^2*d^2*e*(-4*a*c+b^2)^(1/2)+b^3*d
*e^2*(-4*a*c+b^2)^(1/2)-2*b^2*c*d^2*e*(-4*a*c+b^2)^(1/2)+b*c^2*d^3*(-4*a*c+
b^2)^(1/2)-16*a^2*b*e^3*c+28*a^2*c^2*d*e^2+3*a*b^3*e^3+3*a*b^2*c*d*e^2-20*a
*b*c^2*d^2*e+12*a*c^3*d^3-b^4*d*e^2+2*b^3*c*d^2*e-b^2*c^2*d^3)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))-1/8*(10*a^2*c*e^3*(-4*a*c+b^2)^(1/2)-3*a*b^2*e^3*
(-4*a*c+b^2)^(1/2)-a*b*c*d*e^2*(-4*a*c+b^2)^(1/2)+2*a*c^2*d^2*e*(-4*a*c+b^2
)^(1/2)+b^3*d*e^2*(-4*a*c+b^2)^(1/2)-2*b^2*c*d^2*e*(-4*a*c+b^2)^(1/2)+b*c^2
*d^3*(-4*a*c+b^2)^(1/2)+16*a^2*b*e^3*c-28*a^2*c^2*d*e^2-3*a*b^3*e^3-3*a*b^2
*c*d*e^2+20*a*b*c^2*d^2*e-12*a*c^3*d^3+b^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3)
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))+e^4/(a*e^2-b*d*e+c*d^2)^2/(e*d)
^(1/2)*arctan(e*x/(e*d)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\,(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\,(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37254 vs. 2(574) = 1148.

Time = 7.05 (sec) , antiderivative size = 37254, normalized size of antiderivative = 56.45

$$\int \frac{1}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2
*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(d*e)) + 1/16*((2*a^2*b^7*c^8 - 40*a^3*b^
5*c^9 + 224*a^4*b^3*c^10 - 384*a^5*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^8 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^9 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^10 - 2*(b^2 - 4*a*c)*a^2*b^5*c^8 + 32*(b^2 -
 4*a*c)*a^3*b^3*c^9 - 96*(b^2 - 4*a*c)*a^4*b*c^10)*d^11 - 2*(6*a^2*b^8*c^7
- 116*a^3*b^6*c^8 + 640*a^4*b^4*c^9 - 1088*a^5*b^2*c^10 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 58*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 + 6*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 320*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 - 92*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 - 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 + 544*sqrt(2)*sqrt(b^
```

```
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 272*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 + 46*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 - 136*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 6*(b^2 - 4*a*c)*
a^2*b^6*c^7 + 92*(b^2 - 4*a*c)*a^3*b^4*c^8 - 272*(b^2 - 4*a*c)*a^4*b^2*c^9)
*d^10*e + (30*a^2*b^9*c^6 - 542*a^3*b^7*c^7 + 2744*a^4*b^5*c^8 - 3616*a^5*b
^3*c^9 - 2432*a^6*b*c^10 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^9*c^4 + 271*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^7*c^5 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^8*c^5 - 1372*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 422*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 1808*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 + 1056*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 211*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 1216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 608*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 528*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 30*(b^2 - 4*a*c)*a^2*b^7*c^6 + 422*(b^
2 - 4*a*c)*a^3*b^5*c^7 - 1056*(b^2 - 4*a*c)*a^4*b^3*c^8 - 608*(b^2 - 4*a*c)
*a^5*b*c^9)*d^9*e^2 - (40*a^2*b^10*c^5 - 634*a^3*b^8*c^6 + 2448*a^4*b^6*c^7
 + 608*a^5*b^4*c^8 - 11264*a^6*b^2*c^9 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^3 + 317*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^4 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 - 1224*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^5 - 474*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - 304*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^6 + 552*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 + 237*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 + 5632*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^7 + 2816*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 - 276*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 - 1408*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 40*(b^2 - 4*a*c)*a^
2*b^8*c^5 + 474*(b^2 - 4*a*c)*a^3*b^6*c^6 - 552*(b^2 - 4*a*c)*a^4*b^4*c^7 -
 2816*(b^2 - 4*a*c)*a^5*b^2*c^8)*d^8*e^3 + (30*a^2*b^11*c^4 - 356*a^3*b^9*c
^5 + 164*a^4*b^7*c^6 + 7728*a^5*b^5*c^7 - 16960*a^6*b^3*c^8 - 5888*a^7*b*c^
9 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c
^2 + 178*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*
c^3 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10
*c^3 - 82*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7
*c^4 - 236*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
8*c^4 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
```

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9*c^4 - 3864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*
b^5*c^5 - 780*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^6*c^5 + 118*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^7*c^5 + 8480*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^6*b^3*c^6 + 4608*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^5*b^4*c^6 + 390*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^5*c^6 + 2944*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^7*b*c^7 + 1472*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^6*b^2*c^7 - 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^5*b^3*c^7 - 736*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^6*b*c^8 - 30*(b^2 - 4*a*c)*a^2*b^9*c^4 + 236*(b^2 - 4*a*c)*a^3*b^
7*c^5 + 780*(b^2 - 4*a*c)*a^4*b^5*c^6 - 4608*(b^2 - 4*a*c)*a^5*b^3*c^7 - 14
72*(b^2 - 4*a*c)*a^6*b*c^8)*d^7*e^4 - 2*(6*a^2*b^12*c^3 - 22*a^3*b^10*c^4 -
 608*a^4*b^8*c^5 + 3720*a^5*b^6*c^6 - 2624*a^6*b^4*c^7 - 10624*a^7*b^2*c^8
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^12*c +
11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c^2 +
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c^2 +
 304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^3
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^3 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^3 -
 1860*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^4
 - 600*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^
4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^4 +
 1312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^5
 + 1320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c
^5 + 300*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*
c^5 + 5312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^
2*c^6 + 2656*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*
b^3*c^6 - 660*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b^4*c^6 - 1328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^6*b^2*c^7 - 6*(b^2 - 4*a*c)*a^2*b^10*c^3 - 2*(b^2 - 4*a*c)*a^3*b^8*c^4 + 6
00*(b^2 - 4*a*c)*a^4*b^6*c^5 - 1320*(b^2 - 4*a*c)*a^5*b^4*c^6 - 2656*(b^2 -
 4*a*c)*a^6*b^2*c^7)*d^6*e^5 + (2*a^2*b^13*c^2 + 42*a^3*b^11*c^3 - 704*a^4*
b^9*c^4 + 1396*a^5*b^7*c^5 + 8880*a^6*b^5*c^6 - 23872*a^7*b^3*c^7 - 6912*a^
8*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^1
3 - 21*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^11*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^12*c +
 352*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c^2
+ 50*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c^2
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c^2 -
 698*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^3
- 504*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^3
 - 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^3
 - 4440*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c
^4 - 620*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*
```

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c^4 + 252*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7
*c^4 + 11936*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*
b^3*c^5 + 6400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
6*b^4*c^5 + 310*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^5*c^5 + 3456*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^8*b*c^6 + 1728*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^7*b^2*c^6 - 3200*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^6*b^3*c^6 - 864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^7*b*c^7 - 2*(b^2 - 4*a*c)*a^2*b^11*c^2 - 50*(b^2 - 4*a*c)*a^3*b^9*c^3
 + 504*(b^2 - 4*a*c)*a^4*b^7*c^4 + 620*(b^2 - 4*a*c)*a^5*b^5*c^5 - 6400*(b^
2 - 4*a*c)*a^6*b^3*c^6 - 1728*(b^2 - 4*a*c)*a^7*b*c^7)*d^5*e^6 - (14*a^3*b^
12*c^2 - 48*a^4*b^10*c^3 - 1148*a^5*b^8*c^4 + 6784*a^6*b^6*c^5 - 4800*a^7*b
^4*c^6 - 17920*a^8*b^2*c^7 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^12 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*b^10*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^11*c + 574*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*b^8*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^9*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b^10*c^2 - 3392*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^6*b^6*c^3 - 1116*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^7*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^8*c^3 + 2400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^7*b^4*c^4 + 2320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^6*b^5*c^4 + 558*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^5*b^6*c^4 + 8960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^8*b^2*c^5 + 4480*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^5 - 1160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^5 - 2240*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^6 - 14*(b^2 - 4*a*c)*a^3*b^10*c^2 - 8*(b
^2 - 4*a*c)*a^4*b^8*c^3 + 1116*(b^2 - 4*a*c)*a^5*b^6*c^4 - 2320*(b^2 - 4*a*
c)*a^6*b^4*c^5 - 4480*(b^2 - 4*a*c)*a^7*b^2*c^6)*d^4*e^7 + (36*a^4*b^11*c^2
 - 356*a^5*b^9*c^3 + 218*a^6*b^7*c^4 + 5880*a^7*b^5*c^5 - 12448*a^8*b^3*c^6
 - 3968*a^9*b*c^7 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*b^11 + 178*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^5*b^9*c + 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b^10*c - 109*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^6*b^7*c^2 - 212*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^5*b^8*c^2 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^9*c^2 - 2940*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^7*b^5*c^3 - 630*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*b^6*c^3 + 106*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^5*b^7*c^3 + 6224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^8*b^3*c^4 + 3360*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^7*b^4*c^4 + 315*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^6*b^5*c^4 + 1984*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
```

```
2 - 4*a*c)*c)*a^9*b*c^5 + 992*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^8*b^2*c^5 - 1680*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^7*b^3*c^5 - 496*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^8*b*c^6 - 36*(b^2 - 4*a*c)*a^4*b^9*c^2 + 212*(b^2 - 4*a*c
)*a^5*b^7*c^3 + 630*(b^2 - 4*a*c)*a^6*b^5*c^4 - 3360*(b^2 - 4*a*c)*a^7*b^3*
c^5 - 992*(b^2 - 4*a*c)*a^8*b*c^6)*d^3*e^8 - 2*(22*a^5*b^10*c^2 - 270*a^6*b
^8*c^3 + 908*a^7*b^6*c^4 + 64*a^8*b^4*c^5 - 3136*a^9*b^2*c^6 - 11*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^10 + 135*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c + 22*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^9*c - 454*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 182*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 - 11*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^8*c^2 - 32*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 180*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 + 91*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^3 + 1568*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 + 784*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 - 90*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^4 - 392*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^5 - 22*(b^2 - 4
*a*c)*a^5*b^8*c^2 + 182*(b^2 - 4*a*c)*a^6*b^6*c^3 - 180*(b^2 - 4*a*c)*a^7*b
^4*c^4 - 784*(b^2 - 4*a*c)*a^8*b^2*c^5)*d^2*e^9 + (26*a^6*b^9*c^2 - 342*a^7
*b^7*c^3 + 1432*a^8*b^5*c^4 - 1696*a^9*b^3*c^5 - 896*a^10*b*c^6 - 13*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 171*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 26*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 716*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 238*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 13*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 848*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 480*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 119*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 + 448*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 + 224*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 240*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 - 112*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 26*(b^2 -
4*a*c)*a^6*b^7*c^2 + 238*(b^2 - 4*a*c)*a^7*b^5*c^3 - 480*(b^2 - 4*a*c)*a^8*
b^3*c^4 - 224*(b^2 - 4*a*c)*a^9*b*c^5)*d*e^10 - (6*a^7*b^8*c^2 - 80*a^8*b^6
*c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
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sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2
- 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e^11 + 2*(sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^4*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5
- 2*a*b^6*c^5 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 20
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^6 + 28*a^2*b^4*c^6 - 96*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*c^7 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^7 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^7 - 128*a^3*b^2
*c^7 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^8 + 192*a^4*c^8 + 2
*(b^2 - 4*a*c)*a*b^4*c^5 - 20*(b^2 - 4*a*c)*a^2*b^2*c^6 + 48*(b^2 - 4*a*c)*
a^3*c^7)*d^7*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*
c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2
*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4) - 2*(4*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 55*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^5*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4
 - 8*a*b^7*c^4 + 248*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 +
78*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 4*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 + 110*a^2*b^5*c^5 - 368*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 184*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^2*c^6 - 39*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 -
496*a^3*b^3*c^6 + 92*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 + 73
6*a^4*b*c^7 + 8*(b^2 - 4*a*c)*a*b^5*c^4 - 78*(b^2 - 4*a*c)*a^2*b^3*c^5 + 18
4*(b^2 - 4*a*c)*a^3*b*c^6)*d^6*e*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^
3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3
*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^
4) + 4*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 - 38*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^7*c^3 - 6*a*b^8*c^3 + 147*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^4*c^4 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4
+ 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 + 76*a^2*b^6*c^4 - 12
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 - 86*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 - 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c^5 - 294*a^3*b^4*c^5 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^5*c^6 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 + 43
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 240*a^4*b^2*c^6 + 52
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7 + 416*a^5*c^7 + 6*(b^2 - 4
*a*c)*a*b^6*c^3 - 52*(b^2 - 4*a*c)*a^2*b^4*c^4 + 86*(b^2 - 4*a*c)*a^3*b^2*c
^5 + 104*(b^2 - 4*a*c)*a^4*c^6)*d^5*e^2*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 -
 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2
- 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a
^4*c*e^4) - 2*(4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c - 38*sqrt(
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2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^8*c^2 - 8*a*b^9*c^2 + 37*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^5*c^3 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*
c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 + 76*a^2*b^7*c^3
+ 504*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^4 + 102*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^5*c^4 - 74*a^3*b^5*c^4 - 1200*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^5*b*c^5 - 600*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^5 - 51*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 - 1008*a^4*b^
3*c^5 + 300*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 + 2400*a^5*b*
c^6 + 8*(b^2 - 4*a*c)*a*b^7*c^2 - 44*(b^2 - 4*a*c)*a^2*b^5*c^3 - 102*(b^2 -
 4*a*c)*a^3*b^3*c^4 + 600*(b^2 - 4*a*c)*a^4*b*c^5)*d^4*e^3*abs(a*b^2*c^2*d^
4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2
*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3
+ a^3*b^2*e^4 - 4*a^4*c*e^4) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^10 + 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c - 2*a*b^10*c - 129*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^7*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 - 4*a^2*
b^8*c^2 + 646*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 210*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^3 + 6*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^6*c^3 + 258*a^3*b^6*c^3 - 768*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 452*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b^3*c^4 - 105*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 1
292*a^4*b^4*c^4 - 544*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^5 - 272
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^5 + 226*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 1536*a^5*b^2*c^5 + 136*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^5*c^6 + 1088*a^6*c^6 + 2*(b^2 - 4*a*c)*a*b^8*c +
12*(b^2 - 4*a*c)*a^2*b^6*c^2 - 210*(b^2 - 4*a*c)*a^3*b^4*c^3 + 452*(b^2 - 4
*a*c)*a^4*b^2*c^4 + 272*(b^2 - 4*a*c)*a^5*c^5)*d^3*e^4*abs(a*b^2*c^2*d^4 -
4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2
*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^
3*b^2*e^4 - 4*a^4*c*e^4) - 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^9 - 49*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c - 10*a^2*b^9*c + 87*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 58*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^6*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^2
+ 98*a^3*b^7*c^2 + 376*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3
+ 58*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 29*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^3 - 174*a^4*b^5*c^3 - 1040*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 - 520*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*b^2*c^4 - 29*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*
c^4 - 752*a^5*b^3*c^4 + 260*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c
^5 + 2080*a^6*b*c^5 + 10*(b^2 - 4*a*c)*a^2*b^7*c - 58*(b^2 - 4*a*c)*a^3*b^5
*c^2 - 58*(b^2 - 4*a*c)*a^4*b^3*c^3 + 520*(b^2 - 4*a*c)*a^5*b*c^4)*d^2*e^5*
```

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abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a
*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 +
8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4) + 14*(sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^8 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 2*a^3*b^8*c + 4
6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b^6*c^2 + 24*a^4*b^6*c^2 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^6*b^2*c^3 - 28*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 92*a^5*b^4*c^3 - 32*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 - 16*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^6*b*c^4 + 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b^2*c^4 + 96*a^6*b^2*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c
^5 + 64*a^7*c^5 + 2*(b^2 - 4*a*c)*a^3*b^6*c - 16*(b^2 - 4*a*c)*a^4*b^4*c^2
+ 28*(b^2 - 4*a*c)*a^5*b^2*c^3 + 16*(b^2 - 4*a*c)*a^6*c^4)*d*e^6*abs(a*b^2*
c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e
^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*
d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4) - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 6*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 6*a^4*b^7*c + 152*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^5*c^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7
*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 304*a^6*b^3*c^3 + 52*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 416*a^7*b*c^4 + 6*(b^2 - 4*a
*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c^2 + 104*(b^2 - 4*a*c)*a^6*b*c^3)
*e^7*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*
e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e
^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4) + (a*b^2*c^2*d^4 - 4*a^2*
c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c
*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*
e^4 - 4*a^4*c*e^4)^2*(2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*d^3 - 2*(a*b^2*c^2*d^4 - 4*a^2*c
^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*
d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e
^4 - 4*a^4*c*e^4)^2*(2*b^4*c^3 - 10*a*b^2*c^4 + 8*a^2*c^5 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 5*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
```

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b^2 - 4*a*c)*c)*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 2*(b^2 - 4*a*c)*a*c^4)*d^2*e + (
a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4
*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^
3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)^2*(2*b^5*c^2 - 10*a*b^3*c^3 + 8*a^
2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 5
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 2*(b^2
- 4*a*c)*a*b*c^3)*d*e^2 - (a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e
+ 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e
^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)^2*(6*a*
b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*e^3)*arc
tan(2*sqrt(1/2)*x/sqrt((a*b^3*c^2*d^4 - 4*a^2*b*c^3*d^4 - 2*a*b^4*c*d^3*e +
 8*a^2*b^2*c^2*d^3*e + a*b^5*d^2*e^2 - 2*a^2*b^3*c*d^2*e^2 - 8*a^3*b*c^2*d^
2*e^2 - 2*a^2*b^4*d*e^3 + 8*a^3*b^2*c*d*e^3 + a^3*b^3*e^4 - 4*a^4*b*c*e^4 +
 sqrt((a*b^3*c^2*d^4 - 4*a^2*b*c^3*d^4 - 2*a*b^4*c*d^3*e + 8*a^2*b^2*c^2*d^
3*e + a*b^5*d^2*e^2 - 2*a^2*b^3*c*d^2*e^2 - 8*a^3*b*c^2*d^2*e^2 - 2*a^2*b^4
*d*e^3 + 8*a^3*b^2*c*d*e^3 + a^3*b^3*e^4 - 4*a^4*b*c*e^4)^2 - 4*(a^2*b^2*c^
2*d^4 - 4*a^3*c^3*d^4 - 2*a^2*b^3*c*d^3*e + 8*a^3*b*c^2*d^3*e + a^2*b^4*d^2
*e^2 - 2*a^3*b^2*c*d^2*e^2 - 8*a^4*c^2*d^2*e^2 - 2*a^3*b^3*d*e^3 + 8*a^4*b*
c*d*e^3 + a^4*b^2*e^4 - 4*a^5*c*e^4)*(a*b^2*c^3*d^4 - 4*a^2*c^4*d^4 - 2*a*b
^3*c^2*d^3*e + 8*a^2*b*c^3*d^3*e + a*b^4*c*d^2*e^2 - 2*a^2*b^2*c^2*d^2*e^2
- 8*a^3*c^3*d^2*e^2 - 2*a^2*b^3*c*d*e^3 + 8*a^3*b*c^2*d*e^3 + a^3*b^2*c*e^4
 - 4*a^4*c^2*e^4)))/(a*b^2*c^3*d^4 - 4*a^2*c^4*d^4 - 2*a*b^3*c^2*d^3*e + 8*
a^2*b*c^3*d^3*e + a*b^4*c*d^2*e^2 - 2*a^2*b^2*c^2*d^2*e^2 - 8*a^3*c^3*d^2*e
^2 - 2*a^2*b^3*c*d*e^3 + 8*a^3*b*c^2*d*e^3 + a^3*b^2*c*e^4 - 4*a^4*c^2*e^4)
))/((a^3*b^6*c^4 - 12*a^4*b^4*c^5 - 2*a^3*b^5*c^5 + 48*a^5*b^2*c^6 + 16*a^4
*b^3*c^6 + a^3*b^4*c^6 - 64*a^6*c^7 - 32*a^5*b*c^7 - 8*a^4*b^2*c^7 + 16*a^5
*c^8)*d^8*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2
*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^
3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c) - 4*(a^3*b^7*
c^3 - 12*a^4*b^5*c^4 - 2*a^3*b^6*c^4 + 48*a^5*b^3*c^5 + 16*a^4*b^4*c^5 + a^
3*b^5*c^5 - 64*a^6*b*c^6 - 32*a^5*b^2*c^6 - 8*a^4*b^3*c^6 + 16*a^5*b*c^7)*d
^7*e*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*
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e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e
^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c) + 2*(3*a^3*b^8*c^2
 - 34*a^4*b^6*c^3 - 6*a^3*b^7*c^3 + 120*a^5*b^4*c^4 + 44*a^4*b^5*c^4 + 3*a^
3*b^6*c^4 - 96*a^6*b^2*c^5 - 64*a^5*b^3*c^5 - 22*a^4*b^4*c^5 - 128*a^7*c^6
 - 64*a^6*b*c^6 + 32*a^5*b^2*c^6 + 32*a^6*c^7)*d^6*e^2*abs(a*b^2*c^2*d^4 - 4
*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*
b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3
*b^2*e^4 - 4*a^4*c*e^4)*abs(c) - 4*(a^3*b^9*c - 9*a^4*b^7*c^2 - 2*a^3*b^8*c
^2 + 12*a^5*b^5*c^3 + 10*a^4*b^6*c^3 + a^3*b^7*c^3 + 80*a^6*b^3*c^4 + 16*a^
5*b^4*c^4 - 5*a^4*b^5*c^4 - 192*a^7*b*c^5 - 96*a^6*b^2*c^5 - 8*a^5*b^3*c^5
+ 48*a^6*b*c^6)*d^5*e^3*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e
 + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*
e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c)
 + (a^3*b^10 - 2*a^3*b^9*c - 90*a^5*b^6*c^2 - 8*a^4*b^7*c^2 + a^3*b^8*c^2 +
 440*a^6*b^4*c^3 + 148*a^5*b^5*c^3 + 4*a^4*b^6*c^3 - 480*a^7*b^2*c^4 - 288*
a^6*b^3*c^4 - 74*a^5*b^4*c^4 - 384*a^8*c^5 - 192*a^7*b*c^5 + 144*a^6*b^2*c^
5 + 96*a^7*c^6)*d^4*e^4*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e
 + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*
e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c)
 - 4*(a^4*b^9 - 9*a^5*b^7*c - 2*a^4*b^8*c + 12*a^6*b^5*c^2 + 10*a^5*b^6*c^2
 + a^4*b^7*c^2 + 80*a^7*b^3*c^3 + 16*a^6*b^4*c^3 - 5*a^5*b^5*c^3 - 192*a^8*
b*c^4 - 96*a^7*b^2*c^4 - 8*a^6*b^3*c^4 + 48*a^7*b*c^5)*d^3*e^5*abs(a*b^2*c^
2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2
 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*
e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c) + 2*(3*a^5*b^8 - 34*a^6*b^6*c - 6*a
^5*b^7*c + 120*a^7*b^4*c^2 + 44*a^6*b^5*c^2 + 3*a^5*b^6*c^2 - 96*a^8*b^2*c^
3 - 64*a^7*b^3*c^3 - 22*a^6*b^4*c^3 - 128*a^9*c^4 - 64*a^8*b*c^4 + 32*a^7*b
^2*c^4 + 32*a^8*c^5)*d^2*e^6*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*
d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2
*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*a
bs(c) - 4*(a^6*b^7 - 12*a^7*b^5*c - 2*a^6*b^6*c + 48*a^8*b^3*c^2 + 16*a^7*b
^4*c^2 + a^6*b^5*c^2 - 64*a^9*b*c^3 - 32*a^8*b^2*c^3 - 8*a^7*b^3*c^3 + 16*a
^8*b*c^4)*d*e^7*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2
*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*
a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c) + (a^7*
b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^
4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*e^8*abs(a*
b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d
^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*
b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c)) + 1/16*((2*a^2*b^7*c^8 - 40*
a^3*b^5*c^9 + 224*a^4*b^3*c^10 - 384*a^5*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)
```

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*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^8 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^9 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^10 - 2*(b^2 - 4*a*c)*a^2*b^5*c^8 + 32*
(b^2 - 4*a*c)*a^3*b^3*c^9 - 96*(b^2 - 4*a*c)*a^4*b*c^10)*d^11 - 2*(6*a^2*b^
8*c^7 - 116*a^3*b^6*c^8 + 640*a^4*b^4*c^9 - 1088*a^5*b^2*c^10 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 58*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 + 6*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 320*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 - 92*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 - 3*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 + 544*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 272*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 + 46*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 - 136*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 6*(b^2 - 4
*a*c)*a^2*b^6*c^7 + 92*(b^2 - 4*a*c)*a^3*b^4*c^8 - 272*(b^2 - 4*a*c)*a^4*b^
2*c^9)*d^10*e + (30*a^2*b^9*c^6 - 542*a^3*b^7*c^7 + 2744*a^4*b^5*c^8 - 3616
*a^5*b^3*c^9 - 2432*a^6*b*c^10 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 + 271*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - 1372*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 422*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 1808*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 + 1056*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 211*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 1216*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 608*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 528*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 304*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 30*(b^2 - 4*a*c)*a^2*b^7*c^6 + 4
22*(b^2 - 4*a*c)*a^3*b^5*c^7 - 1056*(b^2 - 4*a*c)*a^4*b^3*c^8 - 608*(b^2 -
4*a*c)*a^5*b*c^9)*d^9*e^2 - (40*a^2*b^10*c^5 - 634*a^3*b^8*c^6 + 2448*a^4*b
^6*c^7 + 608*a^5*b^4*c^8 - 11264*a^6*b^2*c^9 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^3 + 317*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^4 + 40*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 - 1224*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^5 - 474*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 - 20*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - 304*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^6 + 552*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 + 237*sqrt(2)*sqrt(b^2
```

```
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 + 5632*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^7 + 2816*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 - 276*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 - 1408*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 40*(b^2 - 4*a
*c)*a^2*b^8*c^5 + 474*(b^2 - 4*a*c)*a^3*b^6*c^6 - 552*(b^2 - 4*a*c)*a^4*b^4
*c^7 - 2816*(b^2 - 4*a*c)*a^5*b^2*c^8)*d^8*e^3 + (30*a^2*b^11*c^4 - 356*a^3
*b^9*c^5 + 164*a^4*b^7*c^6 + 7728*a^5*b^5*c^7 - 16960*a^6*b^3*c^8 - 5888*a^
7*b*c^9 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^11*c^2 + 178*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^9*c^3 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^10*c^3 - 82*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^7*c^4 - 236*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^8*c^4 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^9*c^4 - 3864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^5*b^5*c^5 - 780*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b^6*c^5 + 118*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^3*b^7*c^5 + 8480*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^6*b^3*c^6 + 4608*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^5*b^4*c^6 + 390*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^5*c^6 + 2944*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^7*b*c^7 + 1472*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^6*b^2*c^7 - 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^5*b^3*c^7 - 736*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^6*b*c^8 - 30*(b^2 - 4*a*c)*a^2*b^9*c^4 + 236*(b^2 - 4*a*c)*
a^3*b^7*c^5 + 780*(b^2 - 4*a*c)*a^4*b^5*c^6 - 4608*(b^2 - 4*a*c)*a^5*b^3*c^
7 - 1472*(b^2 - 4*a*c)*a^6*b*c^8)*d^7*e^4 - 2*(6*a^2*b^12*c^3 - 22*a^3*b^10
*c^4 - 608*a^4*b^8*c^5 + 3720*a^5*b^6*c^6 - 2624*a^6*b^4*c^7 - 10624*a^7*b^
2*c^8 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^1
2*c + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^10
*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^11
*c^2 + 304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
8*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9
*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^10
*c^3 - 1860*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b
^6*c^4 - 600*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^7*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8
*c^4 + 1312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b
^4*c^5 + 1320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5
*b^5*c^5 + 300*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
4*b^6*c^5 + 5312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^7*b^2*c^6 + 2656*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^6*b^3*c^6 - 660*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b^4*c^6 - 1328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^6*b^2*c^7 - 6*(b^2 - 4*a*c)*a^2*b^10*c^3 - 2*(b^2 - 4*a*c)*a^3*b^8*c
```

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^4 + 600*(b^2 - 4*a*c)*a^4*b^6*c^5 - 1320*(b^2 - 4*a*c)*a^5*b^4*c^6 - 2656*
(b^2 - 4*a*c)*a^6*b^2*c^7)*d^6*e^5 + (2*a^2*b^13*c^2 + 42*a^3*b^11*c^3 - 70
4*a^4*b^9*c^4 + 1396*a^5*b^7*c^5 + 8880*a^6*b^5*c^6 - 23872*a^7*b^3*c^7 - 6
912*a^8*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^13 - 21*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^11*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
12*c + 352*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
9*c^2 + 50*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
10*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^11
*c^2 - 698*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^
7*c^3 - 504*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b
^8*c^3 - 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^9*c^3 - 4440*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6
*b^5*c^4 - 620*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^6*c^4 + 252*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^7*c^4 + 11936*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^7*b^3*c^5 + 6400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b^4*c^5 + 310*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b^5*c^5 + 3456*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^8*b*c^6 + 1728*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^7*b^2*c^6 - 3200*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^6*b^3*c^6 - 864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^7*b*c^7 - 2*(b^2 - 4*a*c)*a^2*b^11*c^2 - 50*(b^2 - 4*a*c)*a^3*b
^9*c^3 + 504*(b^2 - 4*a*c)*a^4*b^7*c^4 + 620*(b^2 - 4*a*c)*a^5*b^5*c^5 - 64
00*(b^2 - 4*a*c)*a^6*b^3*c^6 - 1728*(b^2 - 4*a*c)*a^7*b*c^7)*d^5*e^6 - (14*
a^3*b^12*c^2 - 48*a^4*b^10*c^3 - 1148*a^5*b^8*c^4 + 6784*a^6*b^6*c^5 - 4800
*a^7*b^4*c^6 - 17920*a^8*b^2*c^7 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^12 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b^10*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^11*c + 574*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*b^8*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^9*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b^10*c^2 - 3392*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^6*b^6*c^3 - 1116*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b^7*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^4*b^8*c^3 + 2400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^7*b^4*c^4 + 2320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^4 + 558*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^4 + 8960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^5 + 4480*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^5 - 1160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^5 - 2240*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^6 - 14*(b^2 - 4*a*c)*a^3*b^10*c^2
- 8*(b^2 - 4*a*c)*a^4*b^8*c^3 + 1116*(b^2 - 4*a*c)*a^5*b^6*c^4 - 2320*(b^2
- 4*a*c)*a^6*b^4*c^5 - 4480*(b^2 - 4*a*c)*a^7*b^2*c^6)*d^4*e^7 + (36*a^4*b^
```

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11*c^2 - 356*a^5*b^9*c^3 + 218*a^6*b^7*c^4 + 5880*a^7*b^5*c^5 - 12448*a^8*b
^3*c^6 - 3968*a^9*b*c^7 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^4*b^11 + 178*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^5*b^9*c + 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^4*b^10*c - 109*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^6*b^7*c^2 - 212*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^5*b^8*c^2 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^9*c^2 - 2940*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^7*b^5*c^3 - 630*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^6*b^6*c^3 + 106*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^5*b^7*c^3 + 6224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^8*b^3*c^4 + 3360*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^7*b^4*c^4 + 315*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^6*b^5*c^4 + 1984*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^9*b*c^5 + 992*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^8*b^2*c^5 - 1680*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^5 - 496*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^8*b*c^6 - 36*(b^2 - 4*a*c)*a^4*b^9*c^2 + 212*(b^2 -
 4*a*c)*a^5*b^7*c^3 + 630*(b^2 - 4*a*c)*a^6*b^5*c^4 - 3360*(b^2 - 4*a*c)*a^
7*b^3*c^5 - 992*(b^2 - 4*a*c)*a^8*b*c^6)*d^3*e^8 - 2*(22*a^5*b^10*c^2 - 270
*a^6*b^8*c^3 + 908*a^7*b^6*c^4 + 64*a^8*b^4*c^5 - 3136*a^9*b^2*c^6 - 11*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^10 + 135*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c + 22*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^9*c - 454*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 182*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 - 11*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^8*c^2 - 32*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 180*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 + 91*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^3 + 1568*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 + 784*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 - 90*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^4 - 392*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^5 - 22*(b
^2 - 4*a*c)*a^5*b^8*c^2 + 182*(b^2 - 4*a*c)*a^6*b^6*c^3 - 180*(b^2 - 4*a*c)
*a^7*b^4*c^4 - 784*(b^2 - 4*a*c)*a^8*b^2*c^5)*d^2*e^9 + (26*a^6*b^9*c^2 - 3
42*a^7*b^7*c^3 + 1432*a^8*b^5*c^4 - 1696*a^9*b^3*c^5 - 896*a^10*b*c^6 - 13*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 171*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 26*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 716*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 238*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 13*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 848*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 480*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 119*sq
```

```
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 + 448*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 + 224*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 240*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 - 112
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 26*(
b^2 - 4*a*c)*a^6*b^7*c^2 + 238*(b^2 - 4*a*c)*a^7*b^5*c^3 - 480*(b^2 - 4*a*c
)*a^8*b^3*c^4 - 224*(b^2 - 4*a*c)*a^9*b*c^5)*d*e^10 - (6*a^7*b^8*c^2 - 80*a
^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56
*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e^11 + 2*(sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 14*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^4*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^5 + 2*a*b^6*c^5 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^
6 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^6 - 28*a^2*b^4*c^6 - 96*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^7 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^7 + 128*a
^3*b^2*c^7 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^8 - 192*a^4*c
^8 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 20*(b^2 - 4*a*c)*a^2*b^2*c^6 - 48*(b^2 - 4
*a*c)*a^3*c^7)*d^7*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*
a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 -
 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4) - 2*(4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 55*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^6*c^4 + 8*a*b^7*c^4 + 248*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*
c^5 + 78*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 4*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 110*a^2*b^5*c^5 - 368*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 184*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^2*c^6 - 39*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^6 + 496*a^3*b^3*c^6 + 92*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
7 - 736*a^4*b*c^7 - 8*(b^2 - 4*a*c)*a*b^5*c^4 + 78*(b^2 - 4*a*c)*a^2*b^3*c^
5 - 184*(b^2 - 4*a*c)*a^3*b*c^6)*d^6*e*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 -
2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 -
 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^
4*c*e^4) + 4*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 - 38*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 6*sqrt(2)*sqrt(b*c - sqrt
```

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(b^2 - 4*a*c)*c)*a*b^7*c^3 + 6*a*b^8*c^3 + 147*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^4*c^4 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
5*c^4 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 76*a^2*b^6*c^
4 - 120*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 - 86*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 - 26*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^5 + 294*a^3*b^4*c^5 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^5*c^6 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^
6 + 43*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 240*a^4*b^2*c^
6 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 416*a^5*c^7 - 6*(b
^2 - 4*a*c)*a*b^6*c^3 + 52*(b^2 - 4*a*c)*a^2*b^4*c^4 - 86*(b^2 - 4*a*c)*a^3
*b^2*c^5 - 104*(b^2 - 4*a*c)*a^4*c^6)*d^5*e^2*abs(a*b^2*c^2*d^4 - 4*a^2*c^3
*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^
2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4
 - 4*a^4*c*e^4) - 2*(4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*c - 38
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^2 - 8*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 + 8*a*b^9*c^2 + 37*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^5*c^3 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^6*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 76*a^2*b^
7*c^3 + 504*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^4 + 102*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 22*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^5*c^4 + 74*a^3*b^5*c^4 - 1200*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^5 - 600*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^2*c^5 - 51*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 1008*
a^4*b^3*c^5 + 300*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 2400*
a^5*b*c^6 - 8*(b^2 - 4*a*c)*a*b^7*c^2 + 44*(b^2 - 4*a*c)*a^2*b^5*c^3 + 102*
(b^2 - 4*a*c)*a^3*b^3*c^4 - 600*(b^2 - 4*a*c)*a^4*b*c^5)*d^4*e^3*abs(a*b^2*
c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e
^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*
d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^10 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c - 2*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*c + 2*a*b^10*c - 129*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^7*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 +
4*a^2*b^8*c^2 + 646*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 2
10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^3 + 6*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 258*a^3*b^6*c^3 - 768*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 452*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^4*b^3*c^4 - 105*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c
^4 + 1292*a^4*b^4*c^4 - 544*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^5
 - 272*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^5 + 226*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 - 1536*a^5*b^2*c^5 + 136*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 1088*a^6*c^6 - 2*(b^2 - 4*a*c)*a*b^
8*c - 12*(b^2 - 4*a*c)*a^2*b^6*c^2 + 210*(b^2 - 4*a*c)*a^3*b^4*c^3 - 452*(b
^2 - 4*a*c)*a^4*b^2*c^4 - 272*(b^2 - 4*a*c)*a^5*c^5)*d^3*e^4*abs(a*b^2*c^2*
d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 -
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 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^
3 + a^3*b^2*e^4 - 4*a^4*c*e^4) - 2*(5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^9 - 49*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c + 10*a^2*b^9*c + 87*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 58*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b^6*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
7*c^2 - 98*a^3*b^7*c^2 + 376*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^
3*c^3 + 58*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 29*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^3 + 174*a^4*b^5*c^3 - 1040*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 - 520*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 29*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
4*b^3*c^4 + 752*a^5*b^3*c^4 + 260*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^5*b*c^5 - 2080*a^6*b*c^5 - 10*(b^2 - 4*a*c)*a^2*b^7*c + 58*(b^2 - 4*a*c)*a
^3*b^5*c^2 + 58*(b^2 - 4*a*c)*a^4*b^3*c^3 - 520*(b^2 - 4*a*c)*a^5*b*c^4)*d^
2*e^5*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3
*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*
e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4) + 14*(sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 2*a^3*b^8
*c + 46*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 16*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b^6*c^2 - 24*a^4*b^6*c^2 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^6*b^2*c^3 - 28*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^
3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 92*a^5*b^4*c^3
 - 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 - 16*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b^2*c^4 - 96*a^6*b^2*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^6*c^5 - 64*a^7*c^5 - 2*(b^2 - 4*a*c)*a^3*b^6*c + 16*(b^2 - 4*a*c)*a^4*b^
4*c^2 - 28*(b^2 - 4*a*c)*a^5*b^2*c^3 - 16*(b^2 - 4*a*c)*a^6*c^4)*d*e^6*abs(
a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4
*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^
3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4) - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c
 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c + 6*a^4*b^7*c + 152*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*b^5*c^2 - 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 + 304*a^6*b^3*c^3 + 52
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 - 416*a^7*b*c^4 - 6*(b^2
 - 4*a*c)*a^4*b^5*c + 50*(b^2 - 4*a*c)*a^5*b^3*c^2 - 104*(b^2 - 4*a*c)*a^6*
b*c^3)*e^7*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^
2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b
^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4) + (a*b^2*c^2*d^4 -
4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2
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*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^
3*b^2*e^4 - 4*a^4*c*e^4)^2*(2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*d^3 - 2*(a*b^2*c^2*d^4 - 4
*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*
b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3
*b^2*e^4 - 4*a^4*c*e^4)^2*(2*b^4*c^3 - 10*a*b^2*c^4 + 8*a^2*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 5*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 2*(b^2 - 4*a*c)*a*c^4)*d^2
*e + (a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e +
 a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3
+ 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)^2*(2*b^5*c^2 - 10*a*b^3*c^3
+ 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^5 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 2
*(b^2 - 4*a*c)*a*b*c^3)*d*e^2 - (a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*
d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2
*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)^2
*(6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*e^
3)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3*c^2*d^4 - 4*a^2*b*c^3*d^4 - 2*a*b^4*c*d
^3*e + 8*a^2*b^2*c^2*d^3*e + a*b^5*d^2*e^2 - 2*a^2*b^3*c*d^2*e^2 - 8*a^3*b*
c^2*d^2*e^2 - 2*a^2*b^4*d*e^3 + 8*a^3*b^2*c*d*e^3 + a^3*b^3*e^4 - 4*a^4*b*c
*e^4 - sqrt((a*b^3*c^2*d^4 - 4*a^2*b*c^3*d^4 - 2*a*b^4*c*d^3*e + 8*a^2*b^2*
c^2*d^3*e + a*b^5*d^2*e^2 - 2*a^2*b^3*c*d^2*e^2 - 8*a^3*b*c^2*d^2*e^2 - 2*a
^2*b^4*d*e^3 + 8*a^3*b^2*c*d*e^3 + a^3*b^3*e^4 - 4*a^4*b*c*e^4)^2 - 4*(a^2*
b^2*c^2*d^4 - 4*a^3*c^3*d^4 - 2*a^2*b^3*c*d^3*e + 8*a^3*b*c^2*d^3*e + a^2*b
^4*d^2*e^2 - 2*a^3*b^2*c*d^2*e^2 - 8*a^4*c^2*d^2*e^2 - 2*a^3*b^3*d*e^3 + 8*
```

```
a^4*b*c*d*e^3 + a^4*b^2*e^4 - 4*a^5*c*e^4)*(a*b^2*c^3*d^4 - 4*a^2*c^4*d^4 -
 2*a*b^3*c^2*d^3*e + 8*a^2*b*c^3*d^3*e + a*b^4*c*d^2*e^2 - 2*a^2*b^2*c^2*d^
2*e^2 - 8*a^3*c^3*d^2*e^2 - 2*a^2*b^3*c*d*e^3 + 8*a^3*b*c^2*d*e^3 + a^3*b^2
*c*e^4 - 4*a^4*c^2*e^4)))/(a*b^2*c^3*d^4 - 4*a^2*c^4*d^4 - 2*a*b^3*c^2*d^3*
e + 8*a^2*b*c^3*d^3*e + a*b^4*c*d^2*e^2 - 2*a^2*b^2*c^2*d^2*e^2 - 8*a^3*c^3
*d^2*e^2 - 2*a^2*b^3*c*d*e^3 + 8*a^3*b*c^2*d*e^3 + a^3*b^2*c*e^4 - 4*a^4*c^
2*e^4)))/((a^3*b^6*c^4 - 12*a^4*b^4*c^5 - 2*a^3*b^5*c^5 + 48*a^5*b^2*c^6 +
16*a^4*b^3*c^6 + a^3*b^4*c^6 - 64*a^6*c^7 - 32*a^5*b*c^7 - 8*a^4*b^2*c^7 +
16*a^5*c^8)*d^8*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2
*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*
a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c) - 4*(a^
3*b^7*c^3 - 12*a^4*b^5*c^4 - 2*a^3*b^6*c^4 + 48*a^5*b^3*c^5 + 16*a^4*b^4*c^
5 + a^3*b^5*c^5 - 64*a^6*b*c^6 - 32*a^5*b^2*c^6 - 8*a^4*b^3*c^6 + 16*a^5*b*
c^7)*d^7*e*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^
2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b
^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c) + 2*(3*a^3*b
^8*c^2 - 34*a^4*b^6*c^3 - 6*a^3*b^7*c^3 + 120*a^5*b^4*c^4 + 44*a^4*b^5*c^4
+ 3*a^3*b^6*c^4 - 96*a^6*b^2*c^5 - 64*a^5*b^3*c^5 - 22*a^4*b^4*c^5 - 128*a^
7*c^6 - 64*a^6*b*c^6 + 32*a^5*b^2*c^6 + 32*a^6*c^7)*d^6*e^2*abs(a*b^2*c^2*d
^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 -
2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3
 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c) - 4*(a^3*b^9*c - 9*a^4*b^7*c^2 - 2*a^3
*b^8*c^2 + 12*a^5*b^5*c^3 + 10*a^4*b^6*c^3 + a^3*b^7*c^3 + 80*a^6*b^3*c^4 +
 16*a^5*b^4*c^4 - 5*a^4*b^5*c^4 - 192*a^7*b*c^5 - 96*a^6*b^2*c^5 - 8*a^5*b^
3*c^5 + 48*a^6*b*c^6)*d^5*e^3*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c
*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^
2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*
abs(c) + (a^3*b^10 - 2*a^3*b^9*c - 90*a^5*b^6*c^2 - 8*a^4*b^7*c^2 + a^3*b^8
*c^2 + 440*a^6*b^4*c^3 + 148*a^5*b^5*c^3 + 4*a^4*b^6*c^3 - 480*a^7*b^2*c^4
- 288*a^6*b^3*c^4 - 74*a^5*b^4*c^4 - 384*a^8*c^5 - 192*a^7*b*c^5 + 144*a^6*
b^2*c^5 + 96*a^7*c^6)*d^4*e^4*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c
*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^
2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*
abs(c) - 4*(a^4*b^9 - 9*a^5*b^7*c - 2*a^4*b^8*c + 12*a^6*b^5*c^2 + 10*a^5*b
^6*c^2 + a^4*b^7*c^2 + 80*a^7*b^3*c^3 + 16*a^6*b^4*c^3 - 5*a^5*b^5*c^3 - 19
2*a^8*b*c^4 - 96*a^7*b^2*c^4 - 8*a^6*b^3*c^4 + 48*a^7*b*c^5)*d^3*e^5*abs(a*
b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d
^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*
b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c) + 2*(3*a^5*b^8 - 34*a^6*b^6*c
 - 6*a^5*b^7*c + 120*a^7*b^4*c^2 + 44*a^6*b^5*c^2 + 3*a^5*b^6*c^2 - 96*a^8*
b^2*c^3 - 64*a^7*b^3*c^3 - 22*a^6*b^4*c^3 - 128*a^9*c^4 - 64*a^8*b*c^4 + 32
*a^7*b^2*c^4 + 32*a^8*c^5)*d^2*e^6*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*
b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a
^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*
e^4)*abs(c) - 4*(a^6*b^7 - 12*a^7*b^5*c - 2*a^6*b^6*c + 48*a^8*b^3*c^2 + 16
```

```
*a^7*b^4*c^2 + a^6*b^5*c^2 - 64*a^9*b*c^3 - 32*a^8*b^2*c^3 - 8*a^7*b^3*c^3
+ 16*a^8*b*c^4)*d*e^7*abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e +
 8*a^2*b*c^2*d^3*e + a*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^
2 - 2*a^2*b^3*d*e^3 + 8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c) +
 (a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 +
a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*e^8*
abs(a*b^2*c^2*d^4 - 4*a^2*c^3*d^4 - 2*a*b^3*c*d^3*e + 8*a^2*b*c^2*d^3*e + a
*b^4*d^2*e^2 - 2*a^2*b^2*c*d^2*e^2 - 8*a^3*c^2*d^2*e^2 - 2*a^2*b^3*d*e^3 +
8*a^3*b*c*d*e^3 + a^3*b^2*e^4 - 4*a^4*c*e^4)*abs(c)) + 1/2*(b*c^2*d*x^3 - b
^2*c*e*x^3 + 2*a*c^2*e*x^3 + b^2*c*d*x - 2*a*c^2*d*x - b^3*e*x + 3*a*b*c*e*
x)/((a*b^2*c*d^2 - 4*a^2*c^2*d^2 - a*b^3*d*e + 4*a^2*b*c*d*e + a^2*b^2*e^2
- 4*a^3*c*e^2)*(c*x^4 + b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 237586, normalized size of antiderivative = 359.98

$$\int \frac{1}{(d+ex^2)\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan(((((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*
c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*
b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 91750
4*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 +
 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c
^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78
848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*
c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4
608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^
5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 68454
4*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c
^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 15
5136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3
*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864
256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7
*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7
 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4
*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*
e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*
a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^1
1*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 -
12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^1
0*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^
```

```
12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6
*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*
d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2
088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^
9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^1
3 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920
*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9
*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7
609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^1
0*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*
e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600
576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c
^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^1
1 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*
a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^
8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 -
 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^1
3*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c
^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^
17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 14
08*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d
*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^
9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^
11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a
^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 -
 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8
+ 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b
^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1
024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b
^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4
*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*
a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 1
92*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3
 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2
*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c
^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*
d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 +
 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a
^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7
*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*
b^3*c^3*d*e^7)) - (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a
^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 2
13*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^
7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*
a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(
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1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6
 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^
4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a
*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4
*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b
^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 1545
6*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^
4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c
^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2
*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^
9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*
a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b
^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^
4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d
^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3
*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*
d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2
*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(
4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^
2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)
^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^
3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(
1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13
*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4
*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c
^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8
 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4
*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^
8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*
c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5
*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 22
40*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e
^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c
^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*
a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6
+ 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5
*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a
^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16
384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3
*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b
^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^1
2*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b
^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^
10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7))^(
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1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^
17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c
^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a
^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 +
 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9
*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*
a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^
11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2
*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2
+ 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*
b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10
*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*
b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^
2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*
a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^
7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 10
8800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^
2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 -
 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*
a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7
*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33
280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2
*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 +
12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*
a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8
*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 -
1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^
14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^
3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 37683
20*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^
7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10
 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^
7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e
^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 7471
1040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b
^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e
^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*
a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12
*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 -
54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9
*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*
e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 1933
3120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10
*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d
^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 +
```

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 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*
a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c
^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^
15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7
*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 +
 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b
*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10
+ 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^1
2*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^
16))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^
8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6
*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2
*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*
e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2
- 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2
 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*
e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d
^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*
c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*
b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512
*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9
*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*
d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5
 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 -
 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))
*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a
^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 -
26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3
*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 38
40*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*
c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5
*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2
)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) -
 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b
^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 103
2*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^
2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^
7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a
^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*
c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2
*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2)
- 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3
*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^
2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10
*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b
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^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*
b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c
^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2
) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a
*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2
*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2
)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^
5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^
10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*
a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^
2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4
*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^
5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^
12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4
*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 -
42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e
^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c
^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^
7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3
+ 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*
d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9
*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5
- 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e
- 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96
*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^
5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*
a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*
a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 4
9152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(626688*a^
10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b
^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^
8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 12697
6*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1
067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2
 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 1
44*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^1
5*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*
c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 -
 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^
8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*
a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*
e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*
b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*
e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3
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*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 +
 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c
^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 -
 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^
6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 -
175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11
*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7
- 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*
b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^
12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a
^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*
e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496
*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8
*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4
739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^1
2*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8
*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*
c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^
16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11
680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*
d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^
7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11
 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*
c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*
b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4
*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e
^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*
c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*
d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4
*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*
c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b
^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*
a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1
536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e
^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 -
1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*
b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c
*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d
^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*
d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6
+ 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*
c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*
b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*
d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*
a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7
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*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a
*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)
^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 6
00*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e
^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^
6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^
5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 -
 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6
*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9
)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5
- 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a
*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*
a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 168
96*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22
400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 -
 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)
^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^
5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) -
42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a
*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*
b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 2
4*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^
8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 614
4*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11
*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^
5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 +
16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2
+ 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^
5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c
^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a
^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 +
 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*
d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*
b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 1
7920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*
d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^
9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*
e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e
 + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6
- 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7
 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d
*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (3269
12*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3
```

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*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a
^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080
*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532
736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*
b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7
*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10
 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3
 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5
*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 234
88*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^
3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^
3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e
^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b
^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7
- 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*
c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 2
36800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c
^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1
106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^
2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^
9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6
*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4
*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b
^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 34
2272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*
e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b
^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*
(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^
5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 2
56*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^
4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 15
36*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3
*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a
^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 12
8*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*
e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*
d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2
*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d
*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e -
4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*
a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7
*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*
b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9
*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^
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8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e +
 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b
^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6
+ 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 2
5*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/
2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*
c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*
d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^
8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168
*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^
2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c
^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*
b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e
^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^
3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e
^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d
^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*
d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e
*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a
*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2
)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4
*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/
2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4
*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8
 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*
c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^
8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6
144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d
^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d
^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^
2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b
^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 145
6*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e
^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c
^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*
a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3
- 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4
*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*
a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*
a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^1
1*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c
*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*
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c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*
c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^1
1*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(22800*a^6*c^9*e^1
3 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15
592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 3030
4*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^
5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4
*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7
 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*
d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a
^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^1
1 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 +
 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8
 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 -
 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^
8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8
*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6
*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^
4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8
+ a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7
*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6
*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^
6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5
*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*
c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6
*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536
*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6
+ 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 102
4*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7
*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^
3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*
e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e
^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3
840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e
^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^1
2*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6
 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4
*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^
3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c
- b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1
/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*
a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2
- 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d
^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b
^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59
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392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-
(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^
4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(
1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 5
1*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^
12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3
*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*
a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400
*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*
b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)
^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(
-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*
a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c
- b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c
^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)
^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a
^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 +
 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a
^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^
4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 +
 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 163
84*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 8
4*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e
^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*
d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*
b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 13
44*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5
*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3
*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 1792
0*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3
*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d
^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3
 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e +
84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 1
5360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 +
5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^
7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*1i - (((((
1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61
440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16
- 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^
12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^1
0*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 -
 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*
c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6
+ 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*
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c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 2457
6*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^1
0*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 -
 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13
*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2
560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c
^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^
5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^
4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6
*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a
^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*
d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 +
4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*
b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^1
0 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^
5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^1
2*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31
076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^
7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^1
1 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6
*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11
*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37
101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^
8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^1
4 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 3436544
0*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c
^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 -
 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*
a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d
^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 -
9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12
*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*
a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 -
 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^
9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c
^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 70451
20*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*
e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*
a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d
*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c
*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*
c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 +
a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d
^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e
^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*
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e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d
^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^
2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b
^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a
^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 +
512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*
a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c
^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*
e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^
3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7
)) + (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 +
 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c
*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b
^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d
^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a
^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*
b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*
c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^
(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 60
0*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^
2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6
*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5
*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 -
59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*
b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)
^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 -
 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*
b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a
^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 1689
6*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 224
00*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 -
4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^
9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5
*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 4
2*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*
c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b
*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24
*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8
 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144
*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*
b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5
 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 1
6384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 +
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 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5
*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^
4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^
7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 +
1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d
^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b
^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17
920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d
^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9
*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e
^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e
+ 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 -
 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7
+ 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*
e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576
*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^
11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 157
2864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*
e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12
*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 2
56*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*
d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 1433
6*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8
*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b
^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12
*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384
*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*
e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^
4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*
d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 +
490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15
*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 -
163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*
b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d
^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 22
79680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*
c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 3
27680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*
b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*
d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10
864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b
^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^
13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*
a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^
11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8
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 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^
7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^
4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728
*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*
c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^
9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*
a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*
d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9
502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*
b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*
e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080
*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3
*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*
e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 292
8640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^1
1*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*
d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 +
 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*
a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*
e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9
*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8
 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^1
1*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e
^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*
b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9
*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^
5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4
 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^
8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*
c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^
7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5
*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*
a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 -
 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e
^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*
d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7
 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3
*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b
^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*
c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c
^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6
 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c
^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^
14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^
8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 106
```

56*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4
*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) +
22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d
^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e
^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4
*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*
b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 3
7632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^
3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(
1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*
c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-
(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1
116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 +
15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e
+ 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5
- 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4
*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*
c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^
(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) +
120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c
- b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*
a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d
^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 2
40*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a
^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^
6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^
6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5
*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c
^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6
*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 -
10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^
2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b
^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 465
92*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*
e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*
b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*
b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4
*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*
e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d
^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d
*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c
^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(626688*a^10*b*c^8*e^15

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 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15
+ 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^1
5 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^
11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^
11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^1
0*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d
^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9
 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2
 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^
7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 7
8496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*
d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^
2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12
*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*
a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*
e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^
11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 +
 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b
^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8
 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^
9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13
 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5
*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e
^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a
^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d
^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 305964
8*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10
*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2
370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^
3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*
a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 60
00*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 +
 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^
12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*
c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601
536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*
e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^
8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/
(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4
*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8
 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12
*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 +
 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*
a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 19
2*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 -
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 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^
5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d
^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c
^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*
b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^
4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e
 - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 10
24*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*
a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27
*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*
c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880
*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*
e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^
4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e
^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6
 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^
(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^
13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c
^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3
*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7
168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4
*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^
2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106
*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*
c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3
*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*
d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^
8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c
^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^
5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 1
1*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c -
b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*
d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^
(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(
-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*
e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b
^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9
*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8
 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^1
4*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^
7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13
*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^
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5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 +
1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^
6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^1
0*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 322
56*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e
^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*
c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 215
04*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 163
84*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*
b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^1
3*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b
^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b
^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*
a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*
e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^1
4 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^
14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9
*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*
d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10
*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16
*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^
3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*
b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6
 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^
7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*
a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^
8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a
^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^
11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*
b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10
+ 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2
*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 -
 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^
3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^
4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*
a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a
*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 2
40*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^1
3 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^
12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*
a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13
 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8
+ 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 +
 a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^
7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3
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*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^
4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*
e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^
5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4
*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*
c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6
*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 204
8*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6
 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*
b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*
d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d
^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^
7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 -
 b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^
5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 +
 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3
*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 +
9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b
^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6
*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^
7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 +
 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180
*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3
 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*
d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5
*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 +
60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 3
9*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*
e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c -
 b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*
b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^
4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a
^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296
*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b
^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^
4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) +
86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*
b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9
)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e
^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13
*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 128
0*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b
^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*
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c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 1638
4*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^
3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1
344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e
^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^
3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^
7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 -
21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d
^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b
^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12
288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*
d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*
e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 51
20*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 2
4576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 4
9152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e
^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^
8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c
^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^
4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^
6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*
c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b
^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 12
54*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^
3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*
c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c
^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*
c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c
^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*
b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*
(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^
5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 2
56*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^
4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 15
36*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3
*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a
^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 12
8*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*
e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*
d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2
*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d
*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e -
4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*
```

a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7
*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*
b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9
*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^
8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e +
 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b
^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6
 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 2
5*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/
2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*
c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*
d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^
8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168
*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^
2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c
^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*
b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e
^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^
3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e
^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d
^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*
d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e
*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a
*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2
)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4
e^2(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/
2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4
*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8
 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*
c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^
8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6
144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d
^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d
^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^
2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b
^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 145
6*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e
^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c
^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*
a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3
 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4
*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*

```
a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*
a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^1
1*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c
*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*
c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*
c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^1
1*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*1i)/((2000*a^4*c^9*e^12
 + 21*a^2*b^4*c^7*e^12 - 520*a^3*b^2*c^8*e^12 + 1296*a^2*c^11*d^4*e^8 + 432
0*a^3*c^10*d^2*e^10 + 25*b^4*c^9*d^4*e^8 - 60*b^5*c^8*d^3*e^9 + 35*b^6*c^7*
d^2*e^10 + 192*a^2*b^2*c^9*d^2*e^10 - 112*a*b^5*c^7*d*e^11 - 4480*a^3*b*c^9
*d*e^11 - 360*a*b^2*c^10*d^4*e^8 + 832*a*b^3*c^9*d^3*e^9 - 362*a*b^4*c^8*d^
2*e^10 - 2880*a^2*b*c^10*d^3*e^9 + 1440*a^2*b^3*c^8*d*e^11)/(8*(a^6*b^8*e^8
 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7
+ a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c
^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^
3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d
^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6
*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d
^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^
4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2
*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^
6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 20
48*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6
 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2
*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c
*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*
d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e
^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((((1048576*a^13*
c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c
^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^1
2*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 5898
24*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^
10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8
*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4
+ 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*
b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 +
 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^1
2*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 -
1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b
^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9
- 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*
c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3
- 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a
^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^
8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 11200
```

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0*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d
^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 60
78464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b
^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e
^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a
^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^
3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 30801
92*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^
5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9
 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^
6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2
*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 80
28160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^
6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^1
2 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*
a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9
*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 -
 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9
*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d
^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 432
5376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*
b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15
 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^
12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*
c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^1
8*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7
012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12
*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600
*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*
d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a
^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*
b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256
*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*
e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536
*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b
^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4
*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*
a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 5
12*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^
4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^
4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c
^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e
^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*
a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^
5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b
```

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^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (x*((27*
a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c
^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*
a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e
 + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4
*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^
6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 +
 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(
1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^1
3*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^
3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*
b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 71
68*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*
e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2
*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*
a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c
*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*
e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d
*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8
*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^
5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5
*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11
*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b
^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d
^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(
1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-
(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/
(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e
^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^
8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*
d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 -
 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14
*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7
*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*
c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5
*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1
456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6
*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10
*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 3225
6*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^
3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c
^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 2150
4*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 1638
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4*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b
^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13
*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^
5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^
7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a
^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^1
7 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^
17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2
*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184
*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^1
1 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c
^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14
336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^
5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^
2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14
*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680
*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5
*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a
^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^
14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 28
67456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^
13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10
 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^
5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13
*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 515
9936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^1
2*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11
 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3
*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13
*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 13
72160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^
10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e
^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6
*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*
d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 -
 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a
^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^
6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 3148
8*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13
*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7
 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*
a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d
^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55
808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^
3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e
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^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 526336
0*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4
*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^1
0 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 150732
80*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8
*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5
*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 83
55840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^1
2*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*
d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024
*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*
e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^1
0*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e
^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 2070937
6*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256
*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*
b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8
 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11
*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4
 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 +
 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3
 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*
e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d
^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*
c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7
*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 102
4*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c
^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e
^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e
 - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3
072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11
*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6
*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072
*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3
 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2
*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^
3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4
*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8
*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*
b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*
b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 28
71*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e
^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*
c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928
*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3
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*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9
)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7
*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)
^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c
^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4
*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^
6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*
b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9
)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(
4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^
3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3
*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/
2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 +
4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^
7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6
*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^
2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e
^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^1
0*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^1
4*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a
^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 -
6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4
*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8
*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504
*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^
4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^
6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a
^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e
^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 -
 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a
^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^
6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*
a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*
a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 +
24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(626688*a^10*b*c^8*e^15 - 784384*a^1
0*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b
^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a
^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 3256
32*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 -
 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 +
240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*
b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^
3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*
b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e
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^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10
*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 92
64*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^
2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296
*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*
d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3
*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^
11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b
^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e
^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4
*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^1
1 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^
5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7
*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*
a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^
2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 1523
2*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7
*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6
873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^
5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^1
3 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^
11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7
*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*
c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*
b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 -
675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^1
2*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304
*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e
^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e
^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^
7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2
*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*
a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6
*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d
^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3
*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*
c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b
^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*
a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 +
2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e
^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a
^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10
*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^
6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d
*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^
```

```
6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a
*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^
6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*
d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6
 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^
5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*
e^6*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528
*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^
2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 +
180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*
e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c
^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*
a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3
 + 60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d
^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*
c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a
^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232
*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 134
4*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23
296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c
- b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2
*e^4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2)
 + 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a
^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2
)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^1
2*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b
^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 -
1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^
9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b
^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 1
6384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6
*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3
+ 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^
4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10
*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080
*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6
 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^
5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^
9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 +
 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c
^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6
*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*
```

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e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 +
 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e
+ 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7
- 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^
3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*e^13 - 241664
*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4
*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a
^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120
*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40
*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^
8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6
*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 -
16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10
*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*
b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 2
6384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*
d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 12643
2*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^
5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^
3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*
e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b
^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9
 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b
^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^1
2 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e
^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8
*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5
*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3
*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3
*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 -
76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d
*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7
*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8
*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*
d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^
8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5
 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*
a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*
b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^
4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 80
0*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 +
 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*
e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4
*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*
c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e
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 - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384
*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5
*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*
b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6
 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*
c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9
*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^
2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6
*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 -
30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b
^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3
 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e
^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2
*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b
^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 168
96*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*
e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3
*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2
*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*
c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2
) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e
 - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5
*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*
e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^
2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2)
- 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c -
b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*
d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1
/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3
*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108
*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9
*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*
b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7
*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 -
1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3
*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^
6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^
6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*
c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6
*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 6
72*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4
*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4
*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720
*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^
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4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*
c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*
a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b
^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*
c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^
6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*
c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c
^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^
11*b^3*c^5*d*e^7)))^(1/2) - (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 -
600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 129
6*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512
*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c
^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10
+ 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e
^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^
7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516
*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 -
41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 -
 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 -
 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 +
 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^1
2 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8
+ 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 +
 a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^
7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3
*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^
4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*
e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^
5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4
*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*
c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6
*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 204
8*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6
- 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*
b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*
d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d
^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^
7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 -
 b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^
5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 +
 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3
*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 +
9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b
^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6
*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^
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7*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 +
 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180
*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3
 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*
d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5
*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 +
60928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 3
9*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*
e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c -
 b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*
b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^
4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a
^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296
*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b
^2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^
4*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) +
86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*
b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9
)^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e
^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13
*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 128
0*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b
^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*
c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 1638
4*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^
3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1
344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e
^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^
3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^
7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 -
21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d
^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b
^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12
288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*
d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*
e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 51
20*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 2
4576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 4
9152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e
^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (((((1048576*a^13*c^8*e^16 + 256*a
^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680
*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 -
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 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10
*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832*a^
11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 +
 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^1
1*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7
 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c
^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198
656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*
c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7
+ 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14
*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 1
06496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^
6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*
e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*
a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d
^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376
*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^
12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6
 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5
*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*
e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^
5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*
d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 +
 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*
b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e
^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184
*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^
10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 -
 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*
b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8
*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 1642
9056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7
*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^
10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 923238
4*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^
9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14
 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^
10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*
b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 35
84*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 +
 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^1
3*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 32448
0*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e
^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 1920204
8*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256
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*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*
b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8
 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11
*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4
 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 +
 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3
 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*
e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d
^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*
c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7
*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 102
4*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c
^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e
^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e
- 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3
072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*((27*a*b^9*c^5*d^6 -
b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5
*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 +
3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*
e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9
*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^
7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*
(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7
*c^8*d^3*e^3 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 +
6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*
a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3
- 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d
^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*
b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 6
0928*a^6*b^3*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39
*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e
 + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c -
b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b
^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4
*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^
5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*
a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^
2)^9)^(1/2) - 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4
*(-(4*a*c - b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 8
6*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b
*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)
^(1/2) - 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^
8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*
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d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280
*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^
8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c
^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384
*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3
*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 13
44*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^
4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3
*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7
*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2
1504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^
4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^
4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 122
88*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d
^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e
^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 512
0*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24
576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49
152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^
5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12
*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12
*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048
576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*
e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13
*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2
048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7
*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 716
8*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e
^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^
3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^
11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176
*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*
e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400
*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c
^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 +
 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^1
6*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2
+ 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*
a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^
8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 +
770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15
*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 -
 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*
a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^
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9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 -
 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6
*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^
14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527
424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^
6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e
^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*
a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d
^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 3112
9600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*
b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*
e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 128614
4*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3
*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 +
 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a
^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^
4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 4980
7360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10
*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*
d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 -
11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000
*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c
^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^
14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 55050
24*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13
*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*
b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16
 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a
^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3
*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 2
56*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16
*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^
2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4
*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*
d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*
d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^
2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^
6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^
6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 51
2*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5
 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*
e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2
*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5
*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d
^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^
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3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*
d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)
^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e +
 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^
6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*
c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^
6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(
1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d
^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 147
1*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4
 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d
^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b
^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47
712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2
*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(
4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^
5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^
4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*
a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*
a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 349
2*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 74
24*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 -
53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*
c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^
(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-
(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12
*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*
c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2
*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^
8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4
*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3
840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^1
0*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^
4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 +
24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 -
140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*
e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5
*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b
^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2
240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6
*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8
*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 2150
4*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*
e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11
*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*
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d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^
4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e
- 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*
e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e
^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c
^5*d*e^7)))^(1/2) + (x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 2
08*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 24
2176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15
+ 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*
e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^1
0*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^1
1*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7
 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*
b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^
3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b
^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 3
2064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*
d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a
^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^
11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736
*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9
 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^
12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4
 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4
*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9
 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b
^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e
^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a
^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*
e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688
*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^
5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 2472
96*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c
^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 +
 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^
12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*
b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*
a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 2
40*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14
- 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^1
3*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 5147
68*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e
^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^
9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8
*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*
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d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^
8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5
 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*
a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*
b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^
4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 80
0*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 +
 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*
e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4
*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*
c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e
 - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384
*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5
*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*
b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6
 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*
c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9
*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^
2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6
*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 -
30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b
^2)^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3
 + b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e
^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2
*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b
^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 168
96*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*
e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3
*c^6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2
*e^4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*
c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2
) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e
 - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5
*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*
e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^
2*e^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2)
- 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c -
b^2)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*
d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1
/2) + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3
*(-(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108
*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9
*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*
b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7
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*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 -
1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3
*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^
6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^
6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*
c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6
*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 6
72*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4
*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4
*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720
*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^
4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*
c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*
a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b
^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*
c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^
6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*
c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c
^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^
11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14
- 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 +
75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14
+ 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^
7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e
^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 -
264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c
^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*
e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2
*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7
- 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6
*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 12505
6*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d
^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a
^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^
12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*
b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11
+ 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^
3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12
- 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^
2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*
c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^
8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^
11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152
*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4
 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^
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5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a
^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12
- 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*
c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6
*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 -
256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^
2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6
+ 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3
+ 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^
4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^
4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^
5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^
6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*
a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a
^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*
d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7
*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 -
 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 +
1024*a^8*b^3*c^3*d*e^7))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4
- 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2
) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a
^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 +
1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)
^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^
4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b
^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(
-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^
9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*
a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 -
 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d
^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*
b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 4
1*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c -
b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*
(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*
a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c
^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*
c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^
8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b
*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^
7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^
3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) +
20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c -
 b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c
^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)
```

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^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2
*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096
*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 2
4*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*
b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^
3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 -
4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^
11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*
b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 67
2*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2
 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*
d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*
b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 2
1504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2
*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^
5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57
344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7
 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e -
4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*
a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^
6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*
a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 153
60*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7
)))^(1/2) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*
e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e
^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^
11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168
*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2
*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^
4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 222
24*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2
*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*
d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10
*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d
^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6
*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*
c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^
8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8
 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b
^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 +
6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9
*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9
*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b
^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a
^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 20
```

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48*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5
 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^
3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7
*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e -
4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^
4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^
8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c
^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 -
b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c -
 b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^
5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b
^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-
(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 302
40*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)
^9)^(1/2) + b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 +
b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5
- 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^
2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*
c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*
a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3
 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^
6*d^2*e^4 - 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^
4*(-(4*a*c - b^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6
*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d
^2*e^4 - 128*a^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) +
 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e -
5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e
- 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5
 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e
^4 - 53760*a^7*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4
*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2
)^9)^(1/2) + 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e
^5*(-(4*a*c - b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-
(4*a*c - b^2)^9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^
2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^
10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^1
2*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^
8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 128
0*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^
16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e
^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e
^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4
*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^
8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*
```

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a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^
4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^
7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^
8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 -
 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7
*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7
*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13
*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d
^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d
^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8
*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*
d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*
b^3*c^5*d*e^7)))^(1/2)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 -
 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2)
 + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^
8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1
504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 + 9*a^2*b^4*e^6*(-(4*a*c - b^2)^
9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4
*e^6 + 44800*a^7*b^3*c^5*e^6 + 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) + b^
2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 + b^6*d^2*e^4*(-
(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9
*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a
^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 -
15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^
2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b
^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 - 41
*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 39*a^3*c^3*d^2*e^4*(-(4*a*c - b
^2)^9)^(1/2) + 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^5*d*e^5*(
-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a
^2*b^12*c*d*e^5 - 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^
3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c
^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8
*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*
c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7
*b^2*c^6*d*e^5 - 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 4*b^5*c*d^3*e^3
*(-(4*a*c - b^2)^9)^(1/2) + 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) + 2
0*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 86*a^3*b*c^2*d*e^5*(-(4*a*c -
b^2)^9)^(1/2) - 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b^3*c^
2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) + 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^
9)^(1/2) + 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 108*a^2*b^2*c^2*d^2*
e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*
a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24
*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b
^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3
*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4
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*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^1
1*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b
^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672
*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2
+ 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d
^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b
^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21
504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*
e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5
*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 573
44*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7
- 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4
*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a
^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6
*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a
^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 1536
0*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)
))^(1/2)*2i - atan((((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 61
44*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 +
983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14
*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^
12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 53
08416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d
^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 1827
84*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5
*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2
*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13
*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 140377
6*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*
d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*
a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^
14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 518
1440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9
*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9
+ 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b
^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^
2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920
*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^
9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 73
4080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^1
2*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 -
 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a
^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9
*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 60
43520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11
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*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^
6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096
*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*
d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 66
1632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b
^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*
e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 31516
16*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3
*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^1
3 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504
*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^
8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*
b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8
960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8
 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d
^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^
7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15
 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b
*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13
 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^
10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*
b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8
 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10
*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e
^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e
^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4
*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3
*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3
*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6
*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 11
52*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 6
4*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11
*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*
d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^
6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5
 + 1024*a^8*b^3*c^3*d*e^7)) - (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d
^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^
9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e +
35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6
*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c
 - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6
*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1
/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^
2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471
*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4
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+ 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^
2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^
7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 477
12*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*
e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4
*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5
*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4
 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a
*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a
^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492
*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 742
4*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 5
3760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c
*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(
1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(
4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*
a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c
 - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*
c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8
 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*
d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 38
40*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10
*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4
*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 2
4576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 1
40*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e
^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*
d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^
11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 22
40*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*
e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*
c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504
*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e
^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*
c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d
^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4
 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e -
 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e
 - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^
3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^
5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10
*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 98304
0*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3
 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^
12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242
```

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880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14
*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2
*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^
8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^
11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 +
 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b
^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9
 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*
c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^
4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144
*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*
d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304
*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^1
2*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5
 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*
a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d
^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*
a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13
*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5
 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400
*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6
*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 +
25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*
c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12
*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 501
26848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*
c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12
 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^
14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^1
1*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11
075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b
^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^
13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*
a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^1
0*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 +
 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*
b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^
8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 1
4974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^
10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^1
0*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^1
3 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 1258291
2*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^
4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16
 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*
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c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 380
10880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^
11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 -
 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*
b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*
a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96
*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c
^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*
a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*
c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6
*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*
b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a
^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2
240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6
 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d
^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^
7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*
a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b
^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c
^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*
c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*
e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b
^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5
+ 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*
c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2
077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800
*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-
(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2
)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2
 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d
^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^
5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 1024
0*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e
^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4
*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2)
 - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^
2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*
e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 -
84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1
030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 -
 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2
+ 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e
^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c -
b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*
d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2
```

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) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-
(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 3
4*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8
 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^
5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 -
 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*
a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^
3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^
4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*
e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^
3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b
^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 179
20*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e
^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*
c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*
a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5
 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*
c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*
b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*
d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d
^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*
e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d
*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c
^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (
x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15
- 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15
 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13
*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12
*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c
^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*
d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e
^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3
200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^1
1*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 7457
6*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6
*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b
^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2
+ 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^
5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7
+ 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^
6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 841
6*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^1
2*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 3
14496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7
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*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62
064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c
^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 -
1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^
8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8
 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*
b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^
13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400
*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*
d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*
b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 48
0*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 -
5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^1
1 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d
^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a
^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14
 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c
^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19
968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e
^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*
d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a
^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6
 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a
^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512
*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1
152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4
 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5
*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2
*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b
^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^
7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e
^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e +
52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072
*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*
a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a
^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 2
13*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^
7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*
a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(
1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6
 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^
4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a
*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4
*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b
^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 1545
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6*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^
4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c
^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2
*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^
9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*
a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b
^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^
4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d
^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3
*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*
d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2
*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(
4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^
2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)
^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^
3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(
1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13
*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4
*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c
^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8
 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4
*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^
8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*
c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5
*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 22
40*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e
^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c
^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*
a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6
+ 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5
*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a
^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16
384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3
*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b
^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^1
2*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b
^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^
10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(
1/2) - (326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^1
4 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^1
4 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11
*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d
^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^1
1*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 +
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 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*
c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c
^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 5
2144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d
^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2
*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^
5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b
^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 -
 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c
^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277
000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*
d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 1596
32*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*
d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*
c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a
*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a
*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 153
6*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2
 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4
*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5
*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 -
113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*
d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*
c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4
*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 +
 a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*
d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*
e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6
*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*
d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c
^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*
b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*
a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 +
 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024
*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*
c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3
*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e
^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^
7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 38
40*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^
6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12
*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6
 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*
b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3
*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c -
```

```
 b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/
2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a
^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 -
 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^
4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^
3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 593
92*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4
*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1
/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51
*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^1
2*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*
b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a
^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*
a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b
^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^
(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-
(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a
*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c -
 b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^
4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^
9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^
8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 +
240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^
8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4
*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 +
6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 1638
4*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84
*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^
3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d
^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b
^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 134
4*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*
e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*
c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920
*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*
e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^
7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3
+ 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 8
4*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15
360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5
120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7
 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(22800
*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c
^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^
```

6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*
e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53
*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*
c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*
a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e
^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2
*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^
12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5
*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^
6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b
*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376
*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8
- 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8
+ 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*
b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 +
1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*
b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^
4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152
*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 -
192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^
3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^
2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*
c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c
*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3
+ 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*
a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^
7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8
*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*
b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*
a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d
*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3
*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2
) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 +
44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d
^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c
- b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^
4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*
c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a
^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 +
10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*
d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^
4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^
(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c
- b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12
*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e

^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*
e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*
e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4
*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^
6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a
*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b
^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)
^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e
^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2
) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^
6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^
10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*
d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 +
3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^
15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d
^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3
*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^
11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*
a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5
+ 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*
d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8
*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 5
7344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^
3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10
*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384
*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^
15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*
c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c
*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*
c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*
b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2
)*1i - (((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c
^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b
^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504
*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 +
10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^
9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 788
48*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c
^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 46
08*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5
*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544
*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^
8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155
136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*

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d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 8642
56*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*
c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7
+ 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*
b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e
^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a
^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11
*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 1
2657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10
*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^1
2 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*
b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d
^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 20
88960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9
*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13
 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*
a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*
d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 76
09856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10
*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e
^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 246005
76*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^
5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11
 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a
^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8
*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 -
4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13
*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^
6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^1
7*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 140
8*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*
e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9
*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^1
1 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^
11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 -
16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 +
 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^
2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 10
24*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^
10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*
b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a
^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 19
2*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3
- 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*
e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^
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3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d
*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 +
64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^
4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*
b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b
^3*c^3*d*e^7)) + (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^
2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 21
3*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7
*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a
^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1
/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6
 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4
*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*
c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*
d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^
7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456
*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4
 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^
5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*
c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9
)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a
*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^
12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4
*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^
5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*
d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d
^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*
c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4
*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2
*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^
9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3
*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1
/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(
-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*
c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*
b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^
8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8
 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*
b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8
*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c
^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*
b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 224
0*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^
5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^
3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a
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^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 +
 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*
d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^
10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 163
84*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*
b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^
9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12
*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^
9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^1
0*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1
/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^1
7 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^
6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^
9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 +
5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*
d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a
^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^1
1*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*
b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 +
 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b
^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*
e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b
^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2
 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a
^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7
*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108
800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2
*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 -
5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a
^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*
d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 332
80*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*
d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 1
2615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a
^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*
d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1
111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^1
4*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3
 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 376832
0*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7
*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10
+ 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7
*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^
15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711
040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^
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6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^
11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a
^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*
d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 5
4886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*
b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e
^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333
120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*
b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^
3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 +
23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a
^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^
9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^1
5 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*
b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 +
25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*
c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 +
 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12
*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^1
6))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8
 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*
d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*
b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e
^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 -
 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2
 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e
^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^
3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c
^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b
^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*
a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*
b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d
^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5
+ 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 -
384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*
((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^
5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 2
6880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*
d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 384
0*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c
^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*
e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)
^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) -
6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^
10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032
```

*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2
 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7
*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^
6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*
d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) -
 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*
b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2
*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*
c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^
2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b
^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^
3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2)
 - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*
c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*
c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)
^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5
e(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1
/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^1
0*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a
^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2
*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*
e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5
*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^1
2*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*
b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 4
2*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^
3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^
6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7
*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 +
 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d
^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*
b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 -
 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e -
 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*
a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5
*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a
^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a
^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49
152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(626688*a^1
0*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^
11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8
*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976
*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 10
67008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2

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- 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 14
4*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15
*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c
^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 -
36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8
*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a
^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e
^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b
^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e
^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*
b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 +
77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^
3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 -
916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6
*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 1
75296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*
c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 -
 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b
^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^1
2 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^
6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e
^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*
a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*
d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 47
39072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12
*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*
d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c
^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^1
6*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 116
80*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d
*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7
*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11
+ 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c
^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b
^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*
b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^
8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c
^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d
^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*
d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c
^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^
7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a
^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 15
36*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^
6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1
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024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b
^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*
d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^
5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d
*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 +
 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c
*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b
^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d
^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a
^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*
b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*
c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^
(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 60
0*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^
2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6
*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5
*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 -
59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*
b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)
^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 +
 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*
b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a
^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 1689
6*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 224
00*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 +
4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^
9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5
*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 4
2*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*
c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b
*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24
*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8
 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144
*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*
b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5
 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 1
6384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 +
 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5
*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^
4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^
7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 +
1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d
^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b
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^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17
920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d
^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9
*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e
^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e
+ 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 -
 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7
+ 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*
e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (32691
2*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*
b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^
6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*
a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 5327
36*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b
^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*
d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10
+ 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3
+ 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*
c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 2348
8*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3
*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3
*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^
8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^
8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 -
 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c
^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 23
6800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^
8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 11
06496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2
 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9
*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*
e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*
d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^
12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342
272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e
^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^
4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(
a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5
*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 25
6*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4
*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 153
6*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*
b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^
4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128
*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
```

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512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e
^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d
^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*
c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*
e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4
*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a
^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*
b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b
^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*
d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8
*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e +
4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^
3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 +
 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25
*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2
) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c
^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d
^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8
*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*
a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2
 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^
7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^
9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b
^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^
6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3
 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^
5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^
5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d
*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*
(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*
b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)
^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2
) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*
a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8
 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c
^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8
 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 61
44*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^
2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^
2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2
*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^
12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456
```

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*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^
2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^
2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a
^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 -
 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*
d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a
^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a
^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11
*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*
d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c
^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c
^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11
*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(22800*a^6*c^9*e^13
 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 155
92*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304
*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5
*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*
e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7
+ 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d
^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^
3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11
 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 +
1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8
+ 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 -
5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8
 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*
d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*
c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4
*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 +
 a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*
d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*
e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6
*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*
d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c
^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*
b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*
a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 +
 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024
*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*
c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3
*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e
^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^
7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 38
40*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^
6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12
```

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*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6
- 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*
b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3
*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c -
 b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/
2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a
^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 -
 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^
4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^
3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 593
92*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4
*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1
/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51
*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^1
2*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*
b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a
^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*
a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b
^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^
(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-
(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a
*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c -
 b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^
4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^
9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^
8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 +
240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^
8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4
*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 +
6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 1638
4*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84
*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^
3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d
^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b
^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 134
4*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*
e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*
c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920
*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*
e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^
7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3
+ 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 8
4*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15
360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5
```

```
120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7
 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*1i)/((2000*
a^4*c^9*e^12 + 21*a^2*b^4*c^7*e^12 - 520*a^3*b^2*c^8*e^12 + 1296*a^2*c^11*d
^4*e^8 + 4320*a^3*c^10*d^2*e^10 + 25*b^4*c^9*d^4*e^8 - 60*b^5*c^8*d^3*e^9 +
 35*b^6*c^7*d^2*e^10 + 192*a^2*b^2*c^9*d^2*e^10 - 112*a*b^5*c^7*d*e^11 - 44
80*a^3*b*c^9*d*e^11 - 360*a*b^2*c^10*d^4*e^8 + 832*a*b^3*c^9*d^3*e^9 - 362*
a*b^4*c^8*d^2*e^10 - 2880*a^2*b*c^10*d^3*e^9 + 1440*a^2*b^3*c^8*d*e^11)/(8*
(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^
5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 2
56*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^
4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 15
36*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3
*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a
^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 12
8*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*
e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*
d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2
*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d
*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e -
4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*
a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7
*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((((1
048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 614
40*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 -
 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^1
2*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10
*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 -
2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c
^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 +
 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c
^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576
*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10
*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 -
798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*
c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 25
60*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^
12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5
 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4
*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*
e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^
4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d
^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4
055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b
^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10
```

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 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5
*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12
*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 310
76352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7
*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11
 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*
b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*
d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 371
01568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8
*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14
 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440
*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^
7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 -
10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a
^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^
2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9
117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*
b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a
*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 -
7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9
 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^
2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 704512
0*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e
^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a
^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*
e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*
e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c
^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a
^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^
6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^
2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e
^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^
6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2
*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^
4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^
7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 5
12*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a
^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^
4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e
^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3
 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)
) - (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 +
3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*
e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^
12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^
```

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6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^
4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b
^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c
 - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(
1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600
*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2
 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*
d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*
b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 5
9392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b
^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^
(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 +
51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b
^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^
3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896
*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 2240
0*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4
*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9
)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*
(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42
*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c
 - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*
c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*
a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8
+ 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*
a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b
^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5
+ 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16
384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 +
84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*
e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4
*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7
*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1
344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^
5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^
3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 179
20*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^
3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*
d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^
3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e +
 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 -
15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 +
 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e
```

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^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*
a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^1
1*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572
864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e
^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*
c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 25
6*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d
^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336
*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*
e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^
10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*
e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*
a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e
^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4
*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d
^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 4
90240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*
c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 1
63840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b
^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^
11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 227
9680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c
^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 32
7680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b
^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d
^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 108
64640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^
12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^1
3 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a
^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^1
1*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8
- 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7
*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4
*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*
a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c
^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9
 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a
^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d
^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 95
02720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b
^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e
^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*
a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*
c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e
^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928
```

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640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11
*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d
^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 +
19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a
^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e
^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*
b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8
- 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11
*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^
16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b
^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*
d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5
*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4
- 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8
*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c
^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7
*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*
b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a
^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 -
128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^
4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d
^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 -
 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*
b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^
3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c
^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^
5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 +
 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^
6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^1
4*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8
*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 1065
6*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*
c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 2
2528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^
4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^
3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*
d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b
^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37
632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3
*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1
/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c
^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(
4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 11
16*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 1
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5232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e +
 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5
 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*
a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c
*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(
1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(
-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 1
20*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c -
 b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7
*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a
^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^
8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 24
0*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^
12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6
 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6
 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*
e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^
2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*
b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 1
0080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2
*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^
6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 4659
2*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e
^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b
^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b
*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*
d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e
^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^
7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*
e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^
7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(626688*a^10*b*c^8*e^15
 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 +
 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15
 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^1
1*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^1
1*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10
*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^
9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9
 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2
 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7
*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78
496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d
^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2
*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*
e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a
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^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e
^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^1
1*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 +
162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^
4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8
- 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9
*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13
+ 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*
b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^
10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^
5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^
6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648
*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*
d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 23
70048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3
*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a
*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 600
0*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 +
1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^1
2 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c
^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 16015
36*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e
^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8
*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(
8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*
a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 -
 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*
d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 +
1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a
^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192
*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 -
128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5
 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^
4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^
5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b
^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4
*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e
- 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 102
4*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a
^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*
a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c
^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*
a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e
 + 4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4
*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^
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6 + 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 -
 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(
1/2) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^1
3*c^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^
3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*
b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 71
68*a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*
e^2 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2
*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e
^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*
a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c
*e^6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*
e^3 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d
*e^5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8
*d^5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^
5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5
*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11
*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b
^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d
^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(
1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-
(4*a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/
(32*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e
^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^
8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*
d^8 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 -
 6144*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14
*d^2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7
*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*
c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5
*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1
456*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6
*e^2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10
*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 3225
6*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^
3 - 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c
^4*d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 2150
4*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 1638
4*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b
^11*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13
*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^
5*c^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^
7*c^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a
^11*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*e
```

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^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14
 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^1
4 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*
e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d
^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*
e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*
b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3
*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b
^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6
 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7
*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a
^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8
*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^
3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^1
1 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b
^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 +
 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*
c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 -
670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3
*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4
*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a
*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*
b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 24
0*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13
 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^1
2*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a
^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13
 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 +
 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 +
a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7
*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*
b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4
*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e
^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5
*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*
d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c
^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*
b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048
*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 -
 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b
^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d
^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^
7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7
 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 -
b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5
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*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 +
3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*
e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9
*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^
7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*
(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7
*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 +
6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*
a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3
- 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d
^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*
b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 6
0928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39
*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e
 + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c -
b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b
^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4
*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^
5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*
a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^
2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4
*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 8
6*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b
*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)
^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^
8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*
d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280
*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^
8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c
^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384
*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3
*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 13
44*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^
4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3
*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7
*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2
1504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^
4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^
4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 122
88*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d
^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e
^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 512
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0*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24
576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49
152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^
5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8
*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^
8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4
*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6
 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c
^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^
3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 125
4*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3
*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c
^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^
11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c
^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^
12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b
^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(
a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5
*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 25
6*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4
*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 153
6*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*
b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^
4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128
*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 +
512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e
^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d
^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*
c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*
e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4
*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a
^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*
b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b
^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*
d^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8
*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e +
4*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^
3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 +
 10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25
*a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2
) + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c
^2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d
^3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8
*c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*
a^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2
```

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 + 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^
7*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^
9)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b
^10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^
6*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3
 + 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^
5 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^
5*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d
*e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*
(-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*
b^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)
^9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2
) - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*
a*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8
 - 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c
^6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8
 + 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 61
44*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^
2*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^
2*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2
*d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^
12*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456
*a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^
2 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^
2*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a
^8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 -
 46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*
d^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a
^10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a
^12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11
*c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*
d^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c
^7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c
^3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11
*b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) + (((((1048576*a^13*c^8*
e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e
^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^
2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a
^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 +
 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^1
1*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 15
4112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13
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*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 332
8*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^
14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291
520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*
c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77
440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*
d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 29
24544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b
^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^
8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^
4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e
^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 607846
4*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c
^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 +
 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b
^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^
13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a
^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^
10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 1
2205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^
10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^1
4 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 802816
0*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^
8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 +
2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*
b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6
*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 168
3456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2
*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e
^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376
*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*
c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 1
28*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e
^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*
d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^
2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 70123
52*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9
*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9
*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^
15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b
^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*
d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5
*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4
- 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8
*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c
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^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7
*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*
b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a
^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 -
128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^
4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d
^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 -
 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*
b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^
3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c
^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*((27*a*b^
9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d
^6 + 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*
b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4
*b^14*c*d^3*e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3
*c^8*d^6 - 9*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 +
10656*a^5*b^7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*
a^4*c^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2)
 + 22528*a^7*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^
2*d^4*e^2 + 6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^
3*e^3 + 180*a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*
c^4*d^3*e^3 - 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a
^4*b^6*c^5*d^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2
+ 37632*a^5*b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7
*d^3*e^3 + 60928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9
)^(1/2) + 39*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^
10*c^4*d^5*e + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6
*(-(4*a*c - b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3
+ 1116*a^2*b^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5
 + 15232*a^4*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5
*e + 1344*a^5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*
e^5 - 23296*a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(
-(4*a*c - b^2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b
^4*c*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^
9)^(1/2) - 86*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2)
 - 120*a^2*b*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a
*c - b^2)^9)^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*
(a^7*b^12*e^8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 -
 4*a^6*b^13*d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^
6*d^8 - 1280*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8
+ 240*a^9*b^8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 614
4*a^12*b^2*c^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2
*e^6 + 16384*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2
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*e^6 + 6*a^3*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*
d^5*e^3 + 1344*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^1
2*c^2*d^4*e^4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*
a^6*b^10*c^3*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2
 - 10080*a^7*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2
*d^2*e^6 - 21504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^
8*b^6*c^5*d^4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 -
46592*a^9*b^4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d
^2*e^6 + 12288*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^
10*b^4*c^5*d^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^
12*b*c^6*d*e^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*
c^4*d^7*e - 12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d
^3*e^5 + 5120*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^
7*d^7*e + 24576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^
3*d*e^7 - 49152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*
b*c^7*d^3*e^5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2)*(1048576*a^15*c^8*e^17 +
256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 -
 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7
*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^1
0*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 +
9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*
d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*
a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^
10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^
19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3
 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3
*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9
*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b
^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e
^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 286745
6*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c
^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 1
3824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^
13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4
 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936
*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^
6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41
216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^1
4*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4
 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 137216
0*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c
^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11
+ 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^1
5*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13
*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 388
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95616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b
^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^
11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^
7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^1
2*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 5
5476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*
b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e
^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*
a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^
11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10
+ 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^
9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2
*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 +
7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a
^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5
*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^1
2 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 835584
0*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^
3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*
e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8
*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6
+ 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^
9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16
- 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^
13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(8*(a^6*b^8*e^8 + 256*a^6
*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*
c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 9
6*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3
*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1
024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*
a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 9
0*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3
+ 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e
^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*
d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3
*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^
6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d
^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 -
 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92
*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*
a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4
*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(
4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6
*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 28
8*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4
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*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^
6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c
 - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3
*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14
*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11
*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a
^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 +
 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*
d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6
*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3
*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1
/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b
^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^
(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d
^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7
*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^
4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^
7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1
/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*
c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*
c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9
)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3
*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) +
 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096
*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 +
a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6
*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^
8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 +
 a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^
9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^
2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b
^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720
*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4
 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4
*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8
*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 -
6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^
4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*
b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 +
 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a
^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b
^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^
7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*
b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10
*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 2457
```

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6*a^11*b^3*c^5*d*e^7)))^(1/2) + (x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^
9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c
^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b
^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a
^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 177
3568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*
b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13
*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^
5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*
c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 -
 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7
*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a
^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^
13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3
*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*
e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9
*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 +
 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c
^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 +
 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7
*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 -
189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^
2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8
 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*
b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^
13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^
6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3
*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 68730
88*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^
7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 -
352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e
^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8
*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*
d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14
*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 6758
40*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^
8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7
*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14
+ 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 +
 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 +
a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7
*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*
b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4
*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e
^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5
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*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*
d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c
^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*
b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048
*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 -
 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b
^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d
^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^
7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7
 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 -
b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5
*d^6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 +
3072*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*
e^3 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9
*a^2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^
7*c^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*
(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7
*c^8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 +
6*a*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*
a^2*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3
- 2871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d
^3*e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*
b^4*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 6
0928*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39
*a^3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e
 + 7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c -
b^2)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b
^8*c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4
*b^4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^
5*b^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*
a^7*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^
2)^9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4
*(-(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 8
6*a^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b
*c^3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)
^(1/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^
8 + 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*
d*e^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280
*a^6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^
8*c^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c
^5*e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384
*a^10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3
*b^14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 13
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44*a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^
4 - 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3
*d^4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7
*b^8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2
1504*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^
4*e^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^
4*c^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 122
88*a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d
^2*e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e
^7 - 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e -
12*a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 512
0*a^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24
576*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49
152*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^
5 + 24576*a^11*b^3*c^5*d*e^7)))^(1/2) - (326912*a^8*c^9*d*e^13 - 241664*a^8
*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9
*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b
^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5
*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5
*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^
9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8
 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b
^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4
 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*
c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384
*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*
e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^
3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^
9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^
9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8
+ 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c
^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 3
88032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c
^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 +
64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 +
 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6
 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9
 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2
*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c
^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 7692
8*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^1
3 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c
^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8
 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8
- 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^
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4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6
*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*
c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*
c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^
8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^
5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 204
8*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5
+ 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3
*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*
d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4
*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4
*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8
*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^
5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b
^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c -
b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5
*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^
7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(
4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 3024
0*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^
9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b
^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 -
 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2
*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c
^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a
^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3
- 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6
*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4
*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*
a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^
2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) +
150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 5
824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e -
 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5
+ 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^
4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*
b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)
^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^
5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(
4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2
*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^1
0*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12
*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8
 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280
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*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^1
6*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^
2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^
2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*
d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8
*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a
^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4
 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7
*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8
*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 -
21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*
d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*
b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*
c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^
4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^
7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*
d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d
^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b
^3*c^5*d*e^7)))^(1/2) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*
a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^
2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5
*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d
^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*
b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 -
 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^
2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3
*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 4108
8*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 260
4*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232
*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416
*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 -
63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 25
6*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2
*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^
8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^1
1*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^
4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2
+ 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^
3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5
*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*
d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5
*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^
7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 10
24*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*
c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*
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e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e
 - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 -
3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*((27*a*b^9*c^5*d^6 - b^1
1*c^4*d^6 - b^15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^
6*(-(4*a*c - b^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 307
2*a^6*c^9*d^5*e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3
 - 288*a^2*b^7*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^
2*b^4*e^6*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c
^3*e^6 - 30240*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(
4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^
8*d^3*e^3 - b^6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a
*b^14*d*e^5 - 1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2
*b^11*c^2*d^2*e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2
871*a^3*b^9*c^3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*
e^3 + 16896*a^4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4
*c^6*d^3*e^3 - 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 6092
8*a^6*b^3*c^6*d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^
3*c^3*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^
9)^(1/2) + 6*a*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e +
7*a*b^13*c*d^2*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2
)^9)^(1/2) + 150*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*
c^5*d^5*e - 5824*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^
4*c^7*d^5*e - 3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b
^6*c^4*d*e^5 + 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7
*b*c^7*d^2*e^4 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^
9)^(1/2) + 4*b^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-
(4*a*c - b^2)^9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a
^3*b*c^2*d*e^5*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^
3*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1
/2) + 108*a^2*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 +
 4096*a^9*c^10*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e
^7 + a^3*b^12*c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^
6*b^6*c^7*d^8 + 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c
^2*e^8 - 1280*a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*
e^8 + a^3*b^16*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^
10*c^9*d^6*e^2 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^
14*c^2*d^6*e^2 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*
a^5*b^10*c^4*d^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 -
 6720*a^6*b^8*c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^
4*e^4 - 672*a^6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^
8*c^4*d^4*e^4 + 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 2150
4*a^8*b^4*c^7*d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e
^4 - 6720*a^8*b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c
^6*d^4*e^4 - 21504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*
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a^10*b^2*c^7*d^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*
e^6 + 96*a^7*b^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7
- 4*a^3*b^13*c^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*
a^4*b^14*c*d^4*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a
^6*b^7*c^6*d^7*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576
*a^8*b^3*c^8*d^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152
*a^10*b*c^8*d^5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 +
 24576*a^11*b^3*c^5*d*e^7)))^(1/2)))*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^
15*d^2*e^4 - 9*a^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 + 9*a*c^5*d^6*(-(4*a*c - b
^2)^9)^(1/2) + 213*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*
e + 35840*a^8*c^7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 - 288*a^2*b^7
*c^6*d^6 + 1504*a^3*b^5*c^7*d^6 - 3840*a^4*b^3*c^8*d^6 - 9*a^2*b^4*e^6*(-(4
*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^6 + 10656*a^5*b^7*c^3*e^6 - 30240
*a^6*b^5*c^4*e^6 + 44800*a^7*b^3*c^5*e^6 - 25*a^4*c^2*e^6*(-(4*a*c - b^2)^9
)^(1/2) - b^2*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 22528*a^7*c^8*d^3*e^3 - b^
6*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*b^13*c^2*d^4*e^2 + 6*a*b^14*d*e^5 -
1471*a^2*b^9*c^4*d^4*e^2 + 600*a^2*b^10*c^3*d^3*e^3 + 180*a^2*b^11*c^2*d^2*
e^4 + 6976*a^3*b^7*c^5*d^4*e^2 - 1032*a^3*b^8*c^4*d^3*e^3 - 2871*a^3*b^9*c^
3*d^2*e^4 - 15456*a^4*b^5*c^6*d^4*e^2 - 7168*a^4*b^6*c^5*d^3*e^3 + 16896*a^
4*b^7*c^4*d^2*e^4 + 10240*a^5*b^3*c^7*d^4*e^2 + 37632*a^5*b^4*c^6*d^3*e^3 -
 47712*a^5*b^5*c^5*d^2*e^4 - 59392*a^6*b^2*c^7*d^3*e^3 + 60928*a^6*b^3*c^6*
d^2*e^4 + 41*a^2*c^4*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 39*a^3*c^3*d^2*e^4*
(-(4*a*c - b^2)^9)^(1/2) - 6*b^4*c^2*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a
*b^5*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 106*a*b^10*c^4*d^5*e + 7*a*b^13*c*d^2
*e^4 - 128*a^2*b^12*c*d*e^5 + 51*a^3*b^2*c*e^6*(-(4*a*c - b^2)^9)^(1/2) + 1
50*a*b^11*c^3*d^4*e^2 - 84*a*b^12*c^2*d^3*e^3 + 1116*a^2*b^8*c^5*d^5*e - 58
24*a^3*b^6*c^6*d^5*e + 1030*a^3*b^10*c^2*d*e^5 + 15232*a^4*b^4*c^7*d^5*e -
3492*a^4*b^8*c^3*d*e^5 - 16896*a^5*b^2*c^8*d^5*e + 1344*a^5*b^6*c^4*d*e^5 +
 7424*a^6*b*c^8*d^4*e^2 + 22400*a^6*b^4*c^5*d*e^5 - 23296*a^7*b*c^7*d^2*e^4
 - 53760*a^7*b^2*c^6*d*e^5 + 4*b^3*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 4*b
^5*c*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 11*a*b^4*c*d^2*e^4*(-(4*a*c - b^2)^
9)^(1/2) - 20*a^2*b^3*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2) - 86*a^3*b*c^2*d*e^5
*(-(4*a*c - b^2)^9)^(1/2) + 42*a*b^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
 12*a*b^3*c^2*d^3*e^3*(-(4*a*c - b^2)^9)^(1/2) - 120*a^2*b*c^3*d^3*e^3*(-(4
*a*c - b^2)^9)^(1/2) - 34*a*b*c^4*d^5*e*(-(4*a*c - b^2)^9)^(1/2) + 108*a^2*
b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^8 + 4096*a^9*c^10
*d^8 + 4096*a^13*c^6*e^8 - 24*a^8*b^10*c*e^8 - 4*a^6*b^13*d*e^7 + a^3*b^12*
c^4*d^8 - 24*a^4*b^10*c^5*d^8 + 240*a^5*b^8*c^6*d^8 - 1280*a^6*b^6*c^7*d^8
+ 3840*a^7*b^4*c^8*d^8 - 6144*a^8*b^2*c^9*d^8 + 240*a^9*b^8*c^2*e^8 - 1280*
a^10*b^6*c^3*e^8 + 3840*a^11*b^4*c^4*e^8 - 6144*a^12*b^2*c^5*e^8 + a^3*b^16
*d^4*e^4 - 4*a^4*b^15*d^3*e^5 + 6*a^5*b^14*d^2*e^6 + 16384*a^10*c^9*d^6*e^2
 + 24576*a^11*c^8*d^4*e^4 + 16384*a^12*c^7*d^2*e^6 + 6*a^3*b^14*c^2*d^6*e^2
 - 140*a^4*b^12*c^3*d^6*e^2 + 84*a^4*b^13*c^2*d^5*e^3 + 1344*a^5*b^10*c^4*d
^6*e^2 - 672*a^5*b^11*c^3*d^5*e^3 - 42*a^5*b^12*c^2*d^4*e^4 - 6720*a^6*b^8*
c^5*d^6*e^2 + 2240*a^6*b^9*c^4*d^5*e^3 + 1456*a^6*b^10*c^3*d^4*e^4 - 672*a^
```

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6*b^11*c^2*d^3*e^5 + 17920*a^7*b^6*c^6*d^6*e^2 - 10080*a^7*b^8*c^4*d^4*e^4
+ 2240*a^7*b^9*c^3*d^3*e^5 + 1344*a^7*b^10*c^2*d^2*e^6 - 21504*a^8*b^4*c^7*
d^6*e^2 - 21504*a^8*b^5*c^6*d^5*e^3 + 32256*a^8*b^6*c^5*d^4*e^4 - 6720*a^8*
b^8*c^3*d^2*e^6 + 57344*a^9*b^3*c^7*d^5*e^3 - 46592*a^9*b^4*c^6*d^4*e^4 - 2
1504*a^9*b^5*c^5*d^3*e^5 + 17920*a^9*b^6*c^4*d^2*e^6 + 12288*a^10*b^2*c^7*d
^4*e^4 + 57344*a^10*b^3*c^6*d^3*e^5 - 21504*a^10*b^4*c^5*d^2*e^6 + 96*a^7*b
^11*c*d*e^7 - 16384*a^9*b*c^9*d^7*e - 16384*a^12*b*c^6*d*e^7 - 4*a^3*b^13*c
^3*d^7*e - 4*a^3*b^15*c*d^5*e^3 + 96*a^4*b^11*c^4*d^7*e - 12*a^4*b^14*c*d^4
*e^4 - 960*a^5*b^9*c^5*d^7*e + 84*a^5*b^13*c*d^3*e^5 + 5120*a^6*b^7*c^6*d^7
*e - 140*a^6*b^12*c*d^2*e^6 - 15360*a^7*b^5*c^7*d^7*e + 24576*a^8*b^3*c^8*d
^7*e - 960*a^8*b^9*c^2*d*e^7 + 5120*a^9*b^7*c^3*d*e^7 - 49152*a^10*b*c^8*d^
5*e^3 - 15360*a^10*b^5*c^4*d*e^7 - 49152*a^11*b*c^7*d^3*e^5 + 24576*a^11*b^
3*c^5*d*e^7)))^(1/2)*2i - ((x*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(2
*a*(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d
*e)) - (c*x^3*(2*a*c*e - b^2*e + b*c*d))/(2*a*(a*b^2*e^2 - 4*a*c^2*d^2 - 4*
a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e)))/(a + b*x^2 + c*x^4) - (ata
n((((((-d*e^7)^(1/2)*((326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a
^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*
a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 1152
0*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 33
6384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 2
16*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^
9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5
*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 +
 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c
^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 1304
0*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e
^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*
b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7
 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7
*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 4
74112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^
9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 1955
84*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*
d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 4889
60*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^
13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d
^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*
d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5
*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b
*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 158
08*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^
13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c
^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 2440
96*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^
```

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8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d
^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^
9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6
+ 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^
2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*
a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 11
52*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4
- 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*
e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*
d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^
4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7
*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^
3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 5
2*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*
a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a
^8*b^3*c^3*d*e^7)) + (((x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14
+ 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 -
 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^
15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d
^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*
c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*
d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*
e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 +
16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12
*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^
2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8
+ 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c
^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 1280
0*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12
*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436
736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*
e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3
*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*
e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*
a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*
e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^
4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^
9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 324096
0*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d
^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442
688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9
*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 2
47296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^
3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^1
3 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6
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*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168
*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 81
92*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10
+ 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^
14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*
c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 5
14768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*
d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792
*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*
c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c
^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96
*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*
e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 10
24*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a
^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90
*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 +
 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^
2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d
^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*
c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6
*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^
7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 -
384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*
a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a
^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (((1048576*a^13*c^8*e^16 + 25
6*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327
680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^1
6 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d
^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832
*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^
2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*
b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*
e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^1
6*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 -
198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b
^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e
^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b
^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12
- 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4
*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^
10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 17320
96*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^
4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229
376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4
*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*
```

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e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*
a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d
^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200
*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^
12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^
7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a
^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^
4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725
184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4
*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^1
0 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a
^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*
d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 1
6429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*
b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6
*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 923
2384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2
*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e
^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10
*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168
*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 +
 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^1
0 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*
c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 32
4480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*
d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 1920
2048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 +
256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a
^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*
d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b
^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*
e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^
2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*
e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d
^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^
6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b
^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*
a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 -
1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^
9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^
4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7
*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7
- 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - (x*(-d*e^7)^(1/2)*(10
48576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 614
40*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17
```

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- 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*
d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880
*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^1
5 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13
*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 -
 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^
3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*
a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8
*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 1
12384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3
*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 3481
60*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10
*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e
^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4
*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^
12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680
*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*
c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8
 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*
b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^1
3 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680
*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*
c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8
 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*
a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d
^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 1422
1312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b
^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^
9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 62003
20*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c
^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 31
45728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8
*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d
^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 16
8960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11
*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^
7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400
*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7
*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 5
58080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^1
0*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8
*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14
- 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 2392064
0*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5
*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e
```

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^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 340
7872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*
d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 2595225
6*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^
9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 167936
0*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^
6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(16
*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d
^3*e^2)*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^
8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6
*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2
*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*
e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2
- 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2
 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*
e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d
^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*
c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*
b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512
*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9
*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*
d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5
 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 -
 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))
*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*
b*d^2*e^3 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*
d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2))))/(2*(c^2*d^5 + a^2
*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)) + (x*(
22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*
b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^
12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11
*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8
 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2
*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 +
1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*
d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^
4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b
^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*
a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b
^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*
a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 +
21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4
*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^
5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256
*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e
```

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^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6
*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 5
12*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 -
 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e
^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d
^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c
^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7
*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*
b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5
*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e
+ 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 30
72*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 102
4*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2)*1i)/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3
*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)) - ((((-d*e^7)^(1/2)*((
326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264
*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 2329
60*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 7
8080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 +
 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 -
464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10
*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*
e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11
*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2
*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 +
 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^
5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 3622
4*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d
^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a
^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*
e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*
b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12
 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b
^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11
 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^1
2*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^1
0*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7
*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12
*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a
^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13
- 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*
d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a
^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/
(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 -
4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8
 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^1
```

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2*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2
+ 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92
*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 1
92*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2
- 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e
^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*
d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*
c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8
*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c
^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*
e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1
024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384
*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) - ((
(x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15
 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^1
5 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^1
3*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^1
2*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*
c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8
*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*
e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 +
3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^
11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 745
76*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^
6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*
b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2
 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b
^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7
 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c
^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 84
16*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^
12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 -
314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^
7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 6
2064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*
c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 -
 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b
^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^
8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6
*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e
^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 282240
0*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9
*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a
*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 4
80*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 -
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 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^
11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*
d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*
a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^1
4 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*
c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 1
9968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*
e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5
*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*
a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^
6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*
a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 51
2*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 -
1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^
4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^
5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^
2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*
b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b
^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*
e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e +
 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 307
2*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024
*a^8*b^3*c^3*d*e^7)) + (((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6
144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 +
 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^1
4*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c
^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5
308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*
d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182
784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^
5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^
2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^1
3*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 14037
76*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6
*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504
*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d
^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 51
81440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^
9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9
 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*
b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e
^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 929792
0*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c
^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 7
34080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^
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12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14
 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*
a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^
9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6
043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^1
1*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e
^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 3226009
6*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7
*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 6
61632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*
b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5
*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151
616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^
3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^
13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 2549350
4*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c
^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a
*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 +
8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^
8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*
d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a
^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15
 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*
b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^1
3 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a
^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3
*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^
8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^1
0*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*
e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*
e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^
4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^
3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^
3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^
6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1
152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e +
64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^1
1*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5
*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e
^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^
5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*(-d*e^7)^(1/2)*(1048576*a^15*c^8*e^17 + 2
56*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 -
327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*
e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10
*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9
```

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437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d
^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a
^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^1
0*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^1
9*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3
 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*
b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*
e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^
18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^
3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456
*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^
6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13
824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^1
3*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4
 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*
a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6
*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 412
16*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14
*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4
 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160
*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^
7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 +
 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15
*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*
e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 3889
5616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^
8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^1
1 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7
*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12
*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55
476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b
^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^
12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a
^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^1
1*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 +
 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9
*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*
e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7
208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^
10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*
d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12
 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840
*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3
*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e
^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*
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b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 +
 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9
*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 -
 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^1
3*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/(16*(c^2*d^5 + a^2*d*e^4 +
 b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)*(a^6*b^8*e^8 +
256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a
^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*
d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b
^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*
e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^
2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*
e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d
^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^
6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b
^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*
a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 -
1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^
9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^
4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7
*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7
- 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2))/(2*(c^
2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e
^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e -
 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2))))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 -
2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)) - (x*(22800*a^6*c^9*e^13 + 36
*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^
5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*
c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10
*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 -
 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 384
0*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^
10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3
*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24
*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*
a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332
*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*
a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 41
20*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^1
2))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8
 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*
d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*
b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e
^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 -
 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2
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- 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e
^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^
3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c
^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b
^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*
a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*
b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d
^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5
+ 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 -
384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*
(-d*e^7)^(1/2)*1i)/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*
a*b*d^2*e^3 + 2*a*c*d^3*e^2)))/(((((-d*e^7)^(1/2)*((326912*a^8*c^9*d*e^13 -
 241664*a^8*b*c^8*e^14 - 48*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13
552*a^4*b^9*c^4*e^14 + 75776*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 3
72736*a^7*b^3*c^7*e^14 + 11520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 +
 197120*a^5*c^12*d^7*e^7 + 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^
11 - 40*b^5*c^12*d^12*e^2 + 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 +
 496*b^8*c^9*d^9*e^5 - 264*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*
c^6*d^6*e^8 + 64*b^12*c^5*d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*
e^11 - 16*b^15*c^2*d^2*e^12 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^
12*d^10*e^4 - 47152*a^2*b^4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 162
72*a^2*b^6*c^9*d^7*e^7 - 13040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*
e^9 - 26384*a^2*b^9*c^6*d^4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^
11*c^4*d^2*e^12 + 125056*a^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6
- 126432*a^3*b^4*c^10*d^7*e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6
*c^8*d^5*e^9 + 125392*a^3*b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 2
5264*a^3*b^9*c^5*d^2*e^12 + 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^
10*d^6*e^8 + 97184*a^4*b^4*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 5605
6*a^4*b^6*c^7*d^3*e^11 + 195584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*
d^5*e^9 + 388032*a^5*b^3*c^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 67048
8*a^5*b^5*c^7*d^2*e^12 - 488960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*
d^2*e^12 + 64*a*b^14*c^2*d*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12
*d^11*e^3 + 2504*a*b^5*c^11*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*
c^9*d^8*e^6 + 3568*a*b^8*c^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*
c^6*d^5*e^9 + 2528*a*b^11*c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b
^13*c^3*d^2*e^12 - 1152*a^2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67
968*a^3*b*c^13*d^10*e^4 + 15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8
*e^6 - 76928*a^4*b^8*c^5*d*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^
6*c^6*d*e^13 - 586368*a^6*b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731
008*a^7*b*c^9*d^2*e^12 - 244096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*
a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b
^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8
+ 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*
d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4
+ 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 +
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52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3
- 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e
^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^
6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c
^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*
b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024
*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^
3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^
4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e -
 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 30
72*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((x*(626688*a^10*b*c^8*e
^15 - 784384*a^10*c^9*d*e^14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^
15 + 47312*a^6*b^9*c^4*e^15 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*
e^15 - 1028096*a^9*b^3*c^7*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14
*d^11*e^4 + 325632*a^6*c^13*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8
*c^11*d^5*e^10 - 1773568*a^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*
c^10*d^12*e^3 + 240*b^10*c^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^
7*d^9*e^6 + 144*b^13*c^6*d^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*
e^9 - 96*b^16*c^3*d^5*e^10 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*
e^2 - 18432*a^2*b^5*c^12*d^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2
*b^7*c^10*d^10*e^5 - 16208*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7
- 78496*a^2*b^10*c^7*d^7*e^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c
^5*d^5*e^10 - 9264*a^2*b^13*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416
*a^2*b^15*c^2*d^2*e^13 - 12800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d
^12*e^3 - 151296*a^3*b^4*c^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 2066
88*a^3*b^6*c^10*d^9*e^6 - 436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d
^7*e^8 + 992*a^3*b^9*c^7*d^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3
*b^11*c^5*d^4*e^11 + 6912*a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^1
3 + 162816*a^4*b^2*c^13*d^11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^
4*b^4*c^11*d^9*e^6 + 1165824*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*
e^8 - 822272*a^4*b^7*c^8*d^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4
*b^9*c^6*d^4*e^11 - 189328*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e
^13 + 1290752*a^5*b^2*c^12*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*
a^5*b^4*c^10*d^7*e^8 + 3240960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^
5*e^10 - 683008*a^5*b^7*c^7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 16411
2*a^5*b^9*c^5*d^2*e^13 + 3442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^1
0*d^6*e^9 + 15232*a^6*b^4*c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 305
9648*a^6*b^6*c^7*d^3*e^12 - 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c
^10*d^5*e^10 - 6873088*a^7*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12
+ 2370048*a^7*b^5*c^7*d^2*e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8
*b^3*c^8*d^2*e^13 - 352*a*b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 48
00*a*b^8*c^10*d^11*e^4 + 5168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 -
 6000*a*b^11*c^7*d^8*e^7 + 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^
9 + 1152*a*b^14*c^4*d^5*e^10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3
*e^12 - 512*a^3*b^14*c^2*d*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^
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12*c^3*d*e^14 - 675840*a^5*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1
601536*a^6*b*c^12*d^8*e^7 + 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d
^6*e^9 - 1278304*a^7*b^6*c^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600
*a^8*b^4*c^7*d*e^14 + 2977792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14
))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8
- 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d
^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b
^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^
2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 -
92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 -
 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^
2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3
*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^
4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^
2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a
^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b
*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^
7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 +
 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 3
84*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) -
(((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 +
 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^
16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14
*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*
a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^1
4 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^
10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e
^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^
15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 2
4576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*
c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^
6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b
^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11
+ 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^
5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11
*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544
*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*
d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 384
0*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^
13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5
 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a
^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*
e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080
*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*
d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 -
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 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6
*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*
e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*
a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c
^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 +
 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7
*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*
e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 3436
5440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^
6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^1
4 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 306216
96*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^
6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13
 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a
^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 35
84*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^
6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7
*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^1
3*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 70
45120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d
^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 49356
80*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^
7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^
6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b
^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8
 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^
7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^
6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d
^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^
5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7
*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^
6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 153
6*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6
 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 10
24*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^
7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d
^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5
*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*
e^7)) - (x*(-d*e^7)^(1/2)*(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 -
6144*a^10*b^10*c^3*e^17 + 61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^1
7 + 983040*a^13*b^4*c^6*e^17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15
*d^14*e^3 - 5242880*a^9*c^14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 524288
0*a^11*c^12*d^8*e^9 + 5242880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^
13 + 5242880*a^14*c^9*d^2*e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12
*c^9*d^14*e^3 + 7168*a^2*b^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 +
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17920*a^2*b^15*c^6*d^11*e^6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c
^4*d^9*e^8 - 2048*a^2*b^18*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a
^3*b^9*c^11*d^15*e^2 + 41984*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d
^13*e^4 + 296192*a^3*b^12*c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267
520*a^3*b^14*c^6*d^10*e^7 - 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^
4*d^8*e^9 + 3328*a^3*b^17*c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960
*a^4*b^7*c^12*d^15*e^2 - 348160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^1
0*d^13*e^4 - 2478080*a^4*b^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6
- 1862144*a^4*b^12*c^7*d^10*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*
b^14*c^5*d^8*e^9 - 108800*a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^
11 + 2304*a^4*b^17*c^2*d^5*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^
5*b^6*c^12*d^14*e^3 - 5447680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10
*d^12*e^5 - 11166720*a^5*b^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 +
 1073920*a^5*b^11*c^7*d^9*e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b
^13*c^5*d^7*e^10 + 33280*a^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^1
2 - 1280*a^5*b^16*c^2*d^4*e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6
*b^4*c^13*d^14*e^3 + 12615680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11
*d^12*e^5 + 19701760*a^6*b^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 -
 15846400*a^6*b^9*c^8*d^9*e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6
*b^11*c^6*d^7*e^10 - 1111040*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^
5*e^12 + 25600*a^6*b^14*c^3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872
*a^7*b^2*c^14*d^14*e^3 - 14221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*
c^12*d^12*e^5 - 3768320*a^7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*
e^7 + 50126848*a^7*b^7*c^9*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104
*a^7*b^9*c^7*d^7*e^10 + 6200320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5
*d^5*e^12 - 228608*a^7*b^12*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 23
04*a^7*b^14*c^2*d^2*e^15 - 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3
*c^12*d^11*e^6 + 74711040*a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9
*e^8 - 11075584*a^8*b^6*c^9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479
360*a^8*b^8*c^7*d^6*e^11 - 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c
^5*d^4*e^13 - 302336*a^8*b^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 -
36962304*a^9*b^2*c^12*d^10*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^
9*b^4*c^10*d^8*e^9 - 54886400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d
^6*e^11 + 5545984*a^9*b^7*c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 135
6800*a^9*b^9*c^5*d^3*e^14 + 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^
2*c^11*d^8*e^9 + 19333120*a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6
*e^11 - 14974976*a^10*b^5*c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2
170880*a^10*b^7*c^6*d^3*e^14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^1
1*b^2*c^10*d^6*e^11 + 23920640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^
8*d^4*e^13 - 4096000*a^11*b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15
+ 12582912*a^12*b^2*c^9*d^4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 1153433
6*a^12*b^4*c^7*d^2*e^15 + 3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^
8*d*e^16 - 262144*a^7*b*c^15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*
a^8*b^13*c^2*d*e^16 + 25952256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e
^16 + 38010880*a^10*b*c^12*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*
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a^11*b*c^11*d^7*e^10 + 1679360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d
^5*e^12 - 5079040*a^12*b^5*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192
000*a^13*b^3*c^7*d*e^16))/(16*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^
4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a
^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3
*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^
8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^1
0*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*
e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*
e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^
4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^
3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^
3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^
6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1
152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e +
64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^1
1*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5
*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e
^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^
5 + 1024*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2
*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(
2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*
d^3*e^2))))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2
*e^3 + 2*a*c*d^3*e^2)) + (x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600
*a^3*b^6*c^6*e^13 + 4313*a^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a
^2*c^13*d^8*e^5 + 9792*a^3*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^
5*c^10*d^2*e^11 + 25*b^4*c^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*
d^6*e^7 - 168*b^7*c^8*d^5*e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4
*b^10*c^5*d^2*e^11 + 6336*a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8
 - 8506*a^2*b^4*c^9*d^4*e^9 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d
^2*e^11 + 22224*a^3*b^2*c^10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^
3*b^4*c^8*d^2*e^11 + 11712*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 410
88*a^5*b*c^9*d*e^12 - 360*a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 26
04*a*b^4*c^10*d^6*e^7 + 1272*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 23
2*a*b^7*c^7*d^3*e^10 - 48*a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 41
6*a^2*b^7*c^6*d*e^12 - 32128*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 -
 63360*a^4*b*c^10*d^3*e^10 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 2
56*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^
2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d
^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^
11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e
^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2
 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e
^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^
5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6
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*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^
5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a
^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1
024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9
*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4
*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*
e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 -
 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2))/(2*(c^2
*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^
2)) - (2000*a^4*c^9*e^12 + 21*a^2*b^4*c^7*e^12 - 520*a^3*b^2*c^8*e^12 + 129
6*a^2*c^11*d^4*e^8 + 4320*a^3*c^10*d^2*e^10 + 25*b^4*c^9*d^4*e^8 - 60*b^5*c
^8*d^3*e^9 + 35*b^6*c^7*d^2*e^10 + 192*a^2*b^2*c^9*d^2*e^10 - 112*a*b^5*c^7
*d*e^11 - 4480*a^3*b*c^9*d*e^11 - 360*a*b^2*c^10*d^4*e^8 + 832*a*b^3*c^9*d^
3*e^9 - 362*a*b^4*c^8*d^2*e^10 - 2880*a^2*b*c^10*d^3*e^9 + 1440*a^2*b^3*c^8
*d*e^11)/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*
c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4
*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 +
 a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*
d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*
e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6
*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*
d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c
^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*
b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*
a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 +
 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024
*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*
c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3
*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e
^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^
7)) + ((((-d*e^7)^(1/2)*((326912*a^8*c^9*d*e^13 - 241664*a^8*b*c^8*e^14 - 4
8*a^2*b^13*c^2*e^14 + 1264*a^3*b^11*c^3*e^14 - 13552*a^4*b^9*c^4*e^14 + 757
76*a^5*b^7*c^5*e^14 - 232960*a^6*b^5*c^6*e^14 + 372736*a^7*b^3*c^7*e^14 + 1
1520*a^3*c^14*d^11*e^3 + 78080*a^4*c^13*d^9*e^5 + 197120*a^5*c^12*d^7*e^7 +
 336384*a^6*c^11*d^5*e^9 + 532736*a^7*c^10*d^3*e^11 - 40*b^5*c^12*d^12*e^2
+ 216*b^6*c^11*d^11*e^3 - 464*b^7*c^10*d^10*e^4 + 496*b^8*c^9*d^9*e^5 - 264
*b^9*c^8*d^8*e^6 + 56*b^10*c^7*d^7*e^7 - 16*b^11*c^6*d^6*e^8 + 64*b^12*c^5*
d^5*e^9 - 96*b^13*c^4*d^4*e^10 + 64*b^14*c^3*d^3*e^11 - 16*b^15*c^2*d^2*e^1
2 + 1536*a^2*b^2*c^13*d^11*e^3 + 14400*a^2*b^3*c^12*d^10*e^4 - 47152*a^2*b^
4*c^11*d^9*e^5 + 52144*a^2*b^5*c^10*d^8*e^6 - 16272*a^2*b^6*c^9*d^7*e^7 - 1
3040*a^2*b^7*c^8*d^6*e^8 + 23488*a^2*b^8*c^7*d^5*e^9 - 26384*a^2*b^9*c^6*d^
4*e^10 + 13824*a^2*b^10*c^5*d^3*e^11 + 256*a^2*b^11*c^4*d^2*e^12 + 125056*a
^3*b^2*c^12*d^9*e^5 - 36224*a^3*b^3*c^11*d^8*e^6 - 126432*a^3*b^4*c^10*d^7*
e^7 + 144848*a^3*b^5*c^9*d^6*e^8 - 114752*a^3*b^6*c^8*d^5*e^9 + 125392*a^3*
b^7*c^7*d^4*e^10 - 53248*a^3*b^8*c^6*d^3*e^11 - 25264*a^3*b^9*c^5*d^2*e^12
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+ 474112*a^4*b^2*c^11*d^7*e^7 - 191104*a^4*b^3*c^10*d^6*e^8 + 97184*a^4*b^4
*c^9*d^5*e^9 - 277000*a^4*b^5*c^8*d^4*e^10 + 56056*a^4*b^6*c^7*d^3*e^11 + 1
95584*a^4*b^7*c^6*d^2*e^12 + 236800*a^5*b^2*c^10*d^5*e^9 + 388032*a^5*b^3*c
^9*d^4*e^10 + 159632*a^5*b^4*c^8*d^3*e^11 - 670488*a^5*b^5*c^7*d^2*e^12 - 4
88960*a^6*b^2*c^9*d^3*e^11 + 1106496*a^6*b^3*c^8*d^2*e^12 + 64*a*b^14*c^2*d
*e^13 + 448*a*b^3*c^13*d^12*e^2 - 1968*a*b^4*c^12*d^11*e^3 + 2504*a*b^5*c^1
1*d^10*e^4 + 768*a*b^6*c^10*d^9*e^5 - 4368*a*b^7*c^9*d^8*e^6 + 3568*a*b^8*c
^8*d^7*e^7 - 520*a*b^9*c^7*d^6*e^8 - 1728*a*b^10*c^6*d^5*e^9 + 2528*a*b^11*
c^5*d^4*e^10 - 1536*a*b^12*c^4*d^3*e^11 + 240*a*b^13*c^3*d^2*e^12 - 1152*a^
2*b*c^14*d^12*e^2 - 1600*a^2*b^12*c^3*d*e^13 - 67968*a^3*b*c^13*d^10*e^4 +
15808*a^3*b^10*c^4*d*e^13 - 342272*a^4*b*c^12*d^8*e^6 - 76928*a^4*b^8*c^5*d
*e^13 - 569088*a^5*b*c^11*d^6*e^8 + 179200*a^5*b^6*c^6*d*e^13 - 586368*a^6*
b*c^10*d^4*e^10 - 113008*a^6*b^4*c^7*d*e^13 - 731008*a^7*b*c^9*d^2*e^12 - 2
44096*a^7*b^2*c^8*d*e^13)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4
*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^
5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256
*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e
^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6
*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 5
12*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 -
 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e
^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d
^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c
^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7
*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*
b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5
*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e
 + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 30
72*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 102
4*a^8*b^3*c^3*d*e^7)) - (((x*(626688*a^10*b*c^8*e^15 - 784384*a^10*c^9*d*e^
14 + 208*a^4*b^13*c^2*e^15 - 4880*a^5*b^11*c^3*e^15 + 47312*a^6*b^9*c^4*e^1
5 - 242176*a^7*b^7*c^5*e^15 + 688640*a^8*b^5*c^6*e^15 - 1028096*a^9*b^3*c^7
*e^15 + 18432*a^4*c^15*d^13*e^2 + 126976*a^5*c^14*d^11*e^4 + 325632*a^6*c^1
3*d^9*e^6 + 139264*a^7*c^12*d^7*e^8 - 1067008*a^8*c^11*d^5*e^10 - 1773568*a
^9*c^10*d^3*e^12 + 16*b^8*c^11*d^13*e^2 - 96*b^9*c^10*d^12*e^3 + 240*b^10*c
^9*d^11*e^4 - 304*b^11*c^8*d^10*e^5 + 144*b^12*c^7*d^9*e^6 + 144*b^13*c^6*d
^8*e^7 - 304*b^14*c^5*d^7*e^8 + 240*b^15*c^4*d^6*e^9 - 96*b^16*c^3*d^5*e^10
 + 16*b^17*c^2*d^4*e^11 + 3200*a^2*b^4*c^13*d^13*e^2 - 18432*a^2*b^5*c^12*d
^12*e^3 + 41024*a^2*b^6*c^11*d^11*e^4 - 36352*a^2*b^7*c^10*d^10*e^5 - 16208
*a^2*b^8*c^9*d^9*e^6 + 74576*a^2*b^9*c^8*d^8*e^7 - 78496*a^2*b^10*c^7*d^7*e
^8 + 32064*a^2*b^11*c^6*d^6*e^9 + 6000*a^2*b^12*c^5*d^5*e^10 - 9264*a^2*b^1
3*c^4*d^4*e^11 + 1472*a^2*b^14*c^3*d^3*e^12 + 416*a^2*b^15*c^2*d^2*e^13 - 1
2800*a^3*b^2*c^14*d^13*e^2 + 73728*a^3*b^3*c^13*d^12*e^3 - 151296*a^3*b^4*c
^12*d^11*e^4 + 78336*a^3*b^5*c^11*d^10*e^5 + 206688*a^3*b^6*c^10*d^9*e^6 -
436736*a^3*b^7*c^9*d^8*e^7 + 324224*a^3*b^8*c^8*d^7*e^8 + 992*a^3*b^9*c^7*d
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^6*e^9 - 158176*a^3*b^10*c^6*d^5*e^10 + 77056*a^3*b^11*c^5*d^4*e^11 + 6912*
a^3*b^12*c^4*d^3*e^12 - 8416*a^3*b^13*c^3*d^2*e^13 + 162816*a^4*b^2*c^13*d^
11*e^4 + 184320*a^4*b^3*c^12*d^10*e^5 - 916608*a^4*b^4*c^11*d^9*e^6 + 11658
24*a^4*b^5*c^10*d^8*e^7 - 314496*a^4*b^6*c^9*d^7*e^8 - 822272*a^4*b^7*c^8*d
^6*e^9 + 919152*a^4*b^8*c^7*d^5*e^10 - 175296*a^4*b^9*c^6*d^4*e^11 - 189328
*a^4*b^10*c^5*d^3*e^12 + 62064*a^4*b^11*c^4*d^2*e^13 + 1290752*a^5*b^2*c^12
*d^9*e^6 - 659456*a^5*b^3*c^11*d^8*e^7 - 1561088*a^5*b^4*c^10*d^7*e^8 + 324
0960*a^5*b^5*c^9*d^6*e^9 - 1964192*a^5*b^6*c^8*d^5*e^10 - 683008*a^5*b^7*c^
7*d^4*e^11 + 1162304*a^5*b^8*c^6*d^3*e^12 - 164112*a^5*b^9*c^5*d^2*e^13 + 3
442688*a^6*b^2*c^11*d^7*e^8 - 3670016*a^6*b^3*c^10*d^6*e^9 + 15232*a^6*b^4*
c^9*d^5*e^10 + 4230144*a^6*b^5*c^8*d^4*e^11 - 3059648*a^6*b^6*c^7*d^3*e^12
- 247296*a^6*b^7*c^6*d^2*e^13 + 4010496*a^7*b^2*c^10*d^5*e^10 - 6873088*a^7
*b^3*c^9*d^4*e^11 + 2822400*a^7*b^4*c^8*d^3*e^12 + 2370048*a^7*b^5*c^7*d^2*
e^13 + 1178624*a^8*b^2*c^9*d^3*e^12 - 4739072*a^8*b^3*c^8*d^2*e^13 - 352*a*
b^6*c^12*d^13*e^2 + 2048*a*b^7*c^11*d^12*e^3 - 4800*a*b^8*c^10*d^11*e^4 + 5
168*a*b^9*c^9*d^10*e^5 - 480*a*b^10*c^8*d^9*e^6 - 6000*a*b^11*c^7*d^8*e^7 +
 8192*a*b^12*c^6*d^7*e^8 - 5040*a*b^13*c^5*d^6*e^9 + 1152*a*b^14*c^4*d^5*e^
10 + 240*a*b^15*c^3*d^4*e^11 - 128*a*b^16*c^2*d^3*e^12 - 512*a^3*b^14*c^2*d
*e^14 - 106496*a^4*b*c^14*d^12*e^3 + 11680*a^4*b^12*c^3*d*e^14 - 675840*a^5
*b*c^13*d^10*e^5 - 108288*a^5*b^10*c^4*d*e^14 - 1601536*a^6*b*c^12*d^8*e^7
+ 514768*a^6*b^8*c^5*d*e^14 - 925696*a^7*b*c^11*d^6*e^9 - 1278304*a^7*b^6*c
^6*d*e^14 + 2457600*a^8*b*c^10*d^4*e^11 + 1385600*a^8*b^4*c^7*d*e^14 + 2977
792*a^9*b*c^9*d^2*e^13 + 19968*a^9*b^2*c^8*d*e^14))/(8*(a^6*b^8*e^8 + 256*a
^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^
8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 +
 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d
^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 +
 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 5
2*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 -
 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^
3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6
*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^
3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b
^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*
a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3
*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4
 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e -
92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 307
2*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (((1048576*a^13*c^8*e^16 +
 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 -
327680*a^10*b^6*c^5*e^16 + 983040*a^11*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*
e^16 - 196608*a^6*c^15*d^14*e^2 - 917504*a^7*c^14*d^12*e^4 - 589824*a^8*c^1
3*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 + 10158080*a^10*c^11*d^6*e^10 + 10616
832*a^11*c^10*d^4*e^12 + 5308416*a^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14
*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a
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^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d
^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*
b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2
 - 198656*a^3*b^7*c^11*d^13*e^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^
3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^
9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^
3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^
12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*
a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9
*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 17
32096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13
*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 +
229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*
b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^
10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 61301
76*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^
5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3
200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3
*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9
*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 1220531
2*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5
*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21
725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*
b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*
e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 211225
6*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^
11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10
- 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a
^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*
d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 -
9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*
b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^
2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b
^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7
168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^
7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*
e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7
*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 -
 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c
^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 1
9202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8
 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7
+ a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c
^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^
3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d
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^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6
*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d
^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^
4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2
*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^
6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 20
48*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6
 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2
*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c
*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*
d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e
^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7)) + (x*(-d*e^7)^(1/2)*
(1048576*a^15*c^8*e^17 + 256*a^9*b^12*c^2*e^17 - 6144*a^10*b^10*c^3*e^17 +
61440*a^11*b^8*c^4*e^17 - 327680*a^12*b^6*c^5*e^17 + 983040*a^13*b^4*c^6*e^
17 - 1572864*a^14*b^2*c^7*e^17 - 1048576*a^8*c^15*d^14*e^3 - 5242880*a^9*c^
14*d^12*e^5 - 9437184*a^10*c^13*d^10*e^7 - 5242880*a^11*c^12*d^8*e^9 + 5242
880*a^12*c^11*d^6*e^11 + 9437184*a^13*c^10*d^4*e^13 + 5242880*a^14*c^9*d^2*
e^15 + 256*a^2*b^11*c^10*d^15*e^2 - 2048*a^2*b^12*c^9*d^14*e^3 + 7168*a^2*b
^13*c^8*d^13*e^4 - 14336*a^2*b^14*c^7*d^12*e^5 + 17920*a^2*b^15*c^6*d^11*e^
6 - 14336*a^2*b^16*c^5*d^10*e^7 + 7168*a^2*b^17*c^4*d^9*e^8 - 2048*a^2*b^18
*c^3*d^8*e^9 + 256*a^2*b^19*c^2*d^7*e^10 - 5120*a^3*b^9*c^11*d^15*e^2 + 419
84*a^3*b^10*c^10*d^14*e^3 - 148736*a^3*b^11*c^9*d^13*e^4 + 296192*a^3*b^12*
c^8*d^12*e^5 - 359680*a^3*b^13*c^7*d^11*e^6 + 267520*a^3*b^14*c^6*d^10*e^7
- 112384*a^3*b^15*c^5*d^9*e^8 + 18176*a^3*b^16*c^4*d^8*e^9 + 3328*a^3*b^17*
c^3*d^7*e^10 - 1280*a^3*b^18*c^2*d^6*e^11 + 40960*a^4*b^7*c^12*d^15*e^2 - 3
48160*a^4*b^8*c^11*d^14*e^3 + 1254400*a^4*b^9*c^10*d^13*e^4 - 2478080*a^4*b
^10*c^9*d^12*e^5 + 2867456*a^4*b^11*c^8*d^11*e^6 - 1862144*a^4*b^12*c^7*d^1
0*e^7 + 490240*a^4*b^13*c^6*d^9*e^8 + 128000*a^4*b^14*c^5*d^8*e^9 - 108800*
a^4*b^15*c^4*d^7*e^10 + 13824*a^4*b^16*c^3*d^6*e^11 + 2304*a^4*b^17*c^2*d^5
*e^12 - 163840*a^5*b^5*c^13*d^15*e^2 + 1474560*a^5*b^6*c^12*d^14*e^3 - 5447
680*a^5*b^7*c^11*d^13*e^4 + 10588160*a^5*b^8*c^10*d^12*e^5 - 11166720*a^5*b
^9*c^9*d^11*e^6 + 5159936*a^5*b^10*c^8*d^10*e^7 + 1073920*a^5*b^11*c^7*d^9*
e^8 - 2279680*a^5*b^12*c^6*d^8*e^9 + 770560*a^5*b^13*c^5*d^7*e^10 + 33280*a
^5*b^14*c^4*d^6*e^11 - 41216*a^5*b^15*c^3*d^5*e^12 - 1280*a^5*b^16*c^2*d^4*
e^13 + 327680*a^6*b^3*c^14*d^15*e^2 - 3276800*a^6*b^4*c^13*d^14*e^3 + 12615
680*a^6*b^5*c^12*d^13*e^4 - 23592960*a^6*b^6*c^11*d^12*e^5 + 19701760*a^6*b
^7*c^10*d^11*e^6 + 1372160*a^6*b^8*c^9*d^10*e^7 - 15846400*a^6*b^9*c^8*d^9*
e^8 + 10864640*a^6*b^10*c^7*d^8*e^9 - 1352960*a^6*b^11*c^6*d^7*e^10 - 11110
40*a^6*b^12*c^5*d^6*e^11 + 273920*a^6*b^13*c^4*d^5*e^12 + 25600*a^6*b^14*c^
3*d^4*e^13 - 1280*a^6*b^15*c^2*d^3*e^14 + 3407872*a^7*b^2*c^14*d^14*e^3 - 1
4221312*a^7*b^3*c^13*d^13*e^4 + 23527424*a^7*b^4*c^12*d^12*e^5 - 3768320*a^
7*b^5*c^11*d^11*e^6 - 38895616*a^7*b^6*c^10*d^10*e^7 + 50126848*a^7*b^7*c^9
*d^9*e^8 - 18362368*a^7*b^8*c^8*d^8*e^9 - 6831104*a^7*b^9*c^7*d^7*e^10 + 62
00320*a^7*b^10*c^6*d^6*e^11 - 726784*a^7*b^11*c^5*d^5*e^12 - 228608*a^7*b^1
2*c^4*d^4*e^13 + 31488*a^7*b^13*c^3*d^3*e^14 + 2304*a^7*b^14*c^2*d^2*e^15 -
```

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 3145728*a^8*b^2*c^13*d^12*e^5 - 31129600*a^8*b^3*c^12*d^11*e^6 + 74711040*
a^8*b^4*c^11*d^10*e^7 - 55476224*a^8*b^5*c^10*d^9*e^8 - 11075584*a^8*b^6*c^
9*d^8*e^9 + 35381248*a^8*b^7*c^8*d^7*e^10 - 14479360*a^8*b^8*c^7*d^6*e^11 -
 168960*a^8*b^9*c^6*d^5*e^12 + 1286144*a^8*b^10*c^5*d^4*e^13 - 302336*a^8*b
^11*c^4*d^3*e^14 - 55808*a^8*b^12*c^3*d^2*e^15 - 36962304*a^9*b^2*c^12*d^10
*e^7 - 9502720*a^9*b^3*c^11*d^9*e^8 + 67174400*a^9*b^4*c^10*d^8*e^9 - 54886
400*a^9*b^5*c^9*d^7*e^10 + 11239424*a^9*b^6*c^8*d^6*e^11 + 5545984*a^9*b^7*
c^7*d^5*e^12 - 5263360*a^9*b^8*c^6*d^4*e^13 + 1356800*a^9*b^9*c^5*d^3*e^14
+ 558080*a^9*b^10*c^4*d^2*e^15 - 49807360*a^10*b^2*c^11*d^8*e^9 + 19333120*
a^10*b^3*c^10*d^7*e^10 + 7208960*a^10*b^4*c^9*d^6*e^11 - 14974976*a^10*b^5*
c^8*d^5*e^12 + 15073280*a^10*b^6*c^7*d^4*e^13 - 2170880*a^10*b^7*c^6*d^3*e^
14 - 2928640*a^10*b^8*c^5*d^2*e^15 - 11796480*a^11*b^2*c^10*d^6*e^11 + 2392
0640*a^11*b^3*c^9*d^5*e^12 - 24576000*a^11*b^4*c^8*d^4*e^13 - 4096000*a^11*
b^5*c^7*d^3*e^14 + 8355840*a^11*b^6*c^6*d^2*e^15 + 12582912*a^12*b^2*c^9*d^
4*e^13 + 19857408*a^12*b^3*c^8*d^3*e^14 - 11534336*a^12*b^4*c^7*d^2*e^15 +
3407872*a^13*b^2*c^8*d^2*e^15 - 5505024*a^14*b*c^8*d*e^16 - 262144*a^7*b*c^
15*d^15*e^2 + 5505024*a^8*b*c^14*d^13*e^4 - 1280*a^8*b^13*c^2*d*e^16 + 2595
2256*a^9*b*c^13*d^11*e^6 + 30976*a^9*b^11*c^3*d*e^16 + 38010880*a^10*b*c^12
*d^9*e^8 - 312320*a^10*b^9*c^4*d*e^16 + 11796480*a^11*b*c^11*d^7*e^10 + 167
9360*a^11*b^7*c^5*d*e^16 - 21233664*a^12*b*c^10*d^5*e^12 - 5079040*a^12*b^5
*c^6*d*e^16 - 20709376*a^13*b*c^9*d^3*e^14 + 8192000*a^13*b^3*c^7*d*e^16))/
(16*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*
c*d^3*e^2)*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c
*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*
c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 +
a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d
^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e
^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*
e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d
^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^
2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b
^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a
^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 +
512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*
a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c
^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a^4*b^9*c*d^3*
e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7*b*c^6*d^5*e^
3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*b^3*c^3*d*e^7
)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2
*a*b*d^2*e^3 + 2*a*c*d^3*e^2)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b
^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2))))/(2*(c^2*d^5 +
a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2)) - (
x*(22800*a^6*c^9*e^13 + 36*a^2*b^8*c^5*e^13 - 600*a^3*b^6*c^6*e^13 + 4313*a
^4*b^4*c^7*e^13 - 15592*a^5*b^2*c^8*e^13 + 1296*a^2*c^13*d^8*e^5 + 9792*a^3
*c^12*d^6*e^7 + 30304*a^4*c^11*d^4*e^9 + 40512*a^5*c^10*d^2*e^11 + 25*b^4*c
```

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^11*d^8*e^5 - 120*b^5*c^10*d^7*e^6 + 214*b^6*c^9*d^6*e^7 - 168*b^7*c^8*d^5*
e^8 + 53*b^8*c^7*d^4*e^9 - 8*b^9*c^6*d^3*e^10 + 4*b^10*c^5*d^2*e^11 + 6336*
a^2*b^2*c^11*d^6*e^7 + 3840*a^2*b^3*c^10*d^5*e^8 - 8506*a^2*b^4*c^9*d^4*e^9
 + 1112*a^2*b^5*c^8*d^3*e^10 + 1254*a^2*b^6*c^7*d^2*e^11 + 22224*a^3*b^2*c^
10*d^4*e^9 + 13824*a^3*b^3*c^9*d^3*e^10 - 9516*a^3*b^4*c^8*d^2*e^11 + 11712
*a^4*b^2*c^9*d^2*e^11 - 24*a*b^9*c^5*d*e^12 - 41088*a^5*b*c^9*d*e^12 - 360*
a*b^2*c^12*d^8*e^5 + 1664*a*b^3*c^11*d^7*e^6 - 2604*a*b^4*c^10*d^6*e^7 + 12
72*a*b^5*c^9*d^5*e^8 + 332*a*b^6*c^8*d^4*e^9 - 232*a*b^7*c^7*d^3*e^10 - 48*
a*b^8*c^6*d^2*e^11 - 5760*a^2*b*c^12*d^7*e^6 + 416*a^2*b^7*c^6*d*e^12 - 321
28*a^3*b*c^11*d^5*e^8 - 4120*a^3*b^5*c^7*d*e^12 - 63360*a^4*b*c^10*d^3*e^10
 + 21376*a^4*b^3*c^8*d*e^12))/(8*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*
c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6
*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 -
256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^
2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6
+ 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3
+ 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^
4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^
4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^
5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^
6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*
a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a
^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*
d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7
*e + 52*a^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 -
 3072*a^7*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 +
1024*a^8*b^3*c^3*d*e^7)))*(-d*e^7)^(1/2))/(2*(c^2*d^5 + a^2*d*e^4 + b^2*d^3
*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3*e^2))))*(-d*e^7)^(1/2)*1i)/(
c^2*d^5 + a^2*d*e^4 + b^2*d^3*e^2 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + 2*a*c*d^3
*e^2)
```

3.275 $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1077

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)^2\,(a+bx^2+cx^4)^2}\,dx &= \frac{e^4x}{2d\,(cd^2-bde+ae^2)^2\,(d+ex^2)} \\
&+ \frac{x(abce(2cd-be)+(b^2-2ac)\,(c^2d^2+b^2e^2-ce(2bd+ae))-c(2b^2cde-4ac^2de-b^3e^2-bc(cd^2-3ae^2)}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)^2\,(a+bx^2+cx^4)} \\
&+ \frac{\sqrt{2}\sqrt{c}e^2\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^3} \\
&+ \frac{\sqrt{c}\left(b^4e^2-b^3e\left(2cd-\sqrt{b^2-4ac}e\right)-4ac^2\left(3cd^2-e\left(\sqrt{b^2-4ac}d+3ae\right)\right)+b^2c\left(cd^2-e\left(2\sqrt{b^2-4ac}d+9\right.\right.\right.}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-} \\
&- \frac{\sqrt{2}\sqrt{c}e^2\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^3} \\
&- \frac{\sqrt{c}\left(b^4e^2-b^3e\left(2cd+\sqrt{b^2-4ac}e\right)+bc\left(3a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-16ae\right)\right)+b^2c\left(cd^2+e\left(2\sqrt{b^2\right.\right.\right.}{2\sqrt{2}a\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-} \\
&+ \frac{2e^{7/2}(2cd-be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^3} + \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\,(cd^2-bde+ae^2)^2}
\end{aligned}
$$

```
[Out] 1/2*e^4*x/d/(a*e^2-b*d*e+c*d^2)^2/(e*x^2+d)+1/2*x*(a*b*c*e*(-b*e+2*c*d)+(-2
*a*c+b^2)*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d))-c*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*
e^2-b*c*(-3*a*e^2+c*d^2))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+
b*x^2+a)+1/2*e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)^
2+2*e^(7/2)*(-b*e+2*c*d)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)^3/d^
(1/2)+e^2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^
```

```
(1/2)*(3*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-c*e*(3*b*d+a*e+2*d*(-4*a*c+b^
2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b
^4*e^2-b^3*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-4*a*c^2*(3*c*d^2-e*(3*a*e+d*(-4*a
*c+b^2)^(1/2)))-b*c*(3*a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(16*a*e+d*(-4*a*c+b^2)^
(1/2)))+b^2*c*(c*d^2-e*(9*a*e+2*d*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)^(3/2
)/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-e^2*arctan(x*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)*(3*c^2*d^2+b*e
^2*(b-(-4*a*c+b^2)^(1/2))-c*e*(3*b*d+a*e-2*d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*
d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4*e^2-b^3*e*(2*c*d+
e*(-4*a*c+b^2)^(1/2))+b*c*(3*a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-16*a*e+d*(-4*a*
c+b^2)^(1/2)))-4*a*c^2*(3*c*d^2+e*(-3*a*e+d*(-4*a*c+b^2)^(1/2)))+b^2*c*(c*d
^2+e*(-9*a*e+2*d*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*
d^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 8.97 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1252, 205, 211, 1192, 1180}

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{xe^4}{2d\left(cd^2-bed+ae^2\right)^2\left(ex^2+d\right)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2-bed+ae^2\right)^2}+\frac{2(2cd-be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\sqrt{d}\left(cd^2-bed+ae^2\right)^3}$$

$$+\frac{\sqrt{2}\sqrt{c}\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)e^2}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^3}$$

$$-\frac{\sqrt{2}\sqrt{c}\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)e^2}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bed+ae^2\right)^3}$$

$$+\frac{\sqrt{c}\left(e^2b^4-e\left(2cd-\sqrt{b^2-4ac}e\right)b^3+c\left(cd^2-e\left(2\sqrt{b^2-4ac}d+9ae\right)\right)b^2-c\left(3a\sqrt{b^2-4ac}e^2-cd(\sqrt{b^2\right.\right.}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}(cd}$$

$$-\frac{\sqrt{c}\left(e^2b^4-e\left(2cd+\sqrt{b^2-4ac}e\right)b^3+c\left(cd^2+e\left(2\sqrt{b^2-4ac}d-9ae\right)\right)b^2+c\left(3a\sqrt{b^2-4ac}e^2-cd(\sqrt{b^2\right.\right.}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}(cd}$$

$$+\frac{x(-c(-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de)x^2+abce(2cd-be)+\left(b^2-2ac\right)\left(c^2d^2+b^2e^2-ce(2\right.}{2a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2\left(cx^4+bx^2+a\right)}$$

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) + (x*(a*b*c*e*(2*c*d -
b*e) + (b^2 - 2*a*c)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e)) - c*(2*b^2*c*d
*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^2))/(2*a*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2*(3*
c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*
d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) + (Sqrt[c
]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) - 4*a*c^2*(3*c*d^2 - e*(Sq
rt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 9*a
*e)) - b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 16*a*e))
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^
2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (
Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d
 - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)^3) - (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + b
*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 16*a*e)) + b^2*c
*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 9*a*e)) - 4*a*c^2*(3*c*d^2 + e*(Sqrt[b
^2 - 4*a*c]*d - 3*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)^2) + (2*e^(7/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^3) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{e^4}{(cd^2-bde+ae^2)^2\,(d+ex^2)^2}-\frac{2e^4(-2cd+be)}{(cd^2-bde+ae^2)^3\,(d+ex^2)}\right.\\&\qquad+\frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)x^2}{(cd^2-bde+ae^2)^2\,(a+bx^2+cx^4)^2}\\&\qquad\left.+\frac{e^2(3c^2d^2+2b^2e^2-ce(5bd+ae)-2ce(2cd-be)x^2)}{(cd^2-bde+ae^2)^3\,(a+bx^2+cx^4)}\right)dx\\&=\frac{e^2\int\frac{3c^2d^2+2b^2e^2-ce(5bd+ae)-2ce(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{(cd^2-bde+ae^2)^3}+\frac{(2e^4(2cd-be))\int\frac{1}{d+ex^2}\,dx}{(cd^2-bde+ae^2)^3}\\&\quad+\frac{\int\frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)x^2}{(a+bx^2+cx^4)^2}\,dx}{(cd^2-bde+ae^2)^2}+\frac{e^4\int\frac{1}{(d+ex^2)^2}\,dx}{(cd^2-bde+ae^2)^2}\end{aligned}$$

$$\begin{aligned}
&= \frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} \\
&+ \frac{x(abce(2cd-be)+(b^2-2ac)(c^2d^2+b^2e^2-ce(2bd+ae))-c(2b^2cde-4ac^2de-b^3e^2-bc(cd^2}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2\left(a+bx^2+cx^4\right)} \\
&+ \frac{2e^{7/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)^3} \\
&- \frac{\int \frac{2b^3cde-10abc^2de-b^4e^2-b^2c\left(cd^2-6ae^2\right)+6ac^2\left(cd^2-ae^2\right)+c\left(2b^2cde-4ac^2de-b^3e^2-bc\left(cd^2-3ae^2\right)\right)x^2}{a+bx^2+cx^4}dx}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2} \\
&+ \frac{e^4\int \frac{1}{d+ex^2}dx}{2d\left(cd^2-bde+ae^2\right)^2} \\
&- \frac{\left(ce^2\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^3} \\
&+ \frac{\left(ce^2\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} \\
&+ \frac{x(abce(2cd-be)+(b^2-2ac)(c^2d^2+b^2e^2-ce(2bd+ae))-c(2b^2cde-4ac^2de-b^3e^2-bc(cd^2}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2\left(a+bx^2+cx^4\right)} \\
&+ \frac{\sqrt{2}\sqrt{c}e^2\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)^3} \\
&- \frac{\sqrt{2}\sqrt{c}e^2\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)^3} \\
&+ \frac{2e^{7/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)^3} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2-bde+ae^2\right)^2} \\
&- \frac{(c(b^4e^2-b^3e(2cd+\sqrt{b^2-4ac}e)+bc(3a\sqrt{b^2-4ac}e^2-cd(\sqrt{b^2-4ac}d-16ae))+b^2c(cd^2+e}{4a\left(b^2-4ac\right)^{3/2}(cd^2-b} \\
&+ \frac{(c(b^4e^2-b^3e(2cd-\sqrt{b^2-4ac}e)-4ac^2(3cd^2-e(\sqrt{b^2-4ac}d+3ae))+b^2c(cd^2-e(2\sqrt{b^2-4}}{4a\left(b^2-4ac\right)^{3/2}(cd^2-b}
\end{aligned}$$

$$= \frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)}$$

$$+ \frac{x(abce(2cd-be)+\left(b^2-2ac\right)\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)-c(2b^2cde-4ac^2de-b^3e^2-bc(cd}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2\left(a+bx^2+cx^4\right)}$$

$$+ \frac{\sqrt{2}\sqrt{c}e^2\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)^3}$$

$$+ \frac{\sqrt{c}\left(b^4e^2-b^3e\left(2cd-\sqrt{b^2-4ac}e\right)-4ac^2\left(3cd^2-e\left(\sqrt{b^2-4ac}d+3ae\right)\right)+b^2c\left(cd^2-e\left(2\sqrt{b^2-\right.\right.\right.}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-}}}$$

$$- \frac{\sqrt{2}\sqrt{c}e^2\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)^3}$$

$$- \frac{\sqrt{c}\left(b^4e^2-b^3e\left(2cd+\sqrt{b^2-4ac}e\right)+bc\left(3a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-16ae\right)\right)+b^2c\left(cd^2+\right.\right.}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-}}}$$

$$+ \frac{2e^{7/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)^3} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2-bde+ae^2\right)^2}$$

Mathematica [A] (verified)

Time = 3.32 (sec) , antiderivative size = 1020, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{4}\left(\frac{2e^4x}{d\left(cd^2+e(-bd+ae)\right)^2\left(d+ex^2\right)}\right.$$

$$- \frac{2x(b^4e^2+b^3ce(-2d+ex^2)+2ac^2(ae^2-cd(d-2ex^2))+b^2c(-4ae^2+cd(d-2ex^2))+bc^2(cd^2x^2-3a}{a\left(-b^2+4ac\right)\left(cd^2+e(-bd+ae)\right)^2\left(a+bx^2+cx^4\right)}$$

$$+ \frac{\sqrt{2}\sqrt{c}\left(b^5de^3+b^3e\left(cd-\sqrt{b^2-4ac}e\right)\left(3cd^2+5ae^2\right)+b^4e^2\left(-3cd^2+e\left(\sqrt{b^2-4ac}d-5ae\right)\right)-4ac^2\left(-3\right.\right.}{}$$

$$- \frac{\sqrt{2}\sqrt{c}\left(b^5de^3+b^3e\left(cd+\sqrt{b^2-4ac}e\right)\left(3cd^2+5ae^2\right)-b^2c\left(c^2d^4+ae^3\left(7\sqrt{b^2-4ac}d-29ae\right)+3cd^2e\left(\sqrt{}\right.\right.\right.}{}$$

$$\left.+ \frac{2e^{7/2}\left(9cd^2+e(-5bd+ae)\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+e(-bd+ae)\right)^3}\right)$$

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*e^4*x)/(d*(c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x^2)) - (2*x*(b^4*e^2 + b
^3*c*e*(-2*d + e*x^2) + 2*a*c^2*(a*e^2 - c*d*(d - 2*e*x^2)) + b^2*c*(-4*a*e
^2 + c*d*(d - 2*e*x^2)) + b*c^2*(c*d^2*x^2 - 3*a*e*(-2*d + e*x^2))))/(a*(-b
```

```
^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sq
rt[c]*(b^5*d*e^3 + b^3*e*(c*d - Sqrt[b^2 - 4*a*c]*e)*(3*c*d^2 + 5*a*e^2) +
b^4*e^2*(-3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - 5*a*e)) - 4*a*c^2*(-3*c^2*d^4
+ c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d + 7
*a*e)) - b*c*(-19*a^2*Sqrt[b^2 - 4*a*c]*e^4 + 2*a*c*d*e^2*(-3*Sqrt[b^2 - 4*
a*c]*d + 26*a*e) + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d + 28*a*e)) + b^2*c*(-(c^2*d
^4) + 3*c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*e^3*(7*Sqrt[b^2 - 4*a*c]*
d + 29*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))^3
) - (Sqrt[2]*Sqrt[c]*(b^5*d*e^3 + b^3*e*(c*d + Sqrt[b^2 - 4*a*c]*e)*(3*c*d^
2 + 5*a*e^2) - b^2*c*(c^2*d^4 + a*e^3*(7*Sqrt[b^2 - 4*a*c]*d - 29*a*e) + 3*
c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b^4*e^2*(3*c*d^2 + e*(Sqrt[b^2 - 4
*a*c]*d + 5*a*e)) + 4*a*c^2*(3*c^2*d^4 + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d - 7*a
*e) + c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)) + b*c*(-19*a^2*Sqrt[b^2 - 4*a
*c]*e^4 + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d - 28*a*e) - 2*a*c*d*e^2*(3*Sqrt[b^2
- 4*a*c]*d + 26*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d
- a*e))^3) + (2*e^(7/2)*(9*c*d^2 + e*(-5*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(d^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^3))/4
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 1250, normalized size of antiderivative = 1.16

method	result	size
default	Expression too large to display	1250
risch	Expression too large to display	79373

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*e^2-b*d*e+c*d^2)^3*((-1/2*c*(3*a^2*b*c*e^4-4*a^2*c^2*d*e^3-a*b^3*e^4-
a*b^2*c*d*e^3+6*a*b*c^2*d^2*e^2-4*a*c^3*d^3*e+b^4*d*e^3-3*b^3*c*d^2*e^2+3*b
^2*c^2*d^3*e-b*c^3*d^4)/a/(4*a*c-b^2)*x^3+1/2*(2*a^3*c^2*e^4-4*a^2*b^2*c*e^
4+4*a^2*b*c^2*d*e^3+a*b^4*e^4+2*a*b^3*c*d*e^3-9*a*b^2*c^2*d^2*e^2+8*a*b*c^3
*d^3*e-2*a*c^4*d^4-b^5*d*e^3+3*b^4*c*d^2*e^2-3*b^3*c^2*d^3*e+b^2*c^3*d^4)/a
/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*c*(1/8*(-19*a^2*b*c*e^4*(-4
*a*c+b^2)^(1/2)+36*a^2*c^2*d*e^3*(-4*a*c+b^2)^(1/2)+5*a*b^3*e^4*(-4*a*c+b^2
)^(1/2)-7*b^2*c*d*e^3*a*(-4*a*c+b^2)^(1/2)-6*b*c^2*d^2*e^2*a*(-4*a*c+b^2)^(
1/2)+4*c^3*d^3*e*a*(-4*a*c+b^2)^(1/2)-b^4*d*e^3*(-4*a*c+b^2)^(1/2)+3*b^3*c*
d^2*e^2*(-4*a*c+b^2)^(1/2)-3*b^2*c^2*d^3*e*(-4*a*c+b^2)^(1/2)+b*c^3*d^4*(-4
*a*c+b^2)^(1/2)-28*a^3*c^2*e^4+29*a^2*b^2*c*e^4-52*a^2*b*c^2*d*e^3+48*a^2*c
^3*d^2*e^2-5*a*b^4*e^4+5*a*b^3*c*d*e^3+12*a*b^2*c^2*d^2*e^2-28*a*b*c^3*d^3*
e+12*a*c^4*d^4+b^5*d*e^3-3*b^4*c*d^2*e^2+3*b^3*c^2*d^3*e-b^2*c^3*d^4)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-19*a^2*b*c*e^4*(-4*a*c+b^2)^(1/2)+36
```

```
*a^2*c^2*d*e^3*(-4*a*c+b^2)^(1/2)+5*a*b^3*e^4*(-4*a*c+b^2)^(1/2)-7*b^2*c*d*
e^3*a*(-4*a*c+b^2)^(1/2)-6*b*c^2*d^2*e^2*a*(-4*a*c+b^2)^(1/2)+4*c^3*d^3*e*a
*(-4*a*c+b^2)^(1/2)-b^4*d*e^3*(-4*a*c+b^2)^(1/2)+3*b^3*c*d^2*e^2*(-4*a*c+b^
2)^(1/2)-3*b^2*c^2*d^3*e*(-4*a*c+b^2)^(1/2)+b*c^3*d^4*(-4*a*c+b^2)^(1/2)+28
*a^3*c^2*e^4-29*a^2*b^2*c*e^4+52*a^2*b*c^2*d*e^3-48*a^2*c^3*d^2*e^2+5*a*b^4
*e^4-5*a*b^3*c*d*e^3-12*a*b^2*c^2*d^2*e^2+28*a*b*c^3*d^3*e-12*a*c^4*d^4-b^5
*d*e^3+3*b^4*c*d^2*e^2-3*b^3*c^2*d^3*e+b^2*c^3*d^4)/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))))+e^4/(a*e^2-b*d*e+c*d^2)^3*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/
(e*x^2+d)+1/2*(a*e^2-5*b*d*e+9*c*d^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65158 vs. $2(954) = 1908$.

Time = 10.36 (sec) , antiderivative size = 65158, normalized size of antiderivative = 60.50

$$\int \frac{1}{(d+ex^2)^2 (a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*((2*a^2*b^7*c^11 - 40*a^3*b^5*c^12 + 224*a^4*b^3*c^13 - 384*a^5*b*c^14
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9 +
20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^10 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^10 -
 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^11
 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^1
1 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^11
+ 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^12
+ 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^12
 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^1
2 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^13
 - 2*(b^2 - 4*a*c)*a^2*b^5*c^11 + 32*(b^2 - 4*a*c)*a^3*b^3*c^12 - 96*(b^2 -
 4*a*c)*a^4*b*c^13)*d^16 - (18*a^2*b^8*c^10 - 344*a^3*b^6*c^11 + 1888*a^4*b
^4*c^12 - 3200*a^5*b^2*c^13 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^8*c^8 + 172*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^6*c^9 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^7*c^9 - 944*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^4*c^10 - 272*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^5*c^10 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^6*c^10 + 1600*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^11 + 800*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^11 + 136*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^11 - 400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^12 - 18*(b^2 - 4*a*c)*a^2*b^6*c^10 +
272*(b^2 - 4*a*c)*a^3*b^4*c^11 - 800*(b^2 - 4*a*c)*a^4*b^2*c^12)*d^15*e + 6
*(12*a^2*b^9*c^9 - 214*a^3*b^7*c^10 + 1096*a^4*b^5*c^11 - 1568*a^5*b^3*c^12
 - 640*a^6*b*c^13 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^9*c^7 + 107*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^7*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^8*c^8 - 548*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b^5*c^9 - 166*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b^6*c^9 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^7*c^9 + 784*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^5*b^3*c^10 + 432*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^4*c^10 + 83*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
```

```
 4*a*c)*c)*a^3*b^5*c^10 + 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^6*b*c^11 + 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^2*c^11 - 216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^4*b^3*c^11 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^5*b*c^12 - 12*(b^2 - 4*a*c)*a^2*b^7*c^9 + 166*(b^2 - 4*a*
c)*a^3*b^5*c^10 - 432*(b^2 - 4*a*c)*a^4*b^3*c^11 - 160*(b^2 - 4*a*c)*a^5*b*
c^12)*d^14*e^2 - 7*(24*a^2*b^10*c^8 - 386*a^3*b^8*c^9 + 1688*a^4*b^6*c^10 -
 1120*a^5*b^4*c^11 - 3968*a^6*b^2*c^12 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^6 + 193*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^7 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^7 - 844*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^8 - 290*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^8 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^8 + 560*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^9 + 528*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^9 + 145*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^9 + 1984*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^10 + 992*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^10 - 264*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^10 - 496*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^11 - 24*(b^2 - 4*a*c)*a
^2*b^8*c^8 + 290*(b^2 - 4*a*c)*a^3*b^6*c^9 - 528*(b^2 - 4*a*c)*a^4*b^4*c^10
 - 992*(b^2 - 4*a*c)*a^5*b^2*c^11)*d^13*e^3 + (252*a^2*b^11*c^7 - 3450*a^3*
b^9*c^8 + 10148*a^4*b^7*c^9 + 19024*a^5*b^5*c^10 - 78656*a^6*b^3*c^11 - 140
80*a^7*b*c^12 - 126*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^11*c^5 + 1725*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^9*c^6 + 252*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^10*c^6 - 5074*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^7*c^7 - 2442*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^8*c^7 - 126*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^9*c^7 - 9512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^5*b^5*c^8 + 380*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^6*c^8 + 1221*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^7*c^8 + 39328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^9 + 20544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^9 - 190*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^9 + 7040*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^10 + 3520*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^10 - 10272*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^10 - 1760*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^11 - 252*(b^2 - 4*a*c)*a^2*b^9*c
^7 + 2442*(b^2 - 4*a*c)*a^3*b^7*c^8 - 380*(b^2 - 4*a*c)*a^4*b^5*c^9 - 20544
*(b^2 - 4*a*c)*a^5*b^3*c^10 - 3520*(b^2 - 4*a*c)*a^6*b*c^11)*d^12*e^4 - 3*(
84*a^2*b^12*c^6 - 866*a^3*b^10*c^7 - 130*a^4*b^8*c^8 + 19288*a^5*b^6*c^9 -
33888*a^6*b^4*c^10 - 29056*a^7*b^2*c^11 - 42*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
```

```
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^12*c^4 + 433*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c^5 + 84*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c^5 + 65*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^6 - 530*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^6 - 42*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^6 - 9644*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^7 - 2250*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^7 + 265*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^7 + 16944*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^8 + 10288*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^8 + 1125*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^8 + 14528*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^9 + 7264*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^9 - 5144*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^9 - 3632*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^10 - 84*(
b^2 - 4*a*c)*a^2*b^10*c^6 + 530*(b^2 - 4*a*c)*a^3*b^8*c^7 + 2250*(b^2 - 4*a
*c)*a^4*b^6*c^8 - 10288*(b^2 - 4*a*c)*a^5*b^4*c^9 - 7264*(b^2 - 4*a*c)*a^6*
b^2*c^10)*d^11*e^5 + (168*a^2*b^13*c^5 - 874*a^3*b^11*c^6 - 9992*a^4*b^9*c^
7 + 64652*a^5*b^7*c^8 - 30480*a^6*b^5*c^9 - 214976*a^7*b^3*c^10 - 25344*a^8
*b*c^11 - 84*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^13*c^3 + 437*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^11*c^4 + 168*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^12*c^4 + 4996*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^9*c^5 - 202*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^10*c^5 - 84*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^11*c^5 - 32326*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^5*b^7*c^6 - 10800*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^8*c^6 + 101*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^9*c^6 + 15240*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^6*b^5*c^7 + 21452*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^5*b^6*c^7 + 5400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^7 + 107488*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^8 + 55328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^8 - 10726*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^8 + 12672*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^9 + 6336*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^9 - 27664*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^9 - 3168*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^10 - 168*(b^2 - 4*a*c)*a^2*b
^11*c^5 + 202*(b^2 - 4*a*c)*a^3*b^9*c^6 + 10800*(b^2 - 4*a*c)*a^4*b^7*c^7 -
 21452*(b^2 - 4*a*c)*a^5*b^5*c^8 - 55328*(b^2 - 4*a*c)*a^6*b^3*c^9 - 6336*(
b^2 - 4*a*c)*a^7*b*c^10)*d^10*e^6 - (72*a^2*b^14*c^4 + 246*a^3*b^12*c^5 - 1
0034*a^4*b^10*c^6 + 29882*a^5*b^8*c^7 + 79240*a^6*b^6*c^8 - 256800*a^7*b^4*
c^9 - 131200*a^8*b^2*c^10 - 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
```

```
2 - 4*a*c)*c)*a^2*b^14*c^2 - 123*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^12*c^3 + 72*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^13*c^3 + 5017*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^4*b^10*c^4 + 534*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b^11*c^4 - 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^12*c^4 - 14941*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^8*c^5 - 7898*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c^5 - 267*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c^5 - 39620*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^6 - 1710*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^6 + 3949*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^6 + 128400*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^7 + 72400*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^7 + 855*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^7 + 65600*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^8 + 32800*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^8 - 36200*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^8 - 16400*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^9 - 72*
(b^2 - 4*a*c)*a^2*b^12*c^4 - 534*(b^2 - 4*a*c)*a^3*b^10*c^5 + 7898*(b^2 - 4
*a*c)*a^4*b^8*c^6 + 1710*(b^2 - 4*a*c)*a^5*b^6*c^7 - 72400*(b^2 - 4*a*c)*a^
6*b^4*c^8 - 32800*(b^2 - 4*a*c)*a^7*b^2*c^9)*d^9*e^7 + 9*(2*a^2*b^15*c^3 +
42*a^3*b^13*c^4 - 476*a^4*b^11*c^5 - 530*a^5*b^9*c^6 + 11816*a^6*b^7*c^7 -
13440*a^7*b^5*c^8 - 30080*a^8*b^3*c^9 - 2560*a^9*b*c^10 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^15*c - 21*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^13*c^2 + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^14*c^2 + 238*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^11*c^3 + 50*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^12*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^13*c^3 + 265*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^9*c^4 - 276*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^10*c^4 - 25*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^11*c^4 - 5908*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^5 - 1634*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^8*c^5 + 138*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c^5 + 6720*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^6 + 5280*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^6 + 817*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^6 + 15040
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^7 + 76
80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^7 -
2640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^7
+ 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^8
+ 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^8
 - 3840*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c
```

```
^8 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^
9 - 2*(b^2 - 4*a*c)*a^2*b^13*c^3 - 50*(b^2 - 4*a*c)*a^3*b^11*c^4 + 276*(b^2
 - 4*a*c)*a^4*b^9*c^5 + 1634*(b^2 - 4*a*c)*a^5*b^7*c^6 - 5280*(b^2 - 4*a*c)
*a^6*b^5*c^7 - 7680*(b^2 - 4*a*c)*a^7*b^3*c^8 - 640*(b^2 - 4*a*c)*a^8*b*c^9
)*d^8*e^8 - (2*a^2*b^16*c^2 + 150*a^3*b^14*c^3 - 362*a^4*b^12*c^4 - 11636*a
^5*b^10*c^5 + 49334*a^6*b^8*c^6 + 46200*a^7*b^6*c^7 - 274400*a^8*b^4*c^8 -
96640*a^9*b^2*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^16 - 75*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^14*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^15*c + 181*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b^12*c^2 + 158*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^13*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^14*c^2 + 5818*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^10*c^3 + 270*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^11*c^3 - 79*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^12*c^3 - 24667*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^6*b^8*c^4 - 10556*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^5*b^9*c^4 - 135*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^10*c^4 - 23100*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^7*b^6*c^5 + 7110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^6*b^7*c^5 + 5278*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^5*b^8*c^5 + 137200*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^6 + 74640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^6 - 3555*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^6 + 48320*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^7 + 24160*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^7 - 37320*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^7 - 12080*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^8 - 2*(b^2 - 4*a*c)*a^2*
b^14*c^2 - 158*(b^2 - 4*a*c)*a^3*b^12*c^3 - 270*(b^2 - 4*a*c)*a^4*b^10*c^4
+ 10556*(b^2 - 4*a*c)*a^5*b^8*c^5 - 7110*(b^2 - 4*a*c)*a^6*b^6*c^6 - 74640*
(b^2 - 4*a*c)*a^7*b^4*c^7 - 24160*(b^2 - 4*a*c)*a^8*b^2*c^8)*d^7*e^9 + (22*
a^3*b^15*c^2 + 336*a^4*b^13*c^3 - 4312*a^5*b^11*c^4 + 56*a^6*b^9*c^5 + 8184
4*a^7*b^7*c^6 - 119280*a^8*b^5*c^7 - 164416*a^9*b^3*c^8 - 7424*a^10*b*c^9 -
 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^15 - 16
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^13*c + 22
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^14*c + 215
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^11*c^2 +
424*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^12*c^2
- 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^13*c^2
 - 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9*c^3
 - 2616*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^10*
c^3 - 212*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^1
1*c^3 - 40922*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7
*b^7*c^4 - 10408*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
```

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a^6*b^8*c^4 + 1308*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^5*b^9*c^4 + 59640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^8*b^5*c^5 + 40212*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^7*b^6*c^5 + 5204*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^6*b^7*c^5 + 82208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^9*b^3*c^6 + 41568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^8*b^4*c^6 - 20106*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^7*b^5*c^6 + 3712*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^10*b*c^7 + 1856*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^7 - 20784*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^7 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^8 - 22*(b^2 - 4*a*c)*a^3*b^13*c^2 - 424*(
b^2 - 4*a*c)*a^4*b^11*c^3 + 2616*(b^2 - 4*a*c)*a^5*b^9*c^4 + 10408*(b^2 - 4
*a*c)*a^6*b^7*c^5 - 40212*(b^2 - 4*a*c)*a^7*b^5*c^6 - 41568*(b^2 - 4*a*c)*a
^8*b^3*c^7 - 1856*(b^2 - 4*a*c)*a^9*b*c^8)*d^6*e^10 - 3*(30*a^4*b^14*c^2 -
26*a^5*b^12*c^3 - 2460*a^6*b^10*c^4 + 10034*a^7*b^8*c^5 + 5544*a^8*b^6*c^6
- 47264*a^9*b^4*c^7 - 8320*a^10*b^2*c^8 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^14 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b^12*c + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^13*c + 1230*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^6*b^10*c^2 + 94*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^11*c^2 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^12*c^2 - 5017*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^8*c^3 - 2084*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9*c^3 - 47*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^10*c^3 - 2772*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c^4 + 1698*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^4 + 1042*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^4 + 23632*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^5 + 12336*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^5 - 849*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^5 + 4160*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^6 + 2080*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^6 - 6168*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^6 - 1040*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^7 - 30*(b^
2 - 4*a*c)*a^4*b^12*c^2 - 94*(b^2 - 4*a*c)*a^5*b^10*c^3 + 2084*(b^2 - 4*a*c
)*a^6*b^8*c^4 - 1698*(b^2 - 4*a*c)*a^7*b^6*c^5 - 12336*(b^2 - 4*a*c)*a^8*b^
4*c^6 - 2080*(b^2 - 4*a*c)*a^9*b^2*c^7)*d^5*e^11 + (190*a^5*b^13*c^2 - 1440
*a^6*b^11*c^3 - 2158*a^7*b^9*c^4 + 35196*a^8*b^7*c^5 - 52304*a^9*b^5*c^6 -
42688*a^10*b^3*c^7 + 3840*a^11*b*c^8 - 95*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b^13 + 720*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^6*b^11*c + 190*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b^12*c + 1079*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^7*b^9*c^2 - 680*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

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c + sqrt(b^2 - 4*a*c)*c)*a^6*b^10*c^2 - 95*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^11*c^2 - 17598*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^7*c^3 - 4878*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^8*c^3 + 340*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9*c^3 + 26152*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c^4 + 15684*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c^4 + 2439*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^4 + 21344*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^3*c^5 + 10432*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^5 - 7842*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^5 - 1920*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^11*b*c^6 - 960*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^6 - 5216*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^6 + 480*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^7 - 190*(b
^2 - 4*a*c)*a^5*b^11*c^2 + 680*(b^2 - 4*a*c)*a^6*b^9*c^3 + 4878*(b^2 - 4*a*
c)*a^7*b^7*c^4 - 15684*(b^2 - 4*a*c)*a^8*b^5*c^5 - 10432*(b^2 - 4*a*c)*a^9*
b^3*c^6 + 960*(b^2 - 4*a*c)*a^10*b*c^7)*d^4*e^12 - (230*a^6*b^12*c^2 - 2502
*a^7*b^10*c^3 + 6718*a^8*b^8*c^4 + 7896*a^9*b^6*c^5 - 39520*a^10*b^4*c^6 +
6784*a^11*b^2*c^7 - 115*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^6*b^12 + 1251*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^7*b^10*c + 230*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*b^11*c - 3359*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^8*b^8*c^2 - 1582*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^7*b^9*c^2 - 115*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^6*b^10*c^2 - 3948*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^9*b^6*c^3 + 390*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^8*b^7*c^3 + 791*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^7*b^8*c^3 + 19760*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^10*b^4*c^4 + 9456*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c^4 - 195*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c^4 - 3392*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^11*b^2*c^5 - 1696*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^3*c^5 - 4728*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^5 + 848*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^6 - 230*(b^2 - 4*a*c)*a^6*b^10*c^
2 + 1582*(b^2 - 4*a*c)*a^7*b^8*c^3 - 390*(b^2 - 4*a*c)*a^8*b^6*c^4 - 9456*(
b^2 - 4*a*c)*a^9*b^4*c^5 + 1696*(b^2 - 4*a*c)*a^10*b^2*c^6)*d^3*e^13 + 3*(5
4*a^7*b^11*c^2 - 680*a^8*b^9*c^3 + 2796*a^9*b^7*c^4 - 3472*a^10*b^5*c^5 - 1
472*a^11*b^3*c^6 + 1280*a^12*b*c^7 - 27*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^7*b^11 + 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^8*b^9*c + 54*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^7*b^10*c - 1398*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^9*b^7*c^2 - 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^8*b^8*c^2 - 27*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
```

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sqrt(b^2 - 4*a*c)*c)*a^7*b^9*c^2 + 1736*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^10*b^5*c^3 + 940*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^9*b^6*c^3 + 232*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^7*c^3 + 736*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^11*b^3*c^4 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^4*c^4 - 470*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c^4 - 640*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^12*b*c^5 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^11*b^2*c^5 - 144*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^3*c^5 + 160*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^11*b*c^6 - 54*(b^2 - 4*a*c)*a^7*b^9*c
^2 + 464*(b^2 - 4*a*c)*a^8*b^7*c^3 - 940*(b^2 - 4*a*c)*a^9*b^5*c^4 - 288*(b
^2 - 4*a*c)*a^10*b^3*c^5 + 320*(b^2 - 4*a*c)*a^11*b*c^6)*d^2*e^14 - (62*a^8
*b^10*c^2 - 834*a^9*b^8*c^3 + 3928*a^10*b^6*c^4 - 7264*a^11*b^4*c^5 + 3712*
a^12*b^2*c^6 - 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^8*b^10 + 417*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^9*b^8*c + 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8
*b^9*c - 1964*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^1
0*b^6*c^2 - 586*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^9*b^7*c^2 - 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^8*b^8*c^2 + 3632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^11*b^4*c^3 + 1584*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^10*b^5*c^3 + 293*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^9*b^6*c^3 - 1856*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^12*b^2*c^4 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^11*b^3*c^4 - 792*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^10*b^4*c^4 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^11*b^2*c^5 - 62*(b^2 - 4*a*c)*a^8*b^8*c^2 + 586*(b^2 - 4*a*
c)*a^9*b^6*c^3 - 1584*(b^2 - 4*a*c)*a^10*b^4*c^4 + 928*(b^2 - 4*a*c)*a^11*b
^2*c^5)*d*e^15 + (10*a^9*b^9*c^2 - 138*a^10*b^7*c^3 + 680*a^11*b^5*c^4 - 13
76*a^12*b^3*c^5 + 896*a^13*b*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^9*b^9 + 69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^10*b^7*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^9*b^8*c - 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^11*b^5*c^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^10*b^6*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^9*b^7*c^2 + 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^12*b^3*c^3 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^11*b^4*c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^10*b^5*c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^13*b*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^12*b^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^11*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^12*b*c^5 - 10*(b^2 - 4*a*c)*a^9*b^7*c^2 + 98*(b^
2 - 4*a*c)*a^10*b^5*c^3 - 288*(b^2 - 4*a*c)*a^11*b^3*c^4 + 224*(b^2 - 4*a*c
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)*a^12*b*c^5)*e^16 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 -
 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 2*a*b^6*c^7 + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^8 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 + 28*a^2*b
^4*c^8 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^9 - 48*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^9 - 128*a^3*b^2*c^9 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^10 + 192*a^4*c^10 + 2*(b^2 - 4*a*c)*a*b^4*c^7 - 20*(b^2 - 4
*a*c)*a^2*b^2*c^8 + 48*(b^2 - 4*a*c)*a^3*c^9)*d^10*abs(a*b^2*c^3*d^6 - 4*a^
2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*
a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*
e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a
^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c
*e^6) - 4*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 - 41*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 6*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^6*c^6 - 6*a*b^7*c^6 + 184*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^3*c^7 + 58*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c
^7 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 82*a^2*b^5*c^7 -
 272*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 136*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 29*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c^8 - 368*a^3*b^3*c^8 + 68*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b*c^9 + 544*a^4*b*c^9 + 6*(b^2 - 4*a*c)*a*b^5*c^6 - 58*(b^2 - 4
*a*c)*a^2*b^3*c^7 + 136*(b^2 - 4*a*c)*a^3*b*c^8)*d^9*e*abs(a*b^2*c^3*d^6 -
4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2
- 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*
d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 -
12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a
^5*c*e^6) + 6*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^4 - 64*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 - 10*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^7*c^5 - 10*a*b^8*c^5 + 258*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^4*c^6 + 88*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^6 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 128*a^2*b^
6*c^6 - 272*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 164*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 44*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 516*a^3*b^4*c^7 - 224*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*c^8 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b*c^8 + 82*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 544*a^4*b
^2*c^8 + 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^9 + 448*a^5*c^9 +
 10*(b^2 - 4*a*c)*a*b^6*c^5 - 88*(b^2 - 4*a*c)*a^2*b^4*c^6 + 164*(b^2 - 4*a
*c)*a^3*b^2*c^7 + 112*(b^2 - 4*a*c)*a^4*c^8)*d^8*e^2abs(a*b^2*c^3*d^6 - 4*
a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 -
9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^
3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12
*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5
```

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*c*e^6) - 40*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^3 - 11*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^8*c^4 - 2*a*b^9*c^4 + 30*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^5*c^5 + 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c
^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 22*a^2*b^7*c^5 + 3
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - 4*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^5*c^6 - 60*a^3*b^5*c^6 - 160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^5*b*c^7 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 64*a^4*b^3*c^7 + 40*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 + 320*a^5*b*c^8 + 2*(b^2 -
 4*a*c)*a*b^7*c^4 - 14*(b^2 - 4*a*c)*a^2*b^5*c^5 + 4*(b^2 - 4*a*c)*a^3*b^3*
c^6 + 80*(b^2 - 4*a*c)*a^4*b*c^7)*d^7*e^3*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6
 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c
^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*
a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^
2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) + 2
*(15*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^2 - 110*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^3 - 30*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^9*c^3 - 30*a*b^10*c^3 - 206*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^6*c^4 + 100*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4
 + 15*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^4 + 220*a^2*b^8*c^4 +
 3012*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 812*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 50*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^6*c^5 + 412*a^3*b^6*c^5 - 5248*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^2*c^6 - 2776*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^3*c^6 - 406*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 - 6024*a
^4*b^4*c^6 - 1216*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^7 - 608*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 + 1388*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 10496*a^5*b^2*c^7 + 304*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^5*c^8 + 2432*a^6*c^8 + 30*(b^2 - 4*a*c)*a*b^8*c^3 -
 100*(b^2 - 4*a*c)*a^2*b^6*c^4 - 812*(b^2 - 4*a*c)*a^3*b^4*c^5 + 2776*(b^2
- 4*a*c)*a^4*b^2*c^6 + 608*(b^2 - 4*a*c)*a^5*c^7)*d^6*e^4*abs(a*b^2*c^3*d^6
 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e
^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3
*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4
 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 -
4*a^5*c*e^6) - 12*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^11*c + sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^10*c^2 - 2*a*b^11*c^2 - 106*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^7*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^8*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^3 - 2*a^2*b^9*c^3
+ 494*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 + 172*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^7*c^4 + 212*a^3*b^7*c^4 - 400*sqrt(2)*sqrt(b*c + sqrt(b^2 -
```

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 4*a*c)*c)*a^5*b^3*c^5 - 300*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
4*c^5 - 86*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 988*a^4*b^
5*c^5 - 800*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 - 400*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 + 150*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^3*c^6 + 800*a^5*b^3*c^6 + 200*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^5*b*c^7 + 1600*a^6*b*c^7 + 2*(b^2 - 4*a*c)*a*b^9*c^2 + 1
0*(b^2 - 4*a*c)*a^2*b^7*c^3 - 172*(b^2 - 4*a*c)*a^3*b^5*c^4 + 300*(b^2 - 4*
a*c)*a^4*b^3*c^5 + 400*(b^2 - 4*a*c)*a^5*b*c^6)*d^5*e^5*abs(a*b^2*c^3*d^6 -
 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2
 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c
*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 -
 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*
a^5*c*e^6) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^12 + 28*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^11*c - 2*a*b^12*c - 346*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^8*c^2 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^2
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^2 - 56*a^2*b^10*c^2 + 72
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 436*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^8*c^3 + 692*a^3*b^8*c^3 + 2300*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^5*b^4*c^4 + 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*
c^4 - 218*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 - 1456*a^4*b^
6*c^4 - 6752*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 - 3448*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 - 144*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 - 4600*a^5*b^4*c^5 - 576*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^7*c^6 - 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^6*b*c^6 + 1724*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 + 135
04*a^6*b^2*c^6 + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^7 + 1152
*a^7*c^7 + 2*(b^2 - 4*a*c)*a*b^10*c + 64*(b^2 - 4*a*c)*a^2*b^8*c^2 - 436*(b
^2 - 4*a*c)*a^3*b^6*c^3 - 288*(b^2 - 4*a*c)*a^4*b^4*c^4 + 3448*(b^2 - 4*a*c
)*a^5*b^2*c^5 + 288*(b^2 - 4*a*c)*a^6*c^6)*d^4*e^6*abs(a*b^2*c^3*d^6 - 4*a^
2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*
a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*
e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a
^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c
*e^6) - 8*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^11 - 7*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^10*c - 4*a^2*b^11*c - 107*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b^7*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2
+ 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^2 + 14*a^3*b^9*c^2 +
646*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + 206*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^7*c^3 + 214*a^4*b^7*c^3 - 800*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^6*b^3*c^4 - 468*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^
4 - 103*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 - 1292*a^5*b^5*
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c^4 - 544*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 - 272*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 234*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^5*b^3*c^5 + 1600*a^6*b^3*c^5 + 136*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^6*b*c^6 + 1088*a^7*b*c^6 + 4*(b^2 - 4*a*c)*a^2*b^9*c + 2*
(b^2 - 4*a*c)*a^3*b^7*c^2 - 206*(b^2 - 4*a*c)*a^4*b^5*c^3 + 468*(b^2 - 4*a*
c)*a^5*b^3*c^4 + 272*(b^2 - 4*a*c)*a^6*b*c^5)*d^3*e^7*abs(a*b^2*c^3*d^6 - 4
*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 -
 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d
^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 1
2*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^
5*c*e^6) + 18*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^10 - 20*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c - 4*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^9*c - 4*a^3*b^10*c + 45*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*b^6*c^2 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*
c^2 + 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 + 40*a^4*b^8*c^
2 + 90*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 6*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^6*c^3 - 90*a^5*b^6*c^3 - 320*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^7*b^2*c^4 - 156*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3
*c^4 - 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 180*a^6*b^4*
c^4 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^5 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 + 78*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^6*b^2*c^5 + 640*a^7*b^2*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^7*c^6 - 64*a^8*c^6 + 4*(b^2 - 4*a*c)*a^3*b^8*c - 24*(b^2 - 4*a*c)*a
^4*b^6*c^2 - 6*(b^2 - 4*a*c)*a^5*b^4*c^3 + 156*(b^2 - 4*a*c)*a^6*b^2*c^4 -
16*(b^2 - 4*a*c)*a^7*c^5)*d^2*e^8*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b
^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e
^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^
2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 -
3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) - 4*(8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9 - 97*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^5*b^7*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
8*c - 16*a^4*b^9*c + 391*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^
2 + 130*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 + 8*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 + 194*a^5*b^7*c^2 - 520*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^3 - 262*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^6*b^4*c^3 - 65*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
^5*c^3 - 782*a^6*b^5*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b
*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 + 131*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 + 1040*a^7*b^3*c^4 + 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 + 32*a^8*b*c^5 + 16*(b^2 - 4*a*c)
*a^4*b^7*c - 130*(b^2 - 4*a*c)*a^5*b^5*c^2 + 262*(b^2 - 4*a*c)*a^6*b^3*c^3
+ 8*(b^2 - 4*a*c)*a^7*b*c^4)*d*e^9*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*
b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*
e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c
```

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^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 -
 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) + 2*(5*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^6*b^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
^7*c - 10*a^5*b^8*c + 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c
^2 + 88*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 + 5*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 + 128*a^6*b^6*c^2 - 496*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^7*b^3*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b
^4*c^3 - 572*a^7*b^4*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*
c^4 + 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^4 + 110*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 + 992*a^8*b^2*c^4 - 56*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^5 - 448*a^9*c^5 + 10*(b^2 - 4*a*c)*a^5
*b^6*c - 88*(b^2 - 4*a*c)*a^6*b^4*c^2 + 220*(b^2 - 4*a*c)*a^7*b^2*c^3 - 112
*(b^2 - 4*a*c)*a^8*c^4)*e^10*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^
2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 -
12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3
*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3
*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) + (a*b^2*c^3*d^6
 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e
^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3
*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4
 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 -
4*a^5*c*e^6)^2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^4 - (a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 -
 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2
*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^
3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*
e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)^2*(6*
b^4*c^4 - 32*a*b^2*c^5 + 32*a^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^5 - 6*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*d^3*e + 3*(a*b^2*c^3*
d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^
4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*
b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*
e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6
 - 4*a^5*c*e^6)^2*(2*b^5*c^3 - 12*a*b^3*c^4 + 16*a^2*b*c^5 - sqrt(2)*sqrt(b
```

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^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 6*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 4*(b^2 - 4*a*c)*a*b*c^4
)*d^2*e^2 - (a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c
^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 -
 a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2
*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*
b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)^2*(2*b^6*c^2 + 6*a*b^4*c^3 - 128*a^2
*b^2*c^4 + 288*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c
+ 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2
+ 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 144*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 72*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 7*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 36*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4
*c^2 - 14*(b^2 - 4*a*c)*a*b^2*c^3 + 72*(b^2 - 4*a*c)*a^2*c^4)*d*e^3 + (a*b^
2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^
4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 -
2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*
c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b
^2*e^6 - 4*a^5*c*e^6)^2*(10*a*b^5*c^2 - 78*a^2*b^3*c^3 + 152*a^3*b*c^4 - 5*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 39*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 10*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 76*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 38*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 5*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 19*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 10*(b^2 - 4*a*c)*a*b^3
*c^2 + 38*(b^2 - 4*a*c)*a^2*b*c^3)*e^4)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3*c^
3*d^6 - 4*a^2*b*c^4*d^6 - 3*a*b^4*c^2*d^5*e + 12*a^2*b^2*c^3*d^5*e + 3*a*b^
5*c*d^4*e^2 - 9*a^2*b^3*c^2*d^4*e^2 - 12*a^3*b*c^3*d^4*e^2 - a*b^6*d^3*e^3
- 2*a^2*b^4*c*d^3*e^3 + 24*a^3*b^2*c^2*d^3*e^3 + 3*a^2*b^5*d^2*e^4 - 9*a^3*
b^3*c*d^2*e^4 - 12*a^4*b*c^2*d^2*e^4 - 3*a^3*b^4*d*e^5 + 12*a^4*b^2*c*d*e^5
 + a^4*b^3*e^6 - 4*a^5*b*c*e^6 + sqrt((a*b^3*c^3*d^6 - 4*a^2*b*c^4*d^6 - 3*
a*b^4*c^2*d^5*e + 12*a^2*b^2*c^3*d^5*e + 3*a*b^5*c*d^4*e^2 - 9*a^2*b^3*c^2*
d^4*e^2 - 12*a^3*b*c^3*d^4*e^2 - a*b^6*d^3*e^3 - 2*a^2*b^4*c*d^3*e^3 + 24*a
^3*b^2*c^2*d^3*e^3 + 3*a^2*b^5*d^2*e^4 - 9*a^3*b^3*c*d^2*e^4 - 12*a^4*b*c^2
*d^2*e^4 - 3*a^3*b^4*d*e^5 + 12*a^4*b^2*c*d*e^5 + a^4*b^3*e^6 - 4*a^5*b*c*e
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^6)^2 - 4*(a^2*b^2*c^3*d^6 - 4*a^3*c^4*d^6 - 3*a^2*b^3*c^2*d^5*e + 12*a^3*b
*c^3*d^5*e + 3*a^2*b^4*c*d^4*e^2 - 9*a^3*b^2*c^2*d^4*e^2 - 12*a^4*c^3*d^4*e
^2 - a^2*b^5*d^3*e^3 - 2*a^3*b^3*c*d^3*e^3 + 24*a^4*b*c^2*d^3*e^3 + 3*a^3*b
^4*d^2*e^4 - 9*a^4*b^2*c*d^2*e^4 - 12*a^5*c^2*d^2*e^4 - 3*a^4*b^3*d*e^5 + 1
2*a^5*b*c*d*e^5 + a^5*b^2*e^6 - 4*a^6*c*e^6)*(a*b^2*c^4*d^6 - 4*a^2*c^5*d^6
 - 3*a*b^3*c^3*d^5*e + 12*a^2*b*c^4*d^5*e + 3*a*b^4*c^2*d^4*e^2 - 9*a^2*b^2
*c^3*d^4*e^2 - 12*a^3*c^4*d^4*e^2 - a*b^5*c*d^3*e^3 - 2*a^2*b^3*c^2*d^3*e^3
 + 24*a^3*b*c^3*d^3*e^3 + 3*a^2*b^4*c*d^2*e^4 - 9*a^3*b^2*c^2*d^2*e^4 - 12*
a^4*c^3*d^2*e^4 - 3*a^3*b^3*c*d*e^5 + 12*a^4*b*c^2*d*e^5 + a^4*b^2*c*e^6 -
4*a^5*c^2*e^6)))/(a*b^2*c^4*d^6 - 4*a^2*c^5*d^6 - 3*a*b^3*c^3*d^5*e + 12*a^
2*b*c^4*d^5*e + 3*a*b^4*c^2*d^4*e^2 - 9*a^2*b^2*c^3*d^4*e^2 - 12*a^3*c^4*d^
4*e^2 - a*b^5*c*d^3*e^3 - 2*a^2*b^3*c^2*d^3*e^3 + 24*a^3*b*c^3*d^3*e^3 + 3*
a^2*b^4*c*d^2*e^4 - 9*a^3*b^2*c^2*d^2*e^4 - 12*a^4*c^3*d^2*e^4 - 3*a^3*b^3*
c*d*e^5 + 12*a^4*b*c^2*d*e^5 + a^4*b^2*c*e^6 - 4*a^5*c^2*e^6)))/((a^3*b^6*c
^6 - 12*a^4*b^4*c^7 - 2*a^3*b^5*c^7 + 48*a^5*b^2*c^8 + 16*a^4*b^3*c^8 + a^3
*b^4*c^8 - 64*a^6*c^9 - 32*a^5*b*c^9 - 8*a^4*b^2*c^9 + 16*a^5*c^10)*d^12*ab
s(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e +
3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*
e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^
3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 +
 a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) - 6*(a^3*b^7*c^5 - 12*a^4*b^5*c^6 - 2*a^
3*b^6*c^6 + 48*a^5*b^3*c^7 + 16*a^4*b^4*c^7 + a^3*b^5*c^7 - 64*a^6*b*c^8 -
32*a^5*b^2*c^8 - 8*a^4*b^3*c^8 + 16*a^5*b*c^9)*d^11*e*abs(a*b^2*c^3*d^6 - 4
*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 -
 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d
^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 1
2*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^
5*c*e^6)*abs(c) + 3*(5*a^3*b^8*c^4 - 58*a^4*b^6*c^5 - 10*a^3*b^7*c^5 + 216*
a^5*b^4*c^6 + 76*a^4*b^5*c^6 + 5*a^3*b^6*c^6 - 224*a^6*b^2*c^7 - 128*a^5*b^
3*c^7 - 38*a^4*b^4*c^7 - 128*a^7*c^8 - 64*a^6*b*c^8 + 64*a^5*b^2*c^8 + 32*a
^6*c^9)*d^10*e^2*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12
*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d
^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2
*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 +
 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) - 10*(2*a^3*b^9*c^3 -
 21*a^4*b^7*c^4 - 4*a^3*b^8*c^4 + 60*a^5*b^5*c^5 + 26*a^4*b^6*c^5 + 2*a^3*b
^7*c^5 + 16*a^6*b^3*c^6 - 16*a^5*b^4*c^6 - 13*a^4*b^5*c^6 - 192*a^7*b*c^7 -
 96*a^6*b^2*c^7 + 8*a^5*b^3*c^7 + 48*a^6*b*c^8)*d^9*e^3*abs(a*b^2*c^3*d^6 -
 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2
 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c
*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 -
 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*
a^5*c*e^6)*abs(c) + 15*(a^3*b^10*c^2 - 8*a^4*b^8*c^3 - 2*a^3*b^9*c^3 + a^5*
b^6*c^4 + 8*a^4*b^7*c^4 + a^3*b^8*c^4 + 116*a^6*b^4*c^5 + 30*a^5*b^5*c^5 -
4*a^4*b^6*c^5 - 208*a^7*b^2*c^6 - 112*a^6*b^3*c^6 - 15*a^5*b^4*c^6 - 64*a^8
```

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*c^7 - 32*a^7*b*c^7 + 56*a^6*b^2*c^7 + 16*a^7*c^8)*d^8*e^4*abs(a*b^2*c^3*d^
6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*
e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^
3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^
4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 -
 4*a^5*c*e^6)*abs(c) - 6*(a^3*b^11*c - 2*a^4*b^9*c^2 - 2*a^3*b^10*c^2 - 62*
a^5*b^7*c^3 - 4*a^4*b^8*c^3 + a^3*b^9*c^3 + 296*a^6*b^5*c^4 + 108*a^5*b^6*c
^4 + 2*a^4*b^7*c^4 - 160*a^7*b^3*c^5 - 160*a^6*b^4*c^5 - 54*a^5*b^5*c^5 - 6
40*a^8*b*c^6 - 320*a^7*b^2*c^6 + 80*a^6*b^3*c^6 + 160*a^7*b*c^7)*d^7*e^5*ab
s(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e +
3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*
e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^
3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 +
 a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) + (a^3*b^12 + 18*a^4*b^10*c - 2*a^3*b^11
*c - 222*a^5*b^8*c^2 - 44*a^4*b^9*c^2 + a^3*b^10*c^2 + 316*a^6*b^6*c^3 + 26
8*a^5*b^7*c^3 + 22*a^4*b^8*c^3 + 2160*a^7*b^4*c^4 + 440*a^6*b^5*c^4 - 134*a
^5*b^6*c^4 - 4800*a^8*b^2*c^5 - 2560*a^7*b^3*c^5 - 220*a^6*b^4*c^5 - 1280*a
^9*c^6 - 640*a^8*b*c^6 + 1280*a^7*b^2*c^6 + 320*a^8*c^7)*d^6*e^6*abs(a*b^2*
c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*
c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*
a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*
d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2
*e^6 - 4*a^5*c*e^6)*abs(c) - 6*(a^4*b^11 - 2*a^5*b^9*c - 2*a^4*b^10*c - 62*
a^6*b^7*c^2 - 4*a^5*b^8*c^2 + a^4*b^9*c^2 + 296*a^7*b^5*c^3 + 108*a^6*b^6*c
^3 + 2*a^5*b^7*c^3 - 160*a^8*b^3*c^4 - 160*a^7*b^4*c^4 - 54*a^6*b^5*c^4 - 6
40*a^9*b*c^5 - 320*a^8*b^2*c^5 + 80*a^7*b^3*c^5 + 160*a^8*b*c^6)*d^5*e^7*ab
s(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e +
3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*
e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^
3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 +
 a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) + 15*(a^5*b^10 - 8*a^6*b^8*c - 2*a^5*b^9
*c + a^7*b^6*c^2 + 8*a^6*b^7*c^2 + a^5*b^8*c^2 + 116*a^8*b^4*c^3 + 30*a^7*b
^5*c^3 - 4*a^6*b^6*c^3 - 208*a^9*b^2*c^4 - 112*a^8*b^3*c^4 - 15*a^7*b^4*c^4
 - 64*a^10*c^5 - 32*a^9*b*c^5 + 56*a^8*b^2*c^5 + 16*a^9*c^6)*d^4*e^8*abs(a*
b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*
b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3
 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^
2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4
*b^2*e^6 - 4*a^5*c*e^6)*abs(c) - 10*(2*a^6*b^9 - 21*a^7*b^7*c - 4*a^6*b^8*c
 + 60*a^8*b^5*c^2 + 26*a^7*b^6*c^2 + 2*a^6*b^7*c^2 + 16*a^9*b^3*c^3 - 16*a^
8*b^4*c^3 - 13*a^7*b^5*c^3 - 192*a^10*b*c^4 - 96*a^9*b^2*c^4 + 8*a^8*b^3*c^
4 + 48*a^9*b*c^5)*d^3*e^9*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d
^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*
a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^
3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^
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3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) + 3*(5*a^7*b
^8 - 58*a^8*b^6*c - 10*a^7*b^7*c + 216*a^9*b^4*c^2 + 76*a^8*b^5*c^2 + 5*a^7
*b^6*c^2 - 224*a^10*b^2*c^3 - 128*a^9*b^3*c^3 - 38*a^8*b^4*c^3 - 128*a^11*c
^4 - 64*a^10*b*c^4 + 64*a^9*b^2*c^4 + 32*a^10*c^5)*d^2*e^10*abs(a*b^2*c^3*d
^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4
*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b
^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e
^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6
- 4*a^5*c*e^6)*abs(c) - 6*(a^8*b^7 - 12*a^9*b^5*c - 2*a^8*b^6*c + 48*a^10*b
^3*c^2 + 16*a^9*b^4*c^2 + a^8*b^5*c^2 - 64*a^11*b*c^3 - 32*a^10*b^2*c^3 - 8
*a^9*b^3*c^3 + 16*a^10*b*c^4)*d*e^11*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*
a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^
4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b
*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4
 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) +
 (a^9*b^6 - 12*a^10*b^4*c - 2*a^9*b^5*c + 48*a^11*b^2*c^2 + 16*a^10*b^3*c^2
 + a^9*b^4*c^2 - 64*a^12*c^3 - 32*a^11*b*c^3 - 8*a^10*b^2*c^3 + 16*a^11*c^4
)*e^12*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3
*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a
*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e
^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*
c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c)) + 1/16*((2*a^2*b^7*c^11 - 40*a
^3*b^5*c^12 + 224*a^4*b^3*c^13 - 384*a^5*b*c^14 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^10 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^10 - 112*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^11 - 32*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^11 + 192*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^12 + 96*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^12 + 16*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^12 - 48*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^13 - 2*(b^2 - 4*a*c)*a^2*b^5*c
^11 + 32*(b^2 - 4*a*c)*a^3*b^3*c^12 - 96*(b^2 - 4*a*c)*a^4*b*c^13)*d^16 - (
18*a^2*b^8*c^10 - 344*a^3*b^6*c^11 + 1888*a^4*b^4*c^12 - 3200*a^5*b^2*c^13
- 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^8 +
 172*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^9
+ 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9
- 944*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^1
0 - 272*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c
^10 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c
^10 + 1600*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^
2*c^11 + 800*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^3*c^11 + 136*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^4*c^11 - 400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
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a^4*b^2*c^12 - 18*(b^2 - 4*a*c)*a^2*b^6*c^10 + 272*(b^2 - 4*a*c)*a^3*b^4*c^
11 - 800*(b^2 - 4*a*c)*a^4*b^2*c^12)*d^15*e + 6*(12*a^2*b^9*c^9 - 214*a^3*b
^7*c^10 + 1096*a^4*b^5*c^11 - 1568*a^5*b^3*c^12 - 640*a^6*b*c^13 - 6*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^7 + 107*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^8 + 12*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^8 - 548*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^9 - 166*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^9 - 6*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9 + 784*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^10 + 432*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^10 + 83*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^10 + 32
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^11 + 16
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^11 -
216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^11
 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^12 -
 12*(b^2 - 4*a*c)*a^2*b^7*c^9 + 166*(b^2 - 4*a*c)*a^3*b^5*c^10 - 432*(b^2 -
 4*a*c)*a^4*b^3*c^11 - 160*(b^2 - 4*a*c)*a^5*b*c^12)*d^14*e^2 - 7*(24*a^2*b
^10*c^8 - 386*a^3*b^8*c^9 + 1688*a^4*b^6*c^10 - 1120*a^5*b^4*c^11 - 3968*a^
6*b^2*c^12 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^10*c^6 + 193*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b^8*c^7 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^9*c^7 - 844*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^6*c^8 - 290*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^3*b^7*c^8 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^8*c^8 + 560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^4*c^9 + 528*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b^5*c^9 + 145*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^6*c^9 + 1984*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^6*b^2*c^10 + 992*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^5*b^3*c^10 - 264*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^4*b^4*c^10 - 496*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^5*b^2*c^11 - 24*(b^2 - 4*a*c)*a^2*b^8*c^8 + 290*(b^2 - 4*a*
c)*a^3*b^6*c^9 - 528*(b^2 - 4*a*c)*a^4*b^4*c^10 - 992*(b^2 - 4*a*c)*a^5*b^2
*c^11)*d^13*e^3 + (252*a^2*b^11*c^7 - 3450*a^3*b^9*c^8 + 10148*a^4*b^7*c^9
+ 19024*a^5*b^5*c^10 - 78656*a^6*b^3*c^11 - 14080*a^7*b*c^12 - 126*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c^5 + 1725*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^6 + 252*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^6 - 5074*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^7 - 2442
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^7 - 12
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^7 - 9
512*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^8 +
 380*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^8
+ 1221*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^
```

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8 + 39328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3
*c^9 + 20544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*
b^4*c^9 - 190*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4
*b^5*c^9 + 7040*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^7*b*c^10 + 3520*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^6*b^2*c^10 - 10272*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^3*c^10 - 1760*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^6*b*c^11 - 252*(b^2 - 4*a*c)*a^2*b^9*c^7 + 2442*(b^2 - 4*a*c)*a^3*
b^7*c^8 - 380*(b^2 - 4*a*c)*a^4*b^5*c^9 - 20544*(b^2 - 4*a*c)*a^5*b^3*c^10
- 3520*(b^2 - 4*a*c)*a^6*b*c^11)*d^12*e^4 - 3*(84*a^2*b^12*c^6 - 866*a^3*b^
10*c^7 - 130*a^4*b^8*c^8 + 19288*a^5*b^6*c^9 - 33888*a^6*b^4*c^10 - 29056*a
^7*b^2*c^11 - 42*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^12*c^4 + 433*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^10*c^5 + 84*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^11*c^5 + 65*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*b^8*c^6 - 530*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*b^9*c^6 - 42*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^10*c^6 - 9644*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^5*b^6*c^7 - 2250*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^4*b^7*c^7 + 265*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b^8*c^7 + 16944*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^6*b^4*c^8 + 10288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^5*c^8 + 1125*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b^6*c^8 + 14528*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^9 + 7264*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^9 - 5144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^9 - 3632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^10 - 84*(b^2 - 4*a*c)*a^2*b^10*c^6 +
530*(b^2 - 4*a*c)*a^3*b^8*c^7 + 2250*(b^2 - 4*a*c)*a^4*b^6*c^8 - 10288*(b^2
 - 4*a*c)*a^5*b^4*c^9 - 7264*(b^2 - 4*a*c)*a^6*b^2*c^10)*d^11*e^5 + (168*a^
2*b^13*c^5 - 874*a^3*b^11*c^6 - 9992*a^4*b^9*c^7 + 64652*a^5*b^7*c^8 - 3048
0*a^6*b^5*c^9 - 214976*a^7*b^3*c^10 - 25344*a^8*b*c^11 - 84*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^13*c^3 + 437*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^11*c^4 + 168*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^12*c^4 + 4996*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c^5 - 202*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c^5 - 84*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c^5 - 32326*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^6 - 10800
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^6 + 10
1*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^6 + 1
5240*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^7
+ 21452*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c
^7 + 5400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7
*c^7 + 107488*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7
```

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*b^3*c^8 + 55328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^6*b^4*c^8 - 10726*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b^5*c^8 + 12672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^8*b*c^9 + 6336*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^7*b^2*c^9 - 27664*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^6*b^3*c^9 - 3168*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^7*b*c^10 - 168*(b^2 - 4*a*c)*a^2*b^11*c^5 + 202*(b^2 - 4*a*c)*
a^3*b^9*c^6 + 10800*(b^2 - 4*a*c)*a^4*b^7*c^7 - 21452*(b^2 - 4*a*c)*a^5*b^5
*c^8 - 55328*(b^2 - 4*a*c)*a^6*b^3*c^9 - 6336*(b^2 - 4*a*c)*a^7*b*c^10)*d^1
0*e^6 - (72*a^2*b^14*c^4 + 246*a^3*b^12*c^5 - 10034*a^4*b^10*c^6 + 29882*a^
5*b^8*c^7 + 79240*a^6*b^6*c^8 - 256800*a^7*b^4*c^9 - 131200*a^8*b^2*c^10 -
36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^14*c^2 -
 123*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^12*c^3
 + 72*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^13*c^
3 + 5017*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^10
*c^4 + 534*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
11*c^4 - 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
^12*c^4 - 14941*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^5*b^8*c^5 - 7898*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^9*c^5 - 267*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^10*c^5 - 39620*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^6*b^6*c^6 - 1710*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^5*b^7*c^6 + 3949*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^4*b^8*c^6 + 128400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^7*b^4*c^7 + 72400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^6*b^5*c^7 + 855*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^5*b^6*c^7 + 65600*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^8 + 32800*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^8 - 36200*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^8 - 16400*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^9 - 72*(b^2 - 4*a*c)*a^2*b^12*c^4 -
 534*(b^2 - 4*a*c)*a^3*b^10*c^5 + 7898*(b^2 - 4*a*c)*a^4*b^8*c^6 + 1710*(b^
2 - 4*a*c)*a^5*b^6*c^7 - 72400*(b^2 - 4*a*c)*a^6*b^4*c^8 - 32800*(b^2 - 4*a
*c)*a^7*b^2*c^9)*d^9*e^7 + 9*(2*a^2*b^15*c^3 + 42*a^3*b^13*c^4 - 476*a^4*b^
11*c^5 - 530*a^5*b^9*c^6 + 11816*a^6*b^7*c^7 - 13440*a^7*b^5*c^8 - 30080*a^
8*b^3*c^9 - 2560*a^9*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^15*c - 21*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^13*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^14*c^2 + 238*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b^11*c^3 + 50*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^12*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^13*c^3 + 265*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^5*b^9*c^4 - 276*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^10*c^4 - 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^11*c^4 - 5908*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
```

```
qrt(b^2 - 4*a*c)*c)*a^6*b^7*c^5 - 1634*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^5*b^8*c^5 + 138*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c^5 + 6720*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^6 + 5280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^6 + 817*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^6 + 15040*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^7 + 7680*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^7 - 2640*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^7 + 1280*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^8 + 640*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^8 - 3840*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^8 - 320*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^9 - 2*(b^2 - 4*a*c)*a^2*b^13
*c^3 - 50*(b^2 - 4*a*c)*a^3*b^11*c^4 + 276*(b^2 - 4*a*c)*a^4*b^9*c^5 + 1634
*(b^2 - 4*a*c)*a^5*b^7*c^6 - 5280*(b^2 - 4*a*c)*a^6*b^5*c^7 - 7680*(b^2 - 4
*a*c)*a^7*b^3*c^8 - 640*(b^2 - 4*a*c)*a^8*b*c^9)*d^8*e^8 - (2*a^2*b^16*c^2
+ 150*a^3*b^14*c^3 - 362*a^4*b^12*c^4 - 11636*a^5*b^10*c^5 + 49334*a^6*b^8*
c^6 + 46200*a^7*b^6*c^7 - 274400*a^8*b^4*c^8 - 96640*a^9*b^2*c^9 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^16 - 75*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^14*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^15*c + 181*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^12*c^2 + 158*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^13*c^2 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^14*c^2 + 5818*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^10*c^3 + 270*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^11*c^3 - 79*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^12*c^3 - 24667*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^4 - 10556*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^9*c^4 - 135
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^10*c^4 - 2
3100*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^5
+ 7110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^
5 + 5278*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^8*
c^5 + 137200*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*
b^4*c^6 + 74640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^7*b^5*c^6 - 3555*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^6*b^6*c^6 + 48320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^9*b^2*c^7 + 24160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^8*b^3*c^7 - 37320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^7*b^4*c^7 - 12080*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^8*b^2*c^8 - 2*(b^2 - 4*a*c)*a^2*b^14*c^2 - 158*(b^2 - 4*a*c)
*a^3*b^12*c^3 - 270*(b^2 - 4*a*c)*a^4*b^10*c^4 + 10556*(b^2 - 4*a*c)*a^5*b^
8*c^5 - 7110*(b^2 - 4*a*c)*a^6*b^6*c^6 - 74640*(b^2 - 4*a*c)*a^7*b^4*c^7 -
24160*(b^2 - 4*a*c)*a^8*b^2*c^8)*d^7*e^9 + (22*a^3*b^15*c^2 + 336*a^4*b^13*
c^3 - 4312*a^5*b^11*c^4 + 56*a^6*b^9*c^5 + 81844*a^7*b^7*c^6 - 119280*a^8*b
```

```
^5*c^7 - 164416*a^9*b^3*c^8 - 7424*a^10*b*c^9 - 11*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^15 - 168*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^13*c + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^14*c + 2156*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^11*c^2 + 424*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^12*c^2 - 11*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^13*c^2 - 28*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9*c^3 - 2616*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^10*c^3 - 212*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^11*c^3 - 40922*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^4 - 10408*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^4 + 1308*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^9*c^4 + 59640*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^5 + 40212*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^5 + 5204*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^5 + 8220
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^6 + 4
1568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^6
 - 20106*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c
^6 + 3712*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*
c^7 + 1856*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^
2*c^7 - 20784*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8
*b^3*c^7 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
9*b*c^8 - 22*(b^2 - 4*a*c)*a^3*b^13*c^2 - 424*(b^2 - 4*a*c)*a^4*b^11*c^3 +
2616*(b^2 - 4*a*c)*a^5*b^9*c^4 + 10408*(b^2 - 4*a*c)*a^6*b^7*c^5 - 40212*(b
^2 - 4*a*c)*a^7*b^5*c^6 - 41568*(b^2 - 4*a*c)*a^8*b^3*c^7 - 1856*(b^2 - 4*a
*c)*a^9*b*c^8)*d^6*e^10 - 3*(30*a^4*b^14*c^2 - 26*a^5*b^12*c^3 - 2460*a^6*b
^10*c^4 + 10034*a^7*b^8*c^5 + 5544*a^8*b^6*c^6 - 47264*a^9*b^4*c^7 - 8320*a
^10*b^2*c^8 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^14 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5
*b^12*c + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^13*c + 1230*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6
*b^10*c^2 + 94*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^11*c^2 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^12*c^2 - 5017*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^7*b^8*c^3 - 2084*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b^9*c^3 - 47*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b^10*c^3 - 2772*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^8*b^6*c^4 + 1698*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^7*b^7*c^4 + 1042*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^6*b^8*c^4 + 23632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^9*b^4*c^5 + 12336*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^8*b^5*c^5 - 849*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^7*b^6*c^5 + 4160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^10*b^2*c^6 + 2080*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
- sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^6 - 6168*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^6 - 1040*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^7 - 30*(b^2 - 4*a*c)*a^4*b^12*c^2 - 94
*(b^2 - 4*a*c)*a^5*b^10*c^3 + 2084*(b^2 - 4*a*c)*a^6*b^8*c^4 - 1698*(b^2 -
4*a*c)*a^7*b^6*c^5 - 12336*(b^2 - 4*a*c)*a^8*b^4*c^6 - 2080*(b^2 - 4*a*c)*a
^9*b^2*c^7)*d^5*e^11 + (190*a^5*b^13*c^2 - 1440*a^6*b^11*c^3 - 2158*a^7*b^9
*c^4 + 35196*a^8*b^7*c^5 - 52304*a^9*b^5*c^6 - 42688*a^10*b^3*c^7 + 3840*a^
11*b*c^8 - 95*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5
*b^13 + 720*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b
^11*c + 190*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b
^12*c + 1079*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*
b^9*c^2 - 680*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6
*b^10*c^2 - 95*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^11*c^2 - 17598*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^8*b^7*c^3 - 4878*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^7*b^8*c^3 + 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^6*b^9*c^3 + 26152*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^9*b^5*c^4 + 15684*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^8*b^6*c^4 + 2439*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^7*b^7*c^4 + 21344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^10*b^3*c^5 + 10432*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^9*b^4*c^5 - 7842*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^5 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^11*b*c^6 - 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^6 - 5216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^6 + 480*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^7 - 190*(b^2 - 4*a*c)*a^5*b^11*c^2 + 6
80*(b^2 - 4*a*c)*a^6*b^9*c^3 + 4878*(b^2 - 4*a*c)*a^7*b^7*c^4 - 15684*(b^2
- 4*a*c)*a^8*b^5*c^5 - 10432*(b^2 - 4*a*c)*a^9*b^3*c^6 + 960*(b^2 - 4*a*c)*
a^10*b*c^7)*d^4*e^12 - (230*a^6*b^12*c^2 - 2502*a^7*b^10*c^3 + 6718*a^8*b^8
*c^4 + 7896*a^9*b^6*c^5 - 39520*a^10*b^4*c^6 + 6784*a^11*b^2*c^7 - 115*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^12 + 1251*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^10*c + 230*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^11*c - 3359*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^8*c^2 - 1582*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^9*c^2 - 115*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^10*c^2 - 394
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^6*c^3 + 3
90*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^7*c^3 +
791*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^8*c^3 +
 19760*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^4*c
^4 + 9456*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^5
*c^4 - 195*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^
6*c^4 - 3392*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11
*b^2*c^5 - 1696*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
```

```
^10*b^3*c^5 - 4728*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^9*b^4*c^5 + 848*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^10*b^2*c^6 - 230*(b^2 - 4*a*c)*a^6*b^10*c^2 + 1582*(b^2 - 4*a*c)*a^7*b
^8*c^3 - 390*(b^2 - 4*a*c)*a^8*b^6*c^4 - 9456*(b^2 - 4*a*c)*a^9*b^4*c^5 + 1
696*(b^2 - 4*a*c)*a^10*b^2*c^6)*d^3*e^13 + 3*(54*a^7*b^11*c^2 - 680*a^8*b^9
*c^3 + 2796*a^9*b^7*c^4 - 3472*a^10*b^5*c^5 - 1472*a^11*b^3*c^6 + 1280*a^12
*b*c^7 - 27*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b
^11 + 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^9
*c + 54*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^10*
c - 1398*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^7*
c^2 - 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^8
*c^2 - 27*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^9
*c^2 + 1736*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*
b^5*c^3 + 940*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9
*b^6*c^3 + 232*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
8*b^7*c^3 + 736*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^11*b^3*c^4 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^10*b^4*c^4 - 470*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^9*b^5*c^4 - 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^12*b*c^5 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^11*b^2*c^5 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^10*b^3*c^5 + 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^11*b*c^6 - 54*(b^2 - 4*a*c)*a^7*b^9*c^2 + 464*(b^2 - 4*a*c)*a^8*b
^7*c^3 - 940*(b^2 - 4*a*c)*a^9*b^5*c^4 - 288*(b^2 - 4*a*c)*a^10*b^3*c^5 + 3
20*(b^2 - 4*a*c)*a^11*b*c^6)*d^2*e^14 - (62*a^8*b^10*c^2 - 834*a^9*b^8*c^3
+ 3928*a^10*b^6*c^4 - 7264*a^11*b^4*c^5 + 3712*a^12*b^2*c^6 - 31*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^10 + 417*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c + 62*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^9*c - 1964*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^6*c^2 - 586*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^7*c^2 - 31*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^8*c^2 + 3632*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^4*c^3 + 1584*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^5*c^3 + 293*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^6*c^3 - 1856*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^2*c^4 - 928*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^3*c^4 - 792
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^4*c^4 + 4
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^2*c^5 -
 62*(b^2 - 4*a*c)*a^8*b^8*c^2 + 586*(b^2 - 4*a*c)*a^9*b^6*c^3 - 1584*(b^2 -
 4*a*c)*a^10*b^4*c^4 + 928*(b^2 - 4*a*c)*a^11*b^2*c^5)*d*e^15 + (10*a^9*b^9
*c^2 - 138*a^10*b^7*c^3 + 680*a^11*b^5*c^4 - 1376*a^12*b^3*c^5 + 896*a^13*b
*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^9
+ 69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^7*c +
 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c - 3
```

```
40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^5*c^2 -
 98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^6*c^2
- 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^7*c^2 +
 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^3*c^3
 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^4*c
^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^5*
c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^13*b*
c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^
2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*
b^3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^1
2*b*c^5 - 10*(b^2 - 4*a*c)*a^9*b^7*c^2 + 98*(b^2 - 4*a*c)*a^10*b^5*c^3 - 28
8*(b^2 - 4*a*c)*a^11*b^3*c^4 + 224*(b^2 - 4*a*c)*a^12*b*c^5)*e^16 + 2*(sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 14*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^7 + 2*a*b^6*c^7 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c
^8 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 28*a^2*b^4*c^8 - 96*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^4*c^9 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^9 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 128*
a^3*b^2*c^9 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^10 - 192*a^4
*c^10 - 2*(b^2 - 4*a*c)*a*b^4*c^7 + 20*(b^2 - 4*a*c)*a^2*b^2*c^8 - 48*(b^2
- 4*a*c)*a^3*c^9)*d^10*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*
e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3
*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 +
 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d
*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) - 4*(3*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 - 41*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^5*c^6 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 6*
a*b^7*c^6 + 184*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 58*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 3*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 82*a^2*b^5*c^7 - 272*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b*c^8 - 136*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^8 - 29*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 368*a^
3*b^3*c^8 + 68*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 544*a^4*
b*c^9 - 6*(b^2 - 4*a*c)*a*b^5*c^6 + 58*(b^2 - 4*a*c)*a^2*b^3*c^7 - 136*(b^2
 - 4*a*c)*a^3*b*c^8)*d^9*e*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*
d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12
*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e
^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b
^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) + 6*(5*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^4 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^6*c^5 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5
 + 10*a*b^8*c^5 + 258*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 +
 88*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 + 5*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 128*a^2*b^6*c^6 - 272*sqrt(2)*sqrt(b*c
```

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 - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 164*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^7 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7
 + 516*a^3*b^4*c^7 - 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^8 -
112*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 + 82*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 544*a^4*b^2*c^8 + 56*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^4*c^9 - 448*a^5*c^9 - 10*(b^2 - 4*a*c)*a*b^6*c^5
+ 88*(b^2 - 4*a*c)*a^2*b^4*c^6 - 164*(b^2 - 4*a*c)*a^3*b^2*c^7 - 112*(b^2 -
 4*a*c)*a^4*c^8)*d^8*e^2*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^
5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a
^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3
 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3
*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) - 40*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*c^3 - 11*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^7*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^4 + 2
*a*b^9*c^4 + 30*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 + 14*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^7*c^5 - 22*a^2*b^7*c^5 + 32*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^4*c^6 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 + 60*a^3*b^5
*c^6 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 - 80*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^3*c^7 + 64*a^4*b^3*c^7 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^4*b*c^8 - 320*a^5*b*c^8 - 2*(b^2 - 4*a*c)*a*b^7*c^4 + 14*(b^2
- 4*a*c)*a^2*b^5*c^5 - 4*(b^2 - 4*a*c)*a^3*b^3*c^6 - 80*(b^2 - 4*a*c)*a^4*b
*c^7)*d^7*e^3*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^
2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*
e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^
4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12
*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) + 2*(15*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^10*c^2 - 110*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^8*c^3 - 30*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*c^3 + 30*a*b^
10*c^3 - 206*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 100*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4 + 15*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^8*c^4 - 220*a^2*b^8*c^4 + 3012*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 812*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b^5*c^5 - 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 - 412
*a^3*b^6*c^5 - 5248*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 2
776*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - 406*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 6024*a^4*b^4*c^6 - 1216*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^7 - 608*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^5*b*c^7 + 1388*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c
^7 - 10496*a^5*b^2*c^7 + 304*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^
8 - 2432*a^6*c^8 - 30*(b^2 - 4*a*c)*a*b^8*c^3 + 100*(b^2 - 4*a*c)*a^2*b^6*c
^4 + 812*(b^2 - 4*a*c)*a^3*b^4*c^5 - 2776*(b^2 - 4*a*c)*a^4*b^2*c^6 - 608*(
b^2 - 4*a*c)*a^5*c^7)*d^6*e^4*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c
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^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 -
 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^
3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^
3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) - 12*(sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^11*c + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^9*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^10*c^2 +
 2*a*b^11*c^2 - 106*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 - 1
0*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^3 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^9*c^3 + 2*a^2*b^9*c^3 + 494*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^4*b^5*c^4 + 172*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b^6*c^4 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4 - 212*
a^3*b^7*c^4 - 400*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 - 300
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 - 86*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 + 988*a^4*b^5*c^5 - 800*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 - 400*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b^2*c^6 + 150*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6
- 800*a^5*b^3*c^6 + 200*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 -
 1600*a^6*b*c^7 - 2*(b^2 - 4*a*c)*a*b^9*c^2 - 10*(b^2 - 4*a*c)*a^2*b^7*c^3
+ 172*(b^2 - 4*a*c)*a^3*b^5*c^4 - 300*(b^2 - 4*a*c)*a^4*b^3*c^5 - 400*(b^2
- 4*a*c)*a^5*b*c^6)*d^5*e^5*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2
*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 1
2*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*
e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*
b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) + 2*(sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^12 + 28*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^10*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^11*c + 2*a*
b^12*c - 346*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 - 64*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^10*c^2 + 56*a^2*b^10*c^2 + 728*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b^6*c^3 + 436*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*b^7*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^3 - 692*a^3
*b^8*c^3 + 2300*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 + 288*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 - 218*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 1456*a^4*b^6*c^4 - 6752*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 - 3448*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^5*b^3*c^5 - 144*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*
c^5 + 4600*a^5*b^4*c^5 - 576*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^
6 - 288*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 + 1724*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 13504*a^6*b^2*c^6 + 144*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^7 - 1152*a^7*c^7 - 2*(b^2 - 4*a*c)*a
*b^10*c - 64*(b^2 - 4*a*c)*a^2*b^8*c^2 + 436*(b^2 - 4*a*c)*a^3*b^6*c^3 + 28
8*(b^2 - 4*a*c)*a^4*b^4*c^4 - 3448*(b^2 - 4*a*c)*a^5*b^2*c^5 - 288*(b^2 - 4
*a*c)*a^6*c^6)*d^4*e^6*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*
e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3
*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 +
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 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d
*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) - 8*(2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^11 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^3*b^9*c - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c + 4*a^2
*b^11*c - 107*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 - 2*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 + 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^9*c^2 - 14*a^3*b^9*c^2 + 646*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*b^5*c^3 + 206*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^6*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 - 214*a^4*
b^7*c^3 - 800*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 - 468*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 103*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 + 1292*a^5*b^5*c^4 - 544*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 - 272*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^6*b^2*c^5 + 234*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 -
1600*a^6*b^3*c^5 + 136*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^6 -
1088*a^7*b*c^6 - 4*(b^2 - 4*a*c)*a^2*b^9*c - 2*(b^2 - 4*a*c)*a^3*b^7*c^2 +
206*(b^2 - 4*a*c)*a^4*b^5*c^3 - 468*(b^2 - 4*a*c)*a^5*b^3*c^4 - 272*(b^2 -
4*a*c)*a^6*b*c^5)*d^3*e^7*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d
^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*
a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^
3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^
3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) + 18*(2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^10 - 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^4*b^8*c - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c + 4
*a^3*b^10*c + 45*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 + 24*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 + 2*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 - 40*a^4*b^8*c^2 + 90*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^5*b^5*c^3 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 90*a
^5*b^6*c^3 - 320*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 - 156*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 - 3*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 + 180*a^6*b^4*c^4 + 32*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^8*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^7*b*c^5 + 78*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 - 640*a^7
*b^2*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^6 + 64*a^8*c^6 -
 4*(b^2 - 4*a*c)*a^3*b^8*c + 24*(b^2 - 4*a*c)*a^4*b^6*c^2 + 6*(b^2 - 4*a*c)
*a^5*b^4*c^3 - 156*(b^2 - 4*a*c)*a^6*b^2*c^4 + 16*(b^2 - 4*a*c)*a^7*c^5)*d^
2*e^8*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*
d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*
b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^
4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c
*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) - 4*(8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^4*b^9 - 97*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c - 16
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c + 16*a^4*b^9*c + 391*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 + 130*sqrt(2)*sqrt(b*c - s
```

```
qrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^7*c^2 - 194*a^5*b^7*c^2 - 520*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^7*b^3*c^3 - 262*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 - 6
5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + 782*a^6*b^5*c^3 - 1
6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^4 - 8*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 + 131*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b^3*c^4 - 1040*a^7*b^3*c^4 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^7*b*c^5 - 32*a^8*b*c^5 - 16*(b^2 - 4*a*c)*a^4*b^7*c + 130*(b^2 - 4*a*
c)*a^5*b^5*c^2 - 262*(b^2 - 4*a*c)*a^6*b^3*c^3 - 8*(b^2 - 4*a*c)*a^7*b*c^4)
*d*e^9*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3
*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a
*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e
^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*
c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6) + 2*(5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^5*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c - 1
0*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c + 10*a^5*b^8*c + 286*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^2 + 88*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^5*b^6*c^2 - 128*a^6*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^8*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^3 - 4
4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 572*a^7*b^4*c^3 + 2
24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*c^4 + 112*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^8*b*c^4 + 110*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^7*b^2*c^4 - 992*a^8*b^2*c^4 - 56*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^8*c^5 + 448*a^9*c^5 - 10*(b^2 - 4*a*c)*a^5*b^6*c + 88*(b^2 - 4*a*c)*a^
6*b^4*c^2 - 220*(b^2 - 4*a*c)*a^7*b^2*c^3 + 112*(b^2 - 4*a*c)*a^8*c^4)*e^10
*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e
 + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d
^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9
*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^
5 + a^4*b^2*e^6 - 4*a^5*c*e^6) + (a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c
^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 -
 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^
3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^
3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)^2*(2*b^3*c^5 -
8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c
)*b*c^5)*d^4 - (a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*
b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^
2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*
d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a
^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)^2*(6*b^4*c^4 - 32*a*b^2*c^5 + 32*
a^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c
```

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^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 - 6*(b^2 - 4*a*c)*b^2*c^4
 + 8*(b^2 - 4*a*c)*a*c^5)*d^3*e + 3*(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^
3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^
2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2
*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3
*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)^2*(2*b^5*c^3
 - 12*a*b^3*c^4 + 16*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^5*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*
(b^2 - 4*a*c)*b^3*c^3 + 4*(b^2 - 4*a*c)*a*b*c^4)*d^2*e^2 - (a*b^2*c^3*d^6 -
 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2
 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c
*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 -
 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*
a^5*c*e^6)^2*(2*b^6*c^2 + 6*a*b^4*c^3 - 128*a^2*b^2*c^4 + 288*a^3*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 3*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 64*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 72*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 - 14*(b^2 - 4*a*c)*a*b^
2*c^3 + 72*(b^2 - 4*a*c)*a^2*c^4)*d*e^3 + (a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 -
3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*
d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3
*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e
^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)^2*(10*
a*b^5*c^2 - 78*a^2*b^3*c^3 + 152*a^3*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
```

```
^2 - 4*a*c)*c)*a*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^3 - 10*(b^2 - 4*a*c)*a*b^3*c^2 + 38*(b^2 - 4*a*c)*a^2*
b*c^3)*e^4)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3*c^3*d^6 - 4*a^2*b*c^4*d^6 - 3*
a*b^4*c^2*d^5*e + 12*a^2*b^2*c^3*d^5*e + 3*a*b^5*c*d^4*e^2 - 9*a^2*b^3*c^2*
d^4*e^2 - 12*a^3*b*c^3*d^4*e^2 - a*b^6*d^3*e^3 - 2*a^2*b^4*c*d^3*e^3 + 24*a
^3*b^2*c^2*d^3*e^3 + 3*a^2*b^5*d^2*e^4 - 9*a^3*b^3*c*d^2*e^4 - 12*a^4*b*c^2
*d^2*e^4 - 3*a^3*b^4*d*e^5 + 12*a^4*b^2*c*d*e^5 + a^4*b^3*e^6 - 4*a^5*b*c*e
^6 - sqrt((a*b^3*c^3*d^6 - 4*a^2*b*c^4*d^6 - 3*a*b^4*c^2*d^5*e + 12*a^2*b^2
*c^3*d^5*e + 3*a*b^5*c*d^4*e^2 - 9*a^2*b^3*c^2*d^4*e^2 - 12*a^3*b*c^3*d^4*e
^2 - a*b^6*d^3*e^3 - 2*a^2*b^4*c*d^3*e^3 + 24*a^3*b^2*c^2*d^3*e^3 + 3*a^2*b
^5*d^2*e^4 - 9*a^3*b^3*c*d^2*e^4 - 12*a^4*b*c^2*d^2*e^4 - 3*a^3*b^4*d*e^5 +
 12*a^4*b^2*c*d*e^5 + a^4*b^3*e^6 - 4*a^5*b*c*e^6)^2 - 4*(a^2*b^2*c^3*d^6 -
 4*a^3*c^4*d^6 - 3*a^2*b^3*c^2*d^5*e + 12*a^3*b*c^3*d^5*e + 3*a^2*b^4*c*d^4
*e^2 - 9*a^3*b^2*c^2*d^4*e^2 - 12*a^4*c^3*d^4*e^2 - a^2*b^5*d^3*e^3 - 2*a^3
*b^3*c*d^3*e^3 + 24*a^4*b*c^2*d^3*e^3 + 3*a^3*b^4*d^2*e^4 - 9*a^4*b^2*c*d^2
*e^4 - 12*a^5*c^2*d^2*e^4 - 3*a^4*b^3*d*e^5 + 12*a^5*b*c*d*e^5 + a^5*b^2*e^
6 - 4*a^6*c*e^6)*(a*b^2*c^4*d^6 - 4*a^2*c^5*d^6 - 3*a*b^3*c^3*d^5*e + 12*a^
2*b*c^4*d^5*e + 3*a*b^4*c^2*d^4*e^2 - 9*a^2*b^2*c^3*d^4*e^2 - 12*a^3*c^4*d^
4*e^2 - a*b^5*c*d^3*e^3 - 2*a^2*b^3*c^2*d^3*e^3 + 24*a^3*b*c^3*d^3*e^3 + 3*
a^2*b^4*c*d^2*e^4 - 9*a^3*b^2*c^2*d^2*e^4 - 12*a^4*c^3*d^2*e^4 - 3*a^3*b^3*
c*d*e^5 + 12*a^4*b*c^2*d*e^5 + a^4*b^2*c*e^6 - 4*a^5*c^2*e^6)))/(a*b^2*c^4*
d^6 - 4*a^2*c^5*d^6 - 3*a*b^3*c^3*d^5*e + 12*a^2*b*c^4*d^5*e + 3*a*b^4*c^2*
d^4*e^2 - 9*a^2*b^2*c^3*d^4*e^2 - 12*a^3*c^4*d^4*e^2 - a*b^5*c*d^3*e^3 - 2*
a^2*b^3*c^2*d^3*e^3 + 24*a^3*b*c^3*d^3*e^3 + 3*a^2*b^4*c*d^2*e^4 - 9*a^3*b^
2*c^2*d^2*e^4 - 12*a^4*c^3*d^2*e^4 - 3*a^3*b^3*c*d*e^5 + 12*a^4*b*c^2*d*e^5
 + a^4*b^2*c*e^6 - 4*a^5*c^2*e^6)))/((a^3*b^6*c^6 - 12*a^4*b^4*c^7 - 2*a^3*
b^5*c^7 + 48*a^5*b^2*c^8 + 16*a^4*b^3*c^8 + a^3*b^4*c^8 - 64*a^6*c^9 - 32*a
^5*b*c^9 - 8*a^4*b^2*c^9 + 16*a^5*c^10)*d^12*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*
d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^
2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 +
24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2
*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*
abs(c) - 6*(a^3*b^7*c^5 - 12*a^4*b^5*c^6 - 2*a^3*b^6*c^6 + 48*a^5*b^3*c^7 +
 16*a^4*b^4*c^7 + a^3*b^5*c^7 - 64*a^6*b*c^8 - 32*a^5*b^2*c^8 - 8*a^4*b^3*c
^8 + 16*a^5*b*c^9)*d^11*e*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d
^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*
a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^
3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^
3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) + 3*(5*a^3*b
^8*c^4 - 58*a^4*b^6*c^5 - 10*a^3*b^7*c^5 + 216*a^5*b^4*c^6 + 76*a^4*b^5*c^6
 + 5*a^3*b^6*c^6 - 224*a^6*b^2*c^7 - 128*a^5*b^3*c^7 - 38*a^4*b^4*c^7 - 128
*a^7*c^8 - 64*a^6*b*c^8 + 64*a^5*b^2*c^8 + 32*a^6*c^9)*d^10*e^2*abs(a*b^2*c
^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c
*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a
^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d
```

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^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*
e^6 - 4*a^5*c*e^6)*abs(c) - 10*(2*a^3*b^9*c^3 - 21*a^4*b^7*c^4 - 4*a^3*b^8*
c^4 + 60*a^5*b^5*c^5 + 26*a^4*b^6*c^5 + 2*a^3*b^7*c^5 + 16*a^6*b^3*c^6 - 16
*a^5*b^4*c^6 - 13*a^4*b^5*c^6 - 192*a^7*b*c^7 - 96*a^6*b^2*c^7 + 8*a^5*b^3*
c^7 + 48*a^6*b*c^8)*d^9*e^3*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2
*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 1
2*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*
e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*
b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) + 15*(a^3*
b^10*c^2 - 8*a^4*b^8*c^3 - 2*a^3*b^9*c^3 + a^5*b^6*c^4 + 8*a^4*b^7*c^4 + a^
3*b^8*c^4 + 116*a^6*b^4*c^5 + 30*a^5*b^5*c^5 - 4*a^4*b^6*c^5 - 208*a^7*b^2*
c^6 - 112*a^6*b^3*c^6 - 15*a^5*b^4*c^6 - 64*a^8*c^7 - 32*a^7*b*c^7 + 56*a^6
*b^2*c^7 + 16*a^7*c^8)*d^8*e^4*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*
c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2
- 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d
^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a
^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) - 6*(a^
3*b^11*c - 2*a^4*b^9*c^2 - 2*a^3*b^10*c^2 - 62*a^5*b^7*c^3 - 4*a^4*b^8*c^3
+ a^3*b^9*c^3 + 296*a^6*b^5*c^4 + 108*a^5*b^6*c^4 + 2*a^4*b^7*c^4 - 160*a^7
*b^3*c^5 - 160*a^6*b^4*c^5 - 54*a^5*b^5*c^5 - 640*a^8*b*c^6 - 320*a^7*b^2*c
^6 + 80*a^6*b^3*c^6 + 160*a^7*b*c^7)*d^7*e^5*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*
d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^
2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 +
24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2
*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*
abs(c) + (a^3*b^12 + 18*a^4*b^10*c - 2*a^3*b^11*c - 222*a^5*b^8*c^2 - 44*a^
4*b^9*c^2 + a^3*b^10*c^2 + 316*a^6*b^6*c^3 + 268*a^5*b^7*c^3 + 22*a^4*b^8*c
^3 + 2160*a^7*b^4*c^4 + 440*a^6*b^5*c^4 - 134*a^5*b^6*c^4 - 4800*a^8*b^2*c^
5 - 2560*a^7*b^3*c^5 - 220*a^6*b^4*c^5 - 1280*a^9*c^6 - 640*a^8*b*c^6 + 128
0*a^7*b^2*c^6 + 320*a^8*c^7)*d^6*e^6*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*
a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^
4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b
*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4
 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) -
 6*(a^4*b^11 - 2*a^5*b^9*c - 2*a^4*b^10*c - 62*a^6*b^7*c^2 - 4*a^5*b^8*c^2
+ a^4*b^9*c^2 + 296*a^7*b^5*c^3 + 108*a^6*b^6*c^3 + 2*a^5*b^7*c^3 - 160*a^8
*b^3*c^4 - 160*a^7*b^4*c^4 - 54*a^6*b^5*c^4 - 640*a^9*b*c^5 - 320*a^8*b^2*c
^5 + 80*a^7*b^3*c^5 + 160*a^8*b*c^6)*d^5*e^7*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*
d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^
2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 +
24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2
*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*
abs(c) + 15*(a^5*b^10 - 8*a^6*b^8*c - 2*a^5*b^9*c + a^7*b^6*c^2 + 8*a^6*b^7
*c^2 + a^5*b^8*c^2 + 116*a^8*b^4*c^3 + 30*a^7*b^5*c^3 - 4*a^6*b^6*c^3 - 208
*a^9*b^2*c^4 - 112*a^8*b^3*c^4 - 15*a^7*b^4*c^4 - 64*a^10*c^5 - 32*a^9*b*c^
```

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5 + 56*a^8*b^2*c^5 + 16*a^9*c^6)*d^4*e^8*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6
- 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^
2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a
^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2
*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(
c) - 10*(2*a^6*b^9 - 21*a^7*b^7*c - 4*a^6*b^8*c + 60*a^8*b^5*c^2 + 26*a^7*b
^6*c^2 + 2*a^6*b^7*c^2 + 16*a^9*b^3*c^3 - 16*a^8*b^4*c^3 - 13*a^7*b^5*c^3 -
 192*a^10*b*c^4 - 96*a^9*b^2*c^4 + 8*a^8*b^3*c^4 + 48*a^9*b*c^5)*d^3*e^9*ab
s(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e +
3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*
e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^
3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 +
 a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) + 3*(5*a^7*b^8 - 58*a^8*b^6*c - 10*a^7*b
^7*c + 216*a^9*b^4*c^2 + 76*a^8*b^5*c^2 + 5*a^7*b^6*c^2 - 224*a^10*b^2*c^3
- 128*a^9*b^3*c^3 - 38*a^8*b^4*c^3 - 128*a^11*c^4 - 64*a^10*b*c^4 + 64*a^9*
b^2*c^4 + 32*a^10*c^5)*d^2*e^10*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3
*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2
 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*
d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*
a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) - 6*(a
^8*b^7 - 12*a^9*b^5*c - 2*a^8*b^6*c + 48*a^10*b^3*c^2 + 16*a^9*b^4*c^2 + a^
8*b^5*c^2 - 64*a^11*b*c^3 - 32*a^10*b^2*c^3 - 8*a^9*b^3*c^3 + 16*a^10*b*c^4
)*d*e^11*abs(a*b^2*c^3*d^6 - 4*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c
^3*d^5*e + 3*a*b^4*c*d^4*e^2 - 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 -
 a*b^5*d^3*e^3 - 2*a^2*b^3*c*d^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2
*e^4 - 9*a^3*b^2*c*d^2*e^4 - 12*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*
b*c*d*e^5 + a^4*b^2*e^6 - 4*a^5*c*e^6)*abs(c) + (a^9*b^6 - 12*a^10*b^4*c -
2*a^9*b^5*c + 48*a^11*b^2*c^2 + 16*a^10*b^3*c^2 + a^9*b^4*c^2 - 64*a^12*c^3
 - 32*a^11*b*c^3 - 8*a^10*b^2*c^3 + 16*a^11*c^4)*e^12*abs(a*b^2*c^3*d^6 - 4
*a^2*c^4*d^6 - 3*a*b^3*c^2*d^5*e + 12*a^2*b*c^3*d^5*e + 3*a*b^4*c*d^4*e^2 -
 9*a^2*b^2*c^2*d^4*e^2 - 12*a^3*c^3*d^4*e^2 - a*b^5*d^3*e^3 - 2*a^2*b^3*c*d
^3*e^3 + 24*a^3*b*c^2*d^3*e^3 + 3*a^2*b^4*d^2*e^4 - 9*a^3*b^2*c*d^2*e^4 - 1
2*a^4*c^2*d^2*e^4 - 3*a^3*b^3*d*e^5 + 12*a^4*b*c*d*e^5 + a^4*b^2*e^6 - 4*a^
5*c*e^6)*abs(c)) + 1/2*(9*c*d^2*e^4 - 5*b*d*e^5 + a*e^6)*arctan(e*x/sqrt(d*
e))/((c^3*d^7 - 3*b*c^2*d^6*e + 3*b^2*c*d^5*e^2 + 3*a*c^2*d^5*e^2 - b^3*d^4
*e^3 - 6*a*b*c*d^4*e^3 + 3*a*b^2*d^3*e^4 + 3*a^2*c*d^3*e^4 - 3*a^2*b*d^2*e^
5 + a^3*d*e^6)*sqrt(d*e)) + 1/2*(b*c^3*d^3*e*x^5 - 2*b^2*c^2*d^2*e^2*x^5 +
4*a*c^3*d^2*e^2*x^5 + b^3*c*d*e^3*x^5 - 3*a*b*c^2*d*e^3*x^5 + a*b^2*c*e^4*x
^5 - 4*a^2*c^2*e^4*x^5 + b*c^3*d^4*x^3 - b^2*c^2*d^3*e*x^3 + 2*a*c^3*d^3*e*
x^3 - b^3*c*d^2*e^2*x^3 + 3*a*b*c^2*d^2*e^2*x^3 + b^4*d*e^3*x^3 - 4*a*b^2*c
*d*e^3*x^3 + 2*a^2*c^2*d*e^3*x^3 + a*b^3*e^4*x^3 - 4*a^2*b*c*e^4*x^3 + b^2*
c^2*d^4*x - 2*a*c^3*d^4*x - 2*b^3*c*d^3*e*x + 6*a*b*c^2*d^3*e*x + b^4*d^2*e
^2*x - 4*a*b^2*c*d^2*e^2*x + 2*a^2*c^2*d^2*e^2*x + a^2*b^2*e^4*x - 4*a^3*c*
e^4*x)/((a*b^2*c^2*d^5 - 4*a^2*c^3*d^5 - 2*a*b^3*c*d^4*e + 8*a^2*b*c^2*d^4*
e + a*b^4*d^3*e^2 - 2*a^2*b^2*c*d^3*e^2 - 8*a^3*c^2*d^3*e^2 - 2*a^2*b^3*d^2
```

```
*e^3 + 8*a^3*b*c*d^2*e^3 + a^3*b^2*d*e^4 - 4*a^4*c*d*e^4)*(c*e*x^6 + c*d*x^
4 + b*e*x^4 + b*d*x^2 + a*e*x^2 + a*d))
```

Mupad [B] (verification not implemented)

Time = 17.55 (sec) , antiderivative size = 97073, normalized size of antiderivative = 90.13

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log(root(128723189760*a^14*b^4*c^9*d^13*e^14*z^6 + 128723189760*a^12
*b^4*c^11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^4*d^11*e^16*z^6 - 84324556
80*a^7*b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b^11*c^5*d^12*e^15*z^6 + 12
673351680*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960*a^12*b^9*c^6*d^12*e^15*z^
6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z^6 - 21048344576*a^9*b^12*c^6*d^15*e^
12*z^6 - 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 - 16609443840*a^10*b^3*c^14*
d^22*e^5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^13*z^6 + 145332633600*a^12*
b^5*c^10*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^8*d^12*e^15*z^6 + 12374035
6608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17*b^5*c^5*d^6*e^21*z^6 + 34
60300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a^15*b^7*c^5*d^8*e^19*z^6 -
 7751073792*a^8*b^7*c^12*d^22*e^5*z^6 + 12041846784*a^14*b^7*c^6*d^10*e^17*
z^6 + 12041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325545099264*a^14*b^3*c^10*d^
14*e^13*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^11*z^6 - 3330539520*a^13*b^
10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^21*e^6*z^6 + 157789716480*
a^12*b^7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^7*c^9*d^16*e^11*z^6 + 3749
2359168*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168*a^9*b^10*c^8*d^17*e^10*z^
6 + 301989888*a^8*b^3*c^16*d^26*e*z^6 - 7266631680*a^17*b^4*c^6*d^7*e^20*z^
6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326592*a^20*b*c^6*d^4*e^23*z^
6 - 188743680*a^7*b^5*c^15*d^26*e*z^6 + 45747339264*a^13*b^8*c^6*d^11*e^16*
z^6 + 45747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 74612736*a^10*b^16*c*d^9*e^18
*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 2768240640*a^7*b^7*c^13*d^24*
e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^19*z^6 + 62914560*a^6*b^7*c^14*d^26*e*
z^6 + 2752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 2752020480*a^7*b^13*c^7*d^18
*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45957120*a^12*b^14*c*d^7*e^2
0*z^6 - 2724986880*a^14*b^9*c^4*d^8*e^19*z^6 - 2724986880*a^7*b^9*c^11*d^22
*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21086208*a^13*b^13*c*d^6*e^2
1*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z^6 - 6438912*a^14*b^12*c*d^5*e^22*z^6
 + 5406720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6*b^20*c*d^13*e^14*z^6 - 15
23712*a^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^11*c*d^4*e^23*z^6 + 1179648
*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^15*e^12*z^6 - 98304*a^16*b^
10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^6 - 49152*a^3*b^13*c^11*d^
26*e*z^6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6 + 6897106944*a^8*b^13*c^6*
d^16*e^11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*z^6 - 2422210560*a^8*b^6*c
```

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^13*d^23*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^15*e^12*z^6 + 41004564480*a
^15*b^4*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*c^12*d^19*e^8*z^6 + 227082
2400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*b^11*c^10*d^22*e^5*z^6 + 2
3677108224*a^14*b^8*c^5*d^9*e^18*z^6 + 23677108224*a^8*b^8*c^11*d^21*e^6*z^
6 + 212600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 212600881152*a^13*b^2*c^12*d
^17*e^10*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17*z^6 + 75157733376*a^10*b^
5*c^12*d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d^13*e^14*z^6 - 25121783808
0*a^11*b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b^10*c^3*d^7*e^20*z^6 - 195
2907264*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*a^13*b^9*c^5*d^10*e^17*z^6
 - 27691057152*a^8*b^9*c^10*d^20*e^7*z^6 - 1902673920*a^8*b^15*c^4*d^14*e^1
3*z^6 - 1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 10465050624*a^10*b^11*c^6*d
^14*e^13*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11*z^6 + 1613905920*a^9*b^14
*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17*e^10*z^6 - 33218887680*a^
17*b*c^9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*d^20*e^7*z^6 + 1524695040*
a^10*b^14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14*c^7*d^19*e^8*z^6 - 147220
0704*a^18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b^4*c^15*d^25*e^2*z^6 - 830
47219200*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200*a^11*b^3*c^13*d^20*e^7*z^
6 + 44291850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291850240*a^11*b^2*c^14*d^21*
e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 - 201326592*a^9*b*c^17*d^26
*e*z^6 + 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 + 48530718720*a^10*b^8*c^9*
d^17*e^10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18*z^6 - 1242644480*a^6*b^12
*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11*e^16*z^6 + 9813196800*a^8
*b^10*c^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*d^14*e^13*z^6 - 9301288550
4*a^14*b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b^4*c^10*d^15*e^12*z^6 + 52
730658816*a^10*b^10*c^7*d^15*e^12*z^6 - 1180106752*a^9*b^15*c^3*d^12*e^15*z
^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*z^6 + 1023672320*a^15*b^9*c^3*d^6*e^2
1*z^6 + 1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 975175680*a^17*b^6*c^4*d^5*e
^22*z^6 + 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 11072962560*a^18*b*c^8*d^8*
e^19*z^6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 + 9412018176*a^18*b^2*c^7*d
^7*e^20*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z^6 + 805306368*a^19*b^2*c^
6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*z^6 - 133809831936*a^14*b^
6*c^7*d^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*d^19*e^8*z^6 - 2214592512*
a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*d^24*e^3*z^6 + 82216747008
*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*b^7*c^10*d^18*e^9*z^6 - 5866
29120*a^12*b^13*c^2*d^8*e^19*z^6 - 586629120*a^5*b^13*c^9*d^22*e^5*z^6 + 56
8565760*a^7*b^16*c^4*d^15*e^12*z^6 - 4844421120*a^16*b^4*c^7*d^9*e^18*z^6 -
 4844421120*a^10*b^4*c^13*d^21*e^6*z^6 + 531210240*a^11*b^14*c^2*d^9*e^18*z
^6 + 531210240*a^5*b^14*c^8*d^21*e^6*z^6 - 527155200*a^11*b^13*c^3*d^10*e^1
7*z^6 - 527155200*a^6*b^13*c^8*d^20*e^7*z^6 + 43470028800*a^11*b^8*c^8*d^15
*e^12*z^6 - 107874877440*a^11*b^9*c^7*d^14*e^13*z^6 - 107874877440*a^10*b^9
*c^8*d^16*e^11*z^6 + 9018408960*a^12*b^11*c^4*d^10*e^17*z^6 + 9018408960*a^
7*b^11*c^9*d^20*e^7*z^6 + 421994496*a^13*b^12*c^2*d^7*e^20*z^6 + 421994496*
a^5*b^12*c^10*d^23*e^4*z^6 - 66437775360*a^16*b*c^10*d^12*e^15*z^6 - 664377
75360*a^13*b*c^13*d^18*e^9*z^6 + 26159874048*a^16*b^5*c^6*d^8*e^19*z^6 + 26
159874048*a^9*b^5*c^13*d^22*e^5*z^6 - 369098752*a^18*b^3*c^6*d^6*e^21*z^6 -
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 369098752*a^9*b^3*c^15*d^24*e^3*z^6 + 351436800*a^8*b^16*c^3*d^13*e^14*z^6
 + 351436800*a^6*b^16*c^5*d^17*e^10*z^6 - 334233600*a^16*b^8*c^3*d^5*e^22*z
^6 - 334233600*a^6*b^8*c^13*d^25*e^2*z^6 + 301989888*a^19*b^3*c^5*d^4*e^23*
z^6 - 266010624*a^10*b^15*c^2*d^10*e^17*z^6 - 266010624*a^5*b^15*c^7*d^20*e
^7*z^6 - 305198530560*a^12*b^6*c^9*d^15*e^12*z^6 - 203292672*a^14*b^11*c^2*
d^6*e^21*z^6 - 203292672*a^5*b^11*c^11*d^24*e^3*z^6 - 188743680*a^18*b^5*c^
4*d^4*e^23*z^6 + 120418467840*a^16*b^2*c^9*d^11*e^16*z^6 + 120418467840*a^1
2*b^2*c^13*d^19*e^8*z^6 - 17293934592*a^10*b^12*c^5*d^13*e^14*z^6 - 1729393
4592*a^8*b^12*c^7*d^17*e^10*z^6 + 104890368*a^8*b^17*c^2*d^12*e^15*z^6 + 10
4890368*a^5*b^17*c^5*d^18*e^9*z^6 + 4390256640*a^15*b^8*c^4*d^7*e^20*z^6 +
4390256640*a^7*b^8*c^12*d^23*e^4*z^6 - 91750400*a^6*b^18*c^3*d^15*e^12*z^6
+ 79134720*a^7*b^17*c^3*d^14*e^13*z^6 + 79134720*a^6*b^17*c^4*d^16*e^11*z^6
 - 74612736*a^4*b^16*c^7*d^21*e^6*z^6 - 72990720*a^7*b^18*c^2*d^13*e^14*z^6
 - 72990720*a^5*b^18*c^4*d^17*e^10*z^6 + 69746688*a^4*b^15*c^8*d^22*e^5*z^6
 + 63700992*a^15*b^10*c^2*d^5*e^22*z^6 + 63700992*a^5*b^10*c^12*d^25*e^2*z^
6 + 62914560*a^17*b^7*c^3*d^4*e^23*z^6 + 55148544*a^4*b^17*c^6*d^20*e^7*z^6
 - 45957120*a^4*b^14*c^9*d^23*e^4*z^6 - 25952256*a^4*b^18*c^5*d^19*e^8*z^6
- 25165824*a^20*b^2*c^5*d^3*e^24*z^6 + 21086208*a^4*b^13*c^10*d^24*e^3*z^6
+ 20643840*a^6*b^19*c^2*d^14*e^13*z^6 + 20643840*a^5*b^19*c^3*d^16*e^11*z^6
 + 15728640*a^19*b^4*c^4*d^3*e^24*z^6 - 11796480*a^16*b^9*c^2*d^4*e^23*z^6
- 6438912*a^4*b^12*c^11*d^25*e^2*z^6 + 5406720*a^4*b^19*c^4*d^18*e^9*z^6 -
5242880*a^18*b^6*c^3*d^3*e^24*z^6 + 3784704*a^3*b^18*c^6*d^21*e^6*z^6 - 324
4032*a^3*b^19*c^5*d^20*e^7*z^6 - 3244032*a^3*b^17*c^7*d^22*e^5*z^6 + 202752
0*a^3*b^20*c^4*d^19*e^8*z^6 + 2027520*a^3*b^16*c^8*d^23*e^4*z^6 - 1622016*a
^9*b^16*c^2*d^11*e^16*z^6 - 1622016*a^5*b^16*c^6*d^19*e^8*z^6 + 1622016*a^4
*b^20*c^3*d^17*e^10*z^6 - 1523712*a^4*b^21*c^2*d^16*e^11*z^6 + 983040*a^17*
b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^21*c^3*d^18*e^9*z^6 - 901120*a^3*b^15*c
^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^2*d^17*e^10*z^6 + 270336*a^3*b^14*c^10*
d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d^15*e^12*z^6 - 38593888256*a^15*b^6*c^6
*d^9*e^18*z^6 - 38593888256*a^9*b^6*c^12*d^21*e^6*z^6 - 210386288640*a^15*b
^3*c^9*d^12*e^15*z^6 - 210386288640*a^12*b^3*c^12*d^18*e^9*z^6 + 1550214758
4*a^15*c^12*d^15*e^12*z^6 + 1107296256*a^19*c^8*d^7*e^20*z^6 + 1107296256*a
^11*c^16*d^23*e^4*z^6 + 13287555072*a^16*c^11*d^13*e^14*z^6 + 13287555072*a
^14*c^13*d^17*e^10*z^6 + 201326592*a^20*c^7*d^5*e^22*z^6 + 201326592*a^10*c
^17*d^25*e^2*z^6 + 16777216*a^21*c^6*d^3*e^24*z^6 + 3784704*a^9*b^18*d^9*e^
18*z^6 - 3244032*a^10*b^17*d^8*e^19*z^6 - 3244032*a^8*b^19*d^10*e^17*z^6 +
2027520*a^11*b^16*d^7*e^20*z^6 + 2027520*a^7*b^20*d^11*e^16*z^6 - 901120*a^
12*b^15*d^6*e^21*z^6 - 901120*a^6*b^21*d^12*e^15*z^6 + 270336*a^13*b^14*d^5
*e^22*z^6 + 270336*a^5*b^22*d^13*e^14*z^6 - 49152*a^14*b^13*d^4*e^23*z^6 -
49152*a^4*b^23*d^14*e^13*z^6 + 4096*a^15*b^12*d^3*e^24*z^6 + 4096*a^3*b^24*
d^15*e^12*z^6 - 25165824*a^8*b^2*c^17*d^27*z^6 + 15728640*a^7*b^4*c^16*d^27
*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6 + 983040*a^5*b^8*c^14*d^27*z^6 - 98304
*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b^12*c^12*d^27*z^6 + 8304721920*a^17*c^1
0*d^11*e^16*z^6 + 8304721920*a^13*c^14*d^19*e^8*z^6 + 3690987520*a^18*c^9*d
^9*e^18*z^6 + 3690987520*a^12*c^15*d^21*e^6*z^6 + 16777216*a^9*c^18*d^27*z^
```

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6 - 8493371392*a^6*b^8*c^9*d^14*e^9*z^4 + 1458044928*a^8*b*c^14*d^17*e^6*z^
4 - 12604538880*a^11*b^4*c^8*d^8*e^15*z^4 - 8303067136*a^9*b^5*c^9*d^11*e^1
2*z^4 - 5588058112*a^13*b*c^9*d^7*e^16*z^4 - 3892838400*a^8*b^2*c^13*d^16*e
^7*z^4 - 3611713536*a^8*b^8*c^7*d^10*e^13*z^4 + 7819006464*a^7*b^9*c^7*d^11
*e^12*z^4 - 7782137856*a^8*b^7*c^8*d^11*e^12*z^4 + 7780433920*a^12*b^2*c^9*
d^8*e^15*z^4 - 12020465664*a^7*b^5*c^11*d^15*e^8*z^4 + 3176792064*a^8*b^3*c
^12*d^15*e^8*z^4 - 322633728*a^15*b*c^7*d^3*e^20*z^4 + 210829312*a^7*b*c^15
*d^19*e^4*z^4 + 15623258112*a^9*b^6*c^8*d^10*e^13*z^4 + 25165824*a^15*b^3*c
^5*d*e^22*z^4 - 15728640*a^14*b^5*c^4*d*e^22*z^4 + 12582912*a^5*b^2*c^16*d^
22*e*z^4 - 11730944*a^4*b^4*c^15*d^22*e*z^4 + 5242880*a^13*b^7*c^3*d*e^22*z
^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4 + 4521984*a^3*b^6*c^14*d^22*e*z^4 + 44
60544*a*b^14*c^8*d^18*e^5*z^4 + 3538944*a^6*b*c^16*d^21*e^2*z^4 + 3108864*a
*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b^13*c^9*d^19*e^4*z^4 - 2345472*a^5*b^17
*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c*d^4*e^19*z^4 + 1824768*a^6*b^16*c*d^6*
e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^20*z^4 + 1419264*a*b^12*c^10*d^20*e^3*z
^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4 - 983040*a^12*b^9*c^2*d*e^22*z^4 + 964
608*a^4*b^18*c*d^8*e^15*z^4 - 866304*a^2*b^8*c^13*d^22*e*z^4 + 703488*a^7*b
^15*c*d^5*e^18*z^4 - 608256*a^10*b^12*c*d^2*e^21*z^4 - 440832*a*b^11*c^11*d
^21*e^2*z^4 + 275968*a*b^19*c^3*d^13*e^10*z^4 - 159744*a^2*b^20*c*d^10*e^13
*z^4 - 153600*a*b^20*c^2*d^12*e^11*z^4 + 64512*a^3*b^19*c*d^9*e^14*z^4 + 19
746062336*a^8*b^6*c^9*d^12*e^11*z^4 - 15333588992*a^10*b^4*c^9*d^10*e^13*z^
4 + 6702170112*a^7*b^4*c^12*d^16*e^7*z^4 + 15167913984*a^10*b^3*c^10*d^11*e
^12*z^4 - 2256638976*a^5*b^11*c^7*d^13*e^10*z^4 + 2254307328*a^5*b^7*c^11*d
^17*e^6*z^4 - 2200633344*a^6*b^5*c^12*d^17*e^6*z^4 + 6457131008*a^11*b^3*c^
9*d^9*e^14*z^4 - 2128785408*a^5*b^8*c^10*d^16*e^7*z^4 - 2126057472*a^6*b^11
*c^6*d^11*e^12*z^4 + 2038349824*a^12*b^5*c^6*d^5*e^18*z^4 + 2037841920*a^5*
b^10*c^8*d^14*e^9*z^4 + 3615621120*a^9*b*c^13*d^15*e^8*z^4 + 1900019712*a^1
1*b^2*c^10*d^10*e^13*z^4 + 1867698432*a^9*b^9*c^5*d^7*e^16*z^4 - 6157369344
*a^9*b^4*c^10*d^12*e^11*z^4 - 1856913408*a^7*b^10*c^6*d^10*e^13*z^4 + 17891
32800*a^6*b^4*c^13*d^18*e^5*z^4 + 6082658304*a^8*b^4*c^11*d^14*e^9*z^4 + 60
29549568*a^11*b^5*c^7*d^7*e^16*z^4 + 6010159104*a^6*b^7*c^10*d^15*e^8*z^4 +
 1703182336*a^7*b^7*c^9*d^13*e^10*z^4 + 1658388480*a^11*b^6*c^6*d^6*e^17*z^
4 + 5917114368*a^10*b^6*c^7*d^8*e^15*z^4 - 1591197696*a^11*b^7*c^5*d^5*e^18
*z^4 - 1526464512*a^8*b^10*c^5*d^8*e^15*z^4 - 5772607488*a^12*b^4*c^7*d^6*e
^17*z^4 - 1423507456*a^13*b^4*c^6*d^4*e^19*z^4 - 1387266048*a^7*b^3*c^13*d^
17*e^6*z^4 + 2976120832*a^10*b*c^12*d^13*e^10*z^4 - 9906946048*a^9*b^2*c^12
*d^14*e^9*z^4 - 18421874688*a^8*b^5*c^10*d^13*e^10*z^4 + 1141217280*a^6*b^1
2*c^5*d^10*e^13*z^4 - 9714364416*a^7*b^8*c^8*d^12*e^11*z^4 - 16777216*a^16*
b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c*d*e^22*z^4 + 81920*a*b^10*c^12*d^22*e*
z^4 + 39168*a*b^21*c*d^11*e^12*z^4 - 1091829760*a^5*b^6*c^12*d^18*e^5*z^4 +
 1046740992*a^14*b^2*c^7*d^4*e^19*z^4 - 6884425728*a^12*b*c^10*d^9*e^14*z^4
 + 987445248*a^4*b^10*c^9*d^16*e^7*z^4 + 984087552*a^5*b^12*c^6*d^12*e^11*z
^4 - 9564585984*a^9*b^7*c^7*d^9*e^14*z^4 - 5266857984*a^10*b^7*c^6*d^7*e^16
*z^4 - 892145664*a^7*b^11*c^5*d^9*e^14*z^4 - 2444623872*a^11*b*c^11*d^11*e^
12*z^4 + 768540672*a^12*b^3*c^8*d^7*e^16*z^4 + 5048322048*a^8*b^9*c^6*d^9*e
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^14*z^4 + 5047612416*a^6*b^9*c^8*d^13*e^10*z^4 - 732492288*a^4*b^11*c^8*d^1
5*e^8*z^4 + 9266921472*a^7*b^6*c^10*d^14*e^9*z^4 - 645857280*a^6*b^6*c^11*d
^16*e^7*z^4 - 623867904*a^4*b^9*c^10*d^17*e^6*z^4 - 622067712*a^6*b^3*c^14*
d^19*e^4*z^4 + 582617088*a^10*b^8*c^5*d^6*e^17*z^4 + 577119744*a^7*b^12*c^4
*d^8*e^15*z^4 + 552566784*a^12*b^6*c^5*d^4*e^19*z^4 + 549224448*a^9*b^8*c^6
*d^8*e^15*z^4 - 526565376*a^9*b^10*c^4*d^6*e^17*z^4 + 511520256*a^10*b^9*c^
4*d^5*e^18*z^4 + 13393723392*a^9*b^3*c^11*d^13*e^10*z^4 - 2066350080*a^14*b
*c^8*d^5*e^18*z^4 + 4718592000*a^13*b^2*c^8*d^6*e^17*z^4 - 314572800*a^7*b^
2*c^14*d^18*e^5*z^4 + 287250432*a^4*b^13*c^6*d^13*e^10*z^4 + 4565827584*a^1
0*b^5*c^8*d^9*e^14*z^4 - 250785792*a^4*b^14*c^5*d^12*e^11*z^4 + 235536384*a
^13*b^3*c^7*d^5*e^18*z^4 - 232683264*a^8*b^11*c^4*d^7*e^16*z^4 - 199627776*
a^5*b^14*c^4*d^10*e^13*z^4 - 190267392*a^12*b^7*c^4*d^3*e^20*z^4 + 18489139
2*a^6*b^10*c^7*d^12*e^11*z^4 + 180502528*a^4*b^7*c^12*d^19*e^4*z^4 + 178877
952*a^3*b^13*c^7*d^15*e^8*z^4 + 172490752*a^14*b^3*c^6*d^3*e^20*z^4 + 16394
6496*a^13*b^5*c^5*d^3*e^20*z^4 + 155839488*a^8*b^12*c^3*d^6*e^17*z^4 + 1550
00832*a^5*b^5*c^13*d^19*e^4*z^4 - 152076288*a^4*b^6*c^13*d^20*e^3*z^4 - 137
592576*a^3*b^12*c^8*d^16*e^7*z^4 - 133693440*a^14*b^4*c^5*d^2*e^21*z^4 - 11
6767488*a^3*b^9*c^11*d^19*e^4*z^4 - 108985344*a^3*b^14*c^6*d^14*e^9*z^4 - 1
06223616*a^6*b^13*c^4*d^9*e^14*z^4 + 106119168*a^3*b^10*c^10*d^18*e^5*z^4 +
 102432768*a^5*b^4*c^14*d^20*e^3*z^4 + 102113280*a^4*b^12*c^7*d^14*e^9*z^4
+ 100674048*a^5*b^9*c^9*d^15*e^8*z^4 + 90439680*a^13*b^6*c^4*d^2*e^21*z^4 -
 86808576*a^6*b^14*c^3*d^8*e^15*z^4 + 86245376*a^6*b^2*c^15*d^20*e^3*z^4 +
79011840*a^4*b^8*c^11*d^18*e^5*z^4 + 78345216*a^4*b^15*c^4*d^11*e^12*z^4 +
78006528*a^11*b^9*c^3*d^3*e^20*z^4 - 73253376*a^9*b^11*c^3*d^5*e^18*z^4 + 6
7524608*a^3*b^8*c^12*d^20*e^3*z^4 + 67108864*a^15*b^2*c^6*d^2*e^21*z^4 - 61
590528*a^10*b^10*c^3*d^4*e^19*z^4 + 61559808*a^5*b^15*c^3*d^9*e^14*z^4 - 59
637760*a^5*b^3*c^15*d^21*e^2*z^4 + 58638336*a^4*b^5*c^14*d^21*e^2*z^4 - 408
28416*a^7*b^13*c^3*d^7*e^16*z^4 - 35639296*a^2*b^12*c^9*d^18*e^5*z^4 - 3129
3440*a^12*b^8*c^3*d^2*e^21*z^4 + 29933568*a^5*b^13*c^5*d^11*e^12*z^4 + 2779
3920*a^2*b^11*c^10*d^19*e^4*z^4 + 27168768*a^2*b^13*c^8*d^17*e^6*z^4 - 2360
2176*a^7*b^14*c^2*d^6*e^17*z^4 - 23248896*a^3*b^7*c^13*d^21*e^2*z^4 + 20929
536*a^3*b^15*c^5*d^13*e^10*z^4 + 18428928*a^9*b^12*c^2*d^4*e^19*z^4 + 18026
496*a^6*b^15*c^2*d^7*e^16*z^4 - 16261632*a^10*b^11*c^2*d^3*e^20*z^4 + 15128
064*a^3*b^16*c^4*d^12*e^11*z^4 - 14060544*a^2*b^10*c^11*d^20*e^3*z^4 + 1317
8880*a^2*b^16*c^5*d^14*e^9*z^4 - 11244288*a^3*b^17*c^3*d^11*e^12*z^4 - 1050
9312*a^2*b^15*c^6*d^15*e^8*z^4 - 7262208*a^4*b^17*c^2*d^9*e^14*z^4 - 704563
2*a^2*b^17*c^4*d^13*e^10*z^4 - 6285312*a^2*b^14*c^7*d^16*e^7*z^4 + 5996544*
a^11*b^10*c^2*d^2*e^21*z^4 + 4558336*a^2*b^9*c^12*d^21*e^2*z^4 + 4478976*a^
11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^4*b^16*c^3*d^10*e^13*z^4 + 2629632*a^3*
b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*b^18*c^2*d^10*e^13*z^4 + 1627136*a^2*b^
18*c^3*d^12*e^11*z^4 + 1605120*a^8*b^13*c^2*d^5*e^18*z^4 + 1483776*a^5*b^16
*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*c^2*d^11*e^12*z^4 - 8542224384*a^10*b^2
*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^12*e^11*z^4 - 3072*b^12*c^11*d^22*e*z^4
 - 1572864*a^6*c^17*d^22*e*z^4 - 4096*a^10*b^13*d*e^22*z^4 - 4096*a*b^22*d^
10*e^13*z^4 - 6144*a^12*b^10*c*e^23*z^4 - 983040*a^5*b*c^17*d^23*z^4 - 6912
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*a*b^9*c^13*d^23*z^4 + 1824522240*a^13*c^10*d^8*e^15*z^4 + 1730150400*a^12*
c^11*d^10*e^13*z^4 + 958922752*a^14*c^9*d^6*e^17*z^4 - 537919488*a^9*c^14*d
^16*e^7*z^4 + 508559360*a^11*c^12*d^12*e^11*z^4 - 500170752*a^10*c^13*d^14*
e^9*z^4 + 246939648*a^15*c^8*d^4*e^19*z^4 - 199229440*a^8*c^15*d^18*e^5*z^4
 - 29884416*a^7*c^16*d^20*e^3*z^4 + 25165824*a^16*c^7*d^2*e^21*z^4 + 236544
*b^17*c^6*d^17*e^6*z^4 - 202752*b^18*c^5*d^16*e^7*z^4 - 202752*b^16*c^7*d^1
8*e^5*z^4 + 126720*b^19*c^4*d^15*e^8*z^4 + 126720*b^15*c^8*d^19*e^4*z^4 - 5
6320*b^20*c^3*d^14*e^9*z^4 - 56320*b^14*c^9*d^20*e^3*z^4 + 16896*b^21*c^2*d
^13*e^10*z^4 + 16896*b^13*c^10*d^21*e^2*z^4 + 110080*a^7*b^16*d^4*e^19*z^4
 + 110080*a^4*b^19*d^7*e^16*z^4 - 75520*a^8*b^15*d^3*e^20*z^4 - 75520*a^3*b^
20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5*e^18*z^4 - 56320*a^5*b^18*d^6*e^17*z^4
 + 25600*a^9*b^14*d^2*e^21*z^4 + 25600*a^2*b^21*d^9*e^14*z^4 - 1572864*a^16
*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*c^4*e^23*z^4 - 327680*a^14*b^6*c^3*e^23
*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 + 983040*a^4*b^3*c^16*d^23*z^4 - 385024*
a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^7*c^14*d^23*z^4 + 256*b^23*d^11*e^12*z^
4 + 1048576*a^17*c^6*e^23*z^4 + 256*b^11*c^12*d^23*z^4 + 256*a^11*b^12*e^23
*z^4 + 948695040*a^8*b*c^10*d^6*e^13*z^2 + 348917760*a^7*b*c^11*d^8*e^11*z^
2 - 125030400*a^9*b*c^9*d^4*e^15*z^2 - 50728960*a^6*b*c^12*d^10*e^9*z^2 - 4
4298240*a^5*b*c^13*d^12*e^7*z^2 - 36495360*a^10*b*c^8*d^2*e^17*z^2 + 296755
20*a^8*b^6*c^5*d*e^18*z^2 - 24170496*a^9*b^4*c^6*d*e^18*z^2 - 17202816*a^7*
b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*c^14*d^14*e^5*z^2 + 5532416*a^6*b^10*c^
3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*d*e^18*z^2 - 2662400*a^3*b*c^15*d^16*e^
3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*z^2 - 1136160*a*b^13*c^5*d^8*e^11*z^2 -
 1017600*a^5*b^12*c^2*d*e^18*z^2 - 744768*a*b^11*c^7*d^10*e^9*z^2 + 607872*
a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b^6*c^12*d^15*e^4*z^2 + 408576*a*b^5*c^1
3*d^16*e^3*z^2 + 361152*a*b^10*c^8*d^11*e^8*z^2 - 287408*a*b^9*c^9*d^12*e^7
*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^2 - 203904*a*b^4*c^14*d^17*e^2*z^2 + 20
0832*a*b^8*c^10*d^13*e^6*z^2 + 126720*a*b^7*c^11*d^14*e^5*z^2 - 123968*a*b^
15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c^2*d^5*e^14*z^2 + 11904*a^2*b^16*c*d^3*
e^16*z^2 + 1824135552*a^7*b^4*c^8*d^5*e^14*z^2 - 1457252352*a^8*b^2*c^9*d^5
*e^14*z^2 - 1405209600*a^7*b^5*c^7*d^4*e^15*z^2 - 184320*a^2*b*c^16*d^18*e*
z^2 + 100608*a^4*b^14*c*d*e^18*z^2 + 53248*a*b^3*c^15*d^18*e*z^2 + 26448*a*
b^17*c*d^4*e^15*z^2 + 1067599872*a^8*b^3*c^8*d^4*e^15*z^2 - 930828288*a^7*b
^3*c^9*d^6*e^13*z^2 + 920760000*a^6*b^4*c^9*d^7*e^12*z^2 - 806639616*a^6*b^
3*c^10*d^8*e^11*z^2 - 791052480*a^6*b^6*c^7*d^5*e^14*z^2 + 772237824*a^6*b^
7*c^6*d^4*e^15*z^2 - 701025408*a^5*b^6*c^8*d^7*e^12*z^2 + 443340288*a^5*b^5
*c^9*d^8*e^11*z^2 + 433047552*a^7*b^6*c^6*d^3*e^16*z^2 + 405741312*a^5*b^7*
c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^10*z^2 - 276962688*a^6*b^8*
c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16*z^2 + 213564384*a^4*b^8*c
^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15*z^2 - 182520896*a^4*b^9*c^
6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9*z^2 - 152151552*a^7*b^2*c^
10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8*z^2 + 108085248*a^9*b^3*c
^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9*z^2 - 98323200*a^6*b^5*c^
8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^2 + 89464512*a^5*b^10*c^4*
d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2 + 68315904*a^5*b^8*c^6*d^5
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*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 62951040*a^9*b^2*c^8*d^3*e^
16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 47614464*a^3*b^8*c^8*d^9*e^10
*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983040*a^3*b^11*c^5*d^6*e^13*
z^2 - 33515520*a^4*b^3*c^12*d^12*e^7*z^2 - 33463808*a^3*b^7*c^9*d^10*e^9*z^
2 - 25128864*a^4*b^4*c^11*d^11*e^8*z^2 - 23193728*a^3*b^10*c^6*d^7*e^12*z^2
 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2 + 19924176*a^4*b^11*c^4*d^4*e^15*z^2 -
 19251216*a^3*b^9*c^7*d^8*e^11*z^2 - 16434048*a^5*b^4*c^10*d^9*e^10*z^2 - 1
6289664*a^3*b^12*c^4*d^5*e^14*z^2 - 15059328*a^4*b^12*c^3*d^3*e^16*z^2 - 10
766016*a^2*b^10*c^7*d^9*e^10*z^2 - 10453632*a^5*b^11*c^3*d^2*e^17*z^2 - 994
0992*a^3*b^3*c^13*d^14*e^5*z^2 + 8373696*a^2*b^11*c^6*d^8*e^11*z^2 + 777676
8*a^3*b^2*c^14*d^15*e^4*z^2 + 7077888*a^3*b^5*c^11*d^12*e^7*z^2 + 6798240*a
^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^2*b^6*c^11*d^13*e^6*z^2 + 3544320*a^3*b
^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b^5*c^12*d^14*e^5*z^2 + 2346336*a^4*b^13
*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*c^9*d^11*e^8*z^2 - 2125824*a^2*b^13*c^4
*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12*d^13*e^6*z^2 + 1927680*a^2*b^7*c^10*d^
12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^5*e^14*z^2 - 1807104*a^2*b^12*c^5*d^7*e
^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e^15*z^2 + 1083456*a^3*b^14*c^2*d^3*e^16
*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*z^2 - 657408*a^2*b^3*c^14*d^16*e^3*z^2
+ 608256*a^7*b^7*c^5*d^2*e^17*z^2 + 595968*a^2*b^2*c^15*d^17*e^2*z^2 - 4986
24*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*b^18*c*d^5*e^14*z^2 - 3840*b^5*c^14*d^1
8*e*z^2 + 2064384*a^11*c^8*d*e^18*z^2 - 4160*a^3*b^16*d*e^18*z^2 - 4160*a*b
^18*d^3*e^16*z^2 - 1290240*a^11*b*c^7*e^19*z^2 - 9840*a^5*b^13*c*e^19*z^2 -
 5760*a*b^2*c^16*d^19*z^2 - 280581120*a^8*c^11*d^7*e^12*z^2 + 110278656*a^9
*c^10*d^5*e^14*z^2 - 89479168*a^7*c^12*d^9*e^10*z^2 + 34464000*a^10*c^9*d^3
*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*z^2 + 54240*b^8*c^11*d^15*e^4*z^2 - 499
20*b^14*c^5*d^9*e^10*z^2 - 49920*b^9*c^10*d^14*e^5*z^2 - 37376*b^16*c^3*d^7
*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*z^2 + 28480*b^13*c^6*d^10*e^9*z^2 + 284
80*b^10*c^9*d^13*e^6*z^2 + 15936*b^17*c^2*d^6*e^13*z^2 + 15936*b^6*c^13*d^1
7*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z^2 - 7920*b^11*c^8*d^12*e^7*z^2 + 74895
36*a^5*c^14*d^13*e^6*z^2 + 6084096*a^6*c^13*d^11*e^8*z^2 + 2280448*a^4*c^15
*d^15*e^4*z^2 + 350208*a^3*c^16*d^17*e^2*z^2 + 11616*a^2*b^17*d^2*e^17*z^2
- 3515904*a^9*b^5*c^5*e^19*z^2 + 3440640*a^10*b^3*c^6*e^19*z^2 + 1870848*a^
8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*c^3*e^19*z^2 + 101856*a^6*b^11*c^2*e^19
*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b^4*c^15*d^19*z^2 + 20736*a^2*c^17*d^19*
z^2 + 400*a^4*b^15*e^19*z^2 - 3969216*a^4*b*c^10*d^3*e^12 - 3001536*a^3*b*c
^11*d^5*e^10 - 419904*a^2*b*c^12*d^7*e^8 + 184608*a^4*b^3*c^8*d*e^14 - 1530
36*a*b^4*c^10*d^6*e^9 + 127008*a*b^3*c^11*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^1
1 - 29160*a*b^2*c^12*d^8*e^7 - 21060*a^3*b^5*c^7*d*e^14 - 21060*a*b^7*c^7*d
^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 - 404544*a^5*b*c^9*d*e^14 + 1251872*a^3*b
^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9*d^2*e^13 + 820512*a^2*b^3*c^10*d^5*e^1
0 + 750672*a^3*b^2*c^10*d^4*e^11 - 657498*a^2*b^4*c^9*d^4*e^11 - 487116*a^3
*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c^11*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^1
3 + 13140*a^2*b^5*c^8*d^3*e^12 + 15286*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e
^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b^8*c^7*d^4*e^11 + 2025*b^4*c^11*d^8*e^7
 + 3367008*a^4*c^11*d^4*e^11 + 1166400*a^3*c^12*d^6*e^9 + 705600*a^5*c^10*d
```

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^2*e^13 + 104976*a^2*c^13*d^8*e^7 - 17640*a^5*b^2*c^8*e^15 + 2025*a^4*b^4*c
^7*e^15 + 38416*a^6*c^9*e^15, z, k)*(root(128723189760*a^14*b^4*c^9*d^13*e^
14*z^6 + 128723189760*a^12*b^4*c^11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^
4*d^11*e^16*z^6 - 8432455680*a^7*b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b
^11*c^5*d^12*e^15*z^6 + 12673351680*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960
*a^12*b^9*c^6*d^12*e^15*z^6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z^6 - 210483
44576*a^9*b^12*c^6*d^15*e^12*z^6 - 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 -
16609443840*a^10*b^3*c^14*d^22*e^5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^1
3*z^6 + 145332633600*a^12*b^5*c^10*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^
8*d^12*e^15*z^6 + 123740356608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17
*b^5*c^5*d^6*e^21*z^6 + 3460300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a
^15*b^7*c^5*d^8*e^19*z^6 - 7751073792*a^8*b^7*c^12*d^22*e^5*z^6 + 120418467
84*a^14*b^7*c^6*d^10*e^17*z^6 + 12041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325
545099264*a^14*b^3*c^10*d^14*e^13*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^1
1*z^6 - 3330539520*a^13*b^10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^
21*e^6*z^6 + 157789716480*a^12*b^7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^
7*c^9*d^16*e^11*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168
*a^9*b^10*c^8*d^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^6 - 726663168
0*a^17*b^4*c^6*d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326
592*a^20*b*c^6*d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^6 + 457473392
64*a^13*b^8*c^6*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 746
12736*a^10*b^16*c*d^9*e^18*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 276
8240640*a^7*b^7*c^13*d^24*e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^19*z^6 + 629
14560*a^6*b^7*c^14*d^26*e*z^6 + 2752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 27
52020480*a^7*b^13*c^7*d^18*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45
957120*a^12*b^14*c*d^7*e^20*z^6 - 2724986880*a^14*b^9*c^4*d^8*e^19*z^6 - 27
24986880*a^7*b^9*c^11*d^22*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21
086208*a^13*b^13*c*d^6*e^21*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z^6 - 643891
2*a^14*b^12*c*d^5*e^22*z^6 + 5406720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6
*b^20*c*d^13*e^14*z^6 - 1523712*a^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^1
1*c*d^4*e^23*z^6 + 1179648*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^1
5*e^12*z^6 - 98304*a^16*b^10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^
6 - 49152*a^3*b^13*c^11*d^26*e*z^6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6
+ 6897106944*a^8*b^13*c^6*d^16*e^11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*
z^6 - 2422210560*a^8*b^6*c^13*d^23*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^1
5*e^12*z^6 + 41004564480*a^15*b^4*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*
c^12*d^19*e^8*z^6 + 2270822400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*
b^11*c^10*d^22*e^5*z^6 + 23677108224*a^14*b^8*c^5*d^9*e^18*z^6 + 2367710822
4*a^8*b^8*c^11*d^21*e^6*z^6 + 212600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 21
2600881152*a^13*b^2*c^12*d^17*e^10*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17
*z^6 + 75157733376*a^10*b^5*c^12*d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d
^13*e^14*z^6 - 251217838080*a^11*b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b
^10*c^3*d^7*e^20*z^6 - 1952907264*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*
a^13*b^9*c^5*d^10*e^17*z^6 - 27691057152*a^8*b^9*c^10*d^20*e^7*z^6 - 190267
3920*a^8*b^15*c^4*d^14*e^13*z^6 - 1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 1
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0465050624*a^10*b^11*c^6*d^14*e^13*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11
*z^6 + 1613905920*a^9*b^14*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17
*e^10*z^6 - 33218887680*a^17*b*c^9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*
d^20*e^7*z^6 + 1524695040*a^10*b^14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14
*c^7*d^19*e^8*z^6 - 1472200704*a^18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b
^4*c^15*d^25*e^2*z^6 - 83047219200*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200
*a^11*b^3*c^13*d^20*e^7*z^6 + 44291850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291
850240*a^11*b^2*c^14*d^21*e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 -
 201326592*a^9*b*c^17*d^26*e*z^6 + 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 +
 48530718720*a^10*b^8*c^9*d^17*e^10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18
*z^6 - 1242644480*a^6*b^12*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11
*e^16*z^6 + 9813196800*a^8*b^10*c^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*
d^14*e^13*z^6 - 93012885504*a^14*b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b
^4*c^10*d^15*e^12*z^6 + 52730658816*a^10*b^10*c^7*d^15*e^12*z^6 - 118010675
2*a^9*b^15*c^3*d^12*e^15*z^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*z^6 + 10236
72320*a^15*b^9*c^3*d^6*e^21*z^6 + 1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 97
5175680*a^17*b^6*c^4*d^5*e^22*z^6 + 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 1
1072962560*a^18*b*c^8*d^8*e^19*z^6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 +
 9412018176*a^18*b^2*c^7*d^7*e^20*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z
^6 + 805306368*a^19*b^2*c^6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*
z^6 - 133809831936*a^14*b^6*c^7*d^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*
d^19*e^8*z^6 - 2214592512*a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*
d^24*e^3*z^6 + 82216747008*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*b^
7*c^10*d^18*e^9*z^6 - 586629120*a^12*b^13*c^2*d^8*e^19*z^6 - 586629120*a^5*
b^13*c^9*d^22*e^5*z^6 + 568565760*a^7*b^16*c^4*d^15*e^12*z^6 - 4844421120*a
^16*b^4*c^7*d^9*e^18*z^6 - 4844421120*a^10*b^4*c^13*d^21*e^6*z^6 + 53121024
0*a^11*b^14*c^2*d^9*e^18*z^6 + 531210240*a^5*b^14*c^8*d^21*e^6*z^6 - 527155
200*a^11*b^13*c^3*d^10*e^17*z^6 - 527155200*a^6*b^13*c^8*d^20*e^7*z^6 + 434
70028800*a^11*b^8*c^8*d^15*e^12*z^6 - 107874877440*a^11*b^9*c^7*d^14*e^13*z
^6 - 107874877440*a^10*b^9*c^8*d^16*e^11*z^6 + 9018408960*a^12*b^11*c^4*d^1
0*e^17*z^6 + 9018408960*a^7*b^11*c^9*d^20*e^7*z^6 + 421994496*a^13*b^12*c^2
*d^7*e^20*z^6 + 421994496*a^5*b^12*c^10*d^23*e^4*z^6 - 66437775360*a^16*b*c
^10*d^12*e^15*z^6 - 66437775360*a^13*b*c^13*d^18*e^9*z^6 + 26159874048*a^16
*b^5*c^6*d^8*e^19*z^6 + 26159874048*a^9*b^5*c^13*d^22*e^5*z^6 - 369098752*a
^18*b^3*c^6*d^6*e^21*z^6 - 369098752*a^9*b^3*c^15*d^24*e^3*z^6 + 351436800*
a^8*b^16*c^3*d^13*e^14*z^6 + 351436800*a^6*b^16*c^5*d^17*e^10*z^6 - 3342336
00*a^16*b^8*c^3*d^5*e^22*z^6 - 334233600*a^6*b^8*c^13*d^25*e^2*z^6 + 301989
888*a^19*b^3*c^5*d^4*e^23*z^6 - 266010624*a^10*b^15*c^2*d^10*e^17*z^6 - 266
010624*a^5*b^15*c^7*d^20*e^7*z^6 - 305198530560*a^12*b^6*c^9*d^15*e^12*z^6
- 203292672*a^14*b^11*c^2*d^6*e^21*z^6 - 203292672*a^5*b^11*c^11*d^24*e^3*z
^6 - 188743680*a^18*b^5*c^4*d^4*e^23*z^6 + 120418467840*a^16*b^2*c^9*d^11*e
^16*z^6 + 120418467840*a^12*b^2*c^13*d^19*e^8*z^6 - 17293934592*a^10*b^12*c
^5*d^13*e^14*z^6 - 17293934592*a^8*b^12*c^7*d^17*e^10*z^6 + 104890368*a^8*b
^17*c^2*d^12*e^15*z^6 + 104890368*a^5*b^17*c^5*d^18*e^9*z^6 + 4390256640*a^
15*b^8*c^4*d^7*e^20*z^6 + 4390256640*a^7*b^8*c^12*d^23*e^4*z^6 - 91750400*a
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^6*b^18*c^3*d^15*e^12*z^6 + 79134720*a^7*b^17*c^3*d^14*e^13*z^6 + 79134720*
a^6*b^17*c^4*d^16*e^11*z^6 - 74612736*a^4*b^16*c^7*d^21*e^6*z^6 - 72990720*
a^7*b^18*c^2*d^13*e^14*z^6 - 72990720*a^5*b^18*c^4*d^17*e^10*z^6 + 69746688
*a^4*b^15*c^8*d^22*e^5*z^6 + 63700992*a^15*b^10*c^2*d^5*e^22*z^6 + 63700992
*a^5*b^10*c^12*d^25*e^2*z^6 + 62914560*a^17*b^7*c^3*d^4*e^23*z^6 + 55148544
*a^4*b^17*c^6*d^20*e^7*z^6 - 45957120*a^4*b^14*c^9*d^23*e^4*z^6 - 25952256*
a^4*b^18*c^5*d^19*e^8*z^6 - 25165824*a^20*b^2*c^5*d^3*e^24*z^6 + 21086208*a
^4*b^13*c^10*d^24*e^3*z^6 + 20643840*a^6*b^19*c^2*d^14*e^13*z^6 + 20643840*
a^5*b^19*c^3*d^16*e^11*z^6 + 15728640*a^19*b^4*c^4*d^3*e^24*z^6 - 11796480*
a^16*b^9*c^2*d^4*e^23*z^6 - 6438912*a^4*b^12*c^11*d^25*e^2*z^6 + 5406720*a^
4*b^19*c^4*d^18*e^9*z^6 - 5242880*a^18*b^6*c^3*d^3*e^24*z^6 + 3784704*a^3*b
^18*c^6*d^21*e^6*z^6 - 3244032*a^3*b^19*c^5*d^20*e^7*z^6 - 3244032*a^3*b^17
*c^7*d^22*e^5*z^6 + 2027520*a^3*b^20*c^4*d^19*e^8*z^6 + 2027520*a^3*b^16*c^
8*d^23*e^4*z^6 - 1622016*a^9*b^16*c^2*d^11*e^16*z^6 - 1622016*a^5*b^16*c^6*
d^19*e^8*z^6 + 1622016*a^4*b^20*c^3*d^17*e^10*z^6 - 1523712*a^4*b^21*c^2*d^
16*e^11*z^6 + 983040*a^17*b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^21*c^3*d^18*e
^9*z^6 - 901120*a^3*b^15*c^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^2*d^17*e^10*z
^6 + 270336*a^3*b^14*c^10*d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d^15*e^12*z^6
- 38593888256*a^15*b^6*c^6*d^9*e^18*z^6 - 38593888256*a^9*b^6*c^12*d^21*e^6
*z^6 - 210386288640*a^15*b^3*c^9*d^12*e^15*z^6 - 210386288640*a^12*b^3*c^12
*d^18*e^9*z^6 + 15502147584*a^15*c^12*d^15*e^12*z^6 + 1107296256*a^19*c^8*d
^7*e^20*z^6 + 1107296256*a^11*c^16*d^23*e^4*z^6 + 13287555072*a^16*c^11*d^1
3*e^14*z^6 + 13287555072*a^14*c^13*d^17*e^10*z^6 + 201326592*a^20*c^7*d^5*e
^22*z^6 + 201326592*a^10*c^17*d^25*e^2*z^6 + 16777216*a^21*c^6*d^3*e^24*z^6
 + 3784704*a^9*b^18*d^9*e^18*z^6 - 3244032*a^10*b^17*d^8*e^19*z^6 - 3244032
*a^8*b^19*d^10*e^17*z^6 + 2027520*a^11*b^16*d^7*e^20*z^6 + 2027520*a^7*b^20
*d^11*e^16*z^6 - 901120*a^12*b^15*d^6*e^21*z^6 - 901120*a^6*b^21*d^12*e^15*
z^6 + 270336*a^13*b^14*d^5*e^22*z^6 + 270336*a^5*b^22*d^13*e^14*z^6 - 49152
*a^14*b^13*d^4*e^23*z^6 - 49152*a^4*b^23*d^14*e^13*z^6 + 4096*a^15*b^12*d^3
*e^24*z^6 + 4096*a^3*b^24*d^15*e^12*z^6 - 25165824*a^8*b^2*c^17*d^27*z^6 +
15728640*a^7*b^4*c^16*d^27*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6 + 983040*a^5
*b^8*c^14*d^27*z^6 - 98304*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b^12*c^12*d^27
*z^6 + 8304721920*a^17*c^10*d^11*e^16*z^6 + 8304721920*a^13*c^14*d^19*e^8*z
^6 + 3690987520*a^18*c^9*d^9*e^18*z^6 + 3690987520*a^12*c^15*d^21*e^6*z^6 +
 16777216*a^9*c^18*d^27*z^6 - 8493371392*a^6*b^8*c^9*d^14*e^9*z^4 + 1458044
928*a^8*b*c^14*d^17*e^6*z^4 - 12604538880*a^11*b^4*c^8*d^8*e^15*z^4 - 83030
67136*a^9*b^5*c^9*d^11*e^12*z^4 - 5588058112*a^13*b*c^9*d^7*e^16*z^4 - 3892
838400*a^8*b^2*c^13*d^16*e^7*z^4 - 3611713536*a^8*b^8*c^7*d^10*e^13*z^4 + 7
819006464*a^7*b^9*c^7*d^11*e^12*z^4 - 7782137856*a^8*b^7*c^8*d^11*e^12*z^4
+ 7780433920*a^12*b^2*c^9*d^8*e^15*z^4 - 12020465664*a^7*b^5*c^11*d^15*e^8*
z^4 + 3176792064*a^8*b^3*c^12*d^15*e^8*z^4 - 322633728*a^15*b*c^7*d^3*e^20*
z^4 + 210829312*a^7*b*c^15*d^19*e^4*z^4 + 15623258112*a^9*b^6*c^8*d^10*e^13
*z^4 + 25165824*a^15*b^3*c^5*d*e^22*z^4 - 15728640*a^14*b^5*c^4*d*e^22*z^4
+ 12582912*a^5*b^2*c^16*d^22*e*z^4 - 11730944*a^4*b^4*c^15*d^22*e*z^4 + 524
2880*a^13*b^7*c^3*d*e^22*z^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4 + 4521984*a^
```

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3*b^6*c^14*d^22*e*z^4 + 4460544*a*b^14*c^8*d^18*e^5*z^4 + 3538944*a^6*b*c^1
6*d^21*e^2*z^4 + 3108864*a*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b^13*c^9*d^19*
e^4*z^4 - 2345472*a^5*b^17*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c*d^4*e^19*z^4
 + 1824768*a^6*b^16*c*d^6*e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^20*z^4 + 1419
264*a*b^12*c^10*d^20*e^3*z^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4 - 983040*a^1
2*b^9*c^2*d*e^22*z^4 + 964608*a^4*b^18*c*d^8*e^15*z^4 - 866304*a^2*b^8*c^13
*d^22*e*z^4 + 703488*a^7*b^15*c*d^5*e^18*z^4 - 608256*a^10*b^12*c*d^2*e^21*
z^4 - 440832*a*b^11*c^11*d^21*e^2*z^4 + 275968*a*b^19*c^3*d^13*e^10*z^4 - 1
59744*a^2*b^20*c*d^10*e^13*z^4 - 153600*a*b^20*c^2*d^12*e^11*z^4 + 64512*a^
3*b^19*c*d^9*e^14*z^4 + 19746062336*a^8*b^6*c^9*d^12*e^11*z^4 - 15333588992
*a^10*b^4*c^9*d^10*e^13*z^4 + 6702170112*a^7*b^4*c^12*d^16*e^7*z^4 + 151679
13984*a^10*b^3*c^10*d^11*e^12*z^4 - 2256638976*a^5*b^11*c^7*d^13*e^10*z^4 +
 2254307328*a^5*b^7*c^11*d^17*e^6*z^4 - 2200633344*a^6*b^5*c^12*d^17*e^6*z^
4 + 6457131008*a^11*b^3*c^9*d^9*e^14*z^4 - 2128785408*a^5*b^8*c^10*d^16*e^7
*z^4 - 2126057472*a^6*b^11*c^6*d^11*e^12*z^4 + 2038349824*a^12*b^5*c^6*d^5*
e^18*z^4 + 2037841920*a^5*b^10*c^8*d^14*e^9*z^4 + 3615621120*a^9*b*c^13*d^1
5*e^8*z^4 + 1900019712*a^11*b^2*c^10*d^10*e^13*z^4 + 1867698432*a^9*b^9*c^5
*d^7*e^16*z^4 - 6157369344*a^9*b^4*c^10*d^12*e^11*z^4 - 1856913408*a^7*b^10
*c^6*d^10*e^13*z^4 + 1789132800*a^6*b^4*c^13*d^18*e^5*z^4 + 6082658304*a^8*
b^4*c^11*d^14*e^9*z^4 + 6029549568*a^11*b^5*c^7*d^7*e^16*z^4 + 6010159104*a
^6*b^7*c^10*d^15*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^13*e^10*z^4 + 165838848
0*a^11*b^6*c^6*d^6*e^17*z^4 + 5917114368*a^10*b^6*c^7*d^8*e^15*z^4 - 159119
7696*a^11*b^7*c^5*d^5*e^18*z^4 - 1526464512*a^8*b^10*c^5*d^8*e^15*z^4 - 577
2607488*a^12*b^4*c^7*d^6*e^17*z^4 - 1423507456*a^13*b^4*c^6*d^4*e^19*z^4 -
1387266048*a^7*b^3*c^13*d^17*e^6*z^4 + 2976120832*a^10*b*c^12*d^13*e^10*z^4
 - 9906946048*a^9*b^2*c^12*d^14*e^9*z^4 - 18421874688*a^8*b^5*c^10*d^13*e^1
0*z^4 + 1141217280*a^6*b^12*c^5*d^10*e^13*z^4 - 9714364416*a^7*b^8*c^8*d^12
*e^11*z^4 - 16777216*a^16*b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c*d*e^22*z^4 +
 81920*a*b^10*c^12*d^22*e*z^4 + 39168*a*b^21*c*d^11*e^12*z^4 - 1091829760*a
^5*b^6*c^12*d^18*e^5*z^4 + 1046740992*a^14*b^2*c^7*d^4*e^19*z^4 - 688442572
8*a^12*b*c^10*d^9*e^14*z^4 + 987445248*a^4*b^10*c^9*d^16*e^7*z^4 + 98408755
2*a^5*b^12*c^6*d^12*e^11*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^14*z^4 - 526685
7984*a^10*b^7*c^6*d^7*e^16*z^4 - 892145664*a^7*b^11*c^5*d^9*e^14*z^4 - 2444
623872*a^11*b*c^11*d^11*e^12*z^4 + 768540672*a^12*b^3*c^8*d^7*e^16*z^4 + 50
48322048*a^8*b^9*c^6*d^9*e^14*z^4 + 5047612416*a^6*b^9*c^8*d^13*e^10*z^4 -
732492288*a^4*b^11*c^8*d^15*e^8*z^4 + 9266921472*a^7*b^6*c^10*d^14*e^9*z^4
- 645857280*a^6*b^6*c^11*d^16*e^7*z^4 - 623867904*a^4*b^9*c^10*d^17*e^6*z^4
 - 622067712*a^6*b^3*c^14*d^19*e^4*z^4 + 582617088*a^10*b^8*c^5*d^6*e^17*z^
4 + 577119744*a^7*b^12*c^4*d^8*e^15*z^4 + 552566784*a^12*b^6*c^5*d^4*e^19*z
^4 + 549224448*a^9*b^8*c^6*d^8*e^15*z^4 - 526565376*a^9*b^10*c^4*d^6*e^17*z
^4 + 511520256*a^10*b^9*c^4*d^5*e^18*z^4 + 13393723392*a^9*b^3*c^11*d^13*e^
10*z^4 - 2066350080*a^14*b*c^8*d^5*e^18*z^4 + 4718592000*a^13*b^2*c^8*d^6*e
^17*z^4 - 314572800*a^7*b^2*c^14*d^18*e^5*z^4 + 287250432*a^4*b^13*c^6*d^13
*e^10*z^4 + 4565827584*a^10*b^5*c^8*d^9*e^14*z^4 - 250785792*a^4*b^14*c^5*d
^12*e^11*z^4 + 235536384*a^13*b^3*c^7*d^5*e^18*z^4 - 232683264*a^8*b^11*c^4
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*d^7*e^16*z^4 - 199627776*a^5*b^14*c^4*d^10*e^13*z^4 - 190267392*a^12*b^7*c
^4*d^3*e^20*z^4 + 184891392*a^6*b^10*c^7*d^12*e^11*z^4 + 180502528*a^4*b^7*
c^12*d^19*e^4*z^4 + 178877952*a^3*b^13*c^7*d^15*e^8*z^4 + 172490752*a^14*b^
3*c^6*d^3*e^20*z^4 + 163946496*a^13*b^5*c^5*d^3*e^20*z^4 + 155839488*a^8*b^
12*c^3*d^6*e^17*z^4 + 155000832*a^5*b^5*c^13*d^19*e^4*z^4 - 152076288*a^4*b
^6*c^13*d^20*e^3*z^4 - 137592576*a^3*b^12*c^8*d^16*e^7*z^4 - 133693440*a^14
*b^4*c^5*d^2*e^21*z^4 - 116767488*a^3*b^9*c^11*d^19*e^4*z^4 - 108985344*a^3
*b^14*c^6*d^14*e^9*z^4 - 106223616*a^6*b^13*c^4*d^9*e^14*z^4 + 106119168*a^
3*b^10*c^10*d^18*e^5*z^4 + 102432768*a^5*b^4*c^14*d^20*e^3*z^4 + 102113280*
a^4*b^12*c^7*d^14*e^9*z^4 + 100674048*a^5*b^9*c^9*d^15*e^8*z^4 + 90439680*a
^13*b^6*c^4*d^2*e^21*z^4 - 86808576*a^6*b^14*c^3*d^8*e^15*z^4 + 86245376*a^
6*b^2*c^15*d^20*e^3*z^4 + 79011840*a^4*b^8*c^11*d^18*e^5*z^4 + 78345216*a^4
*b^15*c^4*d^11*e^12*z^4 + 78006528*a^11*b^9*c^3*d^3*e^20*z^4 - 73253376*a^9
*b^11*c^3*d^5*e^18*z^4 + 67524608*a^3*b^8*c^12*d^20*e^3*z^4 + 67108864*a^15
*b^2*c^6*d^2*e^21*z^4 - 61590528*a^10*b^10*c^3*d^4*e^19*z^4 + 61559808*a^5*
b^15*c^3*d^9*e^14*z^4 - 59637760*a^5*b^3*c^15*d^21*e^2*z^4 + 58638336*a^4*b
^5*c^14*d^21*e^2*z^4 - 40828416*a^7*b^13*c^3*d^7*e^16*z^4 - 35639296*a^2*b^
12*c^9*d^18*e^5*z^4 - 31293440*a^12*b^8*c^3*d^2*e^21*z^4 + 29933568*a^5*b^1
3*c^5*d^11*e^12*z^4 + 27793920*a^2*b^11*c^10*d^19*e^4*z^4 + 27168768*a^2*b^
13*c^8*d^17*e^6*z^4 - 23602176*a^7*b^14*c^2*d^6*e^17*z^4 - 23248896*a^3*b^7
*c^13*d^21*e^2*z^4 + 20929536*a^3*b^15*c^5*d^13*e^10*z^4 + 18428928*a^9*b^1
2*c^2*d^4*e^19*z^4 + 18026496*a^6*b^15*c^2*d^7*e^16*z^4 - 16261632*a^10*b^1
1*c^2*d^3*e^20*z^4 + 15128064*a^3*b^16*c^4*d^12*e^11*z^4 - 14060544*a^2*b^1
0*c^11*d^20*e^3*z^4 + 13178880*a^2*b^16*c^5*d^14*e^9*z^4 - 11244288*a^3*b^1
7*c^3*d^11*e^12*z^4 - 10509312*a^2*b^15*c^6*d^15*e^8*z^4 - 7262208*a^4*b^17
*c^2*d^9*e^14*z^4 - 7045632*a^2*b^17*c^4*d^13*e^10*z^4 - 6285312*a^2*b^14*c
^7*d^16*e^7*z^4 + 5996544*a^11*b^10*c^2*d^2*e^21*z^4 + 4558336*a^2*b^9*c^12
*d^21*e^2*z^4 + 4478976*a^11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^4*b^16*c^3*d^
10*e^13*z^4 + 2629632*a^3*b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*b^18*c^2*d^10
*e^13*z^4 + 1627136*a^2*b^18*c^3*d^12*e^11*z^4 + 1605120*a^8*b^13*c^2*d^5*e
^18*z^4 + 1483776*a^5*b^16*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*c^2*d^11*e^12
*z^4 - 8542224384*a^10*b^2*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^12*e^11*z^4 -
 3072*b^12*c^11*d^22*e*z^4 - 1572864*a^6*c^17*d^22*e*z^4 - 4096*a^10*b^13*d
*e^22*z^4 - 4096*a*b^22*d^10*e^13*z^4 - 6144*a^12*b^10*c*e^23*z^4 - 983040*
a^5*b*c^17*d^23*z^4 - 6912*a*b^9*c^13*d^23*z^4 + 1824522240*a^13*c^10*d^8*e
^15*z^4 + 1730150400*a^12*c^11*d^10*e^13*z^4 + 958922752*a^14*c^9*d^6*e^17*
z^4 - 537919488*a^9*c^14*d^16*e^7*z^4 + 508559360*a^11*c^12*d^12*e^11*z^4 -
 500170752*a^10*c^13*d^14*e^9*z^4 + 246939648*a^15*c^8*d^4*e^19*z^4 - 19922
9440*a^8*c^15*d^18*e^5*z^4 - 29884416*a^7*c^16*d^20*e^3*z^4 + 25165824*a^16
*c^7*d^2*e^21*z^4 + 236544*b^17*c^6*d^17*e^6*z^4 - 202752*b^18*c^5*d^16*e^7
*z^4 - 202752*b^16*c^7*d^18*e^5*z^4 + 126720*b^19*c^4*d^15*e^8*z^4 + 126720
*b^15*c^8*d^19*e^4*z^4 - 56320*b^20*c^3*d^14*e^9*z^4 - 56320*b^14*c^9*d^20*
e^3*z^4 + 16896*b^21*c^2*d^13*e^10*z^4 + 16896*b^13*c^10*d^21*e^2*z^4 + 110
080*a^7*b^16*d^4*e^19*z^4 + 110080*a^4*b^19*d^7*e^16*z^4 - 75520*a^8*b^15*d
^3*e^20*z^4 - 75520*a^3*b^20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5*e^18*z^4 - 5
```

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6320*a^5*b^18*d^6*e^17*z^4 + 25600*a^9*b^14*d^2*e^21*z^4 + 25600*a^2*b^21*d
^9*e^14*z^4 - 1572864*a^16*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*c^4*e^23*z^4
- 327680*a^14*b^6*c^3*e^23*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 + 983040*a^4*b
^3*c^16*d^23*z^4 - 385024*a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^7*c^14*d^23*z
^4 + 256*b^23*d^11*e^12*z^4 + 1048576*a^17*c^6*e^23*z^4 + 256*b^11*c^12*d^2
3*z^4 + 256*a^11*b^12*e^23*z^4 + 948695040*a^8*b*c^10*d^6*e^13*z^2 + 348917
760*a^7*b*c^11*d^8*e^11*z^2 - 125030400*a^9*b*c^9*d^4*e^15*z^2 - 50728960*a
^6*b*c^12*d^10*e^9*z^2 - 44298240*a^5*b*c^13*d^12*e^7*z^2 - 36495360*a^10*b
*c^8*d^2*e^17*z^2 + 29675520*a^8*b^6*c^5*d*e^18*z^2 - 24170496*a^9*b^4*c^6*
d*e^18*z^2 - 17202816*a^7*b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*c^14*d^14*e^5
*z^2 + 5532416*a^6*b^10*c^3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*d*e^18*z^2 -
2662400*a^3*b*c^15*d^16*e^3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*z^2 - 1136160
*a*b^13*c^5*d^8*e^11*z^2 - 1017600*a^5*b^12*c^2*d*e^18*z^2 - 744768*a*b^11*
c^7*d^10*e^9*z^2 + 607872*a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b^6*c^12*d^15*
e^4*z^2 + 408576*a*b^5*c^13*d^16*e^3*z^2 + 361152*a*b^10*c^8*d^11*e^8*z^2 -
 287408*a*b^9*c^9*d^12*e^7*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^2 - 203904*a*
b^4*c^14*d^17*e^2*z^2 + 200832*a*b^8*c^10*d^13*e^6*z^2 + 126720*a*b^7*c^11*
d^14*e^5*z^2 - 123968*a*b^15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c^2*d^5*e^14*z
^2 + 11904*a^2*b^16*c*d^3*e^16*z^2 + 1824135552*a^7*b^4*c^8*d^5*e^14*z^2 -
1457252352*a^8*b^2*c^9*d^5*e^14*z^2 - 1405209600*a^7*b^5*c^7*d^4*e^15*z^2 -
 184320*a^2*b*c^16*d^18*e*z^2 + 100608*a^4*b^14*c*d*e^18*z^2 + 53248*a*b^3*
c^15*d^18*e*z^2 + 26448*a*b^17*c*d^4*e^15*z^2 + 1067599872*a^8*b^3*c^8*d^4*
e^15*z^2 - 930828288*a^7*b^3*c^9*d^6*e^13*z^2 + 920760000*a^6*b^4*c^9*d^7*e
^12*z^2 - 806639616*a^6*b^3*c^10*d^8*e^11*z^2 - 791052480*a^6*b^6*c^7*d^5*e
^14*z^2 + 772237824*a^6*b^7*c^6*d^4*e^15*z^2 - 701025408*a^5*b^6*c^8*d^7*e^
12*z^2 + 443340288*a^5*b^5*c^9*d^8*e^11*z^2 + 433047552*a^7*b^6*c^6*d^3*e^1
6*z^2 + 405741312*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^1
0*z^2 - 276962688*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16
*z^2 + 213564384*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15*
z^2 - 182520896*a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9*
z^2 - 152151552*a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8
*z^2 + 108085248*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9
*z^2 - 98323200*a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^
2 + 89464512*a^5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2
+ 68315904*a^5*b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 6
2951040*a^9*b^2*c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 476
14464*a^3*b^8*c^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983
040*a^3*b^11*c^5*d^6*e^13*z^2 - 33515520*a^4*b^3*c^12*d^12*e^7*z^2 - 334638
08*a^3*b^7*c^9*d^10*e^9*z^2 - 25128864*a^4*b^4*c^11*d^11*e^8*z^2 - 23193728
*a^3*b^10*c^6*d^7*e^12*z^2 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2 + 19924176*a
^4*b^11*c^4*d^4*e^15*z^2 - 19251216*a^3*b^9*c^7*d^8*e^11*z^2 - 16434048*a^5
*b^4*c^10*d^9*e^10*z^2 - 16289664*a^3*b^12*c^4*d^5*e^14*z^2 - 15059328*a^4*
b^12*c^3*d^3*e^16*z^2 - 10766016*a^2*b^10*c^7*d^9*e^10*z^2 - 10453632*a^5*b
^11*c^3*d^2*e^17*z^2 - 9940992*a^3*b^3*c^13*d^14*e^5*z^2 + 8373696*a^2*b^11
*c^6*d^8*e^11*z^2 + 7776768*a^3*b^2*c^14*d^15*e^4*z^2 + 7077888*a^3*b^5*c^1
```

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1*d^12*e^7*z^2 + 6798240*a^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^2*b^6*c^11*d^
13*e^6*z^2 + 3544320*a^3*b^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b^5*c^12*d^14*
e^5*z^2 + 2346336*a^4*b^13*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*c^9*d^11*e^8*
z^2 - 2125824*a^2*b^13*c^4*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12*d^13*e^6*z^2
 + 1927680*a^2*b^7*c^10*d^12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^5*e^14*z^2 -
1807104*a^2*b^12*c^5*d^7*e^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e^15*z^2 + 108
3456*a^3*b^14*c^2*d^3*e^16*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*z^2 - 657408*
a^2*b^3*c^14*d^16*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^17*z^2 + 595968*a^2*b^
2*c^15*d^17*e^2*z^2 - 498624*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*b^18*c*d^5*e^
14*z^2 - 3840*b^5*c^14*d^18*e*z^2 + 2064384*a^11*c^8*d*e^18*z^2 - 4160*a^3*
b^16*d*e^18*z^2 - 4160*a*b^18*d^3*e^16*z^2 - 1290240*a^11*b*c^7*e^19*z^2 -
9840*a^5*b^13*c*e^19*z^2 - 5760*a*b^2*c^16*d^19*z^2 - 280581120*a^8*c^11*d^
7*e^12*z^2 + 110278656*a^9*c^10*d^5*e^14*z^2 - 89479168*a^7*c^12*d^9*e^10*z
^2 + 34464000*a^10*c^9*d^3*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*z^2 + 54240*b
^8*c^11*d^15*e^4*z^2 - 49920*b^14*c^5*d^9*e^10*z^2 - 49920*b^9*c^10*d^14*e^
5*z^2 - 37376*b^16*c^3*d^7*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*z^2 + 28480*b
^13*c^6*d^10*e^9*z^2 + 28480*b^10*c^9*d^13*e^6*z^2 + 15936*b^17*c^2*d^6*e^1
3*z^2 + 15936*b^6*c^13*d^17*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z^2 - 7920*b^1
1*c^8*d^12*e^7*z^2 + 7489536*a^5*c^14*d^13*e^6*z^2 + 6084096*a^6*c^13*d^11*
e^8*z^2 + 2280448*a^4*c^15*d^15*e^4*z^2 + 350208*a^3*c^16*d^17*e^2*z^2 + 11
616*a^2*b^17*d^2*e^17*z^2 - 3515904*a^9*b^5*c^5*e^19*z^2 + 3440640*a^10*b^3
*c^6*e^19*z^2 + 1870848*a^8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*c^3*e^19*z^2
 + 101856*a^6*b^11*c^2*e^19*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b^4*c^15*d^19*
z^2 + 20736*a^2*c^17*d^19*z^2 + 400*a^4*b^15*e^19*z^2 - 3969216*a^4*b*c^10*
d^3*e^12 - 3001536*a^3*b*c^11*d^5*e^10 - 419904*a^2*b*c^12*d^7*e^8 + 184608
*a^4*b^3*c^8*d*e^14 - 153036*a*b^4*c^10*d^6*e^9 + 127008*a*b^3*c^11*d^7*e^8
 + 63108*a*b^6*c^8*d^4*e^11 - 29160*a*b^2*c^12*d^8*e^7 - 21060*a^3*b^5*c^7*
d*e^14 - 21060*a*b^7*c^7*d^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 - 404544*a^5*b*
c^9*d*e^14 + 1251872*a^3*b^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9*d^2*e^13 + 8
20512*a^2*b^3*c^10*d^5*e^10 + 750672*a^3*b^2*c^10*d^4*e^11 - 657498*a^2*b^4
*c^9*d^4*e^11 - 487116*a^3*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c^11*d^6*e^9 +
 58806*a^2*b^6*c^7*d^2*e^13 + 13140*a^2*b^5*c^8*d^3*e^12 + 15286*b^6*c^9*d^
6*e^9 - 9540*b^7*c^8*d^5*e^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b^8*c^7*d^4*e^
11 + 2025*b^4*c^11*d^8*e^7 + 3367008*a^4*c^11*d^4*e^11 + 1166400*a^3*c^12*d
^6*e^9 + 705600*a^5*c^10*d^2*e^13 + 104976*a^2*c^13*d^8*e^7 - 17640*a^5*b^2
*c^8*e^15 + 2025*a^4*b^4*c^7*e^15 + 38416*a^6*c^9*e^15, z, k)*((57344*a^12*
c^9*e^21 - 80*a^5*b^14*c^2*e^21 + 1824*a^6*b^12*c^3*e^21 - 17296*a^7*b^10*c
^4*e^21 + 87520*a^8*b^8*c^5*e^21 - 250880*a^9*b^6*c^6*e^21 + 394240*a^10*b^
4*c^7*e^21 - 290816*a^11*b^2*c^8*e^21 + 18432*a^3*c^18*d^18*e^3 + 210944*a^
4*c^17*d^16*e^5 + 878592*a^5*c^16*d^14*e^7 + 4749312*a^6*c^15*d^12*e^9 + 20
518912*a^7*c^14*d^10*e^11 + 12306432*a^8*c^13*d^8*e^13 - 22743040*a^9*c^12*
d^6*e^15 - 20076544*a^10*c^11*d^4*e^17 - 1425408*a^11*c^10*d^2*e^19 - 80*b^
5*c^16*d^19*e^2 + 704*b^6*c^15*d^18*e^3 - 2688*b^7*c^14*d^17*e^4 + 5824*b^8
*c^13*d^16*e^5 - 7840*b^9*c^12*d^15*e^6 + 6720*b^10*c^11*d^14*e^7 - 3728*b^
11*c^10*d^13*e^8 + 2176*b^12*c^9*d^12*e^9 - 3728*b^13*c^8*d^11*e^10 + 6720*
```

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b^14*c^7*d^10*e^11 - 7840*b^15*c^6*d^9*e^12 + 5824*b^16*c^5*d^8*e^13 - 2688
*b^17*c^4*d^7*e^14 + 704*b^18*c^3*d^6*e^15 - 80*b^19*c^2*d^5*e^16 + 12288*a
^2*b^2*c^17*d^18*e^3 - 1536*a^2*b^3*c^16*d^17*e^4 - 131712*a^2*b^4*c^15*d^1
6*e^5 + 410112*a^2*b^5*c^14*d^15*e^6 - 576576*a^2*b^6*c^13*d^14*e^7 + 34272
0*a^2*b^7*c^12*d^13*e^8 + 298464*a^2*b^8*c^11*d^12*e^9 - 1248672*a^2*b^9*c^
10*d^11*e^10 + 2177920*a^2*b^10*c^9*d^10*e^11 - 2309120*a^2*b^11*c^8*d^9*e^
12 + 1389888*a^2*b^12*c^7*d^8*e^13 - 314048*a^2*b^13*c^6*d^7*e^14 - 120896*
a^2*b^14*c^5*d^6*e^15 + 88128*a^2*b^15*c^4*d^5*e^16 - 14240*a^2*b^16*c^3*d^
4*e^17 - 416*a^2*b^17*c^2*d^3*e^18 + 621568*a^3*b^2*c^16*d^16*e^5 - 953344*
a^3*b^3*c^15*d^15*e^6 + 196224*a^3*b^4*c^14*d^14*e^7 + 1667904*a^3*b^5*c^13
*d^13*e^8 - 3981824*a^3*b^6*c^12*d^12*e^9 + 7617920*a^3*b^7*c^11*d^11*e^10
- 11899456*a^3*b^8*c^10*d^10*e^11 + 11500496*a^3*b^9*c^9*d^9*e^12 - 4599536
*a^3*b^10*c^8*d^8*e^13 - 1951936*a^3*b^11*c^7*d^7*e^14 + 2953152*a^3*b^12*c
^6*d^6*e^15 - 1134960*a^3*b^13*c^5*d^5*e^16 + 98960*a^3*b^14*c^4*d^4*e^17 +
 21920*a^3*b^15*c^3*d^3*e^18 - 416*a^3*b^16*c^2*d^2*e^19 + 4509696*a^4*b^2*
c^15*d^14*e^7 - 6720000*a^4*b^3*c^14*d^13*e^8 + 8231808*a^4*b^4*c^13*d^12*e
^9 - 17138976*a^4*b^5*c^12*d^11*e^10 + 30880320*a^4*b^6*c^11*d^10*e^11 - 24
883456*a^4*b^7*c^10*d^9*e^12 - 6291360*a^4*b^8*c^9*d^8*e^13 + 28429152*a^4*
b^9*c^8*d^7*e^14 - 21523072*a^4*b^10*c^7*d^6*e^15 + 5834928*a^4*b^11*c^6*d^
5*e^16 + 339872*a^4*b^12*c^5*d^4*e^17 - 325216*a^4*b^13*c^4*d^3*e^18 + 1344
*a^4*b^14*c^3*d^2*e^19 + 5483520*a^5*b^2*c^14*d^12*e^9 + 14537472*a^5*b^3*c
^13*d^11*e^10 - 39383680*a^5*b^4*c^12*d^10*e^11 + 5513408*a^5*b^5*c^11*d^9*
e^12 + 84582144*a^5*b^6*c^10*d^8*e^13 - 124246848*a^5*b^7*c^9*d^7*e^14 + 70
979712*a^5*b^8*c^8*d^6*e^15 - 8326320*a^5*b^9*c^7*d^5*e^16 - 7484656*a^5*b^
10*c^6*d^4*e^17 + 2142272*a^5*b^11*c^5*d^3*e^18 + 83520*a^5*b^12*c^4*d^2*e^
19 + 25849856*a^6*b^2*c^13*d^10*e^11 + 67294720*a^6*b^3*c^12*d^9*e^12 - 216
767360*a^6*b^4*c^11*d^8*e^13 + 237211008*a^6*b^5*c^10*d^7*e^14 - 88839360*a
^6*b^6*c^9*d^6*e^15 - 35929920*a^6*b^7*c^8*d^5*e^16 + 37859616*a^6*b^8*c^7*
d^4*e^17 - 6475552*a^6*b^9*c^6*d^3*e^18 - 1055296*a^6*b^10*c^5*d^2*e^19 + 1
90669824*a^7*b^2*c^12*d^8*e^13 - 143425536*a^7*b^3*c^11*d^7*e^14 - 47908992
*a^7*b^4*c^10*d^6*e^15 + 154814400*a^7*b^5*c^9*d^5*e^16 - 83642880*a^7*b^6*
c^8*d^4*e^17 + 4534272*a^7*b^7*c^7*d^3*e^18 + 5525568*a^7*b^8*c^6*d^2*e^19
+ 165122048*a^8*b^2*c^11*d^6*e^15 - 187467264*a^8*b^3*c^10*d^5*e^16 + 66920
064*a^8*b^4*c^9*d^4*e^17 + 21356016*a^8*b^5*c^8*d^3*e^18 - 14644224*a^8*b^6
*c^7*d^2*e^19 + 16114688*a^9*b^2*c^10*d^4*e^17 - 48695936*a^9*b^3*c^9*d^3*e
^18 + 18757632*a^9*b^4*c^8*d^2*e^19 - 8060928*a^10*b^2*c^9*d^2*e^19 + 12574
72*a^11*b*c^9*d*e^20 + 896*a*b^3*c^17*d^19*e^2 - 7040*a*b^4*c^16*d^18*e^3 +
 22080*a*b^5*c^15*d^17*e^4 - 32512*a*b^6*c^14*d^16*e^5 + 12736*a*b^7*c^13*d
^15*e^6 + 31104*a*b^8*c^12*d^14*e^7 - 51472*a*b^9*c^11*d^13*e^8 + 10864*a*b
^10*c^10*d^12*e^9 + 85440*a*b^11*c^9*d^11*e^10 - 186560*a*b^12*c^8*d^10*e^1
1 + 215904*a*b^13*c^7*d^9*e^12 - 151008*a*b^14*c^6*d^8*e^13 + 59776*a*b^15*
c^5*d^7*e^14 - 9408*a*b^16*c^4*d^6*e^15 - 1296*a*b^17*c^3*d^5*e^16 + 496*a*
b^18*c^2*d^4*e^17 - 2304*a^2*b*c^18*d^19*e^2 - 175104*a^3*b*c^17*d^17*e^4 -
 1556480*a^4*b*c^16*d^15*e^6 + 496*a^4*b^15*c^2*d*e^20 - 4746240*a^5*b*c^15
*d^13*e^8 - 10256*a^5*b^13*c^3*d*e^20 - 24033792*a^6*b*c^14*d^11*e^10 + 845
```

12*a^6*b^11*c^4*d*e^20 - 100332544*a^7*b*c^13*d^9*e^12 - 341264*a^7*b^9*c^5
*d*e^20 - 65824768*a^8*b*c^12*d^7*e^14 + 621568*a^8*b^7*c^6*d*e^20 + 397383
68*a^9*b*c^11*d^5*e^16 - 68096*a^9*b^5*c^7*d*e^20 + 27159296*a^10*b*c^10*d^
3*e^18 - 1310720*a^10*b^3*c^8*d*e^20)/(32*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^
8*d^18 - 256*a^6*c^12*d^18 - 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 -
a^2*b^16*d^10*e^8 + 8*a^3*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^1
3*d^7*e^11 - 70*a^6*b^12*d^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*
e^14 + 8*a^9*b^9*d^3*e^15 - a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 71
68*a^8*c^10*d^14*e^4 - 14336*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 1
4336*a^11*c^7*d^8*e^10 - 7168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 -
256*a^14*c^4*d^2*e^16 - 28*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3
- 70*a^2*b^12*c^4*d^14*e^4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^
12*e^6 + 440*a^3*b^8*c^7*d^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10
*c^5*d^14*e^4 - 616*a^3*b^11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*
a^3*b^13*c^2*d^11*e^7 - 2560*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e
^3 - 4060*a^4*b^8*c^6*d^14*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*
c^4*d^12*e^6 - 1360*a^4*b^11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 640
0*a^5*b^4*c^9*d^16*e^2 - 8960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*
e^4 + 9856*a^5*b^7*c^6*d^13*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9
*c^4*d^11*e^7 + 1040*a^5*b^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 51
20*a^6*b^2*c^10*d^16*e^2 + 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d
^13*e^5 + 25088*a^6*b^6*c^6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^
6*b^8*c^4*d^10*e^8 + 5400*a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10
- 35840*a^7*b^2*c^9*d^14*e^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c
^7*d^12*e^6 - 73472*a^7*b^5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 832
0*a^7*b^7*c^4*d^9*e^9 - 13048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e
^11 - 93184*a^8*b^2*c^8*d^12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b
^4*c^6*d^10*e^8 - 73472*a^8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 +
9856*a^8*b^7*c^3*d^7*e^11 - 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*
d^10*e^8 + 71680*a^9*b^3*c^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a
^9*b^5*c^4*d^7*e^11 + 2240*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13
- 93184*a^10*b^2*c^6*d^8*e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b
^4*c^4*d^6*e^12 - 8960*a^10*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 -
35840*a^11*b^2*c^5*d^6*e^12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^
2*d^3*e^15 - 5120*a^12*b^2*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a
^12*b^4*c^2*d^2*e^16 + 256*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e +
8*a^2*b^9*c^7*d^17*e + 8*a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40
*a^3*b^14*c*d^10*e^8 + 768*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 204
8*a^5*b^3*c^10*d^17*e + 168*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 +
14336*a^7*b*c^10*d^15*e^3 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13
*e^5 - 840*a^8*b^9*c*d^5*e^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^
4*e^14 + 71680*a^10*b*c^7*d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*
c^6*d^7*e^11 + 16*a^11*b^6*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^
13*b*c^4*d^3*e^15)) - root(128723189760*a^14*b^4*c^9*d^13*e^14*z^6 + 128723
189760*a^12*b^4*c^11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^4*d^11*e^16*z^6

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 - 8432455680*a^7*b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b^11*c^5*d^12*e^
15*z^6 + 12673351680*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960*a^12*b^9*c^6*d
^12*e^15*z^6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z^6 - 21048344576*a^9*b^12*
c^6*d^15*e^12*z^6 - 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 - 16609443840*a^1
0*b^3*c^14*d^22*e^5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^13*z^6 + 1453326
33600*a^12*b^5*c^10*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^8*d^12*e^15*z^6
 + 123740356608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17*b^5*c^5*d^6*e^
21*z^6 + 3460300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a^15*b^7*c^5*d^8
*e^19*z^6 - 7751073792*a^8*b^7*c^12*d^22*e^5*z^6 + 12041846784*a^14*b^7*c^6
*d^10*e^17*z^6 + 12041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325545099264*a^14*
b^3*c^10*d^14*e^13*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^11*z^6 - 3330539
520*a^13*b^10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^21*e^6*z^6 + 15
7789716480*a^12*b^7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^7*c^9*d^16*e^11
*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168*a^9*b^10*c^8*d
^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^6 - 7266631680*a^17*b^4*c^6*
d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326592*a^20*b*c^6*
d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^6 + 45747339264*a^13*b^8*c^6
*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 74612736*a^10*b^16
*c*d^9*e^18*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 2768240640*a^7*b^7
*c^13*d^24*e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^19*z^6 + 62914560*a^6*b^7*c
^14*d^26*e*z^6 + 2752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 2752020480*a^7*b^
13*c^7*d^18*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45957120*a^12*b^1
4*c*d^7*e^20*z^6 - 2724986880*a^14*b^9*c^4*d^8*e^19*z^6 - 2724986880*a^7*b^
9*c^11*d^22*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21086208*a^13*b^1
3*c*d^6*e^21*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z^6 - 6438912*a^14*b^12*c*d
^5*e^22*z^6 + 5406720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6*b^20*c*d^13*e^
14*z^6 - 1523712*a^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^11*c*d^4*e^23*z^
6 + 1179648*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^15*e^12*z^6 - 98
304*a^16*b^10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^6 - 49152*a^3*b
^13*c^11*d^26*e*z^6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6 + 6897106944*a^
8*b^13*c^6*d^16*e^11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*z^6 - 242221056
0*a^8*b^6*c^13*d^23*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^15*e^12*z^6 + 41
004564480*a^15*b^4*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*c^12*d^19*e^8*z
^6 + 2270822400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*b^11*c^10*d^22*
e^5*z^6 + 23677108224*a^14*b^8*c^5*d^9*e^18*z^6 + 23677108224*a^8*b^8*c^11*
d^21*e^6*z^6 + 212600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 212600881152*a^13
*b^2*c^12*d^17*e^10*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17*z^6 + 75157733
376*a^10*b^5*c^12*d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d^13*e^14*z^6 -
251217838080*a^11*b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b^10*c^3*d^7*e^2
0*z^6 - 1952907264*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*a^13*b^9*c^5*d^
10*e^17*z^6 - 27691057152*a^8*b^9*c^10*d^20*e^7*z^6 - 1902673920*a^8*b^15*c
^4*d^14*e^13*z^6 - 1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 10465050624*a^10
*b^11*c^6*d^14*e^13*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11*z^6 + 16139059
20*a^9*b^14*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17*e^10*z^6 - 332
18887680*a^17*b*c^9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*d^20*e^7*z^6 +
```

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1524695040*a^10*b^14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14*c^7*d^19*e^8*z
^6 - 1472200704*a^18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b^4*c^15*d^25*e^
2*z^6 - 83047219200*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200*a^11*b^3*c^13*
d^20*e^7*z^6 + 44291850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291850240*a^11*b^2
*c^14*d^21*e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 - 201326592*a^9*
b*c^17*d^26*e*z^6 + 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 + 48530718720*a^
10*b^8*c^9*d^17*e^10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18*z^6 - 12426444
80*a^6*b^12*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11*e^16*z^6 + 981
3196800*a^8*b^10*c^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*d^14*e^13*z^6 -
 93012885504*a^14*b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b^4*c^10*d^15*e^
12*z^6 + 52730658816*a^10*b^10*c^7*d^15*e^12*z^6 - 1180106752*a^9*b^15*c^3*
d^12*e^15*z^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*z^6 + 1023672320*a^15*b^9*
c^3*d^6*e^21*z^6 + 1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 975175680*a^17*b^
6*c^4*d^5*e^22*z^6 + 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 11072962560*a^18
*b*c^8*d^8*e^19*z^6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 + 9412018176*a^1
8*b^2*c^7*d^7*e^20*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z^6 + 805306368*
a^19*b^2*c^6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*z^6 - 133809831
936*a^14*b^6*c^7*d^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*d^19*e^8*z^6 -
2214592512*a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*d^24*e^3*z^6 +
82216747008*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*b^7*c^10*d^18*e^9
*z^6 - 586629120*a^12*b^13*c^2*d^8*e^19*z^6 - 586629120*a^5*b^13*c^9*d^22*e
^5*z^6 + 568565760*a^7*b^16*c^4*d^15*e^12*z^6 - 4844421120*a^16*b^4*c^7*d^9
*e^18*z^6 - 4844421120*a^10*b^4*c^13*d^21*e^6*z^6 + 531210240*a^11*b^14*c^2
*d^9*e^18*z^6 + 531210240*a^5*b^14*c^8*d^21*e^6*z^6 - 527155200*a^11*b^13*c
^3*d^10*e^17*z^6 - 527155200*a^6*b^13*c^8*d^20*e^7*z^6 + 43470028800*a^11*b
^8*c^8*d^15*e^12*z^6 - 107874877440*a^11*b^9*c^7*d^14*e^13*z^6 - 1078748774
40*a^10*b^9*c^8*d^16*e^11*z^6 + 9018408960*a^12*b^11*c^4*d^10*e^17*z^6 + 90
18408960*a^7*b^11*c^9*d^20*e^7*z^6 + 421994496*a^13*b^12*c^2*d^7*e^20*z^6 +
 421994496*a^5*b^12*c^10*d^23*e^4*z^6 - 66437775360*a^16*b*c^10*d^12*e^15*z
^6 - 66437775360*a^13*b*c^13*d^18*e^9*z^6 + 26159874048*a^16*b^5*c^6*d^8*e^
19*z^6 + 26159874048*a^9*b^5*c^13*d^22*e^5*z^6 - 369098752*a^18*b^3*c^6*d^6
*e^21*z^6 - 369098752*a^9*b^3*c^15*d^24*e^3*z^6 + 351436800*a^8*b^16*c^3*d^
13*e^14*z^6 + 351436800*a^6*b^16*c^5*d^17*e^10*z^6 - 334233600*a^16*b^8*c^3
*d^5*e^22*z^6 - 334233600*a^6*b^8*c^13*d^25*e^2*z^6 + 301989888*a^19*b^3*c^
5*d^4*e^23*z^6 - 266010624*a^10*b^15*c^2*d^10*e^17*z^6 - 266010624*a^5*b^15
*c^7*d^20*e^7*z^6 - 305198530560*a^12*b^6*c^9*d^15*e^12*z^6 - 203292672*a^1
4*b^11*c^2*d^6*e^21*z^6 - 203292672*a^5*b^11*c^11*d^24*e^3*z^6 - 188743680*
a^18*b^5*c^4*d^4*e^23*z^6 + 120418467840*a^16*b^2*c^9*d^11*e^16*z^6 + 12041
8467840*a^12*b^2*c^13*d^19*e^8*z^6 - 17293934592*a^10*b^12*c^5*d^13*e^14*z^
6 - 17293934592*a^8*b^12*c^7*d^17*e^10*z^6 + 104890368*a^8*b^17*c^2*d^12*e^
15*z^6 + 104890368*a^5*b^17*c^5*d^18*e^9*z^6 + 4390256640*a^15*b^8*c^4*d^7*
e^20*z^6 + 4390256640*a^7*b^8*c^12*d^23*e^4*z^6 - 91750400*a^6*b^18*c^3*d^1
5*e^12*z^6 + 79134720*a^7*b^17*c^3*d^14*e^13*z^6 + 79134720*a^6*b^17*c^4*d^
16*e^11*z^6 - 74612736*a^4*b^16*c^7*d^21*e^6*z^6 - 72990720*a^7*b^18*c^2*d^
13*e^14*z^6 - 72990720*a^5*b^18*c^4*d^17*e^10*z^6 + 69746688*a^4*b^15*c^8*d
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^22*e^5*z^6 + 63700992*a^15*b^10*c^2*d^5*e^22*z^6 + 63700992*a^5*b^10*c^12*
d^25*e^2*z^6 + 62914560*a^17*b^7*c^3*d^4*e^23*z^6 + 55148544*a^4*b^17*c^6*d
^20*e^7*z^6 - 45957120*a^4*b^14*c^9*d^23*e^4*z^6 - 25952256*a^4*b^18*c^5*d^
19*e^8*z^6 - 25165824*a^20*b^2*c^5*d^3*e^24*z^6 + 21086208*a^4*b^13*c^10*d^
24*e^3*z^6 + 20643840*a^6*b^19*c^2*d^14*e^13*z^6 + 20643840*a^5*b^19*c^3*d^
16*e^11*z^6 + 15728640*a^19*b^4*c^4*d^3*e^24*z^6 - 11796480*a^16*b^9*c^2*d^
4*e^23*z^6 - 6438912*a^4*b^12*c^11*d^25*e^2*z^6 + 5406720*a^4*b^19*c^4*d^18
*e^9*z^6 - 5242880*a^18*b^6*c^3*d^3*e^24*z^6 + 3784704*a^3*b^18*c^6*d^21*e^
6*z^6 - 3244032*a^3*b^19*c^5*d^20*e^7*z^6 - 3244032*a^3*b^17*c^7*d^22*e^5*z
^6 + 2027520*a^3*b^20*c^4*d^19*e^8*z^6 + 2027520*a^3*b^16*c^8*d^23*e^4*z^6
- 1622016*a^9*b^16*c^2*d^11*e^16*z^6 - 1622016*a^5*b^16*c^6*d^19*e^8*z^6 +
1622016*a^4*b^20*c^3*d^17*e^10*z^6 - 1523712*a^4*b^21*c^2*d^16*e^11*z^6 + 9
83040*a^17*b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^21*c^3*d^18*e^9*z^6 - 901120
*a^3*b^15*c^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^2*d^17*e^10*z^6 + 270336*a^3
*b^14*c^10*d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d^15*e^12*z^6 - 38593888256*a
^15*b^6*c^6*d^9*e^18*z^6 - 38593888256*a^9*b^6*c^12*d^21*e^6*z^6 - 21038628
8640*a^15*b^3*c^9*d^12*e^15*z^6 - 210386288640*a^12*b^3*c^12*d^18*e^9*z^6 +
 15502147584*a^15*c^12*d^15*e^12*z^6 + 1107296256*a^19*c^8*d^7*e^20*z^6 + 1
107296256*a^11*c^16*d^23*e^4*z^6 + 13287555072*a^16*c^11*d^13*e^14*z^6 + 13
287555072*a^14*c^13*d^17*e^10*z^6 + 201326592*a^20*c^7*d^5*e^22*z^6 + 20132
6592*a^10*c^17*d^25*e^2*z^6 + 16777216*a^21*c^6*d^3*e^24*z^6 + 3784704*a^9*
b^18*d^9*e^18*z^6 - 3244032*a^10*b^17*d^8*e^19*z^6 - 3244032*a^8*b^19*d^10*
e^17*z^6 + 2027520*a^11*b^16*d^7*e^20*z^6 + 2027520*a^7*b^20*d^11*e^16*z^6
- 901120*a^12*b^15*d^6*e^21*z^6 - 901120*a^6*b^21*d^12*e^15*z^6 + 270336*a^
13*b^14*d^5*e^22*z^6 + 270336*a^5*b^22*d^13*e^14*z^6 - 49152*a^14*b^13*d^4*
e^23*z^6 - 49152*a^4*b^23*d^14*e^13*z^6 + 4096*a^15*b^12*d^3*e^24*z^6 + 409
6*a^3*b^24*d^15*e^12*z^6 - 25165824*a^8*b^2*c^17*d^27*z^6 + 15728640*a^7*b^
4*c^16*d^27*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6 + 983040*a^5*b^8*c^14*d^27*
z^6 - 98304*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b^12*c^12*d^27*z^6 + 83047219
20*a^17*c^10*d^11*e^16*z^6 + 8304721920*a^13*c^14*d^19*e^8*z^6 + 3690987520
*a^18*c^9*d^9*e^18*z^6 + 3690987520*a^12*c^15*d^21*e^6*z^6 + 16777216*a^9*c
^18*d^27*z^6 - 8493371392*a^6*b^8*c^9*d^14*e^9*z^4 + 1458044928*a^8*b*c^14*
d^17*e^6*z^4 - 12604538880*a^11*b^4*c^8*d^8*e^15*z^4 - 8303067136*a^9*b^5*c
^9*d^11*e^12*z^4 - 5588058112*a^13*b*c^9*d^7*e^16*z^4 - 3892838400*a^8*b^2*
c^13*d^16*e^7*z^4 - 3611713536*a^8*b^8*c^7*d^10*e^13*z^4 + 7819006464*a^7*b
^9*c^7*d^11*e^12*z^4 - 7782137856*a^8*b^7*c^8*d^11*e^12*z^4 + 7780433920*a^
12*b^2*c^9*d^8*e^15*z^4 - 12020465664*a^7*b^5*c^11*d^15*e^8*z^4 + 317679206
4*a^8*b^3*c^12*d^15*e^8*z^4 - 322633728*a^15*b*c^7*d^3*e^20*z^4 + 210829312
*a^7*b*c^15*d^19*e^4*z^4 + 15623258112*a^9*b^6*c^8*d^10*e^13*z^4 + 25165824
*a^15*b^3*c^5*d*e^22*z^4 - 15728640*a^14*b^5*c^4*d*e^22*z^4 + 12582912*a^5*
b^2*c^16*d^22*e*z^4 - 11730944*a^4*b^4*c^15*d^22*e*z^4 + 5242880*a^13*b^7*c
^3*d*e^22*z^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4 + 4521984*a^3*b^6*c^14*d^22
*e*z^4 + 4460544*a*b^14*c^8*d^18*e^5*z^4 + 3538944*a^6*b*c^16*d^21*e^2*z^4
+ 3108864*a*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b^13*c^9*d^19*e^4*z^4 - 23454
72*a^5*b^17*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c*d^4*e^19*z^4 + 1824768*a^6*
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b^16*c*d^6*e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^20*z^4 + 1419264*a*b^12*c^10
*d^20*e^3*z^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4 - 983040*a^12*b^9*c^2*d*e^2
2*z^4 + 964608*a^4*b^18*c*d^8*e^15*z^4 - 866304*a^2*b^8*c^13*d^22*e*z^4 + 7
03488*a^7*b^15*c*d^5*e^18*z^4 - 608256*a^10*b^12*c*d^2*e^21*z^4 - 440832*a*
b^11*c^11*d^21*e^2*z^4 + 275968*a*b^19*c^3*d^13*e^10*z^4 - 159744*a^2*b^20*
c*d^10*e^13*z^4 - 153600*a*b^20*c^2*d^12*e^11*z^4 + 64512*a^3*b^19*c*d^9*e^
14*z^4 + 19746062336*a^8*b^6*c^9*d^12*e^11*z^4 - 15333588992*a^10*b^4*c^9*d
^10*e^13*z^4 + 6702170112*a^7*b^4*c^12*d^16*e^7*z^4 + 15167913984*a^10*b^3*
c^10*d^11*e^12*z^4 - 2256638976*a^5*b^11*c^7*d^13*e^10*z^4 + 2254307328*a^5
*b^7*c^11*d^17*e^6*z^4 - 2200633344*a^6*b^5*c^12*d^17*e^6*z^4 + 6457131008*
a^11*b^3*c^9*d^9*e^14*z^4 - 2128785408*a^5*b^8*c^10*d^16*e^7*z^4 - 21260574
72*a^6*b^11*c^6*d^11*e^12*z^4 + 2038349824*a^12*b^5*c^6*d^5*e^18*z^4 + 2037
841920*a^5*b^10*c^8*d^14*e^9*z^4 + 3615621120*a^9*b*c^13*d^15*e^8*z^4 + 190
0019712*a^11*b^2*c^10*d^10*e^13*z^4 + 1867698432*a^9*b^9*c^5*d^7*e^16*z^4 -
 6157369344*a^9*b^4*c^10*d^12*e^11*z^4 - 1856913408*a^7*b^10*c^6*d^10*e^13*
z^4 + 1789132800*a^6*b^4*c^13*d^18*e^5*z^4 + 6082658304*a^8*b^4*c^11*d^14*e
^9*z^4 + 6029549568*a^11*b^5*c^7*d^7*e^16*z^4 + 6010159104*a^6*b^7*c^10*d^1
5*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^13*e^10*z^4 + 1658388480*a^11*b^6*c^6*
d^6*e^17*z^4 + 5917114368*a^10*b^6*c^7*d^8*e^15*z^4 - 1591197696*a^11*b^7*c
^5*d^5*e^18*z^4 - 1526464512*a^8*b^10*c^5*d^8*e^15*z^4 - 5772607488*a^12*b^
4*c^7*d^6*e^17*z^4 - 1423507456*a^13*b^4*c^6*d^4*e^19*z^4 - 1387266048*a^7*
b^3*c^13*d^17*e^6*z^4 + 2976120832*a^10*b*c^12*d^13*e^10*z^4 - 9906946048*a
^9*b^2*c^12*d^14*e^9*z^4 - 18421874688*a^8*b^5*c^10*d^13*e^10*z^4 + 1141217
280*a^6*b^12*c^5*d^10*e^13*z^4 - 9714364416*a^7*b^8*c^8*d^12*e^11*z^4 - 167
77216*a^16*b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c*d*e^22*z^4 + 81920*a*b^10*c
^12*d^22*e*z^4 + 39168*a*b^21*c*d^11*e^12*z^4 - 1091829760*a^5*b^6*c^12*d^1
8*e^5*z^4 + 1046740992*a^14*b^2*c^7*d^4*e^19*z^4 - 6884425728*a^12*b*c^10*d
^9*e^14*z^4 + 987445248*a^4*b^10*c^9*d^16*e^7*z^4 + 984087552*a^5*b^12*c^6*
d^12*e^11*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^14*z^4 - 5266857984*a^10*b^7*c
^6*d^7*e^16*z^4 - 892145664*a^7*b^11*c^5*d^9*e^14*z^4 - 2444623872*a^11*b*c
^11*d^11*e^12*z^4 + 768540672*a^12*b^3*c^8*d^7*e^16*z^4 + 5048322048*a^8*b^
9*c^6*d^9*e^14*z^4 + 5047612416*a^6*b^9*c^8*d^13*e^10*z^4 - 732492288*a^4*b
^11*c^8*d^15*e^8*z^4 + 9266921472*a^7*b^6*c^10*d^14*e^9*z^4 - 645857280*a^6
*b^6*c^11*d^16*e^7*z^4 - 623867904*a^4*b^9*c^10*d^17*e^6*z^4 - 622067712*a^
6*b^3*c^14*d^19*e^4*z^4 + 582617088*a^10*b^8*c^5*d^6*e^17*z^4 + 577119744*a
^7*b^12*c^4*d^8*e^15*z^4 + 552566784*a^12*b^6*c^5*d^4*e^19*z^4 + 549224448*
a^9*b^8*c^6*d^8*e^15*z^4 - 526565376*a^9*b^10*c^4*d^6*e^17*z^4 + 511520256*
a^10*b^9*c^4*d^5*e^18*z^4 + 13393723392*a^9*b^3*c^11*d^13*e^10*z^4 - 206635
0080*a^14*b*c^8*d^5*e^18*z^4 + 4718592000*a^13*b^2*c^8*d^6*e^17*z^4 - 31457
2800*a^7*b^2*c^14*d^18*e^5*z^4 + 287250432*a^4*b^13*c^6*d^13*e^10*z^4 + 456
5827584*a^10*b^5*c^8*d^9*e^14*z^4 - 250785792*a^4*b^14*c^5*d^12*e^11*z^4 +
235536384*a^13*b^3*c^7*d^5*e^18*z^4 - 232683264*a^8*b^11*c^4*d^7*e^16*z^4 -
 199627776*a^5*b^14*c^4*d^10*e^13*z^4 - 190267392*a^12*b^7*c^4*d^3*e^20*z^4
 + 184891392*a^6*b^10*c^7*d^12*e^11*z^4 + 180502528*a^4*b^7*c^12*d^19*e^4*z
^4 + 178877952*a^3*b^13*c^7*d^15*e^8*z^4 + 172490752*a^14*b^3*c^6*d^3*e^20*
```

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z^4 + 163946496*a^13*b^5*c^5*d^3*e^20*z^4 + 155839488*a^8*b^12*c^3*d^6*e^17
*z^4 + 155000832*a^5*b^5*c^13*d^19*e^4*z^4 - 152076288*a^4*b^6*c^13*d^20*e^
3*z^4 - 137592576*a^3*b^12*c^8*d^16*e^7*z^4 - 133693440*a^14*b^4*c^5*d^2*e^
21*z^4 - 116767488*a^3*b^9*c^11*d^19*e^4*z^4 - 108985344*a^3*b^14*c^6*d^14*
e^9*z^4 - 106223616*a^6*b^13*c^4*d^9*e^14*z^4 + 106119168*a^3*b^10*c^10*d^1
8*e^5*z^4 + 102432768*a^5*b^4*c^14*d^20*e^3*z^4 + 102113280*a^4*b^12*c^7*d^
14*e^9*z^4 + 100674048*a^5*b^9*c^9*d^15*e^8*z^4 + 90439680*a^13*b^6*c^4*d^2
*e^21*z^4 - 86808576*a^6*b^14*c^3*d^8*e^15*z^4 + 86245376*a^6*b^2*c^15*d^20
*e^3*z^4 + 79011840*a^4*b^8*c^11*d^18*e^5*z^4 + 78345216*a^4*b^15*c^4*d^11*
e^12*z^4 + 78006528*a^11*b^9*c^3*d^3*e^20*z^4 - 73253376*a^9*b^11*c^3*d^5*e
^18*z^4 + 67524608*a^3*b^8*c^12*d^20*e^3*z^4 + 67108864*a^15*b^2*c^6*d^2*e^
21*z^4 - 61590528*a^10*b^10*c^3*d^4*e^19*z^4 + 61559808*a^5*b^15*c^3*d^9*e^
14*z^4 - 59637760*a^5*b^3*c^15*d^21*e^2*z^4 + 58638336*a^4*b^5*c^14*d^21*e^
2*z^4 - 40828416*a^7*b^13*c^3*d^7*e^16*z^4 - 35639296*a^2*b^12*c^9*d^18*e^5
*z^4 - 31293440*a^12*b^8*c^3*d^2*e^21*z^4 + 29933568*a^5*b^13*c^5*d^11*e^12
*z^4 + 27793920*a^2*b^11*c^10*d^19*e^4*z^4 + 27168768*a^2*b^13*c^8*d^17*e^6
*z^4 - 23602176*a^7*b^14*c^2*d^6*e^17*z^4 - 23248896*a^3*b^7*c^13*d^21*e^2*
z^4 + 20929536*a^3*b^15*c^5*d^13*e^10*z^4 + 18428928*a^9*b^12*c^2*d^4*e^19*
z^4 + 18026496*a^6*b^15*c^2*d^7*e^16*z^4 - 16261632*a^10*b^11*c^2*d^3*e^20*
z^4 + 15128064*a^3*b^16*c^4*d^12*e^11*z^4 - 14060544*a^2*b^10*c^11*d^20*e^3
*z^4 + 13178880*a^2*b^16*c^5*d^14*e^9*z^4 - 11244288*a^3*b^17*c^3*d^11*e^12
*z^4 - 10509312*a^2*b^15*c^6*d^15*e^8*z^4 - 7262208*a^4*b^17*c^2*d^9*e^14*z
^4 - 7045632*a^2*b^17*c^4*d^13*e^10*z^4 - 6285312*a^2*b^14*c^7*d^16*e^7*z^4
 + 5996544*a^11*b^10*c^2*d^2*e^21*z^4 + 4558336*a^2*b^9*c^12*d^21*e^2*z^4 +
 4478976*a^11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^4*b^16*c^3*d^10*e^13*z^4 + 2
629632*a^3*b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*b^18*c^2*d^10*e^13*z^4 + 162
7136*a^2*b^18*c^3*d^12*e^11*z^4 + 1605120*a^8*b^13*c^2*d^5*e^18*z^4 + 14837
76*a^5*b^16*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*c^2*d^11*e^12*z^4 - 85422243
84*a^10*b^2*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^12*e^11*z^4 - 3072*b^12*c^11
*d^22*e*z^4 - 1572864*a^6*c^17*d^22*e*z^4 - 4096*a^10*b^13*d*e^22*z^4 - 409
6*a*b^22*d^10*e^13*z^4 - 6144*a^12*b^10*c*e^23*z^4 - 983040*a^5*b*c^17*d^23
*z^4 - 6912*a*b^9*c^13*d^23*z^4 + 1824522240*a^13*c^10*d^8*e^15*z^4 + 17301
50400*a^12*c^11*d^10*e^13*z^4 + 958922752*a^14*c^9*d^6*e^17*z^4 - 537919488
*a^9*c^14*d^16*e^7*z^4 + 508559360*a^11*c^12*d^12*e^11*z^4 - 500170752*a^10
*c^13*d^14*e^9*z^4 + 246939648*a^15*c^8*d^4*e^19*z^4 - 199229440*a^8*c^15*d
^18*e^5*z^4 - 29884416*a^7*c^16*d^20*e^3*z^4 + 25165824*a^16*c^7*d^2*e^21*z
^4 + 236544*b^17*c^6*d^17*e^6*z^4 - 202752*b^18*c^5*d^16*e^7*z^4 - 202752*b
^16*c^7*d^18*e^5*z^4 + 126720*b^19*c^4*d^15*e^8*z^4 + 126720*b^15*c^8*d^19*
e^4*z^4 - 56320*b^20*c^3*d^14*e^9*z^4 - 56320*b^14*c^9*d^20*e^3*z^4 + 16896
*b^21*c^2*d^13*e^10*z^4 + 16896*b^13*c^10*d^21*e^2*z^4 + 110080*a^7*b^16*d^
4*e^19*z^4 + 110080*a^4*b^19*d^7*e^16*z^4 - 75520*a^8*b^15*d^3*e^20*z^4 - 7
5520*a^3*b^20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5*e^18*z^4 - 56320*a^5*b^18*d
^6*e^17*z^4 + 25600*a^9*b^14*d^2*e^21*z^4 + 25600*a^2*b^21*d^9*e^14*z^4 - 1
572864*a^16*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*c^4*e^23*z^4 - 327680*a^14*b
^6*c^3*e^23*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 + 983040*a^4*b^3*c^16*d^23*z^
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4 - 385024*a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^7*c^14*d^23*z^4 + 256*b^23*d
^11*e^12*z^4 + 1048576*a^17*c^6*e^23*z^4 + 256*b^11*c^12*d^23*z^4 + 256*a^1
1*b^12*e^23*z^4 + 948695040*a^8*b*c^10*d^6*e^13*z^2 + 348917760*a^7*b*c^11*
d^8*e^11*z^2 - 125030400*a^9*b*c^9*d^4*e^15*z^2 - 50728960*a^6*b*c^12*d^10*
e^9*z^2 - 44298240*a^5*b*c^13*d^12*e^7*z^2 - 36495360*a^10*b*c^8*d^2*e^17*z
^2 + 29675520*a^8*b^6*c^5*d*e^18*z^2 - 24170496*a^9*b^4*c^6*d*e^18*z^2 - 17
202816*a^7*b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*c^14*d^14*e^5*z^2 + 5532416*
a^6*b^10*c^3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*d*e^18*z^2 - 2662400*a^3*b*c
^15*d^16*e^3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*z^2 - 1136160*a*b^13*c^5*d^8
*e^11*z^2 - 1017600*a^5*b^12*c^2*d*e^18*z^2 - 744768*a*b^11*c^7*d^10*e^9*z^
2 + 607872*a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b^6*c^12*d^15*e^4*z^2 + 40857
6*a*b^5*c^13*d^16*e^3*z^2 + 361152*a*b^10*c^8*d^11*e^8*z^2 - 287408*a*b^9*c
^9*d^12*e^7*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^2 - 203904*a*b^4*c^14*d^17*e
^2*z^2 + 200832*a*b^8*c^10*d^13*e^6*z^2 + 126720*a*b^7*c^11*d^14*e^5*z^2 -
123968*a*b^15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c^2*d^5*e^14*z^2 + 11904*a^2*
b^16*c*d^3*e^16*z^2 + 1824135552*a^7*b^4*c^8*d^5*e^14*z^2 - 1457252352*a^8*
b^2*c^9*d^5*e^14*z^2 - 1405209600*a^7*b^5*c^7*d^4*e^15*z^2 - 184320*a^2*b*c
^16*d^18*e*z^2 + 100608*a^4*b^14*c*d*e^18*z^2 + 53248*a*b^3*c^15*d^18*e*z^2
 + 26448*a*b^17*c*d^4*e^15*z^2 + 1067599872*a^8*b^3*c^8*d^4*e^15*z^2 - 9308
28288*a^7*b^3*c^9*d^6*e^13*z^2 + 920760000*a^6*b^4*c^9*d^7*e^12*z^2 - 80663
9616*a^6*b^3*c^10*d^8*e^11*z^2 - 791052480*a^6*b^6*c^7*d^5*e^14*z^2 + 77223
7824*a^6*b^7*c^6*d^4*e^15*z^2 - 701025408*a^5*b^6*c^8*d^7*e^12*z^2 + 443340
288*a^5*b^5*c^9*d^8*e^11*z^2 + 433047552*a^7*b^6*c^6*d^3*e^16*z^2 + 4057413
12*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^10*z^2 - 2769626
88*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16*z^2 + 21356438
4*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15*z^2 - 182520896
*a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9*z^2 - 152151552
*a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8*z^2 + 10808524
8*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9*z^2 - 98323200
*a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^2 + 89464512*a^
5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2 + 68315904*a^5*
b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 62951040*a^9*b^2
*c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 47614464*a^3*b^8*c
^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983040*a^3*b^11*c^
5*d^6*e^13*z^2 - 33515520*a^4*b^3*c^12*d^12*e^7*z^2 - 33463808*a^3*b^7*c^9*
d^10*e^9*z^2 - 25128864*a^4*b^4*c^11*d^11*e^8*z^2 - 23193728*a^3*b^10*c^6*d
^7*e^12*z^2 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2 + 19924176*a^4*b^11*c^4*d^4
*e^15*z^2 - 19251216*a^3*b^9*c^7*d^8*e^11*z^2 - 16434048*a^5*b^4*c^10*d^9*e
^10*z^2 - 16289664*a^3*b^12*c^4*d^5*e^14*z^2 - 15059328*a^4*b^12*c^3*d^3*e^
16*z^2 - 10766016*a^2*b^10*c^7*d^9*e^10*z^2 - 10453632*a^5*b^11*c^3*d^2*e^1
7*z^2 - 9940992*a^3*b^3*c^13*d^14*e^5*z^2 + 8373696*a^2*b^11*c^6*d^8*e^11*z
^2 + 7776768*a^3*b^2*c^14*d^15*e^4*z^2 + 7077888*a^3*b^5*c^11*d^12*e^7*z^2
+ 6798240*a^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^2*b^6*c^11*d^13*e^6*z^2 + 35
44320*a^3*b^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b^5*c^12*d^14*e^5*z^2 + 23463
36*a^4*b^13*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*c^9*d^11*e^8*z^2 - 2125824*a
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^2*b^13*c^4*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12*d^13*e^6*z^2 + 1927680*a^2*
b^7*c^10*d^12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^5*e^14*z^2 - 1807104*a^2*b^1
2*c^5*d^7*e^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e^15*z^2 + 1083456*a^3*b^14*c
^2*d^3*e^16*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*z^2 - 657408*a^2*b^3*c^14*d^
16*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^17*z^2 + 595968*a^2*b^2*c^15*d^17*e^2
*z^2 - 498624*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*b^18*c*d^5*e^14*z^2 - 3840*b
^5*c^14*d^18*e*z^2 + 2064384*a^11*c^8*d*e^18*z^2 - 4160*a^3*b^16*d*e^18*z^2
 - 4160*a*b^18*d^3*e^16*z^2 - 1290240*a^11*b*c^7*e^19*z^2 - 9840*a^5*b^13*c
*e^19*z^2 - 5760*a*b^2*c^16*d^19*z^2 - 280581120*a^8*c^11*d^7*e^12*z^2 + 11
0278656*a^9*c^10*d^5*e^14*z^2 - 89479168*a^7*c^12*d^9*e^10*z^2 + 34464000*a
^10*c^9*d^3*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*z^2 + 54240*b^8*c^11*d^15*e^
4*z^2 - 49920*b^14*c^5*d^9*e^10*z^2 - 49920*b^9*c^10*d^14*e^5*z^2 - 37376*b
^16*c^3*d^7*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*z^2 + 28480*b^13*c^6*d^10*e^
9*z^2 + 28480*b^10*c^9*d^13*e^6*z^2 + 15936*b^17*c^2*d^6*e^13*z^2 + 15936*b
^6*c^13*d^17*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z^2 - 7920*b^11*c^8*d^12*e^7*
z^2 + 7489536*a^5*c^14*d^13*e^6*z^2 + 6084096*a^6*c^13*d^11*e^8*z^2 + 22804
48*a^4*c^15*d^15*e^4*z^2 + 350208*a^3*c^16*d^17*e^2*z^2 + 11616*a^2*b^17*d^
2*e^17*z^2 - 3515904*a^9*b^5*c^5*e^19*z^2 + 3440640*a^10*b^3*c^6*e^19*z^2 +
 1870848*a^8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*c^3*e^19*z^2 + 101856*a^6*b^
11*c^2*e^19*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b^4*c^15*d^19*z^2 + 20736*a^2
*c^17*d^19*z^2 + 400*a^4*b^15*e^19*z^2 - 3969216*a^4*b*c^10*d^3*e^12 - 3001
536*a^3*b*c^11*d^5*e^10 - 419904*a^2*b*c^12*d^7*e^8 + 184608*a^4*b^3*c^8*d*
e^14 - 153036*a*b^4*c^10*d^6*e^9 + 127008*a*b^3*c^11*d^7*e^8 + 63108*a*b^6*
c^8*d^4*e^11 - 29160*a*b^2*c^12*d^8*e^7 - 21060*a^3*b^5*c^7*d*e^14 - 21060*
a*b^7*c^7*d^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 - 404544*a^5*b*c^9*d*e^14 + 12
51872*a^3*b^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9*d^2*e^13 + 820512*a^2*b^3*c
^10*d^5*e^10 + 750672*a^3*b^2*c^10*d^4*e^11 - 657498*a^2*b^4*c^9*d^4*e^11 -
 487116*a^3*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c^11*d^6*e^9 + 58806*a^2*b^6*
c^7*d^2*e^13 + 13140*a^2*b^5*c^8*d^3*e^12 + 15286*b^6*c^9*d^6*e^9 - 9540*b^
7*c^8*d^5*e^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b^8*c^7*d^4*e^11 + 2025*b^4*c
^11*d^8*e^7 + 3367008*a^4*c^11*d^4*e^11 + 1166400*a^3*c^12*d^6*e^9 + 705600
*a^5*c^10*d^2*e^13 + 104976*a^2*c^13*d^8*e^7 - 17640*a^5*b^2*c^8*e^15 + 202
5*a^4*b^4*c^7*e^15 + 38416*a^6*c^9*e^15, z, k)*(root(128723189760*a^14*b^4*
c^9*d^13*e^14*z^6 + 128723189760*a^12*b^4*c^11*d^17*e^10*z^6 - 8432455680*a
^11*b^12*c^4*d^11*e^16*z^6 - 8432455680*a^7*b^12*c^8*d^19*e^8*z^6 + 1267335
1680*a^11*b^11*c^5*d^12*e^15*z^6 + 12673351680*a^8*b^11*c^8*d^18*e^9*z^6 -
72637480960*a^12*b^9*c^6*d^12*e^15*z^6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z
^6 - 21048344576*a^9*b^12*c^6*d^15*e^12*z^6 - 16609443840*a^17*b^3*c^7*d^8*
e^19*z^6 - 16609443840*a^10*b^3*c^14*d^22*e^5*z^6 + 145332633600*a^13*b^5*c
^9*d^14*e^13*z^6 + 145332633600*a^12*b^5*c^10*d^16*e^11*z^6 + 123740356608*
a^14*b^5*c^8*d^12*e^15*z^6 + 123740356608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460
300800*a^17*b^5*c^5*d^6*e^21*z^6 + 3460300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7
751073792*a^15*b^7*c^5*d^8*e^19*z^6 - 7751073792*a^8*b^7*c^12*d^22*e^5*z^6
+ 12041846784*a^14*b^7*c^6*d^10*e^17*z^6 + 12041846784*a^9*b^7*c^11*d^20*e^
7*z^6 - 325545099264*a^14*b^3*c^10*d^14*e^13*z^6 - 325545099264*a^13*b^3*c^
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11*d^16*e^11*z^6 - 3330539520*a^13*b^10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b
^10*c^10*d^21*e^6*z^6 + 157789716480*a^12*b^7*c^8*d^14*e^13*z^6 + 157789716
480*a^11*b^7*c^9*d^16*e^11*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6 +
37492359168*a^9*b^10*c^8*d^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^6
- 7266631680*a^17*b^4*c^6*d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z
^6 - 201326592*a^20*b*c^6*d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^6
+ 45747339264*a^13*b^8*c^6*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*e^
8*z^6 - 74612736*a^10*b^16*c*d^9*e^18*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^2
1*z^6 - 2768240640*a^7*b^7*c^13*d^24*e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^1
9*z^6 + 62914560*a^6*b^7*c^14*d^26*e*z^6 + 2752020480*a^10*b^13*c^4*d^12*e^
15*z^6 + 2752020480*a^7*b^13*c^7*d^18*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^
17*z^6 - 45957120*a^12*b^14*c*d^7*e^20*z^6 - 2724986880*a^14*b^9*c^4*d^8*e^
19*z^6 - 2724986880*a^7*b^9*c^11*d^22*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^
16*z^6 + 21086208*a^13*b^13*c*d^6*e^21*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z
^6 - 6438912*a^14*b^12*c*d^5*e^22*z^6 + 5406720*a^7*b^19*c*d^12*e^15*z^6 +
1622016*a^6*b^20*c*d^13*e^14*z^6 - 1523712*a^5*b^21*c*d^14*e^13*z^6 + 11796
48*a^15*b^11*c*d^4*e^23*z^6 + 1179648*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4
*b^22*c*d^15*e^12*z^6 - 98304*a^16*b^10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d
^16*e^11*z^6 - 49152*a^3*b^13*c^11*d^26*e*z^6 + 6897106944*a^9*b^13*c^5*d^1
4*e^13*z^6 + 6897106944*a^8*b^13*c^6*d^16*e^11*z^6 - 2422210560*a^16*b^6*c^
5*d^7*e^20*z^6 - 2422210560*a^8*b^6*c^13*d^23*e^4*z^6 + 255785435136*a^14*b
^2*c^11*d^15*e^12*z^6 + 41004564480*a^15*b^4*c^8*d^11*e^16*z^6 + 4100456448
0*a^11*b^4*c^12*d^19*e^8*z^6 + 2270822400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270
822400*a^6*b^11*c^10*d^22*e^5*z^6 + 23677108224*a^14*b^8*c^5*d^9*e^18*z^6 +
 23677108224*a^8*b^8*c^11*d^21*e^6*z^6 + 212600881152*a^15*b^2*c^10*d^13*e^
14*z^6 + 212600881152*a^13*b^2*c^12*d^17*e^10*z^6 + 75157733376*a^15*b^5*c^
7*d^10*e^17*z^6 + 75157733376*a^10*b^5*c^12*d^20*e^7*z^6 - 251217838080*a^1
3*b^6*c^8*d^13*e^14*z^6 - 251217838080*a^11*b^6*c^10*d^17*e^10*z^6 - 195290
7264*a^14*b^10*c^3*d^7*e^20*z^6 - 1952907264*a^6*b^10*c^11*d^23*e^4*z^6 - 2
7691057152*a^13*b^9*c^5*d^10*e^17*z^6 - 27691057152*a^8*b^9*c^10*d^20*e^7*z
^6 - 1902673920*a^8*b^15*c^4*d^14*e^13*z^6 - 1902673920*a^7*b^15*c^5*d^16*e
^11*z^6 + 10465050624*a^10*b^11*c^6*d^14*e^13*z^6 + 10465050624*a^9*b^11*c^
7*d^16*e^11*z^6 + 1613905920*a^9*b^14*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^
14*c^6*d^17*e^10*z^6 - 33218887680*a^17*b*c^9*d^10*e^17*z^6 - 33218887680*a
^12*b*c^14*d^20*e^7*z^6 + 1524695040*a^10*b^14*c^3*d^11*e^16*z^6 + 15246950
40*a^6*b^14*c^7*d^19*e^8*z^6 - 1472200704*a^18*b^4*c^5*d^5*e^22*z^6 - 14722
00704*a^8*b^4*c^15*d^25*e^2*z^6 - 83047219200*a^16*b^3*c^8*d^10*e^17*z^6 -
83047219200*a^11*b^3*c^13*d^20*e^7*z^6 + 44291850240*a^17*b^2*c^8*d^9*e^18*
z^6 + 44291850240*a^11*b^2*c^14*d^21*e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15
*e^12*z^6 - 201326592*a^9*b*c^17*d^26*e*z^6 + 48530718720*a^12*b^8*c^7*d^13
*e^14*z^6 + 48530718720*a^10*b^8*c^9*d^17*e^10*z^6 - 1242644480*a^12*b^12*c
^3*d^9*e^18*z^6 - 1242644480*a^6*b^12*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^
10*c^5*d^11*e^16*z^6 + 9813196800*a^8*b^10*c^9*d^19*e^8*z^6 - 93012885504*a
^15*b*c^11*d^14*e^13*z^6 - 93012885504*a^14*b*c^12*d^16*e^11*z^6 + 17730581
2992*a^13*b^4*c^10*d^15*e^12*z^6 + 52730658816*a^10*b^10*c^7*d^15*e^12*z^6
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- 1180106752*a^9*b^15*c^3*d^12*e^15*z^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*
z^6 + 1023672320*a^15*b^9*c^3*d^6*e^21*z^6 + 1023672320*a^6*b^9*c^12*d^24*e
^3*z^6 + 975175680*a^17*b^6*c^4*d^5*e^22*z^6 + 975175680*a^7*b^6*c^14*d^25*
e^2*z^6 - 11072962560*a^18*b*c^8*d^8*e^19*z^6 - 11072962560*a^11*b*c^15*d^2
2*e^5*z^6 + 9412018176*a^18*b^2*c^7*d^7*e^20*z^6 + 9412018176*a^10*b^2*c^15
*d^23*e^4*z^6 + 805306368*a^19*b^2*c^6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^1
6*d^25*e^2*z^6 - 133809831936*a^14*b^6*c^7*d^11*e^16*z^6 - 133809831936*a^1
0*b^6*c^11*d^19*e^8*z^6 - 2214592512*a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a
^10*b*c^16*d^24*e^3*z^6 + 82216747008*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747
008*a^10*b^7*c^10*d^18*e^9*z^6 - 586629120*a^12*b^13*c^2*d^8*e^19*z^6 - 586
629120*a^5*b^13*c^9*d^22*e^5*z^6 + 568565760*a^7*b^16*c^4*d^15*e^12*z^6 - 4
844421120*a^16*b^4*c^7*d^9*e^18*z^6 - 4844421120*a^10*b^4*c^13*d^21*e^6*z^6
 + 531210240*a^11*b^14*c^2*d^9*e^18*z^6 + 531210240*a^5*b^14*c^8*d^21*e^6*z
^6 - 527155200*a^11*b^13*c^3*d^10*e^17*z^6 - 527155200*a^6*b^13*c^8*d^20*e^
7*z^6 + 43470028800*a^11*b^8*c^8*d^15*e^12*z^6 - 107874877440*a^11*b^9*c^7*
d^14*e^13*z^6 - 107874877440*a^10*b^9*c^8*d^16*e^11*z^6 + 9018408960*a^12*b
^11*c^4*d^10*e^17*z^6 + 9018408960*a^7*b^11*c^9*d^20*e^7*z^6 + 421994496*a^
13*b^12*c^2*d^7*e^20*z^6 + 421994496*a^5*b^12*c^10*d^23*e^4*z^6 - 664377753
60*a^16*b*c^10*d^12*e^15*z^6 - 66437775360*a^13*b*c^13*d^18*e^9*z^6 + 26159
874048*a^16*b^5*c^6*d^8*e^19*z^6 + 26159874048*a^9*b^5*c^13*d^22*e^5*z^6 -
369098752*a^18*b^3*c^6*d^6*e^21*z^6 - 369098752*a^9*b^3*c^15*d^24*e^3*z^6 +
 351436800*a^8*b^16*c^3*d^13*e^14*z^6 + 351436800*a^6*b^16*c^5*d^17*e^10*z^
6 - 334233600*a^16*b^8*c^3*d^5*e^22*z^6 - 334233600*a^6*b^8*c^13*d^25*e^2*z
^6 + 301989888*a^19*b^3*c^5*d^4*e^23*z^6 - 266010624*a^10*b^15*c^2*d^10*e^1
7*z^6 - 266010624*a^5*b^15*c^7*d^20*e^7*z^6 - 305198530560*a^12*b^6*c^9*d^1
5*e^12*z^6 - 203292672*a^14*b^11*c^2*d^6*e^21*z^6 - 203292672*a^5*b^11*c^11
*d^24*e^3*z^6 - 188743680*a^18*b^5*c^4*d^4*e^23*z^6 + 120418467840*a^16*b^2
*c^9*d^11*e^16*z^6 + 120418467840*a^12*b^2*c^13*d^19*e^8*z^6 - 17293934592*
a^10*b^12*c^5*d^13*e^14*z^6 - 17293934592*a^8*b^12*c^7*d^17*e^10*z^6 + 1048
90368*a^8*b^17*c^2*d^12*e^15*z^6 + 104890368*a^5*b^17*c^5*d^18*e^9*z^6 + 43
90256640*a^15*b^8*c^4*d^7*e^20*z^6 + 4390256640*a^7*b^8*c^12*d^23*e^4*z^6 -
 91750400*a^6*b^18*c^3*d^15*e^12*z^6 + 79134720*a^7*b^17*c^3*d^14*e^13*z^6
+ 79134720*a^6*b^17*c^4*d^16*e^11*z^6 - 74612736*a^4*b^16*c^7*d^21*e^6*z^6
- 72990720*a^7*b^18*c^2*d^13*e^14*z^6 - 72990720*a^5*b^18*c^4*d^17*e^10*z^6
 + 69746688*a^4*b^15*c^8*d^22*e^5*z^6 + 63700992*a^15*b^10*c^2*d^5*e^22*z^6
 + 63700992*a^5*b^10*c^12*d^25*e^2*z^6 + 62914560*a^17*b^7*c^3*d^4*e^23*z^6
 + 55148544*a^4*b^17*c^6*d^20*e^7*z^6 - 45957120*a^4*b^14*c^9*d^23*e^4*z^6
- 25952256*a^4*b^18*c^5*d^19*e^8*z^6 - 25165824*a^20*b^2*c^5*d^3*e^24*z^6 +
 21086208*a^4*b^13*c^10*d^24*e^3*z^6 + 20643840*a^6*b^19*c^2*d^14*e^13*z^6
+ 20643840*a^5*b^19*c^3*d^16*e^11*z^6 + 15728640*a^19*b^4*c^4*d^3*e^24*z^6
- 11796480*a^16*b^9*c^2*d^4*e^23*z^6 - 6438912*a^4*b^12*c^11*d^25*e^2*z^6 +
 5406720*a^4*b^19*c^4*d^18*e^9*z^6 - 5242880*a^18*b^6*c^3*d^3*e^24*z^6 + 37
84704*a^3*b^18*c^6*d^21*e^6*z^6 - 3244032*a^3*b^19*c^5*d^20*e^7*z^6 - 32440
32*a^3*b^17*c^7*d^22*e^5*z^6 + 2027520*a^3*b^20*c^4*d^19*e^8*z^6 + 2027520*
a^3*b^16*c^8*d^23*e^4*z^6 - 1622016*a^9*b^16*c^2*d^11*e^16*z^6 - 1622016*a^
```

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5*b^16*c^6*d^19*e^8*z^6 + 1622016*a^4*b^20*c^3*d^17*e^10*z^6 - 1523712*a^4*
b^21*c^2*d^16*e^11*z^6 + 983040*a^17*b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^21
*c^3*d^18*e^9*z^6 - 901120*a^3*b^15*c^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^2*
d^17*e^10*z^6 + 270336*a^3*b^14*c^10*d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d^1
5*e^12*z^6 - 38593888256*a^15*b^6*c^6*d^9*e^18*z^6 - 38593888256*a^9*b^6*c^
12*d^21*e^6*z^6 - 210386288640*a^15*b^3*c^9*d^12*e^15*z^6 - 210386288640*a^
12*b^3*c^12*d^18*e^9*z^6 + 15502147584*a^15*c^12*d^15*e^12*z^6 + 1107296256
*a^19*c^8*d^7*e^20*z^6 + 1107296256*a^11*c^16*d^23*e^4*z^6 + 13287555072*a^
16*c^11*d^13*e^14*z^6 + 13287555072*a^14*c^13*d^17*e^10*z^6 + 201326592*a^2
0*c^7*d^5*e^22*z^6 + 201326592*a^10*c^17*d^25*e^2*z^6 + 16777216*a^21*c^6*d
^3*e^24*z^6 + 3784704*a^9*b^18*d^9*e^18*z^6 - 3244032*a^10*b^17*d^8*e^19*z^
6 - 3244032*a^8*b^19*d^10*e^17*z^6 + 2027520*a^11*b^16*d^7*e^20*z^6 + 20275
20*a^7*b^20*d^11*e^16*z^6 - 901120*a^12*b^15*d^6*e^21*z^6 - 901120*a^6*b^21
*d^12*e^15*z^6 + 270336*a^13*b^14*d^5*e^22*z^6 + 270336*a^5*b^22*d^13*e^14*
z^6 - 49152*a^14*b^13*d^4*e^23*z^6 - 49152*a^4*b^23*d^14*e^13*z^6 + 4096*a^
15*b^12*d^3*e^24*z^6 + 4096*a^3*b^24*d^15*e^12*z^6 - 25165824*a^8*b^2*c^17*
d^27*z^6 + 15728640*a^7*b^4*c^16*d^27*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6 +
 983040*a^5*b^8*c^14*d^27*z^6 - 98304*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b^1
2*c^12*d^27*z^6 + 8304721920*a^17*c^10*d^11*e^16*z^6 + 8304721920*a^13*c^14
*d^19*e^8*z^6 + 3690987520*a^18*c^9*d^9*e^18*z^6 + 3690987520*a^12*c^15*d^2
1*e^6*z^6 + 16777216*a^9*c^18*d^27*z^6 - 8493371392*a^6*b^8*c^9*d^14*e^9*z^
4 + 1458044928*a^8*b*c^14*d^17*e^6*z^4 - 12604538880*a^11*b^4*c^8*d^8*e^15*
z^4 - 8303067136*a^9*b^5*c^9*d^11*e^12*z^4 - 5588058112*a^13*b*c^9*d^7*e^16
*z^4 - 3892838400*a^8*b^2*c^13*d^16*e^7*z^4 - 3611713536*a^8*b^8*c^7*d^10*e
^13*z^4 + 7819006464*a^7*b^9*c^7*d^11*e^12*z^4 - 7782137856*a^8*b^7*c^8*d^1
1*e^12*z^4 + 7780433920*a^12*b^2*c^9*d^8*e^15*z^4 - 12020465664*a^7*b^5*c^1
1*d^15*e^8*z^4 + 3176792064*a^8*b^3*c^12*d^15*e^8*z^4 - 322633728*a^15*b*c^
7*d^3*e^20*z^4 + 210829312*a^7*b*c^15*d^19*e^4*z^4 + 15623258112*a^9*b^6*c^
8*d^10*e^13*z^4 + 25165824*a^15*b^3*c^5*d*e^22*z^4 - 15728640*a^14*b^5*c^4*
d*e^22*z^4 + 12582912*a^5*b^2*c^16*d^22*e*z^4 - 11730944*a^4*b^4*c^15*d^22*
e*z^4 + 5242880*a^13*b^7*c^3*d*e^22*z^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4 +
 4521984*a^3*b^6*c^14*d^22*e*z^4 + 4460544*a*b^14*c^8*d^18*e^5*z^4 + 353894
4*a^6*b*c^16*d^21*e^2*z^4 + 3108864*a*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b^1
3*c^9*d^19*e^4*z^4 - 2345472*a^5*b^17*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c*d
^4*e^19*z^4 + 1824768*a^6*b^16*c*d^6*e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^20
*z^4 + 1419264*a*b^12*c^10*d^20*e^3*z^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4 -
 983040*a^12*b^9*c^2*d*e^22*z^4 + 964608*a^4*b^18*c*d^8*e^15*z^4 - 866304*a
^2*b^8*c^13*d^22*e*z^4 + 703488*a^7*b^15*c*d^5*e^18*z^4 - 608256*a^10*b^12*
c*d^2*e^21*z^4 - 440832*a*b^11*c^11*d^21*e^2*z^4 + 275968*a*b^19*c^3*d^13*e
^10*z^4 - 159744*a^2*b^20*c*d^10*e^13*z^4 - 153600*a*b^20*c^2*d^12*e^11*z^4
 + 64512*a^3*b^19*c*d^9*e^14*z^4 + 19746062336*a^8*b^6*c^9*d^12*e^11*z^4 -
15333588992*a^10*b^4*c^9*d^10*e^13*z^4 + 6702170112*a^7*b^4*c^12*d^16*e^7*z
^4 + 15167913984*a^10*b^3*c^10*d^11*e^12*z^4 - 2256638976*a^5*b^11*c^7*d^13
*e^10*z^4 + 2254307328*a^5*b^7*c^11*d^17*e^6*z^4 - 2200633344*a^6*b^5*c^12*
d^17*e^6*z^4 + 6457131008*a^11*b^3*c^9*d^9*e^14*z^4 - 2128785408*a^5*b^8*c^
```

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10*d^16*e^7*z^4 - 2126057472*a^6*b^11*c^6*d^11*e^12*z^4 + 2038349824*a^12*b
^5*c^6*d^5*e^18*z^4 + 2037841920*a^5*b^10*c^8*d^14*e^9*z^4 + 3615621120*a^9
*b*c^13*d^15*e^8*z^4 + 1900019712*a^11*b^2*c^10*d^10*e^13*z^4 + 1867698432*
a^9*b^9*c^5*d^7*e^16*z^4 - 6157369344*a^9*b^4*c^10*d^12*e^11*z^4 - 18569134
08*a^7*b^10*c^6*d^10*e^13*z^4 + 1789132800*a^6*b^4*c^13*d^18*e^5*z^4 + 6082
658304*a^8*b^4*c^11*d^14*e^9*z^4 + 6029549568*a^11*b^5*c^7*d^7*e^16*z^4 + 6
010159104*a^6*b^7*c^10*d^15*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^13*e^10*z^4
+ 1658388480*a^11*b^6*c^6*d^6*e^17*z^4 + 5917114368*a^10*b^6*c^7*d^8*e^15*z
^4 - 1591197696*a^11*b^7*c^5*d^5*e^18*z^4 - 1526464512*a^8*b^10*c^5*d^8*e^1
5*z^4 - 5772607488*a^12*b^4*c^7*d^6*e^17*z^4 - 1423507456*a^13*b^4*c^6*d^4*
e^19*z^4 - 1387266048*a^7*b^3*c^13*d^17*e^6*z^4 + 2976120832*a^10*b*c^12*d^
13*e^10*z^4 - 9906946048*a^9*b^2*c^12*d^14*e^9*z^4 - 18421874688*a^8*b^5*c^
10*d^13*e^10*z^4 + 1141217280*a^6*b^12*c^5*d^10*e^13*z^4 - 9714364416*a^7*b
^8*c^8*d^12*e^11*z^4 - 16777216*a^16*b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c*d
*e^22*z^4 + 81920*a*b^10*c^12*d^22*e*z^4 + 39168*a*b^21*c*d^11*e^12*z^4 - 1
091829760*a^5*b^6*c^12*d^18*e^5*z^4 + 1046740992*a^14*b^2*c^7*d^4*e^19*z^4
- 6884425728*a^12*b*c^10*d^9*e^14*z^4 + 987445248*a^4*b^10*c^9*d^16*e^7*z^4
 + 984087552*a^5*b^12*c^6*d^12*e^11*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^14*z
^4 - 5266857984*a^10*b^7*c^6*d^7*e^16*z^4 - 892145664*a^7*b^11*c^5*d^9*e^14
*z^4 - 2444623872*a^11*b*c^11*d^11*e^12*z^4 + 768540672*a^12*b^3*c^8*d^7*e^
16*z^4 + 5048322048*a^8*b^9*c^6*d^9*e^14*z^4 + 5047612416*a^6*b^9*c^8*d^13*
e^10*z^4 - 732492288*a^4*b^11*c^8*d^15*e^8*z^4 + 9266921472*a^7*b^6*c^10*d^
14*e^9*z^4 - 645857280*a^6*b^6*c^11*d^16*e^7*z^4 - 623867904*a^4*b^9*c^10*d
^17*e^6*z^4 - 622067712*a^6*b^3*c^14*d^19*e^4*z^4 + 582617088*a^10*b^8*c^5*
d^6*e^17*z^4 + 577119744*a^7*b^12*c^4*d^8*e^15*z^4 + 552566784*a^12*b^6*c^5
*d^4*e^19*z^4 + 549224448*a^9*b^8*c^6*d^8*e^15*z^4 - 526565376*a^9*b^10*c^4
*d^6*e^17*z^4 + 511520256*a^10*b^9*c^4*d^5*e^18*z^4 + 13393723392*a^9*b^3*c
^11*d^13*e^10*z^4 - 2066350080*a^14*b*c^8*d^5*e^18*z^4 + 4718592000*a^13*b^
2*c^8*d^6*e^17*z^4 - 314572800*a^7*b^2*c^14*d^18*e^5*z^4 + 287250432*a^4*b^
13*c^6*d^13*e^10*z^4 + 4565827584*a^10*b^5*c^8*d^9*e^14*z^4 - 250785792*a^4
*b^14*c^5*d^12*e^11*z^4 + 235536384*a^13*b^3*c^7*d^5*e^18*z^4 - 232683264*a
^8*b^11*c^4*d^7*e^16*z^4 - 199627776*a^5*b^14*c^4*d^10*e^13*z^4 - 190267392
*a^12*b^7*c^4*d^3*e^20*z^4 + 184891392*a^6*b^10*c^7*d^12*e^11*z^4 + 1805025
28*a^4*b^7*c^12*d^19*e^4*z^4 + 178877952*a^3*b^13*c^7*d^15*e^8*z^4 + 172490
752*a^14*b^3*c^6*d^3*e^20*z^4 + 163946496*a^13*b^5*c^5*d^3*e^20*z^4 + 15583
9488*a^8*b^12*c^3*d^6*e^17*z^4 + 155000832*a^5*b^5*c^13*d^19*e^4*z^4 - 1520
76288*a^4*b^6*c^13*d^20*e^3*z^4 - 137592576*a^3*b^12*c^8*d^16*e^7*z^4 - 133
693440*a^14*b^4*c^5*d^2*e^21*z^4 - 116767488*a^3*b^9*c^11*d^19*e^4*z^4 - 10
8985344*a^3*b^14*c^6*d^14*e^9*z^4 - 106223616*a^6*b^13*c^4*d^9*e^14*z^4 + 1
06119168*a^3*b^10*c^10*d^18*e^5*z^4 + 102432768*a^5*b^4*c^14*d^20*e^3*z^4 +
 102113280*a^4*b^12*c^7*d^14*e^9*z^4 + 100674048*a^5*b^9*c^9*d^15*e^8*z^4 +
 90439680*a^13*b^6*c^4*d^2*e^21*z^4 - 86808576*a^6*b^14*c^3*d^8*e^15*z^4 +
86245376*a^6*b^2*c^15*d^20*e^3*z^4 + 79011840*a^4*b^8*c^11*d^18*e^5*z^4 + 7
8345216*a^4*b^15*c^4*d^11*e^12*z^4 + 78006528*a^11*b^9*c^3*d^3*e^20*z^4 - 7
3253376*a^9*b^11*c^3*d^5*e^18*z^4 + 67524608*a^3*b^8*c^12*d^20*e^3*z^4 + 67
```

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108864*a^15*b^2*c^6*d^2*e^21*z^4 - 61590528*a^10*b^10*c^3*d^4*e^19*z^4 + 61
559808*a^5*b^15*c^3*d^9*e^14*z^4 - 59637760*a^5*b^3*c^15*d^21*e^2*z^4 + 586
38336*a^4*b^5*c^14*d^21*e^2*z^4 - 40828416*a^7*b^13*c^3*d^7*e^16*z^4 - 3563
9296*a^2*b^12*c^9*d^18*e^5*z^4 - 31293440*a^12*b^8*c^3*d^2*e^21*z^4 + 29933
568*a^5*b^13*c^5*d^11*e^12*z^4 + 27793920*a^2*b^11*c^10*d^19*e^4*z^4 + 2716
8768*a^2*b^13*c^8*d^17*e^6*z^4 - 23602176*a^7*b^14*c^2*d^6*e^17*z^4 - 23248
896*a^3*b^7*c^13*d^21*e^2*z^4 + 20929536*a^3*b^15*c^5*d^13*e^10*z^4 + 18428
928*a^9*b^12*c^2*d^4*e^19*z^4 + 18026496*a^6*b^15*c^2*d^7*e^16*z^4 - 162616
32*a^10*b^11*c^2*d^3*e^20*z^4 + 15128064*a^3*b^16*c^4*d^12*e^11*z^4 - 14060
544*a^2*b^10*c^11*d^20*e^3*z^4 + 13178880*a^2*b^16*c^5*d^14*e^9*z^4 - 11244
288*a^3*b^17*c^3*d^11*e^12*z^4 - 10509312*a^2*b^15*c^6*d^15*e^8*z^4 - 72622
08*a^4*b^17*c^2*d^9*e^14*z^4 - 7045632*a^2*b^17*c^4*d^13*e^10*z^4 - 6285312
*a^2*b^14*c^7*d^16*e^7*z^4 + 5996544*a^11*b^10*c^2*d^2*e^21*z^4 + 4558336*a
^2*b^9*c^12*d^21*e^2*z^4 + 4478976*a^11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^4*
b^16*c^3*d^10*e^13*z^4 + 2629632*a^3*b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*b^
18*c^2*d^10*e^13*z^4 + 1627136*a^2*b^18*c^3*d^12*e^11*z^4 + 1605120*a^8*b^1
3*c^2*d^5*e^18*z^4 + 1483776*a^5*b^16*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*c^
2*d^11*e^12*z^4 - 8542224384*a^10*b^2*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^12
*e^11*z^4 - 3072*b^12*c^11*d^22*e*z^4 - 1572864*a^6*c^17*d^22*e*z^4 - 4096*
a^10*b^13*d*e^22*z^4 - 4096*a*b^22*d^10*e^13*z^4 - 6144*a^12*b^10*c*e^23*z^
4 - 983040*a^5*b*c^17*d^23*z^4 - 6912*a*b^9*c^13*d^23*z^4 + 1824522240*a^13
*c^10*d^8*e^15*z^4 + 1730150400*a^12*c^11*d^10*e^13*z^4 + 958922752*a^14*c^
9*d^6*e^17*z^4 - 537919488*a^9*c^14*d^16*e^7*z^4 + 508559360*a^11*c^12*d^12
*e^11*z^4 - 500170752*a^10*c^13*d^14*e^9*z^4 + 246939648*a^15*c^8*d^4*e^19*
z^4 - 199229440*a^8*c^15*d^18*e^5*z^4 - 29884416*a^7*c^16*d^20*e^3*z^4 + 25
165824*a^16*c^7*d^2*e^21*z^4 + 236544*b^17*c^6*d^17*e^6*z^4 - 202752*b^18*c
^5*d^16*e^7*z^4 - 202752*b^16*c^7*d^18*e^5*z^4 + 126720*b^19*c^4*d^15*e^8*z
^4 + 126720*b^15*c^8*d^19*e^4*z^4 - 56320*b^20*c^3*d^14*e^9*z^4 - 56320*b^1
4*c^9*d^20*e^3*z^4 + 16896*b^21*c^2*d^13*e^10*z^4 + 16896*b^13*c^10*d^21*e^
2*z^4 + 110080*a^7*b^16*d^4*e^19*z^4 + 110080*a^4*b^19*d^7*e^16*z^4 - 75520
*a^8*b^15*d^3*e^20*z^4 - 75520*a^3*b^20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5*e
^18*z^4 - 56320*a^5*b^18*d^6*e^17*z^4 + 25600*a^9*b^14*d^2*e^21*z^4 + 25600
*a^2*b^21*d^9*e^14*z^4 - 1572864*a^16*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*c^
4*e^23*z^4 - 327680*a^14*b^6*c^3*e^23*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 + 9
83040*a^4*b^3*c^16*d^23*z^4 - 385024*a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^7*
c^14*d^23*z^4 + 256*b^23*d^11*e^12*z^4 + 1048576*a^17*c^6*e^23*z^4 + 256*b^
11*c^12*d^23*z^4 + 256*a^11*b^12*e^23*z^4 + 948695040*a^8*b*c^10*d^6*e^13*z
^2 + 348917760*a^7*b*c^11*d^8*e^11*z^2 - 125030400*a^9*b*c^9*d^4*e^15*z^2 -
 50728960*a^6*b*c^12*d^10*e^9*z^2 - 44298240*a^5*b*c^13*d^12*e^7*z^2 - 3649
5360*a^10*b*c^8*d^2*e^17*z^2 + 29675520*a^8*b^6*c^5*d*e^18*z^2 - 24170496*a
^9*b^4*c^6*d*e^18*z^2 - 17202816*a^7*b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*c^
14*d^14*e^5*z^2 + 5532416*a^6*b^10*c^3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*d*
e^18*z^2 - 2662400*a^3*b*c^15*d^16*e^3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*z^
2 - 1136160*a*b^13*c^5*d^8*e^11*z^2 - 1017600*a^5*b^12*c^2*d*e^18*z^2 - 744
768*a*b^11*c^7*d^10*e^9*z^2 + 607872*a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b^6
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*c^12*d^15*e^4*z^2 + 408576*a*b^5*c^13*d^16*e^3*z^2 + 361152*a*b^10*c^8*d^1
1*e^8*z^2 - 287408*a*b^9*c^9*d^12*e^7*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^2
- 203904*a*b^4*c^14*d^17*e^2*z^2 + 200832*a*b^8*c^10*d^13*e^6*z^2 + 126720*
a*b^7*c^11*d^14*e^5*z^2 - 123968*a*b^15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c^2
*d^5*e^14*z^2 + 11904*a^2*b^16*c*d^3*e^16*z^2 + 1824135552*a^7*b^4*c^8*d^5*
e^14*z^2 - 1457252352*a^8*b^2*c^9*d^5*e^14*z^2 - 1405209600*a^7*b^5*c^7*d^4
*e^15*z^2 - 184320*a^2*b*c^16*d^18*e*z^2 + 100608*a^4*b^14*c*d*e^18*z^2 + 5
3248*a*b^3*c^15*d^18*e*z^2 + 26448*a*b^17*c*d^4*e^15*z^2 + 1067599872*a^8*b
^3*c^8*d^4*e^15*z^2 - 930828288*a^7*b^3*c^9*d^6*e^13*z^2 + 920760000*a^6*b^
4*c^9*d^7*e^12*z^2 - 806639616*a^6*b^3*c^10*d^8*e^11*z^2 - 791052480*a^6*b^
6*c^7*d^5*e^14*z^2 + 772237824*a^6*b^7*c^6*d^4*e^15*z^2 - 701025408*a^5*b^6
*c^8*d^7*e^12*z^2 + 443340288*a^5*b^5*c^9*d^8*e^11*z^2 + 433047552*a^7*b^6*
c^6*d^3*e^16*z^2 + 405741312*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2*c
^11*d^9*e^10*z^2 - 276962688*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4*c
^7*d^3*e^16*z^2 + 213564384*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*c^
5*d^4*e^15*z^2 - 182520896*a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c^1
1*d^10*e^9*z^2 - 152151552*a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*c^
12*d^11*e^8*z^2 + 108085248*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*c^
10*d^10*e^9*z^2 - 98323200*a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^9*
d^9*e^10*z^2 + 89464512*a^5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*d^
2*e^17*z^2 + 68315904*a^5*b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8*e
^11*z^2 - 62951040*a^9*b^2*c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e^1
4*z^2 + 47614464*a^3*b^8*c^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^6*
z^2 + 33983040*a^3*b^11*c^5*d^6*e^13*z^2 - 33515520*a^4*b^3*c^12*d^12*e^7*z
^2 - 33463808*a^3*b^7*c^9*d^10*e^9*z^2 - 25128864*a^4*b^4*c^11*d^11*e^8*z^2
 - 23193728*a^3*b^10*c^6*d^7*e^12*z^2 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2 +
 19924176*a^4*b^11*c^4*d^4*e^15*z^2 - 19251216*a^3*b^9*c^7*d^8*e^11*z^2 - 1
6434048*a^5*b^4*c^10*d^9*e^10*z^2 - 16289664*a^3*b^12*c^4*d^5*e^14*z^2 - 15
059328*a^4*b^12*c^3*d^3*e^16*z^2 - 10766016*a^2*b^10*c^7*d^9*e^10*z^2 - 104
53632*a^5*b^11*c^3*d^2*e^17*z^2 - 9940992*a^3*b^3*c^13*d^14*e^5*z^2 + 83736
96*a^2*b^11*c^6*d^8*e^11*z^2 + 7776768*a^3*b^2*c^14*d^15*e^4*z^2 + 7077888*
a^3*b^5*c^11*d^12*e^7*z^2 + 6798240*a^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^2*
b^6*c^11*d^13*e^6*z^2 + 3544320*a^3*b^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b^5
*c^12*d^14*e^5*z^2 + 2346336*a^4*b^13*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*c^
9*d^11*e^8*z^2 - 2125824*a^2*b^13*c^4*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12*d
^13*e^6*z^2 + 1927680*a^2*b^7*c^10*d^12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^5*
e^14*z^2 - 1807104*a^2*b^12*c^5*d^7*e^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e^1
5*z^2 + 1083456*a^3*b^14*c^2*d^3*e^16*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*z^
2 - 657408*a^2*b^3*c^14*d^16*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^17*z^2 + 59
5968*a^2*b^2*c^15*d^17*e^2*z^2 - 498624*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*b^
18*c*d^5*e^14*z^2 - 3840*b^5*c^14*d^18*e*z^2 + 2064384*a^11*c^8*d*e^18*z^2
- 4160*a^3*b^16*d*e^18*z^2 - 4160*a*b^18*d^3*e^16*z^2 - 1290240*a^11*b*c^7*
e^19*z^2 - 9840*a^5*b^13*c*e^19*z^2 - 5760*a*b^2*c^16*d^19*z^2 - 280581120*
a^8*c^11*d^7*e^12*z^2 + 110278656*a^9*c^10*d^5*e^14*z^2 - 89479168*a^7*c^12
*d^9*e^10*z^2 + 34464000*a^10*c^9*d^3*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*z^
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2 + 54240*b^8*c^11*d^15*e^4*z^2 - 49920*b^14*c^5*d^9*e^10*z^2 - 49920*b^9*c
^10*d^14*e^5*z^2 - 37376*b^16*c^3*d^7*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*z^
2 + 28480*b^13*c^6*d^10*e^9*z^2 + 28480*b^10*c^9*d^13*e^6*z^2 + 15936*b^17*
c^2*d^6*e^13*z^2 + 15936*b^6*c^13*d^17*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z^2
 - 7920*b^11*c^8*d^12*e^7*z^2 + 7489536*a^5*c^14*d^13*e^6*z^2 + 6084096*a^6
*c^13*d^11*e^8*z^2 + 2280448*a^4*c^15*d^15*e^4*z^2 + 350208*a^3*c^16*d^17*e
^2*z^2 + 11616*a^2*b^17*d^2*e^17*z^2 - 3515904*a^9*b^5*c^5*e^19*z^2 + 34406
40*a^10*b^3*c^6*e^19*z^2 + 1870848*a^8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*c^
3*e^19*z^2 + 101856*a^6*b^11*c^2*e^19*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b^4
*c^15*d^19*z^2 + 20736*a^2*c^17*d^19*z^2 + 400*a^4*b^15*e^19*z^2 - 3969216*
a^4*b*c^10*d^3*e^12 - 3001536*a^3*b*c^11*d^5*e^10 - 419904*a^2*b*c^12*d^7*e
^8 + 184608*a^4*b^3*c^8*d*e^14 - 153036*a*b^4*c^10*d^6*e^9 + 127008*a*b^3*c
^11*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^11 - 29160*a*b^2*c^12*d^8*e^7 - 21060*a
^3*b^5*c^7*d*e^14 - 21060*a*b^7*c^7*d^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 - 40
4544*a^5*b*c^9*d*e^14 + 1251872*a^3*b^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9*d
^2*e^13 + 820512*a^2*b^3*c^10*d^5*e^10 + 750672*a^3*b^2*c^10*d^4*e^11 - 657
498*a^2*b^4*c^9*d^4*e^11 - 487116*a^3*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c^1
1*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^13 + 13140*a^2*b^5*c^8*d^3*e^12 + 15286
*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b^8
*c^7*d^4*e^11 + 2025*b^4*c^11*d^8*e^7 + 3367008*a^4*c^11*d^4*e^11 + 1166400
*a^3*c^12*d^6*e^9 + 705600*a^5*c^10*d^2*e^13 + 104976*a^2*c^13*d^8*e^7 - 17
640*a^5*b^2*c^8*e^15 + 2025*a^4*b^4*c^7*e^15 + 38416*a^6*c^9*e^15, z, k)*((
1048576*a^17*c^8*d*e^24 - 393216*a^6*c^19*d^23*e^2 - 3407872*a^7*c^18*d^21*
e^4 - 5636096*a^8*c^17*d^19*e^6 + 31457280*a^9*c^16*d^17*e^8 + 175374336*a^
10*c^15*d^15*e^10 + 407371776*a^11*c^14*d^13*e^12 + 556007424*a^12*c^13*d^1
1*e^14 + 481296384*a^13*c^12*d^9*e^16 + 265420800*a^14*c^11*d^7*e^18 + 8886
6816*a^15*c^10*d^5*e^20 + 15859712*a^16*c^9*d^3*e^22 - 5632*a^2*b^8*c^15*d^
23*e^2 + 67584*a^2*b^9*c^14*d^22*e^3 - 368640*a^2*b^10*c^13*d^21*e^4 + 1205
248*a^2*b^11*c^12*d^20*e^5 - 2618880*a^2*b^12*c^11*d^19*e^6 + 3953664*a^2*b
^13*c^10*d^18*e^7 - 4190208*a^2*b^14*c^9*d^17*e^8 + 3041280*a^2*b^15*c^8*d^
16*e^9 - 1368576*a^2*b^16*c^7*d^15*e^10 + 225280*a^2*b^17*c^6*d^14*e^11 + 1
35168*a^2*b^18*c^5*d^13*e^12 - 101376*a^2*b^19*c^4*d^12*e^13 + 28160*a^2*b^
20*c^3*d^11*e^14 - 3072*a^2*b^21*c^2*d^10*e^15 + 49152*a^3*b^6*c^16*d^23*e^
2 - 589824*a^3*b^7*c^15*d^22*e^3 + 3181568*a^3*b^8*c^14*d^21*e^4 - 10121216
*a^3*b^9*c^13*d^20*e^5 + 20854016*a^3*b^10*c^12*d^19*e^6 - 28504064*a^3*b^1
1*c^11*d^18*e^7 + 24727808*a^3*b^12*c^10*d^17*e^8 - 10510336*a^3*b^13*c^9*d
^16*e^9 - 3040768*a^3*b^14*c^8*d^15*e^10 + 7405568*a^3*b^15*c^7*d^14*e^11 -
 4684288*a^3*b^16*c^6*d^13*e^12 + 1314816*a^3*b^17*c^5*d^12*e^13 - 12032*a^
3*b^18*c^4*d^11*e^14 - 86016*a^3*b^19*c^3*d^10*e^15 + 15616*a^3*b^20*c^2*d^
9*e^16 - 212992*a^4*b^4*c^17*d^23*e^2 + 2555904*a^4*b^5*c^16*d^22*e^3 - 135
49568*a^4*b^6*c^15*d^21*e^4 + 41189376*a^4*b^7*c^14*d^20*e^5 - 76867072*a^4
*b^8*c^13*d^19*e^6 + 83304448*a^4*b^9*c^12*d^18*e^7 - 29710336*a^4*b^10*c^1
1*d^17*e^8 - 53473280*a^4*b^11*c^10*d^16*e^9 + 94751744*a^4*b^12*c^9*d^15*e
^10 - 68968448*a^4*b^13*c^8*d^14*e^11 + 20899840*a^4*b^14*c^7*d^13*e^12 + 4
022272*a^4*b^15*c^6*d^12*e^13 - 5248512*a^4*b^16*c^5*d^11*e^14 + 1310720*a^
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4*b^17*c^4*d^10*e^15 + 40960*a^4*b^18*c^3*d^9*e^16 - 45056*a^4*b^19*c^2*d^8
*e^17 + 458752*a^5*b^2*c^18*d^23*e^2 - 5505024*a^5*b^3*c^17*d^22*e^3 + 2821
3248*a^5*b^4*c^16*d^21*e^4 - 77725696*a^5*b^5*c^15*d^20*e^5 + 109985792*a^5
*b^6*c^14*d^19*e^6 - 16252928*a^5*b^7*c^13*d^18*e^7 - 236929024*a^5*b^8*c^1
2*d^17*e^8 + 460423168*a^5*b^9*c^11*d^16*e^9 - 412556800*a^5*b^10*c^10*d^15
*e^10 + 137754624*a^5*b^11*c^9*d^14*e^11 + 80635904*a^5*b^12*c^8*d^13*e^12
- 102774784*a^5*b^13*c^7*d^12*e^13 + 36015104*a^5*b^14*c^6*d^11*e^14 + 1345
536*a^5*b^15*c^5*d^10*e^15 - 3577856*a^5*b^16*c^4*d^9*e^16 + 407552*a^5*b^1
7*c^3*d^8*e^17 + 82432*a^5*b^18*c^2*d^7*e^18 - 21757952*a^6*b^2*c^17*d^21*e
^4 + 39059456*a^6*b^3*c^16*d^20*e^5 + 44351488*a^6*b^4*c^15*d^19*e^6 - 3816
81664*a^6*b^5*c^14*d^18*e^7 + 872808448*a^6*b^6*c^13*d^17*e^8 - 981073920*a
^6*b^7*c^12*d^16*e^9 + 329307136*a^6*b^8*c^11*d^15*e^10 + 558870528*a^6*b^9
*c^10*d^14*e^11 - 809418752*a^6*b^10*c^9*d^13*e^12 + 394459136*a^6*b^11*c^8
*d^12*e^13 + 10594304*a^6*b^12*c^7*d^11*e^14 - 84887552*a^6*b^13*c^6*d^10*e
^15 + 23650304*a^6*b^14*c^5*d^9*e^16 + 2762752*a^6*b^15*c^4*d^8*e^17 - 1268
736*a^6*b^16*c^3*d^7*e^18 - 100352*a^6*b^17*c^2*d^6*e^19 - 192217088*a^7*b^
2*c^16*d^19*e^6 + 514850816*a^7*b^3*c^15*d^18*e^7 - 691208192*a^7*b^4*c^14*
d^17*e^8 + 8388608*a^7*b^5*c^13*d^16*e^9 + 1583054848*a^7*b^6*c^12*d^15*e^1
0 - 2597715968*a^7*b^7*c^11*d^14*e^11 + 1705592832*a^7*b^8*c^10*d^13*e^12 +
 65314816*a^7*b^9*c^9*d^12*e^13 - 792112640*a^7*b^10*c^8*d^11*e^14 + 396832
768*a^7*b^11*c^7*d^10*e^15 + 5305856*a^7*b^12*c^6*d^9*e^16 - 47955968*a^7*b
^13*c^5*d^8*e^17 + 4476416*a^7*b^14*c^4*d^7*e^18 + 1921024*a^7*b^15*c^3*d^6
*e^19 + 82432*a^7*b^16*c^2*d^5*e^20 - 472383488*a^8*b^2*c^15*d^17*e^8 + 155
2941056*a^8*b^3*c^14*d^16*e^9 - 2815066112*a^8*b^4*c^13*d^15*e^10 + 2329542
656*a^8*b^5*c^12*d^14*e^11 + 631472128*a^8*b^6*c^11*d^13*e^12 - 3123511296*
a^8*b^7*c^10*d^12*e^13 + 2406024192*a^8*b^8*c^9*d^11*e^14 - 253763584*a^8*b
^9*c^8*d^10*e^15 - 535957504*a^8*b^10*c^7*d^9*e^16 + 196169728*a^8*b^11*c^6
*d^8*e^17 + 27567104*a^8*b^12*c^5*d^7*e^18 - 13180928*a^8*b^13*c^4*d^6*e^19
 - 1767424*a^8*b^14*c^3*d^5*e^20 - 45056*a^8*b^15*c^2*d^4*e^21 - 26345472*a
^9*b^2*c^14*d^15*e^10 + 1757937664*a^9*b^3*c^13*d^14*e^11 - 4680646656*a^9*
b^4*c^12*d^13*e^12 + 4978376704*a^9*b^5*c^11*d^12*e^13 - 1037008896*a^9*b^6
*c^10*d^11*e^14 - 2360082432*a^9*b^7*c^9*d^10*e^15 + 1791750144*a^9*b^8*c^8
*d^9*e^16 - 76677120*a^9*b^9*c^7*d^8*e^17 - 263758592*a^9*b^10*c^6*d^7*e^18
 + 28357632*a^9*b^11*c^5*d^6*e^19 + 14978560*a^9*b^12*c^4*d^5*e^20 + 102912
0*a^9*b^13*c^3*d^4*e^21 + 15616*a^9*b^14*c^2*d^3*e^22 + 1853358080*a^10*b^2
*c^13*d^13*e^12 + 106430464*a^10*b^3*c^12*d^12*e^13 - 4433149952*a^10*b^4*c
^11*d^11*e^14 + 5213257728*a^10*b^5*c^10*d^10*e^15 - 1239613440*a^10*b^6*c^
9*d^9*e^16 - 1399455744*a^10*b^7*c^8*d^8*e^17 + 721519104*a^10*b^8*c^7*d^7*
e^18 + 92768256*a^10*b^9*c^6*d^6*e^19 - 60235776*a^10*b^10*c^5*d^5*e^20 - 9
616384*a^10*b^11*c^4*d^4*e^21 - 369152*a^10*b^12*c^3*d^3*e^22 - 3072*a^10*b
^13*c^2*d^2*e^23 + 3744333824*a^11*b^2*c^12*d^11*e^14 - 1445986304*a^11*b^3
*c^11*d^10*e^15 - 2945974272*a^11*b^4*c^10*d^9*e^16 + 3180331008*a^11*b^5*c
^9*d^8*e^17 - 344997888*a^11*b^6*c^8*d^7*e^18 - 607715328*a^11*b^7*c^7*d^6*
e^19 + 91261952*a^11*b^8*c^6*d^5*e^20 + 46288896*a^11*b^9*c^5*d^4*e^21 + 36
19072*a^11*b^10*c^4*d^3*e^22 + 73728*a^11*b^11*c^3*d^2*e^23 + 3567255552*a^
```

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12*b^2*c^11*d^9*e^16 - 1152385024*a^12*b^3*c^10*d^8*e^17 - 1550467072*a^12*
b^4*c^9*d^7*e^18 + 1052180480*a^12*b^5*c^8*d^6*e^19 + 114114560*a^12*b^6*c^
7*d^5*e^20 - 115572736*a^12*b^7*c^6*d^4*e^21 - 18767360*a^12*b^8*c^5*d^3*e^
22 - 737280*a^12*b^9*c^4*d^2*e^23 + 1821048832*a^13*b^2*c^10*d^7*e^18 - 236
191744*a^13*b^3*c^9*d^6*e^19 - 544571392*a^13*b^4*c^8*d^5*e^20 + 114688000*
a^13*b^5*c^7*d^4*e^21 + 53821440*a^13*b^6*c^6*d^3*e^22 + 3932160*a^13*b^7*c
^5*d^2*e^23 + 460587008*a^14*b^2*c^9*d^5*e^20 + 57933824*a^14*b^3*c^8*d^4*e
^21 - 78659584*a^14*b^4*c^7*d^3*e^22 - 11796480*a^14*b^5*c^6*d^2*e^23 + 382
07488*a^15*b^2*c^8*d^3*e^22 + 18874368*a^15*b^3*c^7*d^2*e^23 + 256*a*b^10*c
^14*d^23*e^2 - 3072*a*b^11*c^13*d^22*e^3 + 16896*a*b^12*c^12*d^21*e^4 - 563
20*a*b^13*c^11*d^20*e^5 + 126720*a*b^14*c^10*d^19*e^6 - 202752*a*b^15*c^9*d
^18*e^7 + 236544*a*b^16*c^8*d^17*e^8 - 202752*a*b^17*c^7*d^16*e^9 + 126720*
a*b^18*c^6*d^15*e^10 - 56320*a*b^19*c^5*d^14*e^11 + 16896*a*b^20*c^4*d^13*e
^12 - 3072*a*b^21*c^3*d^12*e^13 + 256*a*b^22*c^2*d^11*e^14 + 4718592*a^6*b*
c^18*d^22*e^3 + 38797312*a^7*b*c^17*d^20*e^5 + 77594624*a^8*b*c^16*d^18*e^7
 - 159383552*a^9*b*c^15*d^16*e^9 - 1020264448*a^10*b*c^14*d^14*e^11 - 21286
09280*a^11*b*c^13*d^12*e^13 + 256*a^11*b^12*c^2*d*e^24 - 2451570688*a^12*b*
c^12*d^10*e^15 - 6144*a^12*b^10*c^3*d*e^24 - 1694498816*a^13*b*c^11*d^8*e^1
7 + 61440*a^13*b^8*c^4*d*e^24 - 691535872*a^14*b*c^10*d^6*e^19 - 327680*a^1
4*b^6*c^5*d*e^24 - 149946368*a^15*b*c^9*d^4*e^21 + 983040*a^15*b^4*c^6*d*e^
24 - 12582912*a^16*b*c^8*d^2*e^23 - 1572864*a^16*b^2*c^7*d*e^24)/(32*(16*a^
3*b^6*c^9*d^18 - a^2*b^8*c^8*d^18 - 256*a^6*c^12*d^18 - 96*a^4*b^4*c^10*d^1
8 + 256*a^5*b^2*c^11*d^18 - a^2*b^16*d^10*e^8 + 8*a^3*b^15*d^9*e^9 - 28*a^4
*b^14*d^8*e^10 + 56*a^5*b^13*d^7*e^11 - 70*a^6*b^12*d^6*e^12 + 56*a^7*b^11*
d^5*e^13 - 28*a^8*b^10*d^4*e^14 + 8*a^9*b^9*d^3*e^15 - a^10*b^8*d^2*e^16 -
2048*a^7*c^11*d^16*e^2 - 7168*a^8*c^10*d^14*e^4 - 14336*a^9*c^9*d^12*e^6 -
17920*a^10*c^8*d^10*e^8 - 14336*a^11*c^7*d^8*e^10 - 7168*a^12*c^6*d^6*e^12
- 2048*a^13*c^5*d^4*e^14 - 256*a^14*c^4*d^2*e^16 - 28*a^2*b^10*c^6*d^16*e^2
 + 56*a^2*b^11*c^5*d^15*e^3 - 70*a^2*b^12*c^4*d^14*e^4 + 56*a^2*b^13*c^3*d^
13*e^5 - 28*a^2*b^14*c^2*d^12*e^6 + 440*a^3*b^8*c^7*d^16*e^2 - 840*a^3*b^9*
c^6*d^15*e^3 + 952*a^3*b^10*c^5*d^14*e^4 - 616*a^3*b^11*c^4*d^13*e^5 + 168*
a^3*b^12*c^3*d^12*e^6 + 40*a^3*b^13*c^2*d^11*e^7 - 2560*a^4*b^6*c^8*d^16*e^
2 + 4480*a^4*b^7*c^7*d^15*e^3 - 4060*a^4*b^8*c^6*d^14*e^4 + 1064*a^4*b^9*c^
5*d^13*e^5 + 1372*a^4*b^10*c^4*d^12*e^6 - 1360*a^4*b^11*c^3*d^11*e^7 + 380*
a^4*b^12*c^2*d^10*e^8 + 6400*a^5*b^4*c^9*d^16*e^2 - 8960*a^5*b^5*c^8*d^15*e
^3 + 2240*a^5*b^6*c^7*d^14*e^4 + 9856*a^5*b^7*c^6*d^13*e^5 - 13048*a^5*b^8*
c^5*d^12*e^6 + 5400*a^5*b^9*c^4*d^11*e^7 + 1040*a^5*b^10*c^3*d^10*e^8 - 136
0*a^5*b^11*c^2*d^9*e^9 - 5120*a^6*b^2*c^10*d^16*e^2 + 22400*a^6*b^4*c^8*d^1
4*e^4 - 41216*a^6*b^5*c^7*d^13*e^5 + 25088*a^6*b^6*c^6*d^12*e^6 + 8320*a^6*
b^7*c^5*d^11*e^7 - 17350*a^6*b^8*c^4*d^10*e^8 + 5400*a^6*b^9*c^3*d^9*e^9 +
1372*a^6*b^10*c^2*d^8*e^10 - 35840*a^7*b^2*c^9*d^14*e^4 + 28672*a^7*b^3*c^8
*d^13*e^5 + 30464*a^7*b^4*c^7*d^12*e^6 - 73472*a^7*b^5*c^6*d^11*e^7 + 40544
*a^7*b^6*c^5*d^10*e^8 + 8320*a^7*b^7*c^4*d^9*e^9 - 13048*a^7*b^8*c^3*d^8*e^
10 + 1064*a^7*b^9*c^2*d^7*e^11 - 93184*a^8*b^2*c^8*d^12*e^6 + 71680*a^8*b^3
*c^7*d^11*e^7 + 29120*a^8*b^4*c^6*d^10*e^8 - 73472*a^8*b^5*c^5*d^9*e^9 + 25
```

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088*a^8*b^6*c^4*d^8*e^10 + 9856*a^8*b^7*c^3*d^7*e^11 - 4060*a^8*b^8*c^2*d^6
*e^12 - 125440*a^9*b^2*c^7*d^10*e^8 + 71680*a^9*b^3*c^6*d^9*e^9 + 30464*a^9
*b^4*c^5*d^8*e^10 - 41216*a^9*b^5*c^4*d^7*e^11 + 2240*a^9*b^6*c^3*d^6*e^12
+ 4480*a^9*b^7*c^2*d^5*e^13 - 93184*a^10*b^2*c^6*d^8*e^10 + 28672*a^10*b^3*
c^5*d^7*e^11 + 22400*a^10*b^4*c^4*d^6*e^12 - 8960*a^10*b^5*c^3*d^5*e^13 - 2
560*a^10*b^6*c^2*d^4*e^14 - 35840*a^11*b^2*c^5*d^6*e^12 + 6400*a^11*b^4*c^3
*d^4*e^14 + 768*a^11*b^5*c^2*d^3*e^15 - 5120*a^12*b^2*c^4*d^4*e^14 - 2048*a
^12*b^3*c^3*d^3*e^15 - 96*a^12*b^4*c^2*d^2*e^16 + 256*a^13*b^2*c^3*d^2*e^16
 + 2048*a^6*b*c^11*d^17*e + 8*a^2*b^9*c^7*d^17*e + 8*a^2*b^15*c*d^11*e^7 -
128*a^3*b^7*c^8*d^17*e - 40*a^3*b^14*c*d^10*e^8 + 768*a^4*b^5*c^9*d^17*e +
40*a^4*b^13*c*d^9*e^9 - 2048*a^5*b^3*c^10*d^17*e + 168*a^5*b^12*c*d^8*e^10
- 616*a^6*b^11*c*d^7*e^11 + 14336*a^7*b*c^10*d^15*e^3 + 952*a^7*b^10*c*d^6*
e^12 + 43008*a^8*b*c^9*d^13*e^5 - 840*a^8*b^9*c*d^5*e^13 + 71680*a^9*b*c^8*
d^11*e^7 + 440*a^9*b^8*c*d^4*e^14 + 71680*a^10*b*c^7*d^9*e^9 - 128*a^10*b^7
*c*d^3*e^15 + 43008*a^11*b*c^6*d^7*e^11 + 16*a^11*b^6*c*d^2*e^16 + 14336*a^
12*b*c^5*d^5*e^13 + 2048*a^13*b*c^4*d^3*e^15)) + (root(128723189760*a^14*b^
4*c^9*d^13*e^14*z^6 + 128723189760*a^12*b^4*c^11*d^17*e^10*z^6 - 8432455680
*a^11*b^12*c^4*d^11*e^16*z^6 - 8432455680*a^7*b^12*c^8*d^19*e^8*z^6 + 12673
351680*a^11*b^11*c^5*d^12*e^15*z^6 + 12673351680*a^8*b^11*c^8*d^18*e^9*z^6
- 72637480960*a^12*b^9*c^6*d^12*e^15*z^6 - 72637480960*a^9*b^9*c^9*d^18*e^9
*z^6 - 21048344576*a^9*b^12*c^6*d^15*e^12*z^6 - 16609443840*a^17*b^3*c^7*d^
8*e^19*z^6 - 16609443840*a^10*b^3*c^14*d^22*e^5*z^6 + 145332633600*a^13*b^5
*c^9*d^14*e^13*z^6 + 145332633600*a^12*b^5*c^10*d^16*e^11*z^6 + 12374035660
8*a^14*b^5*c^8*d^12*e^15*z^6 + 123740356608*a^11*b^5*c^11*d^18*e^9*z^6 + 34
60300800*a^17*b^5*c^5*d^6*e^21*z^6 + 3460300800*a^8*b^5*c^14*d^24*e^3*z^6 -
 7751073792*a^15*b^7*c^5*d^8*e^19*z^6 - 7751073792*a^8*b^7*c^12*d^22*e^5*z^
6 + 12041846784*a^14*b^7*c^6*d^10*e^17*z^6 + 12041846784*a^9*b^7*c^11*d^20*
e^7*z^6 - 325545099264*a^14*b^3*c^10*d^14*e^13*z^6 - 325545099264*a^13*b^3*
c^11*d^16*e^11*z^6 - 3330539520*a^13*b^10*c^4*d^9*e^18*z^6 - 3330539520*a^7
*b^10*c^10*d^21*e^6*z^6 + 157789716480*a^12*b^7*c^8*d^14*e^13*z^6 + 1577897
16480*a^11*b^7*c^9*d^16*e^11*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6
+ 37492359168*a^9*b^10*c^8*d^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^
6 - 7266631680*a^17*b^4*c^6*d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4
*z^6 - 201326592*a^20*b*c^6*d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^
6 + 45747339264*a^13*b^8*c^6*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*
e^8*z^6 - 74612736*a^10*b^16*c*d^9*e^18*z^6 - 2768240640*a^16*b^7*c^4*d^6*e
^21*z^6 - 2768240640*a^7*b^7*c^13*d^24*e^3*z^6 + 69746688*a^11*b^15*c*d^8*e
^19*z^6 + 62914560*a^6*b^7*c^14*d^26*e*z^6 + 2752020480*a^10*b^13*c^4*d^12*
e^15*z^6 + 2752020480*a^7*b^13*c^7*d^18*e^9*z^6 + 55148544*a^9*b^17*c*d^10*
e^17*z^6 - 45957120*a^12*b^14*c*d^7*e^20*z^6 - 2724986880*a^14*b^9*c^4*d^8*
e^19*z^6 - 2724986880*a^7*b^9*c^11*d^22*e^5*z^6 - 25952256*a^8*b^18*c*d^11*
e^16*z^6 + 21086208*a^13*b^13*c*d^6*e^21*z^6 - 11796480*a^5*b^9*c^13*d^26*e
*z^6 - 6438912*a^14*b^12*c*d^5*e^22*z^6 + 5406720*a^7*b^19*c*d^12*e^15*z^6
+ 1622016*a^6*b^20*c*d^13*e^14*z^6 - 1523712*a^5*b^21*c*d^14*e^13*z^6 + 117
9648*a^15*b^11*c*d^4*e^23*z^6 + 1179648*a^4*b^11*c^12*d^26*e*z^6 + 442368*a
```

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^4*b^22*c*d^15*e^12*z^6 - 98304*a^16*b^10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c
*d^16*e^11*z^6 - 49152*a^3*b^13*c^11*d^26*e*z^6 + 6897106944*a^9*b^13*c^5*d
^14*e^13*z^6 + 6897106944*a^8*b^13*c^6*d^16*e^11*z^6 - 2422210560*a^16*b^6*
c^5*d^7*e^20*z^6 - 2422210560*a^8*b^6*c^13*d^23*e^4*z^6 + 255785435136*a^14
*b^2*c^11*d^15*e^12*z^6 + 41004564480*a^15*b^4*c^8*d^11*e^16*z^6 + 41004564
480*a^11*b^4*c^12*d^19*e^8*z^6 + 2270822400*a^13*b^11*c^3*d^8*e^19*z^6 + 22
70822400*a^6*b^11*c^10*d^22*e^5*z^6 + 23677108224*a^14*b^8*c^5*d^9*e^18*z^6
 + 23677108224*a^8*b^8*c^11*d^21*e^6*z^6 + 212600881152*a^15*b^2*c^10*d^13*
e^14*z^6 + 212600881152*a^13*b^2*c^12*d^17*e^10*z^6 + 75157733376*a^15*b^5*
c^7*d^10*e^17*z^6 + 75157733376*a^10*b^5*c^12*d^20*e^7*z^6 - 251217838080*a
^13*b^6*c^8*d^13*e^14*z^6 - 251217838080*a^11*b^6*c^10*d^17*e^10*z^6 - 1952
907264*a^14*b^10*c^3*d^7*e^20*z^6 - 1952907264*a^6*b^10*c^11*d^23*e^4*z^6 -
 27691057152*a^13*b^9*c^5*d^10*e^17*z^6 - 27691057152*a^8*b^9*c^10*d^20*e^7
*z^6 - 1902673920*a^8*b^15*c^4*d^14*e^13*z^6 - 1902673920*a^7*b^15*c^5*d^16
*e^11*z^6 + 10465050624*a^10*b^11*c^6*d^14*e^13*z^6 + 10465050624*a^9*b^11*
c^7*d^16*e^11*z^6 + 1613905920*a^9*b^14*c^4*d^13*e^14*z^6 + 1613905920*a^7*
b^14*c^6*d^17*e^10*z^6 - 33218887680*a^17*b*c^9*d^10*e^17*z^6 - 33218887680
*a^12*b*c^14*d^20*e^7*z^6 + 1524695040*a^10*b^14*c^3*d^11*e^16*z^6 + 152469
5040*a^6*b^14*c^7*d^19*e^8*z^6 - 1472200704*a^18*b^4*c^5*d^5*e^22*z^6 - 147
2200704*a^8*b^4*c^15*d^25*e^2*z^6 - 83047219200*a^16*b^3*c^8*d^10*e^17*z^6
- 83047219200*a^11*b^3*c^13*d^20*e^7*z^6 + 44291850240*a^17*b^2*c^8*d^9*e^1
8*z^6 + 44291850240*a^11*b^2*c^14*d^21*e^6*z^6 + 1308131328*a^8*b^14*c^5*d^
15*e^12*z^6 - 201326592*a^9*b*c^17*d^26*e*z^6 + 48530718720*a^12*b^8*c^7*d^
13*e^14*z^6 + 48530718720*a^10*b^8*c^9*d^17*e^10*z^6 - 1242644480*a^12*b^12
*c^3*d^9*e^18*z^6 - 1242644480*a^6*b^12*c^9*d^21*e^6*z^6 + 9813196800*a^12*
b^10*c^5*d^11*e^16*z^6 + 9813196800*a^8*b^10*c^9*d^19*e^8*z^6 - 93012885504
*a^15*b*c^11*d^14*e^13*z^6 - 93012885504*a^14*b*c^12*d^16*e^11*z^6 + 177305
812992*a^13*b^4*c^10*d^15*e^12*z^6 + 52730658816*a^10*b^10*c^7*d^15*e^12*z^
6 - 1180106752*a^9*b^15*c^3*d^12*e^15*z^6 - 1180106752*a^6*b^15*c^6*d^18*e^
9*z^6 + 1023672320*a^15*b^9*c^3*d^6*e^21*z^6 + 1023672320*a^6*b^9*c^12*d^24
*e^3*z^6 + 975175680*a^17*b^6*c^4*d^5*e^22*z^6 + 975175680*a^7*b^6*c^14*d^2
5*e^2*z^6 - 11072962560*a^18*b*c^8*d^8*e^19*z^6 - 11072962560*a^11*b*c^15*d
^22*e^5*z^6 + 9412018176*a^18*b^2*c^7*d^7*e^20*z^6 + 9412018176*a^10*b^2*c^
15*d^23*e^4*z^6 + 805306368*a^19*b^2*c^6*d^5*e^22*z^6 + 805306368*a^9*b^2*c
^16*d^25*e^2*z^6 - 133809831936*a^14*b^6*c^7*d^11*e^16*z^6 - 133809831936*a
^10*b^6*c^11*d^19*e^8*z^6 - 2214592512*a^19*b*c^7*d^6*e^21*z^6 - 2214592512
*a^10*b*c^16*d^24*e^3*z^6 + 82216747008*a^13*b^7*c^7*d^12*e^15*z^6 + 822167
47008*a^10*b^7*c^10*d^18*e^9*z^6 - 586629120*a^12*b^13*c^2*d^8*e^19*z^6 - 5
86629120*a^5*b^13*c^9*d^22*e^5*z^6 + 568565760*a^7*b^16*c^4*d^15*e^12*z^6 -
 4844421120*a^16*b^4*c^7*d^9*e^18*z^6 - 4844421120*a^10*b^4*c^13*d^21*e^6*z
^6 + 531210240*a^11*b^14*c^2*d^9*e^18*z^6 + 531210240*a^5*b^14*c^8*d^21*e^6
*z^6 - 527155200*a^11*b^13*c^3*d^10*e^17*z^6 - 527155200*a^6*b^13*c^8*d^20*
e^7*z^6 + 43470028800*a^11*b^8*c^8*d^15*e^12*z^6 - 107874877440*a^11*b^9*c^
7*d^14*e^13*z^6 - 107874877440*a^10*b^9*c^8*d^16*e^11*z^6 + 9018408960*a^12
*b^11*c^4*d^10*e^17*z^6 + 9018408960*a^7*b^11*c^9*d^20*e^7*z^6 + 421994496*
```

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a^13*b^12*c^2*d^7*e^20*z^6 + 421994496*a^5*b^12*c^10*d^23*e^4*z^6 - 6643777
5360*a^16*b*c^10*d^12*e^15*z^6 - 66437775360*a^13*b*c^13*d^18*e^9*z^6 + 261
59874048*a^16*b^5*c^6*d^8*e^19*z^6 + 26159874048*a^9*b^5*c^13*d^22*e^5*z^6
- 369098752*a^18*b^3*c^6*d^6*e^21*z^6 - 369098752*a^9*b^3*c^15*d^24*e^3*z^6
 + 351436800*a^8*b^16*c^3*d^13*e^14*z^6 + 351436800*a^6*b^16*c^5*d^17*e^10*
z^6 - 334233600*a^16*b^8*c^3*d^5*e^22*z^6 - 334233600*a^6*b^8*c^13*d^25*e^2
*z^6 + 301989888*a^19*b^3*c^5*d^4*e^23*z^6 - 266010624*a^10*b^15*c^2*d^10*e
^17*z^6 - 266010624*a^5*b^15*c^7*d^20*e^7*z^6 - 305198530560*a^12*b^6*c^9*d
^15*e^12*z^6 - 203292672*a^14*b^11*c^2*d^6*e^21*z^6 - 203292672*a^5*b^11*c^
11*d^24*e^3*z^6 - 188743680*a^18*b^5*c^4*d^4*e^23*z^6 + 120418467840*a^16*b
^2*c^9*d^11*e^16*z^6 + 120418467840*a^12*b^2*c^13*d^19*e^8*z^6 - 1729393459
2*a^10*b^12*c^5*d^13*e^14*z^6 - 17293934592*a^8*b^12*c^7*d^17*e^10*z^6 + 10
4890368*a^8*b^17*c^2*d^12*e^15*z^6 + 104890368*a^5*b^17*c^5*d^18*e^9*z^6 +
4390256640*a^15*b^8*c^4*d^7*e^20*z^6 + 4390256640*a^7*b^8*c^12*d^23*e^4*z^6
 - 91750400*a^6*b^18*c^3*d^15*e^12*z^6 + 79134720*a^7*b^17*c^3*d^14*e^13*z^
6 + 79134720*a^6*b^17*c^4*d^16*e^11*z^6 - 74612736*a^4*b^16*c^7*d^21*e^6*z^
6 - 72990720*a^7*b^18*c^2*d^13*e^14*z^6 - 72990720*a^5*b^18*c^4*d^17*e^10*z
^6 + 69746688*a^4*b^15*c^8*d^22*e^5*z^6 + 63700992*a^15*b^10*c^2*d^5*e^22*z
^6 + 63700992*a^5*b^10*c^12*d^25*e^2*z^6 + 62914560*a^17*b^7*c^3*d^4*e^23*z
^6 + 55148544*a^4*b^17*c^6*d^20*e^7*z^6 - 45957120*a^4*b^14*c^9*d^23*e^4*z^
6 - 25952256*a^4*b^18*c^5*d^19*e^8*z^6 - 25165824*a^20*b^2*c^5*d^3*e^24*z^6
 + 21086208*a^4*b^13*c^10*d^24*e^3*z^6 + 20643840*a^6*b^19*c^2*d^14*e^13*z^
6 + 20643840*a^5*b^19*c^3*d^16*e^11*z^6 + 15728640*a^19*b^4*c^4*d^3*e^24*z^
6 - 11796480*a^16*b^9*c^2*d^4*e^23*z^6 - 6438912*a^4*b^12*c^11*d^25*e^2*z^6
 + 5406720*a^4*b^19*c^4*d^18*e^9*z^6 - 5242880*a^18*b^6*c^3*d^3*e^24*z^6 +
3784704*a^3*b^18*c^6*d^21*e^6*z^6 - 3244032*a^3*b^19*c^5*d^20*e^7*z^6 - 324
4032*a^3*b^17*c^7*d^22*e^5*z^6 + 2027520*a^3*b^20*c^4*d^19*e^8*z^6 + 202752
0*a^3*b^16*c^8*d^23*e^4*z^6 - 1622016*a^9*b^16*c^2*d^11*e^16*z^6 - 1622016*
a^5*b^16*c^6*d^19*e^8*z^6 + 1622016*a^4*b^20*c^3*d^17*e^10*z^6 - 1523712*a^
4*b^21*c^2*d^16*e^11*z^6 + 983040*a^17*b^8*c^2*d^3*e^24*z^6 - 901120*a^3*b^
21*c^3*d^18*e^9*z^6 - 901120*a^3*b^15*c^9*d^24*e^3*z^6 + 270336*a^3*b^22*c^
2*d^17*e^10*z^6 + 270336*a^3*b^14*c^10*d^25*e^2*z^6 + 172032*a^5*b^20*c^2*d
^15*e^12*z^6 - 38593888256*a^15*b^6*c^6*d^9*e^18*z^6 - 38593888256*a^9*b^6*
c^12*d^21*e^6*z^6 - 210386288640*a^15*b^3*c^9*d^12*e^15*z^6 - 210386288640*
a^12*b^3*c^12*d^18*e^9*z^6 + 15502147584*a^15*c^12*d^15*e^12*z^6 + 11072962
56*a^19*c^8*d^7*e^20*z^6 + 1107296256*a^11*c^16*d^23*e^4*z^6 + 13287555072*
a^16*c^11*d^13*e^14*z^6 + 13287555072*a^14*c^13*d^17*e^10*z^6 + 201326592*a
^20*c^7*d^5*e^22*z^6 + 201326592*a^10*c^17*d^25*e^2*z^6 + 16777216*a^21*c^6
*d^3*e^24*z^6 + 3784704*a^9*b^18*d^9*e^18*z^6 - 3244032*a^10*b^17*d^8*e^19*
z^6 - 3244032*a^8*b^19*d^10*e^17*z^6 + 2027520*a^11*b^16*d^7*e^20*z^6 + 202
7520*a^7*b^20*d^11*e^16*z^6 - 901120*a^12*b^15*d^6*e^21*z^6 - 901120*a^6*b^
21*d^12*e^15*z^6 + 270336*a^13*b^14*d^5*e^22*z^6 + 270336*a^5*b^22*d^13*e^1
4*z^6 - 49152*a^14*b^13*d^4*e^23*z^6 - 49152*a^4*b^23*d^14*e^13*z^6 + 4096*
a^15*b^12*d^3*e^24*z^6 + 4096*a^3*b^24*d^15*e^12*z^6 - 25165824*a^8*b^2*c^1
7*d^27*z^6 + 15728640*a^7*b^4*c^16*d^27*z^6 - 5242880*a^6*b^6*c^15*d^27*z^6
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 + 983040*a^5*b^8*c^14*d^27*z^6 - 98304*a^4*b^10*c^13*d^27*z^6 + 4096*a^3*b
^12*c^12*d^27*z^6 + 8304721920*a^17*c^10*d^11*e^16*z^6 + 8304721920*a^13*c^
14*d^19*e^8*z^6 + 3690987520*a^18*c^9*d^9*e^18*z^6 + 3690987520*a^12*c^15*d
^21*e^6*z^6 + 16777216*a^9*c^18*d^27*z^6 - 8493371392*a^6*b^8*c^9*d^14*e^9*
z^4 + 1458044928*a^8*b*c^14*d^17*e^6*z^4 - 12604538880*a^11*b^4*c^8*d^8*e^1
5*z^4 - 8303067136*a^9*b^5*c^9*d^11*e^12*z^4 - 5588058112*a^13*b*c^9*d^7*e^
16*z^4 - 3892838400*a^8*b^2*c^13*d^16*e^7*z^4 - 3611713536*a^8*b^8*c^7*d^10
*e^13*z^4 + 7819006464*a^7*b^9*c^7*d^11*e^12*z^4 - 7782137856*a^8*b^7*c^8*d
^11*e^12*z^4 + 7780433920*a^12*b^2*c^9*d^8*e^15*z^4 - 12020465664*a^7*b^5*c
^11*d^15*e^8*z^4 + 3176792064*a^8*b^3*c^12*d^15*e^8*z^4 - 322633728*a^15*b*
c^7*d^3*e^20*z^4 + 210829312*a^7*b*c^15*d^19*e^4*z^4 + 15623258112*a^9*b^6*
c^8*d^10*e^13*z^4 + 25165824*a^15*b^3*c^5*d*e^22*z^4 - 15728640*a^14*b^5*c^
4*d*e^22*z^4 + 12582912*a^5*b^2*c^16*d^22*e*z^4 - 11730944*a^4*b^4*c^15*d^2
2*e*z^4 + 5242880*a^13*b^7*c^3*d*e^22*z^4 - 4561920*a*b^15*c^7*d^17*e^6*z^4
 + 4521984*a^3*b^6*c^14*d^22*e*z^4 + 4460544*a*b^14*c^8*d^18*e^5*z^4 + 3538
944*a^6*b*c^16*d^21*e^2*z^4 + 3108864*a*b^16*c^6*d^16*e^7*z^4 - 3027200*a*b
^13*c^9*d^19*e^4*z^4 - 2345472*a^5*b^17*c*d^7*e^16*z^4 - 2307072*a^8*b^14*c
*d^4*e^19*z^4 + 1824768*a^6*b^16*c*d^6*e^17*z^4 + 1734912*a^9*b^13*c*d^3*e^
20*z^4 + 1419264*a*b^12*c^10*d^20*e^3*z^4 - 1191168*a*b^17*c^5*d^15*e^8*z^4
 - 983040*a^12*b^9*c^2*d*e^22*z^4 + 964608*a^4*b^18*c*d^8*e^15*z^4 - 866304
*a^2*b^8*c^13*d^22*e*z^4 + 703488*a^7*b^15*c*d^5*e^18*z^4 - 608256*a^10*b^1
2*c*d^2*e^21*z^4 - 440832*a*b^11*c^11*d^21*e^2*z^4 + 275968*a*b^19*c^3*d^13
*e^10*z^4 - 159744*a^2*b^20*c*d^10*e^13*z^4 - 153600*a*b^20*c^2*d^12*e^11*z
^4 + 64512*a^3*b^19*c*d^9*e^14*z^4 + 19746062336*a^8*b^6*c^9*d^12*e^11*z^4
 - 15333588992*a^10*b^4*c^9*d^10*e^13*z^4 + 6702170112*a^7*b^4*c^12*d^16*e^7
*z^4 + 15167913984*a^10*b^3*c^10*d^11*e^12*z^4 - 2256638976*a^5*b^11*c^7*d^
13*e^10*z^4 + 2254307328*a^5*b^7*c^11*d^17*e^6*z^4 - 2200633344*a^6*b^5*c^1
2*d^17*e^6*z^4 + 6457131008*a^11*b^3*c^9*d^9*e^14*z^4 - 2128785408*a^5*b^8*
c^10*d^16*e^7*z^4 - 2126057472*a^6*b^11*c^6*d^11*e^12*z^4 + 2038349824*a^12
*b^5*c^6*d^5*e^18*z^4 + 2037841920*a^5*b^10*c^8*d^14*e^9*z^4 + 3615621120*a
^9*b*c^13*d^15*e^8*z^4 + 1900019712*a^11*b^2*c^10*d^10*e^13*z^4 + 186769843
2*a^9*b^9*c^5*d^7*e^16*z^4 - 6157369344*a^9*b^4*c^10*d^12*e^11*z^4 - 185691
3408*a^7*b^10*c^6*d^10*e^13*z^4 + 1789132800*a^6*b^4*c^13*d^18*e^5*z^4 + 60
82658304*a^8*b^4*c^11*d^14*e^9*z^4 + 6029549568*a^11*b^5*c^7*d^7*e^16*z^4 +
 6010159104*a^6*b^7*c^10*d^15*e^8*z^4 + 1703182336*a^7*b^7*c^9*d^13*e^10*z^
4 + 1658388480*a^11*b^6*c^6*d^6*e^17*z^4 + 5917114368*a^10*b^6*c^7*d^8*e^15
*z^4 - 1591197696*a^11*b^7*c^5*d^5*e^18*z^4 - 1526464512*a^8*b^10*c^5*d^8*e
^15*z^4 - 5772607488*a^12*b^4*c^7*d^6*e^17*z^4 - 1423507456*a^13*b^4*c^6*d^
4*e^19*z^4 - 1387266048*a^7*b^3*c^13*d^17*e^6*z^4 + 2976120832*a^10*b*c^12*
d^13*e^10*z^4 - 9906946048*a^9*b^2*c^12*d^14*e^9*z^4 - 18421874688*a^8*b^5*
c^10*d^13*e^10*z^4 + 1141217280*a^6*b^12*c^5*d^10*e^13*z^4 - 9714364416*a^7
*b^8*c^8*d^12*e^11*z^4 - 16777216*a^16*b*c^6*d*e^22*z^4 + 98304*a^11*b^11*c
*d*e^22*z^4 + 81920*a*b^10*c^12*d^22*e*z^4 + 39168*a*b^21*c*d^11*e^12*z^4 -
 1091829760*a^5*b^6*c^12*d^18*e^5*z^4 + 1046740992*a^14*b^2*c^7*d^4*e^19*z^
4 - 6884425728*a^12*b*c^10*d^9*e^14*z^4 + 987445248*a^4*b^10*c^9*d^16*e^7*z
```

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^4 + 984087552*a^5*b^12*c^6*d^12*e^11*z^4 - 9564585984*a^9*b^7*c^7*d^9*e^14
*z^4 - 5266857984*a^10*b^7*c^6*d^7*e^16*z^4 - 892145664*a^7*b^11*c^5*d^9*e^
14*z^4 - 2444623872*a^11*b*c^11*d^11*e^12*z^4 + 768540672*a^12*b^3*c^8*d^7*
e^16*z^4 + 5048322048*a^8*b^9*c^6*d^9*e^14*z^4 + 5047612416*a^6*b^9*c^8*d^1
3*e^10*z^4 - 732492288*a^4*b^11*c^8*d^15*e^8*z^4 + 9266921472*a^7*b^6*c^10*
d^14*e^9*z^4 - 645857280*a^6*b^6*c^11*d^16*e^7*z^4 - 623867904*a^4*b^9*c^10
*d^17*e^6*z^4 - 622067712*a^6*b^3*c^14*d^19*e^4*z^4 + 582617088*a^10*b^8*c^
5*d^6*e^17*z^4 + 577119744*a^7*b^12*c^4*d^8*e^15*z^4 + 552566784*a^12*b^6*c
^5*d^4*e^19*z^4 + 549224448*a^9*b^8*c^6*d^8*e^15*z^4 - 526565376*a^9*b^10*c
^4*d^6*e^17*z^4 + 511520256*a^10*b^9*c^4*d^5*e^18*z^4 + 13393723392*a^9*b^3
*c^11*d^13*e^10*z^4 - 2066350080*a^14*b*c^8*d^5*e^18*z^4 + 4718592000*a^13*
b^2*c^8*d^6*e^17*z^4 - 314572800*a^7*b^2*c^14*d^18*e^5*z^4 + 287250432*a^4*
b^13*c^6*d^13*e^10*z^4 + 4565827584*a^10*b^5*c^8*d^9*e^14*z^4 - 250785792*a
^4*b^14*c^5*d^12*e^11*z^4 + 235536384*a^13*b^3*c^7*d^5*e^18*z^4 - 232683264
*a^8*b^11*c^4*d^7*e^16*z^4 - 199627776*a^5*b^14*c^4*d^10*e^13*z^4 - 1902673
92*a^12*b^7*c^4*d^3*e^20*z^4 + 184891392*a^6*b^10*c^7*d^12*e^11*z^4 + 18050
2528*a^4*b^7*c^12*d^19*e^4*z^4 + 178877952*a^3*b^13*c^7*d^15*e^8*z^4 + 1724
90752*a^14*b^3*c^6*d^3*e^20*z^4 + 163946496*a^13*b^5*c^5*d^3*e^20*z^4 + 155
839488*a^8*b^12*c^3*d^6*e^17*z^4 + 155000832*a^5*b^5*c^13*d^19*e^4*z^4 - 15
2076288*a^4*b^6*c^13*d^20*e^3*z^4 - 137592576*a^3*b^12*c^8*d^16*e^7*z^4 - 1
33693440*a^14*b^4*c^5*d^2*e^21*z^4 - 116767488*a^3*b^9*c^11*d^19*e^4*z^4 -
108985344*a^3*b^14*c^6*d^14*e^9*z^4 - 106223616*a^6*b^13*c^4*d^9*e^14*z^4 +
 106119168*a^3*b^10*c^10*d^18*e^5*z^4 + 102432768*a^5*b^4*c^14*d^20*e^3*z^4
 + 102113280*a^4*b^12*c^7*d^14*e^9*z^4 + 100674048*a^5*b^9*c^9*d^15*e^8*z^4
 + 90439680*a^13*b^6*c^4*d^2*e^21*z^4 - 86808576*a^6*b^14*c^3*d^8*e^15*z^4
+ 86245376*a^6*b^2*c^15*d^20*e^3*z^4 + 79011840*a^4*b^8*c^11*d^18*e^5*z^4 +
 78345216*a^4*b^15*c^4*d^11*e^12*z^4 + 78006528*a^11*b^9*c^3*d^3*e^20*z^4 -
 73253376*a^9*b^11*c^3*d^5*e^18*z^4 + 67524608*a^3*b^8*c^12*d^20*e^3*z^4 +
67108864*a^15*b^2*c^6*d^2*e^21*z^4 - 61590528*a^10*b^10*c^3*d^4*e^19*z^4 +
61559808*a^5*b^15*c^3*d^9*e^14*z^4 - 59637760*a^5*b^3*c^15*d^21*e^2*z^4 + 5
8638336*a^4*b^5*c^14*d^21*e^2*z^4 - 40828416*a^7*b^13*c^3*d^7*e^16*z^4 - 35
639296*a^2*b^12*c^9*d^18*e^5*z^4 - 31293440*a^12*b^8*c^3*d^2*e^21*z^4 + 299
33568*a^5*b^13*c^5*d^11*e^12*z^4 + 27793920*a^2*b^11*c^10*d^19*e^4*z^4 + 27
168768*a^2*b^13*c^8*d^17*e^6*z^4 - 23602176*a^7*b^14*c^2*d^6*e^17*z^4 - 232
48896*a^3*b^7*c^13*d^21*e^2*z^4 + 20929536*a^3*b^15*c^5*d^13*e^10*z^4 + 184
28928*a^9*b^12*c^2*d^4*e^19*z^4 + 18026496*a^6*b^15*c^2*d^7*e^16*z^4 - 1626
1632*a^10*b^11*c^2*d^3*e^20*z^4 + 15128064*a^3*b^16*c^4*d^12*e^11*z^4 - 140
60544*a^2*b^10*c^11*d^20*e^3*z^4 + 13178880*a^2*b^16*c^5*d^14*e^9*z^4 - 112
44288*a^3*b^17*c^3*d^11*e^12*z^4 - 10509312*a^2*b^15*c^6*d^15*e^8*z^4 - 726
2208*a^4*b^17*c^2*d^9*e^14*z^4 - 7045632*a^2*b^17*c^4*d^13*e^10*z^4 - 62853
12*a^2*b^14*c^7*d^16*e^7*z^4 + 5996544*a^11*b^10*c^2*d^2*e^21*z^4 + 4558336
*a^2*b^9*c^12*d^21*e^2*z^4 + 4478976*a^11*b^8*c^4*d^4*e^19*z^4 + 2850816*a^
4*b^16*c^3*d^10*e^13*z^4 + 2629632*a^3*b^11*c^9*d^17*e^6*z^4 + 2503680*a^3*
b^18*c^2*d^10*e^13*z^4 + 1627136*a^2*b^18*c^3*d^12*e^11*z^4 + 1605120*a^8*b
^13*c^2*d^5*e^18*z^4 + 1483776*a^5*b^16*c^2*d^8*e^15*z^4 + 139776*a^2*b^19*
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c^2*d^11*e^12*z^4 - 8542224384*a^10*b^2*c^11*d^12*e^11*z^4 - 3072*b^22*c*d^
12*e^11*z^4 - 3072*b^12*c^11*d^22*e*z^4 - 1572864*a^6*c^17*d^22*e*z^4 - 409
6*a^10*b^13*d*e^22*z^4 - 4096*a*b^22*d^10*e^13*z^4 - 6144*a^12*b^10*c*e^23*
z^4 - 983040*a^5*b*c^17*d^23*z^4 - 6912*a*b^9*c^13*d^23*z^4 + 1824522240*a^
13*c^10*d^8*e^15*z^4 + 1730150400*a^12*c^11*d^10*e^13*z^4 + 958922752*a^14*
c^9*d^6*e^17*z^4 - 537919488*a^9*c^14*d^16*e^7*z^4 + 508559360*a^11*c^12*d^
12*e^11*z^4 - 500170752*a^10*c^13*d^14*e^9*z^4 + 246939648*a^15*c^8*d^4*e^1
9*z^4 - 199229440*a^8*c^15*d^18*e^5*z^4 - 29884416*a^7*c^16*d^20*e^3*z^4 +
25165824*a^16*c^7*d^2*e^21*z^4 + 236544*b^17*c^6*d^17*e^6*z^4 - 202752*b^18
*c^5*d^16*e^7*z^4 - 202752*b^16*c^7*d^18*e^5*z^4 + 126720*b^19*c^4*d^15*e^8
*z^4 + 126720*b^15*c^8*d^19*e^4*z^4 - 56320*b^20*c^3*d^14*e^9*z^4 - 56320*b
^14*c^9*d^20*e^3*z^4 + 16896*b^21*c^2*d^13*e^10*z^4 + 16896*b^13*c^10*d^21*
e^2*z^4 + 110080*a^7*b^16*d^4*e^19*z^4 + 110080*a^4*b^19*d^7*e^16*z^4 - 755
20*a^8*b^15*d^3*e^20*z^4 - 75520*a^3*b^20*d^8*e^15*z^4 - 56320*a^6*b^17*d^5
*e^18*z^4 - 56320*a^5*b^18*d^6*e^17*z^4 + 25600*a^9*b^14*d^2*e^21*z^4 + 256
00*a^2*b^21*d^9*e^14*z^4 - 1572864*a^16*b^2*c^5*e^23*z^4 + 983040*a^15*b^4*
c^4*e^23*z^4 - 327680*a^14*b^6*c^3*e^23*z^4 + 61440*a^13*b^8*c^2*e^23*z^4 +
 983040*a^4*b^3*c^16*d^23*z^4 - 385024*a^3*b^5*c^15*d^23*z^4 + 73728*a^2*b^
7*c^14*d^23*z^4 + 256*b^23*d^11*e^12*z^4 + 1048576*a^17*c^6*e^23*z^4 + 256*
b^11*c^12*d^23*z^4 + 256*a^11*b^12*e^23*z^4 + 948695040*a^8*b*c^10*d^6*e^13
*z^2 + 348917760*a^7*b*c^11*d^8*e^11*z^2 - 125030400*a^9*b*c^9*d^4*e^15*z^2
 - 50728960*a^6*b*c^12*d^10*e^9*z^2 - 44298240*a^5*b*c^13*d^12*e^7*z^2 - 36
495360*a^10*b*c^8*d^2*e^17*z^2 + 29675520*a^8*b^6*c^5*d*e^18*z^2 - 24170496
*a^9*b^4*c^6*d*e^18*z^2 - 17202816*a^7*b^8*c^4*d*e^18*z^2 - 14561280*a^4*b*
c^14*d^14*e^5*z^2 + 5532416*a^6*b^10*c^3*d*e^18*z^2 + 4128768*a^10*b^2*c^7*
d*e^18*z^2 - 2662400*a^3*b*c^15*d^16*e^3*z^2 + 1184512*a*b^12*c^6*d^9*e^10*
z^2 - 1136160*a*b^13*c^5*d^8*e^11*z^2 - 1017600*a^5*b^12*c^2*d*e^18*z^2 - 7
44768*a*b^11*c^7*d^10*e^9*z^2 + 607872*a*b^14*c^4*d^7*e^12*z^2 - 424064*a*b
^6*c^12*d^15*e^4*z^2 + 408576*a*b^5*c^13*d^16*e^3*z^2 + 361152*a*b^10*c^8*d
^11*e^8*z^2 - 287408*a*b^9*c^9*d^12*e^7*z^2 - 260448*a^3*b^15*c*d^2*e^17*z^
2 - 203904*a*b^4*c^14*d^17*e^2*z^2 + 200832*a*b^8*c^10*d^13*e^6*z^2 + 12672
0*a*b^7*c^11*d^14*e^5*z^2 - 123968*a*b^15*c^3*d^6*e^13*z^2 - 39168*a*b^16*c
^2*d^5*e^14*z^2 + 11904*a^2*b^16*c*d^3*e^16*z^2 + 1824135552*a^7*b^4*c^8*d^
5*e^14*z^2 - 1457252352*a^8*b^2*c^9*d^5*e^14*z^2 - 1405209600*a^7*b^5*c^7*d
^4*e^15*z^2 - 184320*a^2*b*c^16*d^18*e*z^2 + 100608*a^4*b^14*c*d*e^18*z^2 +
 53248*a*b^3*c^15*d^18*e*z^2 + 26448*a*b^17*c*d^4*e^15*z^2 + 1067599872*a^8
*b^3*c^8*d^4*e^15*z^2 - 930828288*a^7*b^3*c^9*d^6*e^13*z^2 + 920760000*a^6*
b^4*c^9*d^7*e^12*z^2 - 806639616*a^6*b^3*c^10*d^8*e^11*z^2 - 791052480*a^6*
b^6*c^7*d^5*e^14*z^2 + 772237824*a^6*b^7*c^6*d^4*e^15*z^2 - 701025408*a^5*b
^6*c^8*d^7*e^12*z^2 + 443340288*a^5*b^5*c^9*d^8*e^11*z^2 + 433047552*a^7*b^
6*c^6*d^3*e^16*z^2 + 405741312*a^5*b^7*c^7*d^6*e^13*z^2 + 293652480*a^6*b^2
*c^11*d^9*e^10*z^2 - 276962688*a^6*b^8*c^5*d^3*e^16*z^2 - 247804272*a^8*b^4
*c^7*d^3*e^16*z^2 + 213564384*a^4*b^8*c^7*d^7*e^12*z^2 - 202596816*a^5*b^9*
c^5*d^4*e^15*z^2 - 182520896*a^4*b^9*c^6*d^6*e^13*z^2 - 153489408*a^5*b^3*c
^11*d^10*e^9*z^2 - 152151552*a^7*b^2*c^10*d^7*e^12*z^2 + 115859712*a^5*b^2*
```

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c^12*d^11*e^8*z^2 + 108085248*a^9*b^3*c^7*d^2*e^17*z^2 + 105536256*a^4*b^5*
c^10*d^10*e^9*z^2 - 98323200*a^6*b^5*c^8*d^6*e^13*z^2 - 93564992*a^4*b^6*c^
9*d^9*e^10*z^2 + 89464512*a^5*b^10*c^4*d^3*e^16*z^2 - 75930624*a^8*b^5*c^6*
d^2*e^17*z^2 + 68315904*a^5*b^8*c^6*d^5*e^14*z^2 - 64157184*a^4*b^7*c^8*d^8
*e^11*z^2 - 62951040*a^9*b^2*c^8*d^3*e^16*z^2 + 49056768*a^4*b^10*c^5*d^5*e
^14*z^2 + 47614464*a^3*b^8*c^8*d^9*e^10*z^2 + 35604480*a^4*b^2*c^13*d^13*e^
6*z^2 + 33983040*a^3*b^11*c^5*d^6*e^13*z^2 - 33515520*a^4*b^3*c^12*d^12*e^7
*z^2 - 33463808*a^3*b^7*c^9*d^10*e^9*z^2 - 25128864*a^4*b^4*c^11*d^11*e^8*z
^2 - 23193728*a^3*b^10*c^6*d^7*e^12*z^2 + 21015456*a^6*b^9*c^4*d^2*e^17*z^2
 + 19924176*a^4*b^11*c^4*d^4*e^15*z^2 - 19251216*a^3*b^9*c^7*d^8*e^11*z^2 -
 16434048*a^5*b^4*c^10*d^9*e^10*z^2 - 16289664*a^3*b^12*c^4*d^5*e^14*z^2 -
15059328*a^4*b^12*c^3*d^3*e^16*z^2 - 10766016*a^2*b^10*c^7*d^9*e^10*z^2 - 1
0453632*a^5*b^11*c^3*d^2*e^17*z^2 - 9940992*a^3*b^3*c^13*d^14*e^5*z^2 + 837
3696*a^2*b^11*c^6*d^8*e^11*z^2 + 7776768*a^3*b^2*c^14*d^15*e^4*z^2 + 707788
8*a^3*b^5*c^11*d^12*e^7*z^2 + 6798240*a^2*b^9*c^8*d^10*e^9*z^2 - 3589440*a^
2*b^6*c^11*d^13*e^6*z^2 + 3544320*a^3*b^6*c^10*d^11*e^8*z^2 + 3128064*a^2*b
^5*c^12*d^14*e^5*z^2 + 2346336*a^4*b^13*c^2*d^2*e^17*z^2 - 2261568*a^2*b^8*
c^9*d^11*e^8*z^2 - 2125824*a^2*b^13*c^4*d^6*e^13*z^2 + 2002560*a^3*b^4*c^12
*d^13*e^6*z^2 + 1927680*a^2*b^7*c^10*d^12*e^7*z^2 + 1814784*a^2*b^14*c^3*d^
5*e^14*z^2 - 1807104*a^2*b^12*c^5*d^7*e^12*z^2 + 1637808*a^3*b^13*c^3*d^4*e
^15*z^2 + 1083456*a^3*b^14*c^2*d^3*e^16*z^2 - 792384*a^2*b^4*c^13*d^15*e^4*
z^2 - 657408*a^2*b^3*c^14*d^16*e^3*z^2 + 608256*a^7*b^7*c^5*d^2*e^17*z^2 +
595968*a^2*b^2*c^15*d^17*e^2*z^2 - 498624*a^2*b^15*c^2*d^4*e^15*z^2 - 3840*
b^18*c*d^5*e^14*z^2 - 3840*b^5*c^14*d^18*e*z^2 + 2064384*a^11*c^8*d*e^18*z^
2 - 4160*a^3*b^16*d*e^18*z^2 - 4160*a*b^18*d^3*e^16*z^2 - 1290240*a^11*b*c^
7*e^19*z^2 - 9840*a^5*b^13*c*e^19*z^2 - 5760*a*b^2*c^16*d^19*z^2 - 28058112
0*a^8*c^11*d^7*e^12*z^2 + 110278656*a^9*c^10*d^5*e^14*z^2 - 89479168*a^7*c^
12*d^9*e^10*z^2 + 34464000*a^10*c^9*d^3*e^16*z^2 + 54240*b^15*c^4*d^8*e^11*
z^2 + 54240*b^8*c^11*d^15*e^4*z^2 - 49920*b^14*c^5*d^9*e^10*z^2 - 49920*b^9
*c^10*d^14*e^5*z^2 - 37376*b^16*c^3*d^7*e^12*z^2 - 37376*b^7*c^12*d^16*e^3*
z^2 + 28480*b^13*c^6*d^10*e^9*z^2 + 28480*b^10*c^9*d^13*e^6*z^2 + 15936*b^1
7*c^2*d^6*e^13*z^2 + 15936*b^6*c^13*d^17*e^2*z^2 - 7920*b^12*c^7*d^11*e^8*z
^2 - 7920*b^11*c^8*d^12*e^7*z^2 + 7489536*a^5*c^14*d^13*e^6*z^2 + 6084096*a
^6*c^13*d^11*e^8*z^2 + 2280448*a^4*c^15*d^15*e^4*z^2 + 350208*a^3*c^16*d^17
*e^2*z^2 + 11616*a^2*b^17*d^2*e^17*z^2 - 3515904*a^9*b^5*c^5*e^19*z^2 + 344
0640*a^10*b^3*c^6*e^19*z^2 + 1870848*a^8*b^7*c^4*e^19*z^2 - 572272*a^7*b^9*
c^3*e^19*z^2 + 101856*a^6*b^11*c^2*e^19*z^2 + 400*b^19*d^4*e^15*z^2 + 400*b
^4*c^15*d^19*z^2 + 20736*a^2*c^17*d^19*z^2 + 400*a^4*b^15*e^19*z^2 - 396921
6*a^4*b*c^10*d^3*e^12 - 3001536*a^3*b*c^11*d^5*e^10 - 419904*a^2*b*c^12*d^7
*e^8 + 184608*a^4*b^3*c^8*d*e^14 - 153036*a*b^4*c^10*d^6*e^9 + 127008*a*b^3
*c^11*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^11 - 29160*a*b^2*c^12*d^8*e^7 - 21060
*a^3*b^5*c^7*d*e^14 - 21060*a*b^7*c^7*d^3*e^12 + 5460*a*b^5*c^9*d^5*e^10 -
404544*a^5*b*c^9*d*e^14 + 1251872*a^3*b^3*c^9*d^3*e^12 + 844224*a^4*b^2*c^9
*d^2*e^13 + 820512*a^2*b^3*c^10*d^5*e^10 + 750672*a^3*b^2*c^10*d^4*e^11 - 6
57498*a^2*b^4*c^9*d^4*e^11 - 487116*a^3*b^4*c^8*d^2*e^13 + 160704*a^2*b^2*c
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^11*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^13 + 13140*a^2*b^5*c^8*d^3*e^12 + 152
86*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e^10 - 9540*b^5*c^10*d^7*e^8 + 2025*b
^8*c^7*d^4*e^11 + 2025*b^4*c^11*d^8*e^7 + 3367008*a^4*c^11*d^4*e^11 + 11664
00*a^3*c^12*d^6*e^9 + 705600*a^5*c^10*d^2*e^13 + 104976*a^2*c^13*d^8*e^7 -
17640*a^5*b^2*c^8*e^15 + 2025*a^4*b^4*c^7*e^15 + 38416*a^6*c^9*e^15, z, k)*
x*(1048576*a^8*c^19*d^24*e^3 + 9437184*a^9*c^18*d^22*e^5 + 36700160*a^10*c^
17*d^20*e^7 + 78643200*a^11*c^16*d^18*e^9 + 94371840*a^12*c^15*d^16*e^11 +
44040192*a^13*c^14*d^14*e^13 - 44040192*a^14*c^13*d^12*e^15 - 94371840*a^15
*c^12*d^10*e^17 - 78643200*a^16*c^11*d^8*e^19 - 36700160*a^17*c^10*d^6*e^21
 - 9437184*a^18*c^9*d^4*e^23 - 1048576*a^19*c^8*d^2*e^25 - 256*a^2*b^11*c^1
4*d^25*e^2 + 3072*a^2*b^12*c^13*d^24*e^3 - 16896*a^2*b^13*c^12*d^23*e^4 + 5
6320*a^2*b^14*c^11*d^22*e^5 - 126720*a^2*b^15*c^10*d^21*e^6 + 202752*a^2*b^
16*c^9*d^20*e^7 - 236544*a^2*b^17*c^8*d^19*e^8 + 202752*a^2*b^18*c^7*d^18*e
^9 - 126720*a^2*b^19*c^6*d^17*e^10 + 56320*a^2*b^20*c^5*d^16*e^11 - 16896*a
^2*b^21*c^4*d^15*e^12 + 3072*a^2*b^22*c^3*d^14*e^13 - 256*a^2*b^23*c^2*d^13
*e^14 + 5120*a^3*b^9*c^15*d^25*e^2 - 62464*a^3*b^10*c^14*d^24*e^3 + 346368*
a^3*b^11*c^13*d^23*e^4 - 1152256*a^3*b^12*c^12*d^22*e^5 + 2553600*a^3*b^13*
c^11*d^21*e^6 - 3951360*a^3*b^14*c^10*d^20*e^7 + 4336128*a^3*b^15*c^9*d^19*
e^8 - 3334656*a^3*b^16*c^8*d^18*e^9 + 1700352*a^3*b^17*c^7*d^17*e^10 - 4736
00*a^3*b^18*c^6*d^16*e^11 - 8960*a^3*b^19*c^5*d^15*e^12 + 59136*a^3*b^20*c^
4*d^14*e^13 - 19712*a^3*b^21*c^3*d^13*e^14 + 2304*a^3*b^22*c^2*d^12*e^15 -
40960*a^4*b^7*c^16*d^25*e^2 + 512000*a^4*b^8*c^15*d^24*e^3 - 2872320*a^4*b^
9*c^14*d^23*e^4 + 9519104*a^4*b^10*c^13*d^22*e^5 - 20581120*a^4*b^11*c^12*d
^21*e^6 + 30087680*a^4*b^12*c^11*d^20*e^7 - 29433600*a^4*b^13*c^10*d^19*e^8
 + 17602560*a^4*b^14*c^9*d^18*e^9 - 3798528*a^4*b^15*c^8*d^17*e^10 - 307712
0*a^4*b^16*c^7*d^16*e^11 + 3028480*a^4*b^17*c^6*d^15*e^12 - 1075200*a^4*b^1
8*c^5*d^14*e^13 + 98560*a^4*b^19*c^4*d^13*e^14 + 39424*a^4*b^20*c^3*d^12*e^
15 - 8960*a^4*b^21*c^2*d^11*e^16 + 163840*a^5*b^5*c^17*d^25*e^2 - 2129920*a
^5*b^6*c^16*d^24*e^3 + 12165120*a^5*b^7*c^15*d^23*e^4 - 39997440*a^5*b^8*c^
14*d^22*e^5 + 82611200*a^5*b^9*c^13*d^21*e^6 - 107627520*a^5*b^10*c^12*d^20
*e^7 + 78140160*a^5*b^11*c^11*d^19*e^8 - 6831360*a^5*b^12*c^10*d^18*e^9 - 4
6586880*a^5*b^13*c^9*d^17*e^10 + 47436800*a^5*b^14*c^8*d^16*e^11 - 20088320
*a^5*b^15*c^7*d^15*e^12 + 1128960*a^5*b^16*c^6*d^14*e^13 + 2365440*a^5*b^17
*c^5*d^13*e^14 - 788480*a^5*b^18*c^4*d^12*e^15 + 19200*a^5*b^19*c^3*d^11*e^
16 + 19200*a^5*b^20*c^2*d^10*e^17 - 327680*a^6*b^3*c^18*d^25*e^2 + 4587520*
a^6*b^4*c^17*d^24*e^3 - 27033600*a^6*b^5*c^16*d^23*e^4 + 87162880*a^6*b^6*c
^15*d^22*e^5 - 161996800*a^6*b^7*c^14*d^21*e^6 + 149237760*a^6*b^8*c^13*d^2
0*e^7 + 27202560*a^6*b^9*c^12*d^19*e^8 - 251750400*a^6*b^10*c^11*d^18*e^9 +
 305948160*a^6*b^11*c^10*d^17*e^10 - 160153600*a^6*b^12*c^9*d^16*e^11 + 143
360*a^6*b^13*c^8*d^15*e^12 + 46018560*a^6*b^14*c^7*d^14*e^13 - 21683200*a^6
*b^15*c^6*d^13*e^14 + 1576960*a^6*b^16*c^5*d^12*e^15 + 1305600*a^6*b^17*c^4
*d^11*e^16 - 215040*a^6*b^18*c^3*d^10*e^17 - 23040*a^6*b^19*c^2*d^9*e^18 -
4456448*a^7*b^2*c^18*d^24*e^3 + 28114944*a^7*b^3*c^17*d^23*e^4 - 84869120*a
^7*b^4*c^16*d^22*e^5 + 104366080*a^7*b^5*c^15*d^21*e^6 + 97943552*a^7*b^6*c
^14*d^20*e^7 - 549986304*a^7*b^7*c^13*d^19*e^8 + 841961472*a^7*b^8*c^12*d^1
```

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8*e^9 - 549795840*a^7*b^9*c^11*d^17*e^10 - 68823040*a^7*b^10*c^10*d^16*e^11
 + 375613952*a^7*b^11*c^9*d^15*e^12 - 240167424*a^7*b^12*c^8*d^14*e^13 + 32
840192*a^7*b^13*c^7*d^13*e^14 + 27399680*a^7*b^14*c^6*d^12*e^15 - 10703360*
a^7*b^15*c^5*d^11*e^16 - 81408*a^7*b^16*c^4*d^10*e^17 + 370176*a^7*b^17*c^3
*d^9*e^18 + 10752*a^7*b^18*c^2*d^8*e^19 + 14680064*a^8*b^2*c^17*d^22*e^5 +
80281600*a^8*b^3*c^16*d^21*e^6 - 440401920*a^8*b^4*c^15*d^20*e^7 + 88837324
8*a^8*b^5*c^14*d^19*e^8 - 703266816*a^8*b^6*c^13*d^18*e^9 - 394149888*a^8*b
^7*c^12*d^17*e^10 + 1358438400*a^8*b^8*c^11*d^16*e^11 - 1129891840*a^8*b^9*
c^10*d^15*e^12 + 225189888*a^8*b^10*c^9*d^14*e^13 + 246045184*a^8*b^11*c^8*
d^13*e^14 - 164082688*a^8*b^12*c^7*d^12*e^15 + 18009600*a^8*b^13*c^6*d^11*e
^16 + 10659840*a^8*b^14*c^5*d^10*e^17 - 2099712*a^8*b^15*c^4*d^9*e^18 - 193
536*a^8*b^16*c^3*d^8*e^19 + 10752*a^8*b^17*c^2*d^7*e^20 + 239861760*a^9*b^2
*c^16*d^20*e^7 - 172032000*a^9*b^3*c^15*d^19*e^8 - 704839680*a^9*b^4*c^14*d
^18*e^9 + 2013069312*a^9*b^5*c^13*d^17*e^10 - 2086993920*a^9*b^6*c^12*d^16*
e^11 + 424427520*a^9*b^7*c^11*d^15*e^12 + 1074585600*a^9*b^8*c^10*d^14*e^13
 - 997877760*a^9*b^9*c^9*d^13*e^14 + 234493952*a^9*b^10*c^8*d^12*e^15 + 957
61920*a^9*b^11*c^7*d^11*e^16 - 55288320*a^9*b^12*c^6*d^10*e^17 + 3916800*a^
9*b^13*c^5*d^9*e^18 + 1704960*a^9*b^14*c^4*d^8*e^19 - 250368*a^9*b^15*c^3*d
^7*e^20 - 23040*a^9*b^16*c^2*d^6*e^21 + 857210880*a^10*b^2*c^15*d^18*e^9 -
1036124160*a^10*b^3*c^14*d^17*e^10 - 255590400*a^10*b^4*c^13*d^16*e^11 + 21
95128320*a^10*b^5*c^12*d^15*e^12 - 2422210560*a^10*b^6*c^11*d^14*e^13 + 813
711360*a^10*b^7*c^10*d^13*e^14 + 420372480*a^10*b^8*c^9*d^12*e^15 - 4285952
00*a^10*b^9*c^8*d^11*e^16 + 106106880*a^10*b^10*c^7*d^10*e^17 + 8866560*a^1
0*b^11*c^6*d^9*e^18 - 11074560*a^10*b^12*c^5*d^8*e^19 + 1989120*a^10*b^13*c
^4*d^7*e^20 + 537600*a^10*b^14*c^3*d^6*e^21 + 19200*a^10*b^15*c^2*d^5*e^22
+ 1454899200*a^11*b^2*c^14*d^16*e^11 - 1747845120*a^11*b^3*c^13*d^15*e^12 +
 454164480*a^11*b^4*c^12*d^14*e^13 + 1135411200*a^11*b^5*c^11*d^13*e^14 - 1
286799360*a^11*b^6*c^10*d^12*e^15 + 527155200*a^11*b^7*c^9*d^11*e^16 - 4190
2080*a^11*b^8*c^8*d^10*e^17 - 74849280*a^11*b^9*c^7*d^9*e^18 + 53222400*a^1
1*b^10*c^6*d^8*e^19 - 4023040*a^11*b^11*c^5*d^7*e^20 - 4972800*a^11*b^12*c^
4*d^6*e^21 - 456960*a^11*b^13*c^3*d^5*e^22 - 8960*a^11*b^14*c^2*d^4*e^23 +
1189085184*a^12*b^2*c^13*d^14*e^13 - 1241382912*a^12*b^3*c^12*d^13*e^14 + 6
05552640*a^12*b^4*c^11*d^12*e^15 - 97320960*a^12*b^5*c^10*d^11*e^16 - 14273
7408*a^12*b^6*c^9*d^10*e^17 + 278716416*a^12*b^7*c^8*d^9*e^18 - 144764928*a
^12*b^8*c^7*d^8*e^19 - 28779520*a^12*b^9*c^6*d^7*e^20 + 22077440*a^12*b^10*
c^5*d^6*e^21 + 4456704*a^12*b^11*c^4*d^5*e^22 + 215552*a^12*b^12*c^3*d^4*e^
23 + 2304*a^12*b^13*c^2*d^3*e^24 + 121110528*a^13*b^2*c^12*d^12*e^15 - 1081
34400*a^13*b^3*c^11*d^11*e^16 + 454164480*a^13*b^4*c^10*d^10*e^17 - 5871697
92*a^13*b^5*c^9*d^9*e^18 + 98402304*a^13*b^6*c^8*d^8*e^19 + 184819712*a^13*
b^7*c^7*d^7*e^20 - 39424000*a^13*b^8*c^6*d^6*e^21 - 22471680*a^13*b^9*c^5*d
^5*e^22 - 2151424*a^13*b^10*c^4*d^4*e^23 - 55552*a^13*b^11*c^3*d^3*e^24 - 2
56*a^13*b^12*c^2*d^2*e^25 - 644874240*a^14*b^2*c^11*d^10*e^17 + 339148800*a
^14*b^3*c^10*d^9*e^18 + 371589120*a^14*b^4*c^9*d^8*e^19 - 367689728*a^14*b^
5*c^8*d^7*e^20 - 32112640*a^14*b^6*c^7*d^6*e^21 + 59351040*a^14*b^7*c^6*d^5
*e^22 + 11366400*a^14*b^8*c^5*d^4*e^23 + 558080*a^14*b^9*c^4*d^3*e^24 + 614
```

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4*a^14*b^10*c^3*d^2*e^25 - 578027520*a^15*b^2*c^10*d^8*e^19 + 135331840*a^1
5*b^3*c^9*d^7*e^20 + 217907200*a^15*b^4*c^8*d^6*e^21 - 65372160*a^15*b^5*c^
7*d^5*e^22 - 33259520*a^15*b^6*c^6*d^4*e^23 - 2990080*a^15*b^7*c^5*d^3*e^24
 - 61440*a^15*b^8*c^4*d^2*e^25 - 209715200*a^16*b^2*c^9*d^6*e^21 - 20643840
*a^16*b^3*c^8*d^5*e^22 + 49807360*a^16*b^4*c^7*d^4*e^23 + 9011200*a^16*b^5*
c^6*d^3*e^24 + 327680*a^16*b^6*c^5*d^2*e^25 - 25427968*a^17*b^2*c^8*d^4*e^2
3 - 14483456*a^17*b^3*c^7*d^3*e^24 - 983040*a^17*b^4*c^6*d^2*e^25 + 1572864
*a^18*b^2*c^7*d^2*e^25 + 262144*a^7*b*c^19*d^25*e^2 - 8650752*a^8*b*c^18*d^
23*e^4 - 79953920*a^9*b*c^17*d^21*e^6 - 287047680*a^10*b*c^16*d^19*e^8 - 54
2638080*a^11*b*c^15*d^17*e^10 - 539492352*a^12*b*c^14*d^15*e^12 - 143130624
*a^13*b*c^13*d^13*e^14 + 306708480*a^14*b*c^12*d^11*e^16 + 420741120*a^15*b
*c^11*d^9*e^18 + 250347520*a^16*b*c^10*d^7*e^20 + 76283904*a^17*b*c^9*d^5*e
^22 + 9699328*a^18*b*c^8*d^3*e^24))/(8*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^8*d
^18 - 256*a^6*c^12*d^18 - 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 - a^
2*b^16*d^10*e^8 + 8*a^3*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^13*d
^7*e^11 - 70*a^6*b^12*d^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*e^1
4 + 8*a^9*b^9*d^3*e^15 - a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 7168*
a^8*c^10*d^14*e^4 - 14336*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 1433
6*a^11*c^7*d^8*e^10 - 7168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 - 256
*a^14*c^4*d^2*e^16 - 28*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3 -
70*a^2*b^12*c^4*d^14*e^4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^12*
e^6 + 440*a^3*b^8*c^7*d^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10*c^
5*d^14*e^4 - 616*a^3*b^11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*a^3
*b^13*c^2*d^11*e^7 - 2560*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e^3
 - 4060*a^4*b^8*c^6*d^14*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*c^4
*d^12*e^6 - 1360*a^4*b^11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 6400*a
^5*b^4*c^9*d^16*e^2 - 8960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*e^4
 + 9856*a^5*b^7*c^6*d^13*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9*c^
4*d^11*e^7 + 1040*a^5*b^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 5120*
a^6*b^2*c^10*d^16*e^2 + 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d^13
*e^5 + 25088*a^6*b^6*c^6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^6*b
^8*c^4*d^10*e^8 + 5400*a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10 - 3
5840*a^7*b^2*c^9*d^14*e^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c^7*
d^12*e^6 - 73472*a^7*b^5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 8320*a
^7*b^7*c^4*d^9*e^9 - 13048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e^11
 - 93184*a^8*b^2*c^8*d^12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b^4*
c^6*d^10*e^8 - 73472*a^8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 + 985
6*a^8*b^7*c^3*d^7*e^11 - 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*d^1
0*e^8 + 71680*a^9*b^3*c^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a^9*
b^5*c^4*d^7*e^11 + 2240*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13 -
93184*a^10*b^2*c^6*d^8*e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b^4*
c^4*d^6*e^12 - 8960*a^10*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 - 35
840*a^11*b^2*c^5*d^6*e^12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^2*d
^3*e^15 - 5120*a^12*b^2*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a^12
*b^4*c^2*d^2*e^16 + 256*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e + 8*
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a^2*b^9*c^7*d^17*e + 8*a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40*a^
3*b^14*c*d^10*e^8 + 768*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 2048*a
^5*b^3*c^10*d^17*e + 168*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 + 14
336*a^7*b*c^10*d^15*e^3 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13*e^
5 - 840*a^8*b^9*c*d^5*e^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^4*e
^14 + 71680*a^10*b*c^7*d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*c^6
*d^7*e^11 + 16*a^11*b^6*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^13*
b*c^4*d^3*e^15))) - (x*(49152*a^14*b*c^8*e^23 - 65536*a^14*c^9*d*e^22 + 16*
a^8*b^13*c^2*e^23 - 368*a^9*b^11*c^3*e^23 + 3520*a^10*b^9*c^4*e^23 - 17920*
a^11*b^7*c^5*e^23 + 51200*a^12*b^5*c^6*e^23 - 77824*a^13*b^3*c^7*e^23 + 184
32*a^4*c^19*d^21*e^2 + 243712*a^5*c^18*d^19*e^4 + 1253376*a^6*c^17*d^17*e^6
 + 2252800*a^7*c^16*d^15*e^8 - 7835648*a^8*c^15*d^13*e^10 - 35516416*a^9*c^
14*d^11*e^12 - 50487296*a^10*c^13*d^9*e^14 - 30416896*a^11*c^12*d^7*e^16 -
5797888*a^12*c^11*d^5*e^18 + 522240*a^13*c^10*d^3*e^20 + 16*b^8*c^15*d^21*e
^2 - 160*b^9*c^14*d^20*e^3 + 720*b^10*c^13*d^19*e^4 - 1904*b^11*c^12*d^18*e
^5 + 3200*b^12*c^11*d^17*e^6 - 3312*b^13*c^10*d^16*e^7 + 1440*b^14*c^9*d^15
*e^8 + 1440*b^15*c^8*d^14*e^9 - 3312*b^16*c^7*d^13*e^10 + 3200*b^17*c^6*d^1
2*e^11 - 1904*b^18*c^5*d^11*e^12 + 720*b^19*c^4*d^10*e^13 - 160*b^20*c^3*d^
9*e^14 + 16*b^21*c^2*d^8*e^15 + 3200*a^2*b^4*c^17*d^21*e^2 - 30336*a^2*b^5*
c^16*d^20*e^3 + 123296*a^2*b^6*c^15*d^19*e^4 - 269568*a^2*b^7*c^14*d^18*e^5
 + 295872*a^2*b^8*c^13*d^17*e^6 + 16576*a^2*b^9*c^12*d^16*e^7 - 582688*a^2*
b^10*c^11*d^15*e^8 + 944640*a^2*b^11*c^10*d^14*e^9 - 761856*a^2*b^12*c^9*d^
13*e^10 + 243456*a^2*b^13*c^8*d^12*e^11 + 126048*a^2*b^14*c^7*d^11*e^12 - 1
64096*a^2*b^15*c^6*d^10*e^13 + 58304*a^2*b^16*c^5*d^9*e^14 + 3264*a^2*b^17*
c^4*d^8*e^15 - 7648*a^2*b^18*c^3*d^7*e^16 + 1536*a^2*b^19*c^2*d^6*e^17 - 12
800*a^3*b^2*c^18*d^21*e^2 + 119296*a^3*b^3*c^17*d^20*e^3 - 448896*a^3*b^4*c
^16*d^19*e^4 + 783872*a^3*b^5*c^15*d^18*e^5 - 197504*a^3*b^6*c^14*d^17*e^6
- 1977216*a^3*b^7*c^13*d^16*e^7 + 4413568*a^3*b^8*c^12*d^15*e^8 - 4435520*a
^3*b^9*c^11*d^14*e^9 + 1422432*a^3*b^10*c^10*d^13*e^10 + 1795872*a^3*b^11*c
^9*d^12*e^11 - 2349888*a^3*b^12*c^8*d^11*e^12 + 800352*a^3*b^13*c^7*d^10*e^
13 + 426688*a^3*b^14*c^6*d^9*e^14 - 478112*a^3*b^15*c^5*d^8*e^15 + 145344*a
^3*b^16*c^4*d^7*e^16 - 3104*a^3*b^17*c^3*d^6*e^17 - 4384*a^3*b^18*c^2*d^5*e
^18 + 519680*a^4*b^2*c^17*d^19*e^4 - 122880*a^4*b^3*c^16*d^18*e^5 - 3229184
*a^4*b^4*c^15*d^17*e^6 + 9323008*a^4*b^5*c^14*d^16*e^7 - 11702656*a^4*b^6*c
^13*d^15*e^8 + 3460864*a^4*b^7*c^12*d^14*e^9 + 10917472*a^4*b^8*c^11*d^13*e
^10 - 16615488*a^4*b^9*c^10*d^12*e^11 + 7102272*a^4*b^10*c^9*d^11*e^12 + 58
42272*a^4*b^11*c^8*d^10*e^13 - 8942080*a^4*b^12*c^7*d^9*e^14 + 4203232*a^4*
b^13*c^6*d^8*e^15 - 364736*a^4*b^14*c^5*d^7*e^16 - 309472*a^4*b^15*c^4*d^6*
e^17 + 63136*a^4*b^16*c^3*d^5*e^18 + 6112*a^4*b^17*c^2*d^4*e^19 + 6961152*a
^5*b^2*c^16*d^17*e^6 - 10246144*a^5*b^3*c^15*d^16*e^7 - 747008*a^5*b^4*c^14
*d^15*e^8 + 29979648*a^5*b^5*c^13*d^14*e^9 - 52869952*a^5*b^6*c^12*d^13*e^1
0 + 32791616*a^5*b^7*c^11*d^12*e^11 + 25176960*a^5*b^8*c^10*d^11*e^12 - 629
55552*a^5*b^9*c^9*d^10*e^13 + 45989472*a^5*b^10*c^8*d^9*e^14 - 9362688*a^5*
b^11*c^7*d^8*e^15 - 5824480*a^5*b^12*c^6*d^7*e^16 + 3196768*a^5*b^13*c^5*d^
6*e^17 - 132768*a^5*b^14*c^4*d^5*e^18 - 119680*a^5*b^15*c^3*d^4*e^19 - 4384
```

```
*a^5*b^16*c^2*d^3*e^20 + 32086016*a^6*b^2*c^15*d^15*e^8 - 57880576*a^6*b^3*
c^14*d^14*e^9 + 44683008*a^6*b^4*c^13*d^13*e^10 + 49481984*a^6*b^5*c^12*d^1
2*e^11 - 175788864*a^6*b^6*c^11*d^11*e^12 + 194611968*a^6*b^7*c^10*d^10*e^1
3 - 73867584*a^6*b^8*c^9*d^9*e^14 - 38225280*a^6*b^9*c^8*d^8*e^15 + 4545014
4*a^6*b^10*c^7*d^7*e^16 - 10588672*a^6*b^11*c^6*d^6*e^17 - 2519296*a^6*b^12
*c^5*d^5*e^18 + 864384*a^6*b^13*c^4*d^4*e^19 + 96224*a^6*b^14*c^3*d^3*e^20
+ 1536*a^6*b^15*c^2*d^2*e^21 + 67527680*a^7*b^2*c^14*d^13*e^10 - 181466112*
a^7*b^3*c^13*d^12*e^11 + 278696704*a^7*b^4*c^12*d^11*e^12 - 171431936*a^7*b
^5*c^11*d^10*e^13 - 104909184*a^7*b^6*c^10*d^9*e^14 + 231100032*a^7*b^7*c^9
*d^8*e^15 - 116105856*a^7*b^8*c^8*d^7*e^16 - 5653568*a^7*b^9*c^7*d^6*e^17 +
 19556768*a^7*b^10*c^6*d^5*e^18 - 2291488*a^7*b^11*c^5*d^4*e^19 - 855936*a^
7*b^12*c^4*d^3*e^20 - 35168*a^7*b^13*c^3*d^2*e^21 - 40418304*a^8*b^2*c^13*d
^11*e^12 - 155127808*a^8*b^3*c^12*d^10*e^13 + 421659136*a^8*b^4*c^11*d^9*e^
14 - 366294528*a^8*b^5*c^10*d^8*e^15 + 42953856*a^8*b^6*c^9*d^7*e^16 + 1158
41280*a^8*b^7*c^8*d^6*e^17 - 54301680*a^8*b^8*c^7*d^5*e^18 - 3139616*a^8*b^
9*c^6*d^4*e^19 + 3850352*a^8*b^10*c^5*d^3*e^20 + 333840*a^8*b^11*c^4*d^2*e^
21 - 262465536*a^9*b^2*c^12*d^9*e^14 + 49444864*a^9*b^3*c^11*d^8*e^15 + 255
840768*a^9*b^4*c^10*d^7*e^16 - 241492992*a^9*b^5*c^9*d^6*e^17 + 41574816*a^
9*b^6*c^8*d^5*e^18 + 32344416*a^9*b^7*c^7*d^4*e^19 - 8542208*a^9*b^8*c^6*d^
3*e^20 - 1677872*a^9*b^9*c^5*d^2*e^21 - 270632960*a^10*b^2*c^11*d^7*e^16 +
105492480*a^10*b^3*c^10*d^6*e^17 + 71796864*a^10*b^4*c^9*d^5*e^18 - 6679104
0*a^10*b^5*c^8*d^4*e^19 + 5437088*a^10*b^6*c^7*d^3*e^20 + 4684288*a^10*b^7*
c^6*d^2*e^21 - 105693696*a^11*b^2*c^10*d^5*e^18 + 38220288*a^11*b^3*c^9*d^4
*e^19 + 10967680*a^11*b^4*c^8*d^3*e^20 - 6778368*a^11*b^5*c^7*d^2*e^21 - 15
811072*a^12*b^2*c^9*d^3*e^20 + 3633152*a^12*b^3*c^8*d^2*e^21 - 352*a*b^6*c^
16*d^21*e^2 + 3424*a*b^7*c^15*d^20*e^3 - 14720*a*b^8*c^14*d^19*e^4 + 36048*
a*b^9*c^13*d^18*e^5 - 52384*a*b^10*c^12*d^17*e^6 + 36464*a*b^11*c^11*d^16*e
^7 + 17952*a*b^12*c^10*d^15*e^8 - 75360*a*b^13*c^9*d^14*e^9 + 91104*a*b^14*
c^8*d^13*e^10 - 60992*a*b^15*c^7*d^12*e^11 + 20288*a*b^16*c^6*d^11*e^12 + 1
424*a*b^17*c^5*d^10*e^13 - 4320*a*b^18*c^4*d^9*e^14 + 1648*a*b^19*c^3*d^8*e
^15 - 224*a*b^20*c^2*d^7*e^16 - 169984*a^4*b*c^18*d^20*e^3 - 2076672*a^5*b*
c^17*d^18*e^5 - 9658368*a^6*b*c^16*d^16*e^7 - 16384000*a^7*b*c^15*d^14*e^9
- 224*a^7*b^14*c^2*d*e^22 + 42463232*a^8*b*c^14*d^12*e^11 + 5120*a^8*b^12*c
^3*d*e^22 + 170631168*a^9*b*c^13*d^10*e^13 - 48576*a^9*b^10*c^4*d*e^22 + 19
9843840*a^10*b*c^12*d^8*e^15 + 244480*a^10*b^8*c^5*d*e^22 + 95387648*a^11*b
*c^11*d^6*e^17 - 686080*a^11*b^6*c^6*d*e^22 + 15722496*a^12*b*c^10*d^4*e^19
 + 1007616*a^12*b^4*c^7*d*e^22 + 692224*a^13*b*c^9*d^2*e^21 - 573440*a^13*b
^2*c^8*d*e^22))/(8*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^8*d^18 - 256*a^6*c^12*d
^18 - 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 - a^2*b^16*d^10*e^8 + 8*
a^3*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^13*d^7*e^11 - 70*a^6*b^1
2*d^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*e^14 + 8*a^9*b^9*d^3*e^
15 - a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 7168*a^8*c^10*d^14*e^4 -
14336*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 14336*a^11*c^7*d^8*e^10
- 7168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 - 256*a^14*c^4*d^2*e^16 -
 28*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3 - 70*a^2*b^12*c^4*d^14
```

*e^4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^12*e^6 + 440*a^3*b^8*c^
7*d^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10*c^5*d^14*e^4 - 616*a^3
*b^11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*a^3*b^13*c^2*d^11*e^7 -
2560*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e^3 - 4060*a^4*b^8*c^6*d
^14*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*c^4*d^12*e^6 - 1360*a^4
*b^11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 6400*a^5*b^4*c^9*d^16*e^2
- 8960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*e^4 + 9856*a^5*b^7*c^6*
d^13*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9*c^4*d^11*e^7 + 1040*a^
5*b^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 5120*a^6*b^2*c^10*d^16*e^
2 + 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d^13*e^5 + 25088*a^6*b^6
*c^6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^6*b^8*c^4*d^10*e^8 + 54
00*a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10 - 35840*a^7*b^2*c^9*d^1
4*e^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c^7*d^12*e^6 - 73472*a^7
*b^5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 8320*a^7*b^7*c^4*d^9*e^9 -
13048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e^11 - 93184*a^8*b^2*c^8
*d^12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b^4*c^6*d^10*e^8 - 73472
*a^8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 + 9856*a^8*b^7*c^3*d^7*e^
11 - 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*d^10*e^8 + 71680*a^9*b^
3*c^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a^9*b^5*c^4*d^7*e^11 + 2
240*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13 - 93184*a^10*b^2*c^6*d
^8*e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b^4*c^4*d^6*e^12 - 8960*
a^10*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 - 35840*a^11*b^2*c^5*d^6
*e^12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^2*d^3*e^15 - 5120*a^12*
b^2*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a^12*b^4*c^2*d^2*e^16 +
256*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e + 8*a^2*b^9*c^7*d^17*e +
8*a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40*a^3*b^14*c*d^10*e^8 +
768*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 2048*a^5*b^3*c^10*d^17*e +
168*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 + 14336*a^7*b*c^10*d^15*
e^3 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13*e^5 - 840*a^8*b^9*c*d^
5*e^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^4*e^14 + 71680*a^10*b*c
^7*d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*c^6*d^7*e^11 + 16*a^11*
b^6*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^13*b*c^4*d^3*e^15)))) +
(x*(25*a^4*b^10*c^5*e^19 - 6272*a^9*c^10*e^19 - 440*a^5*b^8*c^6*e^19 + 298
6*a^6*b^6*c^7*e^19 - 9560*a^7*b^4*c^8*e^19 + 13792*a^8*b^2*c^9*e^19 + 1296*
a^2*c^17*d^14*e^5 + 19296*a^3*c^16*d^12*e^7 + 195952*a^4*c^15*d^10*e^9 + 93
8176*a^5*c^14*d^8*e^11 + 1838832*a^6*c^13*d^6*e^13 - 20896*a^7*c^12*d^4*e^1
5 - 57200*a^8*c^11*d^2*e^17 + 25*b^4*c^15*d^14*e^5 - 190*b^5*c^14*d^13*e^6
+ 591*b^6*c^13*d^12*e^7 - 964*b^7*c^12*d^11*e^8 + 952*b^8*c^11*d^10*e^9 - 8
28*b^9*c^10*d^9*e^10 + 952*b^10*c^9*d^8*e^11 - 964*b^11*c^8*d^7*e^12 + 591*
b^12*c^7*d^6*e^13 - 190*b^13*c^6*d^5*e^14 + 25*b^14*c^5*d^4*e^15 + 18816*a^
2*b^2*c^15*d^12*e^7 - 464*a^2*b^3*c^14*d^11*e^8 - 33441*a^2*b^4*c^13*d^10*e
^9 - 9780*a^2*b^5*c^12*d^9*e^10 + 98620*a^2*b^6*c^11*d^8*e^11 - 74420*a^2*b
^7*c^10*d^7*e^12 - 25327*a^2*b^8*c^9*d^6*e^13 + 51944*a^2*b^9*c^8*d^5*e^14
- 19162*a^2*b^10*c^7*d^4*e^15 + 376*a^2*b^11*c^6*d^3*e^16 + 726*a^2*b^12*c^
5*d^2*e^17 + 132104*a^3*b^2*c^14*d^10*e^9 + 202944*a^3*b^3*c^13*d^9*e^10 -

```
496916*a^3*b^4*c^12*d^8*e^11 + 62420*a^3*b^5*c^11*d^7*e^12 + 477560*a^3*b^6
*c^10*d^6*e^13 - 367184*a^3*b^7*c^9*d^5*e^14 + 42920*a^3*b^8*c^8*d^4*e^15 +
 41584*a^3*b^9*c^7*d^3*e^16 - 11716*a^3*b^10*c^6*d^2*e^17 + 774624*a^4*b^2*
c^13*d^8*e^11 + 1091488*a^4*b^3*c^12*d^7*e^12 - 2078409*a^4*b^4*c^11*d^6*e^
13 + 759546*a^4*b^5*c^10*d^5*e^14 + 436579*a^4*b^6*c^9*d^4*e^15 - 373848*a^
4*b^7*c^8*d^3*e^16 + 68053*a^4*b^8*c^7*d^2*e^17 + 2519400*a^5*b^2*c^12*d^6*
e^13 + 1051760*a^5*b^3*c^11*d^5*e^14 - 2494242*a^5*b^4*c^10*d^4*e^15 + 1223
634*a^5*b^5*c^9*d^3*e^16 - 153022*a^5*b^6*c^8*d^2*e^17 + 3717952*a^6*b^2*c^
11*d^4*e^15 - 1366224*a^6*b^3*c^10*d^3*e^16 + 23697*a^6*b^4*c^9*d^2*e^17 +
268408*a^7*b^2*c^10*d^2*e^17 + 43136*a^8*b*c^10*d*e^18 - 360*a*b^2*c^16*d^1
4*e^5 + 2608*a*b^3*c^15*d^13*e^6 - 7218*a*b^4*c^14*d^12*e^7 + 8922*a*b^5*c^
13*d^11*e^8 - 4786*a*b^6*c^12*d^10*e^9 + 4722*a*b^7*c^11*d^9*e^10 - 12250*a
*b^8*c^10*d^8*e^11 + 13434*a*b^9*c^9*d^7*e^12 - 4918*a*b^10*c^8*d^6*e^13 -
1202*a*b^11*c^7*d^5*e^14 + 1308*a*b^12*c^6*d^4*e^15 - 260*a*b^13*c^5*d^3*e^
16 - 8928*a^2*b*c^16*d^13*e^6 - 107360*a^3*b*c^15*d^11*e^8 - 260*a^3*b^11*c
^5*d*e^18 - 846912*a^4*b*c^14*d^9*e^10 + 4518*a^4*b^9*c^6*d*e^18 - 3155136*
a^5*b*c^13*d^7*e^12 - 30034*a^5*b^7*c^7*d*e^18 - 4176736*a^6*b*c^12*d^5*e^1
4 + 92664*a^6*b^5*c^8*d*e^18 - 154080*a^7*b*c^11*d^3*e^16 - 123488*a^7*b^3*
c^9*d*e^18))/(8*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^8*d^18 - 256*a^6*c^12*d^18
 - 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 - a^2*b^16*d^10*e^8 + 8*a^3
*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^13*d^7*e^11 - 70*a^6*b^12*d
^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*e^14 + 8*a^9*b^9*d^3*e^15
- a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 7168*a^8*c^10*d^14*e^4 - 143
36*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 14336*a^11*c^7*d^8*e^10 - 7
168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 - 256*a^14*c^4*d^2*e^16 - 28
*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3 - 70*a^2*b^12*c^4*d^14*e^
4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^12*e^6 + 440*a^3*b^8*c^7*d
^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10*c^5*d^14*e^4 - 616*a^3*b^
11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*a^3*b^13*c^2*d^11*e^7 - 25
60*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e^3 - 4060*a^4*b^8*c^6*d^14
*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*c^4*d^12*e^6 - 1360*a^4*b^
11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 6400*a^5*b^4*c^9*d^16*e^2 - 8
960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*e^4 + 9856*a^5*b^7*c^6*d^1
3*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9*c^4*d^11*e^7 + 1040*a^5*b
^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 5120*a^6*b^2*c^10*d^16*e^2 +
 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d^13*e^5 + 25088*a^6*b^6*c^
6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^6*b^8*c^4*d^10*e^8 + 5400*
a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10 - 35840*a^7*b^2*c^9*d^14*e
^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c^7*d^12*e^6 - 73472*a^7*b^
5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 8320*a^7*b^7*c^4*d^9*e^9 - 13
048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e^11 - 93184*a^8*b^2*c^8*d^
12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b^4*c^6*d^10*e^8 - 73472*a^
8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 + 9856*a^8*b^7*c^3*d^7*e^11
- 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*d^10*e^8 + 71680*a^9*b^3*c
^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a^9*b^5*c^4*d^7*e^11 + 2240
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*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13 - 93184*a^10*b^2*c^6*d^8*
e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b^4*c^4*d^6*e^12 - 8960*a^1
0*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 - 35840*a^11*b^2*c^5*d^6*e^
12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^2*d^3*e^15 - 5120*a^12*b^2
*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a^12*b^4*c^2*d^2*e^16 + 256
*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e + 8*a^2*b^9*c^7*d^17*e + 8*
a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40*a^3*b^14*c*d^10*e^8 + 768
*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 2048*a^5*b^3*c^10*d^17*e + 16
8*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 + 14336*a^7*b*c^10*d^15*e^3
 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13*e^5 - 840*a^8*b^9*c*d^5*e
^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^4*e^14 + 71680*a^10*b*c^7*
d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*c^6*d^7*e^11 + 16*a^11*b^6
*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^13*b*c^4*d^3*e^15))) - (39
20*a^6*b*c^10*e^17 + 32144*a^6*c^11*d*e^16 + 225*a^4*b^5*c^8*e^17 - 1880*a^
5*b^3*c^9*e^17 + 11664*a^2*c^15*d^9*e^8 + 46656*a^3*c^14*d^7*e^10 - 40608*a
^4*c^13*d^5*e^12 + 284224*a^5*c^12*d^3*e^14 + 225*b^4*c^13*d^9*e^8 - 755*b^
5*c^12*d^8*e^9 + 530*b^6*c^11*d^7*e^10 + 530*b^7*c^10*d^6*e^11 - 755*b^8*c^
9*d^5*e^12 + 225*b^9*c^8*d^4*e^13 + 27648*a^2*b^2*c^13*d^7*e^10 + 4576*a^2*
b^3*c^12*d^6*e^11 + 24438*a^2*b^4*c^11*d^5*e^12 - 44262*a^2*b^5*c^10*d^4*e^
13 + 4042*a^2*b^6*c^9*d^3*e^14 + 6534*a^2*b^7*c^8*d^2*e^15 - 23408*a^3*b^2*
c^12*d^5*e^12 + 41872*a^3*b^3*c^11*d^4*e^13 + 100948*a^3*b^4*c^10*d^3*e^14
- 60416*a^3*b^5*c^9*d^2*e^15 - 384384*a^4*b^2*c^11*d^3*e^14 + 165216*a^4*b^
3*c^10*d^2*e^15 - 3240*a*b^2*c^14*d^9*e^8 + 11016*a*b^3*c^13*d^8*e^9 - 8812
*a*b^4*c^12*d^7*e^10 - 1992*a*b^5*c^11*d^6*e^11 + 408*a*b^6*c^10*d^5*e^12 +
 5216*a*b^7*c^9*d^4*e^13 - 2340*a*b^8*c^8*d^3*e^14 - 40176*a^2*b*c^14*d^8*e
^9 - 63360*a^3*b*c^13*d^6*e^11 - 2340*a^3*b^6*c^8*d*e^16 + 120608*a^4*b*c^1
2*d^4*e^13 + 21281*a^4*b^4*c^9*d*e^16 - 114432*a^5*b*c^11*d^2*e^15 - 55656*
a^5*b^2*c^10*d*e^16)/(32*(16*a^3*b^6*c^9*d^18 - a^2*b^8*c^8*d^18 - 256*a^6*
c^12*d^18 - 96*a^4*b^4*c^10*d^18 + 256*a^5*b^2*c^11*d^18 - a^2*b^16*d^10*e^
8 + 8*a^3*b^15*d^9*e^9 - 28*a^4*b^14*d^8*e^10 + 56*a^5*b^13*d^7*e^11 - 70*a
^6*b^12*d^6*e^12 + 56*a^7*b^11*d^5*e^13 - 28*a^8*b^10*d^4*e^14 + 8*a^9*b^9*
d^3*e^15 - a^10*b^8*d^2*e^16 - 2048*a^7*c^11*d^16*e^2 - 7168*a^8*c^10*d^14*
e^4 - 14336*a^9*c^9*d^12*e^6 - 17920*a^10*c^8*d^10*e^8 - 14336*a^11*c^7*d^8
*e^10 - 7168*a^12*c^6*d^6*e^12 - 2048*a^13*c^5*d^4*e^14 - 256*a^14*c^4*d^2*
e^16 - 28*a^2*b^10*c^6*d^16*e^2 + 56*a^2*b^11*c^5*d^15*e^3 - 70*a^2*b^12*c^
4*d^14*e^4 + 56*a^2*b^13*c^3*d^13*e^5 - 28*a^2*b^14*c^2*d^12*e^6 + 440*a^3*
b^8*c^7*d^16*e^2 - 840*a^3*b^9*c^6*d^15*e^3 + 952*a^3*b^10*c^5*d^14*e^4 - 6
16*a^3*b^11*c^4*d^13*e^5 + 168*a^3*b^12*c^3*d^12*e^6 + 40*a^3*b^13*c^2*d^11
*e^7 - 2560*a^4*b^6*c^8*d^16*e^2 + 4480*a^4*b^7*c^7*d^15*e^3 - 4060*a^4*b^8
*c^6*d^14*e^4 + 1064*a^4*b^9*c^5*d^13*e^5 + 1372*a^4*b^10*c^4*d^12*e^6 - 13
60*a^4*b^11*c^3*d^11*e^7 + 380*a^4*b^12*c^2*d^10*e^8 + 6400*a^5*b^4*c^9*d^1
6*e^2 - 8960*a^5*b^5*c^8*d^15*e^3 + 2240*a^5*b^6*c^7*d^14*e^4 + 9856*a^5*b^
7*c^6*d^13*e^5 - 13048*a^5*b^8*c^5*d^12*e^6 + 5400*a^5*b^9*c^4*d^11*e^7 + 1
040*a^5*b^10*c^3*d^10*e^8 - 1360*a^5*b^11*c^2*d^9*e^9 - 5120*a^6*b^2*c^10*d
^16*e^2 + 22400*a^6*b^4*c^8*d^14*e^4 - 41216*a^6*b^5*c^7*d^13*e^5 + 25088*a
```

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^6*b^6*c^6*d^12*e^6 + 8320*a^6*b^7*c^5*d^11*e^7 - 17350*a^6*b^8*c^4*d^10*e^
8 + 5400*a^6*b^9*c^3*d^9*e^9 + 1372*a^6*b^10*c^2*d^8*e^10 - 35840*a^7*b^2*c
^9*d^14*e^4 + 28672*a^7*b^3*c^8*d^13*e^5 + 30464*a^7*b^4*c^7*d^12*e^6 - 734
72*a^7*b^5*c^6*d^11*e^7 + 40544*a^7*b^6*c^5*d^10*e^8 + 8320*a^7*b^7*c^4*d^9
*e^9 - 13048*a^7*b^8*c^3*d^8*e^10 + 1064*a^7*b^9*c^2*d^7*e^11 - 93184*a^8*b
^2*c^8*d^12*e^6 + 71680*a^8*b^3*c^7*d^11*e^7 + 29120*a^8*b^4*c^6*d^10*e^8 -
 73472*a^8*b^5*c^5*d^9*e^9 + 25088*a^8*b^6*c^4*d^8*e^10 + 9856*a^8*b^7*c^3*
d^7*e^11 - 4060*a^8*b^8*c^2*d^6*e^12 - 125440*a^9*b^2*c^7*d^10*e^8 + 71680*
a^9*b^3*c^6*d^9*e^9 + 30464*a^9*b^4*c^5*d^8*e^10 - 41216*a^9*b^5*c^4*d^7*e^
11 + 2240*a^9*b^6*c^3*d^6*e^12 + 4480*a^9*b^7*c^2*d^5*e^13 - 93184*a^10*b^2
*c^6*d^8*e^10 + 28672*a^10*b^3*c^5*d^7*e^11 + 22400*a^10*b^4*c^4*d^6*e^12 -
 8960*a^10*b^5*c^3*d^5*e^13 - 2560*a^10*b^6*c^2*d^4*e^14 - 35840*a^11*b^2*c
^5*d^6*e^12 + 6400*a^11*b^4*c^3*d^4*e^14 + 768*a^11*b^5*c^2*d^3*e^15 - 5120
*a^12*b^2*c^4*d^4*e^14 - 2048*a^12*b^3*c^3*d^3*e^15 - 96*a^12*b^4*c^2*d^2*e
^16 + 256*a^13*b^2*c^3*d^2*e^16 + 2048*a^6*b*c^11*d^17*e + 8*a^2*b^9*c^7*d^
17*e + 8*a^2*b^15*c*d^11*e^7 - 128*a^3*b^7*c^8*d^17*e - 40*a^3*b^14*c*d^10*
e^8 + 768*a^4*b^5*c^9*d^17*e + 40*a^4*b^13*c*d^9*e^9 - 2048*a^5*b^3*c^10*d^
17*e + 168*a^5*b^12*c*d^8*e^10 - 616*a^6*b^11*c*d^7*e^11 + 14336*a^7*b*c^10
*d^15*e^3 + 952*a^7*b^10*c*d^6*e^12 + 43008*a^8*b*c^9*d^13*e^5 - 840*a^8*b^
9*c*d^5*e^13 + 71680*a^9*b*c^8*d^11*e^7 + 440*a^9*b^8*c*d^4*e^14 + 71680*a^
10*b*c^7*d^9*e^9 - 128*a^10*b^7*c*d^3*e^15 + 43008*a^11*b*c^6*d^7*e^11 + 16
*a^11*b^6*c*d^2*e^16 + 14336*a^12*b*c^5*d^5*e^13 + 2048*a^13*b*c^4*d^3*e^15
)))*root(128723189760*a^14*b^4*c^9*d^13*e^14*z^6 + 128723189760*a^12*b^4*c^
11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^4*d^11*e^16*z^6 - 8432455680*a^7*
b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b^11*c^5*d^12*e^15*z^6 + 126733516
80*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960*a^12*b^9*c^6*d^12*e^15*z^6 - 726
37480960*a^9*b^9*c^9*d^18*e^9*z^6 - 21048344576*a^9*b^12*c^6*d^15*e^12*z^6
- 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 - 16609443840*a^10*b^3*c^14*d^22*e^
5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^13*z^6 + 145332633600*a^12*b^5*c^1
0*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^8*d^12*e^15*z^6 + 123740356608*a^
11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17*b^5*c^5*d^6*e^21*z^6 + 346030080
0*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a^15*b^7*c^5*d^8*e^19*z^6 - 775107
3792*a^8*b^7*c^12*d^22*e^5*z^6 + 12041846784*a^14*b^7*c^6*d^10*e^17*z^6 + 1
2041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325545099264*a^14*b^3*c^10*d^14*e^13
*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^11*z^6 - 3330539520*a^13*b^10*c^4*
d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^21*e^6*z^6 + 157789716480*a^12*b^
7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^7*c^9*d^16*e^11*z^6 + 37492359168
*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168*a^9*b^10*c^8*d^17*e^10*z^6 + 301
989888*a^8*b^3*c^16*d^26*e*z^6 - 7266631680*a^17*b^4*c^6*d^7*e^20*z^6 - 726
6631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326592*a^20*b*c^6*d^4*e^23*z^6 - 188
743680*a^7*b^5*c^15*d^26*e*z^6 + 45747339264*a^13*b^8*c^6*d^11*e^16*z^6 + 4
5747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 74612736*a^10*b^16*c*d^9*e^18*z^6 -
2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 2768240640*a^7*b^7*c^13*d^24*e^3*z^6
 + 69746688*a^11*b^15*c*d^8*e^19*z^6 + 62914560*a^6*b^7*c^14*d^26*e*z^6 + 2
752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 2752020480*a^7*b^13*c^7*d^18*e^9*z^
```

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6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45957120*a^12*b^14*c*d^7*e^20*z^6 -
 2724986880*a^14*b^9*c^4*d^8*e^19*z^6 - 2724986880*a^7*b^9*c^11*d^22*e^5*z^
6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21086208*a^13*b^13*c*d^6*e^21*z^6 -
 11796480*a^5*b^9*c^13*d^26*e*z^6 - 6438912*a^14*b^12*c*d^5*e^22*z^6 + 5406
720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6*b^20*c*d^13*e^14*z^6 - 1523712*a
^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^11*c*d^4*e^23*z^6 + 1179648*a^4*b^
11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^15*e^12*z^6 - 98304*a^16*b^10*c*d^
3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^6 - 49152*a^3*b^13*c^11*d^26*e*z^
6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6 + 6897106944*a^8*b^13*c^6*d^16*e^
11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*z^6 - 2422210560*a^8*b^6*c^13*d^2
3*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^15*e^12*z^6 + 41004564480*a^15*b^4
*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*c^12*d^19*e^8*z^6 + 2270822400*a^
13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*b^11*c^10*d^22*e^5*z^6 + 23677108
224*a^14*b^8*c^5*d^9*e^18*z^6 + 23677108224*a^8*b^8*c^11*d^21*e^6*z^6 + 212
600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 212600881152*a^13*b^2*c^12*d^17*e^1
0*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17*z^6 + 75157733376*a^10*b^5*c^12*
d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d^13*e^14*z^6 - 251217838080*a^11*
b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b^10*c^3*d^7*e^20*z^6 - 1952907264
*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*a^13*b^9*c^5*d^10*e^17*z^6 - 2769
1057152*a^8*b^9*c^10*d^20*e^7*z^6 - 1902673920*a^8*b^15*c^4*d^14*e^13*z^6 -
 1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 10465050624*a^10*b^11*c^6*d^14*e^1
3*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11*z^6 + 1613905920*a^9*b^14*c^4*d^
13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17*e^10*z^6 - 33218887680*a^17*b*c^
9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*d^20*e^7*z^6 + 1524695040*a^10*b^
14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14*c^7*d^19*e^8*z^6 - 1472200704*a^
18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b^4*c^15*d^25*e^2*z^6 - 8304721920
0*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200*a^11*b^3*c^13*d^20*e^7*z^6 + 442
91850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291850240*a^11*b^2*c^14*d^21*e^6*z^6
 + 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 - 201326592*a^9*b*c^17*d^26*e*z^6
+ 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 + 48530718720*a^10*b^8*c^9*d^17*e^
10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18*z^6 - 1242644480*a^6*b^12*c^9*d^
21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11*e^16*z^6 + 9813196800*a^8*b^10*c
^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*d^14*e^13*z^6 - 93012885504*a^14*
b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b^4*c^10*d^15*e^12*z^6 + 527306588
16*a^10*b^10*c^7*d^15*e^12*z^6 - 1180106752*a^9*b^15*c^3*d^12*e^15*z^6 - 11
80106752*a^6*b^15*c^6*d^18*e^9*z^6 + 1023672320*a^15*b^9*c^3*d^6*e^21*z^6 +
 1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 975175680*a^17*b^6*c^4*d^5*e^22*z^6
 + 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 11072962560*a^18*b*c^8*d^8*e^19*z^
6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 + 9412018176*a^18*b^2*c^7*d^7*e^20
*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z^6 + 805306368*a^19*b^2*c^6*d^5*e
^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*z^6 - 133809831936*a^14*b^6*c^7*d
^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*d^19*e^8*z^6 - 2214592512*a^19*b*
c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*d^24*e^3*z^6 + 82216747008*a^13*b
^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*b^7*c^10*d^18*e^9*z^6 - 586629120*a
^12*b^13*c^2*d^8*e^19*z^6 - 586629120*a^5*b^13*c^9*d^22*e^5*z^6 + 568565760
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*a^7*b^16*c^4*d^15*e^12*z^6 - 4844421120*a^16*b^4*c^7*d^9*e^18*z^6 - 484442
1120*a^10*b^4*c^13*d^21*e^6*z^6 + 531210240*a^11*b^14*c^2*d^9*e^18*z^6 + 53
1210240*a^5*b^14*c^8*d^21*e^6*z^6 - 527155200*a^11*b^13*c^3*d^10*e^17*z^6 -
 527155200*a^6*b^13*c^8*d^20*e^7*z^6 + 43470028800*a^11*b^8*c^8*d^15*e^12*z
^6 - 107874877440*a^11*b^9*c^7*d^14*e^13*z^6 - 107874877440*a^10*b^9*c^8*d^
16*e^11*z^6 + 9018408960*a^12*b^11*c^4*d^10*e^17*z^6 + 9018408960*a^7*b^11*
c^9*d^20*e^7*z^6 + 421994496*a^13*b^12*c^2*d^7*e^20*z^6 + 421994496*a^5*b^1
2*c^10*d^23*e^4*z^6 - 66437775360*a^16*b*c^10*d^12*e^15*z^6 - 66437775360*a
^13*b*c^13*d^18*e^9*z^6 + 26159874048*a^16*b^5*c^6*d^8*e^19*z^6 + 261598740
48*a^9*b^5*c^13*d^22*e^5*z^6 - 369098752*a^18*b^3*c^6*d^6*e^21*z^6 - 369098
752*a^9*b^3*c^15*d^24*e^3*z^6 + 351436800*a^8*b^16*c^3*d^13*e^14*z^6 + 3514
36800*a^6*b^16*c^5*d^17*e^10*z^6 - 334233600*a^16*b^8*c^3*d^5*e^22*z^6 - 33
4233600*a^6*b^8*c^13*d^25*e^2*z^6 + 301989888*a^19*b^3*c^5*d^4*e^23*z^6 - 2
66010624*a^10*b^15*c^2*d^10*e^17*z^6 - 266010624*a^5*b^15*c^7*d^20*e^7*z^6
 - 305198530560*a^12*b^6*c^9*d^15*e^12*z^6 - 203292672*a^14*b^11*c^2*d^6*e^2
1*z^6 - 203292672*a^5*b^11*c^11*d^24*e^3*z^6 - 188743680*a^18*b^5*c^4*d^4*e
^23*z^6 + 120418467840*a^16*b^2*c^9*d^11*e^16*z^6 + 120418467840*a^12*b^2*c
^13*d^19*e^8*z^6 - 17293934592*a^10*b^12*c^5*d^13*e^14*z^6 - 17293934592*a^
8*b^12*c^7*d^17*e^10*z^6 + 104890368*a^8*b^17*c^2*d^12*e^15*z^6 + 104890368
*a^5*b^17*c^5*d^18*e^9*z^6 + 4390256640*a^15*b^8*c^4*d^7*e^20*z^6 + 4390256
640*a^7*b^8*c^12*d^23*e^4*z^6 - 91750400*a^6*b^18*c^3*d^15*e^12*z^6 + 79134
720*a^7*b^17*c^3*d^14*e^13*z^6 + 79134720*a^6*b^17*c^4*d^16*e^11*z^6 - 7461
2736*a^4*b^16*c^7*d^21*e^6*z^6 - 72990720*a^7*b^18*c^2*d^13*e^14*z^6 - 7299
0720*a^5*b^18*c^4*d^17*e^10*z^6 + 69746688*a^4*b^15*c^8*d^22*e^5*z^6 + 6370
0992*a^15*b^10*c^2*d^5*e^22*z^6 + 63700992*a^5*b^10*c^12*d^25*e^2*z^6 + 629
14560*a^17*b^7*c^3*d^4*e^23*z^6 + 55148544*a^4*b^17*c^6*d^20*e^7*z^6 - 4595
7120*a^4*b^14*c^9*d^23*e^4*z^6 - 25952256*a^4*b^18*c^5*d^19*e^8*z^6 - 25165
824*a^20*b^2*c^5*d^3*e^24*z^6 + 21086208*a^4*b^13*c^10*d^24*e^3*z^6 + 20643
840*a^6*b^19*c^2*d^14*e^13*z^6 + 20643840*a^5*b^19*c^3*d^16*e^11*z^6 + 1572
8640*a^19*b^4*c^4*d^3*e^24*z^6 - 11796480*a^16*b^9*c^2*d^4*e^23*z^6 - 64389
12*a^4*b^12*c^11*d^25*e^2*z^6 + 5406720*a^4*b^19*c^4*d^18*e^9*z^6 - 5242880
*a^18*b^6*c^3*d^3*e^24*z^6 + 3784704*a^3*b^18*c^6*d^21*e^6*z^6 - 3244032*a^
3*b^19*c^5*d^20*e^7*z^6 - 3244032*a^3*b^17*c^7*d^22*e^5*z^6 + 2027520*a^3*b
^20*c^4*d^19*e^8*z^6 + 2027520*a^3*b^16*c^8*d^23*e^4*z^6 - 1622016*a^9*b^16
*c^2*d^11*e^16*z^6 - 1622016*a^5*b^16*c^6*d^19*e^8*z^6 + 1622016*a^4*b^20*c
^3*d^17*e^10*z^6 - 1523712*a^4*b^21*c^2*d^16*e^11*z^6 + 983040*a^17*b^8*c^2
*d^3*e^24*z^6 - 901120*a^3*b^21*c^3*d^18*e^9*z^6 - 901120*a^3*b^15*c^9*d^24
*e^3*z^6 + 270336*a^3*b^22*c^2*d^17*e^10*z^6 + 270336*a^3*b^14*c^10*d^25*e^
2*z^6 + 172032*a^5*b^20*c^2*d^15*e^12*z^6 - 38593888256*a^15*b^6*c^6*d^9*e^
18*z^6 - 38593888256*a^9*b^6*c^12*d^21*e^6*z^6 - 210386288640*a^15*b^3*c^9*
d^12*e^15*z^6 - 210386288640*a^12*b^3*c^12*d^18*e^9*z^6 + 15502147584*a^15*
c^12*d^15*e^12*z^6 + 1107296256*a^19*c^8*d^7*e^20*z^6 + 1107296256*a^11*c^1
6*d^23*e^4*z^6 + 13287555072*a^16*c^11*d^13*e^14*z^6 + 13287555072*a^14*c^1
3*d^17*e^10*z^6 + 201326592*a^20*c^7*d^5*e^22*z^6 + 201326592*a^10*c^17*d^2
5*e^2*z^6 + 16777216*a^21*c^6*d^3*e^24*z^6 + 3784704*a^9*b^18*d^9*e^18*z^6
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- 3244032*a^10*b^17*d^8*e^19*z^6 - 3244032*a^8*b^19*d^10*e^17*z^6 + 2027520
*a^11*b^16*d^7*e^20*z^6 + 2027520*a^7*b^20*d^11*e^16*z^6 - 901120*a^12*b^15
*d^6*e^21*z^6 - 901120*a^6*b^21*d^12*e^15*z^6 + 270336*a^13*b^14*d^5*e^22*z
^6 + 270336*a^5*b^22*d^13*e^14*z^6 - 49152*a^14*b^13*d^4*e^23*z^6 - 49152*a
^4*b^23*d^14*e^13*z^6 + 4096*a^15*b^12*d^3*e^24*z^6 + 4096*a^3*b^24*d^15*e^
12*z^6 - 25165824*a^8*b^2*c^17*d^27*z^6 + 15728640*a^7*b^4*c^16*d^27*z^6 -
5242880*a^6*b^6*c^15*d^27*z^6 + 983040*a^5*b^8*c^14*d^27*z^6 - 98304*a^4*b^
10*c^13*d^27*z^6 + 4096*a^3*b^12*c^12*d^27*z^6 + 8304721920*a^17*c^10*d^11*
e^16*z^6 + 8304721920*a^13*c^14*d^19*e^8*z^6 + 3690987520*a^18*c^9*d^9*e^18
*z^6 + 3690987520*a^12*c^15*d^21*e^6*z^6 + 16777216*a^9*c^18*d^27*z^6 - 849
3371392*a^6*b^8*c^9*d^14*e^9*z^4 + 1458044928*a^8*b*c^14*d^17*e^6*z^4 - 126
04538880*a^11*b^4*c^8*d^8*e^15*z^4 - 8303067136*a^9*b^5*c^9*d^11*e^12*z^4 -
 5588058112*a^13*b*c^9*d^7*e^16*z^4 - 3892838400*a^8*b^2*c^13*d^16*e^7*z^4
- 3611713536*a^8*b^8*c^7*d^10*e^13*z^4 + 7819006464*a^7*b^9*c^7*d^11*e^12*z
^4 - 7782137856*a^8*b^7*c^8*d^11*e^12*z^4 + 7780433920*a^12*b^2*c^9*d^8*e^1
5*z^4 - 12020465664*a^7*b^5*c^11*d^15*e^8*z^4 + 3176792064*a^8*b^3*c^12*d^1
5*e^8*z^4 - 322633728*a^15*b*c^7*d^3*e^20*z^4 + 210829312*a^7*b*c^15*d^19*e
^4*z^4 + 15623258112*a^9*b^6*c^8*d^10*e^13*z^4 + 25165824*a^15*b^3*c^5*d*e^
22*z^4 - 15728640*a^14*b^5*c^4*d*e^22*z^4 + 12582912*a^5*b^2*c^16*d^22*e*z^
4 - 11730944*a^4*b^4*c^15*d^22*e*z^4 + 5242880*a^13*b^7*c^3*d*e^22*z^4 - 45
61920*a*b^15*c^7*d^17*e^6*z^4 + 4521984*a^3*b^6*c^14*d^22*e*z^4 + 4460544*a
*b^14*c^8*d^18*e^5*z^4 + 3538944*a^6*b*c^16*d^21*e^2*z^4 + 3108864*a*b^16*c
^6*d^16*e^7*z^4 - 3027200*a*b^13*c^9*d^19*e^4*z^4 - 2345472*a^5*b^17*c*d^7*
e^16*z^4 - 2307072*a^8*b^14*c*d^4*e^19*z^4 + 1824768*a^6*b^16*c*d^6*e^17*z^
4 + 1734912*a^9*b^13*c*d^3*e^20*z^4 + 1419264*a*b^12*c^10*d^20*e^3*z^4 - 11
91168*a*b^17*c^5*d^15*e^8*z^4 - 983040*a^12*b^9*c^2*d*e^22*z^4 + 964608*a^4
*b^18*c*d^8*e^15*z^4 - 866304*a^2*b^8*c^13*d^22*e*z^4 + 703488*a^7*b^15*c*d
^5*e^18*z^4 - 608256*a^10*b^12*c*d^2*e^21*z^4 - 440832*a*b^11*c^11*d^21*e^2
*z^4 + 275968*a*b^19*c^3*d^13*e^10*z^4 - 159744*a^2*b^20*c*d^10*e^13*z^4 -
153600*a*b^20*c^2*d^12*e^11*z^4 + 64512*a^3*b^19*c*d^9*e^14*z^4 + 197460623
36*a^8*b^6*c^9*d^12*e^11*z^4 - 15333588992*a^10*b^4*c^9*d^10*e^13*z^4 + 670
2170112*a^7*b^4*c^12*d^16*e^7*z^4 + 15167913984*a^10*b^3*c^10*d^11*e^12*z^4
 - 2256638976*a^5*b^11*c^7*d^13*e^10*z^4 + 2254307328*a^5*b^7*c^11*d^17*e^6
*z^4 - 2200633344*a^6*b^5*c^12*d^17*e^6*z^4 + 6457131008*a^11*b^3*c^9*d^9*e
^14*z^4 - 2128785408*a^5*b^8*c^10*d^16*e^7*z^4 - 2126057472*a^6*b^11*c^6*d^
11*e^12*z^4 + 2038349824*a^12*b^5*c^6*d^5*e^18*z^4 + 2037841920*a^5*b^10*c^
8*d^14*e^9*z^4 + 3615621120*a^9*b*c^13*d^15*e^8*z^4 + 1900019712*a^11*b^2*c
^10*d^10*e^13*z^4 + 1867698432*a^9*b^9*c^5*d^7*e^16*z^4 - 6157369344*a^9*b^
4*c^10*d^12*e^11*z^4 - 1856913408*a^7*b^10*c^6*d^10*e^13*z^4 + 1789132800*a
^6*b^4*c^13*d^18*e^5*z^4 + 6082658304*a^8*b^4*c^11*d^14*e^9*z^4 + 602954956
8*a^11*b^5*c^7*d^7*e^16*z^4 + 6010159104*a^6*b^7*c^10*d^15*e^8*z^4 + 170318
2336*a^7*b^7*c^9*d^13*e^10*z^4 + 1658388480*a^11*b^6*c^6*d^6*e^17*z^4 + 591
7114368*a^10*b^6*c^7*d^8*e^15*z^4 - 1591197696*a^11*b^7*c^5*d^5*e^18*z^4 -
1526464512*a^8*b^10*c^5*d^8*e^15*z^4 - 5772607488*a^12*b^4*c^7*d^6*e^17*z^4
 - 1423507456*a^13*b^4*c^6*d^4*e^19*z^4 - 1387266048*a^7*b^3*c^13*d^17*e^6*
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z^4 + 2976120832*a^10*b*c^12*d^13*e^10*z^4 - 9906946048*a^9*b^2*c^12*d^14*e
^9*z^4 - 18421874688*a^8*b^5*c^10*d^13*e^10*z^4 + 1141217280*a^6*b^12*c^5*d
^10*e^13*z^4 - 9714364416*a^7*b^8*c^8*d^12*e^11*z^4 - 16777216*a^16*b*c^6*d
*e^22*z^4 + 98304*a^11*b^11*c*d*e^22*z^4 + 81920*a*b^10*c^12*d^22*e*z^4 + 3
9168*a*b^21*c*d^11*e^12*z^4 - 1091829760*a^5*b^6*c^12*d^18*e^5*z^4 + 104674
0992*a^14*b^2*c^7*d^4*e^19*z^4 - 6884425728*a^12*b*c^10*d^9*e^14*z^4 + 9874
45248*a^4*b^10*c^9*d^16*e^7*z^4 + 984087552*a^5*b^12*c^6*d^12*e^11*z^4 - 95
64585984*a^9*b^7*c^7*d^9*e^14*z^4 - 5266857984*a^10*b^7*c^6*d^7*e^16*z^4 -
892145664*a^7*b^11*c^5*d^9*e^14*z^4 - 2444623872*a^11*b*c^11*d^11*e^12*z^4
+ 768540672*a^12*b^3*c^8*d^7*e^16*z^4 + 5048322048*a^8*b^9*c^6*d^9*e^14*z^4
 + 5047612416*a^6*b^9*c^8*d^13*e^10*z^4 - 732492288*a^4*b^11*c^8*d^15*e^8*z
^4 + 9266921472*a^7*b^6*c^10*d^14*e^9*z^4 - 645857280*a^6*b^6*c^11*d^16*e^7
*z^4 - 623867904*a^4*b^9*c^10*d^17*e^6*z^4 - 622067712*a^6*b^3*c^14*d^19*e^
4*z^4 + 582617088*a^10*b^8*c^5*d^6*e^17*z^4 + 577119744*a^7*b^12*c^4*d^8*e^
15*z^4 + 552566784*a^12*b^6*c^5*d^4*e^19*z^4 + 549224448*a^9*b^8*c^6*d^8*e^
15*z^4 - 526565376*a^9*b^10*c^4*d^6*e^17*z^4 + 511520256*a^10*b^9*c^4*d^5*e
^18*z^4 + 13393723392*a^9*b^3*c^11*d^13*e^10*z^4 - 2066350080*a^14*b*c^8*d^
5*e^18*z^4 + 4718592000*a^13*b^2*c^8*d^6*e^17*z^4 - 314572800*a^7*b^2*c^14*
d^18*e^5*z^4 + 287250432*a^4*b^13*c^6*d^13*e^10*z^4 + 4565827584*a^10*b^5*c
^8*d^9*e^14*z^4 - 250785792*a^4*b^14*c^5*d^12*e^11*z^4 + 235536384*a^13*b^3
*c^7*d^5*e^18*z^4 - 232683264*a^8*b^11*c^4*d^7*e^16*z^4 - 199627776*a^5*b^1
4*c^4*d^10*e^13*z^4 - 190267392*a^12*b^7*c^4*d^3*e^20*z^4 + 184891392*a^6*b
^10*c^7*d^12*e^11*z^4 + 180502528*a^4*b^7*c^12*d^19*e^4*z^4 + 178877952*a^3
*b^13*c^7*d^15*e^8*z^4 + 172490752*a^14*b^3*c^6*d^3*e^20*z^4 + 163946496*a^
13*b^5*c^5*d^3*e^20*z^4 + 155839488*a^8*b^12*c^3*d^6*e^17*z^4 + 155000832*a
^5*b^5*c^13*d^19*e^4*z^4 - 152076288*a^4*b^6*c^13*d^20*e^3*z^4 - 137592576*
a^3*b^12*c^8*d^16*e^7*z^4 - 133693440*a^14*b^4*c^5*d^2*e^21*z^4 - 116767488
*a^3*b^9*c^11*d^19*e^4*z^4 - 108985344*a^3*b^14*c^6*d^14*e^9*z^4 - 10622361
6*a^6*b^13*c^4*d^9*e^14*z^4 + 106119168*a^3*b^10*c^10*d^18*e^5*z^4 + 102432
768*a^5*b^4*c^14*d^20*e^3*z^4 + 102113280*a^4*b^12*c^7*d^14*e^9*z^4 + 10067
4048*a^5*b^9*c^9*d^15*e^8*z^4 + 90439680*a^13*b^6*c^4*d^2*e^21*z^4 - 868085
76*a^6*b^14*c^3*d^8*e^15*z^4 + 86245376*a^6*b^2*c^15*d^20*e^3*z^4 + 7901184
0*a^4*b^8*c^11*d^18*e^5*z^4 + 78345216*a^4*b^15*c^4*d^11*e^12*z^4 + 7800652
8*a^11*b^9*c^3*d^3*e^20*z^4 - 73253376*a^9*b^11*c^3*d^5*e^18*z^4 + 67524608
*a^3*b^8*c^12*d^20*e^3*z^4 + 67108864*a^15*b^2*c^6*d^2*e^21*z^4 - 61590528*
a^10*b^10*c^3*d^4*e^19*z^4 + 61559808*a^5*b^15*c^3*d^9*e^14*z^4 - 59637760*
a^5*b^3*c^15*d^21*e^2*z^4 + 58638336*a^4*b^5*c^14*d^21*e^2*z^4 - 40828416*a
^7*b^13*c^3*d^7*e^16*z^4 - 35639296*a^2*b^12*c^9*d^18*e^5*z^4 - 31293440*a^
12*b^8*c^3*d^2*e^21*z^4 + 29933568*a^5*b^13*c^5*d^11*e^12*z^4 + 27793920*a^
2*b^11*c^10*d^19*e^4*z^4 + 27168768*a^2*b^13*c^8*d^17*e^6*z^4 - 23602176*a^
7*b^14*c^2*d^6*e^17*z^4 - 23248896*a^3*b^7*c^13*d^21*e^2*z^4 + 20929536*a^3
*b^15*c^5*d^13*e^10*z^4 + 18428928*a^9*b^12*c^2*d^4*e^19*z^4 + 18026496*a^6
*b^15*c^2*d^7*e^16*z^4 - 16261632*a^10*b^11*c^2*d^3*e^20*z^4 + 15128064*a^3
*b^16*c^4*d^12*e^11*z^4 - 14060544*a^2*b^10*c^11*d^20*e^3*z^4 + 13178880*a^
2*b^16*c^5*d^14*e^9*z^4 - 11244288*a^3*b^17*c^3*d^11*e^12*z^4 - 10509312*a^
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2*b^15*c^6*d^15*e^8*z^4 - 7262208*a^4*b^17*c^2*d^9*e^14*z^4 - 7045632*a^2*b
^17*c^4*d^13*e^10*z^4 - 6285312*a^2*b^14*c^7*d^16*e^7*z^4 + 5996544*a^11*b^
10*c^2*d^2*e^21*z^4 + 4558336*a^2*b^9*c^12*d^21*e^2*z^4 + 4478976*a^11*b^8*
c^4*d^4*e^19*z^4 + 2850816*a^4*b^16*c^3*d^10*e^13*z^4 + 2629632*a^3*b^11*c^
9*d^17*e^6*z^4 + 2503680*a^3*b^18*c^2*d^10*e^13*z^4 + 1627136*a^2*b^18*c^3*
d^12*e^11*z^4 + 1605120*a^8*b^13*c^2*d^5*e^18*z^4 + 1483776*a^5*b^16*c^2*d^
8*e^15*z^4 + 139776*a^2*b^19*c^2*d^11*e^12*z^4 - 8542224384*a^10*b^2*c^11*d
^12*e^11*z^4 - 3072*b^22*c*d^12*e^11*z^4 - 3072*b^12*c^11*d^22*e*z^4 - 1572
864*a^6*c^17*d^22*e*z^4 - 4096*a^10*b^13*d*e^22*z^4 - 4096*a*b^22*d^10*e^13
*z^4 - 6144*a^12*b^10*c*e^23*z^4 - 983040*a^5*b*c^17*d^23*z^4 - 6912*a*b^9*
c^13*d^23*z^4 + 1824522240*a^13*c^10*d^8*e^15*z^4 + 1730150400*a^12*c^11*d^
10*e^13*z^4 + 958922752*a^14*c^9*d^6*e^17*z^4 - 537919488*a^9*c^14*d^16*e^7
*z^4 + 508559360*a^11*c^12*d^12*e^11*z^4 - 500170752*a^10*c^13*d^14*e^9*z^4
 + 246939648*a^15*c^8*d^4*e^19*z^4 - 199229440*a^8*c^15*d^18*e^5*z^4 - 2988
4416*a^7*c^16*d^20*e^3*z^4 + 25165824*a^16*c^7*d^2*e^21*z^4 + 236544*b^17*c
^6*d^17*e^6*z^4 - 202752*b^18*c^5*d^16*e^7*z^4 - 202752*b^16*c^7*d^18*e^5*z
^4 + 126720*b^19*c^4*d^15*e^8*z^4 + 126720*b^15*c^8*d^19*e^4*z^4 - 56320*b^
20*c^3*d^14*e^9*z^4 - 56320*b^14*c^9*d^20*e^3*z^4 + 16896*b^21*c^2*d^13*e^1
0*z^4 + 16896*b^13*c^10*d^21*e^2*z^4 + 110080*a^7*b^16*d^4*e^19*z^4 + 11008
0*a^4*b^19*d^7*e^16*z^4 - 75520*a^8*b^15*d^3*e^20*z^4 - 75520*a^3*b^20*d^8*
e^15*z^4 - 56320*a^6*b^17*d^5*e^18*z^4 - 56320*a^5*b^18*d^6*e^17*z^4 + 2560
0*a^9*b^14*d^2*e^21*z^4 + 25600*a^2*b^21*d^9*e^14*z^4 - 1572864*a^16*b^2*c^
5*e^23*z^4 + 983040*a^15*b^4*c^4*e^23*z^4 - 327680*a^14*b^6*c^3*e^23*z^4 +
61440*a^13*b^8*c^2*e^23*z^4 + 983040*a^4*b^3*c^16*d^23*z^4 - 385024*a^3*b^5
*c^15*d^23*z^4 + 73728*a^2*b^7*c^14*d^23*z^4 + 256*b^23*d^11*e^12*z^4 + 104
8576*a^17*c^6*e^23*z^4 + 256*b^11*c^12*d^23*z^4 + 256*a^11*b^12*e^23*z^4 +
948695040*a^8*b*c^10*d^6*e^13*z^2 + 348917760*a^7*b*c^11*d^8*e^11*z^2 - 125
030400*a^9*b*c^9*d^4*e^15*z^2 - 50728960*a^6*b*c^12*d^10*e^9*z^2 - 44298240
*a^5*b*c^13*d^12*e^7*z^2 - 36495360*a^10*b*c^8*d^2*e^17*z^2 + 29675520*a^8*
b^6*c^5*d*e^18*z^2 - 24170496*a^9*b^4*c^6*d*e^18*z^2 - 17202816*a^7*b^8*c^4
*d*e^18*z^2 - 14561280*a^4*b*c^14*d^14*e^5*z^2 + 5532416*a^6*b^10*c^3*d*e^1
8*z^2 + 4128768*a^10*b^2*c^7*d*e^18*z^2 - 2662400*a^3*b*c^15*d^16*e^3*z^2 +
 1184512*a*b^12*c^6*d^9*e^10*z^2 - 1136160*a*b^13*c^5*d^8*e^11*z^2 - 101760
0*a^5*b^12*c^2*d*e^18*z^2 - 744768*a*b^11*c^7*d^10*e^9*z^2 + 607872*a*b^14*
c^4*d^7*e^12*z^2 - 424064*a*b^6*c^12*d^15*e^4*z^2 + 408576*a*b^5*c^13*d^16*
e^3*z^2 + 361152*a*b^10*c^8*d^11*e^8*z^2 - 287408*a*b^9*c^9*d^12*e^7*z^2 -
260448*a^3*b^15*c*d^2*e^17*z^2 - 203904*a*b^4*c^14*d^17*e^2*z^2 + 200832*a*
b^8*c^10*d^13*e^6*z^2 + 126720*a*b^7*c^11*d^14*e^5*z^2 - 123968*a*b^15*c^3*
d^6*e^13*z^2 - 39168*a*b^16*c^2*d^5*e^14*z^2 + 11904*a^2*b^16*c*d^3*e^16*z^
2 + 1824135552*a^7*b^4*c^8*d^5*e^14*z^2 - 1457252352*a^8*b^2*c^9*d^5*e^14*z
^2 - 1405209600*a^7*b^5*c^7*d^4*e^15*z^2 - 184320*a^2*b*c^16*d^18*e*z^2 + 1
00608*a^4*b^14*c*d*e^18*z^2 + 53248*a*b^3*c^15*d^18*e*z^2 + 26448*a*b^17*c*
d^4*e^15*z^2 + 1067599872*a^8*b^3*c^8*d^4*e^15*z^2 - 930828288*a^7*b^3*c^9*
d^6*e^13*z^2 + 920760000*a^6*b^4*c^9*d^7*e^12*z^2 - 806639616*a^6*b^3*c^10*
d^8*e^11*z^2 - 791052480*a^6*b^6*c^7*d^5*e^14*z^2 + 772237824*a^6*b^7*c^6*d
```

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^4*e^15*z^2 - 701025408*a^5*b^6*c^8*d^7*e^12*z^2 + 443340288*a^5*b^5*c^9*d^
8*e^11*z^2 + 433047552*a^7*b^6*c^6*d^3*e^16*z^2 + 405741312*a^5*b^7*c^7*d^6
*e^13*z^2 + 293652480*a^6*b^2*c^11*d^9*e^10*z^2 - 276962688*a^6*b^8*c^5*d^3
*e^16*z^2 - 247804272*a^8*b^4*c^7*d^3*e^16*z^2 + 213564384*a^4*b^8*c^7*d^7*
e^12*z^2 - 202596816*a^5*b^9*c^5*d^4*e^15*z^2 - 182520896*a^4*b^9*c^6*d^6*e
^13*z^2 - 153489408*a^5*b^3*c^11*d^10*e^9*z^2 - 152151552*a^7*b^2*c^10*d^7*
e^12*z^2 + 115859712*a^5*b^2*c^12*d^11*e^8*z^2 + 108085248*a^9*b^3*c^7*d^2*
e^17*z^2 + 105536256*a^4*b^5*c^10*d^10*e^9*z^2 - 98323200*a^6*b^5*c^8*d^6*e
^13*z^2 - 93564992*a^4*b^6*c^9*d^9*e^10*z^2 + 89464512*a^5*b^10*c^4*d^3*e^1
6*z^2 - 75930624*a^8*b^5*c^6*d^2*e^17*z^2 + 68315904*a^5*b^8*c^6*d^5*e^14*z
^2 - 64157184*a^4*b^7*c^8*d^8*e^11*z^2 - 62951040*a^9*b^2*c^8*d^3*e^16*z^2
 + 49056768*a^4*b^10*c^5*d^5*e^14*z^2 + 47614464*a^3*b^8*c^8*d^9*e^10*z^2 +
35604480*a^4*b^2*c^13*d^13*e^6*z^2 + 33983040*a^3*b^11*c^5*d^6*e^13*z^2 - 3
3515520*a^4*b^3*c^12*d^12*e^7*z^2 - 33463808*a^3*b^7*c^9*d^10*e^9*z^2 - 251
28864*a^4*b^4*c^11*d^11*e^8*z^2 - 23193728*a^3*b^10*c^6*d^7*e^12*z^2 + 2101
5456*a^6*b^9*c^4*d^2*e^17*z^2 + 19924176*a^4*b^11*c^4*d^4*e^15*z^2 - 192512
16*a^3*b^9*c^7*d^8*e^11*z^2 - 16434048*a^5*b^4*c^10*d^9*e^10*z^2 - 16289664
*a^3*b^12*c^4*d^5*e^14*z^2 - 15059328*a^4*b^12*c^3*d^3*e^16*z^2 - 10766016*
a^2*b^10*c^7*d^9*e^10*z^2 - 10453632*a^5*b^11*c^3*d^2*e^17*z^2 - 9940992*a^
3*b^3*c^13*d^14*e^5*z^2 + 8373696*a^2*b^11*c^6*d^8*e^11*z^2 + 7776768*a^3*b
^2*c^14*d^15*e^4*z^2 + 7077888*a^3*b^5*c^11*d^12*e^7*z^2 + 6798240*a^2*b^9*
c^8*d^10*e^9*z^2 - 3589440*a^2*b^6*c^11*d^13*e^6*z^2 + 3544320*a^3*b^6*c^10
*d^11*e^8*z^2 + 3128064*a^2*b^5*c^12*d^14*e^5*z^2 + 2346336*a^4*b^13*c^2*d^
2*e^17*z^2 - 2261568*a^2*b^8*c^9*d^11*e^8*z^2 - 2125824*a^2*b^13*c^4*d^6*e^
13*z^2 + 2002560*a^3*b^4*c^12*d^13*e^6*z^2 + 1927680*a^2*b^7*c^10*d^12*e^7*
z^2 + 1814784*a^2*b^14*c^3*d^5*e^14*z^2 - 1807104*a^2*b^12*c^5*d^7*e^12*z^2
 + 1637808*a^3*b^13*c^3*d^4*e^15*z^2 + 1083456*a^3*b^14*c^2*d^3*e^16*z^2 -
792384*a^2*b^4*c^13*d^15*e^4*z^2 - 657408*a^2*b^3*c^14*d^16*e^3*z^2 + 60825
6*a^7*b^7*c^5*d^2*e^17*z^2 + 595968*a^2*b^2*c^15*d^17*e^2*z^2 - 498624*a^2*
b^15*c^2*d^4*e^15*z^2 - 3840*b^18*c*d^5*e^14*z^2 - 3840*b^5*c^14*d^18*e*z^2
 + 2064384*a^11*c^8*d*e^18*z^2 - 4160*a^3*b^16*d*e^18*z^2 - 4160*a*b^18*d^3
*e^16*z^2 - 1290240*a^11*b*c^7*e^19*z^2 - 9840*a^5*b^13*c*e^19*z^2 - 5760*a
*b^2*c^16*d^19*z^2 - 280581120*a^8*c^11*d^7*e^12*z^2 + 110278656*a^9*c^10*d
^5*e^14*z^2 - 89479168*a^7*c^12*d^9*e^10*z^2 + 34464000*a^10*c^9*d^3*e^16*z
^2 + 54240*b^15*c^4*d^8*e^11*z^2 + 54240*b^8*c^11*d^15*e^4*z^2 - 49920*b^14
*c^5*d^9*e^10*z^2 - 49920*b^9*c^10*d^14*e^5*z^2 - 37376*b^16*c^3*d^7*e^12*z
^2 - 37376*b^7*c^12*d^16*e^3*z^2 + 28480*b^13*c^6*d^10*e^9*z^2 + 28480*b^10
*c^9*d^13*e^6*z^2 + 15936*b^17*c^2*d^6*e^13*z^2 + 15936*b^6*c^13*d^17*e^2*z
^2 - 7920*b^12*c^7*d^11*e^8*z^2 - 7920*b^11*c^8*d^12*e^7*z^2 + 7489536*a^5*
c^14*d^13*e^6*z^2 + 6084096*a^6*c^13*d^11*e^8*z^2 + 2280448*a^4*c^15*d^15*e
^4*z^2 + 350208*a^3*c^16*d^17*e^2*z^2 + 11616*a^2*b^17*d^2*e^17*z^2 - 35159
04*a^9*b^5*c^5*e^19*z^2 + 3440640*a^10*b^3*c^6*e^19*z^2 + 1870848*a^8*b^7*c
^4*e^19*z^2 - 572272*a^7*b^9*c^3*e^19*z^2 + 101856*a^6*b^11*c^2*e^19*z^2 +
400*b^19*d^4*e^15*z^2 + 400*b^4*c^15*d^19*z^2 + 20736*a^2*c^17*d^19*z^2 + 4
00*a^4*b^15*e^19*z^2 - 3969216*a^4*b*c^10*d^3*e^12 - 3001536*a^3*b*c^11*d^5
```

```
*e^10 - 419904*a^2*b*c^12*d^7*e^8 + 184608*a^4*b^3*c^8*d*e^14 - 153036*a*b^
4*c^10*d^6*e^9 + 127008*a*b^3*c^11*d^7*e^8 + 63108*a*b^6*c^8*d^4*e^11 - 291
60*a*b^2*c^12*d^8*e^7 - 21060*a^3*b^5*c^7*d*e^14 - 21060*a*b^7*c^7*d^3*e^12
 + 5460*a*b^5*c^9*d^5*e^10 - 404544*a^5*b*c^9*d*e^14 + 1251872*a^3*b^3*c^9*
d^3*e^12 + 844224*a^4*b^2*c^9*d^2*e^13 + 820512*a^2*b^3*c^10*d^5*e^10 + 750
672*a^3*b^2*c^10*d^4*e^11 - 657498*a^2*b^4*c^9*d^4*e^11 - 487116*a^3*b^4*c^
8*d^2*e^13 + 160704*a^2*b^2*c^11*d^6*e^9 + 58806*a^2*b^6*c^7*d^2*e^13 + 131
40*a^2*b^5*c^8*d^3*e^12 + 15286*b^6*c^9*d^6*e^9 - 9540*b^7*c^8*d^5*e^10 - 9
540*b^5*c^10*d^7*e^8 + 2025*b^8*c^7*d^4*e^11 + 2025*b^4*c^11*d^8*e^7 + 3367
008*a^4*c^11*d^4*e^11 + 1166400*a^3*c^12*d^6*e^9 + 705600*a^5*c^10*d^2*e^13
 + 104976*a^2*c^13*d^8*e^7 - 17640*a^5*b^2*c^8*e^15 + 2025*a^4*b^4*c^7*e^15
 + 38416*a^6*c^9*e^15, z, k), k, 1, 6) - ((x*(a^2*b^2*e^4 - 4*a^3*c*e^4 - 2
*a*c^3*d^4 + b^2*c^2*d^4 + b^4*d^2*e^2 + 2*a^2*c^2*d^2*e^2 - 2*b^3*c*d^3*e
+ 6*a*b*c^2*d^3*e - 4*a*b^2*c*d^2*e^2))/(2*a*d*(4*a*c^3*d^4 + 4*a^3*c*e^4 -
 a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^
3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2))
 + (x^3*(a*b^3*e^4 + b*c^3*d^4 + b^4*d*e^3 + 2*a^2*c^2*d*e^3 - b^2*c^2*d^3*
e - b^3*c*d^2*e^2 - 4*a^2*b*c*e^4 + 2*a*c^3*d^3*e - 4*a*b^2*c*d*e^3 + 3*a*b
*c^2*d^2*e^2))/(2*a*d*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^
4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b
*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) + (c*e*x^5*(a*b^2*e^3 +
b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b
*c*d*e^2))/(2*a*d*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 -
b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2
*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)))/(a*d + x^2*(a*e + b*d) + x^
4*(b*e + c*d) + c*e*x^6)
```

3.276 $\int (d+ex^2)^{5/2} (a+bx^2+cx^4) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 215

$$\begin{aligned}\int (d+ex^2)^{5/2}(a+bx^2+cx^4)\,dx &= \frac{d^2(3cd^2-10bde+80ae^2)\,x\sqrt{d+ex^2}}{256e^2}\\ &+\frac{d(3cd^2-10bde+80ae^2)\,x(d+ex^2)^{3/2}}{384e^2}+\frac{(3cd^2-10bde+80ae^2)\,x(d+ex^2)^{5/2}}{480e^2}\\ &-\frac{(3cd-10be)x(d+ex^2)^{7/2}}{80e^2}+\frac{cx^3(d+ex^2)^{7/2}}{10e}\\ &+\frac{d^3(3cd^2-10bde+80ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{256e^{5/2}}\end{aligned}$$

```
[Out] 1/384*d*(80*a*e^2-10*b*d*e+3*c*d^2)*x*(e*x^2+d)^(3/2)/e^2+1/480*(80*a*e^2-1
0*b*d*e+3*c*d^2)*x*(e*x^2+d)^(5/2)/e^2-1/80*(-10*b*e+3*c*d)*x*(e*x^2+d)^(7/
2)/e^2+1/10*c*x^3*(e*x^2+d)^(7/2)/e+1/256*d^3*(80*a*e^2-10*b*d*e+3*c*d^2)*a
rctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/256*d^2*(80*a*e^2-10*b*d*e+3*c*
d^2)*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used

$= \{1173, 396, 201, 223, 212\}$

$$\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx = \frac{d^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(80ae^2-10bde+3cd^2\right)}{256e^{5/2}} + \frac{x\left(d+ex^2\right)^{5/2}\left(80ae^2-10bde+3cd^2\right)}{480e^2} + \frac{dx\left(d+ex^2\right)^{3/2}\left(80ae^2-10bde+3cd^2\right)}{384e^2} + \frac{d^2x\sqrt{d+ex^2}\left(80ae^2-10bde+3cd^2\right)}{256e^2} - \frac{x\left(d+ex^2\right)^{7/2}\left(3cd-10be\right)}{80e^2} + \frac{cx^3\left(d+ex^2\right)^{7/2}}{10e}$$

```
[In] Int[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d^2*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*Sqrt[d + e*x^2])/(256*e^2) + (d*(3*c
*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(3/2))/(384*e^2) + ((3*c*d^2 - 10
*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(5/2))/(480*e^2) - ((3*c*d - 10*b*e)*x*(d
+ e*x^2)^(7/2))/(80*e^2) + (c*x^3*(d + e*x^2)^(7/2))/(10*e) + (d^3*(3*c*d^2
 - 10*b*d*e + 80*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(256*e^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
```

```
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{cx^3(d+ex^2)^{7/2}}{10e} + \frac{\int (d+ex^2)^{5/2}\left(10ae-(3cd-10be)x^2\right)\,dx}{10e}\\
&= -\frac{(3cd-10be)x(d+ex^2)^{7/2}}{80e^2} + \frac{cx^3(d+ex^2)^{7/2}}{10e}\\
&\quad - \frac{1}{80}\left(-80a-\frac{d(3cd-10be)}{e^2}\right)\int \left(d+ex^2\right)^{5/2}\,dx\\
&= \frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{5/2} - \frac{(3cd-10be)x(d+ex^2)^{7/2}}{80e^2}\\
&\quad + \frac{cx^3(d+ex^2)^{7/2}}{10e} + \frac{1}{96}\left(d\left(80a+\frac{d(3cd-10be)}{e^2}\right)\right)\int \left(d+ex^2\right)^{3/2}\,dx\\
&= \frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{3/2}\\
&\quad + \frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{5/2} - \frac{(3cd-10be)x(d+ex^2)^{7/2}}{80e^2}\\
&\quad + \frac{cx^3(d+ex^2)^{7/2}}{10e} + \frac{1}{128}\left(d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)\right)\int \sqrt{d+ex^2}\,dx\\
&= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2} + \frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x(d\\
&\quad +ex^2)^{3/2} + \frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{5/2} - \frac{(3cd-10be)x(d+ex^2)^{7/2}}{80e^2}\\
&\quad + \frac{cx^3(d+ex^2)^{7/2}}{10e} + \frac{1}{256}\left(d^3\left(80a+\frac{d(3cd-10be)}{e^2}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2} + \frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x(d\\
&\qquad +ex^2)^{3/2} + \frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{5/2}\\
&\quad - \frac{(3cd-10be)x(d+ex^2)^{7/2}}{80e^2} + \frac{cx^3(d+ex^2)^{7/2}}{10e}\\
&\quad + \frac{1}{256}\left(d^3\left(80a+\frac{d(3cd-10be)}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2} \\
&\quad+\frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{3/2} \\
&\quad+\frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{5/2}-\frac{(3cd-10be)x(d+ex^2)^{7/2}}{80e^2} \\
&\quad+\frac{cx^3(d+ex^2)^{7/2}}{10e}+\frac{d^3(3cd^2-10bde+80ae^2)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{256e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.84

$$
\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx = \sqrt{e}x\sqrt{d+ex^2}(c(-45d^4+30d^3ex^2+744d^2e^2x^4+1008de^3x^6+384e^4x^8)+10e(8ae(33d^2+26de
$$

```
[In] Integrate[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(c*(-45*d^4 + 30*d^3*e*x^2 + 744*d^2*e^2*x^4 + 1
008*d*e^3*x^6 + 384*e^4*x^8) + 10*e*(8*a*e*(33*d^2 + 26*d*e*x^2 + 8*e^2*x^4
) + b*(15*d^3 + 118*d^2*e*x^2 + 136*d*e^2*x^4 + 48*e^3*x^6))) - 15*(3*c*d^5
 - 10*d^3*e*(b*d - 8*a*e))*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(3840*e^(5/
2))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{\frac{5d^{3}\left(a\,e^{2}-\frac{1}{8}bde+\frac{3}{80}c\,d^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)}{16}+\frac{11\left(d^{2}\left(\frac{31}{110}c\,x^{4}+\frac{59}{132}b\,x^{2}+a\right)e^{\frac{5}{2}}+\frac{26d\left(\frac{63}{130}c\,x^{4}+\frac{17}{26}b\,x^{2}+a\right)x^{2}e^{\frac{7}{2}}}{33}+\frac{8\left(\frac{3}{5}c\,x^{4}+\frac{3}{4}b\,x^{2}+a\right)}{33}\right.}{16}}{e^{\frac{5}{2}}}$
risch	$\frac{x\left(384e^{4}c\,x^{8}+480e^{4}b\,x^{6}+1008d\,e^{3}c\,x^{6}+640a\,e^{4}x^{4}+1360bd\,e^{3}x^{4}+744c\,d^{2}e^{2}x^{4}+2080d\,e^{3}a\,x^{2}+1180e^{2}d^{2}b\,x^{2}+30d^{3}ec\,x^{2}+2640\right.}{3840e^{2}}$
default	$a\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}{6}+\frac{5d\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)}{4}\right)}{6}\right)+c\left(\frac{x^{3}\left(e\,x^{2}+d\right)^{\frac{7}{2}}}{10e}-\frac{3d\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{7}{2}}}{8e}\right.}{}\right.$

[In] int((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 11/16/e^(5/2)*(5/11*d^3*(a*e^2-1/8*b*d*e+3/80*c*d^2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+(d^2*(31/110*c*x^4+59/132*b*x^2+a)*e^(5/2)+26/33*d*(63/130*c*x^4+17/26*b*x^2+a)*x^2*e^(7/2)+8/33*(3/5*c*x^4+3/4*b*x^2+a)*x^4*e^(9/2)+5/88*((1/5*c*x^2+b)*e^(3/2)-3/10*c*d*e^(1/2))*d^3)*(e*x^2+d)^(1/2)*x)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.72

$$\int \left(d+ex^2\right)^{5/2} \left(a+bx^2 +cx^4\right) dx = \left[\frac{15\,(3\,cd^5 - 10\,bd^4e + 80\,ad^3e^2)\sqrt{e}\log\left(-2\,ex^2 - 2\,\sqrt{ex^2+d}\sqrt{e}x - d\right) + 2\,(384\,ce^5x^9 + 48\,(21\,c}{} \right.$$

$$-\frac{15\,(3\,cd^5 - 10\,bd^4e + 80\,ad^3e^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - (384\,ce^5x^9 + 48\,(21\,cde^4 + 10\,be^5)x^7 + 8\,(93\,cd^2e^3}{}$$

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*sqrt(e)*log(-2*e*x^2 - 2*
sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(384*c*e^5*x^9 + 48*(21*c*d*e^4 + 10*b*e
^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5)*x^5 + 10*(3*c*d^3*e^2 +
 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e - 10*b*d^3*e^2 - 176*a*d^
2*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/3840*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^
3*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (384*c*e^5*x^9 + 48*(2
1*c*d*e^4 + 10*b*e^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5)*x^5 +
 10*(3*c*d^3*e^2 + 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e - 10*b*
d^3*e^2 - 176*a*d^2*e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(209) = 418$.

Time = 0.52 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.04

$$\int \left(d+ex^2\right)^{5/2} \left(a+bx^2+cx^4\right)\, dx = \begin{cases} \sqrt{d+ex^2}\left(\frac{ce^2x^9}{10} + \frac{x^7\left(be^3+\frac{21cde^2}{10}\right)}{8e} + \frac{x^5\left(ae^3+3bde^2+3cd^2e-\frac{7d\left(be^3+\frac{21cde^2}{10}\right)}{8e}\right)}{6e} + \frac{x^3\cdot\left(3ade^2+3bd^2e+cd^3-\frac{5d\left(ae^3-\right.}{}\right.}{}\right. \\ d^{\frac{5}{2}}\left(ax+\frac{bx^3}{3}+\frac{cx^5}{5}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**(5/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise((sqrt(d + e*x**2)*(c*e**2*x**9/10 + x**7*(b*e**3 + 21*c*d*e**2/10
)/(8*e) + x**5*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e - 7*d*(b*e**3 + 21*c*d*e**
2/10)/(8*e))/(6*e) + x**3*(3*a*d*e**2 + 3*b*d**2*e + c*d**3 - 5*d*(a*e**3 +
 3*b*d*e**2 + 3*c*d**2*e - 7*d*(b*e**3 + 21*c*d*e**2/10)/(8*e))/(6*e))/(4*e
) + x*(3*a*d**2*e + b*d**3 - 3*d*(3*a*d*e**2 + 3*b*d**2*e + c*d**3 - 5*d*(a
*e**3 + 3*b*d*e**2 + 3*c*d**2*e - 7*d*(b*e**3 + 21*c*d*e**2/10)/(8*e))/(6*e
))/(4*e))/(2*e)) + (a*d**3 - d*(3*a*d**2*e + b*d**3 - 3*d*(3*a*d*e**2 + 3*b
*d**2*e + c*d**3 - 5*d*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e - 7*d*(b*e**3 + 21
*c*d*e**2/10)/(8*e))/(6*e))/(4*e))/(2*e))*Piecewise((log(2*sqrt(e)*sqrt(d +
 e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), True)), Ne(e,
 0)), (d**(5/2)*(a*x + b*x**3/3 + c*x**5/5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.91

$$\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx = \frac{1}{3840}\left(2\left(4\left(6\left(8\,ce^2x^2+\frac{21\,cde^9+10\,be^{10}}{e^8}\right)x^2+\frac{93\,cd^2e^8+170\,bde^9+80\,ae^{10}}{e^8}\right)x^2+5\,(3\,cd^3\right.\right.\right.$$

$$-\frac{\left(3\,cd^5-10\,bd^4e+80\,ad^3e^2\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{256\,e^{\frac{5}{2}}}$$

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*c*e^2*x^2 + (21*c*d*e^9 + 10*b*e^10)/e^8)*x^2 + (93*c*d^
2*e^8 + 170*b*d*e^9 + 80*a*e^10)/e^8)*x^2 + 5*(3*c*d^3*e^7 + 118*b*d^2*e^8
+ 208*a*d*e^9)/e^8)*x^2 - 15*(3*c*d^4*e^6 - 10*b*d^3*e^7 - 176*a*d^2*e^8)/e
^8)*sqrt(e*x^2 + d)*x - 1/256*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*log(abs
(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx = \int {\left(e\,x^2+d\right)}^{5/2}\,\left(c\,x^4+b\,x^2+a\right) \,d x$$

```
[In] int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4), x)
```

3.277 $\int \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right) \, dx = \frac{d(3cd^2-8bde+48ae^2)\,x\sqrt{d+ex^2}}{128e^2} + \frac{(3cd^2-8bde+48ae^2)\,x(d+ex^2)^{3/2}}{192e^2} - \frac{(3cd-8be)x(d+ex^2)^{5/2}}{48e^2} + \frac{cx^3(d+ex^2)^{5/2}}{8e} + \frac{d^2(3cd^2-8bde+48ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{5/2}}$$

```
[Out] 1/192*(48*a*e^2-8*b*d*e+3*c*d^2)*x*(e*x^2+d)^(3/2)/e^2-1/48*(-8*b*e+3*c*d)*
x*(e*x^2+d)^(5/2)/e^2+1/8*c*x^3*(e*x^2+d)^(5/2)/e+1/128*d^2*(48*a*e^2-8*b*d
*e+3*c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/128*d*(48*a*e^2-8*
b*d*e+3*c*d^2)*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1173, 396, 201, 223, 212}

$$\int \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right) \, dx = \frac{d^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(48ae^2-8bde+3cd^2)}{128e^{5/2}} + \frac{x(d+ex^2)^{3/2}(48ae^2-8bde+3cd^2)}{192e^2} + \frac{dx\sqrt{d+ex^2}(48ae^2-8bde+3cd^2)}{128e^2} - \frac{x(d+ex^2)^{5/2}(3cd-8be)}{48e^2} + \frac{cx^3(d+ex^2)^{5/2}}{8e}$$

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*Sqrt[d + e*x^2])/(128*e^2) + ((3*c*d^2
- 8*b*d*e + 48*a*e^2)*x*(d + e*x^2)^(3/2))/(192*e^2) - ((3*c*d - 8*b*e)*x*(
d + e*x^2)^(5/2))/(48*e^2) + (c*x^3*(d + e*x^2)^(5/2))/(8*e) + (d^2*(3*c*d^
2 - 8*b*d*e + 48*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{cx^3(d+ex^2)^{5/2}}{8e} + \frac{\int (d+ex^2)^{3/2}\left(8ae-(3cd-8be)x^2\right)\,dx}{8e}$$

$$= -\frac{(3cd-8be)x(d+ex^2)^{5/2}}{48e^2} + \frac{cx^3(d+ex^2)^{5/2}}{8e} - \frac{1}{48}\left(-48a - \frac{d(3cd-8be)}{e^2}\right)\int \left(d+ex^2\right)^{3/2}\,dx$$

$$= \frac{1}{192}\left(48a + \frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2} - \frac{(3cd-8be)x(d+ex^2)^{5/2}}{48e^2} + \frac{cx^3(d+ex^2)^{5/2}}{8e} + \frac{1}{64}\left(d\left(48a + \frac{d(3cd-8be)}{e^2}\right)\right)\int \sqrt{d+ex^2}\,dx$$

$$= \frac{1}{128}d\left(48a + \frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2} + \frac{1}{192}\left(48a + \frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2} - \frac{(3cd-8be)x(d+ex^2)^{5/2}}{48e^2} + \frac{cx^3(d+ex^2)^{5/2}}{8e} + \frac{1}{128}\left(d^2\left(48a + \frac{d(3cd-8be)}{e^2}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{1}{128}d\left(48a + \frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2} + \frac{1}{192}\left(48a + \frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2} - \frac{(3cd-8be)x(d+ex^2)^{5/2}}{48e^2} + \frac{cx^3(d+ex^2)^{5/2}}{8e} + \frac{1}{128}\left(d^2\left(48a + \frac{d(3cd-8be)}{e^2}\right)\right)\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)$$

$$= \frac{1}{128}d\left(48a + \frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2} + \frac{1}{192}\left(48a + \frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2} - \frac{(3cd-8be)x(d+ex^2)^{5/2}}{48e^2} + \frac{cx^3(d+ex^2)^{5/2}}{8e} + \frac{d^2(3cd^2-8bde+48ae^2)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)\,dx = \frac{\sqrt{e}x\sqrt{d+ex^2}(c(-9d^3+6d^2ex^2+72de^2x^4+48e^3x^6)+8e(6ae(5d+2ex^2)+b(3d^2+14dex^2+8}{384e^{5/2}}$$

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(c*(-9*d^3 + 6*d^2*e*x^2 + 72*d*e^2*x^4 + 48*e^3
*x^6) + 8*e*(6*a*e*(5*d + 2*e*x^2) + b*(3*d^2 + 14*d*e*x^2 + 8*e^2*x^4))) -
 3*(3*c*d^4 - 8*d^2*e*(b*d - 6*a*e))*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(
384*e^(5/2))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{3\left(a e^{2}-\frac{1}{6} b d e+\frac{1}{16} c d^{2}\right) d^{2} \operatorname{arctanh}\left(\frac{\sqrt{e x^{2}+d}}{x \sqrt{e}}\right)}{8}+\frac{5\left(d\left(\frac{3}{10} c x^{4}+\frac{7}{15} b x^{2}+a\right) e^{\frac{5}{2}}+\frac{2 x^{2}\left(\frac{1}{2} c x^{4}+\frac{2}{3} b x^{2}+a\right) e^{\frac{7}{2}}}{5}+\frac{\left(\left(\frac{c x^{2}}{4}+b\right) e^{\frac{3}{2}}-\frac{3 c d \sqrt{e}}{8}\right) d^{2}}{10}\right.}{8}}{e^{\frac{5}{2}}}$
risch	$\frac{x\left(48 e^{3} c x^{6}+64 e^{3} b x^{4}+72 d e^{2} c x^{4}+96 a e^{3} x^{2}+112 d e^{2} b x^{2}+6 d^{2} e x^{2} c+240 d e^{2} a+24 d^{2} e b-9 d^{3} c\right) \sqrt{e x^{2}+d}}{384 e^{2}}+\frac{d^{2}\left(48 a e^{2}-8 b d e\right.}{}$
default	$a\left(\frac{x\left(e x^{2}+d\right)^{\frac{3}{2}}}{4}+\frac{3 d\left(\frac{x \sqrt{e x^{2}+d}}{2}+\frac{d \ln\left(x \sqrt{e}+\sqrt{e x^{2}+d}\right)}{2 \sqrt{e}}\right)}{4}\right)+c\left(\frac{x^{3}\left(e x^{2}+d\right)^{\frac{5}{2}}}{8 e}-\frac{3 d\left(\frac{x\left(e x^{2}+d\right)^{\frac{5}{2}}}{6 e}-\frac{d\left(\frac{x\left(e x^{2}+d\right)^{\frac{3}{2}}}{4}\right.}{}\right.}{}\right.$

```
[In] int((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 5/8*(3/5*(a*e^2-1/6*b*d*e+1/16*c*d^2)*d^2*arctanh((e*x^2+d)^(1/2)/x/e^(1/2)
)+(d*(3/10*c*x^4+7/15*b*x^2+a)*e^(5/2)+2/5*x^2*(1/2*c*x^4+2/3*b*x^2+a)*e^(7
/2)+1/10*((1/4*c*x^2+b)*e^(3/2)-3/8*c*d*e^(1/2))*d^2)*(e*x^2+d)^(1/2)*x)/e^
(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.74

$$\int \left(d+ex^{2}\right)^{3/2}\left(a+bx^{2}+cx^{4}\right)\,dx=\left[\frac{3\left(3\,cd^{4}-8\,bd^{3}e+48\,ad^{2}e^{2}\right)\sqrt{e}\log\left(-2\,ex^{2}-2\,\sqrt{ex^{2}+d}\sqrt{e}x-d\right)+2\,(48\,ce^{4}x^{7}+8\,(9\,cde^{3}+}{768\,e^{3}}\right.$$

$$-\frac{3\left(3\,cd^{4}-8\,bd^{3}e+48\,ad^{2}e^{2}\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^{2}+d}}\right)-(48\,ce^{4}x^{7}+8\,(9\,cde^{3}+8\,be^{4})x^{5}+2\,(3\,cd^{2}e^{2}+56\,bde}{384\,e^{3}}$$

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqr
t(e*x^2 + d)*sqrt(e)*x - d) + 2*(48*c*e^4*x^7 + 8*(9*c*d*e^3 + 8*b*e^4)*x^5
 + 2*(3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*x^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2
 - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/384*(3*(3*c*d^4 - 8*b*d^3*e + 48
*a*d^2*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (48*c*e^4*x^7 + 8
*(9*c*d*e^3 + 8*b*e^4)*x^5 + 2*(3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*x^3 -
3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.57

$$\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)\,dx=\begin{cases}\sqrt{d+ex^2}\left(\frac{cex^7}{8}+\frac{x^5\left(be^2+\frac{9cde}{8}\right)}{6e}+\frac{x^3\left(ae^2+2bde+cd^2-\frac{5d\left(be^2+\frac{9cde}{8}\right)}{6e}\right)}{4e}+\frac{x\left(2ade+bd^2-\frac{3d\left(ae^2+2bde+cd^2-\frac{5d\left(be^2+\right.}{6e}\right.}{4e}\right.}{2e}\right. \\ d^{\frac{3}{2}}\left(ax+\frac{bx^3}{3}+\frac{cx^5}{5}\right)\end{cases}$$

```
[In] integrate((e*x**2+d)**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise((sqrt(d + e*x**2)*(c*e*x**7/8 + x**5*(b*e**2 + 9*c*d*e/8)/(6*e) +
 x**3*(a*e**2 + 2*b*d*e + c*d**2 - 5*d*(b*e**2 + 9*c*d*e/8)/(6*e))/(4*e) +
x*(2*a*d*e + b*d**2 - 3*d*(a*e**2 + 2*b*d*e + c*d**2 - 5*d*(b*e**2 + 9*c*d*
e/8)/(6*e))/(4*e))/(2*e)) + (a*d**2 - d*(2*a*d*e + b*d**2 - 3*d*(a*e**2 + 2
*b*d*e + c*d**2 - 5*d*(b*e**2 + 9*c*d*e/8)/(6*e))/(4*e))/(2*e))*Piecewise((
log(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(
e*x**2), True)), Ne(e, 0)), (d**(3/2)*(a*x + b*x**3/3 + c*x**5/5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int (d+ex^2)^{3/2} (a+bx^2$$
$$+cx^4)\ dx = \frac{1}{384} \Bigg(2 \Bigg(4 \Bigg(6\,cex^2 + \frac{9\,cde^6 + 8\,be^7}{e^6}\Bigg)x^2 + \frac{3\,cd^2e^5 + 56\,bde^6 + 48\,ae^7}{e^6}\Bigg)x^2 - \frac{3\,(3\,cd^3e^4 - 8\,bd^2e^5}{e^6}$$
$$- \frac{(3\,cd^4 - 8\,bd^3e + 48\,ad^2e^2)\log\left(\left|-\sqrt{e}x + \sqrt{ex^2+d}\right|\right)}{128\,e^{\frac{5}{2}}}$$

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*c*e*x^2 + (9*c*d*e^6 + 8*b*e^7)/e^6)*x^2 + (3*c*d^2*e^5 + 56
*b*d*e^6 + 48*a*e^7)/e^6)*x^2 - 3*(3*c*d^3*e^4 - 8*b*d^2*e^5 - 80*a*d*e^6)/
e^6)*sqrt(e*x^2 + d)*x - 1/128*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*log(abs
(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^{3/2} (a+bx^2+cx^4)\ dx = \int (e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)\ dx$$

```
[In] int((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4), x)
```

3.278 $\int \sqrt{d+ex^2}(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx=\frac{\left(cd^2-2bde+8ae^2\right)x\sqrt{d+ex^2}}{16e^2}-\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}+\frac{d\left(cd^2-2bde+8ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{5/2}}$$

```
[Out] -1/8*(-2*b*e+c*d)*x*(e*x^2+d)^(3/2)/e^2+1/6*c*x^3*(e*x^2+d)^(3/2)/e+1/16*d*
(8*a*e^2-2*b*d*e+c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/16*(8*
a*e^2-2*b*d*e+c*d^2)*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1173, 396, 201, 223, 212}

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx=\frac{d\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-2bde+cd^2\right)}{16e^{5/2}}+\frac{x\sqrt{d+ex^2}\left(8ae^2-2bde+cd^2\right)}{16e^2}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

```
[In] Int[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - 2*b*d*e + 8*a*e^2)*x*Sqrt[d + e*x^2])/(16*e^2) - ((c*d - 2*b*e)*x
*(d + e*x^2)^(3/2))/(8*e^2) + (c*x^3*(d + e*x^2)^(3/2))/(6*e) + (d*(c*d^2 -
 2*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(16*e^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{cx^3(d+ex^2)^{3/2}}{6e} + \frac{\int \sqrt{d+ex^2}(6ae-3(cd-2be)x^2)\,dx}{6e}$$

$$= -\frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e} + \frac{1}{8}\left(8a+\frac{d(cd-2be)}{e^2}\right)\int \sqrt{d+ex^2}\,dx$$

$$
\begin{aligned}
&= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2}-\frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2} \\
&\quad +\frac{cx^3(d+ex^2)^{3/2}}{6e}+\frac{1}{16}\left(d\left(8a+\frac{d(cd-2be)}{e^2}\right)\right)\int\frac{1}{\sqrt{d+ex^2}}\,dx \\
&= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2}-\frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2}+\frac{cx^3(d+ex^2)^{3/2}}{6e} \\
&\quad +\frac{1}{16}\left(d\left(8a+\frac{d(cd-2be)}{e^2}\right)\right)\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right) \\
&= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2}-\frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2} \\
&\quad +\frac{cx^3(d+ex^2)^{3/2}}{6e}+\frac{d(cd^2-2bde+8ae^2)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$
\begin{aligned}
&\int\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx \\
&= \frac{x\sqrt{d+ex^2}(-3cd^2+6bde+24ae^2+2cdex^2+12be^2x^2+8ce^2x^4)}{48e^2} \\
&\quad -\frac{d(cd^2-2bde+8ae^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{16e^{5/2}}
\end{aligned}
$$

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2]*(-3*c*d^2 + 6*b*d*e + 24*a*e^2 + 2*c*d*e*x^2 + 12*b*e^2*
x^2 + 8*c*e^2*x^4))/(48*e^2) - (d*(c*d^2 - 2*b*d*e + 8*a*e^2)*Log[-(Sqrt[e]
*x) + Sqrt[d + e*x^2]])/(16*e^(5/2))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{d\left(a\,e^{2}-\frac{1}{4}bde+\frac{1}{8}c\,d^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)+\sqrt{e\,x^{2}+d}\left(\left(\frac{1}{3}c\,x^{4}+\frac{1}{2}b\,x^{2}+a\right)e^{\frac{5}{2}}+\frac{d\left(\left(\frac{c\,x^{2}}{3}+b\right)e^{\frac{3}{2}}-\frac{cd\sqrt{e}}{2}\right)}{4}\right)x}{2e^{\frac{5}{2}}}$
risch	$\frac{x\left(8c\,e^{2}x^{4}+12b\,e^{2}x^{2}+2de\,x^{2}c+24a\,e^{2}+6bde-3c\,d^{2}\right)\sqrt{e\,x^{2}+d}}{48e^{2}}+\frac{d\left(8a\,e^{2}-2bde+c\,d^{2}\right)\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{16e^{\frac{5}{2}}}$
default	$a\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)+c\left(\frac{x^{3}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{6e}-\frac{d\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right.}{4e}\right.}{2e}\right.$

```
[In] int((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*(a*e^2-1/4*b*d*e+1/8*c*d^2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+(e*x^
2+d)^(1/2)*((1/3*c*x^4+1/2*b*x^2+a)*e^(5/2)+1/4*d*((1/3*c*x^2+b)*e^(3/2)-1/
2*c*d*e^(1/2)))*x)/e^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.76

$$\int\sqrt{d+ex^{2}}\left(a+bx^{2}+cx^{4}\right)\,dx$$

$$=\left[\frac{3\left(cd^{3}-2\,bd^{2}e+8\,ade^{2}\right)\sqrt{e}\log\left(-2\,ex^{2}-2\,\sqrt{ex^{2}+d}\sqrt{e}x-d\right)+2\left(8\,ce^{3}x^{5}+2\left(cde^{2}+6\,be^{3}\right)x^{3}-3\left(cd\right.\right.}{96\,e^{3}}\right.$$

$$-\frac{3\left(cd^{3}-2\,bd^{2}e+8\,ade^{2}\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^{2}+d}}\right)-\left(8\,ce^{3}x^{5}+2\left(cde^{2}+6\,be^{3}\right)x^{3}-3\left(cd^{2}e-2\,bde^{2}-8\,a\right.\right.}{48\,e^{3}}$$

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^
2 + d)*sqrt(e)*x - d) + 2*(8*c*e^3*x^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c*d
^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/48*(3*(c*d^3 - 2*b*
d^2*e + 8*a*d*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (8*c*e^3*x
^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c*d^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*sqrt(
e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.16

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx$$
$$= \begin{cases} \sqrt{d+ex^2}\left(\frac{cx^5}{6}+\frac{x^3\left(be+\frac{cd}{6}\right)}{4e}+\frac{x\left(ae+bd-\frac{3d\left(be+\frac{cd}{6}\right)}{4e}\right)}{2e}\right)+\left(ad-\frac{d\left(ae+bd-\frac{3d\left(be+\frac{cd}{6}\right)}{4e}\right)}{2e}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} \\ \frac{x\log(x)}{\sqrt{ex^2}}\end{cases}\right. \\ \sqrt{d}\left(ax+\frac{bx^3}{3}+\frac{cx^5}{5}\right) \end{cases}$$

```
[In] integrate((e*x**2+d)**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise((sqrt(d + e*x**2)*(c*x**5/6 + x**3*(b*e + c*d/6)/(4*e) + x*(a*e +
 b*d - 3*d*(b*e + c*d/6)/(4*e))/(2*e)) + (a*d - d*(a*e + b*d - 3*d*(b*e + c
*d/6)/(4*e))/(2*e))*Piecewise((log(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt
(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), True)), Ne(e, 0)), (sqrt(d)*(a*x +
b*x**3/3 + c*x**5/5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx$$
$$= \frac{1}{48}\left(2\left(4\,cx^2+\frac{cde^3+6\,be^4}{e^4}\right)x^2-\frac{3\left(cd^2e^2-2\,bde^3-8\,ae^4\right)}{e^4}\right)\sqrt{ex^2+d}x$$
$$-\frac{\left(cd^3-2\,bd^2e+8\,ade^2\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{16\,e^{\frac{5}{2}}}$$

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*c*x^2 + (c*d*e^3 + 6*b*e^4)/e^4)*x^2 - 3*(c*d^2*e^2 - 2*b*d*e^3
- 8*a*e^4)/e^4)*sqrt(e*x^2 + d)*x - 1/16*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*lo
g(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\,(a+bx^2+cx^4)\ dx = \int \sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)\,dx$$

```
[In] int((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4), x)
```

3.279 $\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e} + \frac{(3cd^2-4bde+8ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{5/2}}$$

```
[Out] 1/8*(8*a*e^2-4*b*d*e+3*c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)-1/
8*(-4*b*e+3*c*d)*x*(e*x^2+d)^(1/2)/e^2+1/4*c*x^3*(e*x^2+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1173, 396, 223, 212}

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(8ae^2-4bde+3cd^2)}{8e^{5/2}} - \frac{x\sqrt{d+ex^2}(3cd-4be)}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

```
[In] Int[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

```
[Out] -1/8*((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/e^2 + (c*x^3*Sqrt[d + e*x^2])/(4*e
) + ((3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8
*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{cx^3\sqrt{d+ex^2}}{4e}+\frac{\int \frac{4ae-(3cd-4be)x^2}{\sqrt{d+ex^2}}\,dx}{4e} \\
&= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}-\frac{1}{8}\left(-8a-\frac{d(3cd-4be)}{e^2}\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx \\
&= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e} \\
&\quad -\frac{1}{8}\left(-8a-\frac{d(3cd-4be)}{e^2}\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right) \\
&= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}+\frac{(3cd^2-4bde+8ae^2)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(-3cdx+4bex+2cex^3)}{8e^2} + \frac{(3cd^2-4bde+8ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{4e^{5/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(-3*c*d*x + 4*b*e*x + 2*c*e*x^3))/(8*e^2) + ((3*c*d^2 - 4*
b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/(-Sqrt[d] + Sqrt[d + e*x^2])])/(4*e^(5
/2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x\left(2c\,x^2e+4be-3cd\right)\sqrt{e\,x^2+d}}{8e^2}+\frac{\left(8a\,e^2-4bde+3c\,d^2\right)\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{8e^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\left(a\,e^2-\frac{1}{2}bde+\frac{3}{8}c\,d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)+\frac{\left(\left(\frac{c\,x^2}{2}+b\right)e^{\frac{3}{2}}-\frac{3cd\sqrt{e}}{4}\right)\sqrt{e\,x^2+d}\,x}{2}}{e^{\frac{5}{2}}}$
default	$\frac{a\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{\sqrt{e}}+c\left(\frac{x^3\sqrt{e\,x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)+b\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e}\right.}{2e^{\frac{3}{2}}}\right.$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(2*c*e*x^2+4*b*e-3*c*d)*(e*x^2+d)^(1/2)/e^2+1/8*(8*a*e^2-4*b*d*e+3*c*
d^2)/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.79

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{(3\,cd^2-4\,bde+8\,ae^2)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(2\,ce^2x^3-(3\,cde-4\,be^2)x)\sqrt{ex^2+d}}{16\,e^3},\right.$$
$$\left.-\frac{(3\,cd^2-4\,bde+8\,ae^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(2\,ce^2x^3-(3\,cde-4\,be^2)x)\sqrt{ex^2+d}}{8\,e^3}\right]$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 +
d)*sqrt(e)*x - d) + 2*(2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 + d)
)/e^3, -1/8*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(
e*x^2 + d)) - (2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$$
$$= \begin{cases} \left(a-\frac{d\left(b-\frac{3cd}{4e}\right)}{2e}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} & \text{for } d\neq 0 \\ \frac{x\log(x)}{\sqrt{ex^2}} & \text{otherwise} \end{cases}\right)+\sqrt{d+ex^2}\left(\frac{cx^3}{4e}+\frac{x\left(b-\frac{3cd}{4e}\right)}{2e}\right) & \text{for } e\neq 0 \\ \frac{ax+\frac{bx^3}{3}+\frac{cx^5}{5}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Piecewise(((a - d*(b - 3*c*d/(4*e))/(2*e))*Piecewise((log(2*sqrt(e)*sqrt(d
+ e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), True)) + sqr
t(d + e*x**2)*(c*x**3/(4*e) + x*(b - 3*c*d/(4*e))/(2*e)), Ne(e, 0)), ((a*x
+ b*x**3/3 + c*x**5/5)/sqrt(d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = \frac{1}{8}\sqrt{ex^2+d}\left(\frac{2\,cx^2}{e}-\frac{3\,cde-4\,be^2}{e^3}\right)x - \frac{(3\,cd^2-4\,bde+8\,ae^2)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{8\,e^{\frac{5}{2}}}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(e*x^2 + d)*(2*c*x^2/e - (3*c*d*e - 4*b*e^2)/e^3)*x - 1/8*(3*c*d^2
- 4*b*d*e + 8*a*e^2)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = \int \frac{c\,x^4+b\,x^2+a}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2), x)
```

3.280 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx=\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}-\frac{(3cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}$$

```
[Out] -1/2*(-2*b*e+3*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+(a+d*(-b*e+c
*d)/e^2)*x/d/(e*x^2+d)^(1/2)+1/2*c*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1171, 396, 223, 212}

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx=\frac{x(ae^2-bde+cd^2)}{de^2\sqrt{d+ex^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(3cd-2be)}{2e^{5/2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(d*e^2*Sqrt[d + e*x^2]) + (c*x*Sqrt[d + e*x^2])
/(2*e^2) - ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(5/2
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{(cd^2-bde+ae^2)\,x}{de^2\sqrt{d+ex^2}} - \frac{\int \frac{\frac{d(cd-be)}{e^2}-\frac{cdx^2}{e}}{\sqrt{d+ex^2}}\,dx}{d}$$

$$= \frac{(cd^2-bde+ae^2)\,x}{de^2\sqrt{d+ex^2}} + \frac{cx\sqrt{d+ex^2}}{2e^2} - \frac{(3cd-2be)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2e^2}$$

$$= \frac{(cd^2-bde+ae^2)\,x}{de^2\sqrt{d+ex^2}} + \frac{cx\sqrt{d+ex^2}}{2e^2} - \frac{(3cd-2be)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2e^2}$$

$$= \frac{(cd^2-bde+ae^2)\,x}{de^2\sqrt{d+ex^2}} + \frac{cx\sqrt{d+ex^2}}{2e^2} - \frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \frac{x(3cd^2-2bde+2ae^2+cdex^2)}{2de^2\sqrt{d+ex^2}} + \frac{(3cd-2be)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{2e^{5/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(3*c*d^2 - 2*b*d*e + 2*a*e^2 + c*d*e*x^2))/(2*d*e^2*Sqrt[d + e*x^2]) + (
(3*c*d - 2*b*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(2*e^(5/2))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{\sqrt{e\,x^2+d}\,d\left(be-\frac{3cd}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)+\left(-\left(-\frac{c\,x^2}{2}+b\right)d\,e^{\frac{3}{2}}+\frac{3c\,d^2\sqrt{e}}{2}+a\,e^{\frac{5}{2}}\right)x}{\sqrt{e\,x^2+d}\,e^{\frac{5}{2}}\,d}$
risch	$\frac{cx\sqrt{e\,x^2+d}}{2e^2}+\frac{\frac{2a\,e^2x}{d\sqrt{e\,x^2+d}}-\frac{cdx}{\sqrt{e\,x^2+d}}+(2b\,e^2-3dce)\left(-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}\right)}{2e^2}$
default	$\frac{ax}{d\sqrt{e\,x^2+d}}+c\left(\frac{x^3}{2e\sqrt{e\,x^2+d}}-\frac{3d\left(-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}\right)}{2e}\right)+b\left(-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}\right)$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(e*x^2+d)^(1/2)/e^(5/2)*((e*x^2+d)^(1/2)*d*(b*e-3/2*c*d)*arctanh((e*x^2+d
)^(1/2)/x/e^(1/2))+(-(-1/2*c*x^2+b)*d*e^(3/2)+3/2*c*d^2*e^(1/2)+a*e^(5/2))*
x)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.80

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \Bigg[-\frac{(3\,cd^3-2\,bd^2e+(3\,cd^2e-2\,bde^2)x^2)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\,(}{4\,(de^4x^2+d^2e^3)}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*c*d^3 - 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(e)*log(-2*e
*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(c*d*e^2*x^3 + (3*c*d^2*e - 2*b
*d*e^2 + 2*a*e^3)*x)*sqrt(e*x^2 + d))/(d*e^4*x^2 + d^2*e^3), 1/2*((3*c*d^3
- 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(
e*x^2 + d)) + (c*d*e^2*x^3 + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*x)*sqrt(e*x^
2 + d))/(d*e^4*x^2 + d^2*e^3)]
```

Sympy [A] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \frac{ax}{d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}} + b\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{\frac{3}{2}}} - \frac{x}{\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right) + c\left(\frac{3\sqrt{d}x}{2e^2\sqrt{1+\frac{ex^2}{d}}} - \frac{3d\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right)$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(3/2),x)
```

```
[Out] a*x/(d**(3/2)*sqrt(1 + e*x**2/d)) + b*(asinh(sqrt(e)*x/sqrt(d))/e**(3/2) -
x/(sqrt(d)*e*sqrt(1 + e*x**2/d))) + c*(3*sqrt(d)*x/(2*e**2*sqrt(1 + e*x**2/
d)) - 3*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(5/2)) + x**3/(2*sqrt(d)*e*sqrt(1
+ e*x**2/d)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \frac{\left(\frac{cx^2}{e}+\frac{3\,cd^2e-2\,bde^2+2\,ae^3}{de^3}\right)x}{2\,\sqrt{ex^2+d}}+\frac{(3\,cd-2\,be)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{2\,e^{\frac{5}{2}}}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2/e + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)/(d*e^3))*x/sqrt(e*x^2 + d)
 + 1/2*(3*c*d - 2*b*e)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \int \frac{c\,x^4+b\,x^2+a}{{\left(e\,x^2+d\right)}^{3/2}} \,dx$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2), x)
```

3.281 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}}-\frac{\left(4cd^2-e(bd+2ae)\right)x}{3d^2e^2\sqrt{d+ex^2}}+\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

```
[Out] 1/3*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^(3/2)+c*arctanh(x*e^(1/2)/(e*x^2+d)^
(1/2))/e^(5/2)-1/3*(4*c*d^2-e*(2*a*e+b*d))*x/d^2/e^2/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1171, 393, 223, 212}

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx=-\frac{x\left(4cd^2-e(2ae+bd)\right)}{3d^2e^2\sqrt{d+ex^2}}+\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^2\right)^{3/2}}+\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^2)^(3/2)) - ((4*c*d^2 - e*(b*
d + 2*a*e))*x)/(3*d^2*e^2*Sqrt[d + e*x^2]) + (c*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]])/e^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^2)^{3/2}}-\frac{\int \frac{-2a+\frac{d(cd-be)}{e^2}-\frac{3cdx^2}{e}}{(d+ex^2)^{3/2}}\,dx}{3d}\\
&= \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^2)^{3/2}}-\frac{(4cd^2-e(bd+2ae))\,x}{3d^2e^2\sqrt{d+ex^2}}+\frac{c\int \frac{1}{\sqrt{d+ex^2}}\,dx}{e^2}\\
&= \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^2)^{3/2}}-\frac{(4cd^2-e(bd+2ae))\,x}{3d^2e^2\sqrt{d+ex^2}}+\frac{c\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{e^2}\\
&= \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^2)^{3/2}}-\frac{(4cd^2-e(bd+2ae))\,x}{3d^2e^2\sqrt{d+ex^2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{-cd^2x(3d+4ex^2)+e^2x(3ad+bdx^2+2aex^2)}{3d^2e^2\left(d+ex^2\right)^{3/2}} - \frac{c\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{e^{5/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(c*d^2*x*(3*d + 4*e*x^2)) + e^2*x*(3*a*d + b*d*x^2 + 2*a*e*x^2))/(3*d^2*e
^2*(d + e*x^2)^(3/2)) - (c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/e^(5/2)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{(e\,x^2+d)^{\frac{3}{2}} c\,d^2 \operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)+x\left(d\left(\frac{b\,x^2}{3}+a\right)e^{\frac{5}{2}}-\frac{4c\,d^2 e^{\frac{3}{2}} x^2}{3}-c\,d^3\sqrt{e}+\frac{2a\,e^{\frac{7}{2}} x^2}{3}\right)}{e^{\frac{5}{2}}(e\,x^2+d)^{\frac{3}{2}} d^2}$
default	$a\left(\frac{x}{3d(e\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{e\,x^2+d}}\right)+c\left(-\frac{x^3}{3e(e\,x^2+d)^{\frac{3}{2}}}+\frac{-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}}{e}\right)+b\left(-\frac{x}{2e(e\,x^2+d)^{\frac{3}{2}}}\right.$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^(5/2)/(e*x^2+d)^(3/2)*((e*x^2+d)^(3/2)*c*d^2*arctanh((e*x^2+d)^(1/2)/x/
e^(1/2))+x*(d*(1/3*b*x^2+a)*e^(5/2)-4/3*c*d^2*e^(3/2)*x^2-c*d^3*e^(1/2)+2/3
*a*e^(7/2)*x^2))/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.86

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx = \left[\frac{3\left(cd^2e^2x^4+2\,cd^3ex^2+cd^4\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\left(\left(4\,cd^2e^2-b\right.\right.}{6\left(d^2e^5x^4+2\,d^3e^4x^2+d^4e^3\right)}\right.$$

$$-\frac{3\left(cd^2e^2x^4+2\,cd^3ex^2+cd^4\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+\left(\left(4\,cd^2e^2-bde^3-2\,ae^4\right)x^3+3\left(cd^3e-ade^3\right)x\right)\sqrt{ex^2}}{3\left(d^2e^5x^4+2\,d^3e^4x^2+d^4e^3\right)}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(e)*log(-2*e*x^2 - 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) - 2*((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 + 3
*(c*d^3*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4
*e^3), -1/3*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(-e)*arctan(sqrt
(-e)*x/sqrt(e*x^2 + d)) + ((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 + 3*(c*d^3
*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(94) = 188$.

Time = 6.51 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.46

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx &= a\left(\frac{3dx}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}\right.\\
&\left.+\frac{2ex^3}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}\right)+\frac{bx^3}{3d^{\frac{5}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{3}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}\\
&+c\left(\frac{3d^{\frac{39}{2}}e^{11}\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}\right.\\
&+\frac{3d^{\frac{37}{2}}e^{12}x^2\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}-\frac{3d^{19}e^{\frac{23}{2}}x}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}\\
&\left.-\frac{4d^{18}e^{\frac{25}{2}}x^3}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}\right)\end{aligned}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(5/2),x)
```

```
[Out] a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**2
/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 +
 e*x**2/d))) + b*x**3/(3*d**(5/2)*sqrt(1 + e*x**2/d) + 3*d**(3/2)*e*x**2*sq
rt(1 + e*x**2/d)) + c*(3*d**(39/2)*e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x
/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)
*x**2*sqrt(1 + e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)*asinh
(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)
*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3*d**(39/2)*e**(
27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) -
 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(3
7/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx = -\frac{x\left(\frac{(4\,cd^2e^2-bde^3-2\,ae^4)x^2}{d^2e^3}+\frac{3\,(cd^3e-ade^3)}{d^2e^3}\right)}{3\,(ex^2+d)^{\frac{3}{2}}} - \frac{c\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{e^{\frac{5}{2}}}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^2/(d^2*e^3) + 3*(c*d^3*e - a*d*
e^3)/(d^2*e^3))/(e*x^2 + d)^(3/2) - c*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d))
)/e^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx = \int \frac{c\,x^4+b\,x^2+a}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2), x)
```

3.282 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx=\frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{\left(3cd^2+2e(bd+4ae)\right)x^5}{15d^3\left(d+ex^2\right)^{5/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(5/2)+1/3*(4*a*e+b*d)*x^3/d^2/(e*x^2+d)^(5/2)+1/15*(3*c*d^2
+2*e*(4*a*e+b*d))*x^5/d^3/(e*x^2+d)^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1169, 1817, 12, 270}

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx=\frac{x^5\left(2e(4ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}+\frac{x^3(4ae+bd)}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(5/2)) + ((b*d + 4*a*e)*x^3)/(3*d^2*(d + e*x^2)^(5/2))
 + ((3*c*d^2 + 2*e*(b*d + 4*a*e))*x^5)/(15*d^3*(d + e*x^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{\int \frac{x^2\left(4ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{7/2}}\,dx}{d}\\
&= \frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{\int \frac{\left(3cd^2+2e(bd+4ae)\right)x^4}{\left(d+ex^2\right)^{7/2}}\,dx}{3d^2}\\
&= \frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{1}{3}\left(3c+\frac{2e(bd+4ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{7/2}}\,dx\\
&= \frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{\left(3cd^2+2e(bd+4ae)\right)x^5}{15d^3\left(d+ex^2\right)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{15ad^2x+5bd^2x^3+20adex^3+3cd^2x^5+2bdex^5+8ae^2x^5}{15d^3\left(d+ex^2\right)^{5/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (15*a*d^2*x + 5*b*d^2*x^3 + 20*a*d*e*x^3 + 3*c*d^2*x^5 + 2*b*d*e*x^5 + 8*a*
e^2*x^5)/(15*d^3*(d + e*x^2)^(5/2))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{5}c\,x^4+\frac{1}{3}b\,x^2+a\right)d^2+\frac{4e\left(\frac{b\,x^2}{10}+a\right)x^2d}{3}+\frac{8a\,e^2x^4}{15}\right)}{\left(e\,x^2+d\right)^{\frac{5}{2}}d^3}$
gosper	$\frac{x\left(8a\,e^2x^4+2bde\,x^4+3c\,d^2x^4+20ade\,x^2+5b\,d^2x^2+15a\,d^2\right)}{15\left(e\,x^2+d\right)^{\frac{5}{2}}d^3}$
trager	$\frac{x\left(8a\,e^2x^4+2bde\,x^4+3c\,d^2x^4+20ade\,x^2+5b\,d^2x^2+15a\,d^2\right)}{15\left(e\,x^2+d\right)^{\frac{5}{2}}d^3}$
default	$a\left(\frac{x}{5d\left(e\,x^2+d\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d\left(e\,x^2+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^2\sqrt{e\,x^2+d}}}{d}\right)+c\left(-\frac{x^3}{2e\left(e\,x^2+d\right)^{\frac{5}{2}}}+\frac{3d\left(-\frac{x}{4e\left(e\,x^2+d\right)^{\frac{5}{2}}}+\frac{d\left(\frac{x}{5d\left(e\,x^2+d\right)^{\frac{5}{2}}}+\right.}{}\right.}{2e}\right.$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(e*x^2+d)^(5/2)*x*((1/5*c*x^4+1/3*b*x^2+a)*d^2+4/3*e*(1/10*b*x^2+a)*x^2*d
+8/15*a*e^2*x^4)/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{((3\,cd^2+2\,bde+8\,ae^2)x^5+15\,ad^2x+5\,(bd^2+4\,ade)x^3)\sqrt{ex^2+d}}{15\,(d^3e^3x^6+3\,d^4e^2x^4+3\,d^5ex^2+d^6)}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*((3*c*d^2 + 2*b*d*e + 8*a*e^2)*x^5 + 15*a*d^2*x + 5*(b*d^2 + 4*a*d*e)*
x^3)*sqrt(e*x^2 + d)/(d^3*e^3*x^6 + 3*d^4*e^2*x^4 + 3*d^5*e*x^2 + d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(80) = 160.

Time = 14.75 (sec) , antiderivative size = 639, normalized size of antiderivative = 7.43

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx &= a\left(\frac{15d^5x}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\right.\\
&+\frac{35d^4ex^3}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\\
&+\frac{28d^3e^2x^5}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\\
&\left.+\frac{8d^2e^3x^7}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\right)\\
&+b\left(\frac{5dx^3}{15d^{\frac{9}{2}}\sqrt{1+\frac{ex^2}{d}}+30d^{\frac{7}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{5}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}\right.\\
&\left.+\frac{2ex^5}{15d^{\frac{9}{2}}\sqrt{1+\frac{ex^2}{d}}+30d^{\frac{7}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{5}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}\right)\\
&+\frac{cx^5}{5d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+10d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+5d^{\frac{3}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}\end{aligned}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(7/2),x)
```

```
[Out] a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1
+ e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3
```

```
*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(15*d**(17/2)*sqrt(1 + e*x**2/d)
 + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 +
 e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2*x**5
/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d)
+ 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1
 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(
15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d)
 + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d))) + b*(5*d*x**3/(15*d**(9/2)*s
qrt(1 + e*x**2/d) + 30*d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**
2*x**4*sqrt(1 + e*x**2/d)) + 2*e*x**5/(15*d**(9/2)*sqrt(1 + e*x**2/d) + 30*
d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**2*x**4*sqrt(1 + e*x**2/
d))) + c*x**5/(5*d**(7/2)*sqrt(1 + e*x**2/d) + 10*d**(5/2)*e*x**2*sqrt(1 +
e*x**2/d) + 5*d**(3/2)*e**2*x**4*sqrt(1 + e*x**2/d))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(76) = 152$.

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.01

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx = & -\frac{cx^3}{2\,(ex^2+d)^{\frac{5}{2}}e} + \frac{8\,ax}{15\,\sqrt{ex^2+d}d^3} \\ & + \frac{4\,ax}{15\,(ex^2+d)^{\frac{3}{2}}d^2} + \frac{ax}{5\,(ex^2+d)^{\frac{5}{2}}d} + \frac{cx}{10\,(ex^2+d)^{\frac{3}{2}}e^2} + \frac{cx}{5\,\sqrt{ex^2+d}de^2} \\ & - \frac{3\,cdx}{10\,(ex^2+d)^{\frac{5}{2}}e^2} - \frac{bx}{5\,(ex^2+d)^{\frac{5}{2}}e} + \frac{2\,bx}{15\,\sqrt{ex^2+d}d^2e} + \frac{bx}{15\,(ex^2+d)^{\frac{3}{2}}de}\end{aligned}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*c*x^3/((e*x^2 + d)^(5/2)*e) + 8/15*a*x/(sqrt(e*x^2 + d)*d^3) + 4/15*a*
x/((e*x^2 + d)^(3/2)*d^2) + 1/5*a*x/((e*x^2 + d)^(5/2)*d) + 1/10*c*x/((e*x^
2 + d)^(3/2)*e^2) + 1/5*c*x/(sqrt(e*x^2 + d)*d*e^2) - 3/10*c*d*x/((e*x^2 +
d)^(5/2)*e^2) - 1/5*b*x/((e*x^2 + d)^(5/2)*e) + 2/15*b*x/(sqrt(e*x^2 + d)*d
^2*e) + 1/15*b*x/((e*x^2 + d)^(3/2)*d*e)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{(3\,cd^2e^2+2\,bde^3+8\,ae^4)x^2}{d^3e^2} + \frac{5\,(bd^2e^2+4\,ade^3)}{d^3e^2}\right) + \frac{15\,a}{d}\right)x}{15\,(ex^2+d)^{\frac{5}{2}}}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*((3*c*d^2*e^2 + 2*b*d*e^3 + 8*a*e^4)*x^2/(d^3*e^2) + 5*(b*d^2*e^2
 + 4*a*d*e^3)/(d^3*e^2)) + 15*a/d)*x/(e*x^2 + d)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.55

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx = \frac{3\,c\,d^4\,x-6\,c\,d^3\,x\,\left(e\,x^2+d\right)-3\,b\,d^3\,e\,x+8\,a\,e^2\,x\,{\left(e\,x^2+d\right)}^2+3\,c\,d^2\,x\,{\left(e\,x^2+d\right)}^2+3\,a}{15\,d^3\,e^2\,{\left(e\,x^2+d\right)}^5}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x)
```

```
[Out] (3*c*d^4*x - 6*c*d^3*x*(d + e*x^2) - 3*b*d^3*e*x + 8*a*e^2*x*(d + e*x^2)^2
+ 3*c*d^2*x*(d + e*x^2)^2 + 3*a*d^2*e^2*x + 4*a*d*e^2*x*(d + e*x^2) + 2*b*d
*e*x*(d + e*x^2)^2 + b*d^2*e*x*(d + e*x^2))/(15*d^3*e^2*(d + e*x^2)^(5/2))
```

3.283 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx = \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}} + \frac{\left(3cd^2+4e(bd+6ae)\right)x^5}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{2e\left(3cd^2+4e(bd+6ae)\right)x^7}{105d^4\left(d+ex^2\right)^{7/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(7/2)+1/3*(6*a*e+b*d)*x^3/d^2/(e*x^2+d)^(7/2)+1/15*(3*c*d^2
+4*e*(6*a*e+b*d))*x^5/d^3/(e*x^2+d)^(7/2)+2/105*e*(3*c*d^2+4*e*(6*a*e+b*d))
*x^7/d^4/(e*x^2+d)^(7/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1169, 1817, 12, 277, 270}

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx = \frac{2ex^7\left(4e(6ae+bd)+3cd^2\right)}{105d^4\left(d+ex^2\right)^{7/2}} + \frac{x^5\left(4e(6ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{x^3(6ae+bd)}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(7/2)) + ((b*d + 6*a*e)*x^3)/(3*d^2*(d + e*x^2)^(7/2))
 + ((3*c*d^2 + 4*e*(b*d + 6*a*e))*x^5)/(15*d^3*(d + e*x^2)^(7/2)) + (2*e*(3
*c*d^2 + 4*e*(b*d + 6*a*e))*x^7)/(105*d^4*(d + e*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{\int\frac{x^2\left(6ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{9/2}}\,dx}{d}\\&=\frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{\int\frac{\left(3cd^2+4e(bd+6ae)\right)x^4}{\left(d+ex^2\right)^{9/2}}\,dx}{3d^2}\\&=\frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{1}{3}\left(3c+\frac{4e(bd+6ae)}{d^2}\right)\int\frac{x^4}{\left(d+ex^2\right)^{9/2}}\,dx\end{aligned}$$

$$
\begin{aligned}
&=\frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{\left(3cd^2+4e(bd+6ae)\right)x^5}{15d^3\left(d+ex^2\right)^{7/2}}\\
&\quad+\frac{\left(2e\left(3cd^2+4e(bd+6ae)\right)\right)\int\frac{x^6}{(d+ex^2)^{9/2}}\,dx}{15d^3}\\
&=\frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{\left(3cd^2+4e(bd+6ae)\right)x^5}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{2e\left(3cd^2+4e(bd+6ae)\right)x^7}{105d^4\left(d+ex^2\right)^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$
\int\frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx=\frac{105ad^3x+35bd^3x^3+210ad^2ex^3+21cd^3x^5+28bd^2ex^5+168ade^2x^5+6cd^2ex^7+8b}{105d^4\left(d+ex^2\right)^{7/2}}
$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (105*a*d^3*x + 35*b*d^3*x^3 + 210*a*d^2*e*x^3 + 21*c*d^3*x^5 + 28*b*d^2*e*x
^5 + 168*a*d*e^2*x^5 + 6*c*d^2*e*x^7 + 8*b*d*e^2*x^7 + 48*a*e^3*x^7)/(105*d
^4*(d + e*x^2)^(7/2))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{5}c\,x^4+\frac{1}{3}b\,x^2+a\right)d^3+2e\left(\frac{1}{35}c\,x^4+\frac{2}{15}b\,x^2+a\right)x^2d^2+\frac{8\left(\frac{b\,x^2}{21}+a\right)e^2x^4d}{5}+\frac{16a\,e^3x^6}{35}\right)x}{\left(e\,x^2+d\right)^{\frac{7}{2}}d^4}$
gosper	$\frac{x\left(48a\,e^3x^6+8bd\,e^2x^6+6c\,d^2e\,x^6+168ad\,e^2x^4+28b\,d^2e\,x^4+21c\,d^3x^4+210a\,d^2e\,x^2+35b\,d^3x^2+105d^3a\right)}{105\left(e\,x^2+d\right)^{\frac{7}{2}}d^4}$
trager	$\frac{x\left(48a\,e^3x^6+8bd\,e^2x^6+6c\,d^2e\,x^6+168ad\,e^2x^4+28b\,d^2e\,x^4+21c\,d^3x^4+210a\,d^2e\,x^2+35b\,d^3x^2+105d^3a\right)}{105\left(e\,x^2+d\right)^{\frac{7}{2}}d^4}$
default	$a\left(\frac{x}{7d\left(e\,x^2+d\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35d\left(e\,x^2+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^2+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^2\sqrt{e\,x^2+d}}\right)}{7d}}{d}\right)+c\left(-\frac{x^3}{4e\left(e\,x^2+d\right)^{\frac{7}{2}}}+\frac{3d\left(-\frac{x}{6e\left(e\,x^2+d\right)^{\frac{7}{2}}}+\right.}{}\right.$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1/5*c*x^4+1/3*b*x^2+a)*d^3+2*e*(1/35*c*x^4+2/15*b*x^2+a)*x^2*d^2+8/5*(1/2
1*b*x^2+a)*e^2*x^4*d+16/35*a*e^3*x^6)/(e*x^2+d)^(7/2)*x/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx = \frac{(2\,(3\,cd^2e+4\,bde^2+24\,ae^3)x^7+7\,(3\,cd^3+4\,bd^2e+24\,ade^2)x^5+105\,ad^3x+35\,(bd^3}{105\,(d^4e^4x^8+4\,d^5e^3x^6+6\,d^6e^2x^4+4\,d^7ex^2+d^8)}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3)*x^7 + 7*(3*c*d^3 + 4*b*d^2*e +
24*a*d*e^2)*x^5 + 105*a*d^3*x + 35*(b*d^3 + 6*a*d^2*e)*x^3)*sqrt(e*x^2 + d)
/(d^4*e^4*x^8 + 4*d^5*e^3*x^6 + 6*d^6*e^2*x^4 + 4*d^7*e*x^2 + d^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1989 vs. 2(119) = 238.

Time = 35.55 (sec) , antiderivative size = 1989, normalized size of antiderivative = 15.79

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(9/2),x)
```

```
[Out] a*(35*d**14*x/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(
1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*
e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) +
 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt
(1 + e*x**2/d)) + 175*d**13*e*x**3/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d
**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**
2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8
*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(
25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 371*d**12*e**2*x**5/(35*d**(37/2)*sq
rt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*
e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) +
 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt
(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 429*d**11*e*
*3*x**7/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*
x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x
**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d
**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e
*x**2/d)) + 286*d**10*e**4*x**9/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(
35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d
) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sq
```

```
rt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/
2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 104*d**9*e**5*x**11/(35*d**(37/2)*sqrt(
1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**
2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 52
5*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1
+ e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 16*d**8*e**6*x*
*13/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2
/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*
sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(2
7/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**
2/d))) + b*(35*d**5*x**3/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420*d**(17/2)*
e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 42
0*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x**8*sqrt(1 +
 e*x**2/d)) + 63*d**4*e*x**5/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420*d**(17
/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x**2/d)
+ 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x**8*sqrt
(1 + e*x**2/d)) + 36*d**3*e**2*x**7/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420
*d**(17/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x
**2/d) + 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x*
*8*sqrt(1 + e*x**2/d)) + 8*d**2*e**3*x**9/(105*d**(19/2)*sqrt(1 + e*x**2/d)
 + 420*d**(17/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1
 + e*x**2/d) + 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e
**4*x**8*sqrt(1 + e*x**2/d))) + c*(7*d*x**5/(35*d**(11/2)*sqrt(1 + e*x**2/d
) + 105*d**(9/2)*e*x**2*sqrt(1 + e*x**2/d) + 105*d**(7/2)*e**2*x**4*sqrt(1
+ e*x**2/d) + 35*d**(5/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 2*e*x**7/(35*d**(
11/2)*sqrt(1 + e*x**2/d) + 105*d**(9/2)*e*x**2*sqrt(1 + e*x**2/d) + 105*d**
(7/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 35*d**(5/2)*e**3*x**6*sqrt(1 + e*x**2/
d)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(112) = 224$.

Time = 0.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx &= -\frac{cx^3}{4\,(ex^2+d)^{\frac{7}{2}}e}+\frac{16\,ax}{35\,\sqrt{ex^2+d}d^4}\\ &\quad+\frac{8\,ax}{35\,(ex^2+d)^{\frac{3}{2}}d^3}+\frac{6\,ax}{35\,(ex^2+d)^{\frac{5}{2}}d^2}+\frac{ax}{7\,(ex^2+d)^{\frac{7}{2}}d}+\frac{3\,cx}{140\,(ex^2+d)^{\frac{5}{2}}e^2}\\ &\quad+\frac{2\,cx}{35\,\sqrt{ex^2+d}d^2e^2}+\frac{cx}{35\,(ex^2+d)^{\frac{3}{2}}de^2}-\frac{3\,cdx}{28\,(ex^2+d)^{\frac{7}{2}}e^2}-\frac{bx}{7\,(ex^2+d)^{\frac{7}{2}}e}\\ &\quad+\frac{8\,bx}{105\,\sqrt{ex^2+d}d^3e}+\frac{4\,bx}{105\,(ex^2+d)^{\frac{3}{2}}d^2e}+\frac{bx}{35\,(ex^2+d)^{\frac{5}{2}}de}\end{aligned}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*c*x^3/((e*x^2 + d)^(7/2)*e) + 16/35*a*x/(sqrt(e*x^2 + d)*d^4) + 8/35*a
*x/((e*x^2 + d)^(3/2)*d^3) + 6/35*a*x/((e*x^2 + d)^(5/2)*d^2) + 1/7*a*x/((e
*x^2 + d)^(7/2)*d) + 3/140*c*x/((e*x^2 + d)^(5/2)*e^2) + 2/35*c*x/(sqrt(e*x
^2 + d)*d^2*e^2) + 1/35*c*x/((e*x^2 + d)^(3/2)*d*e^2) - 3/28*c*d*x/((e*x^2
+ d)^(7/2)*e^2) - 1/7*b*x/((e*x^2 + d)^(7/2)*e) + 8/105*b*x/(sqrt(e*x^2 + d
)*d^3*e) + 4/105*b*x/((e*x^2 + d)^(3/2)*d^2*e) + 1/35*b*x/((e*x^2 + d)^(5/2
)*d*e)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{2\left(3\,cd^2e^4+4\,bde^5+24\,ae^6\right)x^2}{d^4e^3}+\frac{7\left(3\,cd^3e^3+4\,bd^2e^4+24\,ade^5\right)}{d^4e^3}\right)+\frac{35\left(bd^3e^3+6\,ad^2e^4\right)}{d^4e^3}\right)x^2+\frac{105\,a}{d}\right)}{105\left(ex^2+d\right)^{\frac{7}{2}}}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*c*d^2*e^4 + 4*b*d*e^5 + 24*a*e^6)*x^2/(d^4*e^3) + 7*(3*c*
d^3*e^3 + 4*b*d^2*e^4 + 24*a*d*e^5)/(d^4*e^3)) + 35*(b*d^3*e^3 + 6*a*d^2*e^
4)/(d^4*e^3))*x^2 + 105*a/d)*x/(e*x^2 + d)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{x\left(\frac{a}{7\,d}-\frac{d\left(\frac{b}{7\,d}-\frac{c}{7\,e}\right)}{e}\right)}{\left(e\,x^2+d\right)^{7/2}}-\frac{x\left(\frac{c}{5\,e^2}-\frac{-c\,d^2+b\,d\,e+6\,a\,e^2}{35\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{5/2}} + \frac{x\left(3\,c\,d^2+4\,b\,d\,e+24\,a\,e^2\right)}{105\,d^3\,e^2\left(e\,x^2+d\right)^{3/2}}+\frac{x\left(6\,c\,d^2+8\,b\,d\,e+48\,a\,e^2\right)}{105\,d^4\,e^2\sqrt{e\,x^2+d}}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x)
```

```
[Out] (x*(a/(7*d) - (d*(b/(7*d) - c/(7*e)))/e))/(d + e*x^2)^(7/2) - (x*(c/(5*e^2)
 - (6*a*e^2 - c*d^2 + b*d*e)/(35*d^2*e^2)))/(d + e*x^2)^(5/2) + (x*(24*a*e^
2 + 3*c*d^2 + 4*b*d*e))/(105*d^3*e^2*(d + e*x^2)^(3/2)) + (x*(48*a*e^2 + 6*
c*d^2 + 8*b*d*e))/(105*d^4*e^2*(d + e*x^2)^(1/2))
```

3.284 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx=\frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{\left(cd^2+2e(bd+8ae)\right)x^5}{5d^3\left(d+ex^2\right)^{9/2}}+\frac{4e\left(cd^2+2e(bd+8ae)\right)x^7}{35d^4\left(d+ex^2\right)^{9/2}}+\frac{8e^2\left(cd^2+2e(bd+8ae)\right)x^9}{315d^5\left(d+ex^2\right)^{9/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(9/2)+1/3*(8*a*e+b*d)*x^3/d^2/(e*x^2+d)^(9/2)+1/5*(c*d^2+2*
e*(8*a*e+b*d))*x^5/d^3/(e*x^2+d)^(9/2)+4/35*e*(c*d^2+2*e*(8*a*e+b*d))*x^7/d
^4/(e*x^2+d)^(9/2)+8/315*e^2*(c*d^2+2*e*(8*a*e+b*d))*x^9/d^5/(e*x^2+d)^(9/2
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1169, 1817, 12, 277, 270}

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx=\frac{x^5\left(\frac{2e(8ae+bd)}{d^2}+c\right)}{5d\left(d+ex^2\right)^{9/2}}+\frac{8e^2x^9\left(2e(8ae+bd)+cd^2\right)}{315d^5\left(d+ex^2\right)^{9/2}}+\frac{4ex^7\left(2e(8ae+bd)+cd^2\right)}{35d^4\left(d+ex^2\right)^{9/2}}+\frac{x^3(8ae+bd)}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(9/2)) + ((b*d + 8*a*e)*x^3)/(3*d^2*(d + e*x^2)^(9/2))
 + ((c + (2*e*(b*d + 8*a*e))/d^2)*x^5)/(5*d*(d + e*x^2)^(9/2)) + (4*e*(c*d^
```

```
2 + 2*e*(b*d + 8*a*e))*x^7)/(35*d^4*(d + e*x^2)^(9/2)) + (8*e^2*(c*d^2 + 2*
e*(b*d + 8*a*e))*x^9)/(315*d^5*(d + e*x^2)^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{ax}{d\left(d+ex^2\right)^{9/2}} + \frac{\int \frac{x^2\left(8ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{11/2}}\,dx}{d}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}} + \frac{\int \frac{\left(3cd^2+6e(bd+8ae)\right)x^4}{\left(d+ex^2\right)^{11/2}}\,dx}{3d^2}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}} + \left(c+\frac{2e(bd+8ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{11/2}}dx$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}} + \frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\left(d+ex^2\right)^{9/2}} + \frac{\left(4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)\right)\int \frac{x^6}{\left(d+ex^2\right)^{11/2}}dx}{5d}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}} + \frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\left(d+ex^2\right)^{9/2}} + \frac{4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^7}{35d^2\left(d+ex^2\right)^{9/2}} + \frac{\left(8e^2\left(c+\frac{2e(bd+8ae)}{d^2}\right)\right)\int \frac{x^8}{\left(d+ex^2\right)^{11/2}}dx}{35d^2}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}} + \frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\left(d+ex^2\right)^{9/2}} + \frac{4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^7}{35d^2\left(d+ex^2\right)^{9/2}} + \frac{8e^2\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^9}{315d^3\left(d+ex^2\right)^{9/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{11/2}}dx = \frac{a(315d^4x+840d^3ex^3+1008d^2e^2x^5+576de^3x^7+128e^4x^9)+dx^3(cdx^2(63d^2+36dex^2}{315d^5\left(d+ex^2\right)^{9/2}}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*(315*d^4*x + 840*d^3*e*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4*
x^9) + d*x^3*(c*d*x^2*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4) + b*(105*d^3 + 126*
d^2*e*x^2 + 72*d*e^2*x^4 + 16*e^3*x^6)))/(315*d^5*(d + e*x^2)^(9/2))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{5} c\,x^{4}+\frac{1}{3} b\,x^{2}+a\right) d^{4}+\frac{8 e \left(\frac{3}{70} c\,x^{4}+\frac{3}{20} b\,x^{2}+a\right) x^{2} d^{3}}{3}+\frac{16 e^{2} \left(\frac{1}{126} c\,x^{4}+\frac{1}{14} b\,x^{2}+a\right) x^{4} d^{2}}{5}+\frac{64 \left(\frac{b\,x^{2}}{36}+a\right) e^{3} x^{6} d}{35}+\frac{128 a\,e^{4} x^{8}}{315}\right) x}{\left(e\,x^{2}+d\right)^{\frac{9}{2}} d^{5}}$
gosper	$\frac{x \left(128 a\,e^{4} x^{8}+16 b d\,e^{3} x^{8}+8 c\,d^{2} e^{2} x^{8}+576 a d\,e^{3} x^{6}+72 b\,d^{2} e^{2} x^{6}+36 c\,d^{3} e\,x^{6}+1008 a\,d^{2} e^{2} x^{4}+126 b\,d^{3} e\,x^{4}+63 c\,d^{4} x^{4}+840 a\,d^{3} e}{315 \left(e\,x^{2}+d\right)^{\frac{9}{2}} d^{5}}$
trager	$\frac{x \left(128 a\,e^{4} x^{8}+16 b d\,e^{3} x^{8}+8 c\,d^{2} e^{2} x^{8}+576 a d\,e^{3} x^{6}+72 b\,d^{2} e^{2} x^{6}+36 c\,d^{3} e\,x^{6}+1008 a\,d^{2} e^{2} x^{4}+126 b\,d^{3} e\,x^{4}+63 c\,d^{4} x^{4}+840 a\,d^{3} e}{315 \left(e\,x^{2}+d\right)^{\frac{9}{2}} d^{5}}$
default	$a \left(\frac{x}{9 d \left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\frac{\frac{8 x}{63 d \left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{8 \left(\frac{6 x}{35 d \left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 d \left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8 x}{15 d^{2} \sqrt{e\,x^{2}+d}}\right)}{7 d}\right)}{9 d}}{d}\right)+c \left(-\frac{x^{3}}{6 e \left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\right.$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1/5*c*x^4+1/3*b*x^2+a)*d^4+8/3*e*(3/70*c*x^4+3/20*b*x^2+a)*x^2*d^3+16/5*e
^2*(1/126*c*x^4+1/14*b*x^2+a)*x^4*d^2+64/35*(1/36*b*x^2+a)*e^3*x^6*d+128/31
5*a*e^4*x^8)/(e*x^2+d)^(9/2)*x/d^5
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{(8\,(cd^2e^2+2\,bde^3+16\,ae^4)x^9+36\,(cd^3e+2\,bd^2e^2+16\,ade^3)x^7+315\,ad^4x+63\,(cd^4}{315\,(d^5e^5x^{10}+5\,d^6e^4x^8+10\,d^7e^3x^6+10\,d^8e^2x^4}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(8*(c*d^2*e^2 + 2*b*d*e^3 + 16*a*e^4)*x^9 + 36*(c*d^3*e + 2*b*d^2*e^2
 + 16*a*d*e^3)*x^7 + 315*a*d^4*x + 63*(c*d^4 + 2*b*d^3*e + 16*a*d^2*e^2)*x^
5 + 105*(b*d^4 + 8*a*d^3*e)*x^3)*sqrt(e*x^2 + d)/(d^5*e^5*x^10 + 5*d^6*e^4*
x^8 + 10*d^7*e^3*x^6 + 10*d^8*e^2*x^4 + 5*d^9*e*x^2 + d^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5187 vs. $2(160) = 320$.

Time = 80.52 (sec) , antiderivative size = 5187, normalized size of antiderivative = 31.44

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(11/2),x)
```

```
[Out] a*(315*d**30*x/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e*x**2*sq
rt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 37800*d**
(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqrt(1 + e*
x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**(59/2)*e
**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 + e*x**2/d
) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*e**9*x**
18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d)) + 273
0*d**29*e*x**3/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e*x**2*sq
rt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 37800*d**
(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqrt(1 + e*
x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**(59/2)*e
**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 + e*x**2/d
) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*e**9*x**
```

```
18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d)) + 107
73*d**28*e**2*x**5/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e*x**
2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 37800
*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqrt(1
+ e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**(59/
2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 + e*x*
*2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*e**9
*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d)) +
 25524*d**27*e**3*x**7/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e
*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 3
7800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqr
t(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**
(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 +
e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*
e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d
)) + 40229*d**26*e**4*x**9/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/
2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d)
 + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8
*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150
*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(
1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53
/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x*
*2/d)) + 44058*d**25*e**5*x**11/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d*
*(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x*
*2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4
*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) +
66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*
sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d
**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 +
 e*x**2/d)) + 33915*d**24*e**6*x**13/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 31
50*d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 +
 e*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)
*e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/
d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x
**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3
150*d**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqr
t(1 + e*x**2/d)) + 18088*d**23*e**7*x**15/(315*d**(71/2)*sqrt(1 + e*x**2/d)
 + 3150*d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqr
t(1 + e*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(
63/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*
x**2/d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e
**7*x**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d
) + 3150*d**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**2
0*sqrt(1 + e*x**2/d)) + 6384*d**22*e**8*x**17/(315*d**(71/2)*sqrt(1 + e*x**
2/d) + 3150*d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4
```

```
*sqrt(1 + e*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*
d**(63/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1
+ e*x**2/d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/
2)*e**7*x**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x*
*2/d) + 3150*d**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*
x**20*sqrt(1 + e*x**2/d)) + 1344*d**21*e**9*x**19/(315*d**(71/2)*sqrt(1 + e
*x**2/d) + 3150*d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*
x**4*sqrt(1 + e*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66
150*d**(63/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqr
t(1 + e*x**2/d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**
(57/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 +
e*x**2/d) + 3150*d**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e*
*10*x**20*sqrt(1 + e*x**2/d)) + 128*d**20*e**10*x**21/(315*d**(71/2)*sqrt(1
 + e*x**2/d) + 3150*d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e
**2*x**4*sqrt(1 + e*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d)
+ 66150*d**(63/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10
*sqrt(1 + e*x**2/d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800
*d**(57/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(
1 + e*x**2/d) + 3150*d**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2
)*e**10*x**20*sqrt(1 + e*x**2/d))) + b*(105*d**14*x**3/(315*d**(39/2)*sqrt(
1 + e*x**2/d) + 2205*d**(37/2)*e*x**2*sqrt(1 + e*x**2/d) + 6615*d**(35/2)*e
**2*x**4*sqrt(1 + e*x**2/d) + 11025*d**(33/2)*e**3*x**6*sqrt(1 + e*x**2/d)
+ 11025*d**(31/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 6615*d**(29/2)*e**5*x**10*
sqrt(1 + e*x**2/d) + 2205*d**(27/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 315*d**
(25/2)*e**7*x**14*sqrt(1 + e*x**2/d)) + 441*d**13*e*x**5/(315*d**(39/2)*sqr
t(1 + e*x**2/d) + 2205*d**(37/2)*e*x**2*sqrt(1 + e*x**2/d) + 6615*d**(35/2)
*e**2*x**4*sqrt(1 + e*x**2/d) + 11025*d**(33/2)*e**3*x**6*sqrt(1 + e*x**2/d
) + 11025*d**(31/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 6615*d**(29/2)*e**5*x**1
0*sqrt(1 + e*x**2/d) + 2205*d**(27/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 315*d
**(25/2)*e**7*x**14*sqrt(1 + e*x**2/d)) + 765*d**12*e**2*x**7/(315*d**(39/2
)*sqrt(1 + e*x**2/d) + 2205*d**(37/2)*e*x**2*sqrt(1 + e*x**2/d) + 6615*d**(
35/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 11025*d**(33/2)*e**3*x**6*sqrt(1 + e*x
**2/d) + 11025*d**(31/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 6615*d**(29/2)*e**5
*x**10*sqrt(1 + e*x**2/d) + 2205*d**(27/2)*e**6*x**12*sqrt(1 + e*x**2/d) +
315*d**(25/2)*e**7*x**14*sqrt(1 + e*x**2/d)) + 715*d**11*e**3*x**9/(315*d**
(39/2)*sqrt(1 + e*x**2/d) + 2205*d**(37/2)*e*x**2*sqrt(1 + e*x**2/d) + 6615
*d**(35/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 11025*d**(33/2)*e**3*x**6*sqrt(1
+ e*x**2/d) + 11025*d**(31/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 6615*d**(29/2)
*e**5*x**10*sqrt(1 + e*x**2/d) + 2205*d**(27/2)*e**6*x**12*sqrt(1 + e*x**2/
d) + 315*d**(25/2)*e**7*x**14*sqrt(1 + e*x**2/d)) + 390*d**10*e**4*x**11/(3
15*d**(39/2)*sqrt(1 + e*x**2/d) + 2205*d**(37/2)*e*x**2*sqrt(1 + e*x**2/d)
+ 6615*d**(35/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 11025*d**(33/2)*e**3*x**6*s
qrt(1 + e*x**2/d) + 11025*d**(31/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 6615*d**
(29/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 2205*d**(27/2)*e**6*x**12*sqrt(1 + e
*x**2/d) + 315*d**(25/2)*e**7*x**14*sqrt(1 + e*x**2/d)) + 120*d**9*e**5*x**
```

```
13/(315*d**(39/2)*sqrt(1 + e*x**2/d) + 2205*d**(37/2)*e*x**2*sqrt(1 + e*x**
2/d) + 6615*d**(35/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 11025*d**(33/2)*e**3*x
**6*sqrt(1 + e*x**2/d) + 11025*d**(31/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 661
5*d**(29/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 2205*d**(27/2)*e**6*x**12*sqrt(
1 + e*x**2/d) + 315*d**(25/2)*e**7*x**14*sqrt(1 + e*x**2/d)) + 16*d**8*e**6
*x**15/(315*d**(39/2)*sqrt(1 + e*x**2/d) + 2205*d**(37/2)*e*x**2*sqrt(1 + e
*x**2/d) + 6615*d**(35/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 11025*d**(33/2)*e*
*3*x**6*sqrt(1 + e*x**2/d) + 11025*d**(31/2)*e**4*x**8*sqrt(1 + e*x**2/d) +
 6615*d**(29/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 2205*d**(27/2)*e**6*x**12*s
qrt(1 + e*x**2/d) + 315*d**(25/2)*e**7*x**14*sqrt(1 + e*x**2/d))) + c*(63*d
**5*x**5/(315*d**(21/2)*sqrt(1 + e*x**2/d) + 1575*d**(19/2)*e*x**2*sqrt(1 +
 e*x**2/d) + 3150*d**(17/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 3150*d**(15/2)*e
**3*x**6*sqrt(1 + e*x**2/d) + 1575*d**(13/2)*e**4*x**8*sqrt(1 + e*x**2/d) +
 315*d**(11/2)*e**5*x**10*sqrt(1 + e*x**2/d)) + 99*d**4*e*x**7/(315*d**(21/
2)*sqrt(1 + e*x**2/d) + 1575*d**(19/2)*e*x**2*sqrt(1 + e*x**2/d) + 3150*d**
(17/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 3150*d**(15/2)*e**3*x**6*sqrt(1 + e*x
**2/d) + 1575*d**(13/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 315*d**(11/2)*e**5*x
**10*sqrt(1 + e*x**2/d)) + 44*d**3*e**2*x**9/(315*d**(21/2)*sqrt(1 + e*x**2
/d) + 1575*d**(19/2)*e*x**2*sqrt(1 + e*x**2/d) + 3150*d**(17/2)*e**2*x**4*s
qrt(1 + e*x**2/d) + 3150*d**(15/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 1575*d**(
13/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 315*d**(11/2)*e**5*x**10*sqrt(1 + e*x*
*2/d)) + 8*d**2*e**3*x**11/(315*d**(21/2)*sqrt(1 + e*x**2/d) + 1575*d**(19/
2)*e*x**2*sqrt(1 + e*x**2/d) + 3150*d**(17/2)*e**2*x**4*sqrt(1 + e*x**2/d)
+ 3150*d**(15/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 1575*d**(13/2)*e**4*x**8*sq
rt(1 + e*x**2/d) + 315*d**(11/2)*e**5*x**10*sqrt(1 + e*x**2/d)))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.70

$$
\begin{aligned}
\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = & -\frac{cx^3}{6\,(ex^2+d)^{\frac{9}{2}}e}+\frac{128\,ax}{315\,\sqrt{ex^2+d}d^5} \\
& +\frac{64\,ax}{315\,(ex^2+d)^{\frac{3}{2}}d^4}+\frac{16\,ax}{105\,(ex^2+d)^{\frac{5}{2}}d^3}+\frac{8\,ax}{63\,(ex^2+d)^{\frac{7}{2}}d^2}+\frac{ax}{9\,(ex^2+d)^{\frac{9}{2}}d} \\
& +\frac{cx}{126\,(ex^2+d)^{\frac{7}{2}}e^2}+\frac{8\,cx}{315\,\sqrt{ex^2+d}d^3e^2}+\frac{4\,cx}{315\,(ex^2+d)^{\frac{3}{2}}d^2e^2} \\
& +\frac{cx}{105\,(ex^2+d)^{\frac{5}{2}}de^2}-\frac{cdx}{18\,(ex^2+d)^{\frac{9}{2}}e^2}-\frac{bx}{9\,(ex^2+d)^{\frac{9}{2}}e}+\frac{16\,bx}{315\,\sqrt{ex^2+d}d^4e} \\
& +\frac{8\,bx}{315\,(ex^2+d)^{\frac{3}{2}}d^3e}+\frac{2\,bx}{105\,(ex^2+d)^{\frac{5}{2}}d^2e}+\frac{bx}{63\,(ex^2+d)^{\frac{7}{2}}de}
\end{aligned}
$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/6*c*x^3/((e*x^2 + d)^(9/2)*e) + 128/315*a*x/(sqrt(e*x^2 + d)*d^5) + 64/3
15*a*x/((e*x^2 + d)^(3/2)*d^4) + 16/105*a*x/((e*x^2 + d)^(5/2)*d^3) + 8/63*
a*x/((e*x^2 + d)^(7/2)*d^2) + 1/9*a*x/((e*x^2 + d)^(9/2)*d) + 1/126*c*x/((e
*x^2 + d)^(7/2)*e^2) + 8/315*c*x/(sqrt(e*x^2 + d)*d^3*e^2) + 4/315*c*x/((e*
x^2 + d)^(3/2)*d^2*e^2) + 1/105*c*x/((e*x^2 + d)^(5/2)*d*e^2) - 1/18*c*d*x/
((e*x^2 + d)^(9/2)*e^2) - 1/9*b*x/((e*x^2 + d)^(9/2)*e) + 16/315*b*x/(sqrt(
e*x^2 + d)*d^4*e) + 8/315*b*x/((e*x^2 + d)^(3/2)*d^3*e) + 2/105*b*x/((e*x^2
 + d)^(5/2)*d^2*e) + 1/63*b*x/((e*x^2 + d)^(7/2)*d*e)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{\left(\left(\left(4\,x^2\left(\frac{2\,(cd^2e^6+2\,bde^7+16\,ae^8)x^2}{d^5e^4}+\frac{9\,(cd^3e^5+2\,bd^2e^6+16\,ade^7)}{d^5e^4}\right)+\frac{63\,(cd^4e^4+2\,bd^3e^5+16\,ad^2e^6)}{d^5e^4}\right)x^2+}{315\,(ex^2+d)^{\frac{9}{2}}}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((4*x^2*(2*(c*d^2*e^6 + 2*b*d*e^7 + 16*a*e^8)*x^2/(d^5*e^4) + 9*(c*d
^3*e^5 + 2*b*d^2*e^6 + 16*a*d*e^7)/(d^5*e^4)) + 63*(c*d^4*e^4 + 2*b*d^3*e^5
 + 16*a*d^2*e^6)/(d^5*e^4))*x^2 + 105*(b*d^4*e^4 + 8*a*d^3*e^5)/(d^5*e^4))*
x^2 + 315*a/d)*x/(e*x^2 + d)^(9/2)
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx &= \frac{x\left(\frac{a}{9\,d}-\frac{d\left(\frac{b}{9\,d}-\frac{c}{9\,e}\right)}{e}\right)}{(e\,x^2+d)^{9/2}} \\ &\quad - \frac{x\left(\frac{c}{7\,e^2}-\frac{-c\,d^2+b\,d\,e+8\,a\,e^2}{63\,d^2\,e^2}\right)}{(e\,x^2+d)^{7/2}}+\frac{x\,(c\,d^2+2\,b\,d\,e+16\,a\,e^2)}{105\,d^3\,e^2\,(e\,x^2+d)^{5/2}} \\ &\quad + \frac{x\,(4\,c\,d^2+8\,b\,d\,e+64\,a\,e^2)}{315\,d^4\,e^2\,(e\,x^2+d)^{3/2}}+\frac{x\,(8\,c\,d^2+16\,b\,d\,e+128\,a\,e^2)}{315\,d^5\,e^2\,\sqrt{e\,x^2+d}}\end{aligned}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x)
```

```
[Out] (x*(a/(9*d) - (d*(b/(9*d) - c/(9*e)))/e))/(d + e*x^2)^(9/2) - (x*(c/(7*e^2)
 - (8*a*e^2 - c*d^2 + b*d*e)/(63*d^2*e^2)))/(d + e*x^2)^(7/2) + (x*(16*a*e^
2 + c*d^2 + 2*b*d*e))/(105*d^3*e^2*(d + e*x^2)^(5/2)) + (x*(64*a*e^2 + 4*c*
d^2 + 8*b*d*e))/(315*d^4*e^2*(d + e*x^2)^(3/2)) + (x*(128*a*e^2 + 8*c*d^2 +
 16*b*d*e))/(315*d^5*e^2*(d + e*x^2)^(1/2))
```

3.285 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{13/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 210

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{13/2}}\,dx &= \frac{ax}{d\left(d+ex^2\right)^{11/2}}+\frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}}\\ &+\frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}}+\frac{2e\left(3cd^2+8e(bd+10ae)\right)x^7}{35d^4\left(d+ex^2\right)^{11/2}}\\ &+\frac{8e^2\left(3cd^2+8e(bd+10ae)\right)x^9}{315d^5\left(d+ex^2\right)^{11/2}}+\frac{16e^3\left(3cd^2+8e(bd+10ae)\right)x^{11}}{3465d^6\left(d+ex^2\right)^{11/2}}\end{aligned}$$

```
[Out] a*x/d/(e*x^2+d)^(11/2)+1/3*(10*a*e+b*d)*x^3/d^2/(e*x^2+d)^(11/2)+1/15*(3*c*
d^2+8*e*(10*a*e+b*d))*x^5/d^3/(e*x^2+d)^(11/2)+2/35*e*(3*c*d^2+8*e*(10*a*e+
b*d))*x^7/d^4/(e*x^2+d)^(11/2)+8/315*e^2*(3*c*d^2+8*e*(10*a*e+b*d))*x^9/d^5
/(e*x^2+d)^(11/2)+16/3465*e^3*(3*c*d^2+8*e*(10*a*e+b*d))*x^11/d^6/(e*x^2+d)
^(11/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1169, 1817, 12, 277, 270}

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{13/2}}\,dx &= \frac{16e^3x^{11}\left(8e(10ae+bd)+3cd^2\right)}{3465d^6\left(d+ex^2\right)^{11/2}}\\ &+\frac{8e^2x^9\left(8e(10ae+bd)+3cd^2\right)}{315d^5\left(d+ex^2\right)^{11/2}}+\frac{2ex^7\left(8e(10ae+bd)+3cd^2\right)}{35d^4\left(d+ex^2\right)^{11/2}}\\ &+\frac{x^5\left(8e(10ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{11/2}}+\frac{x^3(10ae+bd)}{3d^2\left(d+ex^2\right)^{11/2}}+\frac{ax}{d\left(d+ex^2\right)^{11/2}}\end{aligned}$$

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(11/2)) + ((b*d + 10*a*e)*x^3)/(3*d^2*(d + e*x^2)^(11/
2)) + ((3*c*d^2 + 8*e*(b*d + 10*a*e))*x^5)/(15*d^3*(d + e*x^2)^(11/2)) + (2
*e*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^7)/(35*d^4*(d + e*x^2)^(11/2)) + (8*e^2
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^9)/(315*d^5*(d + e*x^2)^(11/2)) + (16*e^3
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^11)/(3465*d^6*(d + e*x^2)^(11/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{\int \frac{x^2\left(10ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{13/2}}\,dx}{d}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\int \frac{(3cd^2+8e(bd+10ae))x^4}{(d+ex^2)^{13/2}}\,dx}{3d^2}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{1}{3}\left(3c+\frac{8e(bd+10ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{13/2}}\,dx$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}} + \frac{\left(2e(3cd^2+8e(bd+10ae))\right)\int \frac{x^6}{(d+ex^2)^{13/2}}\,dx}{5d^3}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}} + \frac{2e\left(3cd^2+8e(bd+10ae)\right)x^7}{35d^4\left(d+ex^2\right)^{11/2}} + \frac{\left(8e^2(3cd^2+8e(bd+10ae))\right)\int \frac{x^8}{(d+ex^2)^{13/2}}\,dx}{35d^4}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}} + \frac{2e\left(3cd^2+8e(bd+10ae)\right)x^7}{35d^4\left(d+ex^2\right)^{11/2}} + \frac{8e^2\left(3cd^2+8e(bd+10ae)\right)x^9}{315d^5\left(d+ex^2\right)^{11/2}} + \frac{\left(16e^3(3cd^2+8e(bd+10ae))\right)\int \frac{x^{10}}{(d+ex^2)^{13/2}}\,dx}{315d^5}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}} + \frac{2e\left(3cd^2+8e(bd+10ae)\right)x^7}{35d^4\left(d+ex^2\right)^{11/2}} + \frac{8e^2\left(3cd^2+8e(bd+10ae)\right)x^9}{315d^5\left(d+ex^2\right)^{11/2}} + \frac{16e^3\left(3cd^2+8e(bd+10ae)\right)x^{11}}{3465d^6\left(d+ex^2\right)^{11/2}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.80

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{13/2}}\,dx = \frac{5a\left(693d^5x+2310d^4ex^3+3696d^3e^2x^5+3168d^2e^3x^7+1408de^4x^9+256e^5x^{11}\right)+dx^3(}{}$$

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (5*a*(693*d^5*x + 2310*d^4*e*x^3 + 3696*d^3*e^2*x^5 + 3168*d^2*e^3*x^7 + 14
08*d*e^4*x^9 + 256*e^5*x^11) + d*x^3*(3*c*d*x^2*(231*d^3 + 198*d^2*e*x^2 +
88*d*e^2*x^4 + 16*e^3*x^6) + b*(1155*d^4 + 1848*d^3*e*x^2 + 1584*d^2*e^2*x^
4 + 704*d*e^3*x^6 + 128*e^4*x^8)))/(3465*d^6*(d + e*x^2)^(11/2))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{5}c\,x^4+\frac{1}{3}b\,x^2+a\right)d^5+\frac{10e\left(\frac{9}{175}c\,x^4+\frac{4}{25}b\,x^2+a\right)x^2d^4}{3}+\frac{16e^2\left(\frac{1}{70}c\,x^4+\frac{3}{35}b\,x^2+a\right)x^4d^3}{3}+\frac{32e^3x^6\left(\frac{1}{330}c\,x^4+\frac{2}{45}b\,x^2+a\right)d^2}{7}+ 128e^4x}{\left(e\,x^2+d\right)^{\frac{11}{2}}d^6}$
gosper	$\frac{x\left(1280a\,e^5x^{10}+128bd\,e^4x^{10}+48c\,d^2e^3x^{10}+7040ad\,e^4x^8+704b\,d^2e^3x^8+264c\,d^3e^2x^8+15840a\,d^2e^3x^6+1584b\,d^3e^2x^6+594c\,d\right.}{3465\left(e\,x^2+d\right)^{\frac{11}{2}}d^6}$
trager	$\frac{x\left(1280a\,e^5x^{10}+128bd\,e^4x^{10}+48c\,d^2e^3x^{10}+7040ad\,e^4x^8+704b\,d^2e^3x^8+264c\,d^3e^2x^8+15840a\,d^2e^3x^6+1584b\,d^3e^2x^6+594c\,d\right.}{3465\left(e\,x^2+d\right)^{\frac{11}{2}}d^6}$
default	$a\left(\frac{x}{11d\left(e\,x^2+d\right)^{\frac{11}{2}}}+\frac{\frac{10x}{99d\left(e\,x^2+d\right)^{\frac{9}{2}}}+\frac{10\left(\frac{8x}{63d\left(e\,x^2+d\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d\left(e\,x^2+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^2+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^2\sqrt{e\,x^2+d}}\right)}{7d}\right)}{9d}\right)}{11d}}{d}\right)+$

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1/5*c*x^4+1/3*b*x^2+a)*d^5+10/3*e*(9/175*c*x^4+4/25*b*x^2+a)*x^2*d^4+16/3
*e^2*(1/70*c*x^4+3/35*b*x^2+a)*x^4*d^3+32/7*e^3*x^6*(1/330*c*x^4+2/45*b*x^2
+a)*d^2+128/63*e^4*x^8*(1/55*b*x^2+a)*d+256/693*a*e^5*x^10)/(e*x^2+d)^(11/2
)*x/d^6
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.07

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = \frac{(16\,(3\,cd^2e^3+8\,bde^4+80\,ae^5)x^{11}+88\,(3\,cd^3e^2+8\,bd^2e^3+80\,ade^4)x^9+198\,(3\,cd^4e+}{3465\,(d^6e^6x^{12}+6\,d^7e^5x^{10}+15}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(16*(3*c*d^2*e^3 + 8*b*d*e^4 + 80*a*e^5)*x^11 + 88*(3*c*d^3*e^2 + 8*
b*d^2*e^3 + 80*a*d*e^4)*x^9 + 198*(3*c*d^4*e + 8*b*d^3*e^2 + 80*a*d^2*e^3)*
x^7 + 3465*a*d^5*x + 231*(3*c*d^5 + 8*b*d^4*e + 80*a*d^3*e^2)*x^5 + 1155*(b
*d^5 + 10*a*d^4*e)*x^3)*sqrt(e*x^2 + d)/(d^6*e^6*x^12 + 6*d^7*e^5*x^10 + 15
*d^8*e^4*x^8 + 20*d^9*e^3*x^6 + 15*d^10*e^2*x^4 + 6*d^11*e*x^2 + d^12)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11602 vs. $2(206) = 412$.

Time = 154.15 (sec) , antiderivative size = 11602, normalized size of antiderivative = 55.25

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(13/2),x)
```

```
[Out] a*(693*d**55*x/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**2
*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 31531
5*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*sqr
t(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 346846
5*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x**14*
sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 346
8465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)*e**10*x*
*20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1 + e*x**2/d) +
 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)*e**13*x*
*26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x**2/d) + 6
93*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d)) + 9240*d**54*e*x**3/(693*d**(1
```

```
23/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e*x**2/d) + 727
65*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 315315*d**(117/2)*e**3*x**6*sq
rt(1 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 2081079
*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 3468465*d**(111/2)*e**6*x**12*s
qrt(1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 4459
455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3468465*d**(105/2)*e**9*x**1
8*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)*e**10*x**20*sqrt(1 + e*x**2/d) +
945945*d**(101/2)*e**11*x**22*sqrt(1 + e*x**2/d) + 315315*d**(99/2)*e**12*x
**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)*e**13*x**26*sqrt(1 + e*x**2/d) +
10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x**2/d) + 693*d**(93/2)*e**15*x**30*
sqrt(1 + e*x**2/d)) + 57981*d**53*e**2*x**5/(693*d**(123/2)*sqrt(1 + e*x**2
/d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*x*
*4*sqrt(1 + e*x**2/d) + 315315*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 94
5945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x**1
0*sqrt(1 + e*x**2/d) + 3468465*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4
459455*d**(109/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8*x
**16*sqrt(1 + e*x**2/d) + 3468465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d)
 + 2081079*d**(103/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e**
11*x**22*sqrt(1 + e*x**2/d) + 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/
d) + 72765*d**(97/2)*e**13*x**26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**14
*x**28*sqrt(1 + e*x**2/d) + 693*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d)) +
 227238*d**52*e**3*x**7/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/
2)*e*x**2*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d
) + 315315*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*e**4
*x**8*sqrt(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d)
 + 3468465*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e*
*7*x**14*sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2
/d) + 3468465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)
*e**10*x**20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1 + e*
x**2/d) + 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)
*e**13*x**26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x*
*2/d) + 693*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d)) + 622138*d**51*e**4*x
**9/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e
*x**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 315315*d**(117/2
)*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**
2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 3468465*d**(111/2
)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x**14*sqrt(1 + e*
x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3468465*d**(10
5/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)*e**10*x**20*sqrt(1
+ e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1 + e*x**2/d) + 315315*d**
(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)*e**13*x**26*sqrt(1
+ e*x**2/d) + 10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x**2/d) + 693*d**(93/2
)*e**15*x**30*sqrt(1 + e*x**2/d)) + 1260152*d**50*e**5*x**11/(693*d**(123/2
)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e*x**2/d) + 72765*d
**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 315315*d**(117/2)*e**3*x**6*sqrt(1
```

```
 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 2081079*d**
(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 3468465*d**(111/2)*e**6*x**12*sqrt(
1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 4459455*
d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3468465*d**(105/2)*e**9*x**18*sq
rt(1 + e*x**2/d) + 2081079*d**(103/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 9459
45*d**(101/2)*e**11*x**22*sqrt(1 + e*x**2/d) + 315315*d**(99/2)*e**12*x**24
*sqrt(1 + e*x**2/d) + 72765*d**(97/2)*e**13*x**26*sqrt(1 + e*x**2/d) + 1039
5*d**(95/2)*e**14*x**28*sqrt(1 + e*x**2/d) + 693*d**(93/2)*e**15*x**30*sqrt
(1 + e*x**2/d)) + 1949890*d**49*e**6*x**13/(693*d**(123/2)*sqrt(1 + e*x**2/
d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*x**
4*sqrt(1 + e*x**2/d) + 315315*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 945
945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x**10
*sqrt(1 + e*x**2/d) + 3468465*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 44
59455*d**(109/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8*x*
*16*sqrt(1 + e*x**2/d) + 3468465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) +
 2081079*d**(103/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e**1
1*x**22*sqrt(1 + e*x**2/d) + 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d
) + 72765*d**(97/2)*e**13*x**26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**14*
x**28*sqrt(1 + e*x**2/d) + 693*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d)) +
2345412*d**48*e**7*x**15/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121
/2)*e*x**2*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/
d) + 315315*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*e**
4*x**8*sqrt(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d
) + 3468465*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e
**7*x**14*sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**
2/d) + 3468465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2
)*e**10*x**20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1 + e
*x**2/d) + 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2
)*e**13*x**26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x
**2/d) + 693*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d)) + 2209281*d**47*e**8
*x**17/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**2*sqrt(1
+ e*x**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 315315*d**(11
7/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*sqrt(1 + e*
x**2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 3468465*d**(11
1/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x**14*sqrt(1 +
 e*x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3468465*d**
(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)*e**10*x**20*sqrt
(1 + e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1 + e*x**2/d) + 315315*
d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)*e**13*x**26*sqrt
(1 + e*x**2/d) + 10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x**2/d) + 693*d**(9
3/2)*e**15*x**30*sqrt(1 + e*x**2/d)) + 1628256*d**46*e**9*x**19/(693*d**(12
3/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e*x**2/d) + 7276
5*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 315315*d**(117/2)*e**3*x**6*sqr
t(1 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 2081079*
d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 3468465*d**(111/2)*e**6*x**12*sq
```

```
rt(1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 44594
55*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3468465*d**(105/2)*e**9*x**18
*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 9
45945*d**(101/2)*e**11*x**22*sqrt(1 + e*x**2/d) + 315315*d**(99/2)*e**12*x*
*24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)*e**13*x**26*sqrt(1 + e*x**2/d) + 1
0395*d**(95/2)*e**14*x**28*sqrt(1 + e*x**2/d) + 693*d**(93/2)*e**15*x**30*s
qrt(1 + e*x**2/d)) + 930465*d**45*e**10*x**21/(693*d**(123/2)*sqrt(1 + e*x*
*2/d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*
x**4*sqrt(1 + e*x**2/d) + 315315*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) +
945945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x*
*10*sqrt(1 + e*x**2/d) + 3468465*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) +
 4459455*d**(109/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8
*x**16*sqrt(1 + e*x**2/d) + 3468465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d
) + 2081079*d**(103/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e
**11*x**22*sqrt(1 + e*x**2/d) + 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**
2/d) + 72765*d**(97/2)*e**13*x**26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**
14*x**28*sqrt(1 + e*x**2/d) + 693*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d))
 + 404550*d**44*e**11*x**23/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(
121/2)*e*x**2*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x*
*2/d) + 315315*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*
e**4*x**8*sqrt(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**
2/d) + 3468465*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2
)*e**7*x**14*sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*
x**2/d) + 3468465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(10
3/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1
 + e*x**2/d) + 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(9
7/2)*e**13*x**26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**14*x**28*sqrt(1 +
e*x**2/d) + 693*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d)) + 129456*d**43*e*
*12*x**25/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**2*sqrt
(1 + e*x**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 315315*d**
(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*sqrt(1 +
 e*x**2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 3468465*d**
(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x**14*sqrt(
1 + e*x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3468465*
d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)*e**10*x**20*s
qrt(1 + e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1 + e*x**2/d) + 3153
15*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)*e**13*x**26*s
qrt(1 + e*x**2/d) + 10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x**2/d) + 693*d*
*(93/2)*e**15*x**30*sqrt(1 + e*x**2/d)) + 28768*d**42*e**13*x**27/(693*d**(
123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e*x**2/d) + 72
765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 315315*d**(117/2)*e**3*x**6*s
qrt(1 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 208107
9*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 3468465*d**(111/2)*e**6*x**12*
sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 445
9455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3468465*d**(105/2)*e**9*x**
```

```
18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)*e**10*x**20*sqrt(1 + e*x**2/d) +
 945945*d**(101/2)*e**11*x**22*sqrt(1 + e*x**2/d) + 315315*d**(99/2)*e**12*
x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)*e**13*x**26*sqrt(1 + e*x**2/d) +
 10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x**2/d) + 693*d**(93/2)*e**15*x**30
*sqrt(1 + e*x**2/d)) + 3968*d**41*e**14*x**29/(693*d**(123/2)*sqrt(1 + e*x*
*2/d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*
x**4*sqrt(1 + e*x**2/d) + 315315*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) +
945945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x*
*10*sqrt(1 + e*x**2/d) + 3468465*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) +
 4459455*d**(109/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8
*x**16*sqrt(1 + e*x**2/d) + 3468465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d
) + 2081079*d**(103/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e
**11*x**22*sqrt(1 + e*x**2/d) + 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**
2/d) + 72765*d**(97/2)*e**13*x**26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**
14*x**28*sqrt(1 + e*x**2/d) + 693*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d))
 + 256*d**40*e**15*x**31/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121
/2)*e*x**2*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/
d) + 315315*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*e**
4*x**8*sqrt(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d
) + 3468465*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e
**7*x**14*sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**
2/d) + 3468465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2
)*e**10*x**20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1 + e
*x**2/d) + 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2
)*e**13*x**26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x
**2/d) + 693*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d))) + b*(1155*d**30*x**
3/(3465*d**(73/2)*sqrt(1 + e*x**2/d) + 38115*d**(71/2)*e*x**2*sqrt(1 + e*x*
*2/d) + 190575*d**(69/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 571725*d**(67/2)*e*
*3*x**6*sqrt(1 + e*x**2/d) + 1143450*d**(65/2)*e**4*x**8*sqrt(1 + e*x**2/d)
 + 1600830*d**(63/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 1600830*d**(61/2)*e**6
*x**12*sqrt(1 + e*x**2/d) + 1143450*d**(59/2)*e**7*x**14*sqrt(1 + e*x**2/d)
 + 571725*d**(57/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 190575*d**(55/2)*e**9*x
**18*sqrt(1 + e*x**2/d) + 38115*d**(53/2)*e**10*x**20*sqrt(1 + e*x**2/d) +
3465*d**(51/2)*e**11*x**22*sqrt(1 + e*x**2/d)) + 8778*d**29*e*x**5/(3465*d*
*(73/2)*sqrt(1 + e*x**2/d) + 38115*d**(71/2)*e*x**2*sqrt(1 + e*x**2/d) + 19
0575*d**(69/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 571725*d**(67/2)*e**3*x**6*sq
rt(1 + e*x**2/d) + 1143450*d**(65/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 1600830
*d**(63/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 1600830*d**(61/2)*e**6*x**12*sqr
t(1 + e*x**2/d) + 1143450*d**(59/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 571725*
d**(57/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 190575*d**(55/2)*e**9*x**18*sqrt(
1 + e*x**2/d) + 38115*d**(53/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 3465*d**(5
1/2)*e**11*x**22*sqrt(1 + e*x**2/d)) + 29997*d**28*e**2*x**7/(3465*d**(73/2
)*sqrt(1 + e*x**2/d) + 38115*d**(71/2)*e*x**2*sqrt(1 + e*x**2/d) + 190575*d
**(69/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 571725*d**(67/2)*e**3*x**6*sqrt(1 +
 e*x**2/d) + 1143450*d**(65/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 1600830*d**(6
```

```
3/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 1600830*d**(61/2)*e**6*x**12*sqrt(1 +
e*x**2/d) + 1143450*d**(59/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 571725*d**(57
/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 190575*d**(55/2)*e**9*x**18*sqrt(1 + e*
x**2/d) + 38115*d**(53/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 3465*d**(51/2)*e
**11*x**22*sqrt(1 + e*x**2/d)) + 61028*d**27*e**3*x**9/(3465*d**(73/2)*sqrt
(1 + e*x**2/d) + 38115*d**(71/2)*e*x**2*sqrt(1 + e*x**2/d) + 190575*d**(69/
2)*e**2*x**4*sqrt(1 + e*x**2/d) + 571725*d**(67/2)*e**3*x**6*sqrt(1 + e*x**
2/d) + 1143450*d**(65/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 1600830*d**(63/2)*e
**5*x**10*sqrt(1 + e*x**2/d) + 1600830*d**(61/2)*e**6*x**12*sqrt(1 + e*x**2
/d) + 1143450*d**(59/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 571725*d**(57/2)*e*
*8*x**16*sqrt(1 + e*x**2/d) + 190575*d**(55/2)*e**9*x**18*sqrt(1 + e*x**2/d
) + 38115*d**(53/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 3465*d**(51/2)*e**11*x
**22*sqrt(1 + e*x**2/d)) + 82397*d**26*e**4*x**11/(3465*d**(73/2)*sqrt(1 +
e*x**2/d) + 38115*d**(71/2)*e*x**2*sqrt(1 + e*x**2/d) + 190575*d**(69/2)*e*
*2*x**4*sqrt(1 + e*x**2/d) + 571725*d**(67/2)*e**3*x**6*sqrt(1 + e*x**2/d)
+ 1143450*d**(65/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 1600830*d**(63/2)*e**5*x
**10*sqrt(1 + e*x**2/d) + 1600830*d**(61/2)*e**6*x**12*sqrt(1 + e*x**2/d) +
 1143450*d**(59/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 571725*d**(57/2)*e**8*x*
*16*sqrt(1 + e*x**2/d) + 190575*d**(55/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 3
8115*d**(53/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 3465*d**(51/2)*e**11*x**22*
sqrt(1 + e*x**2/d)) + 77658*d**25*e**5*x**13/(3465*d**(73/2)*sqrt(1 + e*x**
2/d) + 38115*d**(71/2)*e*x**2*sqrt(1 + e*x**2/d) + 190575*d**(69/2)*e**2*x*
*4*sqrt(1 + e*x**2/d) + 571725*d**(67/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 114
3450*d**(65/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 1600830*d**(63/2)*e**5*x**10*
sqrt(1 + e*x**2/d) + 1600830*d**(61/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 1143
450*d**(59/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 571725*d**(57/2)*e**8*x**16*s
qrt(1 + e*x**2/d) + 190575*d**(55/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 38115*
d**(53/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 3465*d**(51/2)*e**11*x**22*sqrt(
1 + e*x**2/d)) + 52003*d**24*e**6*x**15/(3465*d**(73/2)*sqrt(1 + e*x**2/d)
+ 38115*d**(71/2)*e*x**2*sqrt(1 + e*x**2/d) + 190575*d**(69/2)*e**2*x**4*sq
rt(1 + e*x**2/d) + 571725*d**(67/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 1143450*
d**(65/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 1600830*d**(63/2)*e**5*x**10*sqrt(
1 + e*x**2/d) + 1600830*d**(61/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 1143450*d
**(59/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 571725*d**(57/2)*e**8*x**16*sqrt(1
 + e*x**2/d) + 190575*d**(55/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 38115*d**(5
3/2)*e**10*x**20*sqrt(1 + e*x**2/d) + 3465*d**(51/2)*e**11*x**22*sqrt(1 + e
*x**2/d)) + 24472*d**23*e**7*x**17/(3465*d**(73/2)*sqrt(1 + e*x**2/d) + 381
15*d**(71/2)*e*x**2*sqrt(1 + e*x**2/d) + 190575*d**(69/2)*e**2*x**4*sqrt(1
+ e*x**2/d) + 571725*d**(67/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 1143450*d**(6
5/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 1600830*d**(63/2)*e**5*x**10*sqrt(1 + e
*x**2/d) + 1600830*d**(61/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 1143450*d**(59
/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 571725*d**(57/2)*e**8*x**16*sqrt(1 + e*
x**2/d) + 190575*d**(55/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 38115*d**(53/2)*
e**10*x**20*sqrt(1 + e*x**2/d) + 3465*d**(51/2)*e**11*x**22*sqrt(1 + e*x**2
/d)) + 7728*d**22*e**8*x**19/(3465*d**(73/2)*sqrt(1 + e*x**2/d) + 38115*d**
```

```
(71/2)*e*x**2*sqrt(1 + e*x**2/d) + 190575*d**(69/2)*e**2*x**4*sqrt(1 + e*x*
*2/d) + 571725*d**(67/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 1143450*d**(65/2)*e
**4*x**8*sqrt(1 + e*x**2/d) + 1600830*d**(63/2)*e**5*x**10*sqrt(1 + e*x**2/
d) + 1600830*d**(61/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 1143450*d**(59/2)*e*
*7*x**14*sqrt(1 + e*x**2/d) + 571725*d**(57/2)*e**8*x**16*sqrt(1 + e*x**2/d
) + 190575*d**(55/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 38115*d**(53/2)*e**10*
x**20*sqrt(1 + e*x**2/d) + 3465*d**(51/2)*e**11*x**22*sqrt(1 + e*x**2/d)) +
 1472*d**21*e**9*x**21/(3465*d**(73/2)*sqrt(1 + e*x**2/d) + 38115*d**(71/2)
*e*x**2*sqrt(1 + e*x**2/d) + 190575*d**(69/2)*e**2*x**4*sqrt(1 + e*x**2/d)
+ 571725*d**(67/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 1143450*d**(65/2)*e**4*x*
*8*sqrt(1 + e*x**2/d) + 1600830*d**(63/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 1
600830*d**(61/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 1143450*d**(59/2)*e**7*x**
14*sqrt(1 + e*x**2/d) + 571725*d**(57/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 19
0575*d**(55/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 38115*d**(53/2)*e**10*x**20*
sqrt(1 + e*x**2/d) + 3465*d**(51/2)*e**11*x**22*sqrt(1 + e*x**2/d)) + 128*d
**20*e**10*x**23/(3465*d**(73/2)*sqrt(1 + e*x**2/d) + 38115*d**(71/2)*e*x**
2*sqrt(1 + e*x**2/d) + 190575*d**(69/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 5717
25*d**(67/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 1143450*d**(65/2)*e**4*x**8*sqr
t(1 + e*x**2/d) + 1600830*d**(63/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 1600830
*d**(61/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 1143450*d**(59/2)*e**7*x**14*sqr
t(1 + e*x**2/d) + 571725*d**(57/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 190575*d
**(55/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 38115*d**(53/2)*e**10*x**20*sqrt(1
 + e*x**2/d) + 3465*d**(51/2)*e**11*x**22*sqrt(1 + e*x**2/d))) + c*(231*d**
14*x**5/(1155*d**(41/2)*sqrt(1 + e*x**2/d) + 9240*d**(39/2)*e*x**2*sqrt(1 +
 e*x**2/d) + 32340*d**(37/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 64680*d**(35/2)
*e**3*x**6*sqrt(1 + e*x**2/d) + 80850*d**(33/2)*e**4*x**8*sqrt(1 + e*x**2/d
) + 64680*d**(31/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 32340*d**(29/2)*e**6*x*
*12*sqrt(1 + e*x**2/d) + 9240*d**(27/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 115
5*d**(25/2)*e**8*x**16*sqrt(1 + e*x**2/d)) + 891*d**13*e*x**7/(1155*d**(41/
2)*sqrt(1 + e*x**2/d) + 9240*d**(39/2)*e*x**2*sqrt(1 + e*x**2/d) + 32340*d*
*(37/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 64680*d**(35/2)*e**3*x**6*sqrt(1 + e
*x**2/d) + 80850*d**(33/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 64680*d**(31/2)*e
**5*x**10*sqrt(1 + e*x**2/d) + 32340*d**(29/2)*e**6*x**12*sqrt(1 + e*x**2/d
) + 9240*d**(27/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 1155*d**(25/2)*e**8*x**1
6*sqrt(1 + e*x**2/d)) + 1375*d**12*e**2*x**9/(1155*d**(41/2)*sqrt(1 + e*x**
2/d) + 9240*d**(39/2)*e*x**2*sqrt(1 + e*x**2/d) + 32340*d**(37/2)*e**2*x**4
*sqrt(1 + e*x**2/d) + 64680*d**(35/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 80850*
d**(33/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 64680*d**(31/2)*e**5*x**10*sqrt(1
+ e*x**2/d) + 32340*d**(29/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 9240*d**(27/2
)*e**7*x**14*sqrt(1 + e*x**2/d) + 1155*d**(25/2)*e**8*x**16*sqrt(1 + e*x**2
/d)) + 1105*d**11*e**3*x**11/(1155*d**(41/2)*sqrt(1 + e*x**2/d) + 9240*d**(
39/2)*e*x**2*sqrt(1 + e*x**2/d) + 32340*d**(37/2)*e**2*x**4*sqrt(1 + e*x**2
/d) + 64680*d**(35/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 80850*d**(33/2)*e**4*x
**8*sqrt(1 + e*x**2/d) + 64680*d**(31/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 32
340*d**(29/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 9240*d**(27/2)*e**7*x**14*sqr
```

```
t(1 + e*x**2/d) + 1155*d**(25/2)*e**8*x**16*sqrt(1 + e*x**2/d)) + 510*d**10
*e**4*x**13/(1155*d**(41/2)*sqrt(1 + e*x**2/d) + 9240*d**(39/2)*e*x**2*sqrt
(1 + e*x**2/d) + 32340*d**(37/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 64680*d**(3
5/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 80850*d**(33/2)*e**4*x**8*sqrt(1 + e*x*
*2/d) + 64680*d**(31/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 32340*d**(29/2)*e**
6*x**12*sqrt(1 + e*x**2/d) + 9240*d**(27/2)*e**7*x**14*sqrt(1 + e*x**2/d) +
 1155*d**(25/2)*e**8*x**16*sqrt(1 + e*x**2/d)) + 136*d**9*e**5*x**15/(1155*
d**(41/2)*sqrt(1 + e*x**2/d) + 9240*d**(39/2)*e*x**2*sqrt(1 + e*x**2/d) + 3
2340*d**(37/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 64680*d**(35/2)*e**3*x**6*sqr
t(1 + e*x**2/d) + 80850*d**(33/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 64680*d**(
31/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 32340*d**(29/2)*e**6*x**12*sqrt(1 + e
*x**2/d) + 9240*d**(27/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 1155*d**(25/2)*e*
*8*x**16*sqrt(1 + e*x**2/d)) + 16*d**8*e**6*x**17/(1155*d**(41/2)*sqrt(1 +
e*x**2/d) + 9240*d**(39/2)*e*x**2*sqrt(1 + e*x**2/d) + 32340*d**(37/2)*e**2
*x**4*sqrt(1 + e*x**2/d) + 64680*d**(35/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 8
0850*d**(33/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 64680*d**(31/2)*e**5*x**10*sq
rt(1 + e*x**2/d) + 32340*d**(29/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 9240*d**
(27/2)*e**7*x**14*sqrt(1 + e*x**2/d) + 1155*d**(25/2)*e**8*x**16*sqrt(1 + e
*x**2/d)))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.60

$$
\begin{aligned}
\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = & -\frac{cx^3}{8\,(ex^2+d)^{\frac{11}{2}}e} + \frac{256\,ax}{693\,\sqrt{ex^2+d}d^6} + \frac{128\,ax}{693\,(ex^2+d)^{\frac{3}{2}}d^5} \\
& + \frac{32\,ax}{231\,(ex^2+d)^{\frac{5}{2}}d^4} + \frac{80\,ax}{693\,(ex^2+d)^{\frac{7}{2}}d^3} + \frac{10\,ax}{99\,(ex^2+d)^{\frac{9}{2}}d^2} + \frac{ax}{11\,(ex^2+d)^{\frac{11}{2}}d} \\
& + \frac{cx}{264\,(ex^2+d)^{\frac{9}{2}}e^2} + \frac{16\,cx}{1155\,\sqrt{ex^2+d}d^4e^2} + \frac{8\,cx}{1155\,(ex^2+d)^{\frac{3}{2}}d^3e^2} + \frac{2\,cx}{385\,(ex^2+d)^{\frac{5}{2}}d^2e^2} \\
& + \frac{cx}{231\,(ex^2+d)^{\frac{7}{2}}de^2} - \frac{3\,cdx}{88\,(ex^2+d)^{\frac{11}{2}}e^2} - \frac{bx}{11\,(ex^2+d)^{\frac{11}{2}}e} + \frac{128\,bx}{3465\,\sqrt{ex^2+d}d^5e} \\
& + \frac{64\,bx}{3465\,(ex^2+d)^{\frac{3}{2}}d^4e} + \frac{16\,bx}{1155\,(ex^2+d)^{\frac{5}{2}}d^3e} + \frac{8\,bx}{693\,(ex^2+d)^{\frac{7}{2}}d^2e} + \frac{bx}{99\,(ex^2+d)^{\frac{9}{2}}de}
\end{aligned}
$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/8*c*x^3/((e*x^2 + d)^(11/2)*e) + 256/693*a*x/(sqrt(e*x^2 + d)*d^6) + 128
/693*a*x/((e*x^2 + d)^(3/2)*d^5) + 32/231*a*x/((e*x^2 + d)^(5/2)*d^4) + 80/
693*a*x/((e*x^2 + d)^(7/2)*d^3) + 10/99*a*x/((e*x^2 + d)^(9/2)*d^2) + 1/11*
a*x/((e*x^2 + d)^(11/2)*d) + 1/264*c*x/((e*x^2 + d)^(9/2)*e^2) + 16/1155*c*
x/(sqrt(e*x^2 + d)*d^4*e^2) + 8/1155*c*x/((e*x^2 + d)^(3/2)*d^3*e^2) + 2/38
```

```
5*c*x/((e*x^2 + d)^(5/2)*d^2*e^2) + 1/231*c*x/((e*x^2 + d)^(7/2)*d*e^2) - 3
/88*c*d*x/((e*x^2 + d)^(11/2)*e^2) - 1/11*b*x/((e*x^2 + d)^(11/2)*e) + 128/
3465*b*x/(sqrt(e*x^2 + d)*d^5*e) + 64/3465*b*x/((e*x^2 + d)^(3/2)*d^4*e) +
16/1155*b*x/((e*x^2 + d)^(5/2)*d^3*e) + 8/693*b*x/((e*x^2 + d)^(7/2)*d^2*e)
 + 1/99*b*x/((e*x^2 + d)^(9/2)*d*e)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = \frac{(((2(4x^2(\frac{2(3cd^2e^8+8bde^9+80ae^{10})x^2}{d^6e^5}+\frac{11(3cd^3e^7+8bd^2e^8+80ade^9)}{d^6e^5})+\frac{99(3cd^4e^6+8bd^3e^7+80ad^2e}{d^6e^5}}{3465(ex^2+d)}$$

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="giac")
```

```
[Out] 1/3465*(((2*(4*x^2*(2*(3*c*d^2*e^8 + 8*b*d*e^9 + 80*a*e^10)*x^2/(d^6*e^5) +
 11*(3*c*d^3*e^7 + 8*b*d^2*e^8 + 80*a*d*e^9)/(d^6*e^5)) + 99*(3*c*d^4*e^6 +
 8*b*d^3*e^7 + 80*a*d^2*e^8)/(d^6*e^5))*x^2 + 231*(3*c*d^5*e^5 + 8*b*d^4*e^
6 + 80*a*d^3*e^7)/(d^6*e^5))*x^2 + 1155*(b*d^5*e^5 + 10*a*d^4*e^6)/(d^6*e^5
))*x^2 + 3465*a/d)*x/(e*x^2 + d)^(11/2)
```

Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = \frac{x\left(\frac{a}{11\,d}-\frac{d\left(\frac{b}{11\,d}-\frac{c}{11\,e}\right)}{e}\right)}{\left(e\,x^2+d\right)^{11/2}}-\frac{x\left(\frac{c}{9\,e^2}-\frac{-c\,d^2+b\,d\,e+10\,a\,e^2}{99\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{9/2}} + \frac{x\left(3\,c\,d^2+8\,b\,d\,e+80\,a\,e^2\right)}{693\,d^3\,e^2\,\left(e\,x^2+d\right)^{7/2}}+\frac{x\left(6\,c\,d^2+16\,b\,d\,e+160\,a\,e^2\right)}{1155\,d^4\,e^2\,\left(e\,x^2+d\right)^{5/2}} + \frac{x\left(24\,c\,d^2+64\,b\,d\,e+640\,a\,e^2\right)}{3465\,d^5\,e^2\,\left(e\,x^2+d\right)^{3/2}}+\frac{x\left(48\,c\,d^2+128\,b\,d\,e+1280\,a\,e^2\right)}{3465\,d^6\,e^2\,\sqrt{e\,x^2+d}}$$

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x)
```

```
[Out] (x*(a/(11*d) - (d*(b/(11*d) - c/(11*e)))/e))/(d + e*x^2)^(11/2) - (x*(c/(9*
e^2) - (10*a*e^2 - c*d^2 + b*d*e)/(99*d^2*e^2)))/(d + e*x^2)^(9/2) + (x*(80
*a*e^2 + 3*c*d^2 + 8*b*d*e))/(693*d^3*e^2*(d + e*x^2)^(7/2)) + (x*(160*a*e^
2 + 6*c*d^2 + 16*b*d*e))/(1155*d^4*e^2*(d + e*x^2)^(5/2)) + (x*(640*a*e^2 +
 24*c*d^2 + 64*b*d*e))/(3465*d^5*e^2*(d + e*x^2)^(3/2)) + (x*(1280*a*e^2 +
48*c*d^2 + 128*b*d*e))/(3465*d^6*e^2*(d + e*x^2)^(1/2))
```

3.286 $\int (7+5x^2)^3 \sqrt{2+3x^2+x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 193

$$\begin{aligned}\int (7+5x^2)^3 \sqrt{2+3x^2+x^4}\,dx &= \frac{577x(2+x^2)}{3\sqrt{2+3x^2+x^4}} + \frac{1}{21}x(2608+757x^2)\sqrt{2+3x^2+x^4}\\ &\quad + \frac{275}{7}x(2+3x^2+x^4)^{3/2} + \frac{125}{9}x^3(2+3x^2+x^4)^{3/2}\\ &\quad - \frac{577\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}\\ &\quad + \frac{2945\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{21\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] 275/7*x*(x^4+3*x^2+2)^(3/2)+125/9*x^3*(x^4+3*x^2+2)^(3/2)+577/3*x*(x^2+2)/(
x^4+3*x^2+2)^(1/2)-577/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1
)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+29
45/21*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2)
)*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/21*x*(757*x^2+2608)
*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{1220, 1693, 1190, 1203, 1113, 1149\}$

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\, dx = \frac{2945\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x), \frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}} - \frac{577\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{275}{7}\left(x^4+3x^2+2\right)^{3/2}x + \frac{1}{21}\left(757x^2+2608\right)\sqrt{x^4+3x^2+2}x + \frac{577(x^2+2)\,x}{3\sqrt{x^4+3x^2+2}} + \frac{125}{9}\left(x^4+3x^2+2\right)^{3/2}x^3$$

```
[In] Int[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (577*x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(2608 + 757*x^2)*Sqrt[2 +
3*x^2 + x^4])/21 + (275*x*(2 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(2 + 3*x^2
+ x^4)^(3/2))/9 - (577*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
```

```
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{9}\int\sqrt{2+3x^2+x^4}\left(3087+5865x^2+2475x^4\right)\,dx\\
&= \frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{63}\int\left(16659+11355x^2\right)\sqrt{2+3x^2+x^4}\,dx\\
&= \frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4}+\frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}\\
&\quad+\frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{945}\int\frac{265050+181755x^2}{\sqrt{2+3x^2+x^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4}+\frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2} \\
&\quad +\frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2}+\frac{577}{3}\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{5890}{21}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{577x(2+x^2)}{3\sqrt{2+3x^2+x^4}}+\frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4} \\
&\quad +\frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2} \\
&\quad -\frac{577\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}+\frac{2945\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{21\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.62

$$
\int\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}\,dx
$$

$$
=\frac{25548x+61214x^3+57312x^5+28496x^7+7725x^9+875x^{11}-12117i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right.}{63\sqrt{2+3x^2+x^4}}
$$

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25548*x + 61214*x^3 + 57312*x^5 + 28496*x^7 + 7725*x^9 + 875*x^11 - (12117
*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5553*
I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(63*Sqrt
[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x\left(875x^6+5100x^4+11446x^2+12774\right)\sqrt{x^4+3x^2+2}}{63}-\frac{2945i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{577i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2}\right.\right.}{6\sqrt{x^4+3x^2+2}}$
default	$\frac{4258x\sqrt{x^4+3x^2+2}}{21}-\frac{2945i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{577i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{6\sqrt{x^4+3x^2+2}}+125$
elliptic	$\frac{4258x\sqrt{x^4+3x^2+2}}{21}-\frac{2945i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{577i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{6\sqrt{x^4+3x^2+2}}+125$

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/63*x*(875*x^6+5100*x^4+11446*x^2+12774)*(x^4+3*x^2+2)^(1/2)-2945/21*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1
/2)*x,2^(1/2))+577/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^
(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.33

$$\int \left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}\,dx$$
$$= \frac{-12117i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+29787i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+\left(875\,x^8+5100\,x^6+11446\,x^4+12774\,x^2+121\right.}{63\,x}$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/63*(-12117*I*x*elliptic_e(arcsin(I/x), 2) + 29787*I*x*elliptic_f(arcsin(I
/x), 2) + (875*x^8 + 5100*x^6 + 11446*x^4 + 12774*x^2 + 12117)*sqrt(x^4 + 3
*x^2 + 2))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}\,dx = \int \sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right)^3\,dx$$

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3\,dx$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx=\int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3\,dx$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx=\int \left(5\,x^2+7\right)^3\,\sqrt{x^4+3\,x^2+2}\,dx$$

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2), x)
```

3.287 $\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\begin{aligned}\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\,dx &= \frac{31x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4} \\ &\quad +\frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2} \\ &\quad -\frac{31\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} \\ &\quad +\frac{472\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{21\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] 25/7*x*(x^4+3*x^2+2)^(3/2)+31*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-31*(x^2+1)^(3/2
)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)
/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+472/21*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*
EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/21*x*(114*x^2+407)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used

$= \{1220, 1190, 1203, 1113, 1149\}$

$$\int \left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}\,dx = \frac{472\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}} - \frac{31\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} + \frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2} + \frac{1}{21}x\left(114x^2+407\right)\sqrt{x^4+3x^2+2} + \frac{31x(x^2+2)}{\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (31*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(407 + 114*x^2)*Sqrt[2 + 3*x^2
+ x^4])/21 + (25*x*(2 + 3*x^2 + x^4)^(3/2))/7 - (31*Sqrt[2]*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (47
2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2
1*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{7}\int\left(293+190x^2\right)\sqrt{2+3x^2+x^4}\,dx\\
&= \frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4}+\frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{105}\int\frac{4720+3255x^2}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4}+\frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}\\
&\quad+31\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{944}{21}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{31x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4}+\frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}\\
&\quad-\frac{31\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{472\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{21\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.69 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.68

$$\int (7+5x^2)^2\sqrt{2+3x^2+x^4}\,dx$$
$$= \frac{1114x+2349x^3+1724x^5+564x^7+75x^9-651i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-293i\sqrt{1+x^2}\sqrt{2}}{21\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (1114*x + 2349*x^3 + 1724*x^5 + 564*x^7 + 75*x^9 - (651*I)*Sqrt[1 + x^2]*Sq
rt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (293*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

method	result
risch	$\frac{x(75x^4+339x^2+557)\sqrt{x^4+3x^2+2}}{21}-\frac{472i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\right.\right.}{2\sqrt{x^4+3x^2+2}}$
default	$\frac{557x\sqrt{x^4+3x^2+2}}{21}-\frac{472i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}+25x^5$
elliptic	$\frac{557x\sqrt{x^4+3x^2+2}}{21}-\frac{472i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}+25x^5$

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*x*(75*x^4+339*x^2+557)*(x^4+3*x^2+2)^(1/2)-472/21*I*2^(1/2)*(2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+
31/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

$$\int (7+5x^2)^2 \sqrt{2+3x^2+x^4}\,dx = \frac{-651i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|2\right)+1595i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|2\right)+(75\,x^6+339\,x^4+557\,x^2+651)\sqrt{x^4+3\,x^2+2}}{21\,x}$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/21*(-651*I*x*elliptic_e(arcsin(I/x), 2) + 1595*I*x*elliptic_f(arcsin(I/x)
, 2) + (75*x^6 + 339*x^4 + 557*x^2 + 651)*sqrt(x^4 + 3*x^2 + 2))/x
```

Sympy [F]

$$\int (7+5x^2)^2 \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{(x^2+1)(x^2+2)}\left(5x^2+7\right)^2\,dx$$

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(1/2),x)
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int (7+5x^2)^2 \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int (7+5x^2)^2 \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\,dx = \int \left(5\,x^2+7\right)^2\,\sqrt{x^4+3\,x^2+2}\,dx$$

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2), x)
```

3.288 $\int \left(7+5 x^2\right) \sqrt{2+3 x^2+x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \left(7+5 x^2\right) \sqrt{2+3 x^2+x^4} \, dx=\frac{5 x \left(2+x^2\right)}{\sqrt{2+3 x^2+x^4}}+\frac{1}{3} x \left(10+3 x^2\right) \sqrt{2+3 x^2+x^4}-\frac{5 \sqrt{2} \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} E\left(\arctan (x)\left|\frac{1}{2}\right.\right)}{\sqrt{2+3 x^2+x^4}}+\frac{11 \sqrt{2} \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} \operatorname{EllipticF}\left(\arctan (x),\frac{1}{2}\right)}{3 \sqrt{2+3 x^2+x^4}}$$

```
[Out] 5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+11/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2
^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/3*x*(3*x^2+10
)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1190, 1203, 1113, 1149}

$$\int \left(7+5 x^2\right) \sqrt{2+3 x^2+x^4} \, dx=\frac{11 \sqrt{2} \left(x^2+1\right) \sqrt{\frac{x^2+2}{x^2+1}} \operatorname{EllipticF}\left(\arctan (x),\frac{1}{2}\right)}{3 \sqrt{x^4+3 x^2+2}}-\frac{5 \sqrt{2} \left(x^2+1\right) \sqrt{\frac{x^2+2}{x^2+1}} E\left(\arctan (x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3 x^2+2}}+\frac{5 x \left(x^2+2\right)}{\sqrt{x^4+3 x^2+2}}+\frac{1}{3} x \left(3 x^2+10\right) \sqrt{x^4+3 x^2+2}$$

```
[In] Int[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(10 + 3*x^2)*Sqrt[2 + 3*x^2 + x^
4])/3 - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
 1/2])/Sqrt[2 + 3*x^2 + x^4] + (11*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x\left(10+3x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{15}\int\frac{110+75x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$
\begin{aligned}
&= \frac{1}{3}x(10+3x^2)\sqrt{2+3x^2+x^4} + 5\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{22}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{5x(2+x^2)}{\sqrt{2+3x^2+x^4}} + \frac{1}{3}x(10+3x^2)\sqrt{2+3x^2+x^4} \\
&\quad - \frac{5\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{11\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.60 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

$$
\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx
$$

$$
= \frac{20x+36x^3+19x^5+3x^7-15i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-7i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\mathrm{ar}\right.}{3\sqrt{2+3x^2+x^4}}
$$

```
[In] Integrate[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x + 36*x^3 + 19*x^5 + 3*x^7 - (15*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipt
icE[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[
I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x(3x^2+10)\sqrt{x^4+3x^2+2}}{3}-\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$
default	$\frac{10x\sqrt{x^4+3x^2+2}}{3}-\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}+x^3\sqrt{x^4}$
elliptic	$\frac{10x\sqrt{x^4+3x^2+2}}{3}-\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}+x^3\sqrt{x^4}$

```
[In] int((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(3*x^2+10)*(x^4+3*x^2+2)^(1/2)-11/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+5/2*I*2^(1/2)
*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)
*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.36

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx = \frac{-15i\,xE\left(\arcsin\left(\frac{i}{x}\right)\mid 2\right)+37i\,xF\left(\arcsin\left(\frac{i}{x}\right)\mid 2\right)+\left(3\,x^4+10\,x^2+15\right)\sqrt{x^4+3\,x^2+2}}{3\,x}$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-15*I*x*elliptic_e(arcsin(I/x), 2) + 37*I*x*elliptic_f(arcsin(I/x), 2)
 + (3*x^4 + 10*x^2 + 15)*sqrt(x^4 + 3*x^2 + 2))/x
```

Sympy [F]

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx = \int \sqrt{(x^2+1)(x^2+2)}\left(5x^2+7\right)\,dx$$

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7), x)
```

Maxima [F]

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)dx$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

Giac [F]

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)dx$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)\sqrt{2+3x^2+x^4}\,dx=\int \left(5\,x^2+7\right)\,\sqrt{x^4+3\,x^2+2}\,dx$$

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2), x)
```

3.289 $\int \sqrt{2+3x^2+x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 141

$$\int \sqrt{2+3x^2+x^4}\,dx = \frac{x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\sqrt{2+3x^2+x^4} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{2\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}$$

```
[Out] x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(
x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/
2)+2/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2
))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/3*x*(x^4+3*x^2+2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1105, 1203, 1113, 1149}

$$\int \sqrt{2+3x^2+x^4}\,dx = \frac{2\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} + \frac{1}{3}\sqrt{x^4+3x^2+2}x+\frac{(x^2+2)\,x}{\sqrt{x^4+3x^2+2}}$$

```
[In] Int[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/3 - (Sqrt[2
]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3
*x^2 + x^4] + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcT
an[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x\sqrt{2+3x^2+x^4}+\frac{1}{3}\int\frac{4+3x^2}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{1}{3}x\sqrt{2+3x^2+x^4}+\frac{4}{3}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx+\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx
\end{aligned}$$

$$= \frac{x(2+x^2)}{\sqrt{2+3x^2+x^4}} + \frac{1}{3}x\sqrt{2+3x^2+x^4} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{2\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.65 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \sqrt{2+3x^2+x^4}\,dx$$

$$= \frac{2x+3x^3+x^5-3i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-i\sqrt{1+x^2}\sqrt{2+x^2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{3\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + x^5 - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[
x/Sqrt[2]], 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2
]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{2i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}F\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	121
risch	$\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{2i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}F\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	121
elliptic	$\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{2i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}F\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	121

```
[In] int((x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(x^4+3*x^2+2)^(1/2)-2/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+
3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*2^(1/2)*(2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-
EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.33

$$\int \sqrt{2+3x^2+x^4}\,dx = \frac{-3i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+7i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+\sqrt{x^4+3\,x^2+2}\,(x^2+3)}{3\,x}$$

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(-3*I*x*elliptic_e(arcsin(I/x), 2) + 7*I*x*elliptic_f(arcsin(I/x), 2) +
 sqrt(x^4 + 3*x^2 + 2)*(x^2 + 3))/x
```

Sympy [F]

$$\int \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3x^2+2}\,dx$$

```
[In] integrate((x**4+3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(x**4 + 3*x**2 + 2), x)
```

Maxima [F]

$$\int \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+3x^2+x^4}\,dx=\int \sqrt{x^4+3\,x^2+2}\,dx$$

```
[In] int((3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(1/2), x)
```

3.290 $\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 178

$$
\begin{aligned}
\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx &= \frac{x(2+x^2)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

```
[Out] 1/5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+3/70*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Elli
pticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^
4+3*x^2+2)^(1/2)-1/5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1
/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5*(x
^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+
2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

$= \{1222, 1203, 1113, 1149, 1228, 1470, 553\}$

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = \frac{4\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{25\sqrt{x^4+3x^2+2}} - \frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}} + \frac{3(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{35\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} + \frac{x(x^2+2)}{5\sqrt{x^4+3x^2+2}}$$

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) - (3*(1 +
x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(25*Sqrt[2]*Sqrt[
2 + 3*x^2 + x^4]) + (4*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
F[ArcTan[x], 1/2])/(25*Sqrt[2 + 3*x^2 + x^4]) + (3*(2 + x^2)*EllipticPi[2/7
, ArcTan[x], 1/2])/(35*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x
^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
```

```
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] && !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{25}\int \frac{-8-5x^2}{\sqrt{2+3x^2+x^4}}\,dx\right)-\frac{6}{25}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx\\ &= -\left(\frac{3}{25}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\right)+\frac{1}{5}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &\quad+\frac{3}{10}\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx+\frac{8}{25}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\end{aligned}$$

$$= \frac{x(2+x^2)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} - \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{4\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{2+3x^2+x^4}} + \frac{\left(3\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{10\sqrt{2+3x^2+x^4}}$$

$$= \frac{x(2+x^2)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} - \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{4\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{2+3x^2+x^4}} + \frac{3(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.53 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

$$\int\frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\Big(35E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+21\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)-6\,\text{EllipticPi}\Big(\frac{10}{7},i\text{arcsinh}}{175\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] ((-1/175*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(35*EllipticE[I*ArcSinh[x/Sqrt[2]],
 2] + 21*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - 6*EllipticPi[10/7, I*ArcSinh[
x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{50\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{10\sqrt{x^4+3x^2+2}}+\frac{6i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}$	138
elliptic	$-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{50\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{10\sqrt{x^4+3x^2+2}}+\frac{6i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}$	138

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] -3/50*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))-1/10*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4
+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+6/175*I*2^(1/2)*(1+1/2*x
^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7
,2^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{(x^2+1)\,(x^2+2)}}{5x^2+7}\,dx$$

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

```
[In] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7),x)
[Out] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7), x)
```

3.291 $\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 209

$$\begin{aligned}\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx = & -\frac{x(2+x^2)}{70\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} \\ & +\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} \\ & +\frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{140\sqrt{2}\sqrt{2+3x^2+x^4}} \\ & -\frac{(2+x^2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{980\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] -1/70*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/1960*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1
)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1
))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/70*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+3/280*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*
2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/14*x*(x^4+3*
x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1240, 1203, 1113, 1149, 1228, 1470, 553}

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx = \frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{980\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} + \frac{\sqrt{x^4+3x^2+2}x}{14\left(5x^2+7\right)} - \frac{(x^2+2)\,x}{70\sqrt{x^4+3x^2+2}}$$

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -1/70*(x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2
])/(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(140*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - ((2 +
x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(980*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)
]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
```

```
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1240

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(d + e*x^2))), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{2+3x^2+x^4}}{14\,(7+5x^2)} + \frac{1}{350}\int \frac{7-5x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{1}{350}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{1}{700}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{1}{280}\int \frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx - \frac{1}{70}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{1}{50}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{x(2+x^2)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{140\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\left(\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{280\sqrt{2+3x^2+x^4}}$$

$$= -\frac{x(2+x^2)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{140\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{980\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx = \frac{350x+525x^3+175x^5+35i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)\,E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\Big|\,2\right)-84i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x}{}$$

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (350*x + 525*x^3 + 175*x^5 + (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)
*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (84*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7
 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (5*I)*x^2*Sqrt[1 + x^2]*S
qrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(2450*(7 + 5*x^2)*S
qrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

method	result
default	$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{140\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\right.}{2450\sqrt{x^4+3x^2+2}}$
elliptic	$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{140\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\right.}{2450\sqrt{x^4+3x^2+2}}$
risch	$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{100\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{140\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}}{2450}$

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-3/175*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/140*I*2^(1
/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/
2)*x,2^(1/2))-1/2450*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx=\int \frac{\sqrt{(x^2+1)\,(x^2+2)}}{(5x^2+7)^2}\,dx$$

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^2,x)
[Out] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^2, x)
```

3.292 $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 237

$$\begin{aligned}\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx = & -\frac{11x(2+x^2)}{11760\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{28\,(7+5x^2)^2} \\ & +\frac{11x\sqrt{2+3x^2+x^4}}{2352\,(7+5x^2)}+\frac{11(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{5880\sqrt{2}\sqrt{2+3x^2+x^4}} \\ & +\frac{81(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{2+3x^2+x^4}} \\ & -\frac{1201(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{164640\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] -11/11760*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1201/329280*(x^2+2)*(1/(x^2+1))^(1/
2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+
2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+11/11760*(x^2+1)^(3/2)*(1/(x^2+1))^(1
/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/
(x^4+3*x^2+2)^(1/2)+81/15680*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x
^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2
)+1/28*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+11/2352*x*(x^4+3*x^2+2)^(1/2)/(5*x
^2+7)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1237, 1710, 1730, 1203, 1113, 1149, 1228, 1470, 553}

$$\begin{aligned}\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx = {} & \frac{81(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}} \\ & + \frac{11(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{5880\sqrt{2}\sqrt{x^4+3x^2+2}} \\ & - \frac{1201(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{164640\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} \\ & + \frac{11\sqrt{x^4+3x^2+2}x}{2352\,(5x^2+7)} + \frac{\sqrt{x^4+3x^2+2}x}{28\,(5x^2+7)^2} - \frac{11(x^2+2)\,x}{11760\sqrt{x^4+3x^2+2}}\end{aligned}$$

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (-11*x*(2 + x^2))/(11760*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])
/(28*(7 + 5*x^2)^2) + (11*x*Sqrt[2 + 3*x^2 + x^4])/(2352*(7 + 5*x^2)) + (11
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5880*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipti
cF[ArcTan[x], 1/2])/(7840*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (1201*(2 + x^2)*
EllipticPi[2/7, ArcTan[x], 1/2])/(164640*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
```

```
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{6}{25\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}}+\frac{1}{25\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\right.\\
&\qquad\left.+\frac{1}{25\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\right)dx\\
&= \frac{1}{25}\int\frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx+\frac{1}{25}\int\frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx\\
&\quad-\frac{6}{25}\int\frac{1}{\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2}-\frac{x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)}+\frac{\int\frac{62+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx}{2100}\\
&\quad-\frac{1}{700}\int\frac{74-10x^2-25x^4}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx\\
&\quad+\frac{1}{50}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx-\frac{1}{20}\int\frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx
\end{aligned}$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{50\sqrt{2}\sqrt{2+3x^2+x^4}}$$

$$- \frac{\int \frac{2838+2310x^2+975x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{58800} - \frac{\int \frac{-175-125x^2}{\sqrt{2+3x^2+x^4}}\,dx}{52500} + \frac{13\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{2100}$$

$$- \frac{\left(\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{20\sqrt{2+3x^2+x^4}}$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{50\sqrt{2}\sqrt{2+3x^2+x^4}}$$

$$- \frac{\left(2+x^2\right)\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{70\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} + \frac{\int \frac{-4725-4875x^2}{\sqrt{2+3x^2+x^4}}\,dx}{1470000}$$

$$+ \frac{1}{420}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{13\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{4200} + \frac{1}{300}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$- \frac{13\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{1680} - \frac{101\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{3920}$$

$$= \frac{x(2+x^2)}{420\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)}$$

$$- \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{210\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{37(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{1400\sqrt{2}\sqrt{2+3x^2+x^4}}$$

$$- \frac{\left(2+x^2\right)\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{70\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} - \frac{9\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{2800} - \frac{13\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx}{3920}$$

$$- \frac{101\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{7840} + \frac{101\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{3136}$$

$$- \frac{\left(13\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{1680\sqrt{2+3x^2+x^4}}$$

$$= -\frac{11x(2+x^2)}{11760\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)}$$

$$+ \frac{11(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5880\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{81(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{2+3x^2+x^4}}$$

$$- \frac{97(2+x^2)\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5880\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} + \frac{\left(101\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{3136\sqrt{2+3x^2+x^4}}$$

$$
\begin{aligned}
&= -\frac{11x(2+x^2)}{11760\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} \\
&\quad + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{11(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5880\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{81(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{1201(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{164640\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.73

$$
\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx
= \frac{\frac{14700x(2+3x^2+x^4)}{(7+5x^2)^2} + \frac{1925x(2+3x^2+x^4)}{7+5x^2} + 385i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 434i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{Ellip}}{411600\sqrt{2+3x^2+x^4}}
$$

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((14700*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (1925*x*(2 + 3*x^2 + x^4))/(7
+ 5*x^2) + (385*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2
]], 2] - (434*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]]
, 2] - (1201*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sq
rt[2]], 2])/(411600*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\sqrt{x^4+3x^2+2}\,x\left(55x^2+161\right)}{2352\left(5x^2+7\right)^2} - \frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{16800\sqrt{x^4+3x^2+2}} - \frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{23520\sqrt{x^4+3x^2+2}} - 1$
default	$\frac{x\sqrt{x^4+3x^2+2}}{28\left(5x^2+7\right)^2} + \frac{11x\sqrt{x^4+3x^2+2}}{2352\left(5x^2+7\right)} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{58800\sqrt{x^4+3x^2+2}} + \frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{23520\sqrt{x^4+3x^2+2}} - 1201i\sqrt{2}$
elliptic	$\frac{x\sqrt{x^4+3x^2+2}}{28\left(5x^2+7\right)^2} + \frac{11x\sqrt{x^4+3x^2+2}}{2352\left(5x^2+7\right)} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{58800\sqrt{x^4+3x^2+2}} + \frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{23520\sqrt{x^4+3x^2+2}} - 1201i\sqrt{2}$

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2352*(x^4+3*x^2+2)^(1/2)*x*(55*x^2+161)/(5*x^2+7)^2-1/16800*I*2^(1/2)*(2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^
(1/2))-11/23520*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)
*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-12
01/411600*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ell
ipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}{\left(5x^2+7\right)^3}\,dx$$

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^3,x)
[Out] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^3, x)
```

3.293 $\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}\,dx=\frac{20884x(2+x^2)}{65\sqrt{2+3x^2+x^4}}$$
$$+\frac{x(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005}$$
$$+\frac{x(208212+65345x^2)\left(2+3x^2+x^4\right)^{3/2}}{3003}+\frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2}$$
$$+\frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2}-\frac{20884\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{65\sqrt{2+3x^2+x^4}}+\frac{1171349\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\,(}{5005\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/3003*x*(65345*x^2+208212)*(x^4+3*x^2+2)^(3/2)+3825/143*x*(x^4+3*x^2+2)^(5
/2)+125/13*x^3*(x^4+3*x^2+2)^(5/2)+20884/65*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2
0884/65*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/
2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1171349/5005*(x^2+1
)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((
x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5005*x*(297911*x^2+1032541)*(x^
4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1190, 1203, 1113, 1149}

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2} dx = \frac{1171349\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{5005\sqrt{x^4+3x^2+2}} - \frac{20884\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{65\sqrt{x^4+3x^2+2}} + \frac{3825}{143}\left(x^4+3x^2+2\right)^{5/2}x + \frac{(65345x^2+208212)\left(x^4+3x^2+2\right)^{3/2}x}{3003} + \frac{(297911x^2+1032541)\sqrt{x^4+3x^2+2}x}{5005} + \frac{20884(x^2+2)\,x}{65\sqrt{x^4+3x^2+2}} + \frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

```
[In] Int[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (20884*x*(2 + x^2))/(65*Sqrt[2 + 3*x^2 + x^4]) + (x*(1032541 + 297911*x^2)*
Sqrt[2 + 3*x^2 + x^4])/5005 + (x*(208212 + 65345*x^2)*(2 + 3*x^2 + x^4)^(3/
2))/3003 + (3825*x*(2 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(2 + 3*x^2 + x^4
)^(5/2))/13 - (20884*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[
ArcTan[x], 1/2])/(65*Sqrt[2 + 3*x^2 + x^4]) + (1171349*Sqrt[2]*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5005*Sqrt[2 + 3*x^2 + x
^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
```

```
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{13}\int\left(2+3x^2+x^4\right)^{3/2}\left(4459+8805x^2+3825x^4\right)\,dx$$

$$\begin{aligned}
&= \frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2} \\
&\quad + \frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2} + \frac{1}{143}\int \left(41399+28005x^2\right)\left(2+3x^2+x^4\right)^{3/2}\,dx \\
&= \frac{x(208212+65345x^2)\left(2+3x^2+x^4\right)^{3/2}}{3003} + \frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2} \\
&\quad + \frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2} + \frac{\int (1322334+893733x^2)\sqrt{2+3x^2+x^4}\,dx}{3003} \\
&= \frac{x(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005} + \frac{x(208212+65345x^2)\left(2+3x^2+x^4\right)^{3/2}}{3003} \\
&\quad + \frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2} + \frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2} + \frac{\int \frac{21084282+14472612x^2}{\sqrt{2+3x^2+x^4}}\,dx}{45045} \\
&= \frac{x(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005} \\
&\quad + \frac{x(208212+65345x^2)\left(2+3x^2+x^4\right)^{3/2}}{3003} + \frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2} \\
&\quad + \frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2} + \frac{20884}{65}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{2342698\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{5005} \\
&= \frac{20884x(2+x^2)}{65\sqrt{2+3x^2+x^4}} + \frac{x(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005} \\
&\quad + \frac{x(208212+65345x^2)\left(2+3x^2+x^4\right)^{3/2}}{3003} + \frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2} \\
&\quad + \frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2} - \frac{20884\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{65\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{1171349\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5005\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.59

$$\int \left(7+5x^2\right)^3\left(2+3x^2+x^4\right)^{3/2}\,dx = \frac{13572486x+40493455x^3+54938052x^5+46218643x^7+25350660x^9+8705725x^{11}+1701000}{}$$

```
[In] Integrate[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (13572486*x + 40493455*x^3 + 54938052*x^5 + 46218643*x^7 + 25350660*x^9 + 8
705725*x^11 + 1701000*x^13 + 144375*x^15 - (4824204*I)*Sqrt[1 + x^2]*Sqrt[2
 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (2203890*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(15015*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.68

method	result
risch	$\frac{x\left(144375x^{10}+1267875x^{8}+4613350x^{6}+8974860x^{4}+10067363x^{2}+6786243\right)\sqrt{x^{4}+3x^{2}+2}}{15015}-\frac{1171349i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\right.}{5005\sqrt{x^{4}+3x^{2}+2}}$
default	$\frac{598324x^{5}\sqrt{x^{4}+3x^{2}+2}}{1001}+\frac{10067363x^{3}\sqrt{x^{4}+3x^{2}+2}}{15015}+\frac{2262081x\sqrt{x^{4}+3x^{2}+2}}{5005}-\frac{1171349i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{5005\sqrt{x^{4}+3x^{2}+2}}+$
elliptic	$\frac{598324x^{5}\sqrt{x^{4}+3x^{2}+2}}{1001}+\frac{10067363x^{3}\sqrt{x^{4}+3x^{2}+2}}{15015}+\frac{2262081x\sqrt{x^{4}+3x^{2}+2}}{5005}-\frac{1171349i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{5005\sqrt{x^{4}+3x^{2}+2}}+$

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15015*x*(144375*x^10+1267875*x^8+4613350*x^6+8974860*x^4+10067363*x^2+678
6243)*(x^4+3*x^2+2)^(1/2)-1171349/5005*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1
/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+10442/65*I*2^(1/
2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/
2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.33

$$\int \left(7+5x^{2}\right)^{3}\left(2+3x^{2}+x^{4}\right)^{3/2}\,dx=\frac{-4824204i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+11852298i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+\left(144375\,x^{12}+1267875\,x^{10}+4\right.}{15015\,x}$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(-4824204*I*x*elliptic_e(arcsin(I/x), 2) + 11852298*I*x*elliptic_f(
arcsin(I/x), 2) + (144375*x^12 + 1267875*x^10 + 4613350*x^8 + 8974860*x^6 +
 10067363*x^4 + 6786243*x^2 + 4824204)*sqrt(x^4 + 3*x^2 + 2))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3\,dx$$

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(3/2),x)
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3\,dx$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3\,dx$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}\,dx=\int {\left(5\,x^2+7\right)}^3\,{\left(x^4+3\,x^2+2\right)}^{3/2}\,dx$$

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2), x)
```

3.294 $\int (7+5x^2)^2 \left(2+3x^2+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\int (7+5x^2)^2 \left(2+3x^2+x^4\right)^{3/2}\,dx = \frac{742x(2+x^2)}{15\sqrt{2+3x^2+x^4}} + \frac{x(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2} + \frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2} - \frac{742\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\middle|\frac{1}{2}\right)}{15\sqrt{2+3x^2+x^4}} + \frac{13879\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{385\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/693*x*(2240*x^2+7281)*(x^4+3*x^2+2)^(3/2)+25/11*x*(x^4+3*x^2+2)^(5/2)+742
/15*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-742/15*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*El
lipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3
*x^2+2)^(1/2)+13879/385*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)
^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/1
155*x*(10643*x^2+36783)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used

$= \{1220, 1190, 1203, 1113, 1149\}$

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2}\, dx = \frac{13879\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x), \frac{1}{2}\right)}{385\sqrt{x^4+3x^2+2}}$$
$$- \frac{742\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{15\sqrt{x^4+3x^2+2}}$$
$$+ \frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2} + \frac{1}{693}x\left(2240x^2+7281\right)\left(x^4+3x^2+2\right)^{3/2}$$
$$+ \frac{x(10643x^2+36783)\sqrt{x^4+3x^2+2}}{1155} + \frac{742x(x^2+2)}{15\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (742*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(36783 + 10643*x^2)*Sqrt[
2 + 3*x^2 + x^4])/1155 + (x*(7281 + 2240*x^2)*(2 + 3*x^2 + x^4)^(3/2))/693
+ (25*x*(2 + 3*x^2 + x^4)^(5/2))/11 - (742*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (13879*
Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(385
*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
```

```
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{11}\int \left(489+320x^2\right)\left(2+3x^2+x^4\right)^{3/2}\,dx \\
&= \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2} \\
&\quad +\frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{231}\int \left(15684+10643x^2\right)\sqrt{2+3x^2+x^4}\,dx \\
&= \frac{x(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155}+\frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2} \\
&\quad +\frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2}+\frac{\int \frac{249822+171402x^2}{\sqrt{2+3x^2+x^4}}\,dx}{3465} \\
&= \frac{x(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155}+\frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2} \\
&\quad +\frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2}+\frac{742}{15}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{27758}{385}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{742x(2+x^2)}{15\sqrt{2+3x^2+x^4}}+\frac{x(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155} \\
&\quad +\frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2}+\frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2} \\
&\quad -\frac{742\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{15\sqrt{2+3x^2+x^4}}+\frac{13879\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{385\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.63

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2} \, dx = \frac{429318x + 1160065x^3 + 1333551x^5 + 892084x^7 + 363480x^9 + 82075x^{11} + 7875x^{13} - 171402i\sqrt{}}{3465\sqrt{2+3}}$$

```
[In] Integrate[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (429318*x + 1160065*x^3 + 1333551*x^5 + 892084*x^7 + 363480*x^9 + 82075*x^1
1 + 7875*x^13 - (171402*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[
x/Sqrt[2]], 2] - (78420*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[
x/Sqrt[2]], 2])/(3465*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x\left(7875x^8+58450x^6+172380x^4+258044x^2+214659\right)\sqrt{x^4+3x^2+2}}{3465}-\frac{13879i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{385\sqrt{x^4+3x^2+2}}+371i\sqrt{2}\,\sqrt{2x^2+4}$
default	$\frac{11492x^5\sqrt{x^4+3x^2+2}}{231}+\frac{258044x^3\sqrt{x^4+3x^2+2}}{3465}+\frac{23851x\sqrt{x^4+3x^2+2}}{385}-\frac{13879i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{385\sqrt{x^4+3x^2+2}}+371i\sqrt{2}$
elliptic	$\frac{11492x^5\sqrt{x^4+3x^2+2}}{231}+\frac{258044x^3\sqrt{x^4+3x^2+2}}{3465}+\frac{23851x\sqrt{x^4+3x^2+2}}{385}-\frac{13879i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{385\sqrt{x^4+3x^2+2}}+371i\sqrt{2}$

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*x*(7875*x^8+58450*x^6+172380*x^4+258044*x^2+214659)*(x^4+3*x^2+2)^(1
/2)-13879/385*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*E
llipticF(1/2*I*2^(1/2)*x,2^(1/2))+371/15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2
*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.34

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2}\,dx = \frac{-171402i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|2\right)+421224i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|2\right)+\left(7875\,x^{10}+58450\,x^8+172380\,x^6\right.}{3465\,x}$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(-171402*I*x*elliptic_e(arcsin(I/x), 2) + 421224*I*x*elliptic_f(arcs
in(I/x), 2) + (7875*x^10 + 58450*x^8 + 172380*x^6 + 258044*x^4 + 214659*x^2
 + 171402)*sqrt(x^4 + 3*x^2 + 2))/x
```

Sympy [**F**]

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2}\,dx = \int \left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2\,dx$$

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

Maxima [**F**]

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2}\,dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Giac [**F**]

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2}\,dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(5\,x^2+7\right)^2\,\left(x^4+3\,x^2+2\right)^{3/2}\,dx$$

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2), x)
```

3.295 $\int (7+5x^2)(2+3x^2+x^4)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 179

$$\int (7+5x^2)\left(2+3x^2+x^4\right)^{3/2}\,dx = \frac{116x(2+x^2)}{15\sqrt{2+3x^2+x^4}} + \frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4} + \frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2} - \frac{116\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{15\sqrt{2+3x^2+x^4}} + \frac{197\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\mathrm{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{35\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/63*x*(35*x^2+108)*(x^4+3*x^2+2)^(3/2)+116/15*x*(x^2+2)/(x^4+3*x^2+2)^(1/2
)-116/15*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+197/35*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2
)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/105*x*(149*x^2+519)*(x^4+3*x^2+2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{1190, 1203, 1113, 1149\}$

$$\int \left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}\,dx = \frac{197\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\mathrm{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}} - \frac{116\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{15\sqrt{x^4+3x^2+2}} + \frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+2\right)^{3/2} + \frac{1}{105}x\left(149x^2+519\right)\sqrt{x^4+3x^2+2} + \frac{116x(x^2+2)}{15\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (116*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(519 + 149*x^2)*Sqrt[2 +
3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)^(3/2))/63 - (116*Sq
rt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sq
rt[2 + 3*x^2 + x^4]) + (197*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{21}\int\left(222+149x^2\right)\sqrt{2+3x^2+x^4}\,dx\\
&= \frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}\\
&\quad+\frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{315}\int\frac{3546+2436x^2}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}\\
&\quad+\frac{116}{15}\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{394}{35}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{116x(2+x^2)}{15\sqrt{2+3x^2+x^4}}+\frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2\right.\\
&\qquad\left.+x^4\right)^{3/2}\\
&\quad-\frac{116\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{15\sqrt{2+3x^2+x^4}}+\frac{197\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{35\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.82 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int\left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}\,dx=\frac{5274x+12745x^3+12018x^5+5962x^7+1590x^9+175x^{11}-2436i\sqrt{1+x^2}\sqrt{2+x^2}E\Big(i\mathrm{arcsin}}{315\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (5274*x + 12745*x^3 + 12018*x^5 + 5962*x^7 + 1590*x^9 + 175*x^11 - (2436*I)
*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1110*I)*
Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(315*Sqrt[2
 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x\left(175x^{6}+1065x^{4}+2417x^{2}+2637\right)\sqrt{x^{4}+3x^{2}+2}}{315}-\frac{197i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^{4}+3x^{2}+2}}+\frac{58i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right.}{15\sqrt{x^{4}+3x^{2}+2}}$
default	$\frac{71x^{5}\sqrt{x^{4}+3x^{2}+2}}{21}+\frac{2417x^{3}\sqrt{x^{4}+3x^{2}+2}}{315}+\frac{293x\sqrt{x^{4}+3x^{2}+2}}{35}-\frac{197i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^{4}+3x^{2}+2}}+\frac{58i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}}}{}$
elliptic	$\frac{71x^{5}\sqrt{x^{4}+3x^{2}+2}}{21}+\frac{2417x^{3}\sqrt{x^{4}+3x^{2}+2}}{315}+\frac{293x\sqrt{x^{4}+3x^{2}+2}}{35}-\frac{197i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^{4}+3x^{2}+2}}+\frac{58i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}}}{}$

```
[In] int((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*x*(175*x^6+1065*x^4+2417*x^2+2637)*(x^4+3*x^2+2)^(1/2)-197/35*I*2^(1/
2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2
)*x,2^(1/2))+58/15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1
/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$\int \left(7+5x^{2}\right)\left(2+3x^{2}+x^{4}\right)^{3/2}\,dx=\frac{-2436i\,xE(\arcsin\left(\frac{i}{x}\right)\,|\,2)+5982i\,xF(\arcsin\left(\frac{i}{x}\right)\,|\,2)+(175\,x^{8}+1065\,x^{6}+2417\,x^{4}+2637\,x^{2}+}{315\,x}$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(-2436*I*x*elliptic_e(arcsin(I/x), 2) + 5982*I*x*elliptic_f(arcsin(I/
x), 2) + (175*x^8 + 1065*x^6 + 2417*x^4 + 2637*x^2 + 2436)*sqrt(x^4 + 3*x^2
 + 2))/x
```

Sympy [F]

$$\int (7+5x^2)\left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\cdot\left(5x^2+7\right)\,dx$$

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(3/2),x)
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7), x)
```

Maxima [F]

$$\int (7+5x^2)\left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Giac [F]

$$\int (7+5x^2)\left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)\left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(5\,x^2+7\right)\,\left(x^4+3\,x^2+2\right)^{3/2}dx$$

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2), x)
```

3.296 $\int \left(2+3 x^2+x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 172

$$\begin{aligned}\int \left(2+3 x^2+x^4\right)^{3/2} \, dx &= \frac{6x(2+x^2)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}\\ &+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}-\frac{6\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{5\sqrt{2+3x^2+x^4}}\\ &+\frac{31\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{35\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] 1/7*x*(x^4+3*x^2+2)^(3/2)+6/5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-6/5*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2
)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+31/35*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*
EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/35*x*(9*x^2+29)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1105, 1190, 1203, 1113, 1149}

$$\begin{aligned}\int \left(2+3 x^2+x^4\right)^{3/2} \, dx &= \frac{31\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}\\ &-\frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{5\sqrt{x^4+3x^2+2}}\\ &+\frac{1}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{35}x\left(9x^2+29\right)\sqrt{x^4+3x^2+2}+\frac{6x(x^2+2)}{5\sqrt{x^4+3x^2+2}}\end{aligned}$$

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (6*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + (x*(29 + 9*x^2)*Sqrt[2 + 3*x^2
+ x^4])/35 + (x*(2 + 3*x^2 + x^4)^(3/2))/7 - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
 x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + (31
*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35
*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
```

```
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{3}{7}\int \left(4+3x^2\right)\sqrt{2+3x^2+x^4}\,dx\\ &= \frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{35}\int \frac{62+42x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}\\ &\quad+\frac{6}{5}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{62}{35}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{6x(2+x^2)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}\\ &\quad-\frac{6\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}}+\frac{31\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2+3x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

$$\int \left(2+3x^2+x^4\right)^{3/2}\,dx=\frac{78x+165x^3+121x^5+39x^7+5x^9-42i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-20i\sqrt{1+x^2}}{35\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (78*x + 165*x^3 + 121*x^5 + 39*x^7 + 5*x^9 - (42*I)*Sqrt[1 + x^2]*Sqrt[2 +
x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (20*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2
]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x(5x^4+24x^2+39)\sqrt{x^4+3x^2+2}}{35}-\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{5\sqrt{x^4+3x^2+2}}$
default	$\frac{x^5\sqrt{x^4+3x^2+2}}{7}+\frac{24x^3\sqrt{x^4+3x^2+2}}{35}+\frac{39x\sqrt{x^4+3x^2+2}}{35}-\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,(F}{5\sqrt{x^4}}$
elliptic	$\frac{x^5\sqrt{x^4+3x^2+2}}{7}+\frac{24x^3\sqrt{x^4+3x^2+2}}{35}+\frac{39x\sqrt{x^4+3x^2+2}}{35}-\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,(F}{5\sqrt{x^4}}$

```
[In] int((x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/35*x*(5*x^4+24*x^2+39)*(x^4+3*x^2+2)^(1/2)-31/35*I*2^(1/2)*(2*x^2+4)^(1/2
)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+3/5*
I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*
I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.34

$$\int (2+3x^2+x^4)^{3/2}\,dx=\frac{-42i\,xE(\arcsin\left(\frac{i}{x}\right)\mid 2)+104i\,xF(\arcsin\left(\frac{i}{x}\right)\mid 2)+(5\,x^6+24\,x^4+39\,x^2+42)\sqrt{x^4+3\,x^2+}}{35\,x}$$

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] 1/35*(-42*I*x*elliptic_e(arcsin(I/x), 2) + 104*I*x*elliptic_f(arcsin(I/x),
2) + (5*x^6 + 24*x^4 + 39*x^2 + 42)*sqrt(x^4 + 3*x^2 + 2))/x
```

Sympy [F]

$$\int (2+3x^2+x^4)^{3/2}\,dx=\int (x^4+3x^2+2)^{\frac{3}{2}}\,dx$$

```
[In] integrate((x**4+3*x**2+2)**(3/2),x)
[Out] Integral((x**4 + 3*x**2 + 2)**(3/2), x)
```

Maxima [F]

$$\int \left(2+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \left(2+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(2+3x^2+x^4\right)^{3/2} \, dx = \int {\left(x^4+3\,x^2+2\right)}^{3/2}\,dx$$

```
[In] int((3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((3*x^2 + x^4 + 2)^(3/2), x)
```

3.297 $\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 207

$$\begin{aligned}\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx &= \frac{24x(2+x^2)}{125\sqrt{2+3x^2+x^4}}\\ &+ \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}}\\ &+ \frac{56\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{375\sqrt{2+3x^2+x^4}}\\ &- \frac{9\sqrt{2}(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] 24/125*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-9/875*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1
)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1
))^(1/2)/(x^4+3*x^2+2)^(1/2)-24/125*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellipti
cE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+
2)^(1/2)+56/375*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1
/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/75*x*(3*x
^2+11)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1222, 1190, 1203, 1113, 1149, 1228, 1470, 553}

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \frac{56\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}} - \frac{24\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{125\sqrt{x^4+3x^2+2}} - \frac{9\sqrt{2}(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} + \frac{24x(x^2+2)}{125\sqrt{x^4+3x^2+2}} + \frac{1}{75}x\left(3x^2+11\right)\sqrt{x^4+3x^2+2}$$

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (24*x*(2 + x^2))/(125*Sqrt[2 + 3*x^2 + x^4]) + (x*(11 + 3*x^2)*Sqrt[2 + 3*x
^2 + x^4])/75 - (24*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[A
rcTan[x], 1/2])/(125*Sqrt[2 + 3*x^2 + x^4]) + (56*Sqrt[2]*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(375*Sqrt[2 + 3*x^2 + x^4]) -
 (9*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
```

```
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{25}\int \left(-8-5x^2\right)\sqrt{2+3x^2+x^4}\,dx\right) - \frac{6}{25}\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{1}{375}\int \frac{-130-90x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{6}{625}\int \frac{-8-5x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{36}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} + \frac{18}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{6}{125}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{9}{125}\int \frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx - \frac{48}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + \frac{6}{25}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{26}{75}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{24x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}} + \frac{56\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{375\sqrt{2+3x^2+x^4}} - \frac{\left(9\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}\left(7+5x^2\right)}\,dx}{125\sqrt{2+3x^2+x^4}}$$

$$= \frac{24x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}} + \frac{56\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{375\sqrt{2+3x^2+x^4}} - \frac{9\sqrt{2}\left(2+x^2\right)\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{875\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.71

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx = \frac{3850x+6825x^3+3500x^5+525x^7-2520i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-}{}$$

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (3850*x + 6825*x^3 + 3500*x^5 + 525*x^7 - (2520*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1022*I)*Sqrt[1 + x^2]*Sqrt[2 + x^
2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (108*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]
*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(13125*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.82

method	result
default	$\frac{x^{3}\sqrt{x^{4}+3x^{2}+2}}{25}+\frac{11x\sqrt{x^{4}+3x^{2}+2}}{75}-\frac{73i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{1875\sqrt{x^{4}+3x^{2}+2}}-\frac{12i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{125\sqrt{x^{4}+3x^{2}+2}}-$ 36$i\sqrt{}$
elliptic	$\frac{x^{3}\sqrt{x^{4}+3x^{2}+2}}{25}+\frac{11x\sqrt{x^{4}+3x^{2}+2}}{75}-\frac{73i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{1875\sqrt{x^{4}+3x^{2}+2}}-\frac{12i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{125\sqrt{x^{4}+3x^{2}+2}}-$ 36$i\sqrt{}$
risch	$\frac{x\left(3x^{2}+11\right)\sqrt{x^{4}+3x^{2}+2}}{75}-\frac{253i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{1875\sqrt{x^{4}+3x^{2}+2}}+\frac{12i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{125\sqrt{x^{4}+3x^{2}+2}}-$

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/25*x^3*(x^4+3*x^2+2)^(1/2)+11/75*x*(x^4+3*x^2+2)^(1/2)-73/1875*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))-12/125*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-36/4375*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{\left(2+3x^{2}+x^{4}\right)^{3/2}}{7+5x^{2}}\,dx=\int \frac{\left(x^{4}+3\,x^{2}+2\right)^{\frac{3}{2}}}{5\,x^{2}+7}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F]

$$\int \frac{\left(2+3x^{2}+x^{4}\right)^{3/2}}{7+5x^{2}}\,dx=\int \frac{\left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}}{5x^{2}+7}\,dx$$

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7), x)
```

Maxima [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Giac [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{3/2}}{5\,x^2+7}\,dx$$

```
[In] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7),x)
[Out] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.298 $\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx = \frac{9x(2+x^2)}{175\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)} - \frac{9\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{175\sqrt{2+3x^2+x^4}} + \frac{59(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{1050\sqrt{2+3x^2+x^4}} + \frac{9(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{2450\sqrt{2+3x^2+x^4}}$$

```
[Out] 9/175*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-9/175*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*E
llipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+
3*x^2+2)^(1/2)+59/1050*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^
(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)+9/2450*(x^
2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((
x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/75*x*(x^4+3*x^2+2)^(1/2)-3/17
5*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.50, number of steps used = 21, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules

used = {1242, 1113, 1149, 1136, 1203, 1237, 1730, 1228, 1470, 553}

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx = \frac{44\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{1875\sqrt{x^4+3x^2+2}} + \frac{81(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{8750\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{9\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{175\sqrt{x^4+3x^2+2}} + \frac{3\sqrt{2}(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} - \frac{39(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{12250\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} - \frac{3\sqrt{x^4+3x^2+2}\,x}{175\left(5x^2+7\right)} + \frac{1}{75}\sqrt{x^4+3x^2+2}\,x + \frac{9(x^2+2)\,x}{175\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/75
- (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(175*Sqrt[2 + 3*x^2 + x^4]
) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(875
0*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (44*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], 1/2])/(1875*Sqrt[2 + 3*x^2 + x^4]) - (39*(2 +
x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(12250*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^
2)]*Sqrt[2 + 3*x^2 + x^4]) + (3*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x]
, 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
```

```
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{52}{625\sqrt{2+3x^2+x^4}}+\frac{16x^2}{125\sqrt{2+3x^2+x^4}}+\frac{x^4}{25\sqrt{2+3x^2+x^4}}\right.\\
&\qquad\left.+\frac{36}{625\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}-\frac{12}{625\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\right)dx\\
&= -\left(\frac{12}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx\right)\\
&\quad+\frac{1}{25}\int \frac{x^4}{\sqrt{2+3x^2+x^4}}\,dx+\frac{36}{625}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx\\
&\quad+\frac{52}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx+\frac{16}{125}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{16x(2+x^2)}{125\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\sqrt{2+3x^2+x^4}-\frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)}\\
&\quad-\frac{16\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}}+\frac{26\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{625\sqrt{2+3x^2+x^4}}\\
&\quad+\frac{3\int \frac{62+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx}{4375}-\frac{6}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\
&\quad-\frac{1}{75}\int \frac{2+6x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{3}{125}\int \frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx
\end{aligned}
$$

$$\begin{aligned}
&= \frac{16x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} \\
&\quad - \frac{16\sqrt{2}(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}E\big(\tan^{-1}(x)|\frac{1}{2}\big)}{125\sqrt{2+3x^2+x^4}} + \frac{23\sqrt{2}(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}F\big(\tan^{-1}(x)|\frac{1}{2}\big)}{625\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{3\int\frac{-175-125x^2}{\sqrt{2+3x^2+x^4}}\,dx}{109375} + \frac{39\int\frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{4375} - \frac{2}{75}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&\quad - \frac{2}{25}\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{\left(3\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{125\sqrt{2+3x^2+x^4}} \\
&= \frac{6x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} \\
&\quad - \frac{6\sqrt{2}(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}E\big(\tan^{-1}(x)|\frac{1}{2}\big)}{125\sqrt{2+3x^2+x^4}} + \frac{44\sqrt{2}(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}F\big(\tan^{-1}(x)|\frac{1}{2}\big)}{1875\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{3\sqrt{2}(2+x^2)\,\Pi\big(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\big)}{875\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} + \frac{3}{875}\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&\quad + \frac{39\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{8750} + \frac{3}{625}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{39\int\frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{3500} \\
&= \frac{9x(2+x^2)}{175\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} \\
&\quad - \frac{9\sqrt{2}(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}E\big(\tan^{-1}(x)|\frac{1}{2}\big)}{175\sqrt{2+3x^2+x^4}} + \frac{81(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}F\big(\tan^{-1}(x)|\frac{1}{2}\big)}{8750\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{44\sqrt{2}(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}F\big(\tan^{-1}(x)|\frac{1}{2}\big)}{1875\sqrt{2+3x^2+x^4}} + \frac{3\sqrt{2}(2+x^2)\,\Pi\big(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\big)}{875\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{\left(39\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{3500\sqrt{2+3x^2+x^4}} \\
&= \frac{9x(2+x^2)}{175\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} \\
&\quad - \frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} - \frac{9\sqrt{2}(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}E\big(\tan^{-1}(x)|\frac{1}{2}\big)}{175\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{81(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}F\big(\tan^{-1}(x)|\frac{1}{2}\big)}{8750\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{44\sqrt{2}(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}F\big(\tan^{-1}(x)|\frac{1}{2}\big)}{1875\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{39(2+x^2)\,\Pi\big(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\big)}{12250\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} + \frac{3\sqrt{2}(2+x^2)\,\Pi\big(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\big)}{875\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \frac{2800x+6650x^3+5075x^5+1225x^7-945i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)\,E\Big(i\operatorname{arcsinh}\Big(}{}$$

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (2800*x + 6650*x^3 + 5075*x^5 + 1225*x^7 - (945*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (182*I)*Sqrt[1 + x^2]*
Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (189*I)*Sqrt
[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (135*I)
*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])
/(18375*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.80

method	result
default	$-\frac{3x\sqrt{x^4+3x^2+2}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+2}}{75}-\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2625\sqrt{x^4+3x^2+2}}-\frac{9i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{350\sqrt{x^4+3x^2+2}}+$
elliptic	$-\frac{3x\sqrt{x^4+3x^2+2}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+2}}{75}-\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2625\sqrt{x^4+3x^2+2}}-\frac{9i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{350\sqrt{x^4+3x^2+2}}+$
risch	$\frac{\sqrt{x^4+3x^2+2}\,x(7x^2+8)}{525x^2+735}-\frac{23i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{750\sqrt{x^4+3x^2+2}}+\frac{9i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{350\sqrt{x^4+3x^2+2}}+$

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/175*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+2)^(1/2)-13/2625*I
*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*
2^(1/2)*x,2^(1/2))-9/350*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2
+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+9/6125*I*2^(1/2)*(1+1/2*x^2)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1
/2))
```

Fricas [F]

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**2,x)
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx=\int \frac{{\left(x^4+3\,x^2+2\right)}^{3/2}}{{\left(5\,x^2+7\right)}^2}\,dx$$

```
[In] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^2,x)
[Out] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.299 $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 231

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3} \, dx = \frac{3x(2+x^2)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\,(7+5x^2)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\,(7+5x^2)} - \frac{3(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{196\sqrt{2+3x^2+x^4}} + \frac{5(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{784\sqrt{2+3x^2+x^4}} + \frac{141(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{27440\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

```
[Out] 3/392*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+141/54880*(x^2+2)*(1/(x^2+1))^(1/2)*(x^
2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^
2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-3/196*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellip
ticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(
1/2)+5/784*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^
(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)-3/350*x*(x^4+3*x^2+2)^
(1/2)/(5*x^2+7)^2+17/9800*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.25, number of steps used = 27, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1242, 1113, 1149, 1237, 1710, 1730, 1203, 1228, 1470, 553}

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{784\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{875\sqrt{x^4+3x^2+2}} - \frac{39(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{24500\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{141(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{27440\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} + \frac{17\sqrt{x^4+3x^2+2}x}{9800\,(5x^2+7)} - \frac{3\sqrt{x^4+3x^2+2}x}{350\,(5x^2+7)^2} + \frac{3(x^2+2)\,x}{392\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2 + x^4])/(
350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) - (39*
(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(24500*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], 1/2])/(875*Sqrt[2 + 3*x^2 + x^4]) + (5*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[2 + 3*x^2
 + x^4]) + (141*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(27440*Sqrt[2]*S
qrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
```

```
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int\left(\frac{9}{625\sqrt{2+3x^2+x^4}}+\frac{x^2}{125\sqrt{2+3x^2+x^4}}+\frac{36}{625\,(7+5x^2)^3\sqrt{2+3x^2+x^4}}\right.\\ &\qquad\left.-\frac{12}{625\,(7+5x^2)^2\sqrt{2+3x^2+x^4}}-\frac{11}{625\,(7+5x^2)\sqrt{2+3x^2+x^4}}\right)dx\\ &= \frac{1}{125}\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{9}{625}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx-\frac{11}{625}\int\frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx\\ &\quad-\frac{12}{625}\int\frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx+\frac{36}{625}\int\frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx\end{aligned}$$

$$
\begin{aligned}
&= \frac{x(2+x^2)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)} \\
&\quad - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}} + \frac{9(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{625\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{\int \frac{62+70x^2+25x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{4375} + \frac{3\int \frac{74-10x^2-25x^4}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx}{8750} \\
&\quad - \frac{11\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{1250} + \frac{11}{500}\int \frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(2+x^2)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} \\
&\quad - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}} + \frac{7(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{1250\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{\int \frac{2838+2310x^2+975x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{245000} + \frac{\int \frac{-175-125x^2}{\sqrt{2+3x^2+x^4}}\,dx}{109375} - \frac{13\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{4375} \\
&\quad + \frac{\left(11\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{500\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(2+x^2)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} \\
&\quad - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}} + \frac{7(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{1250\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{11(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{1750\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} - \frac{\int \frac{-4725-4875x^2}{\sqrt{2+3x^2+x^4}}\,dx}{6125000} \\
&\quad - \frac{1}{875}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{13\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{8750} - \frac{1}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&\quad + \frac{13\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{3500} + \frac{303\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{49000}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6x(2+x^2)}{875\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} \\
&\quad - \frac{6\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{875\sqrt{2+3x^2+x^4}} + \frac{11(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{4375\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{11(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{1750\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} + \frac{27\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{35000} \\
&\quad + \frac{39\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx}{49000} + \frac{303\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{98000} - \frac{303\int\frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{39200} \\
&\quad + \frac{\left(13\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{3500\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3x(2+x^2)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} \\
&\quad - \frac{39(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{24500\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{6\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{875\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{784\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{9(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{1225\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{\left(303\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{39200\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3x(2+x^2)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} \\
&\quad - \frac{39(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{24500\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{6\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{875\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{784\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{141(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{27440\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.75

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx = \frac{-\frac{588x(2+3x^2+x^4)}{(7+5x^2)^2}+\frac{119x(2+3x^2+x^4)}{7+5x^2}-525i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|\,2\right)-4}{}$$

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((-588*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (119*x*(2 + 3*x^2 + x^4))/(7 +
5*x^2) - (525*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]]
, 2] - (406*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]],
2] + (141*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[
2]], 2])/(68600*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.53 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.79

method	result
risch	$\frac{\sqrt{x^4+3x^2+2}\,x\left(17x^2+7\right)}{1960\left(5x^2+7\right)^2}-\frac{19i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2800\sqrt{x^4+3x^2+2}}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{784\sqrt{x^4+3x^2+2}}+$
default	$-\frac{3x\sqrt{x^4+3x^2+2}}{350\left(5x^2+7\right)^2}+\frac{17x\sqrt{x^4+3x^2+2}}{9800\left(5x^2+7\right)}-\frac{29i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9800\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{784\sqrt{x^4+3x^2+2}}+141i$
elliptic	$-\frac{3x\sqrt{x^4+3x^2+2}}{350\left(5x^2+7\right)^2}+\frac{17x\sqrt{x^4+3x^2+2}}{9800\left(5x^2+7\right)}-\frac{29i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9800\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{784\sqrt{x^4+3x^2+2}}+141i$

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1960*(x^4+3*x^2+2)^(1/2)*x*(17*x^2+7)/(5*x^2+7)^2-19/2800*I*2^(1/2)*(2*x^
2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1
/2))+3/784*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ell
ipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))+141/686
00*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi
(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx=\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx=\int \frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**3,x)
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx=\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx=\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx=\int \frac{(x^4+3\,x^2+2)^{3/2}}{(5\,x^2+7)^3}\,dx$$

```
[In] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^3,x)
[Out] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.300 $\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 157

$$\begin{aligned}\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx &= \frac{135x(2+x^2)}{\sqrt{2+3x^2+x^4}} + 75x\sqrt{2+3x^2+x^4} + 25x^3\sqrt{2+3x^2+x^4} \\ &\quad - \frac{135\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2+3x^2+x^4}} \\ &\quad + \frac{193(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\mathrm{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] 135*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+193/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ell
ipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*
x^2+2)^(1/2)-135*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),
1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+75*x*(x^4+
3*x^2+2)^(1/2)+25*x^3*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used

$= \{1220, 1693, 1203, 1113, 1149\}$

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \frac{193(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{135\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} + 75\sqrt{x^4+3x^2+2}x + \frac{135(x^2+2)\,x}{\sqrt{x^4+3x^2+2}} + 25\sqrt{x^4+3x^2+2}x^3$$

```
[In] Int[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (135*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + 75*x*Sqrt[2 + 3*x^2 + x^4] + 25*x
^3*Sqrt[2 + 3*x^2 + x^4] - (135*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]
*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (193*(1 + x^2)*Sqrt[(2
+ x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]
)
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 25x^3\sqrt{2+3x^2+x^4}+\frac{1}{5}\int\frac{1715+2925x^2+1125x^4}{\sqrt{2+3x^2+x^4}}\,dx\\
&= 75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}+\frac{1}{15}\int\frac{2895+2025x^2}{\sqrt{2+3x^2+x^4}}\,dx\\
&= 75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}\\
&\quad+135\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+193\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{135x(2+x^2)}{\sqrt{2+3x^2+x^4}}+75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}\\
&\quad-\frac{135\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{193(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int\frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$$

$$=\frac{25x(6+11x^2+6x^4+x^6)-135i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-58i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\Big(i}{\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(6 + 11*x^2 + 6*x^4 + x^6) - (135*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elli
pticE[I*ArcSinh[x/Sqrt[2]], 2] - (58*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipti
cF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.80

method	result
risch	$25x(x^2+3)\sqrt{x^4+3x^2+2}-\frac{193 i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{135 i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\right.\right.}{2\sqrt{x^4+3x^2+2}}$
default	$-\frac{193 i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+25x^3\sqrt{x^4+3x^2+2}+75x\sqrt{x^4+3x^2+2}+\frac{135 i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\right.}{2\sqrt{x}}$
elliptic	$-\frac{193 i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+25x^3\sqrt{x^4+3x^2+2}+75x\sqrt{x^4+3x^2+2}+\frac{135 i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\right.}{2\sqrt{x}}$

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25*x*(x^2+3)*(x^4+3*x^2+2)^(1/2)-193/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1
/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+135/2*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*
x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.34

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-135 i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+328 i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+5\left(5\,x^4+15\,x^2+27\right)\sqrt{x^4+3\,x^2+2}}{x}$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] (-135*I*x*elliptic_e(arcsin(I/x), 2) + 328*I*x*elliptic_f(arcsin(I/x), 2) +
 5*(5*x^4 + 15*x^2 + 27)*sqrt(x^4 + 3*x^2 + 2))/x
```

Sympy [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5x^2+7)^3}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.301 $\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{20x(2+x^2)}{\sqrt{2+3x^2+x^4}} + \frac{25}{3}x\sqrt{2+3x^2+x^4} - \frac{20\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{97(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] 20*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+97/6*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellip
ticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^
2+2)^(1/2)-20*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2
*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+25/3*x*(x^4+3
*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{1220, 1203, 1113, 1149\}$

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{97(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\mathrm{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{20\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} + \frac{25}{3}\sqrt{x^4+3x^2+2}x + \frac{20(x^2+2)\,x}{\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (25*x*Sqrt[2 + 3*x^2 + x^4])/3 - (
20*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/S
qrt[2 + 3*x^2 + x^4] + (97*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[Ar
cTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
```

```
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{25}{3}x\sqrt{2+3x^2+x^4}+\frac{1}{3}\int\frac{97+60x^2}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{25}{3}x\sqrt{2+3x^2+x^4}+20\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{97}{3}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{20x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{25}{3}x\sqrt{2+3x^2+x^4}\\
&\quad-\frac{20\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{97(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int\frac{\left(7+5x^2\right)^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$=\frac{25x(2+3x^2+x^4)-60i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-37i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}\right.}{3\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(2 + 3*x^2 + x^4) - (60*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*Ar
cSinh[x/Sqrt[2]], 2] - (37*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSi
nh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{97 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{6 \sqrt{x^{4}+3 x^{2}+2}}+\frac{25 x \sqrt{x^{4}+3 x^{2}+2}}{3}+\frac{10 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{\sqrt{x^{4}+3 x^{2}+2}}$	121
risch	$-\frac{97 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{6 \sqrt{x^{4}+3 x^{2}+2}}+\frac{25 x \sqrt{x^{4}+3 x^{2}+2}}{3}+\frac{10 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{\sqrt{x^{4}+3 x^{2}+2}}$	121
elliptic	$-\frac{97 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{6 \sqrt{x^{4}+3 x^{2}+2}}+\frac{25 x \sqrt{x^{4}+3 x^{2}+2}}{3}+\frac{10 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{\sqrt{x^{4}+3 x^{2}+2}}$	121

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -97/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))+25/3*x*(x^4+3*x^2+2)^(1/2)+10*I*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2)
)-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.35

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-60i\,xE(\arcsin\left(\frac{i}{x}\right)\,|2) + 157i\,xF(\arcsin\left(\frac{i}{x}\right)\,|2) + 5\sqrt{x^4+3x^2+2}(5x^2+12)}{3x}$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-60*I*x*elliptic_e(arcsin(I/x), 2) + 157*I*x*elliptic_f(arcsin(I/x), 2
) + 5*sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 12))/x
```

Sympy [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5x^2+7)^2}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.302 $\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{5x(2+x^2)}{\sqrt{2+3x^2+x^4}} - \frac{5\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{7(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] 5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+7/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellipti
cF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+
2)^(1/2)-5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^
(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1203, 1113, 1149}

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{5\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} + \frac{5x(x^2+2)}{\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (7*(1 + x^2)
```

```
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 5\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+7\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{5x(2+x^2)}{\sqrt{2+3x^2+x^4}}-\frac{5\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{7(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$=-\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(5E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+2\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)\right)}{\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(5*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + 2
*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{7 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+\frac{5 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}$	106
elliptic	$-\frac{7 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+\frac{5 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}$	106

```
[In] int((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -7/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(
1/2*I*2^(1/2)*x,2^(1/2))+5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3
*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x
,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.34

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-5i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|2\right)+12i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|2\right)+5\sqrt{x^4+3\,x^2+2}}{x}$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] (-5*I*x*elliptic_e(arcsin(I/x), 2) + 12*I*x*elliptic_f(arcsin(I/x), 2) + 5*
sqrt(x^4 + 3*x^2 + 2))/x
```

Sympy [F]

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{5x^2+7}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral((5*x**2 + 7)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.303 $\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx=\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1113}

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx=\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[In] Int[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

```
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral}=\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx=-\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[
2 + 3*x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}$	46
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}$	46

```
[In] int(1/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(
1/2*I*2^(1/2)*x,2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = -i\,F\!\left(\arcsin\left(\frac{1}{2}i\sqrt{2}x\right)\,|\,2\right)$$

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -I*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3x^2+2}}\,dx$$

```
[In] integrate(1/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+2}} \,d x$$

```
[In] int(1/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.304 $\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{5(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

```
[Out] -5/28*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/
7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/4*(x^2
+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*
((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1228, 1113, 1470, 553}

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{5(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[In] Int[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*
Sqrt[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*
Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{5}{4}\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\left(5\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{4\sqrt{2+3x^2+x^4}}
\end{aligned}$$

$$= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{5(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticPi}\left(\frac{10}{7}, i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right), 2\right)}{7\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-1/7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]]
, 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{7\sqrt{x^4+3x^2+2}}$	47
elliptic	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{7\sqrt{x^4+3x^2+2}}$	47

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
Pi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^6 + 22*x^4 + 31*x^2 + 14), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2+1)\,(x^2+2)}\,(5x^2+7)}\,dx$$

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{\left(5\,x^2+7\right)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.305 $\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 209

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx &= \frac{5x(2+x^2)}{84\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)} \\ &\quad - \frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}} \\ &\quad + \frac{9(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{56\sqrt{2}\sqrt{2+3x^2+x^4}} \\ &\quad - \frac{65(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{1176\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] 5/84*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-65/2352*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1
)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1
))^(1/2)/(x^4+3*x^2+2)^(1/2)-5/84*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+9/112*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*
2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-25/84*x*(x^4+3
*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1237, 1730, 1203, 1113, 1149, 1228, 1470, 553}

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx = {} & \frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{56\sqrt{2}\sqrt{x^4+3x^2+2}} \\ & - \frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+2}} \\ & - \frac{65(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{1176\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} \\ & - \frac{25\sqrt{x^4+3x^2+2}x}{84\,(5x^2+7)} + \frac{5(x^2+2)\,x}{84\sqrt{x^4+3x^2+2}}\end{aligned}$$

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (5*x*(2 + x^2))/(84*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4])/(
84*(7 + 5*x^2)) - (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(42*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(
1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(56*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (
65*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(1176*Sqrt[2]*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
```

```
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
```

```
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)}+\frac{1}{84}\int\frac{62+70x^2+25x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx\\
&= -\frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)}-\frac{\int\frac{-175-125x^2}{\sqrt{2+3x^2+x^4}}\,dx}{2100}+\frac{13}{84}\int\frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx\\
&= -\frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)}+\frac{5}{84}\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{13}{168}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\
&\quad+\frac{1}{12}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx-\frac{65}{336}\int\frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{5x(2+x^2)}{84\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)}-\frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}}\\
&\quad+\frac{9(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{56\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{\left(65\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{336\sqrt{2+3x^2+x^4}}\\
&= \frac{5x(2+x^2)}{84\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)}-\frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}}\\
&\quad+\frac{9(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{56\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{65(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{1176\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int\frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$$

$$=\frac{-350x-525x^3-175x^5-35i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)\,E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-14i\sqrt{1+x^2}\sqrt{2+x^2}(7+}{}$$

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-350*x - 525*x^3 - 175*x^5 - (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2
)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(
7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (91*I)*Sqrt[1 + x^2]*Sqrt[2
 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (65*I)*x^2*Sqrt[1 + x^2
]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(588*(7 + 5*x^2)
*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

method	result
default	$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{84\sqrt{x^4+3x^2+2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}-\frac{13i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi}{588\sqrt{x^4+3x^2}}$
elliptic	$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{84\sqrt{x^4+3x^2+2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}-\frac{13i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi}{588\sqrt{x^4+3x^2}}$
risch	$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{24\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{168\sqrt{x^4+3x^2+2}}-\frac{13i\sqrt{2}}{}$

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/84*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-1/84*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-5/168*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1
/2)*x,2^(1/2))-13/588*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98),
 x)
```

Sympy [F]

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right)^2}\,dx$$

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{{\left(5\,x^2+7\right)}^2\,\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.306 $\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 237

$$
\begin{aligned}
\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = {} & \frac{65x(2+x^2)}{4704\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2} \\
& - \frac{325x\sqrt{2+3x^2+x^4}}{4704\,(7+5x^2)} \\
& - \frac{65(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{2352\sqrt{2}\sqrt{2+3x^2+x^4}} \\
& + \frac{631(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{2+3x^2+x^4}} \\
& - \frac{2525(2+x^2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{65856\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

```
[Out] 65/4704*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2525/131712*(x^2+2)*(1/(x^2+1))^(1/2)
*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)
/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-65/4704*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)
*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^
4+3*x^2+2)^(1/2)+631/18816*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2
+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-
25/168*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2-325/4704*x*(x^4+3*x^2+2)^(1/2)/(5*
x^2+7)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1237, 1710, 1730, 1203, 1113, 1149, 1228, 1470, 553}

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = \frac{631(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{65(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{2352\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{2525(x^2+2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{65856\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} - \frac{325\sqrt{x^4+3x^2+2}x}{4704\,(5x^2+7)} - \frac{25\sqrt{x^4+3x^2+2}x}{168\,(5x^2+7)^2} + \frac{65(x^2+2)\,x}{4704\sqrt{x^4+3x^2+2}}$$

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (65*x*(2 + x^2))/(4704*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4]
)/(168*(7 + 5*x^2)^2) - (325*x*Sqrt[2 + 3*x^2 + x^4])/(4704*(7 + 5*x^2)) -
(65*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(2352*Sq
rt[2]*Sqrt[2 + 3*x^2 + x^4]) + (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(9408*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x
^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(65856*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2
)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2}+\frac{1}{168}\int\frac{74-10x^2-25x^4}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx\\
&= -\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)}+\frac{\int\frac{2838+2310x^2+975x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{14112}\\
&= -\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)}-\frac{\int\frac{-4725-4875x^2}{\sqrt{2+3x^2+x^4}}\,dx}{352800}+\frac{505\int\frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{4704}\\
&= -\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)}+\frac{3}{224}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\
&\quad+\frac{65\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx}{4704}+\frac{505\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{9408}-\frac{2525\int\frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{18816}\\
&= \frac{65x(2+x^2)}{4704\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)}\\
&\quad-\frac{65(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2352\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{631(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{2+3x^2+x^4}}\\
&\quad-\frac{\left(2525\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{18816\sqrt{2+3x^2+x^4}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{65x(2+x^2)}{4704\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2} \\
&\quad - \frac{325x\sqrt{2+3x^2+x^4}}{4704\,(7+5x^2)} - \frac{65(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\big(\tan^{-1}(x)\big|\frac{1}{2}\big)}{2352\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{631(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\big(\tan^{-1}(x)\big|\frac{1}{2}\big)}{9408\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{2525(2+x^2)\,\Pi\big(\frac{2}{7};\tan^{-1}(x)\big|\frac{1}{2}\big)}{65856\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.78

$$
\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx
$$

$$
= \frac{-175x(238+487x^2+314x^4+65x^6) - 455i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)^2\,E\Big(i\operatorname{arcsinh}\Big(\frac{x}{\sqrt{2}}\Big)\Big|\,2\Big) + 14i\sqrt{1+}{32928\,(7}
$$

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-175*x*(238 + 487*x^2 + 314*x^4 + 65*x^6) - (455*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*(7 + 5*x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (14*I)*Sqrt[1 + x^
2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (505*I)
*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticPi[10/7, I*ArcSinh[x/Sqr
t[2]], 2])/(32928*(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{25\sqrt{x^4+3x^2+2}\,x(65x^2+119)}{4704(5x^2+7)^2} - \frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{448\sqrt{x^4+3x^2+2}} + \frac{65i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9408\sqrt{x^4+3x^2+2}}$
default	$-\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2} - \frac{325x\sqrt{x^4+3x^2+2}}{4704(5x^2+7)} + \frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4704\sqrt{x^4+3x^2+2}} - \frac{65i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9408\sqrt{x^4+3x^2+2}} - 505$
elliptic	$-\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2} - \frac{325x\sqrt{x^4+3x^2+2}}{4704(5x^2+7)} + \frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4704\sqrt{x^4+3x^2+2}} - \frac{65i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9408\sqrt{x^4+3x^2+2}} - 505$

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/4704*(x^4+3*x^2+2)^(1/2)*x*(65*x^2+119)/(5*x^2+7)^2-3/448*I*2^(1/2)*(2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^
(1/2))+65/9408*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-505
/32928*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ellipt
icPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 +
2499*x^2 + 686), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2+1)\,(x^2+2)}\,(5x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{(5\,x^2+7)^3\,\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.307 $\int \frac{\left(7+5x^2\right)^5}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{\left(7+5x^2\right)^5}{\left(2+3x^2+x^4\right)^{3/2}}\,dx = \frac{7679x(2+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2+x^4} - \frac{7679(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{15383(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] 7679/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/2*x*(179*x^2+115)/(x^4+3*x^2+2)^(1/2
)-7679/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+15383/6*(x^2+1)^(3
/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+
2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+5000/3*x*(x^4+3*x^2+2)^(1/2)+625*x^3*
(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used

$= \{1219, 1693, 1203, 1113, 1149\}$

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{15383(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\mathrm{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{7679(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{5000}{3}\sqrt{x^4+3x^2+2}x + \frac{7679(x^2+2)\,x}{2\sqrt{x^4+3x^2+2}} - \frac{(179x^2+115)\,x}{2\sqrt{x^4+3x^2+2}} + 625\sqrt{x^4+3x^2+2}x^3$$

```
[In] Int[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (7679*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (x*(115 + 179*x^2))/(2*Sqrt[
2 + 3*x^2 + x^4]) + (5000*x*Sqrt[2 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[2 + 3*x
^2 + x^4] - (7679*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (15383*(1 + x^2)*Sqrt[(2 + x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-16922-35179x^2-25000x^4-6250x^6}{\sqrt{2+3x^2+x^4}}\,dx \\
&= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + 625x^3\sqrt{2+3x^2+x^4} - \frac{1}{10}\int \frac{-84610-138395x^2-50000x^4}{\sqrt{2+3x^2+x^4}}\,dx \\
&= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} \\
&\quad + 625x^3\sqrt{2+3x^2+x^4} - \frac{1}{30}\int \frac{-153830-115185x^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2+x^4} \\
&\quad + \frac{7679}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{15383}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{7679x(2+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2+x^4} \\
&\quad - \frac{7679(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{15383(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.58

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{19655x+36963x^3+21250x^5+3750x^7-23037i\sqrt{1+x^2}\sqrt{2+x^2}E\Bigl(i\operatorname{arcsinh}\Bigl(\frac{x}{\sqrt{}}}{6\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (19655*x + 36963*x^3 + 21250*x^5 + 3750*x^7 - (23037*I)*Sqrt[1 + x^2]*Sqrt[
2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (7729*I)*Sqrt[1 + x^2]*Sqrt[2
 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(6*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.96 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

method	result
risch	$\frac{x(3750x^6+21250x^4+36963x^2+19655)}{6\sqrt{x^4+3x^2+2}}-\frac{15383i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}}+\frac{7679i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,\bigl(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2\left(\frac{179}{4}x^3+\frac{115}{4}x\right)}{\sqrt{x^4+3x^2+2}}+625x^3\sqrt{x^4+3x^2+2}+\frac{5000x\sqrt{x^4+3x^2+2}}{3}-\frac{15383i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}}+7679i$
default	$-\frac{33614\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{15383i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}}+\frac{7679i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$

```
[In] int((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(3750*x^6+21250*x^4+36963*x^2+19655)/(x^4+3*x^2+2)^(1/2)-15383/6*I*2^
(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(
1/2)*x,2^(1/2))+7679/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/
2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.53

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{23037\,(i\,x^5+3i\,x^3+2i\,x)E(\arcsin\left(\frac{i}{x}\right)\,|\,2)+53803\,(-i\,x^5-3i\,x^3-2i\,x)F(\arcsin\left(\frac{i}{x}\right)\,|\,2)-2\,(1875\,x^8+1}{6\,(x^5+3\,x^3+2\,x)}$$

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(23037*(I*x^5 + 3*I*x^3 + 2*I*x)*elliptic_e(arcsin(I/x), 2) + 53803*(-
I*x^5 - 3*I*x^3 - 2*I*x)*elliptic_f(arcsin(I/x), 2) - 2*(1875*x^8 + 10625*x
^6 + 30000*x^4 + 44383*x^2 + 23037)*sqrt(x^4 + 3*x^2 + 2))/(x^5 + 3*x^3 + 2
*x)
```

Sympy [F]

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^5}{((x^2+1)\,(x^2+2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**5/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.308 $\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}} \, dx = \frac{637x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{625}{3}x\sqrt{2+3x^2+x^4} - \frac{637(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{1067\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}$$

```
[Out] 637/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(113*x^2+145)/(x^4+3*x^2+2)^(1/2)
-637/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2
))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1067/3*(x^2+1)^(3/2)
*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/
(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/3*x*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1219, 1693, 1203, 1113, 1149}

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}} \, dx = \frac{1067\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} - \frac{637(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{625}{3}\sqrt{x^4+3x^2+2}x + \frac{637(x^2+2)x}{2\sqrt{x^4+3x^2+2}}+\frac{(113x^2+145)x}{2\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (637*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(145 + 113*x^2))/(2*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/3 - (637*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4
]) + (1067*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x],
 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-2256-3137x^2-1250x^4}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} - \frac{1}{6}\int \frac{-4268-1911x^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} \\
&\quad + \frac{637}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{2134}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{637x(2+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} \\
&\quad - \frac{637(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{1067\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{2935x+4089x^3+1250x^5-1911i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-2357i\sqrt{}}{6\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2935*x + 4089*x^3 + 1250*x^5 - (1911*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipt
icE[I*ArcSinh[x/Sqrt[2]], 2] - (2357*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipti
cF[I*ArcSinh[x/Sqrt[2]], 2])/(6*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

method	result
risch	$\frac{x(1250x^4+4089x^2+2935)}{6\sqrt{x^4+3x^2+2}}-\frac{1067i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{637i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\Big(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2(-\frac{113}{4}x^3-\frac{145}{4}x)}{\sqrt{x^4+3x^2+2}}+\frac{625x\sqrt{x^4+3x^2+2}}{3}-\frac{1067i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{637i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\Big(F\Big(\frac{i\sqrt{2}\,x}{2},\sqrt{2}}{4\sqrt{x^4+3x^2+2}}$
default	$-\frac{4802(-\frac{3}{4}x^3-\frac{5}{4}x)}{\sqrt{x^4+3x^2+2}}-\frac{1067i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{637i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}-$

```
[In] int((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(1250*x^4+4089*x^2+2935)/(x^4+3*x^2+2)^(1/2)-1067/3*I*2^(1/2)*(2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2
))+637/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ellip
ticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.56

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{1911\,(i\,x^5+3i\,x^3+2i\,x)E(\arcsin\left(\frac{i}{x}\right)\,|2)+6179\,(-i\,x^5-3i\,x^3-2i\,x)F(\arcsin\left(\frac{i}{x}\right)\,|2)-2\,(625\,x^6+30}{6\,(x^5+3\,x^3+2\,x)}$$

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(1911*(I*x^5 + 3*I*x^3 + 2*I*x)*elliptic_e(arcsin(I/x), 2) + 6179*(-I*
x^5 - 3*I*x^3 - 2*I*x)*elliptic_f(arcsin(I/x), 2) - 2*(625*x^6 + 3000*x^4 +
 4334*x^2 + 1911)*sqrt(x^4 + 3*x^2 + 2))/(x^5 + 3*x^3 + 2*x)
```

Sympy [F]

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^4}{((x^2+1)\,(x^2+2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**4/(x**4+3*x**2+2)**(3/2),x)
[Out] Integral((5*x**2 + 7)**4/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.309 $\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{x(5-11x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{261x(2+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{261(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{169(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/2*x*(-11*x^2+5)/(x^4+3*x^2+2)^(1/2)+261/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2
61/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))
*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+169/2*(x^2+1)^(3/2)*(1
/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^
2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1219, 1203, 1113, 1149}

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{169(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{261(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}+\frac{261x(x^2+2)}{2\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*x*(2 + x^2))/(2*Sqrt[2 +
3*x^2 + x^4]) - (261*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (169*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(5-11x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-338-261x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x(5-11x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{261}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + 169\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x(5-11x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{261x(2+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{261(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{169(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{-5x+11x^3+261i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+77i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\right.}{2\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/2*(-5*x + 11*x^3 + (261*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSi
nh[x/Sqrt[2]], 2] + (77*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[
x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{x(11x^2-5)}{2\sqrt{x^4+3x^2+2}} - \frac{169i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}} + \frac{261i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2\left(\frac{11}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}} - \frac{169i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}} + \frac{261i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
default	$-\frac{686\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}} - \frac{169i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}} + \frac{261i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}} - \frac{25}{\sqrt{}}$

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*(11*x^2-5)/(x^4+3*x^2+2)^(1/2)-169/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+261/4*I*2^(
```

```
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(
1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.61

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{261\,(i\,x^5+3i\,x^3+2i\,x)E(\arcsin\left(\frac{i}{x}\right)\,|\,2)+599\,(-i\,x^5-3i\,x^3-2i\,x)F(\arcsin\left(\frac{i}{x}\right)\,|\,2)-2\,(125\,x^4+394\,x^2}{2\,(x^5+3\,x^3+2\,x)}$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(261*(I*x^5 + 3*I*x^3 + 2*I*x)*elliptic_e(arcsin(I/x), 2) + 599*(-I*x^
5 - 3*I*x^3 - 2*I*x)*elliptic_f(arcsin(I/x), 2) - 2*(125*x^4 + 394*x^2 + 26
1)*sqrt(x^4 + 3*x^2 + 2))/(x^5 + 3*x^3 + 2*x)
```

Sympy [F]

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^3}{((x^2+1)\,(x^2+2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.310 $\int \frac{\left(7+5x^2\right)^2}{\left(2+3x^2+x^4\right)^{3/2}}\, dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\begin{aligned}\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\, dx &= -\frac{17x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}}\\ &\quad+\frac{17(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{6\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] -17/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(17*x^2+25)/(x^4+3*x^2+2)^(1/2)+1
7/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*
2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+6*(x^2+1)^(3/2)*(1/(x^2
+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))
^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1219, 1203, 1113, 1149}

$$\begin{aligned}\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\, dx &= \frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\\ &\quad+\frac{17(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{17x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}}\end{aligned}$$

```
[In] Int[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (-17*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(25 + 17*x^2))/(2*Sqrt[2 +
 3*x^2 + x^4]) + (17*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x
^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-24+17x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{17}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + 12\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{17x(2+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{17(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{6\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{25x+17x^3+17i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\Big|\,2\right) - 41i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{Elli}}{2\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (25*x + 17*x^3 + (17*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/S
qrt[2]], 2] - (41*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt
[2]], 2])/(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}} - \frac{6i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}} - \frac{17i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2\left(-\frac{17}{4}x^3-\frac{25}{4}x\right)}{\sqrt{x^4+3x^2+2}} - \frac{6i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}} - \frac{17i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
default	$-\frac{98\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}} - \frac{6i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}} - \frac{17i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}} - \frac{50(-\frac{3}{2}x^3}{\sqrt{x^4+3x}}$

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(17*x^2+25)/(x^4+3*x^2+2)^(1/2)-6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-17/4*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*
x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{17\,(i\,x^4+3i\,x^2+2i)E(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)+31\,(i\,x^4+3i\,x^2+2i)F(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)-2\,\sqrt{x^4+3\,x^2}}{4\,(x^4+3\,x^2+2)}$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(17*(I*x^4 + 3*I*x^2 + 2*I)*elliptic_e(arcsin(1/2*I*sqrt(2)*x), 2) + 3
1*(I*x^4 + 3*I*x^2 + 2*I)*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2) - 2*sqrt(x
^4 + 3*x^2 + 2)*(17*x^3 + 25*x))/(x^4 + 3*x^2 + 2)
```

Sympy [F]

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^2}{((x^2+1)\,(x^2+2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx=\int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx=\int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.311 $\int \frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx=-\frac{x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(5+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] -1/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(x^2+5)/(x^4+3*x^2+2)^(1/2)+1/2*(x
^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2
)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/2*(x^2+1)^(3/2)*(1/(x^2+1))
^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/
2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1192, 1203, 1113, 1149}

$$\int \frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx=\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/2*(x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(5 + x^2))/(2*Sqrt[2 + 3*x^2
+ x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(
Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellip
ticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(5+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-2+x^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{x(5+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx
\end{aligned}$$

$$
= -\frac{x(2+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{x(5+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

$$
\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{5x+x^3+i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-3i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}}{2\sqrt{2+3x^2+x^4}}
$$

```
[In] Integrate[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (5*x + x^3 + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]],
2] - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/
(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.87

method	result
risch	$\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2\left(-\frac{1}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
default	$-\frac{14\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}-\frac{10(x^3+\frac{3}{2}}{\sqrt{x^4+3x^2}}$

```
[In] int((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x^2+5)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/
2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-1/4*I*2^(1/2)*(2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2
^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.56

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{(-i\,x^4-3i\,x^2-2i)E(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)-3\,(i\,x^4+3i\,x^2+2i)F(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)}{4\,(x^4+3\,x^2+2)}$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*((-I*x^4 - 3*I*x^2 - 2*I)*elliptic_e(arcsin(1/2*I*sqrt(2)*x), 2) - 3*(I
*x^4 + 3*I*x^2 + 2*I)*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2) + 2*sqrt(x^4 +
 3*x^2 + 2)*(x^3 + 5*x))/(x^4 + 3*x^2 + 2)
```

Sympy [F]

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{5x^2+7}{((x^2+1)\,(x^2+2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{5\,x^2+7}{{\left(x^4+3\,x^2+2\right)}^{3/2}} \,d x$$

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.312 $\int \frac{1}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 149

$$\int \frac{1}{\left(2+3x^2+x^4\right)^{3/2}}\,dx=-\frac{3x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(5+3x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}$$

```
[Out] -3/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(3*x^2+5)/(x^4+3*x^2+2)^(1/2)+3/2*
(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1
/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-(x^2+1)^(3/2)*(1/(x^2+1))^(
1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)
/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1106, 1203, 1113, 1149}

$$\int \frac{1}{\left(2+3x^2+x^4\right)^{3/2}}\,dx=-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{3x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(2 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (-3*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 3*x^2))/(2*Sqrt[2 + 3*
x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1
/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(5+3x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int\frac{4+3x^2}{\sqrt{2+3x^2+x^4}}\,dx\\&=\frac{x(5+3x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{3}{2}\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx-2\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx\end{aligned}$$

$$= -\frac{3x(2+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{x(5+3x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{5x+3x^3+3i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}}{2\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(2 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (5*x + 3*x^3 + (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt
[2]], 2] + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]
)/(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$	128
default	$-\frac{2\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$	129
elliptic	$-\frac{2\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$	129

```
[In] int(1/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(3*x^2+5)/(x^4+3*x^2+2)^(1/2)+I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)
/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-3/4*I*2^(1/2)*(2*x^
2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(
1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{3\,(i\,x^4+3i\,x^2+2i)E(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)+11\,(-i\,x^4-3i\,x^2-2i)F(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)-2\,\sqrt{x^4+3\,x^2}}{4\,(x^4+3\,x^2+2)}$$

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(3*(I*x^4 + 3*I*x^2 + 2*I)*elliptic_e(arcsin(1/2*I*sqrt(2)*x), 2) + 11
*(-I*x^4 - 3*I*x^2 - 2*I)*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2) - 2*sqrt(x
^4 + 3*x^2 + 2)*(3*x^3 + 5*x))/(x^4 + 3*x^2 + 2)
```

Sympy [F]

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

```
[In] int(1/(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int(1/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.313 $\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx = \frac{x}{6\sqrt{2+3x^2+x^4}} + \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}} - \frac{9(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{4\sqrt{2+3x^2+x^4}} + \frac{125(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{84\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/6*x/(x^4+3*x^2+2)^(1/2)+125/168*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticP
i(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x
^2+2)^(1/2)+1/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1
/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-9/4*(x^2+1)
^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(2
*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{1235, 1192, 1203, 1113, 1149, 1228, 1470, 553\}$

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx = -\frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{125(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{84\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} - \frac{x(x^2+2)}{3\sqrt{x^4+3x^2+2}} + \frac{x(2x^2+5)}{6\sqrt{x^4+3x^2+2}}$$

```
[In] Int[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/3*(x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(5 + 2*x^2))/(6*Sqrt[2 + 3*x^
2 + x^4]) + (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) - (9*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]
*EllipticF[ArcTan[x], 1/2])/(4*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (125*(2 + x
^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(84*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{6}\int \frac{-8-5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx\right) - \frac{25}{6}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} + \frac{1}{12}\int \frac{-2-4x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{12}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + \frac{125}{24}\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} - \frac{25(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{12\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{1}{6}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{1}{3}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{\left(125\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{24\sqrt{2+3x^2+x^4}}$$

$$= -\frac{x(2+x^2)}{3\sqrt{2+3x^2+x^4}} + \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} + \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}} - \frac{9(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{125(2+x^2)\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{84\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.80

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx = \frac{35x+14x^3+14i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-7i\sqrt{1+x^2}\sqrt{2}}{}$$

```
[In] Integrate[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (35*x + 14*x^3 + (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/S
qrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[
2]], 2] + (25*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/S
qrt[2]], 2])/(42*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93

method	result
default	$-\frac{2\left(-\frac{1}{6}x^3-\frac{5}{12}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{12\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}}+$
elliptic	$-\frac{2\left(-\frac{1}{6}x^3-\frac{5}{12}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{12\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}}+$
risch	$\frac{x(2x^2+5)}{6\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{12\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{6\sqrt{x^4+3x^2+2}}+$

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/6*x^3-5/12*x)/(x^4+3*x^2+2)^(1/2)-1/12*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/6*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1
/2)*x,2^(1/2))+25/42*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)\,(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^10 + 37*x^8 + 107*x^6 + 151*x^4 + 104*x
^2 + 28), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)\,(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{((x^2+1)\,(x^2+2))^{\frac{3}{2}}\cdot(5x^2+7)}\,dx$$

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)\left(2+3x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)\left(2+3x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\left(2+3x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(5\,x^2+7\right)\,\left(x^4+3\,x^2+2\right)^{3/2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2)), x)
```

3.314 $\int \frac{1}{(7+5x^2)^2(2+3x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 235

$$\int \frac{1}{(7+5x^2)^2(2+3x^2+x^4)^{3/2}}\,dx = -\frac{31x(2+x^2)}{56\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\,(7+5x^2)} + \frac{31(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{28\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{463(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{336\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{375(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{784\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

```
[Out] -31/56*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/36*x*(11*x^2+20)/(x^4+3*x^2+2)^(1/2)
+375/1568*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2
),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+31/5
6*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^
(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-463/672*(x^2+1)^(3/2)*(1/
(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2
+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/504*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1242, 1192, 1203, 1113, 1149, 1237, 1730, 1228, 1470, 553}

$$\int \frac{1}{(7+5x^2)^2\,(2+3x^2+x^4)^{3/2}}\,dx = -\frac{463(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{336\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{31(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{28\sqrt{2}\sqrt{x^4+3x^2+2}} + \frac{375(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{784\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}} + \frac{625\sqrt{x^4+3x^2+2}\,x}{504\,(5x^2+7)} - \frac{31(x^2+2)\,x}{56\sqrt{x^4+3x^2+2}} + \frac{(11x^2+20)\,x}{36\sqrt{x^4+3x^2+2}}$$

```
[In] Int[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/(36*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(7 + 5*x^2)) + (31*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(28*Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4]) - (463*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[A
rcTan[x], 1/2])/(336*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 +
3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
```

```
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
```

```
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{14+5x^2}{36\,(2+3x^2+x^4)^{3/2}} - \frac{25}{6\,(7+5x^2)^2\sqrt{2+3x^2+x^4}} - \frac{25}{36\,(7+5x^2)\sqrt{2+3x^2+x^4}}\right) dx \\
&= \frac{1}{36}\int \frac{14+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx - \frac{25}{36}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx \\
&\quad - \frac{25}{6}\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\,(7+5x^2)} \\
&\quad - \frac{1}{72}\int \frac{26+22x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{504}\int \frac{62+70x^2+25x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx \\
&\quad - \frac{25}{72}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + \frac{125}{144}\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\,(7+5x^2)} - \frac{25(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{72\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{1}{504}\int \frac{-175-125x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{11}{36}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{13}{36}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&\quad - \frac{325}{504}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx + \frac{\left(125\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{144\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{11x(2+x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\,(7+5x^2)} \\
&\quad + \frac{11(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{18\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{17(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{24\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{125(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{504\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} - \frac{125}{504}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&\quad - \frac{325\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{1008} - \frac{25}{72}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + \frac{1625\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{2016}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{31x(2+x^2)}{56\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\,(7+5x^2)} \\
&\quad + \frac{31(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{28\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{463(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{336\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{125(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{504\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} + \frac{\left(1625\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{2016\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{31x(2+x^2)}{56\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{625x\sqrt{2+3x^2+x^4}}{504\,(7+5x^2)} + \frac{31(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{28\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{463(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{336\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{375(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{784\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.89

$$\int \frac{1}{(7+5x^2)^2\,(2+3x^2+x^4)^{3/2}}\,dx = \frac{7490x+10157x^3+3255x^5+651i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)\,E\Big(i\mathrm{arcsin}}{}$$

```
[In] Integrate[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (7490*x + 10157*x^3 + 3255*x^5 + (651*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5
*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (182*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (1575*I)*Sqrt[1 + x^2]
*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (1125*I)*x^2*Sqr
t[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(1176*(
7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.79

method	result
default	$-\frac{2\left(-\frac{11}{72}x^3-\frac{5}{18}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{625x\sqrt{x^4+3x^2+2}}{504(5x^2+7)}+\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}+\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{112\sqrt{x^4+3x^2+2}}+$
elliptic	$-\frac{2\left(-\frac{11}{72}x^3-\frac{5}{18}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{625x\sqrt{x^4+3x^2+2}}{504(5x^2+7)}+\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}+\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{112\sqrt{x^4+3x^2+2}}+$
risch	$\frac{x\left(465x^4+1451x^2+1070\right)}{168(5x^2+7)\sqrt{x^4+3x^2+2}}+\frac{17i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{48\sqrt{x^4+3x^2+2}}-\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{112\sqrt{x^4+3x^2+2}}+$

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-11/72*x^3-5/18*x)/(x^4+3*x^2+2)^(1/2)+625/504*x*(x^4+3*x^2+2)^(1/2)/(5
*x^2+7)+13/168*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+31/112*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+75/392*I*2^(1
/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^
(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^12 + 220*x^10 + 794*x^8 + 1504*x^6 + 1
577*x^4 + 868*x^2 + 196), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{((x^2+1)\,(x^2+2))^{\frac{3}{2}}\,(5x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(7+5x^2\right)^2\left(2+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(5\,x^2+7\right)^2\,\left(x^4+3\,x^2+2\right)^{3/2}} \,dx$$

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2)), x)
```

3.315 $\int \frac{1}{(7+5x^2)^3(2+3x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 263

$$\begin{aligned}\int \frac{1}{(7+5x^2)^3\,(2+3x^2+x^4)^{3/2}}\,dx &= -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}}+\frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}}\\ &\quad+\frac{625x\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2}+\frac{41875x\sqrt{2+3x^2+x^4}}{84672\,(7+5x^2)}+\frac{5797(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{14112\sqrt{2}\sqrt{2+3x^2+x^4}}\\ &\quad-\frac{49907(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\mathrm{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{56448\sqrt{2}\sqrt{2+3x^2+x^4}}\\ &\quad+\frac{192625(2+x^2)\,\mathrm{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{395136\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] -5797/28224*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/216*x*(23*x^2+50)/(x^4+3*x^2+2)
^(1/2)+192625/790272*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(
x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+5797/28224*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2)
,1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-49907/112
896*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/1008*x*(x^4+3*x^2+2
)^(1/2)/(5*x^2+7)^2+41875/84672*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1242, 1192, 1203, 1113, 1149, 1237, 1710, 1730, 1228, 1470, 553}

$$\begin{aligned}\int \frac{1}{(7+5x^2)^3\,(2+3x^2+x^4)^{3/2}}\,dx = &-\frac{49907(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{56448\sqrt{2}\sqrt{x^4+3x^2+2}}\\ &+\frac{5797(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{14112\sqrt{2}\sqrt{x^4+3x^2+2}}\\ &+\frac{192625(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{395136\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{41875\sqrt{x^4+3x^2+2}\,x}{84672\,(5x^2+7)}\\ &+\frac{625\sqrt{x^4+3x^2+2}\,x}{1008\,(5x^2+7)^2}-\frac{5797(x^2+2)\,x}{28224\sqrt{x^4+3x^2+2}}+\frac{(23x^2+50)\,x}{216\sqrt{x^4+3x^2+2}}\end{aligned}$$

```
[In] Int[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x^2))/(216*
Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(1008*(7 + 5*x^2)^2)
 + (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7 + 5*x^2)) + (5797*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[2 +
3*x^2 + x^4]) - (49907*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(56448*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2
 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{-62-35x^2}{216\left(2+3x^2+x^4\right)^{3/2}} - \frac{25}{6\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}} - \frac{25}{36\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}} - \frac{175}{216\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\right) dx$$

$$
\begin{aligned}
&= -\left(\frac{1}{216}\int \frac{-62-35x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx\right) - \frac{25}{36}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx \\
&\quad - \frac{175}{216}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{6}\int \frac{1}{\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} + \frac{625x\sqrt{2+3x^2+x^4}}{3024\left(7+5x^2\right)} \\
&\quad + \frac{1}{432}\int \frac{-38-46x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{25\int \frac{62+70x^2+25x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{3024} - \frac{25\int \frac{74-10x^2-25x^4}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx}{1008} \\
&\quad - \frac{175}{432}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + \frac{875}{864}\int \frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} + \frac{41875x\sqrt{2+3x^2+x^4}}{84672\left(7+5x^2\right)} \\
&\quad - \frac{175(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{432\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{25\int \frac{2838+2310x^2+975x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{84672} \\
&\quad + \frac{\int \frac{-175-125x^2}{\sqrt{2+3x^2+x^4}}\,dx}{3024} - \frac{19}{216}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{23}{216}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&\quad - \frac{325\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{3024} + \frac{\left(875\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{864\sqrt{2+3x^2+x^4}} \\
&= -\frac{23x(2+x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} \\
&\quad + \frac{41875x\sqrt{2+3x^2+x^4}}{84672\left(7+5x^2\right)} + \frac{23(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{108\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{71(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{144\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{125(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{432\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{\int \frac{-4725-4875x^2}{\sqrt{2+3x^2+x^4}}\,dx}{84672} - \frac{125\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx}{3024} \\
&\quad - \frac{325\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{6048} - \frac{25}{432}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\
&\quad + \frac{1625\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{12096} - \frac{12625\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{28224}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{149x(2+x^2)}{1008\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} \\
&\quad + \frac{41875x\sqrt{2+3x^2+x^4}}{84672\left(7+5x^2\right)} + \frac{149(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{504\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{1219(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2016\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{125(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{432\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{25}{448}\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{1625\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx}{28224} - \frac{12625\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{56448} \\
&\quad + \frac{63125\int\frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{112896} + \frac{\left(1625\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{12096\sqrt{2+3x^2+x^4}} \\
&= -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} \\
&\quad + \frac{41875x\sqrt{2+3x^2+x^4}}{84672\left(7+5x^2\right)} + \frac{5797(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{14112\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{49907(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\right.}{56448\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{4625(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{14112\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}} + \frac{\left(63125\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{112896\sqrt{2+3x^2+x^4}} \\
&= -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\left(7+5x^2\right)^2} \\
&\quad + \frac{41875x\sqrt{2+3x^2+x^4}}{84672\left(7+5x^2\right)} + \frac{5797(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{14112\sqrt{2}\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{49907(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{56448\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{192625(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{395136\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.60

$$
\int\frac{1}{\left(7+5x^2\right)^3\left(2+3x^2+x^4\right)^{3/2}}\,dx = \frac{\frac{7x\left(550550+1089803x^2+698290x^4+144925x^6\right)}{\left(7+5x^2\right)^2} + 40579i\sqrt{1+x^2}\sqrt{2+x^2}E\Big(i\mathrm{arcsin}}{}
$$

```
[In] Integrate[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] ((7*x*(550550 + 1089803*x^2 + 698290*x^4 + 144925*x^6))/(7 + 5*x^2)^2 + (40
579*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (74
2*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (3852
5*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])
/(197568*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.51 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.73

method	result
risch	$\frac{x\left(144925x^{6}+698290x^{4}+1089803x^{2}+550550\right)}{28224\left(5x^{2}+7\right)^{2}\sqrt{x^{4}+3x^{2}+2}}+\frac{271i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2688\sqrt{x^{4}+3x^{2}+2}}-$
default	$-\frac{2\left(-\frac{23}{432}x^{3}-\frac{25}{216}x\right)}{\sqrt{x^{4}+3x^{2}+2}}+\frac{625x\sqrt{x^{4}+3x^{2}+2}}{1008\left(5x^{2}+7\right)^{2}}+\frac{41875x\sqrt{x^{4}+3x^{2}+2}}{84672\left(5x^{2}+7\right)}-\frac{53i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{28224\sqrt{x^{4}+3x^{2}+2}}+$
elliptic	$-\frac{2\left(-\frac{23}{432}x^{3}-\frac{25}{216}x\right)}{\sqrt{x^{4}+3x^{2}+2}}+\frac{625x\sqrt{x^{4}+3x^{2}+2}}{1008\left(5x^{2}+7\right)^{2}}+\frac{41875x\sqrt{x^{4}+3x^{2}+2}}{84672\left(5x^{2}+7\right)}-\frac{53i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{28224\sqrt{x^{4}+3x^{2}+2}}+$

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28224*x*(144925*x^6+698290*x^4+1089803*x^2+550550)/(5*x^2+7)^2/(x^4+3*x^2
+2)^(1/2)+271/2688*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1
/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-5797/56448*I*2^(1/2)*(2*x^2+4)^(1/2)
*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-Elli
pticE(1/2*I*2^(1/2)*x,2^(1/2)))+38525/197568*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{1}{\left(7+5x^{2}\right)^{3}\left(2+3x^{2}+x^{4}\right)^{3/2}}\,dx=\int \frac{1}{\left(x^{4}+3\,x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)^{3}}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^14 + 1275*x^12 + 5510*x^10 + 13078*x^
8 + 18413*x^6 + 15379*x^4 + 7056*x^2 + 1372), x)
```

Sympy [F]

$$\int \frac{1}{\left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

Maxima [F]

$$\int \frac{1}{\left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Giac [F]

$$\int \frac{1}{\left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(5\,x^2+7\right)^3\left(x^4+3\,x^2+2\right)^{3/2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2)), x)
```

3.316 $\int (7+5x^2)^4 \sqrt{2+x^2-x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\begin{aligned}\int (7+5x^2)^4 \sqrt{2+x^2-x^4}\,dx &= \frac{1}{231}x(177953+717372x^2)\sqrt{2+x^2-x^4}-\frac{116100}{77}x(2+x^2-x^4)^{3/2}\\&\quad-\frac{14500}{33}x^3(2+x^2-x^4)^{3/2}-\frac{625}{11}x^5(2+x^2-x^4)^{3/2}\\&\quad+\frac{3764813}{231}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{539419}{77}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\end{aligned}$$

```
[Out] -116100/77*x*(-x^4+x^2+2)^(3/2)-14500/33*x^3*(-x^4+x^2+2)^(3/2)-625/11*x^5*
(-x^4+x^2+2)^(3/2)+3764813/231*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-539419/77
*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/231*x*(717372*x^2+177953)*(-x^4+x^2+2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1220, 1693, 1190, 1194, 538, 435, 430}

$$\begin{aligned}\int (7+5x^2)^4 \sqrt{2+x^2-x^4}\,dx = &-\frac{539419}{77}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\\&+\frac{3764813}{231}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\\&-\frac{116100}{77}\left(-x^4+x^2+2\right)^{3/2}x\\&+\frac{1}{231}\left(717372x^2+177953\right)\sqrt{-x^4+x^2+2}x\\&-\frac{625}{11}\left(-x^4+x^2+2\right)^{3/2}x^5-\frac{14500}{33}\left(-x^4+x^2+2\right)^{3/2}x^3\end{aligned}$$

```
[In] Int[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(177953 + 717372*x^2)*Sqrt[2 + x^2 - x^4])/231 - (116100*x*(2 + x^2 - x^
4)^(3/2))/77 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/33 - (625*x^5*(2 + x^2 - x
^4)^(3/2))/11 + (3764813*EllipticE[ArcSin[x/Sqrt[2]], -2])/231 - (539419*El
lipticF[ArcSin[x/Sqrt[2]], -2])/77
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c])))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} \\
&\quad -\frac{1}{11}\int \sqrt{2+x^2-x^4}\left(-26411-75460x^2-87100x^4-43500x^6\right)\,dx \\
&= -\frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} \\
&\quad -\frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2}+\frac{1}{99}\int \sqrt{2+x^2-x^4}\left(237699+940140x^2+1044900x^4\right)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} \\
&\quad - \frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} - \frac{1}{693}\int \left(-3753693-10760580x^2\right)\sqrt{2+x^2-x^4}\,dx \\
&= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} \\
&\quad - \frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} + \frac{\int \frac{96595020+169416585x^2}{\sqrt{2+x^2-x^4}}\,dx}{10395} \\
&= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} \\
&\quad - \frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} + \frac{2\int \frac{96595020+169416585x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{10395} \\
&= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} \\
&\quad - \frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} \\
&\quad - \frac{1078838}{77}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx + \frac{3764813}{231}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx \\
&= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} \\
&\quad - \frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} \\
&\quad + \frac{3764813}{231}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{539419}{77}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$
\int \left(7+5x^2\right)^4\sqrt{2+x^2-x^4}\,dx
$$

$$
= \frac{-1037294x-186503x^3+1125819x^5+231228x^7-105925x^9-75250x^{11}-13125x^{13}+3764813i\sqrt{4+2x}}{231\sqrt{2+x^2-x^4}}
$$

```
[In] Integrate[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-1037294*x - 186503*x^3 + 1125819*x^5 + 231228*x^7 - 105925*x^9 - 75250*x^
11 - 13125*x^13 + (3764813*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x
], -1/2] - (4838091*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2
])/(231*Sqrt[2 + x^2 - x^4])
```

Maple [A] (verified)

Time = 7.91 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{x\left(13125x^8+88375x^6+220550x^4+166072x^2-518647\right)\left(x^4-x^2-2\right)}{231\sqrt{-x^4+x^2+2}}+\frac{1073278\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{231\sqrt{-x^4+x^2+2}}-3764813\sqrt{}$
default	$-\frac{518647x\sqrt{-x^4+x^2+2}}{231}+\frac{1073278\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{231\sqrt{-x^4+x^2+2}}-\frac{3764813\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{}}{2}\right.\right.}{462\sqrt{-x^4+x^2+2}}$
elliptic	$-\frac{518647x\sqrt{-x^4+x^2+2}}{231}+\frac{1073278\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{231\sqrt{-x^4+x^2+2}}-\frac{3764813\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{}}{2}\right.\right.}{462\sqrt{-x^4+x^2+2}}$

```
[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/231*x*(13125*x^8+88375*x^6+220550*x^4+166072*x^2-518647)*(x^4-x^2-2)/(-x
^4+x^2+2)^(1/2)+1073278/231*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^
2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-3764813/462*2^(1/2)*(-2*x^2+4
)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2)
)-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int \left(7+5x^2\right)^4\sqrt{2+x^2-x^4}\,dx$$
$$=\frac{-7529626i\,\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})+8602904i\,\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})+(13125\,x^{10}+88375\,x^{8}+220}{231\,x}$$

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/231*(-7529626*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 8602904*I
*sqrt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) + (13125*x^10 + 88375*x^8 +
220550*x^6 + 166072*x^4 - 518647*x^2 - 3764813)*sqrt(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\,dx=\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^4\,dx$$

```
[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**4, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\,dx=\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^4\,dx$$

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\,dx=\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^4\,dx$$

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\,dx=\int \left(5\,x^2+7\right)^4\sqrt{-x^4+x^2+2}\,dx$$

```
[In] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(1/2), x)
```

3.317 $\int (7+5x^2)^3 \sqrt{2+x^2-x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int (7+5x^2)^3 \sqrt{2+x^2-x^4}\,dx = \frac{1}{63}x(5956+14691x^2)\sqrt{2+x^2-x^4}-\frac{1825}{21}x(2+x^2-x^4)^{3/2} -\frac{125}{9}x^3(2+x^2-x^4)^{3/2}+\frac{79411}{63}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) -\frac{8735}{21}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] -1825/21*x*(-x^4+x^2+2)^(3/2)-125/9*x^3*(-x^4+x^2+2)^(3/2)+79411/63*Ellipti
cE(1/2*x*2^(1/2),I*2^(1/2))-8735/21*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/63
*x*(14691*x^2+5956)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1220, 1693, 1190, 1194, 538, 435, 430}

$$\int (7+5x^2)^3 \sqrt{2+x^2-x^4}\,dx = -\frac{8735}{21}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) +\frac{79411}{63}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1825}{21}(-x^4+x^2+2)^{3/2}x +\frac{1}{63}(14691x^2+5956)\sqrt{-x^4+x^2+2}x -\frac{125}{9}(-x^4+x^2+2)^{3/2}x^3$$

```
[In] Int[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/63 - (1825*x*(2 + x^2 - x^4)^(3/
2))/21 - (125*x^3*(2 + x^2 - x^4)^(3/2))/9 + (79411*EllipticE[ArcSin[x/Sqrt
[2]], -2])/63 - (8735*EllipticF[ArcSin[x/Sqrt[2]], -2])/21
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
```

```
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2}-\frac{1}{9}\int\left(-3087-7365x^2-5475x^4\right)\sqrt{2+x^2-x^4}\,dx\\
&= -\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}-\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2}+\frac{1}{63}\int\left(32559+73455x^2\right)\sqrt{2+x^2-x^4}\,dx\\
&= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4}-\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}\\
&\quad -\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2}-\frac{1}{945}\int\frac{-798090-1191165x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4}-\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}\\
&\quad -\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2}-\frac{2}{945}\int\frac{-798090-1191165x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4}-\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}\\
&\quad -\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2}-\frac{17470}{21}\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{79411}{63}\int\frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4}-\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}-\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2}\\
&\quad +\frac{79411}{63}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{8735}{21}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx$$
$$= \frac{-9988x+9938x^3+21660x^5-1116x^7-3725x^9-875x^{11}+79411i\sqrt{4+2x^2-2x^4}E\left(i\text{arcsinh}(x)\left|-\tfrac{1}{2}\right.\right)-}{63\sqrt{2+x^2-x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-9988*x + 9938*x^3 + 21660*x^5 - 1116*x^7 - 3725*x^9 - 875*x^11 + (79411*I
)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (106014*I)*Sqrt[4
 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(63*Sqrt[2 + x^2 - x^4])
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{x\left(875x^6+4600x^4+7466x^2-4994\right)\left(x^4-x^2-2\right)}{63\sqrt{-x^4+x^2+2}}+\frac{26603\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{63\sqrt{-x^4+x^2+2}}-\frac{79411\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\bigl(F\bigl(\frac{x}{}}{126\sqrt{-x^4+x}}$
default	$-\frac{4994x\sqrt{-x^4+x^2+2}}{63}+\frac{26603\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{63\sqrt{-x^4+x^2+2}}-\frac{79411\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{126\sqrt{-x^4+x^2+2}}$
elliptic	$-\frac{4994x\sqrt{-x^4+x^2+2}}{63}+\frac{26603\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{63\sqrt{-x^4+x^2+2}}-\frac{79411\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{126\sqrt{-x^4+x^2+2}}$

```
[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/63*x*(875*x^6+4600*x^4+7466*x^2-4994)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+266
03/63*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1
/2*x*2^(1/2),I*2^(1/2))-79411/126*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-
x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2
),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx$$
$$= \frac{-158822i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right) |-\frac{1}{2}) + 185425i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right) |-\frac{1}{2}) + (875\,x^8 + 4600\,x^6 + 7466\,x^4 -}{63\,x}$$

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/63*(-158822*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 185425*I*sq
rt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) + (875*x^8 + 4600*x^6 + 7466*x^
4 - 4994*x^2 - 79411)*sqrt(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^3\,dx$$

```
[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^3\,dx$$

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^3 dx$$

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx = \int \left(5\,x^2+7\right)^3 \sqrt{-x^4+x^2+2}\,dx$$

```
[In] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2), x)
```

3.318 $\int (7+5x^2)^2 \sqrt{2+x^2-x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\begin{aligned}\int \left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,dx=&\ \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}\\&-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{2045}{21}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\\&-\frac{79}{7}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\end{aligned}$$

```
[Out] -25/7*x*(-x^4+x^2+2)^(3/2)+2045/21*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-79/7*
EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/21*x*(354*x^2+275)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1220, 1190, 1194, 538, 435, 430}

$$\begin{aligned}\int \left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,dx=&-\frac{79}{7}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\\&+\frac{2045}{21}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}\\&+\frac{1}{21}x\left(354x^2+275\right)\sqrt{-x^4+x^2+2}\end{aligned}$$

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])/21 - (25*x*(2 + x^2 - x^4)^(3/2))/7
 + (2045*EllipticE[ArcSin[x/Sqrt[2]], -2])/21 - (79*EllipticF[ArcSin[x/Sqrt
[2]], -2])/7
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}-\frac{1}{7}\int\left(-393-590x^2\right)\sqrt{2+x^2-x^4}\,dx\\
&= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{1}{105}\int\frac{9040+10225x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{2}{105}\int\frac{9040+10225x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}\\
&\quad-\frac{158}{7}\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{2045}{21}\int\frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}\\
&\quad+\frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{79}{7}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,dx$$
$$=\frac{250x+683x^3+304x^5-204x^7-75x^9+2045i\sqrt{4+2x^2-2x^4}E\left(i\operatorname{arcsinh}(x)\,\middle|-\frac{1}{2}\right)-2949i\sqrt{4+2x^2-2}}{21\sqrt{2+x^2-x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (250*x + 683*x^3 + 304*x^5 - 204*x^7 - 75*x^9 + (2045*I)*Sqrt[4 + 2*x^2 - 2
*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2949*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elli
pticF[I*ArcSinh[x], -1/2])/(21*Sqrt[2 + x^2 - x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(68) = 136.

Time = 2.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

method	result
risch	$-\frac{x\left(75x^4+279x^2+125\right)\left(x^4-x^2-2\right)}{21\sqrt{-x^4+x^2+2}}+\frac{904\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{21\sqrt{-x^4+x^2+2}}-\frac{2045\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\right.}{42\sqrt{-x^4+x^2+2}}$
default	$\frac{125x\sqrt{-x^4+x^2+2}}{21}+\frac{904\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{21\sqrt{-x^4+x^2+2}}-\frac{2045\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{42\sqrt{-x^4+x^2+2}}+25$
elliptic	$\frac{125x\sqrt{-x^4+x^2+2}}{21}+\frac{904\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{21\sqrt{-x^4+x^2+2}}-\frac{2045\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{42\sqrt{-x^4+x^2+2}}+25$

```
[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*x*(75*x^4+279*x^2+125)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+904/21*2^(1/2)*
(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I
*2^(1/2))-2045/42*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)
*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,dx$$
$$=\frac{-4090i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+4994i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+(75\,x^6+279\,x^4+125\,x^2-2045)\sqrt{-}}{21\,x}$$

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-4090*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 4994*I*sqrt(2
)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) + (75*x^6 + 279*x^4 + 125*x^2 - 204
5)*sqrt(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx=\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^2\,dx$$

```
[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx=\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx=\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx=\int \left(5\,x^2+7\right)^2\sqrt{-x^4+x^2+2}\,dx$$

```
[In] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2), x)
```

3.319 $\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx = x(2+x^2)\sqrt{2+x^2-x^4}+7E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + 3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] 7*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+3*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+x
*(x^2+2)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1190, 1194, 538, 435, 430}

$$\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx = 3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) + 7E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+x\sqrt{-x^4+x^2+2}(x^2+2)$$

```
[In] Int[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]], -2] + 3*El
lipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x(2+x^2)\sqrt{2+x^2-x^4}-\frac{1}{15}\int\frac{-150-105x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= x(2+x^2)\sqrt{2+x^2-x^4}-\frac{2}{15}\int\frac{-150-105x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= x(2+x^2)\sqrt{2+x^2-x^4}+6\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+7\int\frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= x(2+x^2)\sqrt{2+x^2-x^4}+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.95 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int \left(7+5x^2\right)\sqrt{2+x^2-x^4}\,dx = \frac{4x+4x^3-x^5-x^7+7i\sqrt{4+2x^2-2x^4}\,E\left(i\text{arcsinh}(x)\left|-\frac{1}{2}\right.\right)-12i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left(i\text{arcsinh}(x),\right.}{\sqrt{2+x^2-x^4}}$$

```
[In] Integrate[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (4*x + 4*x^3 - x^5 - x^7 + (7*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSin
h[x], -1/2] - (12*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])
/Sqrt[2 + x^2 - x^4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(50) = 100$.

Time = 1.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.04

method	result
risch	$-\frac{x\left(x^2+2\right)\left(x^4-x^2-2\right)}{\sqrt{-x^4+x^2+2}}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}$
default	$2x\sqrt{-x^4+x^2+2}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}+x^3$
elliptic	$2x\sqrt{-x^4+x^2+2}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}+x^3$

```
[In] int((5*x^2+7)*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*(x^2+2)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+5*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-7/2*2^(1/2)*(
-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I
*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \left(7+5x^2\right)\sqrt{2+x^2-x^4}\,dx = \frac{-14i\sqrt{2}xE\left(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2}\right)+19i\sqrt{2}xF\left(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2}\right)+\left(x^4+2\,x^2-7\right)\sqrt{-x^4+x^2+2}}{x}$$

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
[Out] (-14*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 19*I*sqrt(2)*x*ellip
tic_f(arcsin(sqrt(2)/x), -1/2) + (x^4 + 2*x^2 - 7)*sqrt(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)\sqrt{2+x^2-x^4}\,dx = \int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)\,dx$$

```
[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7), x)
```

Maxima [F]

$$\int \left(7+5x^2\right)\sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)dx$$

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

Giac [F]

$$\int \left(7+5x^2\right)\sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)dx$$

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)\sqrt{2+x^2-x^4}\,dx=\int \left(5\,x^2+7\right)\,\sqrt{-x^4+x^2+2}\,dx$$

```
[In] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2), x)
```

3.320 $\int \sqrt{2+x^2-x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 44

$$\int \sqrt{2+x^2-x^4}\,dx = \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] 1/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1
/3*x*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1105, 1194, 538, 435, 430}

$$\int \sqrt{2+x^2-x^4}\,dx = \text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) + \frac{1}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{3}\sqrt{-x^4+x^2+2}\,x$$

```
[In] Int[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/3 + EllipticE[ArcSin[x/Sqrt[2]], -2]/3 + EllipticF[
ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}\int \frac{4+x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{2}{3}\int \frac{4+x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+2\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.93 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.05

$$\int \sqrt{2+x^2-x^4}\,dx$$
$$=\frac{2x+x^3-x^5+i\sqrt{4+2x^2-2x^4}E\left(i\text{arcsinh}(x)\,\middle|-\frac{1}{2}\right)-3i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)}{3\sqrt{2+x^2-x^4}}$$

```
[In] Integrate[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (2*x + x^3 - x^5 + I*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2]
- (3*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(3*Sqrt[2 +
x^2 - x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(44) = 88.

Time = 0.64 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.84

method	result	size
default	$\frac{x\sqrt{-x^4+x^2+2}}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{6\sqrt{-x^4+x^2+2}}$	125
elliptic	$\frac{x\sqrt{-x^4+x^2+2}}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{6\sqrt{-x^4+x^2+2}}$	125
risch	$-\frac{x\left(x^4-x^2-2\right)}{3\sqrt{-x^4+x^2+2}}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{6\sqrt{-x^4+x^2+2}}$	135

```
[In] int((-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(-x^4+x^2+2)^(1/2)+2/3*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-1/6*2^(1/2)*(-2*x^2+4)^(1/2)
*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-Ellip
ticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \sqrt{2+x^2-x^4}\,dx = \frac{-2i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) + 4i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) + \sqrt{-x^4+x^2+2}(x^2-1)}{3x}$$

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(-2*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 4*I*sqrt(2)*x*ell
iptic_f(arcsin(sqrt(2)/x), -1/2) + sqrt(-x^4 + x^2 + 2)*(x^2 - 1))/x
```

Sympy [F]

$$\int \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\,dx$$

```
[In] integrate((-x**4+x**2+2)**(1/2),x)
[Out] Integral(sqrt(-x**4 + x**2 + 2), x)
```

Maxima [F]

$$\int \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

Giac [F]

$$\int \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+x^2-x^4}\,dx=\int \sqrt{-x^4+x^2+2}\,dx$$

```
[In] int((x^2 - x^4 + 2)^(1/2),x)
[Out] int((x^2 - x^4 + 2)^(1/2), x)
```

3.321 $\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx=-\frac{1}{5}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{25}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{34}{175}\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] -1/5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+17/25*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-34/175*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1222, 1194, 538, 435, 430, 1226, 551}

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx=\frac{17}{25}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{1}{5}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

```
[Out] -1/5*EllipticE[ArcSin[x/Sqrt[2]], -2] + (17*EllipticF[ArcSin[x/Sqrt[2]], -2
])/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
```

```
d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{25}\int \frac{-12+5x^2}{\sqrt{2+x^2-x^4}}\,dx\right)-\frac{34}{25}\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx\\
&= -\left(\frac{2}{25}\int \frac{-12+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\right)-\frac{68}{25}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7+5x^2)}\,dx\\
&= -\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{5}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{34}{25}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= -\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\right.\\
&\qquad \left.-2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.93 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\begin{aligned}
\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx &= -\frac{1}{175}i\sqrt{2}\left(35E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right)+7\,\text{EllipticF}\left(i\operatorname{arcsinh}(x),-\frac{1}{2}\right)\right.\\
&\qquad \left.-17\,\text{EllipticPi}\left(\frac{5}{7},i\operatorname{arcsinh}(x),-\frac{1}{2}\right)\right)
\end{aligned}$$

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
[Out] (-1/175*I)*Sqrt[2]*(35*EllipticE[I*ArcSinh[x], -1/2] + 7*EllipticF[I*ArcSin
h[x], -1/2] - 17*EllipticPi[5/7, I*ArcSinh[x], -1/2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(47) = 94.

Time = 0.76 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

method	result	siz
default	$\frac{17\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{50\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{10\sqrt{-x^4+x^2+2}}-\frac{34\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\frac{10}{7},i\sqrt{2}\right)}{175\sqrt{-x^4+x^2+2}}$	14
elliptic	$\frac{17\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{50\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{10\sqrt{-x^4+x^2+2}}-\frac{34\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\frac{10}{7},i\sqrt{2}\right)}{175\sqrt{-x^4+x^2+2}}$	14

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] 17/50*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1
/2*x*2^(1/2),I*2^(1/2))-1/10*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-34/175*2^(1/2)*(1-1/2*x^2)^(
1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1
/2))
```

Fricas [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{-(x^2-2)\,(x^2+1)}}{5x^2+7}\,dx$$

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

```
[In] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7),x)
```

```
[Out] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7), x)
```

3.322 $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx = \frac{x\sqrt{2+x^2-x^4}}{14\,(7+5x^2)}+\frac{1}{70}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{6}{175}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) + \frac{99\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{2450}$$

```
[Out] 1/70*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-6/175*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))+99/2450*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/14*x*(-x^4+x^2+2)
^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1240, 1194, 538, 435, 430, 1226, 551}

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx = -\frac{6}{175}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{1}{70}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{99\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{2450}+\frac{\sqrt{-x^4+x^2+2}\,x}{14\,(5x^2+7)}$$

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + EllipticE[ArcSin[x/Sqrt[2]], -2]
/70 - (6*EllipticF[ArcSin[x/Sqrt[2]], -2])/175 + (99*EllipticPi[-10/7, ArcS
in[x/Sqrt[2]], -2])/2450
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1240

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(d + e*x^2))), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\sqrt{2+x^2-x^4}}{14\,(7+5x^2)} - \frac{1}{350}\int \frac{7-5x^2}{\sqrt{2+x^2-x^4}}\,dx + \frac{99}{350}\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x\sqrt{2+x^2-x^4}}{14\,(7+5x^2)} - \frac{1}{175}\int \frac{7-5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx + \frac{99}{175}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7+5x^2)}\,dx \\
&= \frac{x\sqrt{2+x^2-x^4}}{14\,(7+5x^2)} + \frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450} \\
&\quad + \frac{1}{70}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx - \frac{12}{175}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x\sqrt{2+x^2-x^4}}{14\,(7+5x^2)} + \frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) \\
&\quad - \frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx$$

$$= \frac{700x+350x^3-350x^5+70i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right)-21i\sqrt{2}(7+5x^2)\sqrt{2+x^2-}{}$$

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*x + 350*x^3 - 350*x^5 + (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (21*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 -
x^4]*EllipticF[I*ArcSinh[x], -1/2] - (693*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*El
lipticPi[5/7, I*ArcSinh[x], -1/2] - (495*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(4900*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(71) = 142.

Time = 3.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.23

method	result
default	$\frac{x\sqrt{-x^4+x^2+2}}{70x^2+98}-\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{175\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{140\sqrt{-x^4+x^2+2}}+\frac{99\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\right.}{2450\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{x\sqrt{-x^4+x^2+2}}{70x^2+98}-\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{175\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{140\sqrt{-x^4+x^2+2}}+\frac{99\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\right.}{2450\sqrt{-x^4+x^2+2}}$
risch	$-\frac{\left(x^4-x^2-2\right)x}{14(5x^2+7)\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{100\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{140\sqrt{-x^4+x^2+2}}+99$

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-3/175*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/140*2^(1/2)*(
-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*
2^(1/2))+99/2450*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)
*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}{\left(5x^2+7\right)^2}\,dx$$

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^2,x)
[Out] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^2, x)
```

3.323 $\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\begin{aligned}\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^3}\,dx &= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2}-\frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} \\ &\quad -\frac{31E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640}-\frac{269\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{166600} \\ &\quad +\frac{16601\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{2332400}\end{aligned}$$

```
[Out] -31/66640*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-269/166600*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))+16601/2332400*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/2
8*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1242, 1237, 1710, 1730, 1194, 538, 435, 430, 1226, 551}

$$\begin{aligned}\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^3}\,dx &= -\frac{269\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{166600}-\frac{31E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640} \\ &\quad +\frac{16601\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{2332400} \\ &\quad -\frac{31\sqrt{-x^4+x^2+2}\,x}{13328\left(5x^2+7\right)}+\frac{\sqrt{-x^4+x^2+2}\,x}{28\left(5x^2+7\right)^2}\end{aligned}$$

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) - (31*x*Sqrt[2 + x^2 - x^4])/(13
328*(7 + 5*x^2)) - (31*EllipticE[ArcSin[x/Sqrt[2]], -2])/66640 - (269*Ellip
ticF[ArcSin[x/Sqrt[2]], -2])/166600 + (16601*EllipticPi[-10/7, ArcSin[x/Sqr
t[2]], -2])/2332400
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
```

```
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{34}{25\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}+\frac{19}{25\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\right.\\
&\qquad\left.-\frac{1}{25\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\right)dx\\
&= -\left(\frac{1}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx\right)\\
&\quad+\frac{19}{25}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx-\frac{34}{25}\int \frac{1}{\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2}-\frac{19x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}-\frac{1}{700}\int \frac{186-190x^2+25x^4}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx\\
&\quad+\frac{19\int \frac{118-70x^2-25x^4}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{11900}-\frac{2}{25}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx\\
&= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2}-\frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)}-\frac{1}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\\
&\quad-\frac{\int \frac{37698-32690x^2-12525x^4}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{333200}-\frac{19\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}}\,dx}{297500}+\frac{3173\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{11900}\\
&= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2}-\frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)}-\frac{1}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}}\,dx}{8330000}\\
&\quad-\frac{19\int \frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{148750}-\frac{11783\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{66640}+\frac{3173\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)}\,dx}{5950}\\
&= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2}-\frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)}+\frac{2697\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{83300}\\
&\quad+\frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{4165000}-\frac{19\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{2975}-\frac{19\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{2380}\\
&\quad-\frac{11783\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)}\,dx}{33320}\\
&= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2}-\frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)}-\frac{19E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2380}-\frac{19F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5950}\\
&\quad+\frac{16601\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2332400}+\frac{263\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{83300}+\frac{501\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{66640}
\end{aligned}
$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} - \frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{66640} - \frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{166600} + \frac{16601\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{2332400}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.35 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^3}\,dx$$

$$= \frac{181300x - 17850x^3 - 144900x^5 + 54250x^7 - 2170i\sqrt{2}(7+5x^2)^2\sqrt{2+x^2-x^4}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right) + 7021}{}$$

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (181300*x - 17850*x^3 - 144900*x^5 + 54250*x^7 - (2170*I)*Sqrt[2]*(7 + 5*x^
2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (7021*I)*Sqrt[2]*(
7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (813449*I)
*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (1162070
*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (
415025*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/
2])/(4664800*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.85

method	result
default	$\frac{x\sqrt{-x^4+x^2+2}}{28(5x^2+7)^2} - \frac{31x\sqrt{-x^4+x^2+2}}{13328(5x^2+7)} - \frac{269\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{333200\sqrt{-x^4+x^2+2}} - \frac{31\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{133280\sqrt{-x^4+x^2+2}} + 1660$
elliptic	$\frac{x\sqrt{-x^4+x^2+2}}{28(5x^2+7)^2} - \frac{31x\sqrt{-x^4+x^2+2}}{13328(5x^2+7)} - \frac{269\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{333200\sqrt{-x^4+x^2+2}} - \frac{31\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{133280\sqrt{-x^4+x^2+2}} + 1660$
risch	$\frac{(x^4-x^2-2)x(155x^2-259)}{13328(5x^2+7)^2\sqrt{-x^4+x^2+2}} - \frac{99\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{95200\sqrt{-x^4+x^2+2}} + \frac{31\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{133280\sqrt{-x^4+x^2+2}}$

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/2)/(5*x^2+
7)-269/333200*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*x*2^(1/2),I*2^(1/2))-31/133280*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
```

```
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+16601/2332400*2
^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*
2^(1/2),-10/7,I*2^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^3}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{-(x^2-2)(x^2+1)}}{(5x^2+7)^3}\,dx$$

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**3,x)
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^3}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^3}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^3,x)
[Out] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^3, x)
```

3.324 $\int (7+5x^2)^4 \left(2+x^2-x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int (7+5x^2)^4 \left(2+x^2-x^4\right)^{3/2}\,dx = \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005}$$

$$-\frac{x(69817-1581440x^2)\left(2+x^2-x^4\right)^{3/2}}{1001}-\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}$$

$$-\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}+\frac{124141422E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}-\frac{50794416\,\text{EllipticF}\left(\right.}{500}$$

```
[Out] -1/1001*x*(-1581440*x^2+69817)*(-x^4+x^2+2)^(3/2)-132300/143*x*(-x^4+x^2+2)
^(5/2)-11750/39*x^3*(-x^4+x^2+2)^(5/2)-125/3*x^5*(-x^4+x^2+2)^(5/2)+1241414
22/5005*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-50794416/5005*EllipticF(1/2*x*2^
(1/2),I*2^(1/2))+3/5005*x*(7837383*x^2+2193559)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1693, 1190, 1194, 538, 435, 430}

$$\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2}\,dx = -\frac{50794416\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{5005}+\frac{124141422E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005} -\frac{132300}{143}\left(-x^4+x^2+2\right)^{5/2}x-\frac{\left(69817-1581440x^2\right)\left(-x^4+x^2+2\right)^{3/2}x}{1001} +\frac{3(7837383x^2+2193559)\sqrt{-x^4+x^2+2}x}{5005} -\frac{125}{3}\left(-x^4+x^2+2\right)^{5/2}x^5-\frac{11750}{39}\left(-x^4+x^2+2\right)^{5/2}x^3$$

```
[In] Int[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (3*x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 - x^4])/5005 - (x*(69817 - 158144
0*x^2)*(2 + x^2 - x^4)^(3/2))/1001 - (132300*x*(2 + x^2 - x^4)^(5/2))/143 -
 (11750*x^3*(2 + x^2 - x^4)^(5/2))/39 - (125*x^5*(2 + x^2 - x^4)^(5/2))/3 +
 (124141422*EllipticE[ArcSin[x/Sqrt[2]], -2])/5005 - (50794416*EllipticF[Ar
cSin[x/Sqrt[2]], -2])/5005
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2} \\
&\quad -\frac{1}{15}\int \left(2+x^2-x^4\right)^{3/2}\left(-36015-102900x^2-116500x^4-58750x^6\right)\,dx \\
&= -\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2} \\
&\quad +\frac{1}{195}\int \left(2+x^2-x^4\right)^{3/2}\left(468195+1690200x^2+1984500x^4\right)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}\\
&\quad -\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}-\frac{\int \left(-9119145-30499200x^2\right)\left(2+x^2-x^4\right)^{3/2}\,dx}{2145}\\
&= -\frac{x(69817-1581440x^2)\left(2+x^2-x^4\right)^{3/2}}{1001}\\
&\quad -\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}\\
&\quad -\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}+\frac{\int \left(389287620+1058046705x^2\right)\sqrt{2+x^2-x^4}\,dx}{45045}\\
&= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005}\\
&\quad -\frac{x(69817-1581440x^2)\left(2+x^2-x^4\right)^{3/2}}{1001}-\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}\\
&\quad -\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}-\frac{\int \frac{-9901845810-16759091970x^2}{\sqrt{2+x^2-x^4}}\,dx}{675675}\\
&= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005}\\
&\quad -\frac{x(69817-1581440x^2)\left(2+x^2-x^4\right)^{3/2}}{1001}-\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}\\
&\quad -\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}-\frac{2\int \frac{-9901845810-16759091970x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{675675}\\
&= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005}-\frac{x(69817-1581440x^2)\left(2+x^2-x^4\right)^{3/2}}{1001}\\
&\quad -\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}\\
&\quad -\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}-\frac{101588832\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{5005}+\frac{124141422\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{5005}\\
&= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005}-\frac{x(69817-1581440x^2)\left(2+x^2-x^4\right)^{3/2}}{1001}\\
&\quad -\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2}\\
&\quad -\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2}+\frac{124141422E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}-\frac{50794416F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2}\,dx = \frac{-75836958x+48624305x^3+172881581x^5+32834763x^7-36649955x^9-24642275x^{11}-15}{}$$

```
[In] Integrate[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (-75836958*x + 48624305*x^3 + 172881581*x^5 + 32834763*x^7 - 36649955*x^9 -
 24642275*x^11 - 1556625*x^13 + 2646875*x^15 + 625625*x^17 + (372424266*I)*
Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (482444775*I)*Sqrt[
4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(15015*Sqrt[2 + x^2 - x^4
])
```

Maple [A] (verified)

Time = 7.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18

method	result
risch	$\frac{x\left(625625x^{12}+3272500x^{10}+2967125x^{8}-15130150x^{6}-45845855x^{4}-43271392x^{2}+37918479\right)\left(x^{4}-x^{2}-2\right)}{15015\sqrt{-x^{4}+x^{2}+2}}+\frac{36673503\sqrt{2}\,\sqrt{-2x^{2}+}}{5005\sqrt{-}}$
default	$\frac{833561x^{5}\sqrt{-x^{4}+x^{2}+2}}{273}+\frac{43271392x^{3}\sqrt{-x^{4}+x^{2}+2}}{15015}-\frac{12639493x\sqrt{-x^{4}+x^{2}+2}}{5005}+\frac{36673503\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{5005\sqrt{-x^{4}+x^{2}+2}}$
elliptic	$\frac{833561x^{5}\sqrt{-x^{4}+x^{2}+2}}{273}+\frac{43271392x^{3}\sqrt{-x^{4}+x^{2}+2}}{15015}-\frac{12639493x\sqrt{-x^{4}+x^{2}+2}}{5005}+\frac{36673503\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{5005\sqrt{-x^{4}+x^{2}+2}}$

```
[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15015*x*(625625*x^12+3272500*x^10+2967125*x^8-15130150*x^6-45845855*x^4-4
3271392*x^2+37918479)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+36673503/5005*2^(1/2)*
(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I
*2^(1/2))-62070711/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2} dx = \frac{-744848532i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) + 854869041i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) - (625625\,x^{14}}{}$$

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(-744848532*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 85486
9041*I*sqrt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (625625*x^14 + 32725
00*x^12 + 2967125*x^10 - 15130150*x^8 - 45845855*x^6 - 43271392*x^4 + 37918
479*x^2 + 372424266)*sqrt(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2} dx = \int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}} \left(5x^2+7\right)^4 dx$$

```
[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**4, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2} dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^4 dx$$

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2} \, dx=\int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^4 \,dx$$

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2} \, dx=\int {\left(5\,x^2+7\right)}^4\,{\left(-x^4+x^2+2\right)}^{3/2} \,dx$$

```
[In] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(3/2), x)
```

3.325 $\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx = \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015} + \frac{x(33792+374045x^2)\left(2+x^2-x^4\right)^{3/2}}{3003} - \frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2} - \frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2} + \frac{31072528E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{15015} - \frac{3199778\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{5005}$$

```
[Out] 1/3003*x*(374045*x^2+33792)*(-x^4+x^2+2)^(3/2)-7825/143*x*(-x^4+x^2+2)^(5/2
)-125/13*x^3*(-x^4+x^2+2)^(5/2)+31072528/15015*EllipticE(1/2*x*2^(1/2),I*2^
(1/2))-3199778/5005*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/15015*x*(5712051*x
^2+2512273)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1220, 1693, 1190, 1194, 538, 435, 430}

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx = -\frac{3199778\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{5005} + \frac{31072528E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{15015} - \frac{7825}{143}\left(-x^4+x^2+2\right)^{5/2}x + \frac{\left(374045x^2+33792\right)\left(-x^4+x^2+2\right)^{3/2}x}{3003} + \frac{\left(5712051x^2+2512273\right)\sqrt{-x^4+x^2+2x}}{15015} - \frac{125}{13}\left(-x^4+x^2+2\right)^{5/2}x^3$$

```
[In] Int[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(2512273 + 5712051*x^2)*Sqrt[2 + x^2 - x^4])/15015 + (x*(33792 + 374045*
x^2)*(2 + x^2 - x^4)^(3/2))/3003 - (7825*x*(2 + x^2 - x^4)^(5/2))/143 - (12
5*x^3*(2 + x^2 - x^4)^(5/2))/13 + (31072528*EllipticE[ArcSin[x/Sqrt[2]], -2
])/15015 - (3199778*EllipticF[ArcSin[x/Sqrt[2]], -2])/5005
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{1}{13}\int\left(-4459-10305x^2-7825x^4\right)\left(2+x^2-x^4\right)^{3/2}\,dx\\
&= -\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2}\\
&\quad -\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2}+\frac{1}{143}\int\left(64699+160305x^2\right)\left(2+x^2-x^4\right)^{3/2}\,dx\\
&= \frac{x(33792+374045x^2)\left(2+x^2-x^4\right)^{3/2}}{3003}-\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2}\\
&\quad -\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2}-\frac{\int\left(-2649774-5712051x^2\right)\sqrt{2+x^2-x^4}\,dx}{3003}\\
&= \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015}+\frac{x(33792+374045x^2)\left(2+x^2-x^4\right)^{3/2}}{3003}\\
&\quad -\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2}+\frac{\int\frac{64419582+93217584x^2}{\sqrt{2+x^2-x^4}}\,dx}{45045}\\
&= \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015}+\frac{x(33792+374045x^2)\left(2+x^2-x^4\right)^{3/2}}{3003}\\
&\quad -\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2}+\frac{2\int\frac{64419582+93217584x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{45045}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015} \\
&\quad + \frac{x(33792+374045x^2)\left(2+x^2-x^4\right)^{3/2}}{3003} - \frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2} \\
&\quad - \frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2} - \frac{6399556\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{5005} + \frac{31072528\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{15015} \\
&= \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015} + \frac{x(33792+374045x^2)\left(2+x^2-x^4\right)^{3/2}}{3003} \\
&\quad - \frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2} - \frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2} \\
&\quad + \frac{31072528E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{15015} - \frac{3199778F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97

$$
\int \left(7+5x^2\right)^3\left(2+x^2 -x^4\right)^{3/2}\,dx = \frac{-872614x+11078615x^3+13371048x^5-1756521x^7-4448240x^9-1027775x^{11}+388500x^1}{}
$$

```
[In] Integrate[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (-872614*x + 11078615*x^3 + 13371048*x^5 - 1756521*x^7 - 4448240*x^9 - 1027
775*x^11 + 388500*x^13 + 144375*x^15 + (31072528*I)*Sqrt[4 + 2*x^2 - 2*x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (41809125*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Ellip
ticF[I*ArcSinh[x], -1/2])/(15015*Sqrt[2 + x^2 - x^4])
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

method	result
risch	$\frac{x\left(144375x^{10}+532875x^{8}-206150x^{6}-3588640x^{4}-5757461x^{2}+436307\right)\left(x^{4}-x^{2}-2\right)}{15015\sqrt{-x^{4}+x^{2}+2}}+\frac{10736597\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right.}{15015\sqrt{-x^{4}+x^{2}+2}}$
default	$\frac{65248x^{5}\sqrt{-x^{4}+x^{2}+2}}{273}+\frac{5757461x^{3}\sqrt{-x^{4}+x^{2}+2}}{15015}-\frac{436307x\sqrt{-x^{4}+x^{2}+2}}{15015}+\frac{10736597\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{15015\sqrt{-x^{4}+x^{2}+2}}-$
elliptic	$\frac{65248x^{5}\sqrt{-x^{4}+x^{2}+2}}{273}+\frac{5757461x^{3}\sqrt{-x^{4}+x^{2}+2}}{15015}-\frac{436307x\sqrt{-x^{4}+x^{2}+2}}{15015}+\frac{10736597\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{15015\sqrt{-x^{4}+x^{2}+2}}-$

```
[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15015*x*(144375*x^10+532875*x^8-206150*x^6-3588640*x^4-5757461*x^2+436307
)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+10736597/15015*2^(1/2)*(-2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-15536264/
15015*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(
1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2}\,dx = \frac{-62145056i\,\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+72881653i\,\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})-(144375\,x^{12}+}{1501}$$

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(-62145056*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 728816
53*I*sqrt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (144375*x^12 + 532875*
x^10 - 206150*x^8 - 3588640*x^6 - 5757461*x^4 + 436307*x^2 + 31072528)*sqrt
(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2}\,dx = \int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3\,dx$$

```
[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2}\,dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3\,dx$$

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^3 \, dx$$

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(5\,x^2+7\right)^3 \left(-x^4+x^2+2\right)^{3/2} \, dx$$

```
[In] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2), x)
```

3.326 $\int (7+5x^2)^2 (2+x^2-x^4)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int (7+5x^2)^2 (2+x^2-x^4)^{3/2} \, dx = \frac{1}{495}x(11497+14889x^2)\sqrt{2+x^2-x^4} + \frac{1}{99}x(363+920x^2)(2+x^2-x^4)^{3/2} - \frac{25}{11}x(2+x^2-x^4)^{5/2} + \frac{85942}{495}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{3392}{165}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] 1/99*x*(920*x^2+363)*(-x^4+x^2+2)^(3/2)-25/11*x*(-x^4+x^2+2)^(5/2)+85942/49
5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-3392/165*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))+1/495*x*(14889*x^2+11497)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1220, 1190, 1194, 538, 435, 430}

$$\int (7+5x^2)^2 (2+x^2-x^4)^{3/2} \, dx = -\frac{3392}{165}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) + \frac{85942}{495}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{25}{11}x(-x^4+x^2+2)^{5/2} + \frac{1}{99}x(920x^2+363)(-x^4+x^2+2)^{3/2} + \frac{1}{495}x(14889x^2+11497)\sqrt{-x^4+x^2+2}$$

```
[In] Int[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/495 + (x*(363 + 920*x^2)*(2 + x
^2 - x^4)^(3/2))/99 - (25*x*(2 + x^2 - x^4)^(5/2))/11 + (85942*EllipticE[Ar
cSin[x/Sqrt[2]], -2])/495 - (3392*EllipticF[ArcSin[x/Sqrt[2]], -2])/165
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
```

```
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2}-\frac{1}{11}\int\left(-589-920x^2\right)\left(2+x^2-x^4\right)^{3/2}\,dx\\
&= \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2}\\
&\quad-\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2}+\frac{1}{231}\int\left(23044+34741x^2\right)\sqrt{2+x^2-x^4}\,dx\\
&= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2}\\
&\quad-\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2}-\frac{\int\frac{-530362-601594x^2}{\sqrt{2+x^2-x^4}}\,dx}{3465}\\
&= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2}\\
&\quad-\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2}-\frac{2\int\frac{-530362-601594x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{3465}\\
&= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2}\\
&\quad-\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2}-\frac{6784}{165}\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{85942}{495}\int\frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2}\\
&\quad-\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2}+\frac{85942}{495}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{3392}{165}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.01 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$
\int\left(7+5x^2\right)^2\left(2+x^2-x^4\right)^{3/2}\,dx=\frac{21254x+53435x^3+23097x^5-19944x^7-10760x^9+1225x^{11}+1125x^{13}+85942i\sqrt{4+2x^2-}}{495\sqrt{2+x^2-x}}
$$

```
[In] Integrate[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (21254*x + 53435*x^3 + 23097*x^5 - 19944*x^7 - 10760*x^9 + 1225*x^11 + 1125
*x^13 + (85942*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (
123825*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(495*Sqrt[
2 + x^2 - x^4])
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.57

method	result
risch	$\frac{x\left(1125x^{8}+2350x^{6}-6160x^{4}-21404x^{2}-10627\right)\left(x^{4}-x^{2}-2\right)}{495\sqrt{-x^{4}+x^{2}+2}}+\frac{37883\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{495\sqrt{-x^{4}+x^{2}+2}}-\frac{42971\sqrt{2}\,\sqrt{-2x^{2}+4}}{4}$
default	$\frac{112x^{5}\sqrt{-x^{4}+x^{2}+2}}{9}+\frac{21404x^{3}\sqrt{-x^{4}+x^{2}+2}}{495}+\frac{10627x\sqrt{-x^{4}+x^{2}+2}}{495}+\frac{37883\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{495\sqrt{-x^{4}+x^{2}+2}}-42971\sqrt{2}$
elliptic	$\frac{112x^{5}\sqrt{-x^{4}+x^{2}+2}}{9}+\frac{21404x^{3}\sqrt{-x^{4}+x^{2}+2}}{495}+\frac{10627x\sqrt{-x^{4}+x^{2}+2}}{495}+\frac{37883\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{495\sqrt{-x^{4}+x^{2}+2}}-42971\sqrt{2}$

```
[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/495*x*(1125*x^8+2350*x^6-6160*x^4-21404*x^2-10627)*(x^4-x^2-2)/(-x^4+x^2+
2)^(1/2)+37883/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2
)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-42971/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE
(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \left(7+5x^{2}\right)^{2}\left(2+x^{2}-x^{4}\right)^{3/2}\,dx=\frac{-171884i\,\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})+209767i\,\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})-(1125\,x^{10}+2350\,x}{495\,x}$$

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/495*(-171884*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 209767*I*s
qrt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (1125*x^10 + 2350*x^8 - 6160
*x^6 - 21404*x^4 - 10627*x^2 + 85942)*sqrt(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} dx = \int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}} \left(5x^2+7\right)^2 dx$$

```
[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(3/2),x)
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5x^2+7\right)^2 dx$$

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5x^2+7\right)^2 dx$$

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} dx = \int \left(5x^2+7\right)^2 \left(-x^4+x^2+2\right)^{3/2} dx$$

```
[In] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2), x)
```

3.327 $\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx=\frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{4432}{315}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{418}{105}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] 1/63*x*(35*x^2+48)*(-x^4+x^2+2)^(3/2)+4432/315*EllipticE(1/2*x*2^(1/2),I*2^
(1/2))+418/105*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/315*x*(669*x^2+1087)*(-
x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1190, 1194, 538, 435, 430}

$$\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx=\frac{418}{105}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{4432}{315}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{315}x\left(669x^2+1087\right)\sqrt{-x^4+x^2+2}$$

```
[In] Int[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1087 + 669*x^2)*Sqrt[2 + x^2 - x^4])/315 + (x*(48 + 35*x^2)*(2 + x^2 -
x^4)^(3/2))/63 + (4432*EllipticE[ArcSin[x/Sqrt[2]], -2])/315 + (418*Ellipti
cF[ArcSin[x/Sqrt[2]], -2])/105
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}-\frac{1}{21}\int\left(-262-223x^2\right)\sqrt{2+x^2-x^4}\,dx$$

$$
\begin{aligned}
&= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4} \\
&\quad + \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2} + \frac{1}{315}\int \frac{5686+4432x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4} \\
&\quad + \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2} + \frac{2}{315}\int \frac{5686+4432x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2} \\
&\quad + \frac{836}{105}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx + \frac{4432}{315}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx \\
&= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2} \\
&\quad + \frac{4432}{315}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{418}{105}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

$$
\int \left(7+5x^2\right)\left(2+x^2 -x^4\right)^{3/2}\,dx = \frac{3134x+4085x^3-438x^5-1674x^7-110x^9+175x^{11}+4432i\sqrt{4+2x^2-2x^4}E(i\operatorname{arcsinh}(x)\,|}{315\sqrt{2+x^2-x^4}}
$$

```
[In] Integrate[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (3134*x + 4085*x^3 - 438*x^5 - 1674*x^7 - 110*x^9 + 175*x^11 + (4432*I)*Sqr
t[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (7275*I)*Sqrt[4 + 2*x^
2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(315*Sqrt[2 + x^2 - x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(75) = 150.

Time = 1.71 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.88

method	result
risch	$\frac{x(175x^6+65x^4-1259x^2-1567)(x^4-x^2-2)}{315\sqrt{-x^4+x^2+2}}+\frac{2843\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{315\sqrt{-x^4+x^2+2}}-\frac{2216\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,(F(\frac{x\sqrt{2}}{2},i\sqrt{}}{315\sqrt{-x^4+x^2+2}}$
default	$-\frac{13x^5\sqrt{-x^4+x^2+2}}{63}+\frac{1259x^3\sqrt{-x^4+x^2+2}}{315}+\frac{1567x\sqrt{-x^4+x^2+2}}{315}+\frac{2843\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{315\sqrt{-x^4+x^2+2}}-\frac{2216\sqrt{2}\,\sqrt{-2}}{}$
elliptic	$-\frac{13x^5\sqrt{-x^4+x^2+2}}{63}+\frac{1259x^3\sqrt{-x^4+x^2+2}}{315}+\frac{1567x\sqrt{-x^4+x^2+2}}{315}+\frac{2843\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{315\sqrt{-x^4+x^2+2}}-\frac{2216\sqrt{2}\,\sqrt{-2}}{}$

```
[In] int((5*x^2+7)*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*x*(175*x^6+65*x^4-1259*x^2-1567)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+2843/
315*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2
*x*2^(1/2),I*2^(1/2))-2216/315*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4
+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I
*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx=\frac{-8864i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+11707i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})-(175\,x^8+65\,x^6-1259}{315\,x}$$

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(-8864*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 11707*I*sqrt
(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (175*x^8 + 65*x^6 - 1259*x^4 -
1567*x^2 + 4432)*sqrt(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx=\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\cdot\left(5x^2+7\right)\,dx$$

```
[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7), x)
```

Maxima [F]

$$\int \left(7+5x^2\right) \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right) dx$$

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Giac [F]

$$\int \left(7+5x^2\right) \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right) dx$$

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right) \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(5\,x^2+7\right) \,{\left(-x^4+x^2+2\right)}^{3/2} \,d x$$

```
[In] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2), x)
```

3.328 $\int \left(2+x^2-x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \left(2+x^2-x^4\right)^{3/2} \, dx = \frac{1}{35} x \left(19+3 x^2\right) \sqrt{2+x^2-x^4}+\frac{1}{7} x \left(2+x^2-x^4\right)^{3/2} + \frac{34}{35} E\left(\arcsin \left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{48}{35} \operatorname{EllipticF}\left(\arcsin \left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] 1/7*x*(-x^4+x^2+2)^(3/2)+34/35*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+48/35*Ell
ipticF(1/2*x*2^(1/2),I*2^(1/2))+1/35*x*(3*x^2+19)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1105, 1190, 1194, 538, 435, 430}

$$\int \left(2+x^2-x^4\right)^{3/2} \, dx = \frac{48}{35} \operatorname{EllipticF}\left(\arcsin \left(\frac{x}{\sqrt{2}}\right),-2\right) + \frac{34}{35} E\left(\arcsin \left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{7} x \left(-x^4+x^2+2\right)^{3/2}+\frac{1}{35} x \left(3 x^2+19\right) \sqrt{-x^4+x^2+2}$$

```
[In] Int[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/35 + (x*(2 + x^2 - x^4)^(3/2))/7 + (34
*EllipticE[ArcSin[x/Sqrt[2]], -2])/35 + (48*EllipticF[ArcSin[x/Sqrt[2]], -2
])/35
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{3}{7}\int\left(4+x^2\right)\sqrt{2+x^2-x^4}\,dx$$

$$
\begin{aligned}
&= \frac{1}{35}x(19+3x^2)\sqrt{2+x^2-x^4} + \frac{1}{7}x(2+x^2-x^4)^{3/2} - \frac{1}{35}\int \frac{-82-34x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{1}{35}x(19+3x^2)\sqrt{2+x^2-x^4} + \frac{1}{7}x(2+x^2-x^4)^{3/2} - \frac{2}{35}\int \frac{-82-34x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{1}{35}x(19+3x^2)\sqrt{2+x^2-x^4} + \frac{1}{7}x(2+x^2-x^4)^{3/2} \\
&\quad + \frac{34}{35}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{96}{35}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{1}{35}x(19+3x^2)\sqrt{2+x^2-x^4} + \frac{1}{7}x(2+x^2-x^4)^{3/2} \\
&\quad + \frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$
\int (2+x^2 -x^4)^{3/2}\,dx = \frac{58x+45x^3-31x^5-13x^7+5x^9+34i\sqrt{4+2x^2-2x^4}E\left(i\operatorname{arcsinh}(x)\,\middle|-\frac{1}{2}\right)-75i\sqrt{4+2x^2-2}}{35\sqrt{2+x^2-x^4}}
$$

```
[In] Integrate[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (58*x + 45*x^3 - 31*x^5 - 13*x^7 + 5*x^9 + (34*I)*Sqrt[4 + 2*x^2 - 2*x^4]*E
llipticE[I*ArcSinh[x], -1/2] - (75*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*A
rcSinh[x], -1/2])/(35*Sqrt[2 + x^2 - x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(68) = 136.

Time = 0.68 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

method	result
risch	$\frac{x(5x^4-8x^2-29)(x^4-x^2-2)}{35\sqrt{-x^4+x^2+2}}+\frac{41\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{35\sqrt{-x^4+x^2+2}}-\frac{17\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right.}{35\sqrt{-x^4+x^2+2}}$
default	$-\frac{x^5\sqrt{-x^4+x^2+2}}{7}+\frac{8x^3\sqrt{-x^4+x^2+2}}{35}+\frac{29x\sqrt{-x^4+x^2+2}}{35}+\frac{41\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{35\sqrt{-x^4+x^2+2}}-\frac{17\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2}}{3}$
elliptic	$-\frac{x^5\sqrt{-x^4+x^2+2}}{7}+\frac{8x^3\sqrt{-x^4+x^2+2}}{35}+\frac{29x\sqrt{-x^4+x^2+2}}{35}+\frac{41\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{35\sqrt{-x^4+x^2+2}}-\frac{17\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2}}{3}$

```
[In] int((-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*x*(5*x^4-8*x^2-29)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+41/35*2^(1/2)*(-2*x^
2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/
2))-17/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(Ellipt
icF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int (2+x^2 -x^4)^{3/2}\,dx = \frac{-68i\,\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid -\frac{1}{2}) + 109i\,\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid -\frac{1}{2}) - (5\,x^6-8\,x^4-29\,x^2+34)}{35\,x}$$

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(-68*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 109*I*sqrt(2)*x
*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (5*x^6 - 8*x^4 - 29*x^2 + 34)*sqrt(-
x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int (2+x^2-x^4)^{3/2}\,dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(3/2), x)
```

Maxima [F]

$$\int \left(2+x^2-x^4\right)^{3/2}\,dx=\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \left(2+x^2-x^4\right)^{3/2}\,dx=\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(2+x^2-x^4\right)^{3/2}\,dx=\int \left(-x^4+x^2+2\right)^{3/2}\,dx$$

```
[In] int((x^2 - x^4 + 2)^(3/2),x)
[Out] int((x^2 - x^4 + 2)^(3/2), x)
```

3.329 $\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx = \frac{1}{75}x(13-3x^2)\sqrt{2+x^2-x^4}+\frac{92}{375}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{178}{625}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{1156\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

```
[Out] 92/375*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-178/625*EllipticF(1/2*x*2^(1/2),I
*2^(1/2))+1156/4375*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/75*x*(-3*x^
2+13)*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1222, 1190, 1194, 538, 435, 430, 1226, 551}

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx = -\frac{178}{625}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{92}{375}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1156\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}+\frac{1}{75}x\sqrt{-x^4+x^2+2}(13-3x^2)$$

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (x*(13 - 3*x^2)*Sqrt[2 + x^2 - x^4])/75 + (92*EllipticE[ArcSin[x/Sqrt[2]],
-2])/375 - (178*EllipticF[ArcSin[x/Sqrt[2]], -2])/625 + (1156*EllipticPi[-1
0/7, ArcSin[x/Sqrt[2]], -2])/4375
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
```

```
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{25}\int \left(-12+5x^2\right)\sqrt{2+x^2-x^4}\,dx\right)-\frac{34}{25}\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx\\
&= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{375}\int \frac{230-10x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&\quad+\frac{34}{625}\int \frac{-12+5x^2}{\sqrt{2+x^2-x^4}}\,dx+\frac{1156}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{2}{375}\int \frac{230-10x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&\quad+\frac{68}{625}\int \frac{-12+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{2312}{625}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx\\
&= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{1156\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}-\frac{2}{75}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&\quad+\frac{34}{125}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{32}{25}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx-\frac{1156}{625}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\\
&\quad-\frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.81

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx = \frac{4550x+1225x^3-2800x^5+525x^7+3220i\sqrt{4+2x^2-2x^4}\,E\left(i\operatorname{arcsinh}(x)\,\middle|-\tfrac{1}{2}\right)-}{}$$

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (4550*x + 1225*x^3 - 2800*x^5 + 525*x^7 + (3220*I)*Sqrt[4 + 2*x^2 - 2*x^4]*
EllipticE[I*ArcSinh[x], -1/2] - (2961*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[
I*ArcSinh[x], -1/2] - (1734*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticPi[5/7, I*Ar
cSinh[x], -1/2])/(13125*Sqrt[2 + x^2 - x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(69) = 138.

Time = 2.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.40

method	result
default	$-\frac{x^3\sqrt{-x^4+x^2+2}}{25}+\frac{13x\sqrt{-x^4+x^2+2}}{75}-\frac{89\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{625\sqrt{-x^4+x^2+2}}+\frac{46\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{375\sqrt{-x^4+x^2+2}}+$
elliptic	$-\frac{x^3\sqrt{-x^4+x^2+2}}{25}+\frac{13x\sqrt{-x^4+x^2+2}}{75}-\frac{89\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{625\sqrt{-x^4+x^2+2}}+\frac{46\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{375\sqrt{-x^4+x^2+2}}+$
risch	$\frac{x\left(3x^2-13\right)\left(x^4-x^2-2\right)}{75\sqrt{-x^4+x^2+2}}-\frac{37\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{1875\sqrt{-x^4+x^2+2}}-\frac{46\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{375\sqrt{-x^4+x^2+2}}+$

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] -1/25*x^3*(-x^4+x^2+2)^(1/2)+13/75*x*(-x^4+x^2+2)^(1/2)-89/625*2^(1/2)*(-2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))+46/375*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elli
pticE(1/2*x*2^(1/2),I*2^(1/2))+1156/4375*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Fricas [**F**]

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Sympy [**F**]

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{(-(x^2-2)\,(x^2+1))^{\frac{3}{2}}}{5x^2+7}\,dx$$

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7),x)
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7), x)
```

Maxima [**F**]

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Giac [**F**]

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{\left(-x^4+x^2+2\right)^{3/2}}{5\,x^2+7}\,dx$$

```
[In] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7),x)
[Out] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.330 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2} \, dx = -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175(7+5x^2)} - \frac{97}{525}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{458}{875}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - \frac{1241\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{6125}$$

```
[Out] -97/525*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+458/875*EllipticF(1/2*x*2^(1/2),
I*2^(1/2))-1241/6125*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-1/75*x*(-x^4
+x^2+2)^(1/2)-17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1242, 1109, 430, 1146, 507, 435, 1136, 1194, 1237, 1730, 538, 1226, 551}

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2} \, dx = \frac{458}{875}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - \frac{97}{525}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{1241\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{6125} - \frac{17\sqrt{-x^4+x^2+2}x}{175(5x^2+7)} - \frac{1}{75}\sqrt{-x^4+x^2+2}x$$

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] -1/75*(x*Sqrt[2 + x^2 - x^4]) - (17*x*Sqrt[2 + x^2 - x^4])/(175*(7 + 5*x^2)
) - (97*EllipticE[ArcSin[x/Sqrt[2]], -2])/525 + (458*EllipticF[ArcSin[x/Sqr
t[2]], -2])/875 - (1241*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/6125
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
```

```
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1146

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt[
-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] &
& LtQ[c, 0]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
```

```
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{212}{625\sqrt{2+x^2-x^4}} - \frac{24x^2}{125\sqrt{2+x^2-x^4}} + \frac{x^4}{25\sqrt{2+x^2-x^4}} \right.\\
&\qquad \left. + \frac{1156}{625\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}} - \frac{1292}{625\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\right) dx \\
&= \frac{1}{25}\int \frac{x^4}{\sqrt{2+x^2-x^4}}\,dx - \frac{24}{125}\int \frac{x^2}{\sqrt{2+x^2-x^4}}\,dx + \frac{212}{625}\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx \\
&\quad + \frac{1156}{625}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx - \frac{1292}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx \\
&= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} + \frac{17\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{4375} \\
&\quad + \frac{1}{75}\int \frac{2+2x^2}{\sqrt{2+x^2-x^4}}\,dx - \frac{48}{125}\int \frac{x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&\quad + \frac{424}{625}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx - \frac{2584}{625}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx \\
&= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} + \frac{212}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) \\
&\quad - \frac{1292\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375} - \frac{17\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}}\,dx}{109375} + \frac{2}{75}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx \\
&\quad - \frac{24}{125}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{48}{125}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx + \frac{2839\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{4375}
\end{aligned}$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} - \frac{62}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{332}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - \frac{1292\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{4375} - \frac{34\int\frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{109375} + \frac{5678\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)}\,dx}{4375}$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} - \frac{62}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{332}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - \frac{1241\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{6125} - \frac{68\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{4375} - \frac{17}{875}\int\frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} - \frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{458}{875}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - \frac{1241\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{6125}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.16

$$\int\frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx = \frac{-14000x - 11900x^3 + 4550x^5 + 2450x^7 - 6790i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}E(i\text{arc}}{}$$

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (-14000*x - 11900*x^3 + 4550*x^5 + 2450*x^7 - (6790*I)*Sqrt[2]*(7 + 5*x^2)*
Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (567*I)*Sqrt[2]*(7 + 5*
x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (26061*I)*Sqrt[2]*
Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (18615*I)*Sqrt[2]
*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(36750*(7 + 5
*x^2)*Sqrt[2 + x^2 - x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(86) = 172$.

Time = 2.89 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.94

method	result
default	$-\frac{17x\sqrt{-x^4+x^2+2}}{175(5x^2+7)} - \frac{x\sqrt{-x^4+x^2+2}}{75} + \frac{229\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{875\sqrt{-x^4+x^2+2}} - \frac{97\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{1050\sqrt{-x^4+x^2+2}} - 12$
elliptic	$-\frac{17x\sqrt{-x^4+x^2+2}}{175(5x^2+7)} - \frac{x\sqrt{-x^4+x^2+2}}{75} + \frac{229\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{875\sqrt{-x^4+x^2+2}} - \frac{97\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{1050\sqrt{-x^4+x^2+2}} - 12$
risch	$\frac{(x^4-x^2-2)x(7x^2+20)}{105(5x^2+7)\sqrt{-x^4+x^2+2}} + \frac{127\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{750\sqrt{-x^4+x^2+2}} + \frac{97\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{1050\sqrt{-x^4+x^2+2}} -$

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/75*x*(-x^4+x^2+2)^(1/2)+229/875*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))-97/1050*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1241/6125*2^(1/2)*(1-1/2*x^2)^(1/
2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2
))
```

Fricas [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(-(x^2-2)\,(x^2+1))^{\frac{3}{2}}}{(5x^2+7)^2}\,dx$$

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx=\int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx=\int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx=\int \frac{(-x^4+x^2+2)^{3/2}}{(5\,x^2+7)^2}\,dx$$

```
[In] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^2,x)
[Out] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.331 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3} \, dx = -\frac{17x\sqrt{2+x^2-x^4}}{350(7+5x^2)^2}+\frac{563x\sqrt{2+x^2-x^4}}{9800(7+5x^2)} + \frac{191E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}-\frac{1251\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24500} + \frac{9879\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{343000}$$

```
[Out] 191/9800*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1251/24500*EllipticF(1/2*x*2^(1
/2),I*2^(1/2))+9879/343000*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-17/350
*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1242, 1109, 430, 1146, 507, 435, 1237, 1710, 1730, 1194, 538, 1226, 551}

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3} \, dx = -\frac{1251\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24500} + \frac{191E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}+\frac{9879\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{343000} + \frac{563\sqrt{-x^4+x^2+2}x}{9800(5x^2+7)}-\frac{17\sqrt{-x^4+x^2+2}x}{350(5x^2+7)^2}$$

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 + x^2 - x^4
])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]], -2])/9800 - (1251
*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9879*EllipticPi[-10/7, ArcSin[x
/Sqrt[2]], -2])/343000
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1146

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt[
-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] &
& LtQ[c, 0]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{31}{625\sqrt{2+x^2-x^4}}+\frac{x^2}{125\sqrt{2+x^2-x^4}}+\frac{1156}{625\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}\right.\\
&\qquad\left.-\frac{1292}{625\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}+\frac{429}{625\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\right)dx\\
&= \frac{1}{125}\int\frac{x^2}{\sqrt{2+x^2-x^4}}\,dx-\frac{31}{625}\int\frac{1}{\sqrt{2+x^2-x^4}}\,dx+\frac{429}{625}\int\frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx\\
&\quad+\frac{1156}{625}\int\frac{1}{\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}\,dx-\frac{1292}{625}\int\frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2}+\frac{19x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)}+\frac{17\int\frac{186-190x^2+25x^4}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx}{8750}\\
&\quad-\frac{19\int\frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{4375}+\frac{2}{125}\int\frac{x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&\quad-\frac{62}{625}\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{858}{625}\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} - \frac{31}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) \\
&\quad + \frac{429\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{4375} + \frac{\int \frac{37698-32690x^2-12525x^4}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{245000} + \frac{19\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}}\,dx}{109375} \\
&\quad + \frac{1}{125}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx - \frac{2}{125}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx - \frac{3173\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{4375}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{1}{125}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) \\
&\quad - \frac{36}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{429\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{4375} \\
&\quad - \frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}}\,dx}{6125000} + \frac{38\int \frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{109375} \\
&\quad + \frac{11783\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{49000} - \frac{6346\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)}\,dx}{4375}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{1}{125}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) \\
&\quad - \frac{36}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - \frac{34\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{6125} - \frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{3062500} \\
&\quad + \frac{76\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{4375} + \frac{19}{875}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{11783\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)}\,dx}{24500}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{26}{875}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) \\
&\quad - \frac{214F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{4375} + \frac{9879\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{343000} \\
&\quad - \frac{263\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{61250} - \frac{501\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{49000}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{9800} \\
&\quad - \frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{24500} + \frac{9879\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{343000}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.39

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \frac{485100x+636650x^3-45500x^5-197050x^7+13370i\sqrt{2}(7+5x^2)^2\sqrt{2+x^2-x^4}}{}$$

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (485100*x + 636650*x^3 - 45500*x^5 - 197050*x^7 + (13370*I)*Sqrt[2]*(7 + 5*
x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2541*I)*Sqrt[2]
*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (484071*
I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (69153
0*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] -
(246975*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1
/2])/(686000*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.85

method	result
default	$-\frac{17x\sqrt{-x^4+x^2+2}}{350(5x^2+7)^2}+\frac{563x\sqrt{-x^4+x^2+2}}{9800(5x^2+7)}-\frac{1251\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{49000\sqrt{-x^4+x^2+2}}+\frac{191\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{19600\sqrt{-x^4+x^2+2}}$
elliptic	$-\frac{17x\sqrt{-x^4+x^2+2}}{350(5x^2+7)^2}+\frac{563x\sqrt{-x^4+x^2+2}}{9800(5x^2+7)}-\frac{1251\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{49000\sqrt{-x^4+x^2+2}}+\frac{191\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{19600\sqrt{-x^4+x^2+2}}$
risch	$-\frac{(x^4-x^2-2)x(563x^2+693)}{1960(5x^2+7)^2\sqrt{-x^4+x^2+2}}-\frac{221\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{14000\sqrt{-x^4+x^2+2}}-\frac{191\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,(F(\frac{x\sqrt{2}}{2},i\sqrt{2})-E(\frac{x\sqrt{2}}{2},i}{19600\sqrt{-x^4+x^2+2}}$

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -17/350*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^(1/2)/(5*x
^2+7)-1251/49000*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*
EllipticF(1/2*x*2^(1/2),I*2^(1/2))+191/19600*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+9879/343000*
2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x
*2^(1/2),-10/7,I*2^(1/2))
```

Fricas [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(-(x^2-2)\,(x^2+1))^{\frac{3}{2}}}{(5x^2+7)^3}\,dx$$

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**3,x)
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx=\int \frac{{\left(-x^4+x^2+2\right)}^{3/2}}{{\left(5\,x^2+7\right)}^3}\,dx$$

```
[In] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^3,x)
[Out] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.332 $\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}} \, dx = -\frac{625}{3}x\sqrt{2+x^2-x^4} - 25x^3\sqrt{2+x^2-x^4} + \frac{3905}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - 542\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right), -2\right)$$

```
[Out] 3905/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-542*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-625/3*x*(-x^4+x^2+2)^(1/2)-25*x^3*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1220, 1693, 1194, 538, 435, 430}

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}} \, dx = -542\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{3905}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) - \frac{625}{3}\sqrt{-x^4+x^2+2}x - 25\sqrt{-x^4+x^2+2}x^3$$

```
[In] Int[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (3905*Ellipti
cE[ArcSin[x/Sqrt[2]], -2])/3 - 542*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -25x^3\sqrt{2+x^2-x^4}-\frac{1}{5}\int\frac{-1715-4425x^2-3125x^4}{\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{1}{15}\int\frac{11395+19525x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{2}{15}\int\frac{11395+19525x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}\\
&\quad -1084\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{3905}{3}\int\frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}\\
&\quad +\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$
\int\frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx
=\frac{-2500x-1550x^3+1100x^5+150x^7+7810i\sqrt{4+2x^2-2x^4}E\left(i\operatorname{arcsinh}(x)\,\middle|-\frac{1}{2}\right)-10089i\sqrt{4+2x^2-2x^4}}{6\sqrt{2+x^2-x^4}}
$$

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-2500*x - 1550*x^3 + 1100*x^5 + 150*x^7 + (7810*I)*Sqrt[4 + 2*x^2 - 2*x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (10089*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elliptic
F[I*ArcSinh[x], -1/2])/(6*Sqrt[2 + x^2 - x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(63) = 126.

Time = 5.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.18

method	result
default	$\frac{2279\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-25x^3\sqrt{-x^4+x^2+2}-\frac{625x\sqrt{-x^4+x^2+2}}{3}-\frac{3905\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\right.\right.}{6\sqrt{-x^4+}}$
risch	$\frac{25x\left(3x^2+25\right)\left(x^4-x^2-2\right)}{3\sqrt{-x^4+x^2+2}}+\frac{2279\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{3905\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right.\right.}{6\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{2279\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-25x^3\sqrt{-x^4+x^2+2}-\frac{625x\sqrt{-x^4+x^2+2}}{3}-\frac{3905\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\right.\right.}{6\sqrt{-x^4+}}$

```
[In] int((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2279/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(
1/2*x*2^(1/2),I*2^(1/2))-25*x^3*(-x^4+x^2+2)^(1/2)-625/3*x*(-x^4+x^2+2)^(1/
2)-3905/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(Ellipt
icF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{\left(7+5x^2\right)^3}{\sqrt{2+x^2-x^4}}\,dx$$
$$=\frac{-15620i\,\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})+17899i\,\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})-10\left(15\,x^4+125\,x^2+781\right)\sqrt{-x^4}}{6\,x}$$

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(-15620*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 17899*I*sqrt(
2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - 10*(15*x^4 + 125*x^2 + 781)*sqrt
(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5x^2+7)^3}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

```
[In] integrate((5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
[Out] Integral((5*x**2 + 7)**3/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(1/2), x)
```

3.333 $\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = -\frac{25}{3}x\sqrt{2+x^2-x^4}+\frac{260}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - 21\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] 260/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-21*EllipticF(1/2*x*2^(1/2),I*2^(1/
2))-25/3*x*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1220, 1194, 538, 435, 430}

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = -21\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) + \frac{260}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{3}\sqrt{-x^4+x^2+2}x$$

```
[In] Int[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/3 + (260*EllipticE[ArcSin[x/Sqrt[2]], -2])/3 -
21*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25}{3}x\sqrt{2+x^2-x^4}-\frac{1}{3}\int \frac{-197-260x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{25}{3}x\sqrt{2+x^2-x^4}-\frac{2}{3}\int \frac{-197-260x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= -\frac{25}{3}x\sqrt{2+x^2-x^4}-42\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{260}{3}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= -\frac{25}{3}x\sqrt{2+x^2-x^4}+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{-100x-50x^3+50x^5+520i\sqrt{4+2x^2-2x^4}E\left(i\text{arcsinh}(x)\,\middle|-\tfrac{1}{2}\right)-717i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left(i\text{arcsi}\right.}{6\sqrt{2+x^2-x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-100*x - 50*x^3 + 50*x^5 + (520*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*Arc
Sinh[x], -1/2] - (717*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1
/2])/(6*Sqrt[2 + x^2 - x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(46) = 92.

Time = 1.95 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.72

method	result	siz
default	$\frac{197\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{25x\sqrt{-x^4+x^2+2}}{3}-\frac{130\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{3\sqrt{-x^4+x^2+2}}$	125
elliptic	$\frac{197\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{25x\sqrt{-x^4+x^2+2}}{3}-\frac{130\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{3\sqrt{-x^4+x^2+2}}$	125
risch	$\frac{25x\left(x^4-x^2-2\right)}{3\sqrt{-x^4+x^2+2}}+\frac{197\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{130\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{3\sqrt{-x^4+x^2+2}}$	135

```
[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 197/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1
/2*x*2^(1/2),I*2^(1/2))-25/3*x*(-x^4+x^2+2)^(1/2)-130/3*2^(1/2)*(-2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-
EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \frac{-1040i\sqrt{2}xE\left(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}\right)+1237i\sqrt{2}xF\left(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}\right)-10\sqrt{-x^4+x^2+2}(5x^2+52)}{6x}$$

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(-1040*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 1237*I*sqrt(2)
*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - 10*sqrt(-x^4 + x^2 + 2)*(5*x^2 + 5
2))/x
```

Sympy [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5x^2+7)^2}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

```
[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5x^2+7)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(1/2), x)
```

3.334 $\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = 5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] 5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+2*EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1194, 538, 435, 430}

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = 2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[In] Int[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] 5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{7+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= 4\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+5\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\ &= 5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx=\frac{i\left(10E\left(i\operatorname{arcsinh}(x)\,\middle|-\frac{1}{2}\right)-17\operatorname{EllipticF}\left(i\operatorname{arcsinh}(x),-\frac{1}{2}\right)\right)}{\sqrt{2}}$$

```
[In] Integrate[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
[Out] (I*(10*EllipticE[I*ArcSinh[x], -1/2] - 17*EllipticF[I*ArcSinh[x], -1/2]))/S
qrt[2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.

Time = 0.74 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.40

method	result	size
default	$\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}$	110
elliptic	$\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}$	110

```
[In] int((5*x^2+7)/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 7/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2
*x*2^(1/2),I*2^(1/2))-5/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+
2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1
/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.08

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \frac{-20i\,\sqrt{2}xE\left(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|\,-\frac{1}{2}\right)+27i\,\sqrt{2}xF\left(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|\,-\frac{1}{2}\right)-10\,\sqrt{-x^4+x^2+2}}{2\,x}$$

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-20*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 27*I*sqrt(2)*x*e
lliptic_f(arcsin(sqrt(2)/x), -1/2) - 10*sqrt(-x^4 + x^2 + 2))/x
```

Sympy [F]

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{5x^2+7}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}\,dx$$

```
[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

Giac [F]

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(1/2), x)
```

3.335 $\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 10

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = \operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1109, 430}

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = \operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[In] Int[1/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
```

```
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/Sqrt[2 + x^2 - x^4],x]
[Out] ((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(13) = 26.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 4.70

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}}$	47
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}}$	47

```
[In] int(1/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2
*x*2^(1/2),I*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)|-2\right)$$

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] elliptic_f(arcsin(1/2*sqrt(2)*x), -2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] integrate(1/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + x**2 + 2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] int(1/(x^2 - x^4 + 2)^(1/2),x)
[Out] int(1/(x^2 - x^4 + 2)^(1/2), x)
```

3.336 $\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 17

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \frac{1}{7}\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] 1/7*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1226, 551}

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \frac{1}{7}\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[In] Int[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
```

```
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=2\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7+5x^2)}\,dx\\&=\frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int\frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx=-\frac{i\,\text{EllipticPi}\left(\frac{5}{7},i\text{arcsinh}(x),-\frac{1}{2}\right)}{7\sqrt{2}}$$

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] ((-1/7*I)*EllipticPi[5/7, I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.

Time = 0.91 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\frac{10}{7},i\sqrt{2}\right)}{7\sqrt{-x^4+x^2+2}}$	48
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\frac{10}{7},i\sqrt{2}\right)}{7\sqrt{-x^4+x^2+2}}$	48

```
[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1
/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(5*x^6 + 2*x^4 - 17*x^2 - 14), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-(x^2-2)(x^2+1)}\,(5x^2+7)}\,dx$$

```
[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx=\int \frac{1}{(5\,x^2+7)\,\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2)), x)
```

3.337 $\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx &= -\frac{25x\sqrt{2+x^2-x^4}}{476\,(7+5x^2)} - \frac{5}{476}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\\ &\quad -\frac{1}{238}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\\ &\quad +\frac{167\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{3332}\end{aligned}$$

```
[Out] -5/476*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1/238*EllipticF(1/2*x*2^(1/2),I*2
^(1/2))+167/3332*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-25/476*x*(-x^4+x
^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1237, 1730, 1194, 538, 435, 430, 1226, 551}

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx &= -\frac{1}{238}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\\ &\quad -\frac{5}{476}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\\ &\quad +\frac{167\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{3332}\\ &\quad -\frac{25\sqrt{-x^4+x^2+2}x}{476\,(5x^2+7)}\end{aligned}$$

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) - (5*EllipticE[ArcSin[x/Sqrt[
2]], -2])/476 - EllipticF[ArcSin[x/Sqrt[2]], -2]/238 + (167*EllipticPi[-10/
7, ArcSin[x/Sqrt[2]], -2])/3332
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{25x\sqrt{2+x^2-x^4}}{476\,(7+5x^2)}+\frac{1}{476}\int\frac{118-70x^2-25x^4}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476\,(7+5x^2)}-\frac{\int\frac{175+125x^2}{\sqrt{2+x^2-x^4}}\,dx}{11900}+\frac{167}{476}\int\frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476\,(7+5x^2)}-\frac{\int\frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{5950}+\frac{167}{238}\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7+5x^2)}\,dx\\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476\,(7+5x^2)}+\frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}\\
&\quad-\frac{1}{119}\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx-\frac{5}{476}\int\frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476\,(7+5x^2)}-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\\
&\quad-\frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.65

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{-700x-350x^3+350x^5-70i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}E\left(i\operatorname{arcsinh}(x)\,\middle|\,-\frac{1}{2}\right)+119i\sqrt{2}(7+5x^2)\sqrt{2+}}{}$$

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-700*x - 350*x^3 + 350*x^5 - (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4
]*EllipticE[I*ArcSinh[x], -1/2] + (119*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2
- x^4]*EllipticF[I*ArcSinh[x], -1/2] - (1169*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (835*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x
^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(6664*(7 + 5*x^2)*Sqrt[2 + x^2 - x
^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(71) = 142.

Time = 2.95 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.23

method	result
default	$-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}-\frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{476\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{952\sqrt{-x^4+x^2+2}}+\frac{167\sqrt{2}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}}{3332\sqrt{-x^4+}}$
elliptic	$-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}-\frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{476\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{952\sqrt{-x^4+x^2+2}}+\frac{167\sqrt{2}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}}{3332\sqrt{-x^4+}}$
risch	$\frac{25(x^4-x^2-2)x}{476(5x^2+7)\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{136\sqrt{-x^4+x^2+2}}+\frac{5\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{952\sqrt{-x^4+x^2+2}}+$

```
[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/476*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/476*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-5/952*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2)
,I*2^(1/2))+167/3332*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(
1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98), x
)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx=\int \frac{1}{\sqrt{-(x^2-2)(x^2+1)}\,(5x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx=\int \frac{1}{{\left(5\,x^2+7\right)}^2\,\sqrt{-x^4+x^2+2}} \,d x$$

```
[In] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2)), x)
```

3.338 $\int \frac{1}{\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\begin{aligned}\int \frac{1}{\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}\,dx = & -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)} \\ & -\frac{2505E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152} \\ & -\frac{263\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{226576} \\ & +\frac{58915\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{3172064}\end{aligned}$$

```
[Out] -2505/453152*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-263/226576*EllipticF(1/2*x*
2^(1/2),I*2^(1/2))+58915/3172064*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-
25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2+2)^(1/2)/(
5*x^2+7)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{1237, 1710, 1730, 1194, 538, 435, 430, 1226, 551\}$

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = -\frac{263\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{226576} - \frac{2505E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152} + \frac{58915\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{3172064} - \frac{12525\sqrt{-x^4+x^2+2}\,x}{453152\,(5x^2+7)} - \frac{25\sqrt{-x^4+x^2+2}\,x}{952\,(5x^2+7)^2}$$

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) - (12525*x*Sqrt[2 + x^2 - x
^4])/(453152*(7 + 5*x^2)) - (2505*EllipticE[ArcSin[x/Sqrt[2]], -2])/453152
- (263*EllipticF[ArcSin[x/Sqrt[2]], -2])/226576 + (58915*EllipticPi[-10/7,
ArcSin[x/Sqrt[2]], -2])/3172064
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
```

```
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
```

```
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2}+\frac{1}{952}\int\frac{186-190x^2+25x^4}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)}+\frac{\int\frac{37698-32690x^2-12525x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{453152}\\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)}-\frac{\int\frac{75775+62625x^2}{\sqrt{2+x^2-x^4}}\,dx}{11328800}+\frac{58915\int\frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{453152}\\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)}\\
&\quad-\frac{\int\frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{5664400}+\frac{58915\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx}{226576}\\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)}+\frac{58915\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3172064}\\
&\quad-\frac{263\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{113288}-\frac{2505\int\frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{453152}\\
&= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}\\
&\quad-\frac{263F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{226576}+\frac{58915\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3172064}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int\frac{1}{\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}\,dx$$

$$=\frac{\frac{350x\left(-7966-8993x^2+1478x^4+2505x^6\right)}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}-35070i\sqrt{2}E\left(i\text{arcsinh}(x)\,\middle|-\frac{1}{2}\right)+56287i\sqrt{2}\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)-}{6344128}$$

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] ((350*x*(-7966 - 8993*x^2 + 1478*x^4 + 2505*x^6))/((7 + 5*x^2)^2*Sqrt[2 + x
^2 - x^4]) - (35070*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (56287*I)*Sq
rt[2]*EllipticF[I*ArcSinh[x], -1/2] - (58915*I)*Sqrt[2]*EllipticPi[5/7, I*A
rcSinh[x], -1/2])/6344128
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.85

method	result
default	$-\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}-\frac{12525x\sqrt{-x^4+x^2+2}}{453152(5x^2+7)}-\frac{263\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{453152\sqrt{-x^4+x^2+2}}-\frac{2505\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}}{906304\sqrt{-x^4+x^2+2}}$
elliptic	$-\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}-\frac{12525x\sqrt{-x^4+x^2+2}}{453152(5x^2+7)}-\frac{263\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{453152\sqrt{-x^4+x^2+2}}-\frac{2505\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}}{906304\sqrt{-x^4+x^2+2}}$
risch	$\frac{25(x^4-x^2-2)x(2505x^2+3983)}{453152(5x^2+7)^2\sqrt{-x^4+x^2+2}}-\frac{433\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{129472\sqrt{-x^4+x^2+2}}+\frac{2505\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i}{906304\sqrt{-x^4+x^2+2}}$

```
[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2+2)^(1/2)/
(5*x^2+7)-263/453152*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1
/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-2505/906304*2^(1/2)*(-2*x^2+4)^(1/2)
*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+58915/
3172064*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elliptic
Pi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 18
13*x^2 - 686), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^3}\,dx$$

```
[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{(5\,x^2+7)^3\,\sqrt{-x^4+x^2+2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2)), x)
```

3.339 $\int \frac{\left(7+5x^2\right)^5}{\left(2+x^2-x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^5}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}}\\ &+\frac{27500}{3}x\sqrt{2+x^2-x^4}+625x^3\sqrt{2+x^2-x^4}\\ &-\frac{3482293}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{627857}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\end{aligned}$$

```
[Out] -3482293/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+627857/6*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))+1/18*x*(1419793*x^2+1419985)/(-x^4+x^2+2)^(1/2)+27500/3*x*(
-x^4+x^2+2)^(1/2)+625*x^3*(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1219, 1693, 1194, 538, 435, 430}

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^5}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{627857}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\\ &-\frac{3482293}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{27500}{3}\sqrt{-x^4+x^2+2}x\\ &+\frac{\left(1419793x^2+1419985\right)x}{18\sqrt{-x^4+x^2+2}}+625\sqrt{-x^4+x^2+2}x^3\end{aligned}$$

```
[In] Int[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (27500*x*Sqrt[2 + x^
2 - x^4])/3 + 625*x^3*Sqrt[2 + x^2 - x^4] - (3482293*EllipticE[ArcSin[x/Sqr
t[2]], -2])/18 + (627857*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int\frac{1268722+3084793x^2+450000x^4+56250x^6}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}}+625x^3\sqrt{2+x^2-x^4}\\
&\quad+\frac{1}{90}\int\frac{-6343610-15761465x^2-2475000x^4}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}}+\frac{27500}{3}x\sqrt{2+x^2-x^4}\\
&\quad+625x^3\sqrt{2+x^2-x^4}-\frac{1}{270}\int\frac{23980830+52234395x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}}+\frac{27500}{3}x\sqrt{2+x^2-x^4}\\
&\quad+625x^3\sqrt{2+x^2-x^4}-\frac{1}{135}\int\frac{23980830+52234395x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}}+\frac{27500}{3}x\sqrt{2+x^2-x^4}+625x^3\sqrt{2+x^2-x^4}\\
&\quad-\frac{3482293}{18}\int\frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{627857}{3}\int\frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}}+\frac{27500}{3}x\sqrt{2+x^2-x^4}+625x^3\sqrt{2+x^2-x^4}\\
&\quad-\frac{3482293}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{627857}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1749985x + 1607293x^3 - 153750x^5 - 11250x^7 - 3482293i\sqrt{4+2x^2-2x^4}E(i\text{arcs}}{18\sqrt{2+x^2-x}}$$

```
[In] Integrate[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (1749985*x + 1607293*x^3 - 153750*x^5 - 11250*x^7 - (3482293*I)*Sqrt[4 + 2*
x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] + (4281654*I)*Sqrt[4 + 2*x^2 - 2
*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(18*Sqrt[2 + x^2 - x^4])
```

Maple [A] (verified)

Time = 8.85 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.53

method	result
risch	$-\frac{x(11250x^6+153750x^4-1607293x^2-1749985)}{18\sqrt{-x^4+x^2+2}} - \frac{799361\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F(\frac{x\sqrt{2}}{2},i\sqrt{2})}{18\sqrt{-x^4+x^2+2}} + \frac{3482293\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}}{36\sqrt{-x}}$
elliptic	$\frac{\frac{1419793}{18}x^3+\frac{1419985}{18}x}{\sqrt{-x^4+x^2+2}} + 625x^3\sqrt{-x^4+x^2+2} + \frac{27500x\sqrt{-x^4+x^2+2}}{3} - \frac{799361\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F(\frac{x\sqrt{2}}{2},i\sqrt{2})}{18\sqrt{-x^4+x^2+2}} +$
default	$\frac{\frac{84035}{18}x-\frac{16807}{18}x^3}{\sqrt{-x^4+x^2+2}} - \frac{799361\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F(\frac{x\sqrt{2}}{2},i\sqrt{2})}{18\sqrt{-x^4+x^2+2}} + \frac{3482293\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,(F(\frac{x\sqrt{2}}{2},i\sqrt{2})-E(\frac{x\sqrt{2}}{2},i\sqrt{2}))}{36\sqrt{-x^4+x^2+2}}$

```
[In] int((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*x*(11250*x^6+153750*x^4-1607293*x^2-1749985)/(-x^4+x^2+2)^(1/2)-79936
1/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/
2*x*2^(1/2),I*2^(1/2))+3482293/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-
x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2
),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx =$$

$$-\frac{6964586\sqrt{2}(-i\,x^5+i\,x^3+2i\,x)E(\arcsin(\frac{\sqrt{2}}{x})\mid-\frac{1}{2})+7763947\sqrt{2}(i\,x^5-i\,x^3-2i\,x)F(\arcsin(\frac{\sqrt{2}}{x})\mid-}{18\,(x^5-x^3-2\,x)}$$

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
[Out] -1/18*(6964586*sqrt(2)*(-I*x^5 + I*x^3 + 2*I*x)*elliptic_e(arcsin(sqrt(2)/x
), -1/2) + 7763947*sqrt(2)*(I*x^5 - I*x^3 - 2*I*x)*elliptic_f(arcsin(sqrt(2
)/x), -1/2) - 2*(5625*x^8 + 76875*x^6 + 937500*x^4 - 2616139*x^2 - 3482293)
*sqrt(-x^4 + x^2 + 2))/(x^5 - x^3 - 2*x)
```

Sympy [F]

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx=\int \frac{(5x^2+7)^5}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**5/(-x**4+x**2+2)**(3/2),x)
[Out] Integral((5*x**2 + 7)**5/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx=\int \frac{(5\,x^2+7)^5}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx=\int \frac{(5\,x^2+7)^5}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx=\int \frac{{\left(5\,x^2+7\right)}^5}{{\left(-x^4+x^2+2\right)}^{3/2}} \,d x$$

```
[In] int((5*x^2 + 7)^5/(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^5/(x^2 - x^4 + 2)^(3/2), x)
```

3.340 $\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\begin{aligned}\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx &= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}}+\frac{625}{3}x\sqrt{2+x^2-x^4}\\ &\quad -\frac{165239}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{31921}{6}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\end{aligned}$$

```
[Out] -165239/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+31921/6*EllipticF(1/2*x*2^(1/
2),I*2^(1/2))+1/18*x*(83489*x^2+83585)/(-x^4+x^2+2)^(1/2)+625/3*x*(-x^4+x^2
+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1219, 1693, 1194, 538, 435, 430}

$$\begin{aligned}\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx &= \frac{31921}{6}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\\ &\quad -\frac{165239}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{625}{3}\sqrt{-x^4+x^2+2}x+\frac{(83489x^2+83585)\,x}{18\sqrt{-x^4+x^2+2}}\end{aligned}$$

```
[In] Int[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^
4])/3 - (165239*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (31921*EllipticF[Arc
Sin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
```

```
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{61976+157739x^2+11250x^4}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} + \frac{1}{54}\int \frac{-208428-495717x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} + \frac{1}{27}\int \frac{-208428-495717x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} \\
&\quad - \frac{165239}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{31921}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} \\
&\quad - \frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{31921}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \frac{91085x+87239x^3-3750x^5-165239i\sqrt{4+2x^2-2x^4}E\left(i\text{arcsinh}(x)\,\middle|-\frac{1}{2}\right)+19997}{18\sqrt{2+x^2-x^4}}$$

```
[In] Integrate[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (91085*x + 87239*x^3 - 3750*x^5 - (165239*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Ellipt
icE[I*ArcSinh[x], -1/2] + (199977*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*Ar
cSinh[x], -1/2])/(18*Sqrt[2 + x^2 - x^4])
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.85

method	result
risch	$-\frac{x(3750x^4-87239x^2-91085)}{18\sqrt{-x^4+x^2+2}}-\frac{17369\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{165239\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,(F(\frac{x\sqrt{2}}{2},i\sqrt{2})-E(}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{83489}{18}x^3+\frac{83585}{18}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{3}-\frac{17369\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{165239\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,(F(\frac{x\sqrt{2}}{2},i}{36\sqrt{-x^4+x^2+2}}$
default	$\frac{\frac{12005}{18}x-\frac{2401}{18}x^3}{\sqrt{-x^4+x^2+2}}-\frac{17369\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{165239\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+$

```
[In] int((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*x*(3750*x^4-87239*x^2-91085)/(-x^4+x^2+2)^(1/2)-17369/9*2^(1/2)*(-2*x
^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1
/2))+165239/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(E
llipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx =$$

$$-\frac{165239\,\sqrt{2}(-i\,x^5+i\,x^3+2i\,x)E(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid -\frac{1}{2})+182608\,\sqrt{2}(i\,x^5-i\,x^3-2i\,x)F(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid -\frac{1}{2})}{9\,(x^5-x^3-2\,x)}$$

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*(165239*sqrt(2)*(-I*x^5 + I*x^3 + 2*I*x)*elliptic_e(arcsin(sqrt(2)/x),
 -1/2) + 182608*sqrt(2)*(I*x^5 - I*x^3 - 2*I*x)*elliptic_f(arcsin(sqrt(2)/x
), -1/2) - (1875*x^6 + 39000*x^4 - 128162*x^2 - 165239)*sqrt(-x^4 + x^2 + 2
))/(x^5 - x^3 - 2*x)
```

Sympy [F]

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^4}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**4/(-x**4+x**2+2)**(3/2),x)
[Out] Integral((5*x**2 + 7)**4/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(-x^4+x^2+2)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^4/(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^4/(x^2 - x^4 + 2)^(3/2), x)
```

3.341 $\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1763}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] -7147/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1763/6*EllipticF(1/2*x*2^(1/2),
I*2^(1/2))+1/18*x*(4897*x^2+4945)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1219, 1194, 538, 435, 430}

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1763}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - \frac{7147}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}$$

```
[In] Int[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(4945 + 4897*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (7147*EllipticE[ArcSin[x/S
qrt[2]], -2])/18 + (1763*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{1858+7147x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9}\int \frac{1858+7147x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx
\end{aligned}$$

$$= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{1763}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1}{18}\left(\frac{4945x}{\sqrt{2+x^2-x^4}} + \frac{4897x^3}{\sqrt{2+x^2-x^4}} - 7147i\sqrt{2}E\left(i\operatorname{arcsinh}(x)\middle| -\frac{1}{2}\right) + 8076i\sqrt{2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}(x), -\frac{1}{2}\right)\right)$$

```
[In] Integrate[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] ((4945*x)/Sqrt[2 + x^2 - x^4] + (4897*x^3)/Sqrt[2 + x^2 - x^4] - (7147*I)*S
qrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (8076*I)*Sqrt[2]*EllipticF[I*ArcSinh
[x], -1/2])/18
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(53) = 106.

Time = 2.52 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}} - \frac{929\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}} + \frac{7147\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{4897}{18}x^3+\frac{4945}{18}x}{\sqrt{-x^4+x^2+2}} - \frac{929\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}} + \frac{7147\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
default	$\frac{\frac{1715}{18}x-\frac{343}{18}x^3}{\sqrt{-x^4+x^2+2}} - \frac{929\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}} + \frac{7147\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}} + \frac{\frac{625}{9}x}{\sqrt{-x}}$

```
[In] int((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*x*(4897*x^2+4945)/(-x^4+x^2+2)^(1/2)-929/18*2^(1/2)*(-2*x^2+4)^(1/2)*(
x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+7147/36*
2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*
2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.84

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = $$

$$-\frac{14294\,\sqrt{2}(-i\,x^5+i\,x^3+2i\,x)E(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})+15223\,\sqrt{2}(i\,x^5-i\,x^3-2i\,x)F(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})-2}{18\,(x^5-x^3-2\,x)}$$

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/18*(14294*sqrt(2)*(-I*x^5 + I*x^3 + 2*I*x)*elliptic_e(arcsin(sqrt(2)/x),
 -1/2) + 15223*sqrt(2)*(I*x^5 - I*x^3 - 2*I*x)*elliptic_f(arcsin(sqrt(2)/x)
, -1/2) - 2*(1125*x^4 - 6046*x^2 - 7147)*sqrt(-x^4 + x^2 + 2))/(x^5 - x^3 -
 2*x)
```

Sympy [F]

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^3}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(-x^4+x^2+2)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(3/2), x)
```

3.342 $\int \frac{\left(7+5x^2\right)^2}{\left(2+x^2-x^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}} \, dx = \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{281}{18} E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{139}{6} \operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] -281/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+139/6*EllipticF(1/2*x*2^(1/2),I*
2^(1/2))+1/18*x*(281*x^2+305)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1219, 1194, 538, 435, 430}

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}} \, dx = \frac{139}{6} \operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - \frac{281}{18} E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}$$

```
[In] Int[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (281*EllipticE[ArcSin[x/Sqrt
[2]], -2])/18 + (139*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{-136+281x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9}\int \frac{-136+281x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx
\end{aligned}$$

$$= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{281}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{139}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$

$$= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1}{18}\left(\frac{305x}{\sqrt{2+x^2-x^4}} + \frac{281x^3}{\sqrt{2+x^2-x^4}} - 281i\sqrt{2}E\left(i\text{arcsinh}(x)\middle| -\frac{1}{2}\right) + 213i\sqrt{2}\,\text{EllipticF}\left(i\text{arcsinh}(x), -\frac{1}{2}\right)\right)$$

```
[In] Integrate[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] ((305*x)/Sqrt[2 + x^2 - x^4] + (281*x^3)/Sqrt[2 + x^2 - x^4] - (281*I)*Sqrt
[2]*EllipticE[I*ArcSinh[x], -1/2] + (213*I)*Sqrt[2]*EllipticF[I*ArcSinh[x],
 -1/2])/18
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(53) = 106.

Time = 2.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}+\frac{34\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{281}{18}x^3+\frac{305}{18}x}{\sqrt{-x^4+x^2+2}}+\frac{34\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
default	$\frac{\frac{245}{18}x-\frac{49}{18}x^3}{\sqrt{-x^4+x^2+2}}+\frac{34\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+\frac{\frac{25}{9}x^3+\frac{10}{9}}{\sqrt{-x^4+x^2}}$

```
[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*x*(281*x^2+305)/(-x^4+x^2+2)^(1/2)+34/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+281/36*2^(1/
2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/
2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx =$$
$$-\frac{281\,(x^4-x^2-2)E(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|-2)-553\,(x^4-x^2-2)F(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|-2)+2\sqrt{-x^4+x^2+2}}{36\,(x^4-x^2-2)}$$

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*(281*(x^4 - x^2 - 2)*elliptic_e(arcsin(1/2*sqrt(2)*x), -2) - 553*(x^4
 - x^2 - 2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -2) + 2*sqrt(-x^4 + x^2 + 2)*
(281*x^3 + 305*x))/(x^4 - x^2 - 2)
```

Sympy [F]

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^2}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(-x^4+x^2+2)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(3/2), x)
```

3.343 $\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\begin{aligned}\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} \\ &\quad -\frac{13}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\end{aligned}$$

```
[Out] -13/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+17/6*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))+1/18*x*(13*x^2+25)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1192, 1194, 538, 435, 430}

$$\begin{aligned}\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{17}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) \\ &\quad -\frac{13}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}\end{aligned}$$

```
[In] Int[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (13*EllipticE[ArcSin[x/Sqrt[2]
], -2])/18 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{-38+13x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9}\int \frac{-38+13x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} - \frac{13}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{17}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} - \frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1}{18}\left(\frac{25x}{\sqrt{2+x^2-x^4}}+\frac{13x^3}{\sqrt{2+x^2-x^4}} - 13i\sqrt{2}E\left(i\text{arcsinh}(x)\middle|-\frac{1}{2}\right)-6i\sqrt{2}\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)\right)$$

```
[In] Integrate[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] ((25*x)/Sqrt[2 + x^2 - x^4] + (13*x^3)/Sqrt[2 + x^2 - x^4] - (13*I)*Sqrt[2]
*EllipticE[I*ArcSinh[x], -1/2] - (6*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2
])/18
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(53) = 106.

Time = 1.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{13}{18}x^3+\frac{25}{18}x}{\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
default	$\frac{\frac{35}{18}x-\frac{7}{18}x^3}{\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+\frac{\frac{10}{9}x^3-}{\sqrt{-x^4+x}}$

```
[In] int((5*x^2+7)/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*x*(13*x^2+25)/(-x^4+x^2+2)^(1/2)+19/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+13/36*2^(1/2)
*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2)
,I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx = \\ -\frac{13\left(x^4-x^2-2\right)E\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|-2\right)-89\left(x^4-x^2-2\right)F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|-2\right)+2\sqrt{-x^4+x^2+2}(13}{36\left(x^4-x^2-2\right)}$$

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*(13*(x^4 - x^2 - 2)*elliptic_e(arcsin(1/2*sqrt(2)*x), -2) - 89*(x^4 -
 x^2 - 2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -2) + 2*sqrt(-x^4 + x^2 + 2)*(1
3*x^3 + 25*x))/(x^4 - x^2 - 2)
```

Sympy [F]

$$\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx = \int \frac{5x^2+7}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx = \int \frac{5\,x^2+7}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx=\int \frac{5\,x^2+7}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx=\int \frac{5\,x^2+7}{\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(3/2), x)
```

3.344 $\int \frac{1}{(2+x^2-x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{1}{(2+x^2-x^4)^{3/2}} \, dx = \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} + \frac{1}{18} E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{1}{6} \operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right), -2\right)$$

```
[Out] 1/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1/6*EllipticF(1/2*x*2^(1/2),I*2^(1/
2))+1/18*x*(-x^2+5)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1106, 1194, 538, 435, 430}

$$\int \frac{1}{(2+x^2-x^4)^{3/2}} \, dx = \frac{1}{6} \operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right), -2\right) + \frac{1}{18} E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right) + \frac{x(5-x^2)}{18\sqrt{-x^4+x^2+2}}$$

```
[In] Int[(2 + x^2 - x^4)^(-3/2),x]
```

```
[Out] (x*(5 - x^2))/(18*Sqrt[2 + x^2 - x^4]) + EllipticE[ArcSin[x/Sqrt[2]], -2]/1
8 + EllipticF[ArcSin[x/Sqrt[2]], -2]/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{-4-x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9}\int \frac{-4-x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} + \frac{1}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{1}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} + \frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1}{18}\left(\frac{5x}{\sqrt{2+x^2-x^4}} - \frac{x^3}{\sqrt{2+x^2-x^4}} + i\sqrt{2}E\left(i\operatorname{arcsinh}(x)\left|-\frac{1}{2}\right.\right) - 3i\sqrt{2}\,\text{EllipticF}\left(i\operatorname{arcsinh}(x),-\frac{1}{2}\right)\right)$$

```
[In] Integrate[(2 + x^2 - x^4)^(-3/2),x]
```

```
[Out] ((5*x)/Sqrt[2 + x^2 - x^4] - x^3/Sqrt[2 + x^2 - x^4] + I*Sqrt[2]*EllipticE[
I*ArcSinh[x], -1/2] - (3*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2])/18
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(53) = 106.

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.36

method	result	size
risch	$-\frac{x\left(x^{2}-5\right)}{18\sqrt{-x^{4}+x^{2}+2}}+\frac{\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^{4}+x^{2}+2}}-\frac{\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^{4}+x^{2}+2}}$	130
default	$\frac{\frac{5}{18}x-\frac{1}{18}x^{3}}{\sqrt{-x^{4}+x^{2}+2}}+\frac{\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^{4}+x^{2}+2}}-\frac{\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^{4}+x^{2}+2}}$	133
elliptic	$\frac{\frac{5}{18}x-\frac{1}{18}x^{3}}{\sqrt{-x^{4}+x^{2}+2}}+\frac{\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^{4}+x^{2}+2}}-\frac{\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^{4}+x^{2}+2}}$	133

```
[In] int(1/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*x*(x^2-5)/(-x^4+x^2+2)^(1/2)+1/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/
2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-1/36*2^(1/2)*(-2*x
^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \frac{(x^4-x^2-2)E\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|\,-2\right)+7\,(x^4-x^2-2)F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|\,-2\right)+2}{36\,(x^4-x^2-2)}$$

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/36*((x^4 - x^2 - 2)*elliptic_e(arcsin(1/2*sqrt(2)*x), -2) + 7*(x^4 - x^2
- 2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -2) + 2*sqrt(-x^4 + x^2 + 2)*(x^3 -
5*x))/(x^4 - x^2 - 2)
```

Sympy [F]

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(-3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2+x^2-x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

```
[In] int(1/(x^2 - x^4 + 2)^(3/2),x)
[Out] int(1/(x^2 - x^4 + 2)^(3/2), x)
```

3.345 $\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx = \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}}+\frac{8}{153}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{102}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{25}{238}\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] 8/153*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1/102*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))-25/238*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/306*x*(-16*x^2+35
)/(-x^4+x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1235, 1192, 1194, 538, 435, 430, 1226, 551}

$$\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx = \frac{1}{102}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) + \frac{8}{153}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{25}{238}\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}}$$

```
[In] Int[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(35 - 16*x^2))/(306*Sqrt[2 + x^2 - x^4]) + (8*EllipticE[ArcSin[x/Sqrt[2]
], -2])/153 + EllipticF[ArcSin[x/Sqrt[2]], -2]/102 - (25*EllipticPi[-10/7,
ArcSin[x/Sqrt[2]], -2])/238
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{34}\int \frac{-12+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx\right)-\frac{25}{34}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\left(35-16x^2\right)}{306\sqrt{2+x^2-x^4}}+\frac{1}{612}\int \frac{38+32x^2}{\sqrt{2+x^2-x^4}}\,dx-\frac{25}{17}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx\\
&= \frac{x\left(35-16x^2\right)}{306\sqrt{2+x^2-x^4}}-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{306}\int \frac{38+32x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(35-16x^2\right)}{306\sqrt{2+x^2-x^4}}-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\\
&\quad+\frac{1}{51}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{8}{153}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= \frac{x\left(35-16x^2\right)}{306\sqrt{2+x^2-x^4}}+\frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\\
&\quad+\frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}}\,dx=\frac{\frac{490x}{\sqrt{2+x^2-x^4}}-\frac{224x^3}{\sqrt{2+x^2-x^4}}+224i\sqrt{2}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right)-357i\sqrt{2}\,\mathrm{EllipticF}\,(}{4284}$$

```
[In] Integrate[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] ((490*x)/Sqrt[2 + x^2 - x^4] - (224*x^3)/Sqrt[2 + x^2 - x^4] + (224*I)*Sqrt
[2]*EllipticE[I*ArcSinh[x], -1/2] - (357*I)*Sqrt[2]*EllipticF[I*ArcSinh[x],
 -1/2] + (225*I)*Sqrt[2]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/4284
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(69) = 138$.

Time = 1.82 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.28

method	result
default	$\frac{-\frac{8}{153}x^3+\frac{35}{306}x}{\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{204\sqrt{-x^4+x^2+2}}+\frac{4\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{153\sqrt{-x^4+x^2+2}}-\frac{25\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\right.}{238\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{-\frac{8}{153}x^3+\frac{35}{306}x}{\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{204\sqrt{-x^4+x^2+2}}+\frac{4\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{153\sqrt{-x^4+x^2+2}}-\frac{25\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\right.}{238\sqrt{-x^4+x^2+2}}$
risch	$-\frac{x\left(16x^2-35\right)}{306\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{612\sqrt{-x^4+x^2+2}}-\frac{4\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{153\sqrt{-x^4+x^2+2}}-25\sqrt{2}$

```
[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-4/153*x^3+35/612*x)/(-x^4+x^2+2)^(1/2)+1/204*2^(1/2)*(-2*x^2+4)^(1/2)*(
x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+4/153*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(
1/2),I*2^(1/2))-25/238*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)\left(2+x^2-x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^10 - 3*x^8 - 29*x^6 - x^4 + 48*x^2 + 28)
, x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)\left(2+x^2-x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\cdot\left(5x^2+7\right)}\,dx$$

```
[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)\left(2+x^2-x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)\left(2+x^2-x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\left(2+x^2-x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(5\,x^2+7\right)\,\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2)), x)
```

3.346 $\int \frac{1}{\left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2}}\, dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}}\, dx &= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} \\ &+ \frac{625x\sqrt{2+x^2-x^4}}{16184\,(7+5x^2)} + \frac{5143E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656} \\ &+ \frac{89\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24276} - \frac{10825\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{113288}\end{aligned}$$

```
[Out] 5143/145656*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+89/24276*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))-10825/113288*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/10
404*x*(-287*x^2+580)/(-x^4+x^2+2)^(1/2)+625/16184*x*(-x^4+x^2+2)^(1/2)/(5*x
^2+7)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1192, 1194, 538, 435, 430, 1237, 1730, 1226, 551}

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}}\, dx &= \frac{89\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24276} \\ &+ \frac{5143E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656} - \frac{10825\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{113288} \\ &+ \frac{625\sqrt{-x^4+x^2+2}\,x}{16184\,(5x^2+7)} + \frac{(580-287x^2)\,x}{10404\sqrt{-x^4+x^2+2}}\end{aligned}$$

```
[In] Int[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4
])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[2]], -2])/145656 + (
89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 - (10825*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/113288
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{194-95x^2}{1156\left(2+x^2-x^4\right)^{3/2}}-\frac{25}{34\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\right.\\
&\qquad\left.-\frac{475}{1156\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\right)dx\\
&= \frac{\int \frac{194-95x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx}{1156}-\frac{475\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{1156}-\frac{25}{34}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)}-\frac{\int \frac{-586-574x^2}{\sqrt{2+x^2-x^4}}\,dx}{20808}\\
&\quad-\frac{25\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{16184}-\frac{475}{578}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx\\
&= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)}-\frac{475\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{8092}\\
&\quad+\frac{\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}}\,dx}{16184}-\frac{\int \frac{-586-574x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{10404}-\frac{4175\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{16184}\\
&= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)}-\frac{475\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{8092}\\
&\quad+\frac{\int \frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{8092}+\frac{1}{867}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&\quad+\frac{287\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{10404}-\frac{4175\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx}{8092}\\
&= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)}+\frac{287E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{10404}\\
&\quad+\frac{F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{1734}-\frac{10825\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{113288}\\
&\quad+\frac{25\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{4046}+\frac{125\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{16184}\\
&= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)}+\frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}\\
&\quad+\frac{89F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24276}-\frac{10825\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{113288}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.96

$$\int \frac{1}{(7+5x^2)^2\,(2+x^2-x^4)^{3/2}}\,dx = \frac{953260x + 253386x^3 - 360010x^5 + 72002i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}E(i}{}$$

```
[In] Integrate[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (953260*x + 253386*x^3 - 360010*x^5 + (72002*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2
+ x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (111741*I)*Sqrt[2]*(7 + 5*x^2)
*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (681975*I)*Sqrt[2]*Sqr
t[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (487125*I)*Sqrt[2]*x
^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(2039184*(7 + 5
*x^2)*Sqrt[2 + x^2 - x^4])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(93) = 186.

Time = 3.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.88

method	result
default	$\frac{-\frac{287}{10404}x^3+\frac{145}{2601}x}{\sqrt{-x^4+x^2+2}} + \frac{625x\sqrt{-x^4+x^2+2}}{16184(5x^2+7)} + \frac{89\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{48552\sqrt{-x^4+x^2+2}} + \frac{5143\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{291312\sqrt{-x^4+x^2+2}} -$
elliptic	$\frac{-\frac{287}{10404}x^3+\frac{145}{2601}x}{\sqrt{-x^4+x^2+2}} + \frac{625x\sqrt{-x^4+x^2+2}}{16184(5x^2+7)} + \frac{89\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{48552\sqrt{-x^4+x^2+2}} + \frac{5143\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{291312\sqrt{-x^4+x^2+2}} -$
risch	$-\frac{x\left(25715x^4-18099x^2-68090\right)}{145656(5x^2+7)\sqrt{-x^4+x^2+2}} + \frac{811\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{41616\sqrt{-x^4+x^2+2}} - \frac{5143\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E(\frac{x\sqrt{2}}{2},\right.}{291312\sqrt{-x^4+x^2+2}}$

```
[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-287/20808*x^3+145/5202*x)/(-x^4+x^2+2)^(1/2)+625/16184*x*(-x^4+x^2+2)^(
1/2)/(5*x^2+7)+89/48552*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+5143/291312*2^(1/2)*(-2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-108
25/113288*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ellipt
icPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^12 + 20*x^10 - 166*x^8 - 208*x^6 + 233*
x^4 + 476*x^2 + 196), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}(5x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+7)^2 \,(-x^4+x^2+2)^{3/2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2)), x)
```

3.347 $\int \frac{1}{\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 128

$$\begin{aligned}\int \frac{1}{\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}\\ &\quad+\frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)}+\frac{3086453E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{138664512}\\ &\quad+\frac{60409\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{23110752}-\frac{6898575\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{107850176}\end{aligned}$$

```
[Out] 3086453/138664512*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+60409/23110752*Ellipti
cF(1/2*x*2^(1/2),I*2^(1/2))-6898575/107850176*EllipticPi(1/2*x*2^(1/2),-10/
7,I*2^(1/2))+1/353736*x*(-4909*x^2+9830)/(-x^4+x^2+2)^(1/2)+625/32368*x*(-x
^4+x^2+2)^(1/2)/(5*x^2+7)^2+645625/15407168*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1242, 1192, 1194, 538, 435, 430, 1237, 1710, 1730, 1226, 551}

$$\begin{aligned}\int \frac{1}{\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{60409\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{23110752}\\ &\quad+\frac{3086453E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{138664512}-\frac{6898575\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{107850176}\\ &\quad+\frac{645625\sqrt{-x^4+x^2+2}\,x}{15407168\left(5x^2+7\right)}+\frac{625\sqrt{-x^4+x^2+2}\,x}{32368\left(5x^2+7\right)^2}+\frac{\left(9830-4909x^2\right)x}{353736\sqrt{-x^4+x^2+2}}\end{aligned}$$

```
[In] Int[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 -
x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 - x^4])/(15407168*(7 +
 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2]], -2])/138664512 + (60409*El
lipticF[ArcSin[x/Sqrt[2]], -2])/23110752 - (6898575*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/107850176
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{-3278+1635x^2}{39304\left(2+x^2-x^4\right)^{3/2}}-\frac{25}{34\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}\right.\\
&\qquad \left.-\frac{475}{1156\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}-\frac{8175}{39304\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\right) dx\\
&= -\frac{\int \frac{-3278+1635x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx}{39304}-\frac{8175\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{39304}\\
&\quad -\frac{475\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx}{1156}-\frac{25}{34}\int \frac{1}{\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}+\frac{11875x\sqrt{2+x^2-x^4}}{550256\left(7+5x^2\right)}+\frac{\int \frac{9842+9818x^2}{\sqrt{2+x^2-x^4}}\,dx}{707472}\\
&\quad -\frac{25\int \frac{186-190x^2+25x^4}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx}{32368}-\frac{475\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{550256}-\frac{8175\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx}{19652}\\
&= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}+\frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)}\\
&\quad -\frac{8175\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{275128}-\frac{25\int \frac{37698-32690x^2-12525x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{15407168}\\
&\quad +\frac{\int \frac{9842+9818x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{353736}+\frac{19\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}}\,dx}{550256}-\frac{79325\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{550256}\\
&= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2}+\frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)}\\
&\quad -\frac{8175\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{275128}+\frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}}\,dx}{15407168}\\
&\quad +\frac{\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{14739}+\frac{19\int \frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{275128}+\frac{4909\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{353736}\\
&\quad -\frac{1472875\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{15407168}-\frac{79325\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx}{275128}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)} \\
&\quad + \frac{4909E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{353736} + \frac{F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{29478} \\
&\quad - \frac{193775\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3851792} + \frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{7703584} \\
&\quad + \frac{475\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{137564} + \frac{2375\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{550256} - \frac{1472875\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)}\,dx}{7703584} \\
&= \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2} \\
&\quad + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)} + \frac{90101E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4952304} \\
&\quad + \frac{1453F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{825384} - \frac{6898575\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{107850176} \\
&\quad + \frac{6575\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{3851792} + \frac{62625\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx}{15407168} \\
&= \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2} \\
&\quad + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)} + \frac{3086453E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{138664512} \\
&\quad + \frac{60409F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{23110752} - \frac{6898575\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{107850176}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.44 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.91

$$\int \frac{1}{\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}}\,dx = \frac{3857257460x + 3876617542x^3 - 737347940x^5 - 1080258550x^7 + 43210}{}$$

```
[In] Integrate[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (3857257460*x + 3876617542*x^3 - 737347940*x^5 - 1080258550*x^7 + (43210342
*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2]
 - (67352691*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSi
nh[x], -1/2] + (3042271575*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I
*ArcSinh[x], -1/2] + (4346102250*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*Ellipti
```

```
cPi[5/7, I*ArcSinh[x], -1/2] + (1552179375*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^
4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(1941303168*(7 + 5*x^2)^2*Sqrt[2 +
x^2 - x^4])
```

Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{x\left(77161325x^6+52667710x^4-276901253x^2-275518390\right)}{138664512\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}}+\frac{492701\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{39618432\sqrt{-x^4+x^2+2}}-\frac{3086453\sqrt{2}\,\sqrt{-2x^2+4}}{277}$
default	$\frac{-\frac{4909}{353736}x^3+\frac{4915}{176868}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{32368\left(5x^2+7\right)^2}+\frac{645625x\sqrt{-x^4+x^2+2}}{15407168\left(5x^2+7\right)}+\frac{60409\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{46221504\sqrt{-x^4+x^2+2}}+\frac{3086453\sqrt{2}}{27}$
elliptic	$\frac{-\frac{4909}{353736}x^3+\frac{4915}{176868}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{32368\left(5x^2+7\right)^2}+\frac{645625x\sqrt{-x^4+x^2+2}}{15407168\left(5x^2+7\right)}+\frac{60409\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{46221504\sqrt{-x^4+x^2+2}}+\frac{3086453\sqrt{2}}{27}$

```
[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/138664512*x*(77161325*x^6+52667710*x^4-276901253*x^2-275518390)/(5*x^2+7
)^2/(-x^4+x^2+2)^(1/2)+492701/39618432*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/
2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-3086453/277329024*
2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*
2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))-6898575/107850176*2^
(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2
^(1/2),-10/7,I*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^14 + 275*x^12 - 690*x^10 - 2202*x^8 -
291*x^6 + 4011*x^4 + 4312*x^2 + 1372), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

```
[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{\left(5\,x^2+7\right)^3\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2)), x)
```

3.348 $\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 242

$$
\begin{aligned}
\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx = {} & \frac{51665x\sqrt{4+3x^2+x^4}}{33(2+x^2)} \\
& + \frac{1}{33}x(18727+4516x^2)\sqrt{4+3x^2+x^4} \\
& + \frac{3050}{11}x(4+3x^2+x^4)^{3/2} \\
& + \frac{23500}{99}x^3(4+3x^2+x^4)^{3/2} + \frac{625}{11}x^5(4+3x^2+x^4)^{3/2} \\
& - \frac{51665\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{4+3x^2+x^4}} \\
& + \frac{33159(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{11\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

```
[Out] 3050/11*x*(x^4+3*x^2+4)^(3/2)+23500/99*x^3*(x^4+3*x^2+4)^(3/2)+625/11*x^5*(
x^4+3*x^2+4)^(3/2)+51665/33*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/33*x*(4516*x^2+
18727)*(x^4+3*x^2+4)^(1/2)+33159/22*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2
)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2)))
,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-5
1665/33*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1220, 1693, 1190, 1211, 1117, 1209}

$$\begin{aligned}\int \left(7+5x^2\right)^4\sqrt{4+3x^2+x^4}\,dx = & \frac{33159(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{11\sqrt{2}\sqrt{x^4+3x^2+4}} \\ & -\frac{51665\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}} \\ & +\frac{3050}{11}\left(x^4+3x^2+4\right)^{3/2}x \\ & +\frac{1}{33}\left(4516x^2+18727\right)\sqrt{x^4+3x^2+4}x \\ & +\frac{51665\sqrt{x^4+3x^2+4}x}{33\left(x^2+2\right)}+\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5 \\ & +\frac{23500}{99}\left(x^4+3x^2+4\right)^{3/2}x^3\end{aligned}$$

```
[In] Int[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (51665*x*Sqrt[4 + 3*x^2 + x^4])/(33*(2 + x^2)) + (x*(18727 + 4516*x^2)*Sqrt
[4 + 3*x^2 + x^4])/33 + (3050*x*(4 + 3*x^2 + x^4)^(3/2))/11 + (23500*x^3*(4
 + 3*x^2 + x^4)^(3/2))/99 + (625*x^5*(4 + 3*x^2 + x^4)^(3/2))/11 - (51665*S
qrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4]) + (33159*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(11*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2} \\
&\quad +\frac{1}{11}\int \sqrt{4+3x^2+x^4}\left(26411+75460x^2+68350x^4+23500x^6\right)\,dx \\
&= \frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2} \\
&\quad +\frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{99}\int \sqrt{4+3x^2+x^4}\left(237699+397140x^2+192150x^4\right)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2} \\
&\quad +\frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{693}\int\left(895293+474180x^2\right)\sqrt{4+3x^2+x^4}\,dx \\
&= \frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4}+\frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2} \\
&\quad +\frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}+\frac{\int\frac{30121560+16274475x^2}{\sqrt{4+3x^2+x^4}}\,dx}{10395} \\
&= \frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4}+\frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{23500}{99}x^3\left(4+3x^2\right. \\
&\qquad\qquad \left.+x^4\right)^{3/2}+\frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}-\frac{103330}{33}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx \\
&\quad +\frac{66318}{11}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{51665x\sqrt{4+3x^2+x^4}}{33\,(2+x^2)}+\frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4} \\
&\quad +\frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2} \\
&\quad +\frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}-\frac{51665\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{4+3x^2+x^4}} \\
&\quad +\frac{33159(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{11\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.99 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.46

$$
\int\left(7+5x^2\right)^4\sqrt{4+3x^2+x^4}\,dx
$$

$$
= \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(663924+1257535x^2+1217475x^4+712748x^6+264075x^8+57250x^{10}+5625x^{12})-15499}{}
$$

```
[In] Integrate[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(663924 + 1257535*x^2 + 1217475*x^4 + 7127
48*x^6 + 264075*x^8 + 57250*x^10 + 5625*x^12) - 154995*Sqrt[2]*(3*I + Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-36253*I + 51665*Sq
```

```
rt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt
[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(396*Sqrt[(-I)/(-3*I + Sqrt[
7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x\left(5625x^{8}+40375x^{6}+120450x^{4}+189898x^{2}+165981\right)\sqrt{x^{4}+3x^{2}+4}}{99}+\frac{382496\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^{2}}\,F\Big(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}}{33\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+3x^{2}+4}}$
default	$\frac{55327x\sqrt{x^{4}+3x^{2}+4}}{33}+\frac{382496\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^{2}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{33\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+3x^{2}+4}}-\frac{1653280\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}}{}$
elliptic	$\frac{55327x\sqrt{x^{4}+3x^{2}+4}}{33}+\frac{382496\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^{2}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{33\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+3x^{2}+4}}-\frac{1653280\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}}{}$

```
[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/99*x*(5625*x^8+40375*x^6+120450*x^4+189898*x^2+165981)*(x^4+3*x^2+4)^(1/2
)+382496/33/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-
3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7
^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-1653280/33/(-6+2*I*7^(1/2))^(1/2)*
(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+
3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2
+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1
/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.57

$$\int \left(7+5x^{2}\right)^{4}\sqrt{4+3x^{2}+x^{4}}\,dx$$
$$=\frac{154995\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-9\sqrt{2}\left(9253\sqrt{-7}x-75571\,x\right)}{}$$

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/396*(154995*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcs
in(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 9*sqrt(2)*(9253
*sqrt(-7)*x - 75571*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqr
t(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(5625*x^10 + 40375*x^8 + 120450
*x^6 + 189898*x^4 + 165981*x^2 + 154995)*sqrt(x^4 + 3*x^2 + 4))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\,dx=\int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\left(5x^2+7\right)^4\,dx$$

```
[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(1/2),x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**4, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\,dx=\int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^4\,dx$$

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\,dx=\int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^4\,dx$$

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\,dx=\int {\left(5\,x^2+7\right)}^4\,\sqrt{x^4+3\,x^2+4}\,dx$$

```
[In] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.349 $\int (7+5x^2)^3 \sqrt{4+3x^2+x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 221

$$\begin{aligned}\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx &= \frac{4717x\sqrt{4+3x^2+x^4}}{21\left(2+x^2\right)} + \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4} \\ &\quad + \frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2} \\ &\quad - \frac{4717\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{21\sqrt{4+3x^2+x^4}} \\ &\quad + \frac{1301(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 275/7*x*(x^4+3*x^2+4)^(3/2)+125/9*x^3*(x^4+3*x^2+4)^(3/2)+4717/21*x*(x^4+3*
x^2+4)^(1/2)/(x^2+2)+1/21*x*(407*x^2+1708)*(x^4+3*x^2+4)^(1/2)+1301/6*(x^2+
2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elli
pticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(
1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-4717/21*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2
)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1
/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1
/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1220, 1693, 1190, 1211, 1117, 1209}

$$\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx = \frac{1301(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{4717\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{21\sqrt{x^4+3x^2+4}} + \frac{275}{7}\left(x^4+3x^2+4\right)^{3/2}x + \frac{1}{21}\left(407x^2+1708\right)\sqrt{x^4+3x^2+4}x + \frac{4717\sqrt{x^4+3x^2+4}x}{21\left(x^2+2\right)} + \frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

```
[In] Int[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4717*x*Sqrt[4 + 3*x^2 + x^4])/(21*(2 + x^2)) + (x*(1708 + 407*x^2)*Sqrt[4
+ 3*x^2 + x^4])/21 + (275*x*(4 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(4 + 3*x^
2 + x^4)^(3/2))/9 - (4717*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(21*Sqrt[4 + 3*x^2 + x^4]) + (130
1*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{9}\int \sqrt{4+3x^2+x^4}\left(3087+5115x^2+2475x^4\right)\,dx\\
&= \frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{63}\int \left(11709+6105x^2\right)\sqrt{4+3x^2+x^4}\,dx\\
&= \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}\\
&\quad+\frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{945}\int \frac{395100+212265x^2}{\sqrt{4+3x^2+x^4}}\,dx
\end{aligned}
$$

$$= \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}-\frac{9434}{21}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{2602}{3}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{4717x\sqrt{4+3x^2+x^4}}{21\left(2+x^2\right)}+\frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}-\frac{4717\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\mid\frac{1}{8}\right)}{21\sqrt{4+3x^2+x^4}} + \frac{1301(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\mid\frac{1}{8}\right)}{3\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.58

$$\int\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}\,dx$$

$$= \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(60096+93656x^2+71862x^4+30946x^6+7725x^8+875x^{10})-14151\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{}}{-3i}}}{}$$

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(60096 + 93656*x^2 + 71862*x^4 + 30946*x^6
 + 7725*x^8 + 875*x^10) - 14151*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7
] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sq
rt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7
])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-3409*I + 4717*Sqrt[7])*Sqrt[(-3*I + Sqrt[
7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + S
qrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[
7])/(3*I + Sqrt[7])])/(252*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4
])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.09

method	result
risch	$\frac{x\left(875x^{6}+5100x^{4}+12146x^{2}+15024\right)\sqrt{x^{4}+3x^{2}+4}}{63}+\frac{35120\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^{2}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+3x^{2}+4}}-$
default	$\frac{5008x\sqrt{x^{4}+3x^{2}+4}}{21}+\frac{35120\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^{2}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+3x^{2}+4}}-\frac{150944\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1}}{}$
elliptic	$\frac{5008x\sqrt{x^{4}+3x^{2}+4}}{21}+\frac{35120\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^{2}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+3x^{2}+4}}-\frac{150944\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1}}{}$

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/63*x*(875*x^6+5100*x^4+12146*x^2+15024)*(x^4+3*x^2+4)^(1/2)+35120/21/(-6+
2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2
))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2))-150944/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^
(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3
+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.60

$$\int \left(7+5x^{2}\right)^{3}\sqrt{4+3x^{2}+x^{4}}\,dx$$

$$=\frac{14151\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-6\sqrt{2}\left(1261\sqrt{-7}x-10368\,x\right)\sqrt{}}{}$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/252*(14151*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsi
n(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 6*sqrt(2)*(1261*
sqrt(-7)*x - 10368*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt
(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(875*x^8 + 5100*x^6 + 12146*x^4
+ 15024*x^2 + 14151)*sqrt(x^4 + 3*x^2 + 4))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\left(5x^2+7\right)^3\,dx$$

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(1/2),x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^3\,dx$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^3\,dx$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx = \int \left(5\,x^2+7\right)^3\sqrt{x^4+3\,x^2+4}\,dx$$

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.350 $\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx &= \frac{319x\sqrt{4+3x^2+x^4}}{7\left(2+x^2\right)}+\frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4}\\ &\quad+\frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}\\ &\quad-\frac{319\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{7\sqrt{4+3x^2+x^4}}\\ &\quad+\frac{81(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 25/7*x*(x^4+3*x^2+4)^(3/2)+319/7*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/7*x*(38*x^
2+119)*(x^4+3*x^2+4)^(1/2)+81/2*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1
/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4
*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-319/7
*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2))
)*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4
)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1190, 1211, 1117, 1209}

$$\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx = \frac{81(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{319\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{7\sqrt{x^4+3x^2+4}} + \frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2} + \frac{1}{7}x\left(38x^2+119\right)\sqrt{x^4+3x^2+4} + \frac{319x\sqrt{x^4+3x^2+4}}{7\left(x^2+2\right)}$$

```
[In] Int[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (319*x*Sqrt[4 + 3*x^2 + x^4])/(7*(2 + x^2)) + (x*(119 + 38*x^2)*Sqrt[4 + 3*
x^2 + x^4])/7 + (25*x*(4 + 3*x^2 + x^4)^(3/2))/7 - (319*Sqrt[2]*(2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(7*
Sqrt[4 + 3*x^2 + x^4]) + (81*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*
EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
```

```
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{7}\int\left(243+190x^2\right)\sqrt{4+3x^2+x^4}\,dx\\
&= \frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4}+\frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{105}\int\frac{7440+4785x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4}+\frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}\\
&\quad-\frac{638}{7}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+162\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{319x\sqrt{4+3x^2+x^4}}{7\left(2+x^2\right)}+\frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4}\\
&\quad+\frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}-\frac{319\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{7\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{81(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.73

$$\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx$$

$$= \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(876+1109x^2+658x^4+188x^6+25x^8) - 319\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\Big(i}{28\sqrt{}}$$

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(876 + 1109*x^2 + 658*x^4 + 188*x^6 + 25*x
^8) - 319*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I +
 Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*Ar
cSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] +
Sqrt[2]*(-35*I + 319*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(28*S
qrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x(25x^4+113x^2+219)\sqrt{x^4+3x^2+4}}{7}+\frac{1984\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{10208\sqrt{1-\Big(-}{}$
default	$\frac{219x\sqrt{x^4+3x^2+4}}{7}+\frac{1984\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{10208\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-}{}$
elliptic	$\frac{219x\sqrt{x^4+3x^2+4}}{7}+\frac{1984\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{10208\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-}{}$

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x*(25*x^4+113*x^2+219)*(x^4+3*x^2+4)^(1/2)+1984/7/(-6+2*I*7^(1/2))^(1/2
)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^
4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))
^(1/2))-10208/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(
```

```
1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(Ellipt
icF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x
*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\int \left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}\,dx = \frac{319\,\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\,\sqrt{-7}+\frac{1}{8})-3\,\sqrt{2}\left(65\,\sqrt{-7}x-443\,x\right)\sqrt{\sqrt{-7}-}{28\,x}$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/28*(319*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1
/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 3*sqrt(2)*(65*sqrt(
-7)*x - 443*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-
7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(25*x^6 + 113*x^4 + 219*x^2 + 319)*sqrt
(x^4 + 3*x^2 + 4))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}\,dx = \int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\left(5x^2+7\right)^2\,dx$$

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx=\int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx=\int \left(5\,x^2+7\right)^2\,\sqrt{x^4+3\,x^2+4}\,dx$$

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.351 $\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 177

$$\begin{aligned}\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx &= \frac{9x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{1}{3}x(10+3x^2)\sqrt{4+3x^2+x^4} \\ &\quad -\frac{9\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} \\ &\quad +\frac{49(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 9*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/3*x*(3*x^2+10)*(x^4+3*x^2+4)^(1/2)+49/6*(
x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*
EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^
2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-9*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2
)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{1190, 1211, 1117, 1209\}$

$$\int \left(7+5x^2\right)\sqrt{4+3x^2+x^4}\,dx = \frac{49(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{9\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}} + \frac{1}{3}\left(3x^2+10\right)\sqrt{x^4+3x^2+4}x + \frac{9\sqrt{x^4+3x^2+4}x}{x^2+2}$$

```
[In] Int[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*(10 + 3*x^2)*Sqrt[4 + 3*x^2 + x^
4])/3 - (9*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (49*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{15}\int\frac{220+135x^2}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4}-18\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{98}{3}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{9x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4} \\
&\quad -\frac{9\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} \\
&\quad +\frac{49\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.66 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.91

$$\int\left(7+5x^2\right)\sqrt{4+3x^2+x^4}\,dx$$

$$=\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(40+42x^2+19x^4+3x^6\right)-27\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{2i}{-3i+}}\right.\right.}{12\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{}}$$

```
[In] Integrate[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(40 + 42*x^2 + 19*x^4 + 3*x^6) - 27*Sqrt[2
]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[
(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I +
27*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + S
qrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31

method	result
risch	$\frac{x(3x^2+10)\sqrt{x^4+3x^2+4}}{3}+\frac{176\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$ $288\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}$
default	$\frac{10x\sqrt{x^4+3x^2+4}}{3}+\frac{176\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$ $288\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}$
elliptic	$\frac{10x\sqrt{x^4+3x^2+4}}{3}+\frac{176\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$ $288\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}$

```
[In] int((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(3*x^2+10)*(x^4+3*x^2+4)^(1/2)+176/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+
1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^
(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-288
/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7
^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2
*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2
))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69

$$\int \left(7+5x^2\right)\sqrt{4+3x^2+x^4}\,dx$$

$$= \frac{27\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-2\sqrt{2}\left(8\sqrt{-7}x-57\,x\right)\sqrt{\sqrt{-7}-3}}{12\,x}$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(27*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/
2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 2*sqrt(2)*(8*sqrt(-7
)*x - 57*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7)
- 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(3*x^4 + 10*x^2 + 27)*sqrt(x^4 + 3*x^2 + 4
))/x
```

Sympy [F]

$$\int \left(7+5x^2\right) \sqrt{4+3x^2+x^4}\, dx = \int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\left(5x^2+7\right)\, dx$$

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(1/2),x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7), x)
```

Maxima [F]

$$\int \left(7+5x^2\right) \sqrt{4+3x^2+x^4}\, dx = \int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right) dx$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Giac [F]

$$\int \left(7+5x^2\right) \sqrt{4+3x^2+x^4}\, dx = \int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right) dx$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right) \sqrt{4+3x^2+x^4}\, dx = \int \left(5\,x^2+7\right)\,\sqrt{x^4+3\,x^2+4}\, dx$$

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.352 $\int \sqrt{4+3x^2+x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 169

$$\begin{aligned}\int \sqrt{4+3x^2+x^4}\,dx &= \frac{1}{3}x\sqrt{4+3x^2+x^4}+\frac{x\sqrt{4+3x^2+x^4}}{2+x^2}\\ &\quad-\frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\mid\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}\\ &\quad+\frac{7(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 1/3*x*(x^4+3*x^2+4)^(1/2)+x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+7/6*(x^2+2)*(cos(2*
arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(
2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/
2)/(x^4+3*x^2+4)^(1/2)-(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2
*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))
*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{1105, 1211, 1117, 1209\}$

$$\int \sqrt{4+3x^2+x^4}\,dx = \frac{7(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}} + \frac{\sqrt{x^4+3x^2+4}x}{x^2+2} + \frac{1}{3}\sqrt{x^4+3x^2+4}x$$

```
[In] Int[Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/3 + (x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (Sqrt[2
]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2
]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (7*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^
4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
```

```
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x\sqrt{4+3x^2+x^4}+\frac{1}{3}\int \frac{8+3x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{1}{3}x\sqrt{4+3x^2+x^4}-2\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{14}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{1}{3}x\sqrt{4+3x^2+x^4}+\frac{x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{7(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.94 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.96

$$\int \sqrt{4+3x^2+x^4}\,dx$$

$$= \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(4+3x^2+x^4)-3\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{}}{3i+\sqrt{}}\right.}{12\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3}}$$

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 3*Sqrt[2]*(3*I + Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqr
t[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x
], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4
 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.33

method	result
default	$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}}{\sqrt{-6}}$
risch	$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}}{\sqrt{-6}}$
elliptic	$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}}{\sqrt{-6}}$

```
[In] int((x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(x^4+3*x^2+4)^(1/2)+32/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2
))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipt
icF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/(-6+2*I*7^(1
/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^
(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

$$\int \sqrt{4+3x^2+x^4}\,dx$$
$$=\frac{3\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-\sqrt{2}\left(\sqrt{-7}x-15\,x\right)\sqrt{\sqrt{-7}-3}F(\mathrm{arc}}{12\,x}$$

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/2
*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(sqrt(-7)*x -
 15*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/
x), 3/8*sqrt(-7) + 1/8) + 4*sqrt(x^4 + 3*x^2 + 4)*(x^2 + 3))/x
```

Sympy [F]

$$\int \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3x^2+4}\,dx$$

```
[In] integrate((x**4+3*x**2+4)**(1/2),x)
[Out] Integral(sqrt(x**4 + 3*x**2 + 4), x)
```

Maxima [F]

$$\int \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

Giac [F]

$$\int \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\,dx$$

```
[In] int((3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((3*x^2 + x^4 + 4)^(1/2), x)
```

3.353 $\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$
\begin{aligned}
\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx &= \frac{x\sqrt{4+3x^2+x^4}}{5\left(2+x^2\right)} + \frac{1}{5}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) \\
&\quad - \frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{9(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{25\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{11\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{75\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{187(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{525\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

```
[Out] 1/175*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/5*x*(x^4+3*x
^2+4)^(1/2)/(x^2+2)+1/30*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos
(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2
))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+187/1050*(x^
2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*El
lipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x
^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-1/5*(x^2+2)*(cos(2*arctan(1/2*x*
2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*
x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2
+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1222, 1211, 1117, 1209, 1230, 1720}

$$\begin{aligned}\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = {} & \frac{1}{5}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right) \\ & - \frac{11\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}} \\ & + \frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+4}} \\ & - \frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5\sqrt{x^4+3x^2+4}} \\ & + \frac{187(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{525\sqrt{2}\sqrt{x^4+3x^2+4}} \\ & + \frac{\sqrt{x^4+3x^2+4}x}{5\left(x^2+2\right)}\end{aligned}$$

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/(5*(2 + x^2)) + (Sqrt[11/35]*ArcTan[(2*Sqrt[11/35
]*x)/Sqrt[4 + 3*x^2 + x^4]])/5 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5*Sqrt[4 + 3*x^2 + x^4])
 + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(25*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (11*Sqrt[2]*(2 + x^2)*Sq
rt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(75*
Sqrt[4 + 3*x^2 + x^4]) + (187*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(525*Sqrt[2]*Sqrt[4 + 3*x^2
+ x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{25}\int \frac{-8-5x^2}{\sqrt{4+3x^2+x^4}}\,dx\right)+\frac{44}{25}\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$$

$$
\begin{aligned}
&= -\left(\frac{2}{5}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\right) - \frac{44}{75}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx \\
&\quad + \frac{18}{25}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{88}{15}\int \frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{x\sqrt{4+3x^2+x^4}}{5\,(2+x^2)} + \frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) \\
&\quad - \frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{9(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{25\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{11\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{75\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{187(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{525\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.63 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.88

$$
\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx =
$$

$$
-\frac{\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\bigg(35\left(3i+\sqrt{7}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\left(7i-35\sqrt{7}\right)\operatorname{EllipticF}\bigg(i}{350\sqrt{2}\sqrt{-\frac{i}{-3i+\sqrt{7}}}\surd}
$$

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] -1/350*(Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I +
Sqrt[7])]*(35*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[
7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I - 35*Sqrt[7])*EllipticF[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
+ (88*I)*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(Sqrt[2]*Sqrt[(-I)/(-3*I +
Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.20

method	result
default	$\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{\ }}}{4}\right.}{5\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\left(3+i\sqrt{7}\right)}$
elliptic	$\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{\ }}}{4}\right.}{5\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\left(3+i\sqrt{7}\right)}$

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] 32/25/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2
+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/
2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2
-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)
^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1
/2))^(1/2))+32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)
*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*Elli
pticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+44/175/(-3/8+
1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x
^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)
*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2)
)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{(x^2-x+2)\,(x^2+x+2)}}{5x^2+7}\,dx$$

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7),x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

```
[In] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7),x)
[Out] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7), x)
```

3.354 $\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 284

$$\begin{aligned}\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx = & -\frac{x\sqrt{4+3x^2+x^4}}{70\left(2+x^2\right)}+\frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)}+\frac{51\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{280\sqrt{385}}\\ & +\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{4+3x^2+x^4}}\\ & -\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{35\sqrt{2}\sqrt{4+3x^2+x^4}}\\ & +\frac{289(2+x^2)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{9800\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 51/107800*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/70*x*(x^
4+3*x^2+4)^(1/2)/(x^2+2)+1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+1/70*(x^2+2)*
(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipti
cE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)
^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-1/70*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)
^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+28
9/19600*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1240, 1211, 1117, 1209, 1230, 1720}

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \frac{51\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}} - \frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{289(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{9800\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{\sqrt{x^4+3x^2+4}x}{70(x^2+2)} + \frac{\sqrt{x^4+3x^2+4}x}{14(5x^2+7)}$$

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -1/70*(x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*Sqrt[4 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + (51*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(280*Sqr
t[385]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
+ 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2
]*Sqrt[4 + 3*x^2 + x^4]) + (289*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(9800*Sqrt[2]*Sqrt[4 + 3*x
^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1240

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(d + e*x^2))), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{4+3x^2+x^4}}{14\,(7+5x^2)} + \frac{1}{350}\int \frac{7-5x^2}{\sqrt{4+3x^2+x^4}}\,dx + \frac{51}{350}\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)} - \frac{3}{350}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{1}{35}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx$$

$$- \frac{17}{350}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{17}{35}\int \frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{x\sqrt{4+3x^2+x^4}}{70\left(2+x^2\right)} + \frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{280\sqrt{385}}$$

$$+ \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$+ \frac{289(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{9800\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx$$

$$= \frac{700\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(4+3x^2+x^4)+35\left(3i+\sqrt{7}\right)\left(7+5x^2\right)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\Big(E\Big(i\text{arcsinh}\Big(\sqrt{-\frac{2i}{-3i+}}}{}$$

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*(
7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) - (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*
I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
 - (102*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*
I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[
(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(9800*Sqrt[(
-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x\sqrt{x^4+3x^2+4}}{70x^2+98}+\frac{2\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{16\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)}{35\sqrt{-6}}$
default	$\frac{x\sqrt{x^4+3x^2+4}}{70x^2+98}+\frac{2\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{16\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{}}{}\right.}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\left(3\right.}$
elliptic	$\frac{x\sqrt{x^4+3x^2+4}}{70x^2+98}+\frac{2\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{16\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{}}{}\right.}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\left(3\right.}$

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+2/25/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1
/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(
1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+16/3
5/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*
7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+
2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/
2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+51/2450/(-3/8+1/8*I*7^(1/2))^(1/2)*(1
+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+
3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(
1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+4}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{(x^2-x+2)\,(x^2+x+2)}}{(5x^2+7)^2}\,dx$$

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**2,x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^2}\,dx$$

```
[In] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^2,x)
[Out] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^2, x)
```

3.355 $\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 312

$$
\begin{aligned}
\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx = & -\frac{139x\sqrt{4+3x^2+x^4}}{86240\,(2+x^2)}+\frac{x\sqrt{4+3x^2+x^4}}{28\,(7+5x^2)^2} \\
& +\frac{139x\sqrt{4+3x^2+x^4}}{17248\,(7+5x^2)}+\frac{14999\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{344960\sqrt{385}} \\
& +\frac{139(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{43120\sqrt{2}\sqrt{4+3x^2+x^4}} \\
& -\frac{23(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2940\sqrt{2}\sqrt{4+3x^2+x^4}} \\
& +\frac{254983(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{36220800\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

```
[Out] 14999/132809600*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-139/
86240*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/28*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+
139/17248*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+139/86240*(x^2+2)*(cos(2*arctan(1
/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan
(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+
3*x^2+4)^(1/2)-23/5880*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2
*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))
*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+254983/7244160
0*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1242, 1237, 1710, 1728, 1209, 1722, 1117, 1720, 1230}

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx = \frac{14999\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{344960\sqrt{385}} - \frac{23(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2940\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{139(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{43120\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{254983(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{36220800\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{139\sqrt{x^4+3x^2+4}x}{86240\,(x^2+2)} + \frac{139\sqrt{x^4+3x^2+4}x}{17248\,(5x^2+7)} + \frac{\sqrt{x^4+3x^2+4}x}{28\,(5x^2+7)^2}$$

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (-139*x*Sqrt[4 + 3*x^2 + x^4])/(86240*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4]
)/(28*(7 + 5*x^2)^2) + (139*x*Sqrt[4 + 3*x^2 + x^4])/(17248*(7 + 5*x^2)) +
(14999*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(344960*Sqrt[385])
+ (139*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(43120*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (23*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2940*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4]) + (254983*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(36220800*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
```

```
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
```

```
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{44}{25\,(7+5x^2)^3\sqrt{4+3x^2+x^4}}+\frac{1}{25\,(7+5x^2)^2\sqrt{4+3x^2+x^4}}\right.\\
&\qquad\left.+\frac{1}{25\,(7+5x^2)\sqrt{4+3x^2+x^4}}\right)dx\\
&= \frac{1}{25}\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx+\frac{1}{25}\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx\\
&\quad+\frac{44}{25}\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} - \frac{\int \frac{12+70x^2+25x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{15400} \\
&\quad - \frac{1}{700}\int \frac{-76-10x^2-25x^4}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx \\
&\quad - \frac{1}{75}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{2}{15}\int \frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)} + \frac{\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{20\sqrt{385}} \\
&\quad - \frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{150\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{17(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{2100\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{\int \frac{-4412-4690x^2-2775x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{431200} - \frac{\int \frac{410+425x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{77000} + \frac{\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{1540} \\
&= -\frac{x\sqrt{4+3x^2+x^4}}{3080\left(2+x^2\right)} + \frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)} \\
&\quad + \frac{\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{20\sqrt{385}} + \frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{1540\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{150\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{17(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{2100\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{\int \frac{-60910-31775x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{2156000} - \frac{1}{525}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{111\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{43120} \\
&\quad + \frac{37\int \frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{4620}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{139x\sqrt{4+3x^2+x^4}}{86240\,(2+x^2)} + \frac{x\sqrt{4+3x^2+x^4}}{28\,(7+5x^2)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\,(7+5x^2)} \\
&\quad + \frac{653\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{12320\sqrt{385}} + \frac{139(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{43120\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{2\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{525\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{11101(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1293600\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{2450} - \frac{219\int\frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{8624} \\
&= -\frac{139x\sqrt{4+3x^2+x^4}}{86240\,(2+x^2)} + \frac{x\sqrt{4+3x^2+x^4}}{28\,(7+5x^2)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\,(7+5x^2)} \\
&\quad + \frac{14999\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{344960\sqrt{385}} + \frac{139(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{43120\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4900\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{2\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{525\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{254983(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{36220800\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.59 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.99

$$
\int\frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx
$$

$$
= \frac{\frac{700x(1589+695x^2)(4+3x^2+x^4)}{(7+5x^2)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\Bigg(4865(3-i\sqrt{7})\,E\Bigg(i\text{arcsinh}\Bigg(\sqrt{-\frac{2i}{-3i+\sqrt{}}}}{}
$$

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((700*x*(1589 + 695*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I
)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
```

```
 + Sqrt[7])]*(4865*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9597 + (4865*I)*Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
 + Sqrt[7])] - 29998*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*
I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(12073600*Sqrt[
4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.11

method	result
risch	$\frac{\sqrt{x^4+3x^2+4}\,x\left(695x^2+1589\right)}{17248\left(5x^2+7\right)^2}-\frac{51\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{15400\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{139\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}}{}$
default	$\frac{x\sqrt{x^4+3x^2+4}}{28\left(5x^2+7\right)^2}+\frac{139x\sqrt{x^4+3x^2+4}}{17248\left(5x^2+7\right)}-\frac{51\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{15400\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{139\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}}{2695\sqrt{-}}$
elliptic	$\frac{x\sqrt{x^4+3x^2+4}}{28\left(5x^2+7\right)^2}+\frac{139x\sqrt{x^4+3x^2+4}}{17248\left(5x^2+7\right)}-\frac{51\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{15400\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{139\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}}{2695\sqrt{-}}$

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17248*(x^4+3*x^2+4)^(1/2)*x*(695*x^2+1589)/(5*x^2+7)^2-51/15400/(-6+2*I*7
^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^
2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+
6*I*7^(1/2))^(1/2))+139/2695/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))
*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(
1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-El
lipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+14999/3018
400/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x
^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1
/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8
*I*7^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{(x^2-x+2)\,(x^2+x+2)}}{(5x^2+7)^3}\,dx$$

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**3,x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\sqrt{x^4+3\,x^2+4}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^3,x)
[Out] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^3, x)
```

3.356 $\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 268

$$\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2} \, dx = \frac{12665086x\sqrt{4+3x^2+x^4}}{2145(2+x^2)}$$
$$+ \frac{7x(661429+174989x^2)\sqrt{4+3x^2+x^4}}{2145}$$
$$+ \frac{x(452001+131080x^2)(4+3x^2+x^4)^{3/2}}{1287} + \frac{92150}{429}x(4+3x^2+x^4)^{5/2}$$
$$+ \frac{2250}{13}x^3(4+3x^2+x^4)^{5/2} + \frac{125}{3}x^5(4+3x^2+x^4)^{5/2} - \frac{12665086\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2145\sqrt{4+3x^2+x^4}} +$$

```
[Out] 1/1287*x*(131080*x^2+452001)*(x^4+3*x^2+4)^(3/2)+92150/429*x*(x^4+3*x^2+4)^
(5/2)+2250/13*x^3*(x^4+3*x^2+4)^(5/2)+125/3*x^5*(x^4+3*x^2+4)^(5/2)+1266508
6/2145*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+7/2145*x*(174989*x^2+661429)*(x^4+3*x^
2+4)^(1/2)-12665086/2145*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos
(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2
))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+2383556/429*
(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))
*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)
^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1220, 1693, 1190, 1211, 1117, 1209}

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{2383556\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{429\sqrt{x^4+3x^2+4}} - \frac{12665086\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2145\sqrt{x^4+3x^2+4}} + \frac{92150}{429}\left(x^4+3x^2+4\right)^{5/2}x + \frac{(131080x^2+452001)\left(x^4+3x^2+4\right)^{3/2}x}{1287} + \frac{7(174989x^2+661429)\sqrt{x^4+3x^2+4}x}{2145} + \frac{12665086\sqrt{x^4+3x^2+4}x}{2145\left(x^2+2\right)} + \frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5 + \frac{2250}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

```
[In] Int[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (12665086*x*Sqrt[4 + 3*x^2 + x^4])/(2145*(2 + x^2)) + (7*x*(661429 + 174989
*x^2)*Sqrt[4 + 3*x^2 + x^4])/2145 + (x*(452001 + 131080*x^2)*(4 + 3*x^2 + x
^4)^(3/2))/1287 + (92150*x*(4 + 3*x^2 + x^4)^(5/2))/429 + (2250*x^3*(4 + 3*
x^2 + x^4)^(5/2))/13 + (125*x^5*(4 + 3*x^2 + x^4)^(5/2))/3 - (12665086*Sqrt
[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt
[2]], 1/8])/(2145*Sqrt[4 + 3*x^2 + x^4]) + (2383556*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(429*Sq
rt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{125}{3}x^5\left(4+3x^2+x^4\right)^{5/2} \\
&\quad + \frac{1}{15}\int\left(4+3x^2+x^4\right)^{3/2}\left(36015+102900x^2+97750x^4+33750x^6\right)\,dx \\
&= \frac{2250}{13}x^3\left(4+3x^2+x^4\right)^{5/2}+\frac{125}{3}x^5\left(4+3x^2+x^4\right)^{5/2} \\
&\quad + \frac{1}{195}\int\left(4+3x^2+x^4\right)^{3/2}\left(468195+932700x^2+460750x^4\right)\,dx
\end{aligned}$$

$$= \frac{92150}{429} x\left(4+3x^2+x^4\right)^{5/2} + \frac{2250}{13} x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3} x^5\left(4+3x^2+x^4\right)^{5/2} + \frac{\int \left(3307145+1966200x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx}{2145}$$

$$= \frac{x(452001+131080x^2)\left(4+3x^2+x^4\right)^{3/2}}{1287} + \frac{92150}{429} x\left(4+3x^2+x^4\right)^{5/2} + \frac{2250}{13} x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3} x^5\left(4+3x^2+x^4\right)^{5/2} + \frac{\int \left(214520040+128616915x^2\right)\sqrt{4+3x^2+x^4}\,dx}{45045}$$

$$= \frac{7x(661429+174989x^2)\sqrt{4+3x^2+x^4}}{2145} + \frac{x(452001+131080x^2)\left(4+3x^2+x^4\right)^{3/2}}{1287} + \frac{92150}{429} x\left(4+3x^2+x^4\right)^{5/2} + \frac{2250}{13} x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3} x^5\left(4+3x^2+x^4\right)^{5/2} + \frac{\int \frac{7037398620+3989502090x^2}{\sqrt{4+3x^2+x^4}}\,dx}{675675}$$

$$= \frac{7x(661429+174989x^2)\sqrt{4+3x^2+x^4}}{2145} + \frac{x(452001+131080x^2)\left(4+3x^2+x^4\right)^{3/2}}{1287} + \frac{92150}{429} x\left(4+3x^2+x^4\right)^{5/2} + \frac{2250}{13} x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3} x^5\left(4+3x^2+x^4\right)^{5/2} - \frac{25330172\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{2145} + \frac{9534224}{429}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{12665086x\sqrt{4+3x^2+x^4}}{2145\left(2+x^2\right)} + \frac{7x(661429+174989x^2)\sqrt{4+3x^2+x^4}}{2145} + \frac{x(452001+131080x^2)\left(4+3x^2+x^4\right)^{3/2}}{1287} + \frac{92150}{429} x\left(4+3x^2+x^4\right)^{5/2} + \frac{2250}{13} x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3} x^5\left(4+3x^2+x^4\right)^{5/2} - \frac{12665086\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2145\sqrt{4+3x^2+x^4}} + 2383556\sqrt{2}(2+$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.37 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.36

$$\int \left(7+5x^2\right)^4 \left(4+3x^2 +x^4\right)^{3/2}\,dx = \frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(180184116 + 391419623x^2 + 472235001x^4 + 377574349x^6 + 212188905x^8 + 83}{}$$

```
[In] Integrate[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(180184116 + 391419623*x^2 + 472235001*x^4
 + 377574349*x^6 + 212188905*x^8 + 83076275*x^10 + 21862875*x^12 + 3526875*
x^14 + 268125*x^16) - 18997629*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7]
 - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqr
t[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7]
)/(3*I + Sqrt[7])] + 21*Sqrt[2]*(-477617*I + 904649*Sqrt[7])*Sqrt[(-3*I + S
qrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I
 + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - S
qrt[7])/(3*I + Sqrt[7])])/(12870*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2
 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.96

method	result
risch	$\frac{x\left(268125x^{12}+2722500x^{10}+12622875x^{8}+34317650x^{6}+58744455x^{4}+64070384x^{2}+45046029\right)\sqrt{x^{4}+3x^{2}+4}}{6435}+\frac{89363792\sqrt{1-\left(-\frac{3}{8}\right.}}{}$
default	$\frac{356027x^{5}\sqrt{x^{4}+3x^{2}+4}}{39}+\frac{64070384x^{3}\sqrt{x^{4}+3x^{2}+4}}{6435}+\frac{15015343x\sqrt{x^{4}+3x^{2}+4}}{2145}+\frac{89363792\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)}}{2145\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+}}$
elliptic	$\frac{356027x^{5}\sqrt{x^{4}+3x^{2}+4}}{39}+\frac{64070384x^{3}\sqrt{x^{4}+3x^{2}+4}}{6435}+\frac{15015343x\sqrt{x^{4}+3x^{2}+4}}{2145}+\frac{89363792\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)}}{2145\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+}}$

```
[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6435*x*(268125*x^12+2722500*x^10+12622875*x^8+34317650*x^6+58744455*x^4+6
4070384*x^2+45046029)*(x^4+3*x^2+4)^(1/2)+89363792/2145/(-6+2*I*7^(1/2))^(1
```

```
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2
))^(1/2))-405282752/2145/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2
)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2)
)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-Ellipt
icE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.55

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2} dx = \frac{37995258\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right) \mid \frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-21\sqrt{2}\left(1011407\right.}{}$$

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/25740*(37995258*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(
arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 21*sqrt(2)*
(1011407*sqrt(-7)*x - 7821567*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*s
qrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(268125*x^14 + 272250
0*x^12 + 12622875*x^10 + 34317650*x^8 + 58744455*x^6 + 64070384*x^4 + 45046
029*x^2 + 37995258)*sqrt(x^4 + 3*x^2 + 4))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2} dx = \int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}} \left(5x^2+7\right)^4\, dx$$

```
[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**4, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^4 dx$$

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^4 dx$$

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(5\,x^2+7\right)^4 \left(x^4+3\,x^2+4\right)^{3/2} dx$$

```
[In] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.357 $\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 247

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2} \, dx = \frac{4525662x\sqrt{4+3x^2+x^4}}{5005\left(2+x^2\right)}$$
$$+\frac{x(1653701+435441x^2)\sqrt{4+3x^2+x^4}}{5005}$$
$$+\frac{x(53504+15365x^2)\left(4+3x^2+x^4\right)^{3/2}}{1001}+\frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2}$$
$$+\frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2}-\frac{4525662\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5005\sqrt{4+3x^2+x^4}}+\frac{121826\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+}{(2+x^2)}}}{143\sqrt{4}}$$

```
[Out] 1/1001*x*(15365*x^2+53504)*(x^4+3*x^2+4)^(3/2)+3825/143*x*(x^4+3*x^2+4)^(5/
2)+125/13*x^3*(x^4+3*x^2+4)^(5/2)+4525662/5005*x*(x^4+3*x^2+4)^(1/2)/(x^2+2
)+1/5005*x*(435441*x^2+1653701)*(x^4+3*x^2+4)^(1/2)-4525662/5005*(x^2+2)*(c
os(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE
(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2
)^(1/2)/(x^4+3*x^2+4)^(1/2)+121826/143*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2
)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1190, 1211, 1117, 1209}

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{121826\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{143\sqrt{x^4+3x^2+4}} - \frac{4525662\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5005\sqrt{x^4+3x^2+4}} + \frac{3825}{143}\left(x^4+3x^2+4\right)^{5/2}x + \frac{(15365x^2+53504)\left(x^4+3x^2+4\right)^{3/2}x}{1001} + \frac{(435441x^2+1653701)\sqrt{x^4+3x^2+4}x}{5005} + \frac{4525662\sqrt{x^4+3x^2+4}x}{5005\,(x^2+2)} + \frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

```
[In] Int[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4525662*x*Sqrt[4 + 3*x^2 + x^4])/(5005*(2 + x^2)) + (x*(1653701 + 435441*x
^2)*Sqrt[4 + 3*x^2 + x^4])/5005 + (x*(53504 + 15365*x^2)*(4 + 3*x^2 + x^4)^
(3/2))/1001 + (3825*x*(4 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(4 + 3*x^2 +
x^4)^(5/2))/13 - (4525662*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5005*Sqrt[4 + 3*x^2 + x^4]) + (1
21826*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Arc
Tan[x/Sqrt[2]], 1/8])/(143*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2}+\frac{1}{13}\int\left(4+3x^2+x^4\right)^{3/2}\left(4459+8055x^2+3825x^4\right)\,dx\\ &= \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2}\\ &\quad+\frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2}+\frac{1}{143}\int\left(33749+19755x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx\end{aligned}$$

$$= \frac{x(53504+15365x^2)\left(4+3x^2+x^4\right)^{3/2}}{1001} + \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{\int (2192868+1306323x^2)\sqrt{4+3x^2+x^4}\,dx}{3003}$$

$$= \frac{x(1653701+435441x^2)\sqrt{4+3x^2+x^4}}{5005} + \frac{x(53504+15365x^2)\left(4+3x^2+x^4\right)^{3/2}}{1001} + \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{\int \frac{72038844+40730958x^2}{\sqrt{4+3x^2+x^4}}\,dx}{45045}$$

$$= \frac{x(1653701+435441x^2)\sqrt{4+3x^2+x^4}}{5005} + \frac{x(53504+15365x^2)\left(4+3x^2+x^4\right)^{3/2}}{1001} + \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2} - \frac{9051324\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{5005} + \frac{487304}{143}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{4525662x\sqrt{4+3x^2+x^4}}{5005\,(2+x^2)} + \frac{x(1653701+435441x^2)\sqrt{4+3x^2+x^4}}{5005} + \frac{x(53504+15365x^2)\left(4+3x^2+x^4\right)^{3/2}}{1001} + \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2} - \frac{4525662\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5005\sqrt{4+3x^2+x^4}} + \frac{121826\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{143\sqrt{4+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.79 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.45

$$\int \left(7+5x^2\right)^3\left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(19463124+36710547x^2+37166164x^4+24107711x^6+10713970x^8+3158575}{}$$

```
[In] Integrate[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(19463124 + 36710547*x^2 + 37166164*x^4 +
24107711*x^6 + 10713970*x^8 + 3158575*x^10 + 567000*x^12 + 48125*x^14) - 22
62831*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
```

```
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt
[2]*(-1215823*I + 2262831*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I
+ Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*A
rcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/
(10010*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x\left(48125x^{10}+422625x^8+1698200x^6+3928870x^4+5528301x^2+4865781\right)\sqrt{x^4+3x^2+4}}{5005}+\frac{32017264\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{5005\sqrt{-6+2i\sqrt{7}}\,\sqrt{x}}$
default	$\frac{71434x^5\sqrt{x^4+3x^2+4}}{91}+\frac{5528301x^3\sqrt{x^4+3x^2+4}}{5005}+\frac{4865781x\sqrt{x^4+3x^2+4}}{5005}+\frac{32017264\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F}{5005\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+}}$
elliptic	$\frac{71434x^5\sqrt{x^4+3x^2+4}}{91}+\frac{5528301x^3\sqrt{x^4+3x^2+4}}{5005}+\frac{4865781x\sqrt{x^4+3x^2+4}}{5005}+\frac{32017264\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F}{5005\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+}}$

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5005*x*(48125*x^10+422625*x^8+1698200*x^6+3928870*x^4+5528301*x^2+4865781
)*(x^4+3*x^2+4)^(1/2)+32017264/5005/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7
^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*E
llipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-144821184/
5005/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8
*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.58

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}\,dx=\frac{4525662\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-\sqrt{2}\left(2524583\sqrt{-7}\right.}{}$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/20020*(4525662*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(a
rcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(252
4583*sqrt(-7)*x - 19580223*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt
(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(48125*x^12 + 422625*x^1
0 + 1698200*x^8 + 3928870*x^6 + 5528301*x^4 + 4865781*x^2 + 4525662)*sqrt(x
^4 + 3*x^2 + 4))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}\, dx = \int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}} \left(5x^2+7\right)^3\, dx$$

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(3/2),x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}\, dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^3 dx$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}\, dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^3 dx$$

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(5\,x^2+7\right)^3\,\left(x^4+3\,x^2+4\right)^{3/2}\,dx$$

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.358 $\int (7+5x^2)^2 \left(4+3x^2+x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 226

$$\begin{aligned}\int (7+5x^2)^2 \left(4+3x^2+x^4\right)^{3/2} \, dx &= \frac{175346x\sqrt{4+3x^2+x^4}}{1155\,(2+x^2)} \\ &+ \frac{x(64533+18253x^2)\sqrt{4+3x^2+x^4}}{1155} + \frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2} \\ &+ \frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2} - \frac{175346\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1155\sqrt{4+3x^2+x^4}} \\ &+ \frac{4628\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{33\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 1/693*x*(2240*x^2+6831)*(x^4+3*x^2+4)^(3/2)+25/11*x*(x^4+3*x^2+4)^(5/2)+175
346/1155*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/1155*x*(18253*x^2+64533)*(x^4+3*x^
2+4)^(1/2)-175346/1155*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2
*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))
*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4628/33*(x^2+2
)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellip
ticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1
/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1190, 1211, 1117, 1209}

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2} \, dx = \frac{4628\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}$$
$$-\frac{175346\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1155\sqrt{x^4+3x^2+4}}+\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}$$
$$+\frac{1}{693}x\left(2240x^2+6831\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{x(18253x^2+64533)\sqrt{x^4+3x^2+4}}{1155}+\frac{175346x\sqrt{x^4+3x^2+4}}{1155\,(x^2+2)}$$

```
[In] Int[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (175346*x*Sqrt[4 + 3*x^2 + x^4])/(1155*(2 + x^2)) + (x*(64533 + 18253*x^2)*
Sqrt[4 + 3*x^2 + x^4])/1155 + (x*(6831 + 2240*x^2)*(4 + 3*x^2 + x^4)^(3/2))
/693 + (25*x*(4 + 3*x^2 + x^4)^(5/2))/11 - (175346*Sqrt[2]*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(1155*Sq
rt[4 + 3*x^2 + x^4]) + (4628*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
```

```
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2}+\frac{1}{11}\int\left(439+320x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx\\
&= \frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2}\\
&\quad+\frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2}+\frac{1}{231}\int\left(27768+18253x^2\right)\sqrt{4+3x^2+x^4}\,dx\\
&= \frac{x(64533+18253x^2)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2}\\
&\quad+\frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2}+\frac{\int\frac{891684+526038x^2}{\sqrt{4+3x^2+x^4}}\,dx}{3465}\\
&= \frac{x(64533+18253x^2)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2}\\
&\quad+\frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2}-\frac{350692\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{1155}+\frac{18512}{33}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{175346x\sqrt{4+3x^2+x^4}}{1155\,(2+x^2)} + \frac{x(64533+18253x^2)\sqrt{4+3x^2+x^4}}{1155} \\
&\quad + \frac{1}{693}x(6831+2240x^2)\left(4+3x^2+x^4\right)^{3/2} \\
&\quad + \frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2} - \frac{175346\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1155\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{4628\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.57

$$
\int \left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(1824876+2932753x^2+2435811x^4+1229714x^6+408480x^8+82075x^{10}+7875}{}
$$

```
[In] Integrate[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1824876 + 2932753*x^2 + 2435811*x^4 + 122
9714*x^6 + 408480*x^8 + 82075*x^10 + 7875*x^12) - 263019*Sqrt[2]*(3*I + Sqr
t[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[
7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-34209*I + 87673*
Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sq
rt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
 Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6930*Sqrt[(-I)/(-3*I + Sq
rt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.09

method	result
risch	$\frac{x(7875x^8+58450x^6+201630x^4+391024x^2+456219)\sqrt{x^4+3x^2+4}}{3465}+\frac{396304\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\Big(}{385\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
default	$\frac{1222x^5\sqrt{x^4+3x^2+4}}{21}+\frac{391024x^3\sqrt{x^4+3x^2+4}}{3465}+\frac{50691x\sqrt{x^4+3x^2+4}}{385}+\frac{396304\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\Big(}{385\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{1222x^5\sqrt{x^4+3x^2+4}}{21}+\frac{391024x^3\sqrt{x^4+3x^2+4}}{3465}+\frac{50691x\sqrt{x^4+3x^2+4}}{385}+\frac{396304\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\Big(}{385\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*x*(7875*x^8+58450*x^6+201630*x^4+391024*x^2+456219)*(x^4+3*x^2+4)^(1
/2)+396304/385/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1
-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*
I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-5611072/1155/(-6+2*I*7^(1/2))^(
1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/
(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1
/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I
*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.61

$$\int \left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}\,dx=\frac{526038\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-3\sqrt{2}\left(101039\sqrt{-7}\right.}{}$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13860*(526038*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(ar
csin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 3*sqrt(2)*(10
1039*sqrt(-7)*x - 748959*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2
)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(7875*x^10 + 58450*x^8 + 2
01630*x^6 + 391024*x^4 + 456219*x^2 + 526038)*sqrt(x^4 + 3*x^2 + 4))/x
```

Sympy [F]

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2\,dx$$

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(3/2),x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2\,dx$$

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(5\,x^2+7\right)^2\left(x^4+3\,x^2+4\right)^{3/2}\,dx$$

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.359 $\int \left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 207

$$\int \left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{2798x\sqrt{4+3x^2+x^4}}{105\left(2+x^2\right)} + \frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4} + \frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2} - \frac{2798\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{105\sqrt{4+3x^2+x^4}} + \frac{74\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{4+3x^2+x^4}}$$

```
[Out] 1/63*x*(35*x^2+108)*(x^4+3*x^2+4)^(3/2)+2798/105*x*(x^4+3*x^2+4)^(1/2)/(x^2
+2)+1/105*x*(289*x^2+1029)*(x^4+3*x^2+4)^(1/2)-2798/105*(x^2+2)*(cos(2*arct
an(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*ar
ctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)+74/3*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(
2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2)
)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1190, 1211, 1117, 1209}

$$\int \left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{74\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}} - \frac{2798\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{105\sqrt{x^4+3x^2+4}} + \frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+4\right)^{3/2} + \frac{1}{105}x\left(289x^2+1029\right)\sqrt{x^4+3x^2+4} + \frac{2798x\sqrt{x^4+3x^2+4}}{105\left(x^2+2\right)}$$

```
[In] Int[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2798*x*Sqrt[4 + 3*x^2 + x^4])/(105*(2 + x^2)) + (x*(1029 + 289*x^2)*Sqrt[4
 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(4 + 3*x^2 + x^4)^(3/2))/63 - (279
8*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[
x/Sqrt[2]], 1/8])/(105*Sqrt[4 + 3*x^2 + x^4]) + (74*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt
[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{21}\int\left(444+289x^2\right)\sqrt{4+3x^2+x^4}\,dx\\
&= \frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}\\
&\quad+\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{315}\int\frac{14292+8394x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}\\
&\quad-\frac{5596}{105}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{296}{3}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{2798x\sqrt{4+3x^2+x^4}}{105\left(2+x^2\right)}+\frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}\\
&\quad+\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}\\
&\quad-\frac{2798\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{105\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{74\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.69

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx = 2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(20988+28489x^2+19068x^4+7082x^6+1590x^8+175x^{10})-4197\sqrt{2}(3i+\sqrt{7})$$

```
[In] Integrate[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(20988 + 28489*x^2 + 19068*x^4 + 7082*x^6
+ 1590*x^8 + 175*x^10) - 4197*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7]
- (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt
[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])
/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-567*I + 1399*Sqrt[7])*Sqrt[(-3*I + Sqrt[7]
- (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt
[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])
/(3*I + Sqrt[7])])/(630*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.16

method	result
risch	$\frac{x\left(175x^6+1065x^4+3187x^2+5247\right)\sqrt{x^4+3x^2+4}}{315}+\frac{6352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-8953$
default	$\frac{71x^5\sqrt{x^4+3x^2+4}}{21}+\frac{3187x^3\sqrt{x^4+3x^2+4}}{315}+\frac{583x\sqrt{x^4+3x^2+4}}{35}+\frac{6352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\sqrt{}\right.}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{71x^5\sqrt{x^4+3x^2+4}}{21}+\frac{3187x^3\sqrt{x^4+3x^2+4}}{315}+\frac{583x\sqrt{x^4+3x^2+4}}{35}+\frac{6352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\sqrt{}\right.}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

```
[In] int((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*x*(175*x^6+1065*x^4+3187*x^2+5247)*(x^4+3*x^2+4)^(1/2)+6352/35/(-6+2*
I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))
*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2))-89536/105/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1
```

```
/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I
*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
)-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.64

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{8394\sqrt{2}(\sqrt{-7}x-3\,x)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-3\sqrt{2}(1607\sqrt{-7}x-}{}$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1260*(8394*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsi
n(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 3*sqrt(2)*(1607*
sqrt(-7)*x - 11967*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt
(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(175*x^8 + 1065*x^6 + 3187*x^4 +
 5247*x^2 + 8394)*sqrt(x^4 + 3*x^2 + 4))/x
```

Sympy [F]

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx = \int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\cdot\left(5x^2+7\right)\,dx$$

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7), x)
```

Maxima [F]

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

Giac [F]

$$\int \left(7+5x^2\right) \left(4+3x^2+x^4\right)^{3/2} \, dx=\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right) dx$$

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right) \left(4+3x^2+x^4\right)^{3/2} \, dx=\int \left(5\,x^2+7\right)\,\left(x^4+3\,x^2+4\right)^{3/2} dx$$

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.360 $\int \left(4+3x^2+x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 198

$$\int \left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{138x\sqrt{4+3x^2+x^4}}{35\left(2+x^2\right)} + \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4} + \frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2} - \frac{138\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{4+3x^2+x^4}} + \frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}$$

```
[Out] 1/7*x*(x^4+3*x^2+4)^(3/2)+138/35*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/35*x*(9*x^
2+49)*(x^4+3*x^2+4)^(1/2)-138/35*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(
1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/
4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4*(x
^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*E
llipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2
)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used

= {1105, 1190, 1211, 1117, 1209}

$$\int \left(4+3x^2+x^4\right)^{3/2} dx = \frac{4\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}} - \frac{138\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{x^4+3x^2+4}} + \frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2} + \frac{1}{35}x\left(9x^2+49\right)\sqrt{x^4+3x^2+4} + \frac{138x\sqrt{x^4+3x^2+4}}{35\left(x^2+2\right)}$$

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (138*x*Sqrt[4 + 3*x^2 + x^4])/(35*(2 + x^2)) + (x*(49 + 9*x^2)*Sqrt[4 + 3*x
^2 + x^4])/35 + (x*(4 + 3*x^2 + x^4)^(3/2))/7 - (138*Sqrt[2]*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sq
rt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{3}{7}\int\left(8+3x^2\right)\sqrt{4+3x^2+x^4}\,dx\\
&= \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{35}\int\frac{284+138x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}\\
&\quad-\frac{276}{35}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+16\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{138x\sqrt{4+3x^2+x^4}}{35\left(2+x^2\right)}+\frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}\\
&\quad+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}-\frac{138\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.78 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.73

$$\int\left(4+3x^2\right.$$
$$\left.+x^4\right)^{3/2}\,dx=\frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(276+303x^2+161x^4+39x^6+5x^8)-69\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2}{3i+\sqrt{7}}}}{}$$

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(276 + 303*x^2 + 161*x^4 + 39*x^6 + 5*x^8)
 - 69*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt
[2]*(-77*I + 69*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])
]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqr
t[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(70*Sqrt[(
-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x\left(5x^4+24x^2+69\right)\sqrt{x^4+3x^2+4}}{35}+\frac{1136\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{4416\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right.}}{}$
default	$\frac{x^5\sqrt{x^4+3x^2+4}}{7}+\frac{24x^3\sqrt{x^4+3x^2+4}}{35}+\frac{69x\sqrt{x^4+3x^2+4}}{35}+\frac{1136\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i}}{4}\right.}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{x^5\sqrt{x^4+3x^2+4}}{7}+\frac{24x^3\sqrt{x^4+3x^2+4}}{35}+\frac{69x\sqrt{x^4+3x^2+4}}{35}+\frac{1136\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i}}{4}\right.}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

```
[In] int((x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*x*(5*x^4+24*x^2+69)*(x^4+3*x^2+4)^(1/2)+1136/35/(-6+2*I*7^(1/2))^(1/2)
*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4
+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^
(1/2))-4416/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1
-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(Ellipti
cF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*
(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\int (4+3x^2+x^4)^{3/2}\,dx = \frac{138\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-\sqrt{2}\left(67\sqrt{-7}x-627\,x\right.}{1}$$

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/140*(138*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(
1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(67*sqrt(-
7)*x - 627*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7
) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(5*x^6 + 24*x^4 + 69*x^2 + 138)*sqrt(x^4
 + 3*x^2 + 4))/x
```

Sympy [F]

$$\int (4+3x^2+x^4)^{3/2}\,dx = \int \left(x^4+3x^2+4\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 4)**(3/2), x)
```

Maxima [F]

$$\int (4+3x^2+x^4)^{3/2}\,dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

Giac [F]

$$\int \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+4\right)^{3/2}\,dx$$

```
[In] int((3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((3*x^2 + x^4 + 4)^(3/2), x)
```

3.361 $\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 284

$$\begin{aligned}
\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx &= \frac{94x\sqrt{4+3x^2+x^4}}{125\,(2+x^2)} \\
&+ \frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4} + \frac{44}{125}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) \\
&- \frac{94\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{4+3x^2+x^4}} \\
&+ \frac{54\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{125\sqrt{4+3x^2+x^4}} \\
&+ \frac{4114\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{4+3x^2+x^4}}
\end{aligned}$$

```
[Out] 44/4375*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+94/125*x*(x^
4+3*x^2+4)^(1/2)/(x^2+2)+1/75*x*(3*x^2+11)*(x^4+3*x^2+4)^(1/2)-94/125*(x^2+
2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elli
pticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2
+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+54/125*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)
))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/
2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/
2)+4114/13125*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1
/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*
((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1222, 1190, 1211, 1117, 1209, 1230, 1720}

$$\begin{aligned}\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx &= \frac{44}{125}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)\\ &+\frac{54\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}\\ &-\frac{94\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}\\ &+\frac{4114\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}\\ &+\frac{1}{75}(3x^2+11)\sqrt{x^4+3x^2+4}x+\frac{94\sqrt{x^4+3x^2+4}x}{125(x^2+2)}\end{aligned}$$

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (94*x*Sqrt[4 + 3*x^2 + x^4])/(125*(2 + x^2)) + (x*(11 + 3*x^2)*Sqrt[4 + 3*x
^2 + x^4])/75 + (44*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x
^4]])/125 - (94*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticE[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (54*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (4114*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(13125*
Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
```

```
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{25}\int\left(-8-5x^2\right)\sqrt{4+3x^2+x^4}\,dx\right)+\frac{44}{25}\int\frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx\\
&= \frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{1}{375}\int\frac{-260-150x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&\quad -\frac{44}{625}\int\frac{-8-5x^2}{\sqrt{4+3x^2+x^4}}\,dx+\frac{1936}{625}\int\frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{88}{125}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\\
&\quad -\frac{4}{5}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{1936\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{1875}+\frac{792}{625}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&\quad +\frac{112}{75}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{3872}{375}\int\frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{94x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)}+\frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}\\
&\quad +\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)\\
&\quad -\frac{94\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{4+3x^2+x^4}}\\
&\quad +\frac{54\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{4+3x^2+x^4}}\\
&\quad +\frac{4114\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.56 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.68

$$\int\frac{\left(4+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx=\frac{350\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(44+45x^2+20x^4+3x^6)-4935\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+}{}}}{}$$

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (350*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(44 + 45*x^2 + 20*x^4 + 3*x^6) - 4935*Sq
rt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*S
qrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*Sqrt[2]*(-
241*I + 705*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sq
rt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-
2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (5808*I)*Sqrt
[2]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh
[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(26250
*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x(3x^2+11)\sqrt{x^4+3x^2+4}}{75}+\frac{9424\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{1875\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$
default	$\frac{x^3\sqrt{x^4+3x^2+4}}{25}+\frac{11x\sqrt{x^4+3x^2+4}}{75}+\frac{9424\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{1875\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$
elliptic	$\frac{x^3\sqrt{x^4+3x^2+4}}{25}+\frac{11x\sqrt{x^4+3x^2+4}}{75}+\frac{9424\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{1875\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/75*x*(3*x^2+11)*(x^4+3*x^2+4)^(1/2)+9424/1875/(-6+2*I*7^(1/2))^(1/2)*(1-(
-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^
2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
)-3008/125/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3
/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1
/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+
2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+1936/4375/(-3/8+1/8*I*7^(1/2
))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^
(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/
8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="fricas")
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{((x^2-x+2)\,(x^2+x+2))^{\frac{3}{2}}}{5x^2+7}\,dx$$

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7),x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7), x)
```

Maxima [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Giac [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{(x^4+3\,x^2+4)^{3/2}}{5\,x^2+7}\,dx$$

```
[In] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7),x)
```

```
[Out] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7), x)
```

3.362 $\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 305

$$\begin{aligned}\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx &= \frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{4x\sqrt{4+3x^2+x^4}}{175\,(2+x^2)}+\frac{22x\sqrt{4+3x^2+x^4}}{175\,(7+5x^2)}\\
&+\frac{13}{350}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)-\frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{175\sqrt{4+3x^2+x^4}}\\
&+\frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{175\sqrt{4+3x^2+x^4}}\\
&+\frac{2431(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{36750\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 13/12250*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/75*x*(x^4
+3*x^2+4)^(1/2)+4/175*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+22/175*x*(x^4+3*x^2+4)^
(1/2)/(5*x^2+7)+2431/73500*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/c
os(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-4/
175*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/
2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^
2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4/175*(x^2+2)*(cos(2*arctan(1/2*x
*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2
*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^
2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.22, number of steps used = 19, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1242, 1117, 1153, 1209, 1136, 1211, 1237, 1728, 1722, 1720, 1230}

$$\begin{aligned}\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx &= \frac{13}{350}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)\\ &+ \frac{4\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}\\ &- \frac{4\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}\\ &+ \frac{187\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}\\ &+ \frac{6919(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{183750\sqrt{2}\sqrt{x^4+3x^2+4}}\\ &+ \frac{4\sqrt{x^4+3x^2+4}x}{175\,(x^2+2)} + \frac{22\sqrt{x^4+3x^2+4}x}{175\,(5x^2+7)} + \frac{1}{75}\sqrt{x^4+3x^2+4}x\end{aligned}$$

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(2 + x^2))
+ (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*Sqrt[11/35]*ArcTan[(
2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350 - (4*Sqrt[2]*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt
[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (6919
*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[
x/Sqrt[2]], 1/8])/(183750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (187*Sqrt[2]*(2
+ x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sq
rt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
```

```
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
```

```
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{152}{625\sqrt{4+3x^2+x^4}}+\frac{16x^2}{125\sqrt{4+3x^2+x^4}}+\frac{x^4}{25\sqrt{4+3x^2+x^4}}\right.\\
&\qquad\left.+\frac{1936}{625\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}+\frac{88}{625\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\right)dx\\
&= \frac{1}{25}\int \frac{x^4}{\sqrt{4+3x^2+x^4}}\,dx+\frac{16}{125}\int \frac{x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&\quad+\frac{88}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx+\frac{152}{625}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&\quad+\frac{1936}{625}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}\\
&\quad+\frac{38\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{625\sqrt{4+3x^2+x^4}}-\frac{22\int \frac{12+70x^2+25x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{4375}\\
&\quad-\frac{1}{75}\int \frac{4+6x^2}{\sqrt{4+3x^2+x^4}}\,dx-\frac{88\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{1875}+\frac{32}{125}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&\quad-\frac{32}{125}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{176}{375}\int \frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{16x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)}\\
&\quad+\frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+\frac{2}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)\\
&\quad-\frac{16\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{212\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1875\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{187\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{4+3x^2+x^4}}-\frac{22\int \frac{410+425x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{21875}\\
&\quad+\frac{44}{875}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{4}{25}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{16}{75}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{75} x\sqrt{4+3x^2+x^4} + \frac{4x\sqrt{4+3x^2+x^4}}{175\,(2+x^2)} \\
&\quad + \frac{22x\sqrt{4+3x^2+x^4}}{175\,(7+5x^2)} + \frac{2}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) \\
&\quad - \frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{175\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{112\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1875\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{187\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{1936\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{13125} + \frac{1628\int\frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{2625} \\
&= \frac{1}{75} x\sqrt{4+3x^2+x^4} + \frac{4x\sqrt{4+3x^2+x^4}}{175\,(2+x^2)} \\
&\quad + \frac{22x\sqrt{4+3x^2+x^4}}{175\,(7+5x^2)} + \frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) \\
&\quad - \frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{175\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{175\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{6919(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{183750\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{187\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.01

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \frac{\frac{175x(23+7x^2)(4+3x^2+x^4)}{7+5x^2} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\Big(105\big(3-i\sqrt{7}\big)\,E\Big(ia}{}$$

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] ((175*x*(23 + 7*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2) - I*Sqrt[6 + (2*I)*Sqrt
[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqr
t[7])]*(105*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(158 + (15*I)*Sqrt[7])*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])] + 429*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(18375*Sqrt[4 + 3*x^2 + x^
4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.13

method	result
risch	$\frac{\sqrt{x^4+3x^2+4}\,x(7x^2+23)}{525x^2+735}+\frac{232\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{375\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-128\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1}$
default	$\frac{22x\sqrt{x^4+3x^2+4}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+4}}{75}+\frac{232\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{375\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{128\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}}{175\sqrt{-}}$
elliptic	$\frac{22x\sqrt{x^4+3x^2+4}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+4}}{75}+\frac{232\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{375\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{128\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}}{175\sqrt{-}}$

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(x^4+3*x^2+4)^(1/2)*x*(7*x^2+23)/(5*x^2+7)+232/375/(-6+2*I*7^(1/2))^(
1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/
(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))-128/175/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2
)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(Ell
```

```
ipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/
4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+286/6125/(-3/8+1/8*I
*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^
(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5
/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/
2))
```

Fricas [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx=\int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx=\int \frac{((x^2-x+2)\,(x^2+x+2))^{\frac{3}{2}}}{(5x^2+7)^2}\,dx$$

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**2,x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

Maxima [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx=\int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

Giac [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{\left(x^4+3\,x^2+4\right)^{3/2}}{\left(5\,x^2+7\right)^2}\,dx$$

```
[In] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^2,x)
[Out] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

3.363 $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 440

$$
\begin{aligned}
\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3} \, dx &= \frac{9x\sqrt{4+3x^2+x^4}}{1960\,(2+x^2)} + \frac{11x\sqrt{4+3x^2+x^4}}{175\,(7+5x^2)^2} \\
&+ \frac{167x\sqrt{4+3x^2+x^4}}{9800\,(7+5x^2)} + \frac{1347\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{7840\sqrt{385}} \\
&+ \frac{111(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{24500\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&- \frac{6\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{4+3x^2+x^4}} \\
&- \frac{817(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{91875\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&- \frac{22\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{4+3x^2+x^4}} \\
&+ \frac{7633(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{274400\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

```
[Out] 1347/3018400*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+9/1960*
x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+11/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+167/
9800*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-9/1960*(x^2+2)*(cos(2*arctan(1/2*x*2^(
1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2
^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)
```

```
^(1/2)-3/490*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/
2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x
^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+7633/548800*(x^2+2)*(cos
(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(
sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^
(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1117, 1153, 1209, 1237, 1710, 1728, 1722, 1720, 1230}

$$
\begin{aligned}
\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx &= \frac{1347\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{7840\sqrt{385}} \\
&- \frac{22\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}} \\
&- \frac{817(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{91875\sqrt{2}\sqrt{x^4+3x^2+4}} \\
&- \frac{6\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{x^4+3x^2+4}} \\
&+ \frac{111(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{24500\sqrt{2}\sqrt{x^4+3x^2+4}} \\
&+ \frac{7633(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{274400\sqrt{2}\sqrt{x^4+3x^2+4}} \\
&+ \frac{9\sqrt{x^4+3x^2+4}x}{1960(x^2+2)} + \frac{167\sqrt{x^4+3x^2+4}x}{9800(5x^2+7)} + \frac{11\sqrt{x^4+3x^2+4}x}{175(5x^2+7)^2}
\end{aligned}
$$

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]

[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x^2 + x^4])
/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) + (
1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(7840*Sqrt[385]) + (1
11*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[
2]], 1/8])/(24500*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(875*
Sqrt[4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
 - (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Ar
```

```
cTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4]) + (7633*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8
])/(274400*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
```

```
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{9}{625\sqrt{4+3x^2+x^4}}+\frac{x^2}{125\sqrt{4+3x^2+x^4}}+\frac{1936}{625\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\right.\\
&\qquad\left.+\frac{88}{625\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}+\frac{89}{625\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\right)dx\\
&= \frac{1}{125}\int\frac{x^2}{\sqrt{4+3x^2+x^4}}\,dx+\frac{9}{625}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{88}{625}\int\frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx\\
&\quad+\frac{89}{625}\int\frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx+\frac{1936}{625}\int\frac{1}{\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2}+\frac{x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+\frac{9(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1250\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad-\frac{\int\frac{12+70x^2+25x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{4375}-\frac{11\int\frac{-76-10x^2-25x^4}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx}{4375}+\frac{2}{125}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&\quad-\frac{2}{125}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{89\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{1875}+\frac{178}{375}\int\frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)}+\frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2}+\frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)}\\
&\quad+\frac{89\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{500\sqrt{385}}-\frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{4+3x^2+x^4}}\\
&\quad-\frac{8\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1875\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{1513(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{52500\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{\int\frac{-4412-4690x^2-2775x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{245000}-\frac{\int\frac{410+425x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{21875}+\frac{2}{875}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx
\end{aligned}$$

$$= \frac{6x\sqrt{4+3x^2+x^4}}{875\left(2+x^2\right)} + \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)}$$

$$+ \frac{89\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{500\sqrt{385}} - \frac{6\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{4+3x^2+x^4}}$$

$$- \frac{8\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1875\sqrt{4+3x^2+x^4}}$$

$$+ \frac{1513(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{52500\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{\int\frac{-60910-31775x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{1225000}$$

$$+ \frac{111\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{24500} - \frac{88\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{13125} + \frac{74\int\frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{2625}$$

$$= \frac{9x\sqrt{4+3x^2+x^4}}{1960\left(2+x^2\right)} + \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2}$$

$$+ \frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)} + \frac{3}{175}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)$$

$$+ \frac{111(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{24500\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{6\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{4+3x^2+x^4}}$$

$$- \frac{26\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4375\sqrt{4+3x^2+x^4}}$$

$$+ \frac{187(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6125\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{22\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{30625} - \frac{219\int\frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{4900}$$

$$= \frac{9x\sqrt{4+3x^2+x^4}}{1960\left(2+x^2\right)} + \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)}$$

$$+ \frac{3}{175}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) - \frac{657\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{39200\sqrt{385}}$$

$$+ \frac{111(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{24500\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{6\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{4+3x^2+x^4}}$$

$$- \frac{11(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{30625\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{26\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4375\sqrt{4+3x^2+x^4}}$$

$$+ \frac{7633(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{274400\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.55 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.70

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx = \frac{\frac{140x(357+167x^2)(4+3x^2+x^4)}{(7+5x^2)^2} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\Big(315(3-i\sqrt{7})\,E}{}$$

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((140*x*(357 + 167*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 - I*Sqrt[6 + (2*I)
*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
+ Sqrt[7])]*(315*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(103 + (45*I)*Sqrt[7])*Ell
ipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + S
qrt[7])] + 2694*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-
3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(274400*Sqrt[4 + 3*x
^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.79

method	result
risch	$\frac{\sqrt{x^4+3x^2+4}\,x\left(167x^2+357\right)}{1960\left(5x^2+7\right)^2}+\frac{17\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{350\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{36\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{}}{}$
default	$\frac{11x\sqrt{x^4+3x^2+4}}{175\left(5x^2+7\right)^2}+\frac{167x\sqrt{x^4+3x^2+4}}{9800\left(5x^2+7\right)}+\frac{17\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{350\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{36\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}}{245\sqrt{}}$
elliptic	$\frac{11x\sqrt{x^4+3x^2+4}}{175\left(5x^2+7\right)^2}+\frac{167x\sqrt{x^4+3x^2+4}}{9800\left(5x^2+7\right)}+\frac{17\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{350\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{36\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}}{245\sqrt{}}$

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1960*(x^4+3*x^2+4)^(1/2)*x*(167*x^2+357)/(5*x^2+7)^2+17/350/(-6+2*I*7^(1/
2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(
1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*
7^(1/2))^(1/2))-36/245/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^
(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*
(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-Elliptic
E(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+1347/68600/(-3/8
+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*
x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2
)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2
))^(1/2))
```

Fricas [F]

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**3,x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

Maxima [F]

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

Giac [F]

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx=\int \frac{\left(x^4+3\,x^2+4\right)^{3/2}}{\left(5\,x^2+7\right)^3}\,dx$$

```
[In] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^3,x)
[Out] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

3.364 $\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\begin{aligned}\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx &= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}-\frac{15x\sqrt{4+3x^2+x^4}}{2+x^2}\\ &\quad+\frac{15\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}\\ &\quad+\frac{13(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 75*x*(x^4+3*x^2+4)^(1/2)+25*x^3*(x^4+3*x^2+4)^(1/2)-15*x*(x^4+3*x^2+4)^(1/2
)/(x^2+2)+13/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(
1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3
*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+15*(x^2+2)*(cos(2*arct
an(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*ar
ctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used

$= \{1220, 1693, 1211, 1117, 1209\}$

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = \frac{13(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{15\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}} - \frac{15\sqrt{x^4+3x^2+4}x}{x^2+2} + 75\sqrt{x^4+3x^2+4}x + 25\sqrt{x^4+3x^2+4}x^3$$

```
[In] Int[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] 75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] - (15*x*Sqrt[4 +
3*x^2 + x^4])/(2 + x^2) + (15*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
 x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (13*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
```

```
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 25x^3\sqrt{4+3x^2+x^4}+\frac{1}{5}\int\frac{1715+2175x^2+1125x^4}{\sqrt{4+3x^2+x^4}}\,dx\\
&= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}+\frac{1}{15}\int\frac{645-225x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}+13\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx+30\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\\
&= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}-\frac{15x\sqrt{4+3x^2+x^4}}{2+x^2}\\
&\quad+\frac{15\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\mid\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{13(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\mid\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.34 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.80

$$\int\frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{100\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(12+13x^2+6x^4+x^6)+15\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\Big(i\operatorname{arcsinh}\Big(\sqrt{-\frac{2i}{-3i+}}}{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{}}$$

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (100*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(12 + 13*x^2 + 6*x^4 + x^6) + 15*Sqrt[2]
*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(
3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)
/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(131*I +
15*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(4*Sqrt[(-I)/(-3*I + Sq
rt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.22

method	result
risch	$25x(x^2+3)\sqrt{x^4+3x^2+4}+\frac{172\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{480\sqrt{1-\left(-\frac{3}{8}\right.}}{}$
default	$\frac{172\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+25x^3\sqrt{x^4+3x^2+4}+75x\sqrt{x^4+3x^2+}$
elliptic	$\frac{172\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+25x^3\sqrt{x^4+3x^2+4}+75x\sqrt{x^4+3x^2+}$

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25*x*(x^2+3)*(x^4+3*x^2+4)^(1/2)+172/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*
7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*
EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+480/(-6+2
*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2)
)*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(
1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/
2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = -\frac{60\,\sqrt{2}(\sqrt{-7}x-3\,x)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-\sqrt{2}(103\,\sqrt{-7}x-51\,x)\sqrt{\sqrt{-7}-}}{16\,x}$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(60*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1
/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(103*sqrt(-
7)*x - 51*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7)
 - 3)/x), 3/8*sqrt(-7) + 1/8) - 80*(5*x^4 + 15*x^2 - 3)*sqrt(x^4 + 3*x^2 +
4))/x
```

Sympy [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5x^2+7)^3}{\sqrt{(x^2-x+2)\,(x^2+x+2)}}\,dx$$

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.365 $\int \frac{\left(7+5x^2\right)^2}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^2}{\sqrt{4+3x^2+x^4}}\,dx &= \frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{20x\sqrt{4+3x^2+x^4}}{2+x^2}\\ &\quad-\frac{20\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}\\ &\quad+\frac{167(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 25/3*x*(x^4+3*x^2+4)^(1/2)+20*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+167/12*(x^2+2)*
(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipti
cF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2
)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-20*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(
1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/
4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{1220, 1211, 1117, 1209\}$

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx = \frac{167(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{20\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}} + \frac{20\sqrt{x^4+3x^2+4}\,x}{x^2+2} + \frac{25}{3}\sqrt{x^4+3x^2+4}\,x$$

```
[In] Int[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (20*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (
20*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (167*(2 + x^2)*Sqrt[(4 + 3*x^2 +
 x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 +
 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
```

```
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{1}{3}\int\frac{47+60x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{25}{3}x\sqrt{4+3x^2+x^4}-40\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{167}{3}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{20x\sqrt{4+3x^2+x^4}}{2+x^2}\\
&\quad-\frac{20\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\mid\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{167(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\mid\frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.95

$$\int\frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx$$

$$=\frac{50\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(4+3x^2+x^4)-30\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)\mid\frac{3i-}{3i+}\right.}{6\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3}}$$

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (50*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 30*Sqrt[2]*(3*I + Sqr
t[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[
7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(43*I + 30*Sqrt[7])*
Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (
2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])
]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt
[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.32

method	result
default	$\frac{188\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{25x\sqrt{x^4+3x^2+4}}{3}-\frac{640\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\right.}$
risch	$\frac{188\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{25x\sqrt{x^4+3x^2+4}}{3}-\frac{640\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\right.}$
elliptic	$\frac{188\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{25x\sqrt{x^4+3x^2+4}}{3}-\frac{640\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\right.}$

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 188/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/
8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+25/3*x*(x^4+3*x^2+4)^(1/2)-640/(-6+2*I*7
^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^
2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1
/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx$$
$$=\frac{240\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-\sqrt{2}\left(193\sqrt{-7}x-861\,x\right)\sqrt{\sqrt{-7}}}{48\,x}$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(240*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1
/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(193*sqrt(-
7)*x - 861*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7
) - 3)/x), 3/8*sqrt(-7) + 1/8) + 80*sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 12))/x
```

Sympy [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5x^2+7)^2}{\sqrt{(x^2-x+2)(x^2+x+2)}}\,dx$$

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.366 $\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx=\frac{5x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{5\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}+\frac{17(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

```
[Out] 5*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+17/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^
2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))
),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-
5*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1211, 1117, 1209}

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx=\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{5\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{5\sqrt{x^4+3x^2+4}\,x}{x^2+2}$$

```
[In] Int[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (5*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 +
 x^4] + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTa
n[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(10\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\right)+17\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{5x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{5\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}\\ &\quad+\frac{17(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.42

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(-5\left(3i+\sqrt{7}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\left(i+5\sqrt{7}\right)\text{EllipticF}\left(i\text{arcs}\right.\right.}{2\sqrt{2}\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

```
[In] Integrate[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7]
)]*(-5*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (I + 5*Sqrt[7])*EllipticF[I*ArcSinh[Sq
rt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(2*Sqrt[
2]*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.38

method	result
default	$\frac{28\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{160\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(F\left(\frac{x\sqrt{-6}}{4}\right.\right.}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x}}$
elliptic	$\frac{28\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{160\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(F\left(\frac{x\sqrt{-6}}{4}\right.\right.}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x}}$

```
[In] int((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 28/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I
*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))-160/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7
^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(
3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1
/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \frac{20\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right) \mid \frac{3}{8}\sqrt{-7}+\frac{1}{8}) - \sqrt{2}\left(13\sqrt{-7}x-81\,x\right)\sqrt{\sqrt{-7}-3}F}{16\,x}$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(20*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/
2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(13*sqrt(-7)
*x - 81*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) -
 3)/x), 3/8*sqrt(-7) + 1/8) + 80*sqrt(x^4 + 3*x^2 + 4))/x
```

Sympy [F]

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{5x^2+7}{\sqrt{(x^2-x+2)\,(x^2+x+2)}}\,dx$$

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

Maxima [F]

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

Giac [F]

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx=\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx=\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.367 $\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx=\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

```
[Out] 1/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/
2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1117}

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx=\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[In] Int[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]
], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\text{integral} = \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= -\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}x\right),\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

```
[In] Integrate[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[7])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7])]*x], (-3 - I*Sqrt[7])/(-3 + I
*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$	85
elliptic	$\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$	85

```
[In] int(1/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*
7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1
/2),1/4*(2+6*I*7^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = -\frac{1}{16}\,\sqrt{2}\left(\sqrt{-7}+3\right)\sqrt{\sqrt{-7}-3}F(\arcsin\left(\frac{1}{4}\,\sqrt{2}x\sqrt{\sqrt{-7}-3}\right) \mid \frac{3}{8}\,\sqrt{-7}+\frac{1}{8})$$

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*(sqrt(-7) + 3)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/4*sqrt(
2)*x*sqrt(sqrt(-7) - 3)), 3/8*sqrt(-7) + 1/8)
```

Sympy [F]

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3x^2+4}}\,dx$$

```
[In] integrate(1/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+4}} \,dx$$

```
[In] int(1/(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.368 $\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$
\begin{aligned}
&\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{1}{4}\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) - \frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{17\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{84\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

```
[Out] 1/308*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/12*(x^2+2)*(
cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elliptic
F(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)
*2^(1/2)/(x^4+3*x^2+4)^(1/2)+17/168*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2
)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))
),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)
^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{1230,\ 1117,\ 1720\}$

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{1}{4}\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right) - \frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}$$

$$+ \frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{84\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[In] Int[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/4 - ((2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(
6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(84*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\right)+\frac{10}{3}\int \frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)-\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}}\\ &\quad+\frac{17(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{84\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\text{EllipticPi}\left(-\frac{5}{14}\left(-3-i\sqrt{7}\right),i\text{arcsinh}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}x\right),\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{7\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((-1/7*I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[
7])]*EllipticPi[(-5*(-3 - I*Sqrt[7]))/14, I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7]
)]*x], (-3 - I*Sqrt[7])/(-3 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^
(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\Pi\left(\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,x,-\frac{5}{7\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)},\frac{\sqrt{-\frac{3}{8}-\frac{i\sqrt{7}}{8}}}{\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}}\right)}{7\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,\sqrt{x^4+3x^2+4}}$	107
elliptic	$\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\Pi\left(\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,x,-\frac{5}{7\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)},\frac{\sqrt{-\frac{3}{8}-\frac{i\sqrt{7}}{8}}}{\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}}\right)}{7\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,\sqrt{x^4+3x^2+4}}$	107

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x
^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1
/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8
*I*7^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^6 + 22*x^4 + 41*x^2 + 28), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+2)\,(x^2+x+2)}\,(5x^2+7)}\,dx$$

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{(5\,x^2+7)\,\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2)), x)
```

3.369 $\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 286

$$\begin{aligned}
\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx
&= -\frac{5x\sqrt{4+3x^2+x^4}}{616\,(2+x^2)}+\frac{25x\sqrt{4+3x^2+x^4}}{616\,(7+5x^2)}+\frac{37\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{2464}\\
&\quad+\frac{5(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{308\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad-\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{42\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{629(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{51744\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

```
[Out] 37/189728*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-5/616*x*(x
^4+3*x^2+4)^(1/2)/(x^2+2)+25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+5/616*(x^2
+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ell
ipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-1/84*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2
)+629/103488*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/
2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*(
(x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1237, 1728, 1209, 1722, 1117, 1720}

$$\begin{aligned}&\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx\\ &=\frac{37\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{2464}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+4}}\\ &\quad+\frac{5(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{308\sqrt{2}\sqrt{x^4+3x^2+4}}\\ &\quad+\frac{629(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{51744\sqrt{2}\sqrt{x^4+3x^2+4}}\\ &\quad-\frac{5\sqrt{x^4+3x^2+4}x}{616(x^2+2)}+\frac{25\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}\end{aligned}$$

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (-5*x*Sqrt[4 + 3*x^2 + x^4])/(616*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 + x^4])
/(616*(7 + 5*x^2)) + (37*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2
 + x^4]])/2464 + (5*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(308*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/
(42*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (629*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(51744*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\text{integral} = \frac{25x\sqrt{4+3x^2+x^4}}{616\,(7+5x^2)} - \frac{1}{616}\int \frac{12+70x^2+25x^4}{(7+5x^2)\,\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{616\,(7+5x^2)} - \frac{\int \frac{410+425x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{3080} + \frac{5}{308}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{5x\sqrt{4+3x^2+x^4}}{616\,(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{616\,(7+5x^2)} + \frac{5(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{308\sqrt{2}\sqrt{4+3x^2+x^4}} - \frac{1}{21}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{185}{924}\int \frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{5x\sqrt{4+3x^2+x^4}}{616\,(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{616\,(7+5x^2)} + \frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{2464} + \frac{5(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{308\sqrt{2}\sqrt{4+3x^2+x^4}} - \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{42\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{629(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{51744\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.75 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.68

$$\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx = \frac{700\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(4+3x^2+x^4)+35\left(3i+\sqrt{7}\right)(7+5x^2)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(E\left(i\text{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{}}}\right.\right.\right.}{}$$

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*(
7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) + (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*
I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
```

```
 - (74*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(17248*Sqrt[(
-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.19

method	result
risch	$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)}-\frac{\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{22\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{20\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\right.}}{77\sqrt{}}$
default	$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{22\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{20\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F(}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{22\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{20\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F(}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-1/22/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8
+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)
^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+20
/77/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*
I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-
6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(
1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+37/4312/(-3/8+1/8*I*7^(1/2))^(1/2)*
(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^
4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7
^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196)
, x)
```

Sympy [F]

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\left(5x^2+7\right)^2}\,dx$$

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{{\left(5\,x^2+7\right)}^2\,\sqrt{x^4+3\,x^2+4}} \,d x$$

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2)), x)
```

3.370 $\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 314

$$\begin{aligned}\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx &= -\frac{555x\sqrt{4+3x^2+x^4}}{758912\,(2+x^2)}+\frac{25x\sqrt{4+3x^2+x^4}}{1232\,(7+5x^2)^2}+\frac{2775x\sqrt{4+3x^2+x^4}}{758912\,(7+5x^2)} \\ &\quad -\frac{3285\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{3035648}+\frac{555(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{4+3x^2+x^4}} \\ &\quad -\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{8624\sqrt{2}\sqrt{4+3x^2+x^4}} \\ &\quad -\frac{18615(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{21249536\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] -3285/233744896*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-555/
758912*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+25/1232*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7
)^2+2775/758912*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+555/758912*(x^2+2)*(cos(2*a
rctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2
*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2
)/(x^4+3*x^2+4)^(1/2)-1/17248*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2
)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2
^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-18615/4
2499072*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1237, 1710, 1728, 1209, 1722, 1117, 1720}

$$\begin{aligned}
&\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx \\
&= -\frac{3285\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{3035648} - \frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{8624\sqrt{2}\sqrt{x^4+3x^2+4}} \\
&\quad + \frac{555(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}} \\
&\quad - \frac{18615(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{21249536\sqrt{2}\sqrt{x^4+3x^2+4}} \\
&\quad - \frac{555\sqrt{x^4+3x^2+4}x}{758912(x^2+2)} + \frac{2775\sqrt{x^4+3x^2+4}x}{758912(5x^2+7)} + \frac{25\sqrt{x^4+3x^2+4}x}{1232(5x^2+7)^2}
\end{aligned}$$

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (-555*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 +
x^4])/(1232*(7 + 5*x^2)^2) + (2775*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(7 + 5*
x^2)) - (3285*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/3
035648 + (555*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*Arc
Tan[x/Sqrt[2]], 1/8])/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(86
24*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (18615*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)
/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(21249536*Sqrt[
2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
```

```
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} - \frac{\int \frac{-76-10x^2-25x^4}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx}{1232} \\
&= \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)} + \frac{\int \frac{-4412-4690x^2-2775x^4}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{758912} \\
&= \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)} + \frac{\int \frac{-60910-31775x^2}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{3794560} + \frac{555\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{379456} \\
&= -\frac{555x\sqrt{4+3x^2+x^4}}{758912\left(2+x^2\right)} + \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)} \\
&\quad + \frac{555(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{4312} - \frac{5475\int \frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{379456} \\
&= -\frac{555x\sqrt{4+3x^2+x^4}}{758912\left(2+x^2\right)} + \frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)} \\
&\quad - \frac{3285\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{3035648} + \frac{555(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{8624\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{18615(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{21249536\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.72 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.98

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \frac{\frac{700x(1393+555x^2)(4+3x^2+x^4)}{(7+5x^2)^2}+i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(3885\left(3-i\sqrt{7}\right)E\left(i\text{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\right.\right.\right.}{}$$

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((700*x*(1393 + 555*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I
)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(3885*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9401 + (3885*I)*Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
 + Sqrt[7])] + 6570*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(21249536*Sqrt[4
 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.10

method	result
risch	$\frac{25\sqrt{x^4+3x^2+4}\,x(555x^2+1393)}{758912(5x^2+7)^2}-\frac{23\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{27104\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}+\frac{555\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)}}{}$
default	$\frac{25x\sqrt{x^4+3x^2+4}}{1232(5x^2+7)^2}+\frac{2775x\sqrt{x^4+3x^2+4}}{758912(5x^2+7)}-\frac{23\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{27104\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}+\frac{555\sqrt{1+\frac{3x^2}{8}-}}{23716}$
elliptic	$\frac{25x\sqrt{x^4+3x^2+4}}{1232(5x^2+7)^2}+\frac{2775x\sqrt{x^4+3x^2+4}}{758912(5x^2+7)}-\frac{23\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{27104\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}+\frac{555\sqrt{1+\frac{3x^2}{8}-}}{23716}$

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/758912*(x^4+3*x^2+4)^(1/2)*x*(555*x^2+1393)/(5*x^2+7)^2-23/27104/(-6+2*I
*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*
x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(
```

```
2+6*I*7^(1/2))^(1/2))+555/23716/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/
2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*
7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-3285/53
12384/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8
*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^
(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1
/8*I*7^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 +
3969*x^2 + 1372), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+2)\,(x^2+x+2)}\,(5x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

Giac [F]

$$\int \frac{1}{\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\left(5\,x^2+7\right)^3\sqrt{x^4+3\,x^2+4}}\,dx$$

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2)), x)
```

3.371 $\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\begin{aligned}\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx &= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{5000}{3}x\sqrt{4+3x^2+x^4}\\ &\quad+625x^3\sqrt{4+3x^2+x^4}-\frac{220779x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}\\ &\quad+\frac{220779(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}\\ &\quad-\frac{130729(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 1/28*x*(45779*x^2+99493)/(x^4+3*x^2+4)^(1/2)+5000/3*x*(x^4+3*x^2+4)^(1/2)+6
25*x^3*(x^4+3*x^2+4)^(1/2)-220779/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+220779/2
8*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-130729/24*(x^2+2)*(cos(2*arctan(1/2
*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1
/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*
x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1219, 1693, 1211, 1117, 1209}

$$\begin{aligned}\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = &-\frac{130729(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}\\ &+\frac{220779(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{220779\sqrt{x^4+3x^2+4}x}{28\,(x^2+2)}\\ &+\frac{5000}{3}\sqrt{x^4+3x^2+4}x+\frac{(45779x^2+99493)\,x}{28\sqrt{x^4+3x^2+4}}+625\sqrt{x^4+3x^2+4}x^3\end{aligned}$$

```
[In] Int[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (5000*x*Sqrt[4 + 3*x^2
 + x^4])/3 + 625*x^3*Sqrt[4 + 3*x^2 + x^4] - (220779*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (220779*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*E
llipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (1
30729*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int\frac{18156+269221x^2+350000x^4+87500x^6}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}}+625x^3\sqrt{4+3x^2+x^4}+\frac{1}{140}\int\frac{90780+296105x^2+700000x^4}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{5000}{3}x\sqrt{4+3x^2+x^4}\\
&\quad+625x^3\sqrt{4+3x^2+x^4}+\frac{1}{420}\int\frac{-2527660-3311685x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{5000}{3}x\sqrt{4+3x^2+x^4}+625x^3\sqrt{4+3x^2+x^4}\\
&\quad+\frac{220779}{14}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{130729}{6}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{5000}{3}x\sqrt{4+3x^2+x^4} + 625x^3\sqrt{4+3x^2+x^4} \\
&\quad - \frac{220779x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)} + \frac{220779(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{130729(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.55

$$
\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(858479+767337x^2+297500x^4+52500x^6)+662337\sqrt{2}(3i+\sqrt{7})}{}
$$

```
[In] Integrate[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(858479 + 767337*x^2 + 297500*x^4 + 52500*
x^6) + 662337*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3
*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[
I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])
] - Sqrt[2]*(975947*I + 662337*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(
-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])])/(336*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x(52500x^6+297500x^4+767337x^2+858479)}{84\sqrt{x^4+3x^2+4}} - \frac{505532\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{17662}{}$
elliptic	$-\frac{2\left(-\frac{45779}{56}x^3-\frac{99493}{56}x\right)}{\sqrt{x^4+3x^2+4}} + 625x^3\sqrt{x^4+3x^2+4} + \frac{5000x\sqrt{x^4+3x^2+4}}{3} - \frac{505532\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x}}$
default	$-\frac{33614\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}} - \frac{505532\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{1766232\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x}}{}$

```
[In] int((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84*x*(52500*x^6+297500*x^4+767337*x^2+858479)/(x^4+3*x^2+4)^(1/2)-505532/
21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I
*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1766232/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1
/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(
1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{662337\sqrt{2}\left(3x^5+9x^3-\sqrt{-7}(x^5+3x^3+4x)+12x\right)\sqrt{\sqrt{-7}-3}\,E(\arcsin\Big(\frac{\sqrt{2}}{}}{}$$

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/336*(662337*sqrt(2)*(3*x^5 + 9*x^3 - sqrt(-7)*(x^5 + 3*x^3 + 4*x) + 12*x)
*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/
8*sqrt(-7) + 1/8) - 2*sqrt(2)*(1183080*x^5 + 3549240*x^3 - 267977*sqrt(-7)*
(x^5 + 3*x^3 + 4*x) + 4732320*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*s
qrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 16*(13125*x^8 + 74375*x
^6 + 26250*x^4 - 282133*x^2 - 662337)*sqrt(x^4 + 3*x^2 + 4))/(x^5 + 3*x^3 +
 4*x)
```

Sympy [F]

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^5}{\left((x^2-x+2)\,(x^2+x+2)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**5/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.372 $\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 200

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{14523x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)} - \frac{14523(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{4243(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{4+3x^2+x^4}}$$

```
[Out] 1/28*x*(-4023*x^2+2719)/(x^4+3*x^2+4)^(1/2)+625/3*x*(x^4+3*x^2+4)^(1/2)+145
23/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-14523/28*(x^2+2)*(cos(2*arctan(1/2*x*2^
(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*
2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)+4243/24*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan
(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used

$= \{1219, 1693, 1211, 1117, 1209\}$

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{4243(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}$$
$$-\frac{14523(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$
$$+\frac{14523\sqrt{x^4+3x^2+4}x}{28\,(x^2+2)}+\frac{625}{3}\sqrt{x^4+3x^2+4}x+\frac{(2719-4023x^2)\,x}{28\sqrt{x^4+3x^2+4}}$$

```
[In] Int[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (625*x*Sqrt[4 + 3*x^2 +
x^4])/3 + (14523*x*Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (14523*(2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(
14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (4243*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^
2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
```

```
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{1}{28}\int \frac{14088+49523x^2+17500x^4}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{1}{84}\int \frac{-27736+43569x^2}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} \\
&\quad + \frac{4243}{6}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx - \frac{14523}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{14523x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)} \\
&\quad - \frac{14523(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{4243(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.66

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(78157+40431x^2+17500x^4)-43569\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\;}}{}$$

```
[In] Integrate[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(78157 + 40431*x^2 + 17500*x^4) - 43569*Sq
rt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*S
qrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(186
179*I + 43569*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*
Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[
(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(336*Sqrt[(-
I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.18

method	result
risch	$\frac{x(17500x^4+40431x^2+78157)}{84\sqrt{x^4+3x^2+4}} - \frac{27736\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}, \frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} - \frac{116184\sqrt{1-\left(-\frac{3}{8}+\right.}}{}$
elliptic	$-\frac{2\left(\frac{4023}{56}x^3-\frac{2719}{56}x\right)}{\sqrt{x^4+3x^2+4}} + \frac{625x\sqrt{x^4+3x^2+4}}{3} - \frac{27736\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}, \frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} - \frac{11618}{}$
default	$-\frac{4802\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}} - \frac{27736\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}, \frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} - \frac{116184\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{\;}}{}$

```
[In] int((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84*x*(17500*x^4+40431*x^2+78157)/(x^4+3*x^2+4)^(1/2)-27736/21/(-6+2*I*7^(
1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)
^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*
I*7^(1/2))^(1/2))-116184/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x
^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/
2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-Elli
pticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{43569\,\sqrt{2}\left(3\,x^5+9\,x^3-\sqrt{-7}(x^5+3\,x^3+4\,x)+12\,x\right)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-}{}$$

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/336*(43569*sqrt(2)*(3*x^5 + 9*x^3 - sqrt(-7)*(x^5 + 3*x^3 + 4*x) + 12*x)
*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/
8*sqrt(-7) + 1/8) - sqrt(2)*(109905*x^5 + 329715*x^3 - 50503*sqrt(-7)*(x^5
+ 3*x^3 + 4*x) + 439620*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)
*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 16*(4375*x^6 + 21000*x^4 + 52
216*x^2 + 43569)*sqrt(x^4 + 3*x^2 + 4))/(x^5 + 3*x^3 + 4*x)
```

Sympy [F]

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^4}{\left((x^2-x+2)\,(x^2+x+2)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**4/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.373 $\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 181

$$\begin{aligned}\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx &= -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{4449x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}\\ &\quad -\frac{4449(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}\\ &\quad +\frac{973(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] -1/28*x*(949*x^2+2323)/(x^4+3*x^2+4)^(1/2)+4449/28*x*(x^4+3*x^2+4)^(1/2)/(x
^2+2)-4449/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1
/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*
((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+973/8*(x^2+2)*(cos(2*ar
ctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*
arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)
/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{1219, 1211, 1117, 1209\}$

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{973(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{4449(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{4449\sqrt{x^4+3x^2+4}x}{28(x^2+2)} - \frac{(949x^2+2323)\,x}{28\sqrt{x^4+3x^2+4}}$$

```
[In] Int[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/28*(x*(2323 + 949*x^2))/Sqrt[4 + 3*x^2 + x^4] + (4449*x*Sqrt[4 + 3*x^2 +
 x^4])/(28*(2 + x^2)) - (4449*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) +
(973*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqr
t[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
```

```
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int\frac{4724+4449x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{4449}{14}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{973}{2}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{4449x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}\\
&\quad -\frac{4449(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad +\frac{973(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.81

$$\int\frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx=\frac{-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(2323+949x^2)-4449\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E}{}$$

```
[In] Integrate[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(2323 + 949*x^2) - 4449*Sqrt[2]*(3*I + Sq
rt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt
[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(3899*I + 4449*Sqrt
[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7
] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7]
)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{x(949x^2+2323)}{28\sqrt{x^4+3x^2+4}}+\frac{4724\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$ $35592\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-($
elliptic	$-\frac{2\left(\frac{949}{56}x^3+\frac{2323}{56}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{4724\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$ $35592\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-$
default	$-\frac{686\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{4724\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$ $35592\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-$

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/28*x*(949*x^2+2323)/(x^4+3*x^2+4)^(1/2)+4724/7/(-6+2*I*7^(1/2))^(1/2)*(1
-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*
x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2))-35592/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-
3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6
+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{4449\sqrt{2}\left(3x^5+9x^3-\sqrt{-7}(x^5+3x^3+4x)+12x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-2\sqrt{}}{}$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/112*(4449*sqrt(2)*(3*x^5 + 9*x^3 - sqrt(-7)*(x^5 + 3*x^3 + 4*x) + 12*x)*
sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8
*sqrt(-7) + 1/8) - 2*sqrt(2)*(8445*x^5 + 25335*x^3 - 1634*sqrt(-7)*(x^5 + 3
*x^3 + 4*x) + 33780*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqr
t(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 16*(875*x^4 + 2756*x^2 + 4449)*sq
rt(x^4 + 3*x^2 + 4))/(x^5 + 3*x^3 + 4*x)
```

Sympy [F]

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^3}{((x^2-x+2)\,(x^2+x+2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral((5*x**2 + 7)**3/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.374 $\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 181

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx = -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}} - \frac{113x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)} + \frac{113(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{9(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}$$

```
[Out] -1/28*x*(-113*x^2+9)/(x^4+3*x^2+4)^(1/2)-113/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+
2)+113/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x
*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^
4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+9/8*(x^2+2)*(cos(2*arctan(1
/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan
(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+
3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{1219,\ 1211,\ 1117,\ 1209\}$

$$\int \frac{\left(7+5x^2\right)^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx = \frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{113(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{113\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)} - \frac{\left(9-113x^2\right)x}{28\sqrt{x^4+3x^2+4}}$$

```
[In] Int[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/28*(x*(9 - 113*x^2))/Sqrt[4 + 3*x^2 + x^4] - (113*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (113*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Elli
pticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (9*(2
 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
```

```
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int\frac{352-113x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{9}{2}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{113}{14}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{113x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}\\
&\quad+\frac{113(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{9(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.82

$$\int\frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx=\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(-9+113x^2)+113\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\Big(i\text{arcsi}}{}$$

```
[In] Integrate[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(-9 + 113*x^2) + 113*Sqrt[2]*(3*I + Sqrt[7
])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
+ (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[
7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(1043*I + 113*Sqrt[7])*
Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (
2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])
]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sq
rt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.28

method	result
risch	$\frac{x(113x^2-9)}{28\sqrt{x^4+3x^2+4}}+\frac{352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{904\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{7}$
elliptic	$-\frac{2\left(-\frac{113}{56}x^3+\frac{9}{56}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{904\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(\right.}}{}$
default	$-\frac{98\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{904\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x*(113*x^2-9)/(x^4+3*x^2+4)^(1/2)+352/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/
8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+9
04/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8
*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.88

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{113\sqrt{2}\left(3x^4+9x^2-\sqrt{-7}(x^4+3x^2+4)+12\right)\sqrt{\sqrt{-7}-3}\,E\left(\arcsin\left(\frac{1}{4}\sqrt{2}x\sqrt{\sqrt{-7}-3}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)+}{}$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/448*(113*sqrt(2)*(3*x^4 + 9*x^2 - sqrt(-7)*(x^4 + 3*x^2 + 4) + 12)*sqrt(
sqrt(-7) - 3)*elliptic_e(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)), 3/8*sqrt
(-7) + 1/8) + 3*sqrt(2)*(239*x^4 + 717*x^2 + 155*sqrt(-7)*(x^4 + 3*x^2 + 4)
 + 956)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) -
3)), 3/8*sqrt(-7) + 1/8) - 16*sqrt(x^4 + 3*x^2 + 4)*(113*x^3 - 9*x))/(x^4 +
 3*x^2 + 4)
```

Sympy [F]

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx=\int \frac{(5x^2+7)^2}{((x^2-x+2)\,(x^2+x+2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral((5*x**2 + 7)**2/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx=\int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Giac [F]

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx=\int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx=\int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.375 $\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 181

$$\begin{aligned}\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x(53+19x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{19x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}\\ &+\frac{19(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}\\ &-\frac{3(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] 1/28*x*(19*x^2+53)/(x^4+3*x^2+4)^(1/2)-19/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+
19/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(
1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*
x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-3/8*(x^2+2)*(cos(2*arctan(1/2*x
*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2
*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^
2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{1192, 1211, 1117, 1209\}$

$$\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx = -\frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{19(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{19\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)} + \frac{(19x^2+53)\,x}{28\sqrt{x^4+3x^2+4}}$$

```
[In] Int[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (19*x*Sqrt[4 + 3*x^2 + x^4])
/(28*(2 + x^2)) + (19*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3*(2 +
x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8
])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(53+19x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int\frac{-4-19x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{x(53+19x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{19}{14}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{3}{2}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= \frac{x(53+19x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{19x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}+\frac{19(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad-\frac{3(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.82

$$\int\frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx=\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(53+19x^2)+19\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\Big(i\operatorname{arcsin}}{}$$

```
[In] Integrate[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(53 + 19*x^2) + 19*Sqrt[2]*(3*I + Sqrt[7])
*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] +
(2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(49*I + 19*Sqrt[7])*Sqrt[
(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*
x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x],
 (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4
+ 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.28

method	result
risch	$\frac{x\left(19x^2+53\right)}{28\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{152\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}}{7\sqrt{-}}$
elliptic	$-\frac{2\left(-\frac{19}{56}x^3-\frac{53}{56}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{152\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{7}$
default	$-\frac{14\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{152\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{7\sqrt{}}$

```
[In] int((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x*(19*x^2+53)/(x^4+3*x^2+4)^(1/2)-4/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+
1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^
(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+152
/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I
*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6
+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1
/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.88

$$\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx =$$

$$-\frac{19\sqrt{2}\left(3x^4+9x^2-\sqrt{-7}\left(x^4+3x^2+4\right)+12\right)\sqrt{\sqrt{-7}-3}\,E\left(\arcsin\left(\frac{1}{4}\sqrt{2}x\sqrt{\sqrt{-7}-3}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-3}{}$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/448*(19*sqrt(2)*(3*x^4 + 9*x^2 - sqrt(-7)*(x^4 + 3*x^2 + 4) + 12)*sqrt(s
qrt(-7) - 3)*elliptic_e(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)), 3/8*sqrt(
-7) + 1/8) - 3*sqrt(2)*(23*x^4 + 69*x^2 - 5*sqrt(-7)*(x^4 + 3*x^2 + 4) + 92
)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)), 3
/8*sqrt(-7) + 1/8) - 16*sqrt(x^4 + 3*x^2 + 4)*(19*x^3 + 53*x))/(x^4 + 3*x^2
 + 4)
```

Sympy [F]

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{5x^2+7}{((x^2-x+2)\,(x^2+x+2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral((5*x**2 + 7)/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Giac [F]

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.376 $\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 181

$$\begin{aligned}\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = & -\frac{x(1+3x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{3x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)} \\ & -\frac{3(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} \\ & +\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

```
[Out] -1/28*x*(3*x^2+1)/(x^4+3*x^2+4)^(1/2)+3/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-3/
28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2
)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2
+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+1/8*(x^2+2)*(cos(2*arctan(1/2*x*2^
(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*
2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4
)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{1106, 1211, 1117, 1209\}$

$$\int \frac{1}{\left(4+3x^2+x^4\right)^{3/2}}\,dx = \frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{3\sqrt{x^4+3x^2+4}x}{28\left(x^2+2\right)} - \frac{\left(3x^2+1\right)x}{28\sqrt{x^4+3x^2+4}}$$

```
[In] Int[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] -1/28*(x*(1 + 3*x^2))/Sqrt[4 + 3*x^2 + x^4] + (3*x*Sqrt[4 + 3*x^2 + x^4])/(
28*(2 + x^2)) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + ((2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4
*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

```
c/a]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x(1+3x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int\frac{8+3x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x(1+3x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{3}{14}\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{1}{2}\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x(1+3x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{3x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}-\frac{3(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.20 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.81

$$\int\frac{1}{(4+3x^2+x^4)^{3/2}}\,dx=\frac{-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(1+3x^2)-3\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\Big(i\operatorname{arcsinh}\Big(}{}$$

```
[In] Integrate[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1 + 3*x^2) - 3*Sqrt[2]*(3*I + Sqrt[7])*S
qrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2
*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]
*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqrt[(-3
*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2
)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3
*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3
*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{x(3x^2+1)}{28\sqrt{x^4+3x^2+4}}+\frac{8\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{24\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{7}$
default	$-\frac{2\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{8\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{24\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{7}$
elliptic	$-\frac{2\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{8\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{24\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{7}$

```
[In] int(1/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/28*x*(3*x^2+1)/(x^4+3*x^2+4)^(1/2)+8/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1
/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(
1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-24/7
/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7
^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2
*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2
))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.87

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx=\frac{3\sqrt{2}\left(3x^4+9x^2-\sqrt{-7}(x^4+3x^2+4)+12\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{1}{4}\sqrt{2}x\sqrt{\sqrt{-}\right.\right.}{}$$

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/448*(3*sqrt(2)*(3*x^4 + 9*x^2 - sqrt(-7)*(x^4 + 3*x^2 + 4) + 12)*sqrt(sqr
t(-7) - 3)*elliptic_e(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)), 3/8*sqrt(-7
) + 1/8) - sqrt(2)*(33*x^4 + 99*x^2 + 5*sqrt(-7)*(x^4 + 3*x^2 + 4) + 132)*s
qrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)), 3/8*
sqrt(-7) + 1/8) - 16*sqrt(x^4 + 3*x^2 + 4)*(3*x^3 + x))/(x^4 + 3*x^2 + 4)
```

Sympy [F]

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral((x**4 + 3*x**2 + 4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

```
[In] int(1/(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int(1/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.377 $\int \frac{1}{(7+5x^2)\left(4+3x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 284

$$
\begin{aligned}
\int \frac{1}{(7+5x^2)\left(4+3x^2+x^4\right)^{3/2}}\,dx = {} & -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{x\sqrt{4+3x^2+x^4}}{77\left(2+x^2\right)} \\
& + \frac{25}{176}\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) - \frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{77\sqrt{4+3x^2+x^4}} \\
& - \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{4+3x^2+x^4}} \\
& + \frac{425(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\mathrm{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3696\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

```
[Out] 25/13552*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/308*x*(4*
x^2+13)/(x^4+3*x^2+4)^(1/2)+1/77*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-1/24*(x^2+2)
*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipt
icF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/
2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+425/7392*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)
))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1
/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^
2+4)^(1/2)-1/77*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan
(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2
)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1235, 1192, 1211, 1117, 1209, 1230, 1720}

$$\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx = \frac{25}{176}\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right) - \frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{77\sqrt{x^4+3x^2+4}} + \frac{425(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3696\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{\sqrt{x^4+3x^2+4}x}{77(x^2+2)} - \frac{(4x^2+13)x}{308\sqrt{x^4+3x^2+4}}$$

```
[In] Int[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/308*(x*(13 + 4*x^2))/Sqrt[4 + 3*x^2 + x^4] + (x*Sqrt[4 + 3*x^2 + x^4])/(
77*(2 + x^2)) + (25*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^
4]])/176 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(77*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[
2]*Sqrt[4 + 3*x^2 + x^4]) + (425*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)
^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(3696*Sqrt[2]*Sqrt[4 + 3*
x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
```

```
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{44}\int \frac{-8-5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx+\frac{25}{44}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x\left(13+4x^2\right)}{308\sqrt{4+3x^2+x^4}}+\frac{\int \frac{-4+16x^2}{\sqrt{4+3x^2+x^4}}\,dx}{1232}\\
&\quad -\frac{25}{132}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{125}{66}\int \frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x\left(13+4x^2\right)}{308\sqrt{4+3x^2+x^4}}+\frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)\\
&\quad -\frac{25\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{264\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad +\frac{425\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3696\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad +\frac{1}{44}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx-\frac{2}{77}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x\left(13+4x^2\right)}{308\sqrt{4+3x^2+x^4}}+\frac{x\sqrt{4+3x^2+x^4}}{77\left(2+x^2\right)}+\frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)\\
&\quad -\frac{\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{77\sqrt{4+3x^2+x^4}}\\
&\quad -\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad +\frac{425\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3696\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.70

$$\int \frac{1}{(7+5x^2)\left(4+3x^2+x^4\right)^{3/2}}\,dx = \frac{-26\sqrt{-\frac{i}{-3i+\sqrt{7}}}x-8\sqrt{-\frac{i}{-3i+\sqrt{7}}}x^3-2\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}}{}$$

```
[In] Integrate[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-26*Sqrt[(-I)/(-3*I + Sqrt[7])]*x - 8*Sqrt[(-I)/(-3*I + Sqrt[7])]*x^3 - 2*
Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]
*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt
[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(7
*I + 2*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3
*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/
(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (25*I)*Sqrt[2]*Sqr
t[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(616*Sqrt[(-I
)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{x(4x^2+13)}{308\sqrt{x^4+3x^2+4}}-\frac{\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$
default	$-\frac{2\left(\frac{1}{154}x^3+\frac{13}{616}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$
elliptic	$-\frac{2\left(\frac{1}{154}x^3+\frac{13}{616}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/308*x*(4*x^2+13)/(x^4+3*x^2+4)^(1/2)-1/77/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/
8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-3
2/77/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8
*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+25/308/(-3/8+1/8*I*7^(1/2))^(1/2)*
(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^
4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7
^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)\left(4+3x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{{\left(x^{4} + 3\,x^{2} + 4\right)}^{\frac{3}{2}} {\left(5\,x^{2} + 7\right)}}\,{d x}$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^10 + 37*x^8 + 127*x^6 + 239*x^4 + 248*x
^2 + 112), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)\left(4+3x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(\left(x^{2} - x + 2\right)\left(x^{2} + x + 2\right)\right)^{\frac{3}{2}} \cdot \left(5x^{2} + 7\right)}\,dx$$

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)\left(4+3x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{{\left(x^{4} + 3\,x^{2} + 4\right)}^{\frac{3}{2}} {\left(5\,x^{2} + 7\right)}}\,{d x}$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)\left(4+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\left(4+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(5\,x^2+7\right)\,\left(x^4+3\,x^2+4\right)^{3/2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2)), x)
```

3.378 $\int \frac{1}{\left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 312

$$
\begin{aligned}
\int \frac{1}{\left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} - \frac{199x\sqrt{4+3x^2+x^4}}{27104\,(2+x^2)} \\
&+ \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)} + \frac{575\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{108416} \\
&+ \frac{199(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&- \frac{2\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{231\sqrt{4+3x^2+x^4}} \\
&+ \frac{9775(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2276736\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

```
[Out] 575/8348032*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/13552*
x*(37*x^2+24)/(x^4+3*x^2+4)^(1/2)-199/27104*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+6
25/27104*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+199/27104*(x^2+2)*(cos(2*arctan(1/
2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(
1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3
*x^2+4)^(1/2)+9775/4553472*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/c
os(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-2/
231*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/
2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1242, 1192, 1211, 1117, 1209, 1237, 1728, 1722, 1720, 1230}

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2\,(4+3x^2+x^4)^{3/2}}\,dx = {} & \frac{575\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{108416} \\ & - \frac{2\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{231\sqrt{x^4+3x^2+4}} \\ & + \frac{199(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\sqrt{x^4+3x^2+4}} \\ & + \frac{9775(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2276736\sqrt{2}\sqrt{x^4+3x^2+4}} \\ & - \frac{199\sqrt{x^4+3x^2+4}x}{27104\,(x^2+2)} + \frac{625\sqrt{x^4+3x^2+4}x}{27104\,(5x^2+7)} + \frac{(37x^2+24)\,x}{13552\sqrt{x^4+3x^2+4}}\end{aligned}$$

```
[In] Int[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 + 3*x^2 + x
^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(7 + 5*x^2))
+ (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/108416 +
 (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(13552*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2
31*Sqrt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
```

```
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{-36+5x^2}{1936\left(4+3x^2+x^4\right)^{3/2}} + \frac{25}{44\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}} - \frac{25}{1936\left(7+5x^2\right)\sqrt{4+3x^2+x^4}} \right) dx$$

$$= \frac{\int \frac{-36+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx}{1936} - \frac{25\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{1936} + \frac{25}{44}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)} + \frac{\int \frac{-348-148x^2}{\sqrt{4+3x^2+x^4}}\,dx}{54208}$$

$$- \frac{25\int \frac{12+70x^2+25x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{27104} + \frac{25\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{5808} - \frac{125\int \frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{2904}$$

$$= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)} - \frac{25\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{7744}$$

$$+ \frac{25(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{11616\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{425(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{162624\sqrt{2}\sqrt{4+3x^2+x^4}} - \frac{5\int \frac{410+425x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{27104}$$

$$+ \frac{37\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{6776} + \frac{125\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{13552} - \frac{23\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{1936}$$

$$= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} - \frac{199x\sqrt{4+3x^2+x^4}}{27104\,(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)}$$

$$- \frac{25\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{7744} + \frac{199(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{264\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{425(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{162624\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{25}{924}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{4625\int \frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{40656}$$

$$= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} - \frac{199x\sqrt{4+3x^2+x^4}}{27104\,(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)}$$

$$+ \frac{575\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{108416} + \frac{199(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$- \frac{2\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{231\sqrt{4+3x^2+x^4}}$$

$$+ \frac{9775(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2276736\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int \frac{1}{(7+5x^2)^2\,(4+3x^2+x^4)^{3/2}}\,dx = \frac{28x(2836+2633x^2+995x^4)+i\sqrt{6+2i\sqrt{7}}(7+5x^2)\,\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}}{}$$

```
[In] Integrate[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(2836 + 2633*x^2 + 995*x^4) + I*Sqrt[6 + (2*I)*Sqrt[7]]*(7 + 5*x^2)*S
qrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]
*(1393*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(101 + (199*I)*Sqrt[7])*EllipticF[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
- 1150*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(758912*(7 + 5*x^2)*Sqrt[4 +
3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

method	result
risch	$\frac{x(995x^4+2633x^2+2836)}{27104(5x^2+7)\sqrt{x^4+3x^2+4}}-\frac{349\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{6776\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+$
default	$\frac{625x\sqrt{x^4+3x^2+4}}{27104(5x^2+7)}-\frac{2\left(-\frac{37}{27104}x^3-\frac{3}{3388}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{349\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{6776\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+$
elliptic	$\frac{625x\sqrt{x^4+3x^2+4}}{27104(5x^2+7)}-\frac{2\left(-\frac{37}{27104}x^3-\frac{3}{3388}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{349\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{6776\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+$

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27104*x*(995*x^4+2633*x^2+2836)/(5*x^2+7)/(x^4+3*x^2+4)^(1/2)-349/6776/(-
6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1
/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),
1/4*(2+6*I*7^(1/2))^(1/2))+199/847/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^
(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3
```

```
+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+575/
189728/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/
8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7
^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+
1/8*I*7^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)^2\,(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^12 + 220*x^10 + 894*x^8 + 2084*x^6 + 2
913*x^4 + 2296*x^2 + 784), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)^2\,(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{\left((x^2-x+2)\,(x^2+x+2)\right)^{\frac{3}{2}}\,(5x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)^2\,(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)^2(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^2(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+7)^2\,(x^4+3\,x^2+4)^{3/2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2)), x)
```

3.379 $\int \frac{1}{\left(7+5x^2\right)^3\left(4+3x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 340

$$
\begin{aligned}
\int \frac{1}{\left(7+5x^2\right)^3\left(4+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} \\
&- \frac{18159x\sqrt{4+3x^2+x^4}}{33392128\,(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208\,(7+5x^2)^2} + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128\,(7+5x^2)} \\
&- \frac{529425\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{133568512} + \frac{18159(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{16696064\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&+ \frac{843(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&- \frac{3000075(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\mathrm{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{934979584\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

```
[Out] -529425/10284775424*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+
1/596288*x*(139*x^2+548)/(x^4+3*x^2+4)^(1/2)-18159/33392128*x*(x^4+3*x^2+4)
^(1/2)/(x^2+2)+625/54208*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+51875/33392128*x
*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+18159/33392128*(x^2+2)*(cos(2*arctan(1/2*x*2
^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x
*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+
4)^(1/2)+843/758912*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*ar
ctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((
x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-3000075/186995916
8*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1242, 1192, 1211, 1117, 1209, 1237, 1710, 1728, 1722, 1720, 1230}

$$\begin{aligned}
\int \frac{1}{(7+5x^2)^3(4+3x^2+x^4)^{3/2}}\,dx = & -\frac{529425\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{133568512} \\
& +\frac{843(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}} \\
& +\frac{18159(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{16696064\sqrt{2}\sqrt{x^4+3x^2+4}} \\
& -\frac{3000075(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{934979584\sqrt{2}\sqrt{x^4+3x^2+4}} \\
& -\frac{18159\sqrt{x^4+3x^2+4x}}{33392128\,(x^2+2)}+\frac{51875\sqrt{x^4+3x^2+4x}}{33392128\,(5x^2+7)} \\
& +\frac{625\sqrt{x^4+3x^2+4x}}{54208\,(5x^2+7)^2}+\frac{(139x^2+548)\,x}{596288\sqrt{x^4+3x^2+4}}
\end{aligned}$$

```
[In] Int[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqrt[4 + 3*x^
2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(54208*(7 +
5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(7 + 5*x^2)) - (52942
5*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/133568512 + (
18159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3794
56*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^
4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(934979584*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
```

```
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
```

```
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
```

```
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{388+215x^2}{85184\left(4+3x^2+x^4\right)^{3/2}}+\frac{25}{44\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\right.\\
&\qquad\left.-\frac{25}{1936\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}-\frac{1075}{85184\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\right)dx\\
&=\frac{\int \frac{388+215x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx}{85184}-\frac{1075\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{85184}\\
&\quad-\frac{25\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx}{1936}+\frac{25}{44}\int \frac{1}{\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\,dx\\
&=\frac{x\left(548+139x^2\right)}{596288\sqrt{4+3x^2+x^4}}+\frac{625x\sqrt{4+3x^2+x^4}}{54208\left(7+5x^2\right)^2}-\frac{625x\sqrt{4+3x^2+x^4}}{1192576\left(7+5x^2\right)}\\
&\quad+\frac{\int \frac{524-556x^2}{\sqrt{4+3x^2+x^4}}\,dx}{2385152}+\frac{25\int \frac{12+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{1192576}-\frac{25\int \frac{-76-10x^2-25x^4}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx}{54208}\\
&\quad+\frac{1075\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{255552}-\frac{5375\int \frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{127776}\\
&=\frac{x\left(548+139x^2\right)}{596288\sqrt{4+3x^2+x^4}}+\frac{625x\sqrt{4+3x^2+x^4}}{54208\left(7+5x^2\right)^2}+\frac{51875x\sqrt{4+3x^2+x^4}}{33392128\left(7+5x^2\right)}\\
&\quad-\frac{1075\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{340736}+\frac{1075\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{511104\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad-\frac{18275\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{7155456\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{25\int \frac{-4412-4690x^2-2775x^4}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{33392128}+\frac{5\int \frac{410+425x^2}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{1192576}\\
&\quad-\frac{125\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{596288}-\frac{21\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{85184}+\frac{139\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{298144}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{153x\sqrt{4+3x^2+x^4}}{1192576\,(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208\,(7+5x^2)^2} \\
&\quad + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128\,(7+5x^2)} - \frac{1075\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{340736} \\
&\quad + \frac{153(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{596288\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{23(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{11616\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{18275(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{7155456\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{5\int\frac{-60910-31775x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{33392128} \\
&\quad + \frac{25\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{40656} + \frac{13875\int\frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx}{16696064} - \frac{4625\int\frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{1788864} \\
&= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128\,(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208\,(7+5x^2)^2} \\
&\quad + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128\,(7+5x^2)} - \frac{15975\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{4770304} \\
&\quad + \frac{18159(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{16696064\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{31(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{90525(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33392128\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{25\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx}{189728} - \frac{136875\int\frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{16696064}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128\,(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208\,(7+5x^2)^2} \\
&\quad + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128\,(7+5x^2)} - \frac{529425\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{133568512} \\
&\quad + \frac{18159(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{16696064\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad + \frac{843(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad - \frac{3000075(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{934979584\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.68 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.94

$$
\int \frac{1}{(7+5x^2)^3\,(4+3x^2+x^4)^{3/2}}\,dx = \frac{28x(4496212+5811451x^2+2838330x^4+453975x^6)+3i\sqrt{6+2i\sqrt{7}}(7}{}
$$

```
[In] Integrate[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(4496212 + 5811451*x^2 + 2838330*x^4 + 453975*x^6) + (3*I)*Sqrt[6 + (
2*I)*Sqrt[7]]*(7 + 5*x^2)^2*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 +
 ((2*I)*x^2)/(3*I + Sqrt[7])]*(42371*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sq
rt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I)*(2
3633*I + 6053*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 352950*EllipticPi[(5*(3 + I*Sqrt[7]))/
14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt
[7])]))/(934979584*(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.05

method	result
risch	$\frac{x(453975x^6+2838330x^4+5811451x^2+4496212)}{33392128(5x^2+7)^2\sqrt{x^4+3x^2+4}}+\frac{1173\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{1192576\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}+$
default	$\frac{625x\sqrt{x^4+3x^2+4}}{54208(5x^2+7)^2}+\frac{51875x\sqrt{x^4+3x^2+4}}{33392128(5x^2+7)}-\frac{2(-\frac{139}{1192576}x^3-\frac{137}{298144}x)}{\sqrt{x^4+3x^2+4}}+\frac{1173\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F}{1192576\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{625x\sqrt{x^4+3x^2+4}}{54208(5x^2+7)^2}+\frac{51875x\sqrt{x^4+3x^2+4}}{33392128(5x^2+7)}-\frac{2(-\frac{139}{1192576}x^3-\frac{137}{298144}x)}{\sqrt{x^4+3x^2+4}}+\frac{1173\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F}{1192576\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/33392128*x*(453975*x^6+2838330*x^4+5811451*x^2+4496212)/(5*x^2+7)^2/(x^4+
3*x^2+4)^(1/2)+1173/1192576/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*
x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF
(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+18159/1043504/(-6+
2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2
))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1
/2),1/4*(2+6*I*7^(1/2))^(1/2)))-529425/233744896/(-3/8+1/8*I*7^(1/2))^(1/2)
*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x
^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*
7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(7+5x^2)^3\,(4+3x^2+x^4)^{3/2}}\,dx=\int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^14 + 1275*x^12 + 6010*x^10 + 16678*x^
8 + 29153*x^6 + 31871*x^4 + 19992*x^2 + 5488), x)
```

Sympy [F]

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}} (5x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

Maxima [F]

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

Giac [F]

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)^3}\,dx$$

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+7)^3\,(x^4+3\,x^2+4)^{3/2}}\,dx$$

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2)), x)
```

3.380 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 467

$$\begin{aligned}\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}\\ &+\frac{e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}\\ &-\frac{\sqrt[4]{a}e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right.}{15c^{11/4}\sqrt{a+bx^2+cx^4}}\\ &+\frac{\sqrt[4]{a}\left(\frac{\sqrt{c}(15c^2d^3-15acde^2+4abe^3)}{\sqrt{a}}+e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{Elliptic}}{30c^{11/4}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

```
[Out] 1/15*e^2*(-4*b*e+15*c*d)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*e^3*x^3*(c*x^4+b*x
^2+a)^(1/2)/c+1/15*e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))*x*(c*x^4+b
*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))-1/15*a^(1/4)*e*(45*c^2*d^2+8*b^
2*e^2-3*c*e*(3*a*e+10*b*d))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/
2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*a^(1/4)*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(
1/2)+x^2*c^(1/2))*(e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))+(4*a*b*e^3
-15*a*c*d*e^2+15*c^2*d^3)*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^
(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1220, 1693, 1211, 1117, 1209}

$$\begin{aligned}
&\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \\
&-\frac{\sqrt[4]{a}e(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2)\,E\Big(2\arctan\Big(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\Big)|\frac{1}{4}\Big(2-\frac{b}{\sqrt{a}\sqrt{c}}\Big)}{15c^{11/4}\sqrt{a+bx^2+cx^4}} \\
&+\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Big(e(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2)+\frac{\sqrt{c}(4abe^3-15acde^2+15c^2d^3)}{\sqrt{a}}\Big)\,\text{Elliptic}}{30c^{11/4}\sqrt{a+bx^2+cx^4}} \\
&+\frac{ex\sqrt{a+bx^2+cx^4}(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&+\frac{e^2x\sqrt{a+bx^2+cx^4}(15cd-4be)}{15c^2}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}
\end{aligned}$$

```
[In] Int[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e^2*(15*c*d - 4*b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (e^3*x^3*Sqrt[a
 + b*x^2 + c*x^4])/(5*c) + (e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a
*e))*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(
1/4)*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a +
 b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^
3))/Sqrt[a] + e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4
)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
```

```
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}+\frac{\int \frac{5cd^3+3e\left(5cd^2-ae^2\right)x^2+e^2(15cd-4be)x^4}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}\\
&= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}\\
&\quad+\frac{\int \frac{15c^2d^3-15acde^2+4abe^3+e\left(45c^2d^2+8b^2e^2-3ce(10bd+3ae)\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^2}
\end{aligned}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$-\frac{\left(\sqrt{a}e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{5/2}}$$

$$+\frac{\left(15c^2d^3-15acde^2+4abe^3+\frac{\sqrt{a}e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))}{\sqrt{c}}\right)\int\frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^2}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$+\frac{e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{\sqrt[4]{a}e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\Bigl(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\bigl(2-}{15c^{11/4}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\left(15c^2d^3-15acde^2+4abe^3+\frac{\sqrt{a}e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\Bigl(2\tan}{30\sqrt[4]{a}c^{9/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.96 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.25

$$\int\frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$=\frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}e^2x(a+bx^2+cx^4)\left(-4be+3c(5d+ex^2)\right)+i\left(-b+\sqrt{b^2-4ac}\right)e(45c^2d^2+8b^2e^2-3ce(10bd}{}$$

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4)*(-4*b*e + 3*
c*(5*d + e*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*e*(45*c^2*d^2 + 8*b^2*e^2 - 3
*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(30*c^3*d^3 + 8*b^2*(-b + Sq
rt[b^2 - 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2 - 4*a*c]*d - 2*a*e)
+ c*e^2*(30*b^2*d - 30*b*Sqrt[b^2 - 4*a*c]*d + 17*a*b*e - 9*a*Sqrt[b^2 - 4*
a*c]*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
```

```
c])/(b - Sqrt[b^2 - 4*a*c])]])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 7.42 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.04

method	result
elliptic	$\frac{e^3x^3\sqrt{cx^4+bx^2+a}}{5c}+\frac{\left(3de^2-\frac{4e^3b}{5c}\right)x\sqrt{cx^4+bx^2+a}}{3c}+\frac{\left(d^3-\frac{\left(3de^2-\frac{4e^3b}{5c}\right)a}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b\right.}}{a}}}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4}}$
risch	$-\frac{e^2x\left(-3cx^2e+4be-15cd\right)\sqrt{cx^4+bx^2+a}}{15c^2}+\frac{\frac{15c^2d^3\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}}{}$
default	Expression too large to display

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e^3*x^3*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(3*d*e^2-4/5*e^3/c*b)/c*x*(c*x^4+b*
x^2+a)^(1/2)+1/4*(d^3-1/3*(3*d*e^2-4/5*e^3/c*b)/c*a)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-1/2*(3*d^2*e-3/5*e^3/c*a-2/3*(3*d*e^2-4/5*e^3/c*b)/c*b)*a*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+
2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)
^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.21

$$\int\frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$=\frac{\sqrt{\frac{1}{2}}\Big(\left(\left(45\,ac^3d^2e-30\,abc^2de^2+\left(8\,ab^2c-9\,a^2c^2\right)e^3\right)x\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(45\,abc^2d^2e-30\,ab^2cde^2+\left(8\,ab^3-9\,a^2b\right.\right.\right.}{}$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(sqrt(1/2)*((45*a*c^3*d^2*e - 30*a*b*c^2*d*e^2 + (8*a*b^2*c - 9*a^2*c^
2)*e^3)*x*sqrt((b^2 - 4*a*c)/c^2) - (45*a*b*c^2*d^2*e - 30*a*b^2*c*d*e^2 +
(8*a*b^3 - 9*a^2*b*c)*e^3)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/
c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x),
1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((15*c^4
*d^3 - 45*a*c^3*d^2*e + 15*(2*a*b*c^2 - a*c^3)*d*e^2 - (8*a*b^2*c - (9*a^2
+ 4*a*b)*c^2)*e^3)*x*sqrt((b^2 - 4*a*c)/c^2) + (15*b*c^3*d^3 + 45*a*b*c^2*d
^2*e - 15*(2*a*b^2*c + a*b*c^2)*d*e^2 + (8*a*b^3 - (9*a^2*b - 4*a*b^2)*c)*e
^3)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sq
rt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*
a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(3*a*c^3*e^3*x^4 + 45*a*c^3*d^2*e - 30*
a*b*c^2*d*e^2 + (8*a*b^2*c - 9*a^2*c^2)*e^3 + (15*a*c^3*d*e^2 - 4*a*b*c^2*e
^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*c^4*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{{\left(e\,x^2+d\right)}^3}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.381 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 356

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c} + \frac{2e(3cd-be)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2\sqrt[4]{a}e(3cd-be)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(2e(3cd-be)+\frac{\sqrt{c}(3cd^2-ae^2)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/3*e^2*x*(c*x^4+b*x^2+a)^(1/2)/c+2/3*e*(-b*e+3*c*d)*x*(c*x^4+b*x^2+a)^(1/2
)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-2/3*a^(1/4)*e*(-b*e+3*c*d)*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^
2+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b
/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*e*(-b*e+3*c*d)+(-a*e^2+3*
c*d^2)*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(
7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1220, 1211, 1117, 1209}

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}(3cd^2-ae^2)}{\sqrt{a}}+2e(3cd-be)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{2\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(3cd-be)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{2ex\sqrt{a+bx^2+cx^4}(3cd-be)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}$$

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (2*e*(3*c*d - b*e)*x*Sqrt[a + b*x^2
 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*e*(3*c*d - b*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3
*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*e*(3*c*d - b*e) + (Sqrt[c]*
(3*c*d^2 - a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\text{integral} = \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c} + \frac{\int \frac{3cd^2-ae^2+2e(3cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c} - \frac{\left(2\sqrt{a}e(3cd-be)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} + \frac{\left(3cd^2-ae^2+\frac{2\sqrt{a}e(3cd-be)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c} + \frac{2e(3cd-be)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2\sqrt[4]{a}e(3cd-be)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(3cd^2-ae^2+\frac{2\sqrt{a}e(3cd-be)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.10 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}e^2x(a+bx^2+cx^4)-i\left(-b+\sqrt{b^2-4ac}\right)e(-3cd+be)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E}{}$$

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4) - I*(-b + Sq
rt[b^2 - 4*a*c])*e*(-3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-3*c^2*d^2 + b*(
-b + Sqrt[b^2 - 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 - 4*a*c]*d + a*e))*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2
*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
c*x^4])
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{e^{2} x \sqrt{c\,x^{4}+b\,x^{2}+a}}{3 c}+\frac{\left(d^{2}-\frac{a\,e^{2}}{3 c}\right) \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4+\frac{2 b \left(b+\sqrt{-4 a c}\right.}{a c}}}{2}\right.}{4 \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}\, \sqrt{c\,x^{4}+b\,x^{2}+a}}$
risch	$\frac{e^{2} x \sqrt{c\,x^{4}+b\,x^{2}+a}}{3 c}-\frac{\frac{a\,e^{2} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4+\frac{2 b \left(b+\sqrt{-4 a c+b^{2}}\right)}{a c}}}{2}\right)}{4 \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}\, \sqrt{c\,x^{4}+b\,x^{2}+a}}\quad 3 -}{}$
default	$\frac{d^{2} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4+\frac{2 b \left(b+\sqrt{-4 a c+b^{2}}\right)}{a c}}}{2}\right)}{4 \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}\, \sqrt{c\,x^{4}+b\,x^{2}+a}}+e^{2} \left(\frac{x \sqrt{c\,x^{4}+b\,x^{2}+}}{3 c}\right.$

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*x*(c*x^4+b*x^2+a)^(1/2)/c+1/4*(d^2-1/3*a/c*e^2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))-1/2*(2*e*d-2/3*b/c*e^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{\frac{1}{2}}\left(\left(3\,ac^2de-abce^2\right)x\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(3\,abcde-ab^2e^2\right)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid \frac{bc}{}\right.}{}$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(1/2)*((3*a*c^2*d*e - a*b*c*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (3*
a*b*c*d*e - a*b^2*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*e
lliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*
(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((3*c^3*d^2
- 6*a*c^2*d*e + (2*a*b*c - a*c^2)*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) + (3*b*c^2
*d^2 + 6*a*b*c*d*e - (2*a*b^2 + a*b*c)*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 -
4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/
c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2
*(a*c^2*e^2*x^2 + 6*a*c^2*d*e - 2*a*b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(a*c^
3*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{{\left(e\,x^2+d\right)}^2}{\sqrt{c\,x^4+b\,x^2+a}} \,d x$$

```
[In] int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.382 $\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 283

$$\begin{aligned}
\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx
&= \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

```
[Out] e*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))
*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1211, 1117, 1209}

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}} \\ &\quad - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} \\ &\quad + \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}\end{aligned}$$

```
[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\left(\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}}+\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx\\ &= \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}\\ &\quad+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\left(-b+\sqrt{b^2-4ac}\right)eE\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(-}{2\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+}}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.28

method	result
default	$\frac{d\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}-\frac{ea\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-\right.}{}}}{}$
elliptic	$\frac{d\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}-\frac{ea\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-\right.}{}}}{}$

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*e*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.05

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{cx^4+bx^2+a}\,ace+\sqrt{\frac{1}{2}}\left(acex\sqrt{\frac{b^2-4\,ac}{c^2}}-abex\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,a}{c^2}}}{2}\right.}{}$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(c*x^4 + b*x^2 + a)*a*c*e + sqrt(1/2)*(a*c*e*x*sqrt((b^2 - 4*a*c
)/c^2) - a*b*e*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_
e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqr
t((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((c^2*d - a*c*e)*x*s
```

```
qrt((b^2 - 4*a*c)/c^2) + (b*c*d + a*b*e)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a
*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2
) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)))/(a*c^
2*x)
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((d + e*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d + e*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.383 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 401

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{e}\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2-bde+ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}} - \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/2*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2
))*e^(1/2)/d^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(
1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d
*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)-1/4*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4
)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2-b/a^
(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c*x^4
+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(c*x^4+b*
x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1230, 1117, 1720}

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx =$$

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2 \text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de}, 2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\sqrt{a+bx^2+cx^4}\,(cd^2-ae^2)}$$

$$+\frac{\sqrt{e}\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{ae^2-bde+cd^2}}$$

$$+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b
*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[c*d^2 - b*d*e + a*e^2]) + (c^(1/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)
*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/S
qrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d
*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4
)*d*(c*d^2 - a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e} - \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e}\\
&= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2-bde+ae^2}}\\
&\quad + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}\\
&\quad - \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.53

$$
\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticPi}\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd}, i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right), \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d\sqrt{a+bx^2+cx^4}}
$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{1+\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\Pi\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},-\frac{2ae}{\left(-b+\sqrt{-4ac+b^{2}}\right)d},\frac{\sqrt{-\frac{b+\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{2}}{\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	200
elliptic	$\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{1+\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\Pi\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},-\frac{2ae}{\left(-b+\sqrt{-4ac+b^{2}}\right)d},\frac{\sqrt{-\frac{b+\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{2}}{\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	200

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-
4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/
(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^{2})\sqrt{a+bx^{2}+cx^{4}}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^{2})\sqrt{a+bx^{2}+cx^{4}}}\,dx=\int \frac{1}{(d+ex^{2})\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.384 $\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 718

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx=-\frac{\sqrt{c}ex\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{\sqrt{e}(3cd^2-e(2bd-ae))\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\left(cd^2-bde+ae^2\right)^{3/2}}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(3cd^2-e(2bd-ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/4*(3*c*d^2-e*(-a*e+2*b*d))*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(
1/2)/(c*x^4+b*x^2+a)^(1/2))*e^(1/2)/d^(3/2)/(a*e^2-b*d*e+c*d^2)^(3/2)+1/2*e
^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/2*e*x*c^(1/2)*
(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(a^(1/2)+x^2*c^(1/2))+1/2*a^(1/
4)*c^(1/4)*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
```

```
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(-
e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)-1/8*(3*c*d^2-e*(-a*e+2*b*d))*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e
/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/
c^(1/4)/d^2/(a*e^2-b*d*e+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2
)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1237, 1728, 1209, 1722, 1117, 1720}

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\left(3cd^2-e(2bd-ae)\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2-bde+cd^2\right)}$$

$$+\frac{\sqrt{e}\left(3cd^2-e(2bd-ae)\right)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\left(ae^2-bde+cd^2\right)^{3/2}}$$

$$+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$$+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}ex\sqrt{a+bx^2+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2-bde+cd^2\right)}$$

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(Sqrt[c]*e*x*Sqrt[a + b*x^2 + c*x^4])/(d*(c*d^2 - b*d*e + a*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x^2)) + (Sqrt[e]*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*
d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3
/2)*(c*d^2 - b*d*e + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e +
a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
```

```
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e
)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 - e*(2*b*d -
 a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(1/4)*c^(1/
4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}-\frac{\int \frac{-2cd^2+e(2bd-ae)+2cdex^2+ce^2x^4}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{2d\left(cd^2-bde+ae^2\right)}\\
&= \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}\\
&\quad -\frac{\int \frac{\sqrt{a}c^{3/2}de^2+ce\left(-2cd^2+e(2bd-ae)\right)+\left(2c^2de^2-ce^2\left(cd-\sqrt{a}\sqrt{c}e\right)\right)x^2}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{2cde\left(cd^2-bde+ae^2\right)}\\
&\quad +\frac{\left(\sqrt{a}\sqrt{c}e\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{2d\left(cd^2-bde+ae^2\right)}\\
&= -\frac{\sqrt{c}ex\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}\\
&\quad +\frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}\\
&\quad +\frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{d\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}e\left(3cd^2-e(2bd-ae)\right)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{2d\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)}
\end{aligned}$$

$$= -\frac{\sqrt{c}ex\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}$$
$$+ \frac{\sqrt{e}(3cd^2-e(2bd-ae))\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\left(cd^2-bde+ae^2\right)^{3/2}}$$
$$+ \frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}$$
$$+ \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}$$
$$- \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(3cd^2-e(2bd-ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.51 (sec) , antiderivative size = 1069, normalized size of antiderivative = 1.49

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de^2x(a+bx^2+cx^4)+i\sqrt{2}\left(b-\sqrt{b^2-4ac}\right)de\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}(d+ex^2)\left(E\right.}{}$$

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*x*(a + b*x^2 + c*x^4) + I*Sqrt[2]*
(b - Sqrt[b^2 - 4*a*c])*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*(E
llipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
]) + (2*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (6*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] + (4*I)*Sqrt[2]*b*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c]
```

```
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])] - (2*I)*Sqrt[2]*a*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(
d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])])/(8*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*(c*d^3 + d*e*(-(b*d) + a*
e))*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 1279, normalized size of antiderivative = 1.78

method	result	size
default	Expression too large to display	1279
elliptic	Expression too large to display	1279

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/8*c/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x
^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/d/(a*e^2-b*d
*e+c*d^2)*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/
a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/
(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
1/4*c*e/d/(a*e^2-b*d*e+c*d^2)*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)
*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*
c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*(-4*a
*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+
1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellip
ticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(
1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2))*a-1/(a*e^2-b*d*e+c*d^2)*e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2
)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*
x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*
a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2))*b+3/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))
^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/
```

```
2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-
1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2))*c
```

Fricas [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c*e^2*x^8 + (2*c*d*e + b*e^2)*x^6 + (c*d^
2 + 2*b*d*e + a*e^2)*x^4 + a*d^2 + (b*d^2 + 2*a*d*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{\left(e\,x^2+d\right)^2\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.385 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 553

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

$$-\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e(45c^2d^2+8b^2e^2+3ce(10bd+3ae))\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$

$$+\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\left(\frac{2c(15c^2d^3+15acde^2+4abe^3)}{b-\sqrt{b^2+4ac}}+e(45c^2d^2+8b^2e^2+3ce(10bd+3ae))\right)\sqrt{1-}}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -1/15*e^2*(4*b*e+15*c*d)*x*(-c*x^4+b*x^2+a)^(1/2)/c^2-1/5*e^3*x^3*(-c*x^4+b
*x^2+a)^(1/2)/c-1/60*e*(45*c^2*d^2+8*b^2*e^2+3*c*e*(3*a*e+10*b*d))*Elliptic
E(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(
4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(
1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^
(1/2)/c^(7/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/60*EllipticF(x*2^(1/2)*c^(1/
2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2))
)^(1/2))*(e*(45*c^2*d^2+8*b^2*e^2+3*c*e*(3*a*e+10*b*d))+2*c*(4*a*b*e^3+15*a
*c*d*e^2+15*c^2*d^3)/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^
2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4
*a*c+b^2)^(1/2)))^(1/2)/c^(7/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1220, 1693, 1216, 538, 435, 430}

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}(3ce(3ae+10bd)+8b^2e^2+45c^2d^2)\,E}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$

$$+\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{2c(4abe^3+15acde^2+15c^2d^3)}{b-\sqrt{4ac+b^2}}+e(3ce(3ae+1\right.}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$

$$-\frac{e^2x\sqrt{a+bx^2-cx^4}(4be+15cd)}{15c^2}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

```
[In] Int[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/15*(e^2*(15*c*d + 4*b*e)*x*Sqrt[a + b*x^2 - c*x^4])/c^2 - (e^3*x^3*Sqrt[
a + b*x^2 - c*x^4])/(5*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 -
c*x^4]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*((2*c*(15*c^
2*d^3 + 15*a*c*d*e^2 + 4*a*b*e^3))/(b - Sqrt[b^2 + 4*a*c]) + e*(45*c^2*d^2
+ 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e)))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sq
rt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}-\frac{\int \frac{-5cd^3-3e\left(5cd^2+ae^2\right)x^2-e^2(15cd+4be)x^4}{\sqrt{a+bx^2-cx^4}}\,dx}{5c}\\ &= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}\\ &\quad+\frac{\int \frac{15c^2d^3+15acde^2+4abe^3+e\left(45c^2d^2+8b^2e^2+3ce(10bd+3ae)\right)x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{15c^2}\end{aligned}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} + \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{15c^2d^3+15acde^2+4abe^3+e\left(45c^2d^2+8b^2e^2+3ce(10bd+3ae)\right)x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{15c^2\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right)e(45c^2d^2+8b^2e^2+3ce(10bd+3ae))\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\sqrt{1-}}{\sqrt{1-}}}{30c^3\sqrt{a+bx^2-cx^4}} + \frac{\left(\left(b-\sqrt{b^2+4ac}\right)\left(\frac{2c(15c^2d^3+15acde^2+4abe^3)}{b-\sqrt{b^2+4ac}}+e(45c^2d^2+8b^2e^2+3ce(10bd+3ae))\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4}}}}{30c^3\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e(45c^2d^2+8b^2e^2+3ce(10bd+3ae))\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2}{b+\sqrt{}}}}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\left(\frac{2c(15c^2d^3+15acde^2+4abe^3)}{b-\sqrt{b^2+4ac}}+e(45c^2d^2+8b^2e^2+3ce(10bd+3ae))\right)}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.72 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{-4c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}e^2x(a+bx^2-cx^4)\left(4be+3c(5d+ex^2)\right)-i\sqrt{2}\left(-b+\sqrt{b^2+4ac}\right)e(45c^2d^2+8b^2e^2+3ce}{}$$

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-4*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(a + b*x^2 - c*x^4)*(4*b*e +
 3*c*(5*d + e*x^2)) - I*Sqrt[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(45*c^2*d^2 + 8*
b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b
 + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b
^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))
]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-30*c^3
```

```
*d^3 + 8*b^2*(-b + Sqrt[b^2 + 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2
 + 4*a*c]*d - 2*a*e) + c*e^2*(-30*b^2*d + 30*b*Sqrt[b^2 + 4*a*c]*d - 17*a*b
*e + 9*a*Sqrt[b^2 + 4*a*c]*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2
+ 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x
], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(60*c^3*Sqrt[-(c/(b +
Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] (verified)

Time = 6.41 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.88

method	result
elliptic	$-\frac{e^3x^3\sqrt{-cx^4+bx^2+a}}{5c}-\frac{\left(3de^2+\frac{4e^3b}{5c}\right)x\sqrt{-cx^4+bx^2+a}}{3c}+\frac{\left(d^3+\frac{a\left(3de^2+\frac{4e^3b}{5c}\right)}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{4a}\right.}{a}}}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-}}$
risch	$-\frac{e^2x\left(3cx^2e+4be+15cd\right)\sqrt{-cx^4+bx^2+a}}{15c^2}+\frac{\frac{15c^2d^3\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\sqrt{-4}\right.}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-cx^4+bx^2+a}}}{}$
default	Expression too large to display

```
[In] int((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*e^3*x^3*(-c*x^4+b*x^2+a)^(1/2)/c-1/3*(3*d*e^2+4/5*e^3/c*b)/c*x*(-c*x^4
+b*x^2+a)^(1/2)+1/4*(d^3+1/3*a/c*(3*d*e^2+4/5*e^3/c*b))*2^(1/2)/((-b+(4*a*c
+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*
((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/
2))-1/2*(3*d^2*e+3/5*e^3/c*a+2/3*b/c*(3*d*e^2+4/5*e^3/c*b))*a*2^(1/2)/((-b+
(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(
b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/
2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(
b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.04

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left((45\,ac^3d^2e+30\,abc^2de^2+(8\,ab^2c+9\,a^2c^2)e^3)\sqrt{-c}x\sqrt{\frac{b^2+4\,ac}{c^2}}+(45\,abc^2d^2e+30\,ab^2cde^2+(8\,ab^3+\right.}{}$$

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(sqrt(1/2)*((45*a*c^3*d^2*e + 30*a*b*c^2*d*e^2 + (8*a*b^2*c + 9*a^2*c
^2)*e^3)*sqrt(-c)*x*sqrt((b^2 + 4*a*c)/c^2) + (45*a*b*c^2*d^2*e + 30*a*b^2*
c*d*e^2 + (8*a*b^3 + 9*a^2*b*c)*e^3)*sqrt(-c)*x)*sqrt((c*sqrt((b^2 + 4*a*c)
/c^2) + b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) +
 b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 - 2*a*c)/(a*c)) - sqrt(1/
2)*((15*c^4*d^3 + 45*a*c^3*d^2*e + 15*(2*a*b*c^2 + a*c^3)*d*e^2 + (8*a*b^2*
c + (9*a^2 + 4*a*b)*c^2)*e^3)*sqrt(-c)*x*sqrt((b^2 + 4*a*c)/c^2) - (15*b*c^
3*d^3 - 45*a*b*c^2*d^2*e - 15*(2*a*b^2*c - a*b*c^2)*d*e^2 - (8*a*b^3 + (9*a
^2*b - 4*a*b^2)*c)*e^3)*sqrt(-c)*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)
*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x), 1/
2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 - 2*a*c)/(a*c)) + 2*(3*a*c^3*e^3*x^4 +
 45*a*c^3*d^2*e + 30*a*b*c^2*d*e^2 + (8*a*b^2*c + 9*a^2*c^2)*e^3 + (15*a*c^
3*d*e^2 + 4*a*b*c^2*e^3)*x^2)*sqrt(-c*x^4 + b*x^2 + a))/(a*c^4*x)
```

Sympy [F]

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{(ex^2+d)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{(ex^2+d)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{(e\,x^2+d)^3}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((d + e*x^2)^3/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^3/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.386 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 454

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$
$$-\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\right|}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$
$$+\frac{\sqrt{b+\sqrt{b^2+4ac}}\left(3c^2d^2+b\left(b-\sqrt{b^2+4ac}\right)e^2+ce\left(3bd-3\sqrt{b^2+4ac}d+ae\right)\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-}}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -1/3*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/c-1/6*e*(b*e+3*c*d)*EllipticE(x*2^(1/2)*c
^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1
/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*
(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)
*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/6*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b
^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(3*c^
2*d^2+b*e^2*(b-(4*a*c+b^2)^(1/2))+c*e*(3*b*d+a*e-3*d*(4*a*c+b^2)^(1/2)))*(1
-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^
2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1220, 1216, 538, 435, 430}

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}(be+3cd)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b}{b}}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

$$+\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

$$-\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/3*(e^2*x*Sqrt[a + b*x^2 - c*x^4])/c - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b +
Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c
])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b
^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt
[b^2 + 4*a*c]]*(3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*
Sqrt[b^2 + 4*a*c]*d + a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqr
t[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*
a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\text{integral} = -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\int \frac{-3cd^2-ae^2-2e(3cd+be)x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3c}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{-3cd^2-ae^2-2e(3cd+be)x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{3c\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

$$-\frac{\left(\left(b-\sqrt{b^2+4ac}\right)e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{3c^2\sqrt{a+bx^2-cx^4}}$$

$$+\frac{\left(\left(b-\sqrt{b^2+4ac}\right)\left(2e(3cd+be)-\frac{2c\left(-3cd^2-ae^2\right)}{b-\sqrt{b^2+4ac}}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4a}}}}}{6c^2\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{b^2+}}}\right.\right.}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\left(3c^2d^2+b\left(b-\sqrt{b^2+4ac}\right)e^2+ce\left(3bd-3\sqrt{b^2+4ac}d+ae\right)\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.91 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

$$= \frac{2c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}e^2x(-a-bx^2+cx^4) - i\sqrt{2}\left(-b+\sqrt{b^2+4ac}\right)e(3cd+be)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2}{-b+\sqrt{b^2+4ac}}}}{}$$

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(-a - b*x^2 + c*x^4) - I*Sqrt
[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 + 4*a*c] -
2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(
-b + Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2
 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[
2]*(-3*c^2*d^2 + b*(-b + Sqrt[b^2 + 4*a*c])*e^2 - c*e*(3*b*d - 3*Sqrt[b^2 +
 4*a*c]*d + a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*
a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a
*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.91

method	result
elliptic	$-\frac{e^{2} x \sqrt{-c\,x^{4}+b\,x^{2}+a}}{3 c}+\frac{\left(d^{2}+\frac{a\,e^{2}}{3 c}\right) \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4-\frac{2 b \left(b+\sqrt{4 a c+b}\right.}{a c}}}{2}\right.}{4 \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}\, \sqrt{-c\,x^{4}+b\,x^{2}+a}}$
risch	$-\frac{e^{2} x \sqrt{-c\,x^{4}+b\,x^{2}+a}}{3 c}+\frac{\frac{a\,e^{2} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4-\frac{2 b \left(b+\sqrt{4 a c+b^{2}}\right)}{a c}}}{2}\right)}{4 \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}\, \sqrt{-c\,x^{4}+b\,x^{2}+a}}+\frac{3 c\,a}{}}{}$
default	$\frac{d^{2} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4-\frac{2 b \left(b+\sqrt{4 a c+b^{2}}\right)}{a c}}}{2}\right)}{4 \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}\, \sqrt{-c\,x^{4}+b\,x^{2}+a}}+e^{2} \left(-\frac{x \sqrt{-c\,x^{4}+b\,x^{2}+a}}{3 c}\right.$

```
[In] int((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/4*(d^2+1/3*a/c*e^2)*2^(1/2)/((-b+(4*a
*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2
)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(
1/2))-1/2*(2*e*d+2/3*b/c*e^2)*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(
(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2
))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+
(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{2\sqrt{\frac{1}{2}}\left(\left(3\,ac^2de+abce^2\right)\sqrt{-c}x\sqrt{\frac{b^2+4\,ac}{c^2}}+\left(3\,abcde+ab^2e^2\right)\sqrt{-c}x\right)\sqrt{\frac{c\sqrt{\frac{b^2+4\,ac}{c^2}}+b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2+4\,ac}{c^2}}}{c}}}{x}\right.}{}$$

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(1/2)*((3*a*c^2*d*e + a*b*c*e^2)*sqrt(-c)*x*sqrt((b^2 + 4*a*c)/
c^2) + (3*a*b*c*d*e + a*b^2*e^2)*sqrt(-c)*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2
) + b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/
c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*(
(3*c^3*d^2 + 6*a*c^2*d*e + (2*a*b*c + a*c^2)*e^2)*sqrt(-c)*x*sqrt((b^2 + 4*
a*c)/c^2) - (3*b*c^2*d^2 - 6*a*b*c*d*e - (2*a*b^2 - a*b*c)*e^2)*sqrt(-c)*x)
*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((
c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^
2 - 2*a*c)/(a*c)) + 2*(a*c^2*e^2*x^2 + 6*a*c^2*d*e + 2*a*b*c*e^2)*sqrt(-c*x
^4 + b*x^2 + a))/(a*c^3*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{(ex^2+d)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{(ex^2+d)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{{\left(e\,x^2+d\right)}^2}{\sqrt{-c\,x^4+b\,x^2+a}} \,d x$$

```
[In] int((d + e*x^2)^2/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^2/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.387 $\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 385

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+}}}\right.\right.}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] 1/4*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)
^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d+e*(b-(4*a*c+b^2)^(1/2)))*(1-2*
c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(
b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)-1/4*e*El
lipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2)
)/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+
b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1
/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

$= \{1216, 538, 435, 430\}$

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$$
$$= \frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\right.}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$
$$- \frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

```
[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x
^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*El
lipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c
*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*S
qrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2
 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]
]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sq
rt[a + b*x^2 - c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c])))))
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{d+ex^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\left(\left(b-\sqrt{b^2+4ac}\right)\left(-\frac{2cd}{b-\sqrt{b^2+4ac}}-e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+}}}}}{2c\sqrt{a+bx^2-cx^4}}$$

$$-\frac{\left(\left(b-\sqrt{b^2+4ac}\right)e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{2c\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b}}{b-\sqrt{b}}\right.}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

$$+\frac{\sqrt{b+\sqrt{b^2+4ac}}\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4}}}\right.\right.}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.76

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(\left(-b+\sqrt{b^2+4ac}\right)eE\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)-\right.}{2\sqrt{2}c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+b}}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(
b + Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*EllipticE[I*ArcSinh[Sqr
```

```
t[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sq
rt[b^2 + 4*a*c])] + (2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b
- Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[
a + b*x^2 - c*x^4])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.95

method	result
default	$\frac{d\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}-\frac{ea\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{}$
elliptic	$\frac{d\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}-\frac{ea\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{}$

```
[In] int((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4
*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)
^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*
b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.81

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{2\sqrt{-cx^4+bx^2+a}\,ace+\sqrt{\frac{1}{2}}\left(a\sqrt{-c}\,cex\sqrt{\frac{b^2+4\,ac}{c^2}}+ab\sqrt{-c}\,ex\right)\sqrt{\frac{c\sqrt{\frac{b^2+4\,ac}{c^2}}+b}{c}}\,E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2+4\,ac}{c^2}}+b}{c}}}{x}\right)}{}$$

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(-c*x^4 + b*x^2 + a)*a*c*e + sqrt(1/2)*(a*sqrt(-c)*c*e*x*sqrt((
b^2 + 4*a*c)/c^2) + a*b*sqrt(-c)*e*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/
c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x),
1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((c^2*d
+ a*c*e)*sqrt(-c)*x*sqrt((b^2 + 4*a*c)/c^2) - (b*c*d - a*b*e)*sqrt(-c)*x)*s
qrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*
sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2
- 2*a*c)/(a*c)))/(a*c^2*x)
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4+bx^2+a}}\,dx$$

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((d + e*x^2)/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.388 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 197

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\Bigg(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4}}{b-\sqrt{b^2+4}}}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

```
[Out] 1/2*EllipticPi(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),-1/2*e*(b+(4*a
*c+b^2)^(1/2))/c/d,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(1-
2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2
/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1234, 551}

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\text{EllipticPi}\Bigg(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4}}{b-\sqrt{b^2+4}}}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 +
```

```
4*a*c])*e)/(c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]],
(b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[a
 + b*x^2 - c*x^4])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int\frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)}\,dx}{\sqrt{a+bx^2-cx^4}}$$

$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04

$$\int\frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right),-\frac{b+\sqrt{b^2+4a}}{-b+\sqrt{b^2+4a}}\right.}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}d\sqrt{a+bx^2-cx^4}}$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e)/(c*d), I*Ar
```

```
cSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*
c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]
*d*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1+\frac{b x^2}{2a}-\frac{x^2\sqrt{4ac+b^2}}{2a}}\,\sqrt{1+\frac{b x^2}{2a}+\frac{x^2\sqrt{4ac+b^2}}{2a}}\,\Pi\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},-\frac{2ae}{\left(-b+\sqrt{4ac+b^2}\right)d},\frac{\sqrt{-\frac{b+\sqrt{4ac+b^2}}{2a}}\,\sqrt{2}}{\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{4ac+b^2}}{a}}\,\sqrt{-c x^4+b x^2+a}}$	201
elliptic	$\frac{\sqrt{2}\,\sqrt{1+\frac{b x^2}{2a}-\frac{x^2\sqrt{4ac+b^2}}{2a}}\,\sqrt{1+\frac{b x^2}{2a}+\frac{x^2\sqrt{4ac+b^2}}{2a}}\,\Pi\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},-\frac{2ae}{\left(-b+\sqrt{4ac+b^2}\right)d},\frac{\sqrt{-\frac{b+\sqrt{4ac+b^2}}{2a}}\,\sqrt{2}}{\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{4ac+b^2}}{a}}\,\sqrt{-c x^4+b x^2+a}}$	201

```
[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(4*
a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(-c
*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/
2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(
1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 - c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{(e\,x^2+d)\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(1/((d + e*x^2)*(a + b*x^2 - c*x^4)^(1/2)),x)
[Out] int(1/((d + e*x^2)*(a + b*x^2 - c*x^4)^(1/2)), x)
```

3.389 $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 718

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx &= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\,(cd^2+bde-ae^2)\,(d+ex^2)} \\
&+ \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\,(cd^2+e(bd-ae))\sqrt{a+bx^2-cx^4}} \\
&- \frac{\sqrt{b+\sqrt{b^2+4ac}}\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\,\Bigg(\arcsin\Bigg(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+}}}}{4\sqrt{2}\sqrt{c}d\,(cd^2+e(bd-ae))\sqrt{a+bx^2-cx^4}} \\
&+ \frac{\sqrt{b+\sqrt{b^2+4ac}}\,(3cd^2+e(2bd-ae))\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\,\Bigg(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\arcsin}{2\sqrt{2}\sqrt{c}d^2\,(cd^2+e(bd-ae))\sqrt{a+bx^2-cx^4}}
\end{aligned}$$

```
[Out] -1/2*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/d/(-a*e^2+b*d*e+c*d^2)/(e*x^2+d)+1/4*(3*c
*d^2+e*(-a*e+2*b*d))*EllipticPi(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/
2),-1/2*e*(b+(4*a*c+b^2)^(1/2))/c/d,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(
1/2)))^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))
^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d^2/(c*d^2+e*(-a*e+b*d))*2^(
1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)-1/8*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a
*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(
2*c*d+e*(b-(4*a*c+b^2)^(1/2)))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(
4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d/(c*d^2+e*
(-a*e+b*d))*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/8*e*EllipticE(x*2^(1/2
)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)
^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/
2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d/(c
*d^2+e*(-a*e+b*d))*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1237, 1730, 1216, 538, 435, 430, 1234, 551}

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4a}}}\right.\right.}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}$$

$$+\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}$$

$$+\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e(2bd-ae)+3cd^2\right)\text{EllipticPi}\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\right.\right.}{2\sqrt{2}\sqrt{c}d^2\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}$$

$$-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -1/2*(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(d*(c*d^2 + e*(b*d - a*e))*(d + e*x^2)
) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^
2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ell
ipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b
^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d
 - a*e))*Sqrt[a + b*x^2 - c*x^4]) - (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (
b - Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[
1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*
c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4])
 + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1 - (2*c*x
^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*El
lipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e)/(c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*
c])])/(2*Sqrt[2]*Sqrt[c]*d^2*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4
])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
```

```
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)(d+ex^2)}+\frac{\int \frac{2cd^2+e(2bd-ae)-2cdex^2-ce^2x^4}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx}{2d\left(cd^2+e(bd-ae)\right)}\\
&= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)(d+ex^2)}-\frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{2de^2\left(cd^2+e(bd-ae)\right)}\\
&\quad+\frac{\left(3cd^2+e(2bd-ae)\right)\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx}{2d\left(cd^2+e(bd-ae)\right)}\\
&= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)(d+ex^2)}\\
&\quad-\frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{cde^2+ce^3x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{2de^2\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}}\\
&\quad+\frac{\left(\left(3cd^2+e(2bd-ae)\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)}\,dx}{2d\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}}
\end{aligned}
$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)\left(d+ex^2\right)} + \frac{\sqrt{b+\sqrt{b^2+4ac}}(3cd^2+e(2bd-ae))\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\Bigg(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\Big(}{2\sqrt{2}\sqrt{c}d^2\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}} + \frac{\left(\left(b-\sqrt{b^2+4ac}\right)e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{4d\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}} - \frac{\left(\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int\frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{4d\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+e(bd-ae)\right)\left(d+ex^2\right)} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\Big(\sin^{-1}\Big(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\Big)\,|\,\frac{b+\sqrt{b}}{b-\sqrt{b}}}{4\sqrt{2}\sqrt{c}d\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}} - \frac{\sqrt{b+\sqrt{b^2+4ac}}\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}F\Big(\sin^{-1}\Big(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4}}}}{4\sqrt{2}\sqrt{c}d\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}(3cd^2+e(2bd-ae))\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\Bigg(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\Big(}{2\sqrt{2}\sqrt{c}d^2\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.81 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.65

$$\int\frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{a+bx^2-cx^4}\Bigg(4de^2x+\frac{i\sqrt{2+\frac{4cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)\Big(\left(-b+\sqrt{b^2+4ac}\right)deE\Big(i\operatorname{arcsinh}\Big(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\Big)}{}}{}$$

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -1/8*(Sqrt[a + b*x^2 - c*x^4]*(4*d*e^2*x + (I*Sqrt[2 + (4*c*x^2)/(-b + Sqrt
[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(d + e*x^2)*((-
b + Sqrt[b^2 + 4*a*c])*d*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b
^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + d*(2*
```

```
c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b +
Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] +
 2*(-3*c*d^2 + e*(-2*b*d + a*e))*EllipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e
)/(c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt
[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c])
)]*(-a - b*x^2 + c*x^4))))/(d^2*(c*d^2 + e*(b*d - a*e))*(d + e*x^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(613) = 1226$.

Time = 1.62 (sec) , antiderivative size = 1293, normalized size of antiderivative = 1.80

method	result	size
default	Expression too large to display	1293
elliptic	Expression too large to display	1293

```
[In] int(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^2/(a*e^2-b*d*e-c*d^2)/d*x*(-c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)+1/8*c/(a*e
^2-b*d*e-c*d^2)*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x
^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)
/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*c*e/(a*e^2-b*d*e-c*
d^2)/d*a*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(4
*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^
4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/
(a*e^2-b*d*e-c*d^2)/d*a*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x
^2/a-2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(4*a*c+b^2)^(1/2
))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/
2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^
(1/2))+1/2/(a*e^2-b*d*e-c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))
^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2
*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(
1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2
*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*a
-1/(a*e^2-b*d*e-c*d^2)*e/d*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/
2*b*x^2/a-1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(4*a*
c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4
*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(4*a*c+b
^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*b-3/2/(a*e^2-
b*d*e-c*d^2)*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*
x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(4*a*c+b^2)^(1/2))^
(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2
```

```
))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^
(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$$

```
[In] integrate(1/(e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 - c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{1}{\left(e\,x^2+d\right)^2\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 - c*x^4)^(1/2)),x)
[Out] int(1/((d + e*x^2)^2*(a + b*x^2 - c*x^4)^(1/2)), x)
```

3.390 $\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 479

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx &= \frac{\left(b-\sqrt{b^2+4ac}\right)ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} \\ &\quad-\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}} \\ &\quad+\frac{\sqrt{b+\sqrt{b^2+4ac}}d\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}}\end{aligned}$$

```
[Out] 1/2*e*x*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))/c/(c*x^4+b*
x^2-a)^(1/2)+1/2*d*(1/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(
b+(4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2
))^(1/2)/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(
4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b+(4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^
2)^(1/2)))/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)-1/4*e*(1/(1+2*c*x^2/(b+
(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*Elliptic
E(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(4*a*c+b^2)^(
1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2
/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))*(b+(4*a*c+b^2)^(1/2))^(1/2)/c
^(3/2)*2^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))/(1+
2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1216, 545, 429, 506, 422}

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx =$$

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}$$

$$+\frac{d\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}$$

$$+\frac{ex\left(b-\sqrt{4ac+b^2}\right)\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)}{2c\sqrt{-a+bx^2+cx^4}}$$

```
[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((b - Sqrt[b^2 + 4*a*c])*e*x*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])))/(2*c*
Sqrt[-a + b*x^2 + c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*EllipticE[ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^
2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]
))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sq
rt[b + Sqrt[b^2 + 4*a*c]]*d*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*Ellipti
cF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b -
 Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x
^2 + c*x^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{d+ex^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{-a+bx^2+cx^4}}\\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{-a+bx^2+cx^4}}\\
&\quad+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{x^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{-a+bx^2+cx^4}}
\end{aligned}
$$

$$= \frac{\left(b-\sqrt{b^2+4ac}\right) ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}d\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right) F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right) e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\left(1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}\right)^{3/2}}dx}{2c\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\left(b-\sqrt{b^2+4ac}\right) ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right) E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}d\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right) F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.63

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$$

$$= \frac{i\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\left(\left(-b+\sqrt{b^2+4ac}\right)eE\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+(-2c\right.}{2\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{-a+bx^2+c}}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 + 4*
a*c])/(b - Sqrt[b^2 + 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 + 4*a*c]
)]*Sqrt[-a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.74

method	result
default	$\frac{d\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}+\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}}{}$
elliptic	$\frac{d\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}+\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}}{}$

```
[In] int((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*Ellip
ticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(
1/2))/a/c)^(1/2))+e*a/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^
2-a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*(-
2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1
/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.63

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$$

$$=\frac{2\sqrt{cx^4+bx^2-a}\,ace+\sqrt{\frac{1}{2}}\left(acex\sqrt{\frac{b^2+4\,ac}{c^2}}-abex\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2+4\,ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2+4\,ac}{c^2}}-b}{c}}}{x}\right)\,\middle|\,-\frac{bc\sqrt{b^2}}{}\right.}{}$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(c*x^4 + b*x^2 - a)*a*c*e + sqrt(1/2)*(a*c*e*x*sqrt((b^2 + 4*a*c
)/c^2) - a*b*e*x)*sqrt(c)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) - b)/c)*elliptic_
e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) - b)/c)/x), -1/2*(b*c*sq
rt((b^2 + 4*a*c)/c^2) + b^2 + 2*a*c)/(a*c)) - sqrt(1/2)*((c^2*d + a*c*e)*x*
```

```
sqrt((b^2 + 4*a*c)/c^2) + (b*c*d - a*b*e)*x)*sqrt(c)*sqrt((c*sqrt((b^2 + 4*
a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^
2) - b)/c)/x), -1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) + b^2 + 2*a*c)/(a*c)))/(a*
c^2*x)
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4+bx^2-a}}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{cx^4+bx^2-a}}\,dx$$

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

```
[In] int((d + e*x^2)/(b*x^2 - a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(b*x^2 - a + c*x^4)^(1/2), x)
```

3.391 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\sqrt{-b+\sqrt{b^2+4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2+4ac}}}\right),\frac{b-\sqrt{b^2+}}{b+\sqrt{b^2+}}\right.}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

```
[Out] 1/2*EllipticPi(x*2^(1/2)*c^(1/2)/(-b+(4*a*c+b^2)^(1/2))^(1/2),1/2*e*(b-(4*a
*c+b^2)^(1/2))/c/d,((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2)^(1/2)))^(1/2))*(1+
2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(-b+(4*a*c+b^2)^(1/2))^(1/2)*(1+2*c*x^
2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1234, 551}

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right),\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right.}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[((b - Sqrt[b^2 + 4*a*
```

```
c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (
b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[-a
 + b*x^2 + c*x^4])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)}\,dx}{\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\sqrt{-b+\sqrt{b^2+4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\Bigg(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2+4ac}}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left(\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}d\sqrt{-a+bx^2+cx^4}}$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*EllipticPi[((b + Sqrt[b^2 + 4*a*c])*
```

```
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt
[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*
a*c])]*d*Sqrt[-a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{1-\frac{b x^{2}}{2a}+\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\sqrt{1-\frac{b x^{2}}{2a}-\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\Pi\left(\sqrt{-\frac{-b+\sqrt{4ac+b^{2}}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{4ac+b^{2}}\right)d},\frac{\sqrt{2}\,\sqrt{\frac{b+\sqrt{4ac+b^{2}}}{a}}}{2\sqrt{-\frac{-b+\sqrt{4ac+b^{2}}}{2a}}}\right)}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{4ac+b^{2}}}{2a}}\,\sqrt{c x^{4}+b x^{2}-a}}$	198
elliptic	$\frac{\sqrt{1-\frac{b x^{2}}{2a}+\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\sqrt{1-\frac{b x^{2}}{2a}-\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\Pi\left(\sqrt{-\frac{-b+\sqrt{4ac+b^{2}}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{4ac+b^{2}}\right)d},\frac{\sqrt{2}\,\sqrt{\frac{b+\sqrt{4ac+b^{2}}}{a}}}{2\sqrt{-\frac{-b+\sqrt{4ac+b^{2}}}{2a}}}\right)}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{4ac+b^{2}}}{2a}}\,\sqrt{c x^{4}+b x^{2}-a}}$	198

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(1/2*b/a-1/2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a+1/2*x^2/a*(4*a*c
+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a-1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,2/(-b+(4
*a*c+b^2)^(1/2))*a*e/d,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^(1/2)/(-1/2*(-
b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

```
[In] int(1/((d + e*x^2)*(b*x^2 - a + c*x^4)^(1/2)),x)
[Out] int(1/((d + e*x^2)*(b*x^2 - a + c*x^4)^(1/2)), x)
```

3.392 $\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 293

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx &= -\frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ &\quad - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}} \\ &\quad + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}}\end{aligned}$$

```
[Out] -e*x*(-c*x^4+b*x^2-a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*
arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipt
icE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1
/2)+x^2*c^(1/2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-
c*x^4+b*x^2-a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/
2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*x^4+b*x^2-a
)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1211, 1117, 1209}

$$
\begin{aligned}
\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx
&= \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}} \\
&\quad - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}} \\
&\quad - \frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}
\end{aligned}
$$

```
[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] -((e*x*Sqrt[-a + b*x^2 - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1
/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(3/4)*Sqrt[-a + b*x^2 - c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
c^(3/4)*Sqrt[-a + b*x^2 - c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\left(\sqrt{a}e\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}}+\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int\frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx\\
&= -\frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&\quad-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}}\\
&\quad+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{-a+bx^2-cx^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.01

$$\int\frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx=$$

$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\left(\left(-b+\sqrt{b^2-4ac}\right)eE\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right.}{2\sqrt{2}c\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{-a+}}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] + (2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[
-a + b*x^2 - c*x^4])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.22

method	result
default	$\frac{d\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-c\,x^4+b\,x^2-a}}+\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+\right.}}{a}}}{}$
elliptic	$\frac{d\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-c\,x^4+b\,x^2-a}}+\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+\right.}}{a}}}{}$

```
[In] int((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)*E
llipticF(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))+e*a/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-
c*x^4+b*x^2-a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ellip
ticE(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.06

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx =$$

$$-\frac{2\sqrt{-cx^4+bx^2-a}ace+\sqrt{\frac{1}{2}}\left(a\sqrt{-c}cex\sqrt{\frac{b^2-4\,ac}{c^2}}+ab\sqrt{-c}ex\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}}{x}\right)\right.}{}$$

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(-c*x^4 + b*x^2 - a)*a*c*e + sqrt(1/2)*(a*sqrt(-c)*c*e*x*sqrt((
b^2 - 4*a*c)/c^2) + a*b*sqrt(-c)*e*x)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/
c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)/x),
-1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2*a*c)/(a*c)) + sqrt(1/2)*((c^2*d
```

```
 - a*c*e)*sqrt(-c)*x*sqrt((b^2 - 4*a*c)/c^2) - (b*c*d + a*b*e)*sqrt(-c)*x)*
sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c
*sqrt((b^2 - 4*a*c)/c^2) + b)/c)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^
2 + 2*a*c)/(a*c)))/(a*c^2*x)
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx=\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$$

```
[In] integrate((e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{-c\,x^4+b\,x^2-a}}\,dx$$

```
[In] int((d + e*x^2)/(b*x^2 - a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(b*x^2 - a - c*x^4)^(1/2), x)
```

3.393 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 412

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \frac{\sqrt{e}\arctan\left(\frac{\sqrt{-cd^2-e(bd+ae)}x}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-e(bd+ae)}}$$
$$+\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a+bx^2-cx^4}}$$
$$-\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\,(cd^2-ae^2)\sqrt{-a+bx^2-cx^4}}$$

```
[Out] 1/2*arctan(x*(-a*e^2-b*d*e-c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(-c*x^4+b*x^2-a)^(1
/2))*e^(1/2)/d^(1/2)/(-a*e^2-b*d*e-c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*
c^(1/2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2
)+d*c^(1/2))/(-c*x^4+b*x^2-a)^(1/2)-1/4*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^
(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2+
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c
*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(-c*x
^4+b*x^2-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1230, 1117, 1720}

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = -\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2 \text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de}, 2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right), \frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{4\sqrt[4]{c}d\sqrt{-a+bx^2-cx^4}\,(cd^2-ae^2)} + \frac{\sqrt{e}\arctan\left(\frac{x\sqrt{-e(ae+bd)-cd^2}}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\sqrt{-e(ae+bd)-cd^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right), \frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt{-a+bx^2-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - e*(b*d + a*e)]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a
 + b*x^2 - c*x^4])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - e*(b*d + a*e)]) + (c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (a^(3/4)*((Sqrt
[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*S
qrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(
4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[-a + b*x^2 - c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c}\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e} - \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e} \\
&= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{-cd^2-e(bd+ae)}x}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-e(bd+ae)}} \\
&\quad + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a+bx^2-cx^4}} \\
&\quad - \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a+bx^2-cx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.50

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{EllipticPi}\left(-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd}, i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right), -\frac{b+\sqrt{b^2-4a}}{-b+\sqrt{b^2-4}}\right.}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}d\sqrt{-a+bx^2-cx^4}}$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 - 4*a*c])*e)/(c*d), I*Ar
cSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], -((b + Sqrt[b^2 - 4*a*
c])/(-b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]
*d*Sqrt[-a + b*x^2 - c*x^4])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\sqrt{1-\frac{b x^2}{2a}+\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\sqrt{1-\frac{b x^2}{2a}-\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\Pi\left(\sqrt{-\frac{-b+\sqrt{-4ac+b^2}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{-4ac+b^2}\right)d},\frac{\sqrt{2}\,\sqrt{\frac{b+\sqrt{-4ac+b^2}}{a}}}{2\sqrt{-\frac{-b+\sqrt{-4ac+b^2}}{2a}}}\right)}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{-4ac+b^2}}{2a}}\,\sqrt{-c x^4+b x^2-a}}$	199
elliptic	$\frac{\sqrt{1-\frac{b x^2}{2a}+\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\sqrt{1-\frac{b x^2}{2a}-\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\Pi\left(\sqrt{-\frac{-b+\sqrt{-4ac+b^2}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{-4ac+b^2}\right)d},\frac{\sqrt{2}\,\sqrt{\frac{b+\sqrt{-4ac+b^2}}{a}}}{2\sqrt{-\frac{-b+\sqrt{-4ac+b^2}}{2a}}}\right)}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{-4ac+b^2}}{2a}}\,\sqrt{-c x^4+b x^2-a}}$	199

```
[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(1/2*b/a-1/2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a+1/2*x^2/a*(-4*a
*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(-c
*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,2/(
-b+(-4*a*c+b^2)^(1/2))*a*e/d,1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(
-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 - c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2-a}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{(e\,x^2+d)\,\sqrt{-c\,x^4+b\,x^2-a}}\,dx$$

```
[In] int(1/((d + e*x^2)*(b*x^2 - a - c*x^4)^(1/2)),x)
[Out] int(1/((d + e*x^2)*(b*x^2 - a - c*x^4)^(1/2)), x)
```

3.394 $\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 229

$$\begin{aligned}\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx &= \frac{3e(5d^2-10de+6e^2)\,x(2+x^2)}{5\sqrt{2+3x^2+x^4}} \\ &\quad + \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4} + \frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4} \\ &\quad - \frac{3\sqrt{2}e(5d^2-10de+6e^2)\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} \\ &\quad + \frac{(5d^3-10de^2+8e^3)\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{5\sqrt{2}\sqrt{2+3x^2+x^4}}\end{aligned}$$

```
[Out] 3/5*e*(5*d^2-10*d*e+6*e^2)*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/10*(5*d^3-10*d*e
^2+8*e^3)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(
1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-3/5*e*(5*d^2-10*d
*e+6*e^2)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(
1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5*(5*d-4*e)*e^2
*x*(x^4+3*x^2+2)^(1/2)+1/5*e^3*x^3*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1220, 1693, 1203, 1113, 1149}

$$\begin{aligned}\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = {} & \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^3-10de^2+8e^3)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{5\sqrt{2}\sqrt{x^4+3x^2+2}} \\ & - \frac{3\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(5d^2-10de+6e^2)\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}} \\ & + \frac{3e(x^2+2)\,x(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}} \\ & + \frac{1}{5}e^2\sqrt{x^4+3x^2+2}x(5d-4e)+\frac{1}{5}e^3\sqrt{x^4+3x^2+2}x^3\end{aligned}$$

```
[In] Int[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (3*e*(5*d^2 - 10*d*e + 6*e^2)*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*
d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^4])/5 + (e^3*x^3*Sqrt[2 + 3*x^2 + x^4])/5
 - (3*Sqrt[2]*e*(5*d^2 - 10*d*e + 6*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2
 + 8*e^3)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5
*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}+\frac{1}{5}\int\frac{5d^3+3e(5d^2-2e^2)\,x^2+3(5d-4e)e^2x^4}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}+\frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}\\
&\quad+\frac{1}{15}\int\frac{3(5d^3-10de^2+8e^3)+9e(5d^2-10de+6e^2)\,x^2}{\sqrt{2+3x^2+x^4}}\,dx\\
&= \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}+\frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}\\
&\quad+\frac{1}{5}\big(3e\big(5d^2-10de+6e^2\big)\big)\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx\\
&\quad+\frac{1}{5}\big(5d^3-10de^2+8e^3\big)\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&=\frac{3e(5d^2-10de+6e^2)\,x(2+x^2)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}\\
&\quad+\frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}-\frac{3\sqrt{2}e(5d^2-10de+6e^2)\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}}\\
&\quad+\frac{(5d^3-10de^2+8e^3)\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{2}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.67

$$
\int\frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx
$$

$$
=\frac{e^2x(2+3x^2+x^4)\,(5d+e(-4+x^2))-3ie(5d^2-10de+6e^2)\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-5i(}{5\sqrt{2+3x^2+x^4}}
$$

```
[In] Integrate[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4)*(5*d + e*(-4 + x^2)) - (3*I)*e*(5*d^2 - 10*d*e + 6
*e^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5*I
)*(d^3 - 3*d^2*e + 4*d*e^2 - 2*e^3)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I
*ArcSinh[x/Sqrt[2]], 2])/(5*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.80

method	result
elliptic	$\frac{e^3x^3\sqrt{x^4+3x^2+2}}{5}+\left(d\,e^2-\frac{4}{5}e^3\right)x\sqrt{x^4+3x^2+2}-\frac{i\left(d^3-2d\,e^2+\frac{8}{5}e^3\right)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+ i(3d^2e+\frac{18}{5}$
risch	$\frac{x\,e^2\left(e\,x^2+5d-4e\right)\sqrt{x^4+3x^2+2}}{5}-\frac{id^3\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{4ie^3\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{5\sqrt{x^4+3x^2+2}}+ id\,e^2\sqrt{2}\,\sqrt{2x$
default	$-\frac{id^3\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+e^3\left(\frac{x^3\sqrt{x^4+3x^2+2}}{5}-\frac{4x\sqrt{x^4+3x^2+2}}{5}-\frac{4i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{5\sqrt{x^4+3x^2+2}}+ 9i\right.$

```
[In] int((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e^3*x^3*(x^4+3*x^2+2)^(1/2)+(d*e^2-4/5*e^3)*x*(x^4+3*x^2+2)^(1/2)-1/2*I
*(d^3-2*d*e^2+8/5*e^3)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^
```

```
(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*(3*d^2*e+18/5*e^3-6*d*e^2)*2
^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2
^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-3i\,(5\,d^2e-10\,de^2+6\,e^3)xE(\arcsin\left(\frac{i}{x}\right)\,|\,2)+i\,(5\,d^3+15\,d^2e-40\,de^2+26\,e^3)xF(\arcsin\left(\frac{i}{x}\right)\,|\,2)+(e^3x}{5\,x}$$

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*(5*d^2*e - 10*d*e^2 + 6*e^3)*x*elliptic_e(arcsin(I/x), 2) + I*(5*
d^3 + 15*d^2*e - 40*d*e^2 + 26*e^3)*x*elliptic_f(arcsin(I/x), 2) + (e^3*x^4
 + 15*d^2*e - 30*d*e^2 + 18*e^3 + (5*d*e^2 - 4*e^3)*x^2)*sqrt(x^4 + 3*x^2 +
 2))/x
```

Sympy [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(d+ex^2)^3}{\sqrt{(x^2+1)\,(x^2+2)}}\,dx$$

```
[In] integrate((e*x**2+d)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{(ex^2+d)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{(e\,x^2+d)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int((d + e*x^2)^3/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((d + e*x^2)^3/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.395 $\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{2(d-e)ex(2+x^2)}{\sqrt{2+3x^2+x^4}} + \frac{1}{3}e^2x\sqrt{2+3x^2+x^4} - \frac{2\sqrt{2}(d-e)e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{(3d^2-2e^2)(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] 2*(d-e)*e*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/6*(3*d^2-2*e^2)*(x^2+1)^(3/2)*(1/
(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/
2)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-2*(d-e)*e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*El
lipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3
*x^2+2)^(1/2)+1/3*e^2*x*(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{1220, 1203, 1113, 1149\}$

$$\begin{aligned}\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = & \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(3d^2-2e^2)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}} \\ & - \frac{2\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(d-e)E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} \\ & + \frac{2ex(x^2+2)\,(d-e)}{\sqrt{x^4+3x^2+2}} + \frac{1}{3}e^2x\sqrt{x^4+3x^2+2}\end{aligned}$$

```
[In] Int[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (2*(d - e)*e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (e^2*x*Sqrt[2 + 3*x^2 + x
^4])/3 - (2*Sqrt[2]*(d - e)*e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE
[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((3*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(
2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 +
x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}e^2x\sqrt{2+3x^2+x^4}+\frac{1}{3}\int\frac{3d^2-2e^2+6(d-e)ex^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{1}{3}e^2x\sqrt{2+3x^2+x^4}+(2(d-e)e)\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{1}{3}\left(3d^2-2e^2\right)\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{2(d-e)ex(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}e^2x\sqrt{2+3x^2+x^4}\\ &\quad-\frac{2\sqrt{2}(d-e)e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}\\ &\quad+\frac{\left(3d^2-2e^2\right)(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+3x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.76

$$\int\frac{\left(d+ex^2\right)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{e^2x(2+3x^2+x^4)-6i(d-e)e\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-i\left(3d^2-6de+4e^2\right)\sqrt{1+x^2}\sqrt{2+}}{3\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4) - (6*I)*(d - e)*e*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elli
pticE[I*ArcSinh[x/Sqrt[2]], 2] - I*(3*d^2 - 6*d*e + 4*e^2)*Sqrt[1 + x^2]*Sq
rt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.61 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

method	result
elliptic	$\frac{e^{2} x \sqrt{x^{4}+3 x^{2}+2}}{3}-\frac{i\left(d^{2}-\frac{2 e^{2}}{3}\right) \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+\frac{i\left(2 e d-2 e^{2}\right) \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right.\right.}{2 \sqrt{x^{4}+3 x^{2}+2}}$
risch	$\frac{e^{2} x \sqrt{x^{4}+3 x^{2}+2}}{3}-\frac{i d^{2} \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+\frac{i e^{2} \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{3 \sqrt{x^{4}+3 x^{2}+2}}+\frac{i\left(6 e d-6 e^{2}\right) \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}}}{6 \sqrt{}}$
default	$-\frac{i d^{2} \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+e^{2}\left(\frac{x \sqrt{x^{4}+3 x^{2}+2}}{3}+\frac{i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{3 \sqrt{x^{4}+3 x^{2}+2}}-\frac{i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\right.}{\sqrt{x^{4}}}\right.$

```
[In] int((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*x*(x^4+3*x^2+2)^(1/2)-1/2*I*(d^2-2/3*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(
x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*(
2*d*e-2*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ell
ipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$=\frac{-6i\left(de-e^2\right)xE\left(\arcsin\left(\frac{i}{x}\right)\mid 2\right)+i\left(3\,d^2+6\,de-8\,e^2\right)xF\left(\arcsin\left(\frac{i}{x}\right)\mid 2\right)+\left(e^2x^2+6\,de-6\,e^2\right)\sqrt{x^4+3\,x^2}}{3\,x}$$

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-6*I*(d*e - e^2)*x*elliptic_e(arcsin(I/x), 2) + I*(3*d^2 + 6*d*e - 8*e
^2)*x*elliptic_f(arcsin(I/x), 2) + (e^2*x^2 + 6*d*e - 6*e^2)*sqrt(x^4 + 3*x
^2 + 2))/x
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(d+ex^2)^2}{\sqrt{(x^2+1)\,(x^2+2)}}\,dx$$

```
[In] integrate((e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral((d + e*x**2)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(e\,x^2+d)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int((d + e*x^2)^2/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((d + e*x^2)^2/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.396 $\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{ex(2+x^2)}{\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{d(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] e*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*d*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellip
ticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x^
2+2)^(1/2)-e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*
2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1203, 1113, 1149}

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}} + \frac{ex(x^2+2)}{\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*e*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (d*(1 + x^2)
```

```
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + e\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{ex(2+x^2)}{\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{d(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\begin{aligned}
&\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&= -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(eE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+(d-e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)\right)}{\sqrt{2+3x^2+x^4}}
\end{aligned}$$

```
[In] Integrate[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (
d - e)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{id\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^{4}+3x^{2}+2}}+\frac{ie\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^{4}+3x^{2}+2}}$	108
elliptic	$-\frac{id\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^{4}+3x^{2}+2}}+\frac{ie\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^{4}+3x^{2}+2}}$	108

```
[In] int((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
F(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x
^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/
2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.37

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-i\,exE(\arcsin\left(\frac{i}{x}\right)\,|2)+i\,(d+e)xF(\arcsin\left(\frac{i}{x}\right)\,|2)+\sqrt{x^4+3\,x^2+2}e}{x}$$

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*e*x*elliptic_e(arcsin(I/x), 2) + I*(d + e)*x*elliptic_f(arcsin(I/x), 2)
 + sqrt(x^4 + 3*x^2 + 2)*e)/x
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{d+ex^2}{\sqrt{(x^2+1)\,(x^2+2)}}\,dx$$

```
[In] integrate((e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral((d + e*x**2)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{ex^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{ex^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int((d + e*x^2)/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((d + e*x^2)/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.397 $\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}(d-e)\sqrt{2+3x^2+x^4}} - \frac{e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticPi}\left(1-\frac{e}{d},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}d(d-e)\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
((x^2+2)/(x^2+1))^(1/2)/(d-e)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-1/2*e*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-e/d,1/2*2^(1/2))*((x^2+2)
/(x^2+1))^(1/2)/d/(d-e)*2^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1228, 1113, 1470, 553}

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)} - \frac{e(x^2+2)\,\text{EllipticPi}\left(1-\frac{e}{d},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}d\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}(d-e)}$$

```
[In] Int[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*(d
 - e)*Sqrt[2 + 3*x^2 + x^4]) - (e*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x],
1/2])/(Sqrt[2]*d*(d - e)*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{d-e} - \frac{e\int \frac{2+2x^2}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx}{2(d-e)}\\ &= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}(d-e)\sqrt{2+3x^2+x^4}} - \frac{\left(e\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(d+ex^2)}\,dx}{2(d-e)\sqrt{2+3x^2+x^4}}\end{aligned}$$

$$=\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}(d-e)\sqrt{2+3x^2+x^4}}-\frac{e(2+x^2)\,\Pi\left(1-\frac{e}{d};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}d(d-e)\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.48

$$\int\frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx=-\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticPi}\left(\frac{2e}{d},i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{d\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[(2*e)/d, I*ArcSinh[x/Sqrt[2]],
 2])/(d*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{i\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{2e}{d},\sqrt{2}\right)}{d\sqrt{x^4+3x^2+2}}$	55
elliptic	$-\frac{i\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{2e}{d},\sqrt{2}\right)}{d\sqrt{x^4+3x^2+2}}$	55

```
[In] int(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi
(1/2*I*2^(1/2)*x,2*e/d,2^(1/2))
```

Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(e*x^6 + (d + 3*e)*x^4 + (3*d + 2*e)*x^2 + 2
*d), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2+1)\,(x^2+2)}\,(d+ex^2)}\,dx$$

```
[In] integrate(1/(e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(ex^2+d)}\,dx$$

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{(e\,x^2+d)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int(1/((d + e*x^2)*(3*x^2 + x^4 + 2)^(1/2)),x)
[Out] int(1/((d + e*x^2)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.398 $\int \frac{1}{\left(d+ex^2\right)^2\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 316

$$
\begin{aligned}
&\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx \\
&= -\frac{ex(2+x^2)}{2d\left(d^2-3de+2e^2\right)\sqrt{2+3x^2+x^4}}+\frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} \\
&\quad+\frac{e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}d(d-2e)(d-e)\sqrt{2+3x^2+x^4}} \\
&\quad+\frac{(2d-e)\left(1+x^2\right)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2d(d-e)^2\sqrt{2+3x^2+x^4}} \\
&\quad-\frac{e(3d^2-6de+2e^2)\left(2+x^2\right)\text{EllipticPi}\left(1-\frac{e}{d},\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}d^2(d-2e)(d-e)^2\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

```
[Out] -1/2*e*x*(x^2+2)/d/(d^2-3*d*e+2*e^2)/(x^4+3*x^2+2)^(1/2)-1/4*e*(3*d^2-6*d*e
+2*e^2)*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),
1-e/d,1/2*2^(1/2))/d^2/(d-2*e)/(d-e)^2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/2*e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(
1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)/d/(d-2*e)/(d-e)*2^(1/2)/(x^4+3*x^
2+2)^(1/2)+1/2*(2*d-e)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^
(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/d/(d-e)^2/(x^4+3*x^2+2)^(1/2)+
1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1237, 1730, 1203, 1113, 1149, 1228, 1470, 553}

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx &= \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(3d^2-6de+2e^2)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}d\sqrt{x^4+3x^2+2}(d-2e)(d-e)^2} \\ &\quad - \frac{e(x^2+2)\,(3d^2-6de+2e^2)\,\text{EllipticPi}\left(1-\frac{e}{d},\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}d^2\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}(d-2e)(d-e)^2} \\ &\quad - \frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}(d-2e)(d-e)} + \frac{e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}d\sqrt{x^4+3x^2+2}(d-2e)(d-e)} \\ &\quad + \frac{e^2x\sqrt{x^4+3x^2+2}}{2d\,(d^2-3de+2e^2)\,(d+ex^2)} - \frac{ex(x^2+2)}{2d\sqrt{x^4+3x^2+2}\,(d^2-3de+2e^2)}\end{aligned}$$

```
[In] Int[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] -1/2*(e*x*(2 + x^2))/(d*(d^2 - 3*d*e + 2*e^2)*Sqrt[2 + 3*x^2 + x^4]) + (e^2
*x*Sqrt[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (e*(1 +
 x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*d*(d -
2*e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]) - ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*(d - 2*e)*(d - e)*Sqrt[2 + 3*x^2 + x^
4]) + ((3*d^2 - 6*d*e + 2*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
F[ArcTan[x], 1/2])/(2*Sqrt[2]*d*(d - 2*e)*(d - e)^2*Sqrt[2 + 3*x^2 + x^4])
- (e*(3*d^2 - 6*d*e + 2*e^2)*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x], 1/2])
/(2*Sqrt[2]*d^2*(d - 2*e)*(d - e)^2*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^
2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
```

```
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
```

```
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}-\frac{\int\frac{-2\left(d^2-3de+e^2\right)+2dex^2+e^2x^4}{\left(d+ex^2\right)\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)}\\
&= \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}+\frac{\int\frac{-de^2-e^3x^2}{\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)e^2}\\
&\quad+\frac{\left(3d^2-6de+2e^2\right)\int\frac{1}{\left(d+ex^2\right)\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)}\\
&= \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}-\frac{\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{2(d-2e)(d-e)}\\
&\quad+\frac{\left(3d^2-6de+2e^2\right)\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)^2}\\
&\quad-\frac{\left(e\left(3d^2-6de+2e^2\right)\right)\int\frac{2+2x^2}{\left(d+ex^2\right)\sqrt{2+3x^2+x^4}}\,dx}{4d(d-2e)(d-e)^2}-\frac{e\int\frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx}{2d\left(d^2-3de+2e^2\right)}\\
&= -\frac{ex\left(2+x^2\right)}{2d\left(d^2-3de+2e^2\right)\sqrt{2+3x^2+x^4}}+\frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}\\
&\quad+\frac{e\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}d(d-2e)(d-e)\sqrt{2+3x^2+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}(d-2e)(d-e)\sqrt{2+3x^2+x^4}}\\
&\quad+\frac{\left(3d^2-6de+2e^2\right)\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}d(d-2e)(d-e)^2\sqrt{2+3x^2+x^4}}\\
&\quad-\frac{\left(e\left(3d^2-6de+2e^2\right)\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int\frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}\left(d+ex^2\right)}\,dx}{4d(d-2e)(d-e)^2\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ex(2+x^2)}{2d\left(d^2-3de+2e^2\right)\sqrt{2+3x^2+x^4}} + \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} \\
&\quad + \frac{e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}d(d-2e)(d-e)\sqrt{2+3x^2+x^4}} - \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}(d-2e)(d-e)\sqrt{2+3x^2+x^4}} \\
&\quad + \frac{\left(3d^2-6de+2e^2\right)(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}d(d-2e)(d-e)^2\sqrt{2+3x^2+x^4}} \\
&\quad - \frac{e\left(3d^2-6de+2e^2\right)(2+x^2)\,\Pi\left(1-\frac{e}{d};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}d^2(d-2e)(d-e)^2\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.55

$$
\int \frac{1}{\left(d+ex^2\right)^2\sqrt{2+3x^2+x^4}}\,dx
$$

$$
= \frac{\frac{e^2x\left(2+3x^2+x^4\right)}{\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} + \frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(deE\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+d(d-e)\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)+\left(-3d^2+6de-2e^2\right)\,\text{EllipticPi}\left(\right.}{d(d-2e)(d-e)}}{2d\sqrt{2+3x^2+x^4}}
$$

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((e^2*x*(2 + 3*x^2 + x^4))/((d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (I*Sqrt[1
+ x^2]*Sqrt[2 + x^2]*(d*e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + d*(d - e)*El
lipticF[I*ArcSinh[x/Sqrt[2]], 2] + (-3*d^2 + 6*d*e - 2*e^2)*EllipticPi[(2*e
)/d, I*ArcSinh[x/Sqrt[2]], 2]))/(d*(d - 2*e)*(d - e)))/(2*d*Sqrt[2 + 3*x^2
+ x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.40

method	result
default	$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d(d^2-3ed+2e^2)(e\,x^2+d)} + \frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4(d^2-3ed+2e^2)\sqrt{x^4+3x^2+2}} - \frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4(d^2-3ed+2e^2)d\sqrt{x^4+3x^2+2}} + \frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}}{4(d^2-3ed+2e^2)d}$
elliptic	$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d(d^2-3ed+2e^2)(e\,x^2+d)} + \frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4(d^2-3ed+2e^2)\sqrt{x^4+3x^2+2}} - \frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4(d^2-3ed+2e^2)d\sqrt{x^4+3x^2+2}} + \frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}}{4(d^2-3ed+2e^2)d}$

```
[In] int(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)+1/4*I/(d^2-3*d*
e+2*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
F(1/2*I*2^(1/2)*x,2^(1/2))-1/4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/
4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-3/2*I/(d^2-3*d*e+2*e^2)*2^(1/2)
*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/
2)*x,2*e/d,2^(1/2))+3*I/(d^2-3*d*e+2*e^2)*e/d*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^
2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,2*e/d,2^(1/2))-I/
(d^2-3*d*e+2*e^2)/d^2*e^2*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^
2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,2*e/d,2^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2+1)(x^2+2)}\,(d+ex^2)^2}\,dx$$

```
[In] integrate(1/(e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(ex^2+d)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{\left(e\,x^2+d\right)^2\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int(1/((d + e*x^2)^2*(3*x^2 + x^4 + 2)^(1/2)),x)
[Out] int(1/((d + e*x^2)^2*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.399 $\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \operatorname{Int}\left(\left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p, x\right)$$

```
[Out] Unintegrable((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx$$

```
[In] Int[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] Defer[Int][(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Rubi steps

$$\text{integral} = \int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx=\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx$$

```
[In] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
[Out] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(e\,x^2+c\right)^q \left(b\,x^4+c\,x^2+a\right)^p dx$$

```
[In] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
[Out] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx=\int \left(bx^4+cx^2+a\right)^p \left(ex^2+c\right)^q \, dx$$

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx=\text{Timed out}$$

```
[In] integrate((e*x**2+c)**q*(b*x**4+c*x**2+a)**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \int \left(bx^4+cx^2+a\right)^p \left(ex^2+c\right)^q \, dx$$

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

Giac [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \int \left(bx^4+cx^2+a\right)^p \left(ex^2+c\right)^q \, dx$$

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

Mupad [N/A]

Not integrable

Time = 8.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \int {\left(e\,x^2+c\right)}^q\,{\left(b\,x^4+c\,x^2+a\right)}^p \,dx$$

```
[In] int((c + e*x^2)^q*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)^q*(a + b*x^4 + c*x^2)^p, x)
```

3.400 $\int (c+ex^2)^3 (a+cx^2+bx^4)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 498

$$\begin{aligned}
&\int (c+ex^2)^3 (a+cx^2+bx^4)^p \, dx \\
&= \frac{ce^2(21b-5e+12bp-2ep)x(a+cx^2+bx^4)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)} \\
&\quad + \frac{c(ae^3(5+2p)-3abe^2(7+4p)+b^2c^2(35+48p+16p^2))\,x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+}{b^2(5+4p)(7+4p)} \\
&\quad + \frac{e(c^2e^2(15+16p+4p^2)+3b^2c^2(35+48p+16p^2)-3be(ae(5+4p)+c^2(21+26p+8p^2)))\,x^3\left(1+\right.}{3b^2(5+4p)}
\end{aligned}$$

```
[Out] -c*e^2*(e*(5+2*p)-3*b*(7+4*p))*x*(b*x^4+c*x^2+a)^(p+1)/b^2/(16*p^2+48*p+35)
+e^3*x^3*(b*x^4+c*x^2+a)^(p+1)/b/(7+4*p)+c*(a*e^3*(5+2*p)-3*a*b*e^2*(7+4*p)
+b^2*c^2*(16*p^2+48*p+35))*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*
x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/b^2/(5+4*p)/(7+
4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/
2)))^p)+1/3*e*(c^2*e^2*(4*p^2+16*p+15)+3*b^2*c^2*(16*p^2+48*p+35)-3*b*e*(a*
e*(5+4*p)+c^2*(8*p^2+26*p+21)))*x^3*(b*x^4+c*x^2+a)^p*AppellF1(3/2,-p,-p,5/
2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/b^2/(5+4
*p)/(7+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c
^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1220, 1693, 1217, 1119, 440, 1155, 524}

$$\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx$$

$$= \frac{ex^3(-3be(ae(4p+5)+c^2(8p^2+26p+21))+3b^2c^2(16p^2+48p+35)+c^2e^2(4p^2+16p+15))\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}\right.}{3b^2(4p+5)(4p+}$$

$$+\frac{cx(-3abe^2(4p+7)+ae^3(2p+5)+b^2c^2(16p^2+48p+35))\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}(a+bx^4+cx^2)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}}\right.}{b^2(4p+5)(4p+7)}$$

$$+\frac{ce^2x(12bp+21b-2ep-5e)\left(a+bx^4+cx^2\right)^{p+1}}{b^2(4p+5)(4p+7)}+\frac{e^3x^3(a+bx^4+cx^2)^{p+1}}{b(4p+7)}$$

```
[In] Int[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (c*e^2*(21*b - 5*e + 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p))/(b^2*(5
 + 4*p)*(7 + 4*p)) + (e^3*x^3*(a + c*x^2 + b*x^4)^(1 + p))/(b*(7 + 4*p)) +
(c*(a*e^3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b^2*c^2*(35 + 48*p + 16*p^2))*x
*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b^2*(5 + 4*p)*(7 + 4*p)*
(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b
+ c^2]))^p) + (e*(c^2*e^2*(15 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p + 16*p
^2) - 3*b*e*(a*e*(5 + 4*p) + c^2*(21 + 26*p + 8*p^2)))*x^3*(a + c*x^2 + b*x
^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b
*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1217

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{e^3x^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)} + \frac{\int (a+cx^2+bx^4)^p \left(bc^3(7+4p)-3e(ae^2-bc^2(7+4p))\,x^2+ce^2(21b-5e+12bp-2ep)x^4\right)\,dx}{b(7+4p)}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x(a+cx^2+bx^4)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)} + \frac{\int (c(ae^3(5+2p)-3abe^2(7+4p)+b^2c^2(35+48p+16p^2))+e(c^2e^2(15+16p+4p^2)+3b^2c^2(35}{b^2(5+4p)(7+4p}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x(a+cx^2+bx^4)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)} + \frac{\int (c(ae^3(5+2p)-3abe^2(7+4p)+b^2c^2(35+48p+16p^2))\,(a+cx^2+bx^4)^p+e(c^2e^2(15+16p+}{b^2(5+4p}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x(a+cx^2+bx^4)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)} + \frac{(c(ae^3(5+2p)-3abe^2(7+4p)+b^2c^2(35+48p+16p^2)))\int (a+cx^2+bx^4)^p\,dx}{b^2(5+4p)(7+4p)} + \frac{(e(c^2e^2(15+16p+4p^2)+3b^2c^2(35+48p+16p^2)-3be(ae(5+4p)+c^2(21+26p+8p^2))))\int x^2}{b^2(5+4p)(7+4p)}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x(a+cx^2+bx^4)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)} + \frac{\Big(c(ae^3(5+2p)-3abe^2(7+4p)+b^2c^2(35+48p+16p^2))\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)}{b^2(5+4p)(7+4p)} + \frac{\Big(e(c^2e^2(15+16p+4p^2)+3b^2c^2(35+48p+16p^2)-3be(ae(5+4p)+c^2(21+26p+8p^2)))\Big(1}{b^2(5+}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x(a+cx^2+bx^4)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)} + \frac{c(ae^3(5+2p)-3abe^2(7+4p)+b^2c^2(35+48p+16p^2))\,x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-}}{b^2(5+4p)(7+4p)} + \frac{e(c^2e^2(15+16p+4p^2)+3b^2c^2(35+48p+16p^2)-3be(ae(5+4p)+c^2(21+26p+8p^2)))\,x^3\Big(1}{3b^2(5+}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.75

$$\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx$$

$$= \frac{1}{35}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\Big(a+cx^2$$

$$+bx^4\Big)^p\Bigg(35c^3\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)$$

$$+ex^2\Bigg(35c^2\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)$$

$$+ex^2\Bigg(21c\operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+5ex^2\operatorname{AppellF1}\Bigg(\frac{7}{2},-p,-p$$

```
[In] Integrate[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(35*c^3*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
 Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(35*c^2*
AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(
-c + Sqrt[-4*a*b + c^2])] + e*x^2*(21*c*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^
2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + 5*e*x^2
*AppellF1[7/2, -p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/
(-c + Sqrt[-4*a*b + c^2])]))))/(35*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c -
 Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*
b + c^2]))^p)
```

Maple [F]

$$\int \left(e\,x^{2}+c\right)^{3} \left(b\,x^{4}+c\,x^{2}+a\right)^{p}dx$$

```
[In] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

Fricas [F]

$$\int \left(c+ex^{2}\right)^{3} \left(a+cx^{2}+bx^{4}\right)^{p} dx = \int \left(ex^{2}+c\right)^{3}\left(bx^{4}+cx^{2}+a\right)^{p} dx$$

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + c*x^2 + a)^p,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+ex^{2}\right)^{3} \left(a+cx^{2}+bx^{4}\right)^{p} dx = \text{Timed out}$$

```
[In] integrate((e*x**2+c)**3*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(c+ex^{2}\right)^{3} \left(a+cx^{2}+bx^{4}\right)^{p} dx = \int \left(ex^{2}+c\right)^{3}\left(bx^{4}+cx^{2}+a\right)^{p} dx$$

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx=\int \left(ex^2+c\right)^3 \left(bx^4+cx^2+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx=\int {\left(e\,x^2+c\right)}^3\,{\left(b\,x^4+c\,x^2+a\right)}^p \,d x$$

```
[In] int((c + e*x^2)^3*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)^3*(a + b*x^4 + c*x^2)^p, x)
```

3.401 $\int (c+ex^2)^2 (a+cx^2+bx^4)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 358

$$\int (c+ex^2)^2 (a+cx^2+bx^4)^p \, dx = \frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)}$$

$$-\frac{(ae^2-bc^2(5+4p))\,x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\,\mathrm{AppellF1}\,\Big(\frac{1}{2},-p,-p,\frac{3}{2},-}{b(5+4p)}$$

$$+\frac{ce(10b-3e+8bp-2ep)x^3\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\,\mathrm{AppellF1}\,\Big(\frac{3}{2},-p,-}{3b(5+4p)}$$

```
[Out] e^2*x*(b*x^4+c*x^2+a)^(p+1)/b/(5+4*p)-(a*e^2-b*c^2*(5+4*p))*x*(b*x^4+c*x^2+
a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4
*a*b+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*
x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/3*c*e*(8*b*p-2*e*p+10*b-3*e)*x^3*(b*x^4+c*
x^2+a)^p*AppellF1(3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c
+(-4*a*b+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+
2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{1220, 1217, 1119, 440, 1155, 524\}$

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = x\left(c^2-\frac{ae^2}{4bp+5b}\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+\frac{1}{3}cex^3\left(2-\frac{e(2p+3)}{b(4p+5)}\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+\frac{e^2x\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}$$

```
[In] Int[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b +
4*b*p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (
c*e*(2 - (e*(3 + 2*p))/(b*(5 + 4*p)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/
2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-
4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2
)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1217

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)} \\
&\quad + \frac{\int \left(-ae^2+bc^2(5+4p)+ce(10b-3e+8bp-2ep)x^2\right)\left(a+cx^2+bx^4\right)^p\,dx}{b(5+4p)} \\
&= \frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)} \\
&\quad + \frac{\int \left(-ae^2\left(1-\frac{bc^2(5+4p)}{ae^2}\right)(a+cx^2+bx^4)^p-ce(-10b+3e-8bp+2ep)x^2(a+cx^2+bx^4)^p\right)dx}{b(5+4p)}
\end{aligned}
$$

$$\begin{aligned}
&= \frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)}+\left(ce\left(2-\frac{e(3+2p)}{b(5+4p)}\right)\right)\int x^2\left(a+cx^2+bx^4\right)^p\,dx \\
&\quad -\left(-c^2+\frac{ae^2}{5b+4bp}\right)\int \left(a+cx^2+bx^4\right)^p\,dx \\
&= \frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)} \\
&\quad +\left(ce\left(2-\frac{e(3+2p)}{b(5+4p)}\right)\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a\right.\right. \\
&\qquad \left.\left.+cx^2+bx^4\right)^p\right)\int x^2\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^p\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^p dx \\
&\quad -\left(\left(-c^2+\frac{ae^2}{5b+4bp}\right)\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a\right.\right. \\
&\qquad \left.\left.+cx^2+bx^4\right)^p\right)\int \left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^p\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^p dx \\
&= \frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)} \\
&\quad +\left(c^2-\frac{ae^2}{5b+4bp}\right)x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a\right. \\
&\qquad \left.+cx^2+bx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right) \\
&\quad +\frac{1}{3}ce\left(2-\frac{e(3+2p)}{b(5+4p)}\right)x^3\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a\right. \\
&\qquad \left.+cx^2+bx^4\right)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \left(c+ex^2\right)^2\left(a+cx^2+bx^4\right)^p\,dx \\
&= \frac{1}{15}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2\right. \\
&\quad \left.+bx^4\right)^p\left(15c^2\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right. \\
&\quad +ex^2\left(10c\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right. \\
&\quad \left.\left.+3ex^2\operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)\right)
\end{aligned}$$

```
[In] Integrate[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(15*c^2*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
 Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(10*c*Ap
pellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c
 + Sqrt[-4*a*b + c^2])] + 3*e*x^2*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^2)/(c
+ Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])))/(15*((c - Sq
rt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b +
 c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F]

$$\int \left(e\,x^{2}+c\right)^{2} \left(b\,x^{4}+c\,x^{2}+a\right)^{p}dx$$

```
[In] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

Fricas [F]

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \,dx=\int \left(ex^{2}+c\right)^{2}{\left(bx^{4}+cx^{2}+a\right)}^{p}\,dx$$

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+c)**2*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^2 \left(bx^4+cx^2+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^2 \left(bx^4+cx^2+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = \int \left(e\,x^2+c\right)^2 \left(b\,x^4+c\,x^2+a\right)^p \, dx$$

```
[In] int((c + e*x^2)^2*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)^2*(a + b*x^4 + c*x^2)^p, x)
```

3.402 $\int (c+ex^2)(a+cx^2+bx^4)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int (c+ex^2)\left(a+cx^2+bx^4\right)^p\,dx = cx\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)+\frac{1}{3}ex^3\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)$$

```
[Out] c*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)
),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/(
(1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/3*e*x^3*(b*x^4+c*x^2+a)^p*AppellF1(
3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2
)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2
)))^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1217, 1119, 440, 1155, 524}

$$\int \left(c+ex^2\right)\left(a+cx^2+bx^4\right)^p\,dx=\frac{1}{3}ex^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+cx\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

```
[In] Int[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[
-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*x^3
*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1217

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(c\left(a+cx^2+bx^4\right)^p+ex^2\left(a+cx^2+bx^4\right)^p\right)\,dx \\
&= c\int \left(a+cx^2+bx^4\right)^p\,dx+e\int x^2\left(a+cx^2+bx^4\right)^p\,dx \\
&= \left(c\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\right)\int \left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^p\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^p\,dx \\
&\quad +\left(e\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\right)\int x^2\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^p\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^p\,dx
\end{aligned}
$$

$$= cx\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)+\frac{1}{3}ex^3\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)$$

Mathematica [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.85

$$\int (c+ex^2)\left(a+cx^2+bx^4\right)^p\,dx = \frac{1}{3}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\left(3c\,\mathrm{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+ex^2\,\mathrm{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)$$

```
[In] Integrate[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(3*c*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sq
rt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*AppellF1[3/
2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-
4*a*b + c^2])]))/(3*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b +
c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F]

$$\int \left(e\,x^2+c\right)\left(b\,x^4+c\,x^2+a\right)^p dx$$

```
[In] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

Fricas [F]

$$\int \left(c+ex^2\right) \left(a+cx^2+bx^4\right)^p \, dx=\int \left(ex^2+c\right) \left(bx^4+cx^2+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F]

$$\int \left(c+ex^2\right) \left(a+cx^2+bx^4\right)^p \, dx=\int \left(c+ex^2\right) \left(a+bx^4+cx^2\right)^p \, dx$$

```
[In] integrate((e*x**2+c)*(b*x**4+c*x**2+a)**p,x)
[Out] Integral((c + e*x**2)*(a + b*x**4 + c*x**2)**p, x)
```

Maxima [F]

$$\int \left(c+ex^2\right) \left(a+cx^2+bx^4\right)^p \, dx=\int \left(ex^2+c\right) \left(bx^4+cx^2+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(c+ex^2\right) \left(a+cx^2+bx^4\right)^p \, dx=\int \left(ex^2+c\right) \left(bx^4+cx^2+a\right)^p \, dx$$

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+ex^2\right) \left(a+cx^2+bx^4\right)^p \, dx = \int \left(e\,x^2+c\right)\,\left(b\,x^4+c\,x^2+a\right)^p \,dx$$

```
[In] int((c + e*x^2)*(a + b*x^4 + c*x^2)^p,x)
```

```
[Out] int((c + e*x^2)*(a + b*x^4 + c*x^2)^p, x)
```

3.403 $\int (a+cx^2+bx^4)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 133

$$\int (a+cx^2+bx^4)^p \, dx = x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)$$

```
[Out] x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),
-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1
+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1119, 440}

$$\int (a+cx^2+bx^4)^p \, dx = x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}(a+bx^4+cx^2)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

```
[In] Int[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4
*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\right)\int\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^p\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^p dx\\
&= x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int\left(a+cx^2+bx^4\right)^p\,dx = x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)$$

```
[In] Integrate[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4
*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/(((c - Sqrt[-4*a*b + c^
2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^
2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F]

$$\int \left(b\,x^{4}+c\,x^{2}+a\right)^{p}dx$$

```
[In] int((b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((b*x^4+c*x^2+a)^p,x)
```

Fricas [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(bx^4+cx^2+a\right)^p\,dx$$

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(a+bx^4+cx^2\right)^p\,dx$$

```
[In] integrate((b*x**4+c*x**2+a)**p,x)
```

```
[Out] Integral((a + b*x**4 + c*x**2)**p, x)
```

Maxima [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(bx^4+cx^2+a\right)^p\,dx$$

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(bx^4+cx^2+a\right)^p\,dx$$

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(b\,x^4+c\,x^2+a\right)^p\,dx$$

```
[In] int((a + b*x^4 + c*x^2)^p,x)
[Out] int((a + b*x^4 + c*x^2)^p, x)
```

3.404 $\int \frac{(a+cx^2+bx^4)^p}{c+ex^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2} \, dx = \text{Int}\left(\frac{(a+cx^2+bx^4)^p}{c+ex^2}, x\right)$$

```
[Out] Unintegrable((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2} \, dx = \int \frac{(a+cx^2+bx^4)^p}{c+ex^2} \, dx$$

```
[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]
[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+cx^2+bx^4)^p}{c+ex^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx=\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

```
[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]
[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(b\,x^{4}+c\,x^{2}+a\right)^{p}}{e\,x^{2}+c}d x$$

```
[In] int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx=\int \frac{(bx^4+cx^2+a)^p}{ex^2+c}\,dx$$

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="fricas")
[Out] integral((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx=\int \frac{(bx^4+cx^2+a)^p}{ex^2+c}\,dx$$

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx=\int \frac{(bx^4+cx^2+a)^p}{ex^2+c}\,dx$$

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

Mupad [N/A]

Not integrable

Time = 7.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx=\int \frac{(b\,x^4+c\,x^2+a)^p}{e\,x^2+c}\,dx$$

```
[In] int((a + b*x^4 + c*x^2)^p/(c + e*x^2),x)
[Out] int((a + b*x^4 + c*x^2)^p/(c + e*x^2), x)
```

3.405 $\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx = \operatorname{Int}\left(\frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2},x\right)$$

```
[Out] Unintegrable((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx = \int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx$$

```
[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]
```

```
[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx$$

```
[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]
[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(b\,x^4+c\,x^2+a)^p}{(e\,x^2+c)^2}dx$$

```
[In] int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2}\,dx$$

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
[Out] integral((b*x^4 + c*x^2 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2}\,dx$$

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2}\,dx$$

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 8.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(b\,x^4+c\,x^2+a)^p}{(e\,x^2+c)^2}\,dx$$

```
[In] int((a + b*x^4 + c*x^2)^p/(c + e*x^2)^2,x)
[Out] int((a + b*x^4 + c*x^2)^p/(c + e*x^2)^2, x)
```

3.406 $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 446

$$\begin{aligned}\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx &= \frac{(ef-dg)\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{(ef-dg)\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} \\ &\quad + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\ &\quad - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}\end{aligned}$$

```
[Out] 1/2*(-d*g+e*f)*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c
*d^4)^(1/2)-1/2*(-d*g+e*f)*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c
*x^4+a)^(1/2))/(a*e^4+c*d^4)^(1/2)-1/4*(-d*g+e*f)*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2)
,1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/e/(e^2*a^(1/2)+d^2*c^(1/2))
/(c*x^4+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(e*g*a^(1/2)+d*f*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*
c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1756, 12, 1262, 739, 212, 1723, 226, 1721}

$$\begin{aligned}\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = {} & \frac{(ef-dg)\arctan\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}} \\ & + \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} \\ & - \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)} \\ & - \frac{(ef-dg)\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}\end{aligned}$$

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*S
qrt[-(c*d^4) - a*e^4]) - ((e*f - d*g)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d
^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sq
rt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt
[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f
 - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1756

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\text{integral} = \int \frac{(-ef+dg)x}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx + \int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx$$

$$
\begin{aligned}
&= \frac{\left(\sqrt{a}de(ef-dg)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2} \\
&\quad + (-ef+dg)\int \frac{x}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} \\
&\quad + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\
&\quad - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\
&\quad + \frac{1}{2}(-ef+dg)\text{Subst}\left(\int \frac{1}{(d^2-e^2x)\sqrt{a+cx^2}}\,dx,x,x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} \\
&\quad + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\
&\quad - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\
&\quad + \frac{1}{2}(ef-dg)\text{Subst}\left(\int \frac{1}{cd^4+ae^4-x^2}\,dx,x,\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} \\
&\quad + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\
&\quad - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.75 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.61

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \frac{-\frac{ig\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}+\frac{(ef-dg)\Big(\sqrt[4]{c}de\sqrt{a+cx^4}\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)-\sqrt[4]{-1}\sqrt[4]{a}\sqrt{-cd^4-ae^4}}{\sqrt[4]{c}d\sqrt{-cd^4-ae^4}}}{e\sqrt{a+cx^4}}$$

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (((-I)*g*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*
x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + ((e*f - d*g)*(c^(1/4)*d*e*Sqrt[a + c*x
^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) -
a*e^4]] - (-1)^(1/4)*a^(1/4)*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*Ell
ipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4
)], -1]))/(c^(1/4)*d*Sqrt[-(c*d^4) - a*e^4]))/(e*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.56

method	result
default	$\frac{g\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{(-dg+ef)\Bigg(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\Big(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}}{e^2}$
elliptic	$\frac{g\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{(dg-ef)\Bigg(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\Big(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}}{e^2}$

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g/e/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

```
+(-d*g+e*f)/e^2*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+2*a)
/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*e/d*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*
EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^2,(-I/a^(1/
2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{f+gx}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+a)**(1/2),x)
[Out] Integral((f + g*x)/(sqrt(a + c*x**4)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{gx+f}{\sqrt{cx^4+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx=\int \frac{f+g\,x}{\sqrt{c\,x^4+a}\,(d+e\,x)}\,dx$$

```
[In] int((f + g*x)/((a + c*x^4)^(1/2)*(d + e*x)),x)
```

```
[Out] int((f + g*x)/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

3.407 $\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 218

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \frac{(ef-dg)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cd^4-ae^4}\sqrt{-a+cx^4}}\right)}{2\sqrt{cd^4-ae^4}} + \frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}(ef-dg)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{-a+cx^4}}$$

```
[Out] 1/2*(-d*g+e*f)*arctanh((-c*d^2*x^2+a*e^2)/(-a*e^4+c*d^4)^(1/2)/(c*x^4-a)^(1
/2))/(-a*e^4+c*d^4)^(1/2)+a^(1/4)*g*EllipticF(c^(1/4)*x/a^(1/4),I)*(1-c*x^4
/a)^(1/2)/c^(1/4)/e/(c*x^4-a)^(1/2)+a^(1/4)*(-d*g+e*f)*EllipticPi(c^(1/4)*x
/a^(1/4),e^2*a^(1/2)/d^2/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/d/e/(c*x^4-a)
^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules

used = {1756, 12, 1262, 739, 212, 1725, 230, 227, 1233, 1232}

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(ef-dg)\,\mathrm{EllipticPi}\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{cx^4-a}} + \frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}} + \frac{(ef-dg)\mathrm{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqrt[-a + c*x
^4])])/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[A
rcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*Sqrt[-a + c*x^4]) + (a^(1/4)*(e
*f - d*g)*Sqrt[1 - (c*x^4)/a]*EllipticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1725

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + c*x^4], x], x] + Dist[(e*A - d*B)/
e, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B},
 x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 1756

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(-ef+dg)x}{(d^2-e^2x^2)\sqrt{-a+cx^4}}\,dx + \int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{-a+cx^4}}\,dx \\
&= \frac{g\int \frac{1}{\sqrt{-a+cx^4}}\,dx}{e} + \frac{(d(ef-dg))\int \frac{1}{(d^2-e^2x^2)\sqrt{-a+cx^4}}\,dx}{e} \\
&\quad + (-ef+dg)\int \frac{x}{(d^2-e^2x^2)\sqrt{-a+cx^4}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}(-ef+dg)\text{Subst}\left(\int \frac{1}{(d^2-e^2x)\sqrt{-a+cx^2}}\,dx, x, x^2\right) \\
&\quad + \frac{\left(g\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{e\sqrt{-a+cx^4}} + \frac{\left(d(ef-dg)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{(d^2-e^2x^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{e\sqrt{-a+cx^4}} \\
&= \frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}e\sqrt{-a+cx^4}} \\
&\quad + \frac{\sqrt[4]{a}(ef-dg)\sqrt{1-\frac{cx^4}{a}}\Pi\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}de\sqrt{-a+cx^4}} \\
&\quad + \frac{1}{2}(ef-dg)\text{Subst}\left(\int \frac{1}{cd^4-ae^4-x^2}\,dx, x, \frac{ae^2-cd^2x^2}{\sqrt{-a+cx^4}}\right) \\
&= \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cd^4-ae^4}\sqrt{-a+cx^4}}\right)}{2\sqrt{cd^4-ae^4}} + \frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}e\sqrt{-a+cx^4}} \\
&\quad + \frac{\sqrt[4]{a}(ef-dg)\sqrt{1-\frac{cx^4}{a}}\Pi\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}de\sqrt{-a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.11 (sec) , antiderivative size = 719, normalized size of antiderivative = 3.30

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx$$

$$= -\frac{ig\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}e} + \frac{if\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2\sqrt{-\frac{(1-i)\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{i\sqrt[4]{a}+\sqrt[4]{c}x}}\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+i\sqrt[4]{c}x\right)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2}}\Big(\Big(}{}$$

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (((-I)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x
], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*e) + (I*f*(a^(1/4) - I*c^(1/4)*x)^2*Sqrt[
((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqrt[((1 + I)*(a^
(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*((-(
c^(1/4)*d) + a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x
))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] - (1 - I)*a^(1/4)*e*EllipticPi[((1 -
```

```
I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin[Sqrt[((1 + I
)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))/(a^(1/4)*(-(
c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)) + (d*g*(a^(1/4) - I*c^(1
/4)*x)^2*Sqrt[((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqr
t[((1 + I)*(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1
/4)*x)^2]*(I*(c^(1/4)*d - a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4
) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] + (1 + I)*a^(1/4)*e*Ell
ipticPi[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin
[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))
/(a^(1/4)*e*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)))/Sqrt[-a
+ c*x^4]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.13

method	result
default	$\frac{g\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{(-dg+ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}-2a}{2\sqrt{\frac{c\,d^4}{e^4}-a}\,\sqrt{c\,x^4-a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}-a}}+\frac{e\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},-\frac{e^2\sqrt{c}}{d^2\sqrt{c}}\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4-a}}\right.}{e^2}$
elliptic	$\frac{g\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}-\frac{(dg-ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}-2a}{2\sqrt{\frac{c\,d^4}{e^4}-a}\,\sqrt{c\,x^4-a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}-a}}+\frac{e\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},-\frac{e^2\sqrt{a}}{d^2\sqrt{c}},\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4-a}}\right.}{e^2}$

```
[In] int((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g/e/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),
I)+(-d*g+e*f)/e^2*(-1/2/(c/e^4*d^4-a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2-2*
a)/(c/e^4*d^4-a)^(1/2)/(c*x^4-a)^(1/2))+1/(-1/a^(1/2)*c^(1/2))^(1/2)*e/d*(1
+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/
2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/2),-e^2*a^(1/2)/d^2/c^(1/2),(1/a^(1
/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx=\text{Timed out}$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx=\int \frac{f+gx}{\sqrt{-a+cx^4}\,(d+ex)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4-a)**(1/2),x)
[Out] Integral((f + g*x)/(sqrt(-a + c*x**4)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4-a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4-a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx=\int \frac{f+g\,x}{\sqrt{c\,x^4-a}\,\left(d+e\,x\right)}\,dx$$

```
[In] int((f + g*x)/((c*x^4 - a)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)/((c*x^4 - a)^(1/2)*(d + e*x)), x)
```

3.408 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 65

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

```
[Out] 1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/
2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1754, 213}

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{2\sqrt{3}-3}\text{arctanh}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

integral =

$$-\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+4x^2}\,dx,x,\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)\right)$$

$$=\frac{1}{3}\sqrt{-3+2\sqrt{3}}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int\frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$$

$$=\frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\sqrt{9+6\sqrt{3}}\sqrt{-4+4\sqrt{3}x^2+x^4}}{2+\left(-2-2\sqrt{3}\right)x+\left(2+\sqrt{3}\right)x^2}\right)$$

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-4 + 4*Sqrt[3]*x^2
+ x^4])/(2 + (-2 - 2*Sqrt[3])*x + (2 + Sqrt[3])*x^2)])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.54 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.03

method	result
elliptic	$\frac{\sqrt{1-\left(\frac{\sqrt{3}}{2}-1\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\,F\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4x^2\sqrt{3}}}-2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{4\sqrt{3}\left(-1-\sqrt{3}\right)^2-8+4x^2\sqrt{3}+2x^2(}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4\sqrt{3}\left(-1-\sqrt{3}\right)^2-4}}\right.}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4\sqrt{3}\left(-1-\sqrt{3}\right)}}\right.$

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^
2)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*(
1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3^(1/
2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*x^2*3^
(1/2)+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/
2)/(-4+x^4+4*x^2*3^(1/2))^(1/2))-1/(1/2*3^(1/2)-1)^(1/2)/(-1-3^(1/2))*(1-(1
/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*x^2*3^(1/2
))^(1/2)*EllipticPi((1/2*3^(1/2)-1)^(1/2)*x,1/(1/2*3^(1/2)-1)/(-1-3^(1/2))^
2,(1+1/2*3^(1/2))^(1/2)/(1/2*3^(1/2)-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(47) = 94.

Time = 0.40 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.97

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$$

$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37\,x^{12}-204\,x^{11}+804\,x^{10}-2408\,x^{9}+3708\,x^{8}-5472\,x^{7}+6432\,x^{6}+10944\,x^{5}+14}{}\right.$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
```

```
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*x**2*3**(1/2))**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\,\sqrt{x^4+4\,\sqrt{3}\,x^2-4}}\,dx$$

```
[In] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),x
)
```

```
[Out] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

3.409 $\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 63

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

```
[Out] -1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/2
))*(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1754, 209}

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]
)]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

integral =

$$-\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\left(1+\sqrt{3}\right)^4+4x^2}\,dx,x,\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)\right)$$

$$=-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int\frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$$

$$=-\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-4-4\sqrt{3}x^2+x^4}}{-2+\left(2-2\sqrt{3}\right)x+\left(-2+\sqrt{3}\right)x^2}\right)$$

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-4 - 4*Sqrt[3]*
x^2 + x^4])/(-2 + (2 - 2*Sqrt[3])*x + (-2 + Sqrt[3])*x^2)])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.42 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.94

method	result
elliptic	$\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{\sqrt{3}}{2}+1\right)x^{2}}\,F\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right),i\sqrt{1-4\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^{4}-4x^{2}\sqrt{3}}}+2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{-4\sqrt{3}\left(\sqrt{3}-1\right)^{2}-8-4x^{2}\sqrt{}}{2\sqrt{\left(\sqrt{3}-1\right)^{4}-4\sqrt{3}\left(\sqrt{3}-1\right)^{2}-}}\right)}{2\sqrt{\left(\sqrt{3}-1\right)^{4}-4\sqrt{3}\left(\sqrt{3}\right.}}\right.$

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*
x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I
*(1-4*3^(1/2)*(-1/2*3^(1/2)+1))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^(
1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*x^2*3
^(1/2)+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)
/(-4+x^4-4*x^2*3^(1/2))^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-
1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2
))^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/2)-1)
^2,(-1/2*3^(1/2)+1)^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(45) = 90.

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^{2}+x^{4}}}\,dx$$

$$=\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^{4}-30x^{3}+18x^{2}-2\sqrt{3}(2x^{4}-10x^{3}+3x^{2}-10x+2)+24\right)\sqrt{x^{4}-4\sqrt{3}x}}{11x^{6}-42x^{5}+66x^{4}-176x^{3}-132x^{2}-168x-88}\right.$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*x**2*3**(1/2))**(1/2),x)
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x, algor
ithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x, algor
ithm="giac")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

```
[In] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),x
)
[Out] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

3.410 $\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 72

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-1+4\sqrt{3}x^2+4x^4}}\right)$$

```
[Out] 1/3*arctanh(1/2*(1+2*x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-1+4*x^4+4*x^2*3^(1
/2))^(1/2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1754, 213}

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \frac{1}{3}\sqrt{2\sqrt{3}-3}\text{arctanh}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)$$

```
[In] Int[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^
4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3*(-3 + 2*Sqrt[
3])]*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \\
&-\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)^4+12\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+2x^2}\,dx, x, \frac{\left(1-\sqrt{3}+2x\right)^2}{\sqrt{-1+4\sqrt{3}x^2+4x^4}}\right)\right)\\
&= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-1+4\sqrt{3}x^2+4x^4}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 8.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\text{arctanh}\left(\frac{\sqrt{9+6\sqrt{3}}\sqrt{-1+4\sqrt{3}x^2+4x^4}}{1+\left(-2-2\sqrt{3}\right)x+\left(4+2\sqrt{3}\right)x^2}\right)$$

```
[In] Integrate[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2
+ 4*x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-1 + 4*Sqrt[3]*x^2
+ 4*x^4])/(1 + (-2 - 2*Sqrt[3])*x + (4 + 2*Sqrt[3])*x^2)])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.38 (sec) , antiderivative size = 336, normalized size of antiderivative = 4.67

method	result
elliptic	$\frac{\sqrt{1-\left(2\sqrt{3}-4\right)x^2}\,\sqrt{1-\left(2\sqrt{3}+4\right)x^2}\,F\left(x\left(i\sqrt{3}-i\right),i\sqrt{1+\sqrt{3}\left(2\sqrt{3}+4\right)}\right)}{\left(i\sqrt{3}-i\right)\sqrt{-1+4x^4+4x^2\sqrt{3}}}-\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{4\sqrt{3}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2-2+4x^2\sqrt{3}+}{2\sqrt{4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^4+4\sqrt{3}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2}}\right.}{2\sqrt{4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^4+4\sqrt{3}\,(-\frac{1}{2}-}}\right.$

```
[In] int((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/(I*3^(1/2)-I)*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)^(1/2)/(-1
+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I*3^(1/2)-I),I*(1+3^(1/2)*(2*3^(1/
2)+4))^(1/2))-3^(1/2)*(-1/2/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2*3^(
1/2))^2-1)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-2+4*x^2*3^(1/2
)+8*x^2*(-1/2-1/2*3^(1/2))^2)/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2*3
^(1/2))^2-1)^(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2))-1/(2*3^(1/2)-4)^(1/2)/(-
1/2-1/2*3^(1/2))*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)^(1/2)/(-
1+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticPi((2*3^(1/2)-4)^(1/2)*x,1/(2*3^(1/2)-
4)/(-1/2-1/2*3^(1/2))^2,(2*3^(1/2)+4)^(1/2)/(2*3^(1/2)-4)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(52) = 104.

Time = 0.39 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.56

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx$$
$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{2368\,x^{12}-6528\,x^{11}+12864\,x^{10}-19264\,x^{9}+14832\,x^{8}-10944\,x^{7}+6432\,x^{6}+547}{}\right.$$

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(2368*x^12 - 6528*x^11 + 12864*x^10 - 19264*x
^9 + 14832*x^8 - 10944*x^7 + 6432*x^6 + 5472*x^5 + 3708*x^4 + 2408*x^3 + 80
4*x^2 + (1728*x^10 - 4800*x^9 + 8208*x^8 - 8928*x^7 + 6048*x^6 - 3024*x^5 -
 504*x^4 - 504*x^3 - 324*x^2 + 2*sqrt(3)*(496*x^10 - 1408*x^9 + 2304*x^8 -
2640*x^7 + 1848*x^6 - 504*x^5 + 336*x^4 + 204*x^3 + 63*x^2 + 26*x + 4) - 72
```

```
*x - 15)*sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(4
48*x^12 - 1280*x^11 + 2560*x^10 - 3200*x^9 + 3696*x^8 - 1920*x^7 - 960*x^5
- 924*x^4 - 400*x^3 - 160*x^2 - 40*x - 7) + 204*x + 37)/(64*x^12 + 384*x^11
 + 768*x^10 + 320*x^9 - 720*x^8 - 576*x^7 + 384*x^6 + 288*x^5 - 180*x^4 - 4
0*x^3 + 48*x^2 - 12*x + 1))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2x-\sqrt{3}+1}{\left(2x+1+\sqrt{3}\right)\sqrt{4x^4+4\sqrt{3}x^2-1}}\,dx$$

```
[In] integrate((1+2*x-3**(1/2))/(1+2*x+3**(1/2))/(-1+4*x**4+4*x**2*3**(1/2))**(1
/2),x)
```

```
[Out] Integral((2*x - sqrt(3) + 1)/((2*x + 1 + sqrt(3))*sqrt(4*x**4 + 4*sqrt(3)*x
**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^4+4\,\sqrt{3}x^2-1}\left(2\,x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(
3) + 1)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^4+4\,\sqrt{3}x^2-1}\left(2\,x+\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(
3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^4+4\,\sqrt{3}\,x^2-1}\,\left(2\,x+\sqrt{3}+1\right)}\,dx$$

```
[In] int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2) +
 1)),x)
[Out] int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2) +
 1)), x)
```

3.411 $\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 70

$$\begin{aligned}&\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx\\&=-\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-1-4\sqrt{3}x^2+4x^4}}\right)\end{aligned}$$

```
[Out] -1/3*arctan(1/2*(1+2*x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-1+4*x^4-4*x^2*3^(1/
2))^(1/2))*(3+2*3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1754, 209}

$$\begin{aligned}&\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx\\&=-\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}x^2-1}}\right)\end{aligned}$$

```
[In] Int[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^
4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 + 2*Sqr
t[3])]*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

integral =

$$-\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{12\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+6\left(1+\sqrt{3}\right)^4+2x^2}\,dx,x,\frac{\left(1+\sqrt{3}+2x\right)^2}{\sqrt{-1-4\sqrt{3}x^2+4x^4}}\right)\right)$$

$$=-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-1-4\sqrt{3}x^2+4x^4}}\right)$$

Mathematica [A] (verified)

Time = 8.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int\frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx$$

$$=-\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-1-4\sqrt{3}x^2+4x^4}}{-1+\left(2-2\sqrt{3}\right)x+\left(-4+2\sqrt{3}\right)x^2}\right)$$

```
[In] Integrate[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2
+ 4*x^4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-1 - 4*Sqrt[3]*
x^2 + 4*x^4])/(-1 + (2 - 2*Sqrt[3])*x + (-4 + 2*Sqrt[3])*x^2)])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.35 (sec) , antiderivative size = 336, normalized size of antiderivative = 4.80

method	result
elliptic	$\frac{\sqrt{1-\left(-2\sqrt{3}-4\right)x^{2}}\,\sqrt{1-\left(-2\sqrt{3}+4\right)x^{2}}\,F\left(x\left(i+i\sqrt{3}\right),i\sqrt{1-\sqrt{3}\left(-2\sqrt{3}+4\right)}\right)}{\left(i+i\sqrt{3}\right)\sqrt{-1+4x^{4}-4x^{2}\sqrt{3}}}+\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{-4\sqrt{3}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^{2}-2-4x^{2}\sqrt{}}{2\sqrt{4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^{4}-4\sqrt{3}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)}}\right.}{2\sqrt{4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^{4}-4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right.}}\right.$

```
[In] int((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/(I+I*3^(1/2))*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2)/(
-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I+I*3^(1/2)),I*(1-3^(1/2)*(-2*3^
(1/2)+4))^(1/2))+3^(1/2)*(-1/2/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/2
)-1/2)^2-1)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-2-4*x^2*3^(1/
2)+8*x^2*(1/2*3^(1/2)-1/2)^2)/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/2)
-1/2)^2-1)^(1/2)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2))-1/(-2*3^(1/2)-4)^(1/2)/(1/
2*3^(1/2)-1/2)*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2)/(-
1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticPi((-2*3^(1/2)-4)^(1/2)*x,1/(-2*3^(1/2
)-4)/(1/2*3^(1/2)-1/2)^2,(-2*3^(1/2)+4)^(1/2)/(-2*3^(1/2)-4)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(50) = 100$.

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx$$

$$=\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(36x^4-60x^3+18x^2-\sqrt{3}(16x^4-40x^3+6x^2-10x+1)+6\right)\sqrt{4x^4-4\sqrt{3}x}}{88x^6-168x^5+132x^4-176x^3-66x^2-42x-11}\right.$$

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(36*x^4 - 60*x^3 + 18*x^2 - sqrt(3)*(16*x^4
 - 40*x^3 + 6*x^2 - 10*x + 1) + 6)*sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*sqrt(2*s
qrt(3) + 3)/(88*x^6 - 168*x^5 + 132*x^4 - 176*x^3 - 66*x^2 - 42*x - 11))
```

Sympy [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2x+1+\sqrt{3}}{\left(2x-\sqrt{3}+1\right)\sqrt{4x^4-4\sqrt{3}x^2-1}}\,dx$$

```
[In] integrate((1+2*x+3**(1/2))/(1+2*x-3**(1/2))/(-1+4*x**4-4*x**2*3**(1/2))**(1
/2),x)
```

```
[Out] Integral((2*x + 1 + sqrt(3))/((2*x - sqrt(3) + 1)*sqrt(4*x**4 - 4*sqrt(3)*x
**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}x^2-1}\left(2\,x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(
3) + 1)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}x^2-1}\left(2\,x-\sqrt{3}+1\right)}\,dx$$

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(
3) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}\,x^2-1}\,\left(2\,x-\sqrt{3}+1\right)}\,dx$$

```
[In] int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2) +
 1)),x)
```

```
[Out] int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2) +
 1)), x)
```

3.412 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 560

$$\begin{aligned}
&\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx \\
&= \frac{(ef-dg)\arctan\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} - \frac{(ef-dg)\text{arctanh}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2+ae^4}} \\
&\quad + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}} \\
&\quad - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\Bigg(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\Big(2-}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

```
[Out] 1/2*(-d*g+e*f)*arctan(x*(-a*e^4-b*d^2*e^2-c*d^4)^(1/2)/d/e/(c*x^4+b*x^2+a)^
(1/2))/(-a*e^4-b*d^2*e^2-c*d^4)^(1/2)-1/2*(-d*g+e*f)*arctanh(1/2*(b*d^2+2*a
*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^(1/
2))/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)-1/4*(-d*g+e*f)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),
1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^
(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/e/
(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x
/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*g*a^(1/2)+d*f*c^(1
/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
a^(1/4)/c^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1755, 12, 1261, 738, 212, 1722, 1117, 1720}

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$

$$- \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\,\text{EllipticPi}\Bigg(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\Big(2-}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$

$$+ \frac{(ef-dg)\arctan\left(\frac{x\sqrt{-ae^4-bd^2e^2-cd^4}}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-e^2\left(ae^2+bd^2\right)-cd^4}} - \frac{(ef-dg)\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - b*d^2*e^2 - a*e^4]*x)/(d*e*Sqrt[a + b*
x^2 + c*x^4])])/(2*Sqrt[-(c*d^4) - e^2*(b*d^2 + a*e^2)]) - ((e*f - d*g)*Arc
Tanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2 +
a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((S
qrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[
a + b*x^2 + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[
(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c
]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1755

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
```

```
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rubi steps

$$\text{integral} = \int \frac{(-ef+dg)x}{(d^2-e^2x^2)\sqrt{a+bx^2+cx^4}}\,dx + \int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}de(ef-dg)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d^2-e^2x^2)\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2} + (-ef+dg)\int \frac{x}{(d^2-e^2x^2)\sqrt{a+bx^2+cx^4}}\,dx + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}}\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}} + \frac{1}{2}(-ef+dg)\text{Subst}\left(\int \frac{1}{(d^2-e^2x)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}}\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}} + (ef-dg)\text{Subst}\left(\int \frac{1}{4cd^4+4bd^2e^2+4ae^4-x^2}\,dx,x,\frac{-bd^2-2ae^2-\left(2cd^2+be^2\right)x^2}{\sqrt{a+bx^2+cx^4}}\right)$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2+ae^4}}$$
$$+ \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}$$
$$- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.74 (sec) , antiderivative size = 3652, normalized size of antiderivative = 6.52

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2
- 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2
]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*e*Sqrt[a + b*x^2 + c*x^4]) + (2*(Sqrt
[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]
/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sqr
t[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]
) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b
^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)
)]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 -
 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b -
 Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - S
qrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]
+ 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/
c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt[b
^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^2
- 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2] - Sqrt[2]*Sqrt[(-b - Sq
rt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[
2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d + (Sqrt[-(b/c) - Sqrt[b
```

```
^2 - 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
 + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2
 - 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] -
 Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c
] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c]
)/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt
[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^
2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2]))/(Sqrt[(-b - Sqrt[b^
2 - 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/
Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*Sqrt[a + b*x^
2 + c*x^4]) - (2*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c)
+ Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*d*g*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/S
qrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt
[b^2 - 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 -
 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b
/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - S
qrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]
 - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])
/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*
c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b
/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - S
qrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b
 - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(
-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x
))]], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])
^2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2]
 - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sq
rt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d
 + (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[
b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d -
 (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b -
 Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt
[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2
*x))]], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c
])^2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^
2]))/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*e*(-d - (Sqrt[-(b/c)
 - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]
*e)/Sqrt[2])*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.78

method	result
default	$\frac{g\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{(-dg+ef)\left(-\frac{\mathrm{arcta}}{}\right.}{}$
elliptic	$\frac{g\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{(dg-ef)\left(-\frac{\mathrm{arctanh}}{}\right.}{}$

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*g/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c/e^4*d^4+b/e^2*d^
2+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+b/e^2*d^2+b*x^2+2*a)/(c/e^4*d^4+b/e
^2*d^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*e/d*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1+1/2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^(1/2))*a*e^2/d^2,(-1/2*(b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^{2}+cx^{4}}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4+bx^2+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4+bx^2+a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((f + g*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((f + g*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.413 $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 527

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{(ef-dg)\operatorname{arctanh}\left(\frac{bd^2-2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2-ae^4}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2-ae^4}}$$
$$+\frac{\sqrt{b+\sqrt{b^2+4ac}}\,g\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}}$$
$$+\frac{\sqrt{-b+\sqrt{b^2+4ac}}(ef-dg)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\operatorname{EllipticPi}\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2},\arcsin\left(\frac{\sqrt{2}}{\sqrt{-b+}}\right.\right.}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}}$$

```
[Out] -1/2*(-d*g+e*f)*arctanh(1/2*(b*d^2-2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(-a*e^4+b*d
^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2-a)^(1/2))/(-a*e^4+b*d^2*e^2+c*d^4)^(1/2)+1
/2*g*(1/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^
(1/2)))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)/(1+2*
c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(4*a*c+b^2)^(1/
2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b+(4*a*c+b^2)^(1/2))^(1/2)/e
*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))/(
1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)+1/2*(-d*g+e*f)*EllipticPi(x*2^(1/2)
*c^(1/2)/(-b+(4*a*c+b^2)^(1/2))^(1/2),-1/2*e^2*(b-(4*a*c+b^2)^(1/2))/c/d^2,
((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b
^2)^(1/2)))^(1/2)*(-b+(4*a*c+b^2)^(1/2))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^(1
/2)))^(1/2)/d/e*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {1755, 12, 1261, 738, 212, 1724, 1118, 429, 1234, 551}

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$$
$$= \frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}(ef-dg)\operatorname{EllipticPi}\Bigg(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}}$$
$$+\frac{g\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}$$
$$-\frac{(ef-dg)\operatorname{arctanh}\left(\frac{-2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*((e*f - d*g)*ArcTanh[(b*d^2 - 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt
[c*d^4 + b*d^2*e^2 - a*e^4]*Sqrt[-a + b*x^2 + c*x^4])])/Sqrt[c*d^4 + b*d^2*
e^2 - a*e^4] + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*g*(1 + (2*c*x^2)/(b - Sqrt[b^2
+ 4*a*c]))*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]
], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*e*Sqrt
[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a
*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*(e*f - d*g
)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*EllipticPi[-1/2*((b - Sqrt[b^2 + 4*a*c])*e^2)/(c*d^2), ArcSi
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2 + 4*a*c]
)/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*e*Sqrt[-a + b*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1724

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
```

```
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 1755

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(-ef+dg)x}{(d^2-e^2x^2)\sqrt{-a+bx^2+cx^4}}\,dx + \int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{-a+bx^2+cx^4}}\,dx \\
&= \frac{g\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx}{e} + \frac{(d(ef-dg))\int \frac{1}{(d^2-e^2x^2)\sqrt{-a+bx^2+cx^4}}\,dx}{e} \\
&\quad + (-ef+dg)\int \frac{x}{(d^2-e^2x^2)\sqrt{-a+bx^2+cx^4}}\,dx \\
&= \frac{1}{2}(-ef+dg)\text{Subst}\left(\int \frac{1}{(d^2-e^2x)\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right) \\
&\quad + \frac{\left(g\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{e\sqrt{-a+bx^2+cx^4}} \\
&\quad + \frac{\left(d(ef-dg)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d^2-e^2x^2)}\,dx}{e\sqrt{-a+bx^2+cx^4}} \\
&= \frac{\sqrt{b+\sqrt{b^2+4ac}}g\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}} \\
&\quad + \frac{\sqrt{-b+\sqrt{b^2+4ac}}(ef-dg)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2+}}}\right.\right.}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}} \\
&\quad + (ef-dg)\text{Subst}\left(\int \frac{1}{4cd^4+4bd^2e^2-4ae^4-x^2}\,dx, x, \frac{-bd^2+2ae^2-(2cd^2+be^2)x^2}{\sqrt{-a+bx^2+cx^4}}\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(ef-dg)\tanh^{-1}\left(\frac{bd^2-2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2-ae^4}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2-ae^4}} \\
&\quad+\frac{\sqrt{b+\sqrt{b^2+4ac}}g\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}} \\
&\quad+\frac{\sqrt{-b+\sqrt{b^2+4ac}}(ef-dg)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2}}}\right.\right.}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.68 (sec) , antiderivative size = 3658, normalized size of antiderivative = 6.94

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b
 + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2
+ 4*a*c]))]*x], (-b - Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2
]*Sqrt[-(c/(-b - Sqrt[b^2 + 4*a*c]))]*e*Sqrt[-a + b*x^2 + c*x^4]) + (2*(Sqr
t[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c
]/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sq
rt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2
]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[
b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x
))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]
/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c
) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqr
t[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2
+ 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b
- Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(
-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]
/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] -
Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
 + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])
/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt[
b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^2
 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2] - Sqrt[2]*Sqrt[(-b - S
qrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
```

```
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d + (Sqrt[-(b/c) - Sqrt[
b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]
) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^
2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
- Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/
c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c
])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqr
t[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2]))/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
 Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)
/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*Sqrt[-a + b*
x^2 + c*x^4]) - (2*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c
) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*d*g*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]
/Sqrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sq
rt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2
 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-
(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c
]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) -
 Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/
c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c
])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*
a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-
(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b -
 Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt
[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2
*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c
])^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^
2] - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) -
Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*
(d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqr
t[b^2 + 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d
 - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b
 - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[
(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sq
rt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] -
 2*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])
/c])^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c]
)^2]))/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c
]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*e*(-d - (Sqrt[-(b/c
) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/
c]*e)/Sqrt[2])*Sqrt[-a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.83

method	result
default	$\frac{g\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{2e\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}} + \frac{(-dg+ef)\left(-\operatorname{arctanh}\left(\right.\right.}{}$
elliptic	$\frac{g\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{2e\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}} - \frac{(dg-ef)\left(-\operatorname{arctanh}\left(\right.\right.}{}$

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*g/e/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*Ell
ipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)
^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c/e^4*d^4+b/e^2*d^2-a)^(1/2)*arct
anh(1/2*(2*c*x^2/e^2*d^2+b/e^2*d^2+b*x^2-2*a)/(c/e^4*d^4+b/e^2*d^2-a)^(1/2)
/(c*x^4+b*x^2-a)^(1/2))+1/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*e/d*(1+1/2*
(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1-1/2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x
,-2/(-b+(4*a*c+b^2)^(1/2))*a*e^2/d^2,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^
(1/2)/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 - a)*(g*x + f)/(c*e*x^5 + c*d*x^4 + b*e*x^3 + b
*d*x^2 - a*e*x - a*d), x)
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{f+gx}{(d+ex)\,\sqrt{-a+bx^2+cx^4}}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2-a)**(1/2),x)
[Out] Integral((f + g*x)/((d + e*x)*sqrt(-a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

```
[In] int((f + g*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)),x)
[Out] int((f + g*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of op
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != −1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2∗leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2∗leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```